ECON5026 Mid-term *Answers*
Strategic Business Relationships Practice Test

1. Two employees, Anna and Bert, are assigned to a team. If they both work hard, their productivity is enhanced, and at the end of each week they get rewarded with a bonus. However, both Anna and Bert dislike working hard, and prefer to surf the internet. The following matrix summarises the payoffs to both players each week as a function of their strategies (Work or Shirk).

		Bert: Shirk	Bert: Work
Anna	Shirk	$1000, 1000$	$5000, 0$
	Work	$0, 4000$	$2000, 2000$

   (a) Suppose Anna and Bert are assigned to work together for exactly two weeks. What equilibrium outcomes do you predict? Explain. (Use the equilibrium concepts we have discussed in class.) [5 marks]

   ANS: Notice that in the game, each player has a dominant strategy to Shirk. If Anna and Bert know that their relationship will end in two weeks, they will solve a two-period game by backward induction. In the second week, each player has a dominant strategy to Shirk. In the first week, both players know that their rival will Shirk in the second week, no matter what happens in the first week. Therefore, both players also have a dominant strategy to Shirk in the first week. The only subgame perfect Nash equilibrium involves both players choosing Shirk in weeks 1 and 2.

   (b) Suppose that Anna and Bert are assigned together for an extended period. Each week, with probability p, they continue to work together. Under what conditions is it possible for Anna and Bert to sustain a subgame perfect equilibrium in which they both work each week? [6 marks]

   ANS: To cooperate, Anna and Bert need to use strategies that will reward cooperation and punish selfish behaviour. Consider the following grim trigger strategy:

   - Work in week 1, and continue to work as long as both players have always worked.
   - Shirk if either player has ever chosen Shirk in any previous week.

   To check whether this strategy could sustain cooperation, suppose Bert adopts the grim trigger strategy, and examine the incentives for Anna to also adopt this strategy. If she cooperates by choosing Work every period, she receives payoffs with value

$$V_c = 2000 + 2000p + 2000p^2 + \cdots = \frac{2000}{1-p}.$$

If instead she deviates by choosing Shirk, she obtains the value

$$V_d = 5000 + 1000p + 1000p^2 + \cdots = 5000 + p\frac{1000}{1-p}.$$

Anna prefers to cooperate if

$$\begin{aligned} V_c &\geq V_d \\ 2000 &\geq 5000(1-p) + 1000p \\ 4000p &\geq 3000 \\ p &\geq 3/4. \end{aligned}$$

Bert has value of deviation of

$$V_d = 4000 + p\frac{1000}{10p}.$$

Note that this is less than Anna's value of deviation, so Bert has no incentive to deviate if Anna does not. To check this, we could calculate the condition required for cooperation for Bert as well. Bert cooperates if

$$\begin{aligned} 2000 &\geq 4000(1-p) + 1000p \\ 3000p &\geq 2000 \\ p &\geq 2/3. \end{aligned}$$

Therefore, if $p \geq 3/4$, playing grim trigger is a best response for both Bert and Anna, and there is a SPNE in which both Bert and Anna play the grim trigger strategy. Intuitively, if the relationship is more likely to continue, Bert and Anna will be motivated to work hard to sustain the relationship.

(c) Suppose Anna and Bert are paid every two weeks, and are only able to observe the strategy of their partner once every two weeks. Explain what effect this will have on the sustainability of cooperation. [4 marks]

ANS: This change will make monitoring more difficult/slower. It will be possible to get away with cheating on the relationship (by Shirking) for two weeks instead of one week before punishment is triggered. This increases the value of deviation relative to cooperation, and therefore will make cooperation more difficult.

2. Wanda sells bottled water in the town of Freshwaters. Demand for bottled water is give by $Q(P) = 240 - 40P$ per day, where Q is the market output, and P is the market price. The cost of bottling and distributing water is determined by the cost function $C(q) = 2q$.

   (a) Suppose Wanda is the only seller in Freshwater.

      i. What price should she charge? What profits does she earn? [3 marks]
      ANS: Wanda has profits of

$$\pi(P) = Q(P) \times (P-2) = (240-40P)(P-2).$$

      To maximise profits, she solves the first order conditions:

$$\frac{d\pi}{dP} = 0 = 240 - 40P - 40(P-2)$$
$$80P = 320$$
$$P = 4.$$

      Wanda's profits are

$$\pi(P) = (240-40P)(P-2) = (240-160) \times (4-2) = 160.$$

      ii. Discuss possible barriers to entry that might sustain the profitability of Wanda's business. [3 marks]
      ANS: In class we discussed many possible barriers to entry, including:

      - large minimum efficient scale relative to industry size
      - a saturated product space
      - consumer switching costs
      - brands and reputation
      - limited access to distribution channels
      - limited access to raw materials
      - government regulation and intellectual property

      Some of these do not seem to apply here. For example, the question specifies constant marginal costs to production and no fixed costs, so minimum efficient scale is not large relative to industry size. It is also unlikely that consumer switching costs are important in the bottled water market. Perhaps Wanda has developed a popular brand of bottled water, or has tied up the distribution channels for bottled water in Freshwater.

   (b) Suppose Frank also produces identical bottled water in the Freshwater market. Frank and Wanda simultaneously choose the price of bottled water. Consumers are perfectly informed about the prices of bottled water.

i. Derive the reaction functions for Wanda and Frank. [3 marks]
ANS: With two firms producing identical products and competing by price, we have the Bertrand model. Because the products are identical, both Wanda and Frank have an incentive to marginally undercut the price of their rival. However, if their rival sets a price above the monopoly price of \$4, the best response is to undercut down to the monopoly price rather than just marginally undercut. Finally, if one's rival sets a price below the marginal cost of \$2, the best response is to set a higher price than one's rival. Together, this defines the reaction function for both Wanda and Frank.

ii. What is the Nash equilibrium in this market. Explain carefully. [3 marks]
ANS: In the unique Nash equilibrium, both Wanda and Frank set a price equal to marginal cost of \$2. At this price, both are playing a best response to their rival's price, and therefore this represents a Nash equilibrium.

iii. In the Nash equilibrium you described, do Frank and Wanda play dominant strategies? Explain. [3 marks]
ANS: No, Wanda and Frank are not playing dominant strategies. As discussed in part 2(b)i above, both Wanda and Frank have an incentive to change their strategy if their rival sets a different price. This means that they do not have a dominant strategy.

3. Jessica has retired from her job as an economics professor at Harvard, and she now gives freelance economics lectures in her spare time. Two types of students are interested in her lectures. There are 100 Type A consumers who each have demand

$$p(Q) = 100 - 5Q,$$

where $p(Q)$ is the willingness to pay for the Q^{th} lecture. There are 100 Type B consumers who each have demand

$$p(Q) = 100 - 10Q.$$

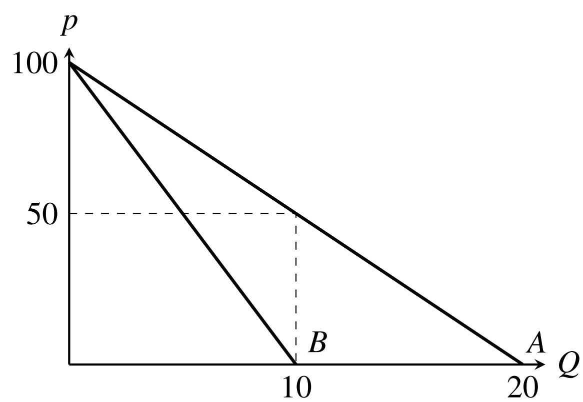
Jessica offers the following two packages to prospective students:

- *Basic*: 10 lectures for \$500.
- *Premium*: 20 lectures for \$1000.

It costs Jessica \$2000 to provide each lecture, including room booking, preparation, and lecture delivery. If all students purchase the basic package, she needs to prepare 10 lectures. If any students purchase the premium package, she must prepare 20 lectures.

(a) If she offers these two packages, what will each type of consumer buy? [4 marks]

ANS: Type A and Type B demand curves are shown below:



A Type B consumer derives consumer surplus of 500 by attending 10 lectures, so they are just willing to purchase the *Basic* package. A Type A consumer gets consumer surplus of 750 by attending 10 lectures and 1000 by attending 20 lectures. At a price of \$500 for *Basic* and \$1000 for *Premium*, Type A consumers would prefer to purchase the *Basic* package.

(b) Jessica suspects that she could increase her profits by changing the price of the premium package. What is the profit-maximising price for the Premium package? What profits does she earn? [6 marks]

ANS: At current prices, both consumer types purchase the *Basic* package. Jessica could increase profits by inducing Type A consumers to purchase the *Premium* package.

If she sets a price of \$750 for the *Premium* package, Type A consumers would be indifferent. Suppose she sets a price of \$750 for premium and Type A consumers purchase premium, while Type B consumers buy the basic package (we assume that Type A consumers buy the premium package if they are indifferent – alternatively, Jessica could set a price marginally below, say at \$749.99). Then, Jessica earns revenue of $100 \times \$500 + 100 \times \$750 = \$125,000$ and incurs costs of $20 \times 2,000$ for profits of \$85,000.

Notice that this is better than the profits she would obtain at the original prices. In this case, all students purchased the basic package for profits of $200 \times \$500 - 10 \times \$2000 = \$80,000$.

(c) What conditions are required for Jessica to practice third degree price discrimination. Suppose Jessica can use third degree price discrimination. What package prices should she set, and what would be her profits. [5 marks]

ANS: To practice third degree price discrimination, Jessica needs to be able to distinguish between Type A and B consumers, and prevent resale between them. The optimal prices are the ones in the original question: sell the basic package to Type B consumers and the premium package to Type A consumers. A price of \$500 for the basic package extracts all consumer surplus from Type B consumers. A price of \$1000 for the premium package extracts all consumer surplus from Type A consumers.

Notice that this also achieves perfect (first-degree) price discrimination – Jessica is able to extract all consumer surplus from all consumers.