Tutorial 10 – Solutions.

1. There are three short articles that discuss incentive schemes at various organisations (linked in Canvas). Read each of the articles and describe why the incentives schemes identified are likely to work well or not.

   Answer: To supplement your understanding of the articles, you should refer back to the lecture notes and the discussion in Brickley about Lincoln Electric and what made an incentive scheme 'work well'. The notes below are simply some key points:

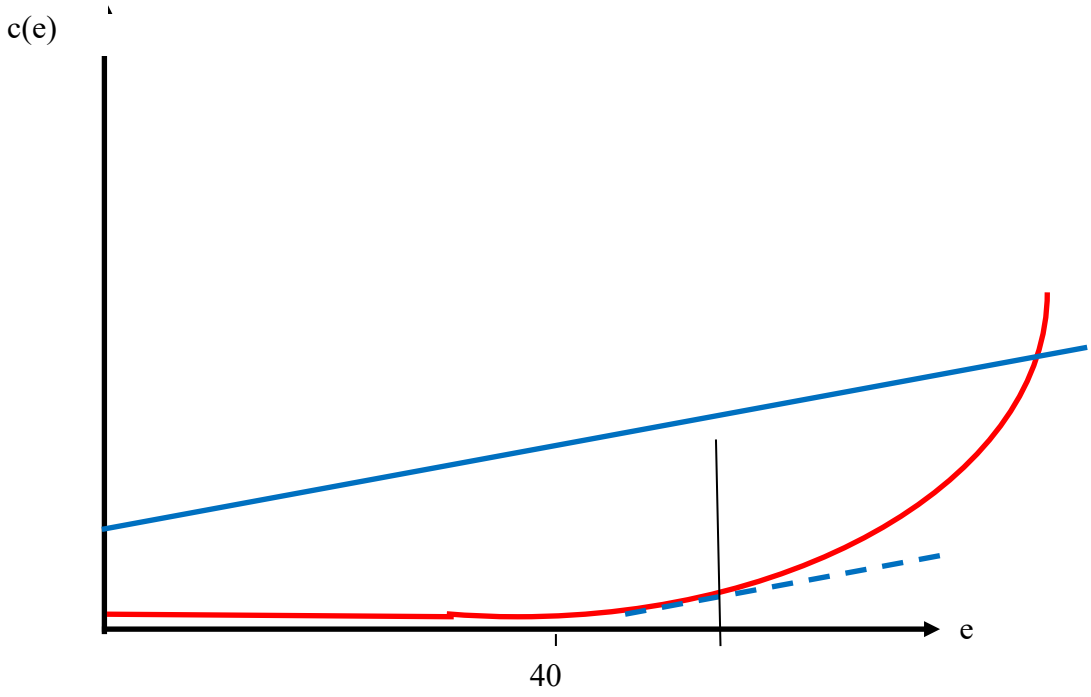
   - *For the QANTAS scheme there is a disconnect between the effort of the employee and the reward. It appears that any payment associated with good performance may not be received for many years, if at all. Moreover, it appears to be contingent on other actions, such as signing a new enterprise agreement, rather than just current performance. In this sense, it could be argued that it will not incentivise workers by linking pay and performance. Finally note that it appears that it may be illegal to make the signing of a new workplace agreement a condition for receiving the bonus payment.*
   - *In the Forbes article, note the discussion around subjective performance measures and the problem of measurement. Further, they note how free riding can potentially weaken the incentive provided by any remuneration scheme. For One Week Bath, however, the payment scheme appears to be very successful. Why? Perhaps because it is transparent and visible to workers. They know here they stand and what they need to do to get a bonus. Importantly, it is generous (so provides a meaningful incentive) and from the company's point of view, self-funding. The clear message from the article is that this is a successful scheme that works well for the workers and the firm.*
   - *The discussion in the article that deals with NAB highlights the problems with incentive schemes that potentially incentivises the wrong type of behaviour. Lenders really multitask – you want them to make more loans (quantity) and good loans (quality). If you incentivise or reward just one task, that is what they focus on. It would appear that NAB has realised the potential pitfalls of such an approach and redesigned incentive schemes to avoid incentive structure 'linked solely to sales'. This has the potential to create problems for institutions such as the NAB both in the short and long term through bad loans and reputational effects.*

2. Consider the principal-agent problem we examined in lectures last week. Assume that an individual has a cost of effort function of the following form:

$$C(e)=\begin{cases}0 & e<40\\ (e-40)^2 & e\geq 40\end{cases}$$

Interpret.

Why is what happens at the margin important for understanding the principal-agent problem?



Solution: The cost of effort function is shown in the diagram above in red. For all values of e less than 40 it is equal to zero. Above 40, it increases. For example, when e=42 then the cost of effort equals 1 (i.e. [42-40]^2 = 4; etc).

A linear payment schedule is given by the straight blue line. The vertical intercept is the fixed payment and the slope gives the commission rate. Note that it is what happens at the margin that is important because the agent choose a level of effort where the marginal cost of the effort (the slope of the red line or cost of effort function) equals the marginal benefit from extra effort (which is equal to the slope of the blue line). This was discussed in lecture 10.

3. Consider a standard principal-agent problem in the context of a computer salesperson. Performance of the salesperson is measured by the number of computers they sell, Q

where $Q = e$ (for the moment we will ignore any measurement error associated with the relationship between output and effort).

Assume that the disutility or cost of effort (e) is given by the following:

$$C(e) = 2e^2$$

You should assume that the salesperson is risk neutral and so only cares about the expected values of his or her remuneration.

If the firm is to offer a linear payment contract of the following form to the salesperson:

$$Pay = a + bQ$$

What is the value of b, e and a? Note, for the purpose of this question assume that each unit of effort produces an extra \$1 in profit – that is, net revenue from each extra unit of effort equals \$1.

Interpret your answer.

ANSWER:

For the worker they face the following problem of maximising their net payoff:

$$\max_e \; a + bQ - C(e) \quad s.t. \quad C(e) = 2e^2$$

$$\max_e \; a + bQ - 2e^2$$

FOC:

$$b - 4e = 0$$

$$e = \frac{b}{4}$$

For the firm, they will try to maximise profits by choosing parameters of the compensation scheme:

$$\max_{a,b} \; \pi = Q - compensation \qquad s.t. \qquad a + bQ \geq C(e)$$

In general, we would expect the constraint requiring that they pay the worker enough to compensate him or her for their effort to be satisfied with an equality. Hence using the fact that $Q = e$:

$$a + bQ = C(e)$$

$$a = C(e) - bQ$$

$$a = 2e^2 - be$$

$$a = 2\left(\frac{b}{4}\right)^2 - b\frac{b}{4} = \frac{b^2}{8} - \frac{b^2}{4} = -\frac{b^2}{8}$$

We can rewrite the firm's problem as:

$$\max_{a,b} \pi = Q - compensation \qquad s.t. \qquad a + bQ \geq C(e)$$

$$\max_{a,b} Q - [a + bQ] \qquad s.t. \qquad a + bQ = 2e^2,\ e = \frac{b}{4},\ a = -\frac{b^2}{8},\ Q = e = \frac{b}{4}$$

$$\max_{b} \frac{b}{4} - \left[-\frac{b^2}{8} + b\frac{b}{4}\right]$$

First order condition:

$$\frac{1}{4} - \frac{b}{4} = 0$$

$$b = 1$$

This implies that:

$$e = \frac{b}{4} = \frac{1}{4}$$

and

$$a = -\frac{b^2}{8} = -\frac{1}{8}$$

What is this saying?

The commission component of the linear compensation scheme should be equal to 1 (a 100% commission). That is, the agent should be paid or receive all the sales s/he makes. Note that in this case this provides the highest possible incentives to the agent because they keep all the additional revenue that any extra effort generates.

The fixed component should be negative – that is effectively the individual should be 'sold' the right to keep all the proceeds from his/ her efforts for a price of 1/8.

4. Reconsider the problem described in Question 3. Suppose now that output is an imprecise measure of effort so that:

$$q = e + \varepsilon$$

What will be the variance of pay now?

If the disutility from riskiness of pay is given by $0.5R\sigma_{pay}^2$ where R is a risk aversion parameter that captures how risk averse the worker is, write out the workers utility

maximization problem. Assuming risk neutrality on the part of the worker, how will your answer to question 1 change?

Solution:

First, note for the worker that the variance of their pay is given by the following:

$$Var(pay) = var(a + bQ) = var\big(a + b(e + \varepsilon)\big) = var(b\varepsilon) = b^2\sigma^2$$

Hence their problem is as follows:

$$\max_{e} \quad a + bQ - C(e) - 0.5Rb^2\sigma^2 \qquad s.t. \quad C(e) = 2e^2$$

$$\max_{e} a + bQ - 2e^2 \ - 0.5Rb^2\sigma^2$$

FOC:

$$b - 4e = 0$$

$$e = \frac{b}{4}$$

This is the same result as before. Effort is not affected by the risk.

For the firm, they will try to maximise profits by choosing parameters of the compensation scheme:

$$\max_{a,b} \pi = Q - compensation \qquad s.t. \qquad a + bQ - 0.5Rb^2\sigma^2 \geq C(e)$$

In general, we would expect the constraint requiring that they pay the worker enough to compensate him or her for their effort to be satisfied with an equality. Hence using the fact that Q = e:

$$a + bQ - 0.5Rb^2\sigma^2 = C(e)$$

$$a = C(e) + 0.5Rb^2\sigma^2 - bQ$$

$$a = 2e^2 + 0.5Rb^2\sigma^2 - be$$

$$\begin{aligned} a &= 2\left(\frac{b}{4}\right)^2 + 0.5Rb^2\sigma^2 - b\left(\frac{b}{4}\right) \\ &= \frac{b^2}{8} + 0.5Rb^2\sigma^2 - \frac{b^2}{4} \\ &= \left(0.5R\sigma^2 - \frac{1}{8}\right)b^2 \end{aligned}$$

Additional payment must be made to compensate for the risk relative to that for a risk neutral worker.