## Tutorial 2 – Game Theory

1. Consider the following simple advertising games played between Coke and Pepsi. Find the Nash Equilibrium assuming advertising choices are made simultaneously. Explain the meaning of the Nash Equilibrium and the outcome observed.

| | | Pepsi's budget | | |
|---|---|---|---|---|
| | | Low | Medium | High |
| Coke's budget | Low | $60, $45 | $57.50, $50.50 | $45, $35 |
| | Medium | $50, $35 | $65, $30 | $30, $25 |
| | High | $45, $10 | $60, $20 | $50, $40 |

*Payoffs are in $m, with the row player (Coke) being the first payoff in each cell.

2. Consider the following simple bargaining games played between management and a union. The rules of bargaining require that each party nominate a specified amount of the $100 surplus that is available. If the amount nominated in total exceeds $100 no agreement is reached and each party simply incurs a small cost associated with bargaining. Find the Nash Equilibria and explain what you might think will happen.

| | | Union | | |
|---|---|---|---|---|
| | | 0 | 50 | 100 |
| Management | 0 | $0, $0 | $0, $50 | $0, $100 |
| | 50 | $50, $0 | $50, $50 | -$1, -$1 |
| | 100 | $100, $0 | -$1, -$1 | -$1, -$1 |

3. Consider the following pricing problem for two Pizza chains, Domino's and Crust, which are considering offering a new lunch special. First, eliminate any dominated strategies. Next, find the Nash Equilibrium if both chains make their choice simultaneously.

| | | Domino's | | |
|---|---|---|---|---|
| | | High ($10) | Medium ($8) | Low ($6) |
| Crust | High ($10) | $1000, $1,000 | $900, $1,100 | $500, $1,200 |
| | Medium ($8) | $1,100, $400 | $800, $800 | $450, $500 |
| | Low ($6) | $1,200, $300 | $500, $350 | $400, $400 |

*Payoffs are profit per week, with the row player (Crust) being the first payoff in each cell.

Now, consider what the outcome of the game will look like if Crust announces its menu first using Facebook and Domino's makes its' choice later. Does Crust have a first or second mover advantage?

4. In business, it is sometimes said that limiting your options or tying one's hands can actually improve your situation. Why is this the case and how might a firm do this?

5. It is easy to think of situations where it advantageous to be a first mover when decisions are being made sequentially. Will it ever be advantageous to be a second mover? Why?

6. Consider the following game between a firm and a customer. The firm may offer a high-quality product or a low-quality product. High quality products cost more to make compared to low quality products and hence profits will be higher if the firm sells a low-quality product. Consumers prefer high quality products but cannot determine quality prior to purchase. The payoffs for the firm and consumer are presented below. Find the Nash Equilibrium if the parties play the game once.

| | | Firm | |
|---|---|---|---|
| | | Low quality | High Quality |
| Customer | Don't buy | \$0, \$0 | \$0, -\$10 |
| | Buy | -\$10, \$10 | \$1, \$1 |

How might your answer change if the game is repeated 'infinitely' and the customer offers to keep buying in the future?

***7.*** Suppose that you are considering purchasing a car. You believe it is valued between \$1000 and \$5000, with an equal probability that it has a value at any point in this range. that is, you believe it is uniformly valued between \$1000 and \$5000. The current owner knows the car and its true value. Assume that because you are a mechanical genius, whatever the car is worth to the seller, it is worth 1.33 times that to you (so a car worth \$2400 to the seller is actually worth \$3200 to you).

(i) What is your expected profit if you offer \$3000?

(ii) What offer should you make to ensure that you will not lose money?

8. **[NOTE THIS IS A CHALLENGING QUESTION FROM THE LECTURES THAT WE WILL ALSO WORK THROUGH DURING THE TUTORIAL**] Consider the following game played between Anne and Bert. Each are part of team that is assigned to work on a project, and both have a choice to work or shirk. They get a \$1000 base salary. If they both work then they develop a good project and get a bonus. However, working creates disutility and this reduces the net payoff. If only Anne or Bert work hard, the project will still be successful and they both get the bonus. However, in this situation the effort of the worker results in a net payoff of \$0 to the individual who put in the effort.

| | | Bert | |
|---|---|---|---|
| | | Shirk | Work |
| Anne | Shirk | \$1,000, \$1,000 | \$3,000, \$0 |
| | Work | \$0, \$3,000 | \$2,000, \$2,000 |

Find the Nash Equilibrium if Anne and Bert expect to be assigned to the same team just once.

Suppose now that Anne and Bert might be assigned to work together in future periods with a probability of p. For example, the probability that they are assigned to work together for the second period is *p*, the probability they are assigned to work together for the third period is $p^2$, etc.

Assume that Anne and Bert choose to shirk in every period they work together. Write out an expression for their expected payoff.

Now suppose that after the work is done, they can observe if the other person actually put in effort or simply shirked. Further, assume that if Anne or Bert observes the other has shirked, they will punish them by shirking in the future. In that case we can write out the payoff matrix for Anne and Bert as follows:

| | | **Bert** | |
|---|---|---|---|
| | | **Always shirk** | **Work then punish** |
| *Anne* | *Always shirk* | $\left(\frac{1000}{(1-p)}, \frac{1000}{(1-p)}\right)$ | $\left(2000+\frac{1000}{(1-p)}, -1000+\frac{1000}{(1-p)}\right)$ |
| | *Work then punish* | $\left(-1000+\frac{1000}{(1-p)}, 2000+\frac{1000}{(1-p)}\right)$ | $\left(\frac{2000}{(1-p)}, \frac{2000}{(1-p)}\right)$ |

In this case, will Anne 'Always shirk'?