Tutorial 2 – Game Theory Solutions

1. Consider the following simple advertising games played between Coke and Pepsi. Find the Nash Equilibrium assuming advertising choices are made simultaneously. Explain the meaning of the Nash Equilibrium and the outcome observed.

		Pepsi's budget		
		Low	Medium	High
	Low	\$60, \$45	\$57.50, \$50.50	\$45, \$35
Coke's budget	Medium	\$50, \$35	\$65, \$30	\$30, \$25
	High	\$45, \$10	\$60, \$20	\$50, \$40

*Payoffs are in \$m, with the row player (Coke) being the first payoff in each cell.

Answer: Best responses are circled below. Note that the NE is an outcome such that given the behavior of the other player there is no unilateral incentive to change your behavior. Note that the non-cooperative NE here results in total profits not being maximized. Further note that neither player has a dominant strategy.

		Pepsi's budget		
		Low	Medium	High
	Low	\$60, \$45	\$57.50, \$50.50	\$45, \$35
Coke's budget	Medium	~~\$50~~, \$35	\$65, ~~\$30~~	~~\$30~~, ~~\$25~~
	High	\$45, ~~\$10~~	~~\$60~~, \$20	\$50, \$40

*Payoffs are in \$m, with the row player (Coke) being the first payoff in each cell.

2. Consider the following simple bargaining games played between management and a union. The rules of bargaining require that each party nominate a specified amount of the \$100 surplus that is available. If the amount nominated in total exceeds \$100 no agreement is reached and each party simply incurs a small cost associated with bargaining. Find the Nash Equilibria and explain what you might think will happen.

		Union		
		0	50	100
	0	\$0, \$0	\$0, \$50	\$0, \$100
Management	50	\$50, \$0	\$50, \$50	-\$1, -\$1
	100	\$100, \$0	-\$1, -\$1	-\$1, -\$1

Answer: Best responses are circled below. Note that there are three NE here. Which one might we observe? There might be a focal point – an understanding that any surplus from bargaining is split 50:50 (it doesn't have to be this, but an assumption is usually made that there surplus is shared equally).

		Union		
		0	50	100
Management	0	\$0, \$0	\$0, \$50	\$0, \$100
	50	\$50, \$0	\$50, \$50	-\$1, -\$1
	100	\$100, \$0	-\$1, -\$1	-\$1, -\$1

3. Consider the following pricing problem for two Pizza chains, Domino's and Crust, which are considering offering a new lunch special. First, eliminate any dominated strategies. Next, find the Nash Equilibrium if both chains make their choice simultaneously.

		Domino's		
		High (\$10)	Medium (\$8)	Low (\$6)
Crust	High (\$10)	\$1000, \$1,000	\$900, \$1,100	\$500, \$1,200
	Medium (\$8)	\$1,100, \$400	\$800, \$800	\$450, \$500
	Low (\$6)	\$1,200, \$300	\$500, \$350	\$400, \$400

*Payoffs are profit per week, with the row player (Crust) being the first payoff in each cell.

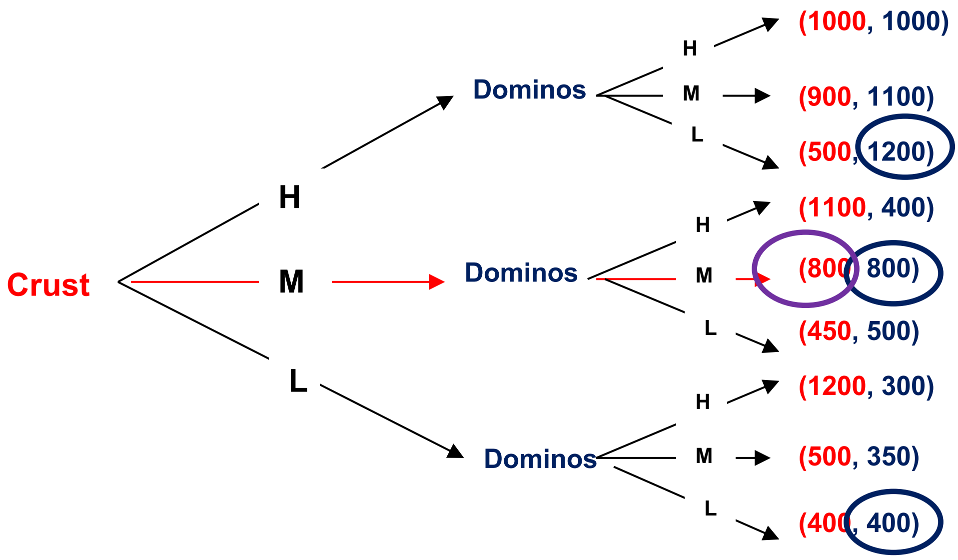
Now, consider what the outcome of the game will look like if Crust announces its menu first using Facebook and Domino's makes its' choice later. Does Crust have a first or second mover advantage?

Answer: We can eliminate dominated strategies. First, eliminate the high strategy for Dominos. Then you can eliminate the low strategy for Crust. This leaves you with a simple game in which the NE is for Crust to choose high and Domino low (the best responses are circled)

		Domino's		
		High (\$10)	Medium (\$8)	Low (\$6)
Crust	High (\$10)	\$1000, \$1,000	\$900, \$1,100	\$500, \$1,200
	Medium (\$8)	\$1,100, \$400	\$800, \$800	\$450, \$500
	Low (\$6)	\$1,200, \$300	~~\$500, \$350~~	~~\$400, \$400~~

We can also ask what happens if the game is played sequentially with Crust making the first move. The game tree for this game is presented below. I have indicated the solution. Remember that we solve using backwards induction. That is start at the end of the game and work backwards in time. For any given choice of Crust, I have circled the best response of Dominos. In turn, given that Crust can anticipate how Dominos will respond

I have indicated in red Crust's best choice. The new outcome of for both to choose medium in which case Crust now gets profits of $800 rather than $500. That is it has a first mover advantage.



4. In business, it is sometimes said that limiting your options or tying one's hands can actually improve your situation. Why is this the case and how might a firm do this?

   Answer: Please refer to the lecture discussion. Economists usually think about more choices as being better. In some cases, however, limiting your options (effectively restricting the set of choices you can make) may actually be better. Consider for example the example of Crust above. If they can credibly commit to a particular strategy (in this case offering the M lunch special, then they limit their choices, but it is to their advantage because Dominos must respond with a choice of M of their own. This is a better outcome then if they make their choices simultaneously. For this to work 'tying one's hands' must be a credible strategy. For example, if Crust can undertake an advertising campaign, they effectively commit to offering a particular menu.

5. It is easy to think of situations where it advantageous to be a first mover when decisions are being made sequentially. Will it ever be advantageous to be a second mover? Why?

   Answer: A first mover advantage example is shown in the example above. It is difficult to think about a second mover advantage but think about situations where the first mover has to make investments in the infrastructure, the development of a market or R&D. It may be possible in those cases for the second mover to enter or act later and take advantage of the infrastructure established by the first mover. In effect the second mover free rides on the investment of the first mover. An example where one player attempted to do this *(and failed) is provided here: https://www.afr.com/technology/web/vodafone-loses-bid-to-access-telstra-and-optus-regional-networks-20171221-h08hfm . The challenge of course, is to try and take advantage of the other players investments – a non-trivial exercise.*

6. Consider the following game between a firm and a customer. The firm may offer a high-quality product or a low-quality product. High quality products cost more to make compared to low quality products and hence profits will be higher if the firm sells a low-quality product. Consumers prefer high quality products but cannot determine quality prior to purchase. The payoffs for the firm and consumer are presented below. Find the Nash Equilibrium if the parties play the game once.

		Firm	
		Low quality	High Quality
Customer	Don't buy	\$0, \$0	\$0, -\$10
	Buy	-\$10, \$10	\$1, \$1

How might your answer change if the game is repeated 'infinitely' and the customer offers to keep buying in the future?

Solution: If the game is played once then the best responses are circled.

		Firm	
		Low quality	High Quality
Customer	Don't buy	\$0, \$0	\$0, -\$10
	Buy	-\$10, \$10	\$1, \$1

In this case there is no transaction – basically the buyer does not trust the seller to sell a high-quality product as it always in the interest of the latter to supply a low-quality product. Potential surplus is then lost.

However, suppose that the game is played an infinite number of times the outcome might be different. In that case let's suppose that if as a buyer you are sold a low-quality product then you stop purchasing. However, if you get a high-quality product then you keep buying. In this case the payoffs for the firm are as follows:

Payoff from selling low quality product: 10
Payoff from continuing to sell high quality product: 1 + 1 + 1 +…

The question then is what payoff stream is the greatest. You would expect the firm to sell high-quality goods as long as the net present value of the payoff steam from selling high quality goods exceeds the one-off payment stream from selling a low quality good and not getting any repeat customers:

$$1+\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\frac{1}{(1+r)^3}+...>10$$

7. Suppose that you are considering purchasing a car. You believe it is valued between \$1000 and \$5000, with an equal probability that it has a value at any point in this range. that is, you believe it is uniformly valued between \$1000 and \$5000. The current owner knows the car and its true value. Assume that because you are a mechanical genius, whatever the car is worth to the seller, it is worth 1.33 times that to you (so a car worth \$2400 to the seller is actually worth \$3200 to you).

(i) *What is your expected profit if you offer \$3000?*

(ii) *What offer should you make to ensure that you will not lose money?*

Solution: Note that you as the buyer effectively believe that the car's value is uniformly distributed over the range [1000, 5000].

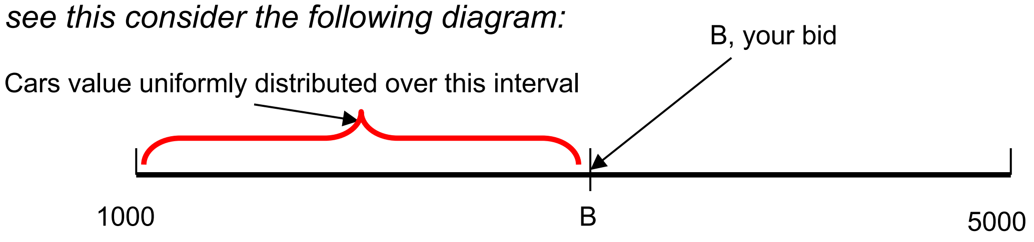
Suppose that you offer \$3000, this means that the seller will sell it as long as they value it \$3000 or less. If this is the case, there is a 1 in 2000 chance (strictly a one in 2001 chance) that its value is \$1000; a 1 in 2000 chance that its value is \$1001; a one in 2000 chance its value if \$1002 etc. The expected value to the seller given that they sell it to you with an offer of \$3000 is \$2000. Hence, the expected value to you if (4/3)(\$2000) or \$2667. Hence your expected profit with an offer of \$3000 is -\$333.*

So what should you do to ensure you don't make a loss? Suppose you make an offer of B. For B to be acceptable, the car must be worth somewhere between [1000, B] to the seller. In this case, the expected value to the owner will be:

$$1000+\frac{B-1000}{2}=500+\frac{B}{2}$$

That is, it is equal to \$1000 plus midway point between \$1000 and your bid (B).

To see this consider the following diagram:



SO, the expected value of the car to you will be $\left(\frac{4}{3}\right)\left(500+\frac{B}{2}\right)$

Your net payoff from purchasing the car will be:

$$\left(\frac{4}{3}\right)\left(500+\frac{B}{2}\right)-B$$

Hence, to ensure you don't lose you want:

$$\left(\frac{4}{3}\right)\left(500+\frac{B}{2}\right)-B>0 \qquad \Rightarrow B<2000$$

8. **[NOTE THIS IS A CHALLENGING QUESTION FROM THE LECTURES THAT WE WILL ALSO WORK THROUGH DURING THE TUTORIAL**] Consider the following game played between Anne and Bert. Each are part of team that is assigned to work on a project, and both have a choice to work or shirk. They get a $1000 base salary. If they both work then they develop a good project and get a bonus. However, working creates disutility and this reduces the net payoff. If only Anne or Bert work hard, the project will still be successful and they both get the bonus. However, in this situation the effort of the worker results in a net payoff of $0 to the individual who put in the effort.

		Bert	
		Shirk	Work
Anne	Shirk	$1,000, $1,000	$3,000, $0
	Work	$0, $3,000	$2,000, $2,000

Find the Nash Equilibrium if Anne and Bert expect to be assigned to the same team just once.

Suppose now that Anne and Bert might be assigned to work together in future periods with a probability of p. For example, the probability that they are assigned to work together for the second period is *p*, the probability they are assigned to work together for the third period is p^2, etc.

Assume that Anne and Bert choose to shirk in every period they work together. Write out an expression for their expected payoff.

Now suppose that after the work is done, they can observe if the other person actually put in effort or simply shirked. Further, assume that if Anne or Bert observes the other has shirked, they will punish them by shirking in the future. In that case we can write out the payoff matrix for Anne and Bert as follows:

		Bert	
		Always shirk	**Work then punish**
Anne	*Always shirk*	$\left(\frac{1000}{(1-p)}, \frac{1000}{(1-p)}\right)$	$\left(2000+\frac{1000}{(1-p)}, -1000+\frac{1000}{(1-p)}\right)$
	Work then punish	$\left(-1000+\frac{1000}{(1-p)}, 2000+\frac{1000}{(1-p)}\right)$	$\left(\frac{2000}{(1-p)}, \frac{2000}{(1-p)}\right)$

In this case, will Anne 'Always shirk'?

Solution: First note that the NE from the one-shot game is for both Anne and Bert to shirk. Bets responses are shown in the payoff matrix below. In fact, they both have a dominant strategy to shirk.

		Bert	
		Shirk	Work
Anne	Shirk	\$1,000, \$1,000	\$3,000, \$0
	Work	\$0, \$3,000	\$2,000, \$2,000

Now consider what happens if they play the game more than once. The way to think about it is that there is a probability that having paired this period you will be paired the following period with probability p, the probability they are assigned to work together for the third period is p^2 etc. Now consider the payoff from always working. If they both always work, they get \$1000 this period, an expected payoff of \$1000p next period, an expected payoff of \$1000 p^2 the following period etc. The value of this is:

$$2000 + 2000p + 2000p^2 + 2000p^3 + \ldots = \frac{2000}{(1-p)}$$

using the formula for the sum of a geometric series. Similarly, the value of always shirking if the other person also shirks is:

$$1000 + 1000p + 1000p^2 + 1000p^3 + \ldots = \frac{1000}{(1-p)}$$

Now consider what happens if you work but in the first period but the other person shirks. In the future periods you never trust them again and will always shirk in the future. In this case your payoff is \$0 this period and \$1000 in future periods. We can write this as follows:

$$0 + 1000p + 1000p^2 + 1000p^3 + \ldots = -1000 + 1000 + 1000p + 1000p^2 + 1000p^3 + \ldots = -1000 + \frac{1000}{(1-p)}$$

You should be able to see this payoff in the payoff matrix.

Now consider what happens if you shirk but in the first period but the other person works. In the future periods they never trust you again and will always shirk in the future. In this case your payoff is $3000 this period and $1000 in future periods. We can write this as follows:

$$3000 + 1000p + 1000p^2 + 1000p^3 + ... = 2000 + 1000 + 1000p + 1000p^2 + 1000p^3 + ... = 2000 + \frac{1000}{(1-p)}$$

Now we can put these payoffs into the payoff matrix to give the following:

		Bert	
		Always shirk	**Work then punish**
Anne	*Always shirk*	$\left(\frac{1000}{(1-p)}, \frac{1000}{(1-p)}\right)$	$\left(2000 + \frac{1000}{(1-p)}, -1000 + \frac{1000}{(1-p)}\right)$
	Work then punish	$\left(-1000 + \frac{1000}{(1-p)}, 2000 + \frac{1000}{(1-p)}\right)$	$\left(\frac{2000}{(1-p)}, \frac{2000}{(1-p)}\right)$

Now consider Anne, suppose that she believes Bert will always shirk (so we are in the left-hand column). In that case her best response is to always shirk. Conversely, suppose that she believes that Bert might work (so we are in the right-hand column). In that case Anne best response is to work if:

$\frac{2000}{(1-p)} > 2000 + \frac{1000}{(1-p)}$*, or*

$$2000 > 2000(1-p) + 1000 \;\Rightarrow\; 2000p > 1000 \;\Rightarrow\; p > 0.5$$

So, if you think there is a high chance (more than 0.5 or 50 per cent) that you will work with the other person next period, then it will be in your interest to work rather than shirk.

So now we can think about two possibilities. The first is that there is a small chance you will be paired with the other person next period ($p = 1/3$*). The second is s large chance that you will be paired with the same person next period (*$p = 3/4$*). With these probabilities we can put the probabilities into the payoff matrix to get the following payoffs. I have circled best responses. In the low probability case the NE is for both Bert and Anne to always shirk. In the high probability case there are two NEs, one where they work/ cooperate, the other where they shirk*

		Bert	
		Always shirk	**Work then punish**
Anne	*Always shirk*	($1,500, $1,500)	($3,500, $500)
	Work then punish	($500, $3,500)	($3,000, $3,000)

		Bert	
		Always shirk	**Work then punish**
Anne	*Always shirk*	($4,000, $4,000)	($6,000, $3,000)
	Work then punish	($3,000, $6,000)	($8,000, $8,000)

The lesson here is that if you want to generate cooperation among team members then you might want to make it clear that the same teams are likely to be used again in the next period. In this case, you have an incentive to work hard and create trust that you won't shirk. This will lead to the cooperative outcome over time. Moreover, you might want to structure payoffs so as to make it worthwhile to cooperate more generally, and punish shirking.

The key lesson here. Cooperative outcomes can be sustained when there is an ongoing relationship (or likely to be one).