TUTORIAL 5

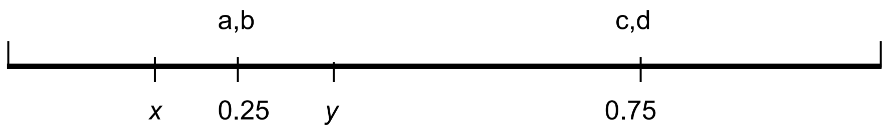
PRODUCT DIFFERENTIATION

QUESTION 1

Consider a market in which firms choose a point on product space of length 1. If the firms have fixed prices but can choose where to locate, where we would expect four firms to locate? Why?

QUESTION 1

Here, let's assume that there are four firms (*a*, *b*, *c* and *d*) with two locating at ¼ and 2 at ¾. What we will do is show that no firm has an incentive to move. Consider:



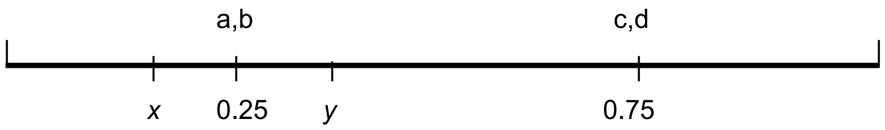
Initially the firms each get one quarter of the market. For example, a and b share the area to the left of 0.25 (so each get 1/8 each) and all four firms share the space between 0.25 and 0.75 and each get 1/8. Hence, each firm gets ¼ in total.

QUESTION 1

Suppose that *a* makes a move to the left. In particular, suppose *a* ‘jumps to x’. In that case *a* will now get all the space to its left (=*x*) and half the distance between x and 0.25 which it now shares with *b* (this is equal to [0.25-*x*]/2). Hence a’s new market share is:

$$x + \frac{(0.25 - x)}{2} = \frac{(0.25 + x)}{2} < 0.25$$

Hence it would not make sense for a to make such a move as it loses market share.



QUESTION 1

Suppose that *a* makes a move to the right. In particular, suppose *a* 'jumps to *y*'. In that case it will now share the space to its left (*y-0.25*) with *b* and get half the distance between *y* and 0.75 which it now shares with *c* & *d* (this is equal to [0.75-*y*]/2). Hence a's new market share is:

$$\frac{(y-0.25)}{2}+\frac{(0.75-y)}{2}=0.25$$

Hence it would not make sense for a to make such a move as generates no extra profit.

A jump to the right of where *c* and *d* are currently located (0.75) has the same outcome as a jump to x – it leads to lower profits.

Hence, for any firm there is no incentive to move away from where they are currently located.

QUESTION 2

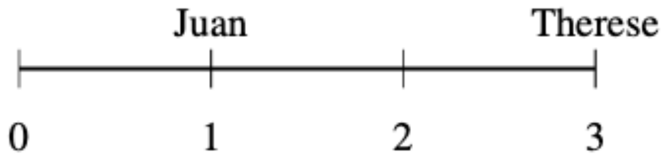
Juan and Therese each own a Taco restaurant in the town of Burritoville. The town is 3km long. Juan is located 1km along the town, and Therese is at 3km along (see the figure below). 300 consumers are uniformly located along the town (between 0 and 3). Consumer *i*'s utility derived from dining at restaurant *j* is given by:

$$u_{ij} = \bar{u} - t|x_i - y_i| - p_j$$

where j = 1, 2 indicate the two restaurants, t is the per unit cost of travelling along the town, x_i is the location of consumer i, y_j is the location of restaurant j, and p_j is the price of restaurant j.

QUESTION 2

Each consumer eats a meal at exactly one restaurant. Restaurants compete with each other by simultaneously choosing prices. Each restaurant has constant marginal costs of *c* and no fixed costs.



(a) Calculate the demand for each restaurant in terms market prices and transport costs.

(b) Find the reaction function for each restaurant.

(c) Find the Nash equilibrium prices and quantities for each restaurant.

(d) Explain why Juan sets higher prices than Therese in equilibrium

QUESTION 2(A)

(a) Calculate the demand for each restaurant in terms market prices and transport costs.

To find demand, we find the location of the indifferent consumer. Let *x* be the location of the indifferent consumer. Then,

$$t|x-3|+p_2=t|x-1|+p_1$$

$$t(3-x)-t(x-1)=p_1-p_2$$

$$2tx=p_2-p_1+4t$$

$$x=\frac{p_2-p_1+4t}{2t}$$

QUESTION 2(A)

The market share of firm 1 is *x*/3 and the demand for firm 1 is:

$$q_1 = 300x/3 = 100x = 100\frac{p_2 - p_1 + 4t}{2t}$$

The market share of firm 2 is (3 − x)/3 and the demand for firm 2 is:

$$q_2 = 300(3 - x)/3 = 100(3 - x) = 100\frac{p_1 - p_2 + 2t}{2t}$$

QUESTION 2(B)

(b) Find the reaction function for each restaurant.

Firm 1 solves the problem:

$$\max_{p_1} \pi_1 = 100\frac{p_2 - p_1 + 4t}{2t}(p_1 - c)$$

yielding first order conditions:

$$p_2 - 2p_1 + 4t + c = 0$$

and the reaction function:

$$p_1 = \frac{p_2 + 4t + c}{2}$$

QUESTION 2(B)

Firm 2 solves the problem:

$$\max_{p_2} \pi_2 = 100 \frac{p_1 - p_2 + 2t}{2t}(p_2 - c)$$

yielding first order conditions:

$$p_1 - 2p_2 + 2t + c = 0$$

and the reaction function:

$$p_2 = \frac{p_1 + 2t + c}{2}$$

QUESTION 2(C)

(c) Find the Nash equilibrium prices and quantities for each restaurant.

In a Nash equilibrium, both firms operate on their reaction function. Therefore:

$$p_1 = \frac{p_2 + 4t + c}{2}$$

$$p_1 = \frac{p_1 + 2t + c}{4} + \frac{4t + c}{2}$$

$$\frac{3}{4}p_1 = \frac{10t + 3c}{4}$$

$$p_1 = \frac{10t}{3} + c$$

QUESTION 2(C)

$$p_2 = \frac{p_1 + 2t + c}{2}$$

$$p_2 = \frac{5t}{3} + \frac{c}{2} + \frac{2t + c}{2}$$

$$p_2 = \frac{8t}{3} + c$$

QUESTION 2(C)

Substitute these prices into the demand functions to obtain equilibrium quantities. Observe that $p1 - p2 = 2t/3$. Therefore:

$$q_1 = 100\frac{4t - 2t/3}{2t} = \frac{500}{3}$$

$$q_2 = 100\frac{2t + 2t/3}{2t} = \frac{400}{3}$$

QUESTION 2(D)

(d) Explain why Juan sets higher prices than Therese in equilibrium.

Juan is at a location advantage relative to Therese, because all consumers to the left of the 2km line in the city are closer to Juan, while those to the right of the line are closer to Therese. This means that 2/3 of consumers are closer to Juan. With more local consumers, Juan has greater market power, and therefore an incentive to set higher prices to take advantage of this market power. Notice also that, despite the fact that Juan sets a higher price, he has more customers in equilibrium.

2019 MID-SEMESTER EXAM Q2

The demand for good X is given by $P(Q) = 128 - 2Q$, where Q is the market quantity, and P is the market price. Production of good X involves costs of $C(q) = 200 + 8q$, where q is firm output.

(a) Suppose a single firm operates in the market. Find the profit-maximising price and quantity of the monopolist.

(b) Suppose two firms engage in simultaneous quantity competition in a single period.

i. Find the reaction function for each firm.

ii. Find the Nash equilibrium outputs of both firms.

(c) Suppose that two firms operate in the market. The firms engage in Stackelberg competition. Firm 1 chooses its output first, then Firm 2 chooses its output. Find the output of each firm in the subgame perfect Nash equilibrium.

2019 MID-SEMESTER EXAM Q2

a) Suppose a single firm operates in the market. Find the profit-maximising price and quantity of the monopolist.

Monopoly profits are:

$$\pi \ = \ Q(128 - 2Q) - 8Q - 200$$

The first order conditions for a maximum yield:

$$0 \ = \ 128 - 4Q - 8$$
$$Q = 30$$

The market price is then determined by the demand curve: $P = 128 - 2Q = 68$.

2019 MID-SEMESTER EXAM Q2

(b) Suppose two firms engage in simultaneous quantity competition in a single period.

i. Find the reaction function for each firm.

The profits of firm *i* are:

$$\pi_i = q_i(128 - 2q_i - 2q_j) - 200 - 8q_i$$

The first order conditions for a maximum are:

$$0 = 128 - 4q_i - 2q_j - 8$$

$$4q_i = 120 - 2q_j$$

$$q_i = 30 - q_j/2 \equiv R_i(q_j)$$

This is the reaction function for firm *i* as a function of the output of firm *j*, for *i* = 1,2,and *j* ≠ *i*.

2019 MID-SEMESTER EXAM Q2

ii. Find the Nash equilibrium outputs of both firms.

In the Nash equilibrium, both firms operate on their reaction functions. Thus:

$$q_i = 30 - \frac{30 - \frac{q_i}{2}}{2}$$

$$(3/4)q_i = 15$$

$$q_i = 20$$

Thus, both firms produce q = 20 in the Nash equilibrium.

2019 MID-SEMESTER EXAM Q2

(c) Suppose that two firms operate in the market. The firms engage in Stackelberg competition. Firm 1 chooses its output first, then Firm 2 chooses its output. Find the output of each firm in the subgame perfect Nash equilibrium.

We solve by backward induction. In stage 2, Firm 2 chooses the optimal output given the output of Firm 1. This is the same as the definition of the reaction function in the Cournot problem. Hence, Firm 2 chooses the output q_2 = *R*2(q_1) = 30− q_1/2. In stage 1, Firm 1 anticipates the behaviour of Firm 2. We therefore substitute Firm 2's reaction function into the profit function of Firm 1.

$$\begin{aligned}\pi_1 \;&=\; q_1\,(128 - 2q_1 - 2(30 - q_1/2)) - 200 - 8q_1 \\ &= q_1(120 - 2q_1 - (60 - q_1)) - 200 \\ &= q_1(60 - q_1) - 200.\end{aligned}$$

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The first order conditions are then:

$$0 \;=\; 60 - q_1 - q_1$$
$$q_1 \;=\; 30$$

Firm 2 then chooses an output according to her reaction function:

$$q_2 = \; 30 \;-\; q_1\,/2 \;=\; 15$$

2019 MID-SEMESTER PRACTICE EXAM Q2

Wanda sells bottled water in the town of Freshwaters. Demand for bottled water is give by $Q(P) = 240 - 40P$ per day, where Q is the market output, and P is the market price. The cost of bottling and distributing water is determined by the cost function $C(q) = 2q$.

(a) Suppose Wanda is the only seller in Freshwater.

i. What price should she charge? What profits does she earn?

ii. Discuss possible barriers to entry that might sustain the profitability of Wanda's business.

2019 MID-SEMESTER PRACTICE EXAM Q2

(b) Suppose Frank also produces identical bottled water in the Fresh water market. Frank and Wanda simultaneously choose the price of bottled water. Consumers are perfectly informed about the prices of bottled water.

i. Derive the reaction functions for Wanda and Frank.

ii. What is the Nash equilibrium in this market. Explain carefully.

iii. In the Nash equilibrium you described, do Frank and Wanda play dominant strategies? Explain.

2019 MID-SEMESTER PRACTICE EXAM Q2

(a) Suppose Wanda is the only seller in Freshwater.

i. What price should she charge? What profits does she earn?

Wanda has profits of:

$$\pi(P) = Q(P)\times(P-2) = (240-40P)(P-2)$$

To maximise profits, she solves the first order conditions:

$$\frac{d\pi}{dP} = 0 = 240-40P-40(P-2)$$

$$80P = 320$$

$$P = 4$$

Wanda's profits are:

$$\pi(P) = (240-40P)(P-2) = (240-160)\times(4-2) = 160$$

2019 MID-SEMESTER PRACTICE EXAM Q2

ii. Discuss possible barriers to entry that might sustain the profitability of Wanda's business.

Possible barriers to entry include:

- large minimum efficient scale relative to industry size
- a saturated product space
- consumer switching costs
- brands and reputation
- limited access to distribution channels
- limited access to raw materials
- government regulation and intellectual property

Some of these do not seem to apply here. For example, the question specifies constant marginal costs to production and no fixed costs, so minimum efficient scale is not large relative to industry size. It is also unlikely that consumer switching costs are important in the bottled water market. Perhaps Wanda has developed a popular brand of bottled water, or has tied up the distribution channels for bottled water in Freshwater.

2019 MID-SEMESTER PRACTICE EXAM Q2

(b) Suppose Frank also produces identical bottled water in the Freshwater market. Frank and Wanda simultaneously choose the price of bottled water. Consumers are perfectly informed about the prices of bottled water.

i. Derive the reaction functions for Wanda and Frank.

With two firms producing identical products and competing by price, we have the Bertrand model. Because the products are identical, both Wanda and Frank have an incentive to marginally undercut the price of their rival. However, if their rival sets a price above the monopoly price of $4, the best response is to undercut down to the monopoly price rather than just marginally undercut. Finally, if one's rival sets a price below the marginal cost of $2, the best response is to set a higher price than one's rival. Together, this defines the reaction function for both Wanda and Frank.

2019 MID-SEMESTER PRACTICE EXAM Q2

ii. What is the Nash equilibrium in this market. Explain carefully.

In the unique Nash equilibrium, both Wanda and Frank set a price equal to marginal cost of $2. At this price, both are playing a best response to their rival's price, and therefore this represents a Nash equilibrium.

2019 MID-SEMESTER PRACTICE EXAM Q2

iii. In the Nash equilibrium you described, do Frank and Wanda play dominant strategies? Explain.

No, Wanda and Frank are not playing dominant strategies. As discussed in part 2(b)i above, both Wanda and Frank have an incentive to change their strategy if their rival sets a different price. This means that they do not have a dominant strategy.