ECON5026	Mid-semester Test *Answers*
Strategic Business Relationships	September, 2019

1. Bubbles and Crazy Juices are two rival bottled drink manufacturers. Both are considering launching a new carbonated guava juice drink on the Sydney University campus. There is sufficient demand to sustain only one carbonated guava drink. Bubbles and Crazy must simultaneously decide whether to launch or stay out of the market. The matrix below summarises payoffs.

		Crazy	
		Launch	Out
Bubbles	Launch	$-60, -80$	$120, 0$
	Out	$0, 120$	$0, 0$

   (a) Identify all Nash equilibria to this game. [8 marks]

   ANS: There are two pure strategy Nash equilibria in which one firm Launches and the other stays Out. For example, if Bubbles Launches, then the best response for Crazy is Out; and if Crazy chooses Out, the best response for Bubbles is to Launch. Therefore, (Launch, Out) is a Nash equilibrium. By similar logic, (Out, Launch) is also a Nash equilibrium.

   There is also a mixed strategy Nash equilibrium in which both Bubbles and Crazy randomise between Launch and Out. In a mixed strategy NE, each player has no incentive to change strategy. It follows that the payoffs to Out and Launch must be the same for each player.

   Let p be the probability that Bubbles chooses Launch, and q be the probability that Crazy chooses Launch. Crazy is indifferent if

$$-80p + 120(1-p) = 0 \Rightarrow p = 3/5.$$

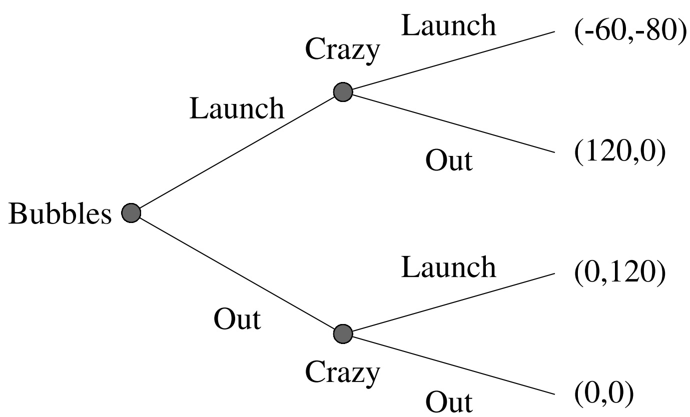
   Bubbles is indifferent if

$$-60q + 120(1-q) = 0 \Rightarrow q = 2/3.$$

   Therefore, there is a mixed strategy NE in which Bubbles chooses Launch with probability $p = 3/5$ and Crazy chooses Launch with probability $q = 2/3$.

   (b) The CEO at Crazy Juices thinks it would be a good idea to wait to gather information before deciding whether to launch. Consider a sequential moves game in which Bubbles makes their launch decision first. Crazy observes this decision and then chooses whether to launch. Illustrate this game with a diagram and solve for the subgame perfect Nash equilibrium to the game. Would you advise the CEO at Crazy to wait? Explain. [7 marks]

   ANS: The figure below illustrates the sequential game.



Solve by backward induction. Consider first Crazy's choices in stage 2. If Bubbles Launches, Crazy is better off choosing Out. If Bubbles chooses Out, Crazy is better off choosing Launch. In the first stage of the game, Bubbles anticipates the reactions of Crazy. If Bubbles chooses Launch, they anticipate that Crazy will choose out, and therefore Bubbles gets a payoff of 120. If instead Bubbles chooses Out, they expect Crazy to Launch, and therefore Bubbles gets a payoff of 0. Therefore, Bubbles is better off choosing Launch. Thus, in the SPNE, Bubbles chooses Launch, and Crazy chooses Out if they observe that Bubbles has Launched, and Launch if they observe that Bubbles has chosen Out.

It follows that the best advice for the CEO at Crazy is not to wait. If the CEO has the opportunity to preempt by choosing Launch before Bubbles chooses, this would be even better.

2. The demand for good X is given by $P(Q) = 128 - 2Q$, where Q is the market quantity, and P is the market price. Production of good X involves costs of $C(q) = 200 + 8q$, where q is firm output.

   (a) Suppose a single firm operates in the market. Find the profit-maximising price and quantity of the monopolist. [3 marks]

   ANS: Monopoly profits are

$$\pi = Q(128 - 2Q) - 8Q - 200.$$

   The first order conditions for a maximum yield

$$\begin{aligned} 0 &= 128 - 4Q - 8 \\ Q &= 30. \end{aligned}$$

   The market price is then determined by the demand curve: $P = 128 - 2Q = 68$.

   (b) Suppose two firms engage in simultaneous quantity competition in a single period.

      i. Find the reaction function for each firm. [4 marks]

      ANS: The profits of firm i are

$$\pi_i = q_i(128 - 2q_i - 2q_j) - 200 - 8q_i.$$

      The first order conditions for a maximum are

$$\begin{aligned} 0 &= 128 - 4q_i - 2q_j - 8 \\ 4q_i &= 120 - 2q_j \\ q_i &= 30 - q_j/2 \equiv R_i(q_j) \end{aligned}$$

      This is the reaction function for firm i as a function of the output of firm j, for $i = 1, 2$, and $j \neq i$.

      ii. Find the Nash equilibrium outputs of both firms. [3 marks]

   ANS: In the Nash equilibrium, both firms operate on their reaction functions. Thus,

$$\begin{aligned} q_i &= 30 - (30 - q_i/2)/2 \\ (3/4)q_i &= 15 \\ q_i &= 20. \end{aligned}$$

   Thus, both firms produce $q = 20$ in the Nash equilibrium.

(c) Suppose that two firms operate in the market. The firms engage in Stackelberg competition. Firm 1 chooses its output first, then Firm 2 chooses its output. Find the output of each firm in the subgame perfect Nash equilibrium. [5 marks]

ANS: We solve by backward induction. In stage 2, Firm 2 chooses the optimal output given the output of Firm 1. This is the same as the definition of the reaction function in the Cournot problem. Hence, Firm 2 chooses the output $q_2 = R_2(q_1) = 30 - q_1/2$. In stage 1, Firm 1 anticipates the behaviour of Firm 2. We therefore substitute Firm 2's reaction function into the profit function of Firm 1.

$$\begin{aligned}\pi_1 &= q_1(128 - 2q_1 - 2(30 - q_1/2)) - 200 - 8q_1 \\ &= q_1(120 - 2q_1 - (60 - q_1)) - 200 \\ &= q_1(60 - q_1) - 200.\end{aligned}$$

The first order conditions are then

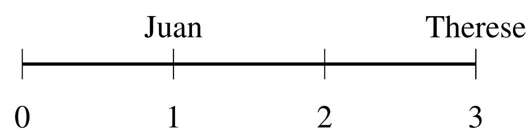
$$\begin{aligned}0 &= 60 - q_1 - q_1 \\ q_1 &= 30.\end{aligned}$$

Firm 2 then chooses an output according to her reaction function, $q_2 = 30 - q_1/2 = 15$.

3. Juan and Therese each own a Taco restaurant in the town of Burritoville. The town is 3km long. Juan is located 1km along the town, and Therese is at 3km along (see the figure below). 300 Consumers are uniformly located along the town (between 0 and 3). Consumer i's utility derived from dining at restaurant j is given by

$$u_{ij} = \overline{u} - t|x_i - y_j| - p_j,$$

where $j = 1,2$ indicate the two restaurants, t is the cost of travelling along the town per km, x_i is the location of consumer i, y_j is the location of restaurant j, and p_j is the price of restaurant j. Each consumer eats a meal at exactly one restaurant. Restaurants compete with each other by simultaneously choosing prices. Each restaurant has constant marginal costs of c and no fixed costs.



(a) Calculate the demand for each restaurant in terms market prices and transport costs. [4 marks]

ANS: To find demand, we find the location of the indifferent consumer. Let x be the location of the indifferent consumer. Then,

$$\begin{aligned} t|x-3| + p_2 &= t|x-1| + p_1 \\ t(3-x) - t(x-1) &= p_1 - p_2 \\ 2tx &= p_2 - p_1 + 4t \\ x &= \frac{p_2 - p_1 + 4t}{2t} \end{aligned}$$

The market share of firm 1 is $x/3$ and the demand for firm 1 is

$$q_1 = 300x/3 = 100x = 100\frac{p_2 - p_1 + 4t}{2t}.$$

The market share of firm 2 is $(3-x)/3$ and the demand for firm 2 is

$$q_2 = 300(3-x)/3 = 100(3-x) = 100\frac{p_1 - p_2 + 2t}{2t}.$$

(b) Find the reaction function for each restaurant. [4 marks]

ANS: Firm 1 solves the problem

$$\max_{p_1} \pi_1 = 100\frac{p_2 - p_1 + 4t}{2t}(p_1 - c)$$

yielding first order conditions:

$$p_2 - 2p_1 + 4t + c = 0$$

and the reaction function

$$p_1 = \frac{p_2 + 4t + c}{2}.$$

Firm 2 solves the problem

$$\max_{p_2} \pi_2 = 100\frac{p_1 - p_2 + 2t}{2t}(p_2 - c)$$

yielding first order conditions:

$$p_1 - 2p_2 + 2t + c = 0$$

and the reaction function

$$p_2 = \frac{p_1 + 2t + c}{2}.$$

(c) Find the Nash equilibrium prices and quantities for each restaurant. [4 marks]

ANS: In a Nash equilibrium, both firms operate on their reaction function. Therefore

$$\begin{aligned}
p_1 &= \frac{4t+c}{2} + \frac{p_1 + 2t + c}{4} \\
\frac{3}{4}p_1 &= \frac{10t + 3c}{4} \\
p_1 &= 10t/3 + c. \\
p_2 &= \frac{2t+c}{2} + \frac{5t}{3} + \frac{c}{2} \\
p_2 &= 8t/3 + c.
\end{aligned}$$

Substitute these prices into the demand functions to obtain equilibrium quantities. Observe that $p_1 - p_2 = 2t/3$. Therefore

$$\begin{aligned}
q_1 &= 100\frac{4t - 2t/3}{2t} = \frac{500}{3}, \\
q_2 &= 100\frac{2t + 2t/3}{2t} = \frac{400}{3}.
\end{aligned}$$

(d) Explain why Juan sets higher prices than Therese in equilibrium. [3 marks]

ANS: Juan is at a location advantage relative to Therese, because all consumers to the left of the 2km line in the city are closer to Juan, while those to the right of the line are closer to Therese. This means that 2/3 of consumers are closer to Juan. With more local consumers, Juan has greater market power, and therefore an incentive to set higher prices to take advantage of this market power. Notice also that, despite the fact that Juan sets a higher price, he has more customers in equilibrium.