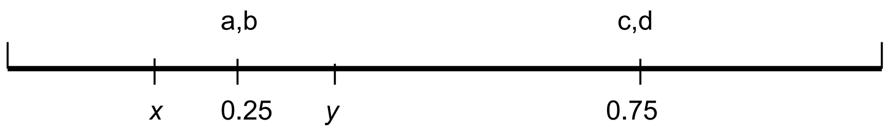
**Tutorial 5, Week 7.**

1. Consider a market in which firms choose a point on product space of length 1. If the firms have fixed prices but can choose where to locate, where we would expect four firms to locate? Why?

Solution: Here, let's assume that there are four firms (*a*, *b*, *c* and *d*) with two locating at ¼ and 2 at ¾. What we will do is show that no firm has an incentive to move. Consider:




Initially the firms each get one quarter of the market. For example, a and b share the area to the left of 0.25 (so each get 1/8 each) and all four firms share the space between 0.25 and 0.75 and each get 1/8. Hence, each firm gets ¼ in total.

Suppose that *a* makes a move to the left. In particular suppose it 'jumps to x'. In that case it will now get all the space to its left (=x) and half the distance between x and 0.25 which it now shares with b (this is equal to [0.25-x]/2). Hence a's new market share is:

$$\mathrm{x}+\frac{(0.25-\mathrm{x})}{2}=\frac{(0.25+\mathrm{x})}{2}<0.25$$

Hence it would not make sense for a to make such a move as it loses market share.

Suppose that a makes a move to the right. In particular suppose it 'jumps to y'. In that case it will now share the space to its left (y-0.25) with *b* and get half the distance between y and 0.75 which it now shares with *c* & *d* (this is equal to [0.75-y]/2). Hence a's new market share is:

$$\frac{(\mathrm{y}-0.25)}{2}+\frac{(0.75-\mathrm{y})}{2}=0.25$$

Hence it would not make sense for a to make such a move as generates no extra profit.

A jump to the right of where *c* and *d* are currently located (0.75) has the same outcome as a jump to x – it leads to lower profits.

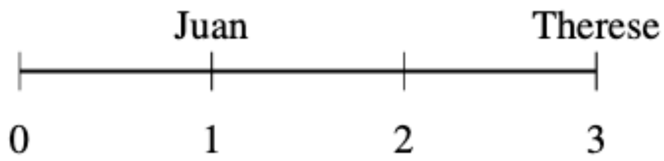
Hence, for any firm there is no incentive to move away from where they are currently located.

2. Juan and Therese each own a Taco restaurant in the town of Burritoville. The town is 3km long. Juan is located 1km along the town, and Therese is at 3km along (see the figure below). 300 consumers are uniformly located along the town (between 0 and 3). Consumer *i*'s utility derived from dining at restaurant *j* is given by:

$$u_{ij} = \bar{u} - t|x_i - y_i| - p_j$$

where *j* = 1, 2 indicate the two restaurants, *t* is the per unit cost of travelling along the town, $x_i$ is the location of consumer *i*, $y_j$ is the location of restaurant *j*, and $p_j$ is the price of restaurant *j*.

Each consumer eats a meal at exactly one restaurant. Restaurants compete with each other by simultaneously choosing prices. Each restaurant has constant marginal costs of *c* and no fixed costs.




(a) Calculate the demand for each restaurant in terms market prices and transport costs.
(b) Find the reaction function for each restaurant.
(c) Find the Nash equilibrium prices and quantities for each restaurant.
(d) Explain why Juan sets higher prices than Therese in equilibrium.

(a) Calculate the demand for each restaurant in terms market prices and transport costs. [4 marks]

ANS: To find demand, we find the location of the indifferent consumer. Let *x* be the location of the indifferent consumer. Then,

$$t|x-3| + p_2 = t|x-1| + p_1$$

$$t(3-x) - t(x-1) = p_1 - p_2$$

$$2tx = p_2 - p_1 + 4t$$

$$x = \frac{p_2 - p_1 + 4t}{2t}$$

The market share of firm 1 is *x*/3 and the demand for firm 1 is:

$$q_1 = 300x/3 = 100x = 100\frac{p_2 - p_1 + 4t}{2t}$$

The market share of firm 2 is (3 − x)/3 and the demand for firm 2 is:

$$q_1 = 300(3 - x)/3 = 100(3 - x) = 100\frac{p_1 - p_2 + 2t}{2t}$$

(b) Find the reaction function for each restaurant.

ANS: Firm 1 solves the problem:

$$\max_{p_1} \pi_1 = 100\frac{p_2 - p_1 + 4t}{2t}(p_1 - c)$$

yielding first order conditions:

$$p_2 - 2p_1 + 4t + c = 0$$

and the reaction function:

$$p_1 = \frac{p_2 + 4t + c}{2}$$

Firm 2 solves the problem:

$$\max_{p_2} \pi_2 = 100\frac{p_1 - p_2 + 2t}{2t}(p_2 - c)$$

yielding first order conditions:

$$p_1 - 2p_2 + 2t + c = 0$$

and the reaction function:

$$p_2 = \frac{p_1 + 2t + c}{2}$$

(c) Find the Nash equilibrium prices and quantities for each restaurant.

ANS: In a Nash equilibrium, both firms operate on their reaction function. Therefore:

$$p_1 = \frac{p_1 + 2t + c}{4} + \frac{4t + c}{2}$$

$$\frac{3}{4}p_1 = \frac{10t + 3c}{4}$$

$$p_1 = \frac{10t}{3} + c$$

$$p_2 = \frac{5t}{3} + \frac{c}{2} + \frac{2t + c}{2}$$

$$p_2 = \frac{8t}{3} + c$$

Substitute these prices into the demand functions to obtain equilibrium quantities. Observe that $p_1 - p_2 = 2t/3$. Therefore:

$$q_1 = 100\frac{4t - 2t/3}{2t} = \frac{500}{3}$$

$$q_2 = 100\frac{2t + 2t/3}{2t} = \frac{400}{3}$$

(d) Explain why Juan sets higher prices than Therese in equilibrium.

ANS: Juan is at a location advantage relative to Therese, because all consumers to the left of the 2km line in the city are closer to Juan, while those to the right of the line are closer to Therese. This means that 2/3 of consumers are closer to Juan. With more local consumers, Juan has greater market power, and therefore an incentive to set higher prices to take advantage of this market power. Notice also that, despite the fact that Juan sets a higher price, he has more customers in equilibrium.