LECTURE 2.0

GAME THEORY

GAME THEORY - READING

Chapter 14, “Game Theory and Strategic Behavior” in Besanko and Braeutigam (2002) *Microeconomics – An Integrated Approach* (Available on Canvas)

WHY STUDY GAME THEORY?

Most relationships within markets and within firms are strategic. Consider the following:

- the firm and its competitors in an oligopoly (e.g. pricing)
- the firm and its suppliers (e.g. contract negotiation)
- a manager and her subordinates (e.g. wage negotiations)
- members of a production team (e.g. allocation of effort)

In each case, the relationships are strategic:

- all parties can impact others
- all parties recognise they are impacted by others
- it matters what I do, and I care about what you do. I need to consider your response.

WHY STUDY GAME THEORY?

Game theory studies the strategic interaction between players.

We can solve strategic problems using the tools of game theory.

This week we will examine some different types of games and approaches to solving them. We will also identify some real-world examples where game theory provides insight into behaviour and outcomes.

GAME THEORY FOUNDATIONS

Our players will have the following features:

- Decision makers are rational optimisers
- Decision makers understand the game they are playing
- Decision makers need to anticipate choices of rivals
- Decision makers presume that rivals are also rational optimisers who understand the game

In this unit we will focus on non-cooperative games: negotiation and enforcement of binding contracts is not possible.

Cooperative game theory enables players or agents to negotiate binding contracts that allow them to plan and implement joint strategies. Cooperative game theory is beyond the scope of this course.

GAME THEORY- LECTURES

2.0 Why study game theory?

2.1 Single period games: simultaneous moves

2.2 Mixed strategies

2.3 Single period games: sequential interactions

2.4 Repeated games

2.5 Auctions

LECTURE 2.1

SINGLE PERIOD GAMES:

SIMULTANEOUS MOVES

SIMULTANEOUS MOVE, ONE-SHOT GAMES

In simultaneous move, one-shot games, you make decisions without knowing the action of your rival. This can be interpreted as:

- Players make decisions at the same time
- Players make decisions before knowing the decisions of their rivals.

Usually write out in what is called “strategic” or “normal” form:

- Payoffs are represented in a matrix
- Payoffs include all the benefits to a player (monetary, non-monetary)

SIMULTANEOUS MOVE, ONE-SHOT GAMES

Consider the following pricing game for Boeing and Airbus. Numbers represent profits.

The number left number in each cell is the payoff to the left player (Boeing).

		Airbus	
		Low price	High price
Boeing	Low price	$500, $500	$1000, $0
	High price	$0, $1000	$750, $750

SIMULTANEOUS MOVE, ONE-SHOT GAMES

Consider the following pricing game for Boeing and Airbus. Numbers represent profits.

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Airbus

Boeing		Low price	High price
	Low price	$500, $500	$1000, $0
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SIMULTANEOUS MOVE, ONE-SHOT GAMES

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SIMULTANEOUS MOVE, ONE-SHOT GAMES

This Airbus-Boeing game is a rebadged Prisoners dilemma in which the (strictly) dominant strategy is to expand.

Notice how this does not maximise surplus from the view of the players. How can firms such as these get to a better outcome?

In this game we assume that players make their decisions simultaneously. Consider what happens when players make decisions sequentially. We'll come to that in another section.

COMPONENTS OF A GAME

- Players: $i = 1,...,n$
- Actions: (e.g. prices or quantities or advertising)
- Strategies: complete contingent plan of action
- Information available to players: we will assume perfect information
- Rules of the game
- Payoffs: a complete summary of the value to each player

DOMINANT STRATEGIES

A strategy is (strictly) **dominant** if it gives a (strictly) higher payoff than every other strategy, for every strategy that your rivals play.

- If you have a strictly dominant strategy, you should play it for sure.
- In a *dominant strategy equilibrium*, all players choose a dominant strategy.

DOMINANT STRATEGIES

Reconsider the pricing game between Boeing and Airbus. Does either player have a dominant strategy?

		Airbus	
		Low price	High price
Boeing	Low price	$500, $500	$1000, $0
	High price	$0, $1000	$750, $750

DOMINANT STRATEGIES

Consider a revised pricing game for Boeing & Airbus where the US Government guarantees Boeing's survival. Can you see any dominant strategies?

		Airbus	
		Low price	High price
Boeing	Low price	$500, $500	$1000, $0
	High price	$600, $1000	$750, $750

EXAMPLE: CIGARETTE ADVERTISING

Consider the problem faced by the major tobacco companies in the 1970s.

- The firms could advertise or not.
- If you don't advertise but your rivals do, their profits increase and yours fall.

		BAT	
		Don't Ad	Advertise
Imperial	Don't advertise	\$50, \$50	\$20, \$60
	Advertise	\$60, \$20	\$27, \$27

EXAMPLE: CIGARETTE ADVERTISING

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EXAMPLE: CIGARETTE ADVERTISING

The US government imposed an advertising ban on TV (amongst other health related measures).

Big 4 tobacco companies spent $315m on advertising in 1970, and $252m in 1971. Profits rose by $91m. The dominant strategy equilibrium was removed, benefitting the tobacco firms

BAT

Imperial

	Don't Ad	~~Advertise~~
Don't advertise	$50, $50	~~$20, $60~~
~~Advertise~~	~~$60, $20~~	~~$27, $27~~

DOMINATED STRATEGIES

A strategy is (strictly) dominated if the player has another strategy that gives a (strictly) higher payoff no matter what her rivals do.

Consider the following game. Does either player have a dominated strategy?

		Toyota		
		Large	**Small**	**Don't build**
	Large	0,0	12,8	18,9
Honda	**Small**	8,12	16,16	20,15
	Don't build	9,18	15,20	18,18

DOMINATED STRATEGIES

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Consider the following game. Does either player have a dominated strategy?

		Toyota		
		~~**Large**~~	**Small**	**Don't build**
	~~**Large**~~	~~0,0~~	~~12,8~~	~~18,9~~
Honda	**Small**	~~8,12~~	16,16	20,15
	Don't build	~~9,18~~	15,20	18,18

BEST RESPONSES

Best response function (reaction function): best choice of strategy for any choice of rival.

A best response is determined by asking:

- “Suppose I know Bob will play ‘Left’. What is my best choice?”
- “Suppose I know Bob will play Right. What is my best response?”

With more players, to determine a best response, you would need to know what every other player would play.

- I’m a member of OPEC. Suppose I know that Kuwait plans to export 20m barrels of oil. Saudi Arabia plans to export 30m barrels, and Nigeria plans to export... How much should I produce?
- Now suppose I know Kuwait plans to export 25m, Saudi Arabia plans to export 32m, and Nigeria plans... How much should I produce now?

NASH EQUILIBRIUM

A set of strategies is a Nash equilibrium if every player is playing a best response to their rivals' strategies. No one has an incentive to change strategy

- A Nash equilibrium is self-enforcing or stable
- Nash equilibrium is a weaker concept than dominant strategy equilibrium

NASH EQUILIBRIUM

Nash equilibrium is a self-fulfilling agreement. If we agree to play a certain way, we'll both go through with it. Unilateral deviations are not worthwhile.

A Nash equilibrium could also be an outcome that we've settled on after repeating a situation many many times, or an outcome we expect given repeated observation of similar situations

What I choose to do depends on what I expect the other player to do – it's an inherently strategic concept.

NASH EQUILIBRIUM

A procedure for finding Nash Equilibrium in two person games in five steps.

- If both players have a dominant strategy – these constitute their Nash equilibrium strategies.
- If one player has a dominant strategy this is their Nash equilibrium strategy, then find the other players best response to identify NE.
- If neither player has a dominant strategy, eliminate dominated strategies.
- Identify best responses for each player.
- Look for an equilibrium in mixed strategies – see later.

NASH EQUILIBRIUM

Recall this example from earlier. Both players have a dominant strategy, which gives the Nash equilibrium.

		Airbus	
		Low price	High price
Boeing	Low price	$500, $500	$1000, $0
	High price	$0, $1000	$750, $750

NASH EQUILIBRIUM

A procedure for finding Nash Equilibrium in two person games in five steps.

- If both players have a dominant strategy – these constitute their Nash equilibrium strategies.
- If one player has a dominant strategy, this is their Nash equilibrium strategy. Then find the other players best response to identify Nash equilibrium.
- If neither player has a dominant strategy, eliminate dominated strategies.
- Identify best responses for each player.
- Look for an equilibrium in mixed strategies – see later.

NASH EQUILIBRIUM

Recall this revised pricing example from earlier. Airbus has a dominant strategy. We can then determine the best response of Boeing to locate the Nash equlilibrium.

		Airbus	
		Low price	High price
Boeing	Low price	$500, $500	$1000, $0
	High price	$0, $1000	$750, $750

NASH EQUILIBRIUM

A procedure for finding Nash Equilibrium in two person games in five steps.

- If both players have a dominant strategy – these constitute their Nash equilibrium strategies.
- If one player has a dominant strategy this is their Nash equilibrium strategy, then find the other players best response to identify NE.
- If neither player has a dominant strategy, eliminate dominated strategies.
- Identify best responses for each player.
- Look for an equilibrium in mixed strategies – see later.

NASH EQUILIBRIUM

Recall this game from before. We effectively used these steps: eliminated the dominated strategies to identify the Nash equilibrium of Small/Small.

		Toyota		
		Large	Small	Don't build
	~~Large~~	~~0,0~~	~~12,8)~~	~~18,9~~
Honda	Small	8,12	16,16	20,15
	Don't build	9,18	15,20	18,18

COORDINATION GAMES

In a competitive setting coordination can often be very profitable. Consider this game between Boeing & Airbus making a decision about using a common communications technology

		Airbus: Alpha	Airbus: Beta
Boeing	Alpha	\$100, \$100	\$50, \$50
Boeing	Beta	\$50, \$50	\$100, \$100

COORDINATION GAMES

There are two Nash equilibria, and no focal point to help coordinate.

		Airbus	
		Alpha	Beta
Boeing	Alpha	\$100, \$100	\$50, \$50
	Beta	\$50, \$50	\$100, \$100

COORDINATION GAMES

Consider this revised technology game. Again there are two Nash equilibria.

How could Boeing get its preferred Nash equilibria? Could Boeing pre-commit to one option? If so, pre-commitment must be credible (e.g. public sign agreement with parts supplier).

		Airbus	
		Alpha	**Beta**
Boeing	**Alpha**	$100, $50	$40, $40
	Beta	$25, $25	$50, $100

LECTURE 2.2

MIXED STRATEGIES

MIXED STRATEGY EQUILIBRIA

To this point considered pure strategy equilibria.

Sometimes randomization can be best strategy. Consider the following game:

		Bob		
		Rock	**Paper**	**Scissors**
	Rock	0,0	-1,1	1,-1
Bill	**Paper**	1,-1	0,0	-1,1
	Scissors	-1,1	1,-1	0,0

PURE VERSUS MIXED STRATEGY NASH EQUILIBRIUM

Rock-Paper-Scissors has no “pure” strategy (non random) Nash equilibrium.

It has a mixed/random-strategy Nash equilibrium: each player chooses actions randomly (i.e. choose each action with probability 1/3).

Similar outcomes in:

- games where each player would like to “out guess” the other
- “zero-sum” games where payoffs are negatively correlated (i.e. I must be doing worse when you are doing better).

PURE VERSUS MIXED STRATEGY NASH EQUILIBRIUM

In a mixed strategy Nash equilibrium:

- players randomise over pure strategies
- as in a pure strategy Nash equilibrium, no one has an incentive to change strategy
- logically, this means that every pure strategy a player randomises over must be equally profitable.

MIXED STRATEGY EQUILIBRIA

Think about a decision about whether to commit to a negative or positive advertising campaign.

There is no dominant strategy and no Nash equilibrium in pure strategies. We have a zero-sum game: if Boeing gains $10, Airbus loses $10, and vice versa.

		Airbus	
		Negative	**Positive**
Boeing	**Negative**	**$10, -$10**	**-$10, $10**
	Positive	**-$10, $10**	**$10, -$10**

MIXED STRATEGY EQUILIBRIA

Suppose Airbus uses negative advertising with probability *p*, and Boeing negative advertising with probability *q*. Then Boeing receives payoffs of:

$$\begin{cases} 10p - 10(1-p) & \text{if they advertise negatively} \\ -10p + 10(1-p) & \text{if they advertise positively} \end{cases}$$

Boeing is indifferent and has no incentive to change strategy if:

$$10p - 10(1-p) = -10p + 10(1-p)$$
$$p = 0.5$$

We can go through the same exercise to show that Airbus has no incentive to change if *q*=0.5.

Therefore, in the mixed strategy equilibrium, both players advertise negatively with probability 0.5.

MIXED STRATEGY EQUILIBRIA

Let's think about Boeings choice in a different way.

The only way they can achieve a payoff that is independent of Airbus's action is to randomise with probability q=0.50. Consider if Airbus chooses a positive campaign with probability p:

- 0.5p becomes the probability that Boeing matches Airbus's positive campaign.
- 0.5(1-p) becomes the probability that Boeing matches Airbus's negative.
- Probability of Boeing winning equals 0.5p + 0.5(1-p) = 0.5.
- Probability of Boeing losing equals 0.5p + 0.5(1-p) = 0.5.
- Therefore Boeing's payoff is independent of what Airbus does.

EXAMPLE: PRICING

Consider a town with:

- two stores each with 1,000 loyal customers (who always shop there)
- an additional 1,000 shoppers who go where prices are lowest.

Think of these as informed and uninformed customers.

Shoppers are willing to pay up to $2 for paper towels for which the marginal cost is $1.

The two stores simultaneously choose a price (between $1 and $2)

What should the firms do?

Let’s look for a Nash equilibrium to this game.

EXAMPLE: PRICING

First, observe that there cannot be a pure strategy Nash equilibrium.

- Suppose otherwise that both firms choose some price *p* between \$1 and \$2
- For any price *p* set by Firm 1, Firm 2 could do better by setting a price just a little bit lower

So, we we will look for a mixed strategy Nash equilibrium

- Let F(*p*) be the probability that each firm sets a price lower than *p*

To describe the mixed strategy equilibrium, we need to find the highest and lowest price set in equilibrium, and we need to find F(p).

- Let $\overline{p}$ be the highest price
- Let $\underline{p}$ be the lowest price

Recall that in a mixed strategy Nash equilibrium, players must be indifferent between each possible pure strategy

- Hence, $\pi(\overline{p}) \; = \; \pi(\underline{p}) \; = \; \pi(p)$, where $\pi(p)$ is the expected profits to each firm from setting price *p*, for *p* between $\underline{p}$ and $\overline{p}$

EXAMPLE: PRICING

To find the highest possible price, observe that, by setting the highest price, a firm only sells to its loyal customers:

$$\pi(p) = 1000(\overline{p} - 1)$$

Therefore, it makes sense to set $\overline{p}$ = 2, the highest possible price, making $\pi(\overline{p})\ = 1000$.

Similarly, by setting the lowest possible price, a firm is guaranteed to capture all the informed consumers:

$$\pi(p) = 2000\left(\underline{p} - 1\right)$$

In equilibrium, this must give the same profits as the highest price: 2000($\underline{p}$ − 1) = 1000 ⇒ $\underline{p}$ = 1.5

EXAMPLE: PRICING

The final piece of the puzzle is to find F(p). Suppose Firm 1 sets price p between $\underline{p}$ and $\overline{p}$. How many customers do they sell to?

- They have 1000 loyal customers
- With probability 1 − F (p), they have the cheapest price, and they also sell to the 1000 informed consumers

Therefore, expected profits are

$$\pi(p) = \left(1000 + 1000\big(1 - F(p)\big)\right)(p - 1)$$
$$= 1000\big(2 - F(p)\big)(p - 1)$$

In equilibrium, this must give the same profits as the highest or lowest price.

$$1000\big(2 - F(p)\big)(p - 1) = 1000$$

$$2 - F(p) = \frac{1}{p - 1} \Rightarrow F(p) = \frac{2p - 3}{p - 1}$$

EXAMPLE: PRICING

In summary, we can show that the firms should choose a price of between \$1.50 and \$2.00 where the probability the price is less than p is given by the following:

$$\mathrm{F}(p) = \frac{2p - 3}{p - 1}$$

So the probability the price is less than \$1.75 is equal to:

$$\mathrm{F}(p) = \frac{2 * 1.75 - 3}{1.75 - 1} = 0.66$$

EXAMPLE: PRICING

So what is the intuition and lesson here?

- We are thinking about a game where the firms are randomly choosing a price – in effect randomly choosing to have a sale. They might have a sale one week in every three and so set the price less than $1.75 once every three weeks.
- The question is whether this is the best strategy and what does it mean for the other firm – will they respond and how to do they respond?
- ***What we showed was the equilibrium set of strategies***.

EXAMPLE: MARKET ENTRY

Consider the market entry game in which firms try to decide whether to enter a market.

		Pure Water Co	
		Enter	**Don't enter**
Eau Claire	**Enter**	**-\$50, -\$50**	**\$100, \$0**
	Don't enter	**\$0, \$100**	**\$0, \$0**

EXAMPLE: MARKET ENTRY

There are two pure strategy equilibria. But is there a mixed strategy equilibrium?

		Pure Water Co	
		Enter	Don't enter
Eau Claire	Enter	-$50, -$50	$100, $0
	Don't enter	$0, $100	$0, $0

EXAMPLE: MARKET ENTRY

The key to solving this is to note that:

"Every pure strategy that is played as part of a mixed strategy Nash equilibrium has the same expected value"

Let *p* be probability that Eau Claire enters, and 1-*p* be probability that Eau Claire does not enter.

Let *q* be probability that Pure Water Co enters, and 1-*q* be probability that Pure Water Co does not enter.

Eau Claire's profit if it chooses the pure strategy of entering while Pure Water Co uses a mixed strategy:

$$EV_1(enter) = -50q + (1-q)100$$

Suppose Eau Claire chooses the pure strategy of not entering while 2 uses a mixed strategy. This gives a payoff of 0.

EXAMPLE: MARKET ENTRY

Now, we want to set the payoffs for Eau Claire under both pure strategies equal to each other:

$$EV_1(enter) = EV_1(stay\ out)\ ,\ or$$

$$-50q + (1-q)100 = 0$$

$$q = 2/3$$

Note Eau Claire's expected payoff:

$$EV_1(enter) = 2/3*(-50)+1/3*(100) = EV_1(stay\ out) = 0$$

So applying the same approach to Pure Water Co, the equilibrium strategy is 2/3 probability on entering market.

EXAMPLE: MARKET NICHE

Consider the market niche game in which firms try to carve out a market niche. Note that this is no longer symmetric.

Again, two pure strategy equilibria.

		Pure Water Co	
		Enter	Don’t enter
Eau Claire	Enter	-$50, -$50	$150, $0
	Don’t enter	$0, $100	$0, $0

EXAMPLE: MARKET NICHE

Solving again:

Let *p* be probability that Eau Claire enters, and 1-*p* be probability that Eau Claire does not enter.

Let *q* be probability that Pure Water Co enters, and 1-*q* be probability that Pure Water Co does not enter.

Eau Claire's profit if it chooses the pure strategy of entering while Pure Water Co uses a mixed strategy:

$$EV_1(enter) = -50q + (1-q)150$$

Suppose Eau Claire chooses the pure strategy of not entering while Pure Water Co uses a mixed strategy. This gives a payoff of 0.

EXAMPLE: MARKET NICHE

Now, we want to set the payoffs for Eau Claire under both pure strategies equal:

$$EV_1(enter) = EV_1(stay\ out)\ ,\ or$$

$$-50q + (1\text{-}q)150 = 0$$

$$q=3/4$$

Note Eau Claire's expected payoff:

$$EV_1(enter) = 3/4*(-50)+1/4*(100) = EV_1(stay\ out) = 0$$

Applying the same approach to Pure Water Co, the equilibrium strategy for Eau Claire is 2/3 probability on entering market (as for the first case).

EXAMPLE: SALES

Think about a choice between everyday low pricing versus sales. There are two pure strategy equilibria.

		Big W	
		No sale	Sale
Target	No sale	\$7500, \$7500	\$7500, \$8500
	Sale	\$8500, \$7500	\$5500, \$5500

EXAMPLE: SALES

What of a mixed strategy?

Let *p* be probability Target charges normal price and 1-*p* be probability charging sale price.

Let *q* be probability Big W charges normal price and 1-*q* be probability charging sale price.

Target's profit if it chooses the pure strategy of charging a sale price while Big W uses a mixed strategy:

$$EV\,(SP) = 8500q + 5500(1\text{-}q)$$

Suppose Target chooses the pure strategy of normal pricing while Big W uses a mixed strategy. This gives a payoff of $7500.

EXAMPLE: SALES

We want to set the payoffs for Target under both pure strategies equal :

$$EV\,(NP) = EV\,(SP) = 7500,\ or$$

$$8500q + 5500(1-q)=7500$$

$$\text{Solving: } q=2/3$$

Note Target's expected payoff:

$$EV\,(SP) = 1/3^*(5500)+2/3^*(8500) = EV_1(NP) = 7500$$

Applying the same approach to Big W, the equilibrium strategy for Target is 1/3 probability of having a sale.

EXAMPLE: SALES

We an also approach this using best response curves:

		Big W	
		No sale	Sale
Target	No sale	\$7500, \$7500	\$7500, \$8500
	Sale	\$8500, \$7500	\$5500, \$5500

EXAMPLE: SALES

Let *p* be probability that Target plays no sale, and 1-*p* be probability it plays sale

Let *q* be probability that BigW plays no sale, and 1-*q* be probability it plays sale.

Combination	Probability	Payoff to Target
No sale, no sale	*pq*	7500
Sale, no sale	(1-p)q	8500
No sale, Sale	p(1-q)	7500
Sale, Sale	(1-p)(1-q)	5500

EXAMPLE: SALES

So:

$$\textit{Target's payoff} = 7500pq + 8500(1-p)q + 7500p(1-q) + 5500(1-p)(1-q)$$

$$= 5500 + 2000p - 3000q - 3000pq$$

So if Target considers increasing *p* by Δp the change in payoff is given by:

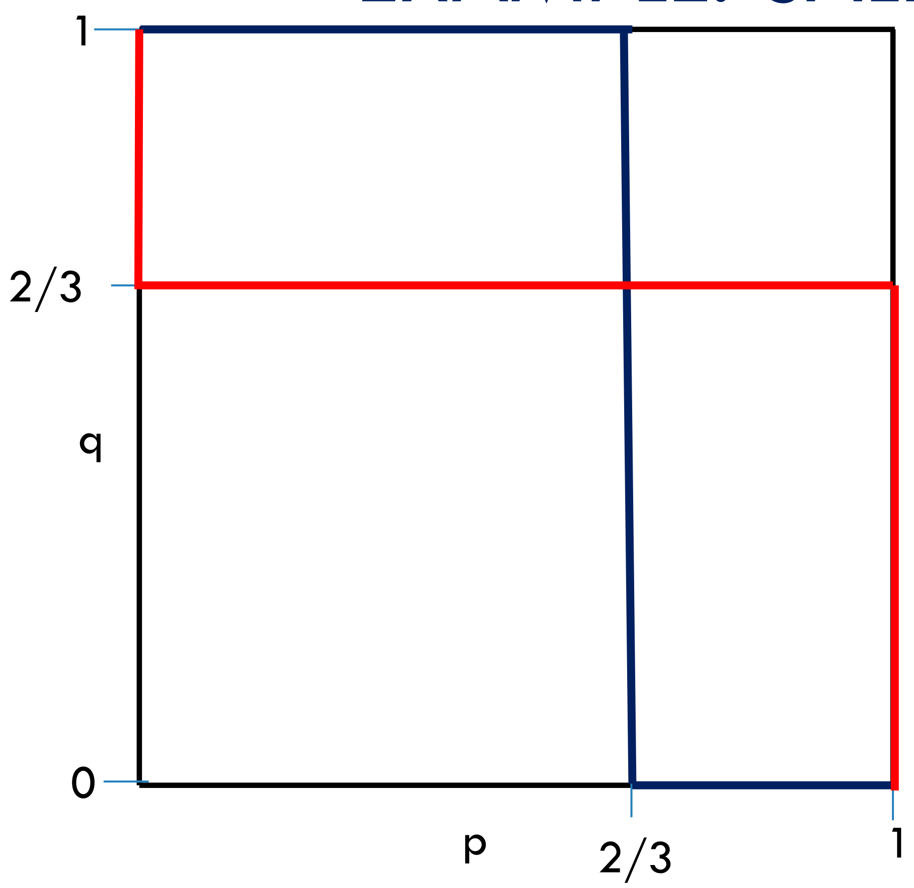
$$\Delta \textit{Target's payoff} = 2000\,\Delta p - 3000\,\Delta pq = (2000-3000q)\,\Delta p$$

This is positive if $q<2/3$.

Hence Target will want to increase *p* when $q<2/3$ and decrease *p* when $q>2/3$, and be happy with any value of *p* when $q=2/3$.

We can use an analogous argument for Big W to show that it will want to increase *q* when $p<2/3$, decrease *q* when $p>2/3$ and be happy with any value of *q* as when $p=2/3$.

EXAMPLE: SALES



We can identify Big W's best response curve. Recall it will want to increase q when $p<2/3$, decrease q when $p>2/3$ and be happy with any value of q as when $p=2/3$

We can identify Targets best response curve, recall it will want to increase p when $q<2/3$, decrease p when $q>2/3$ and be happy with any value of p when $q=2/3$.

Where the curves cross correspond to the pure strategy Nash equilibrium, and the mixed strategy Nash equilibrium.

EXAMPLE: SALES

In each case the reaction functions simply told us what the best response for each of the firms was to the choice of the other firm.

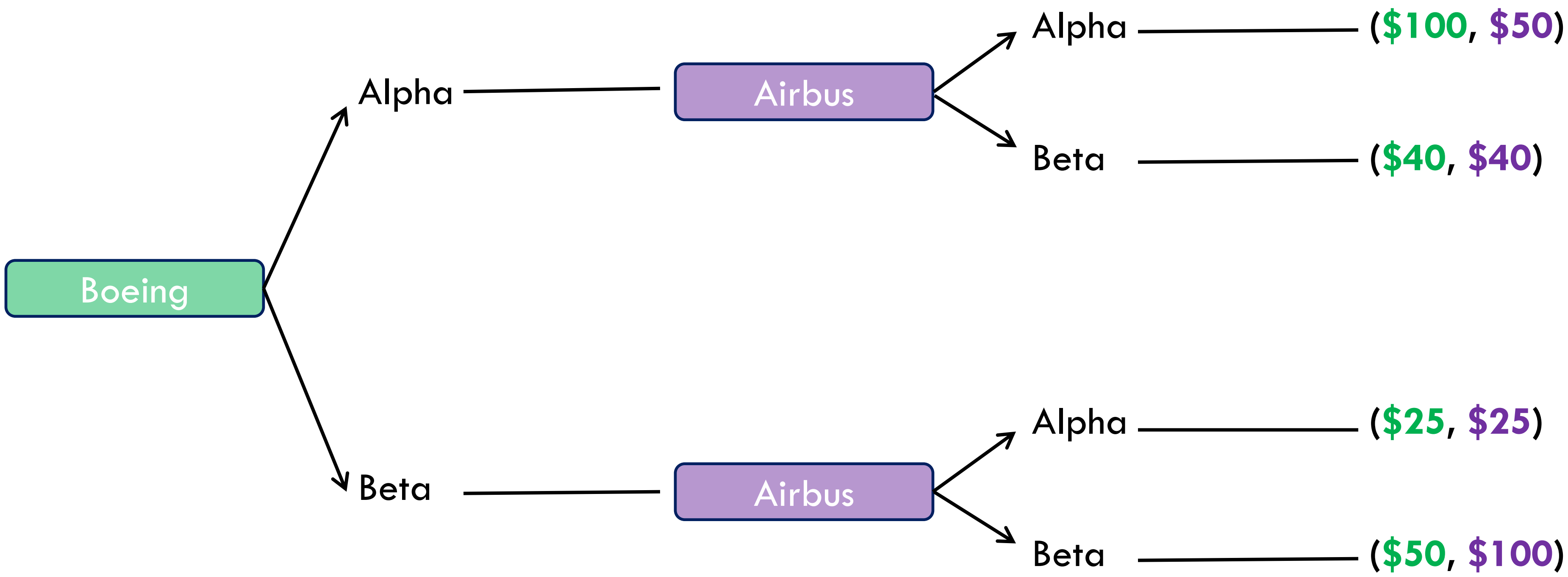
When those best response curves intersected the choices were consistent and none of the players had any incentive to unilaterally change their behavior.

LECTURE 2.3

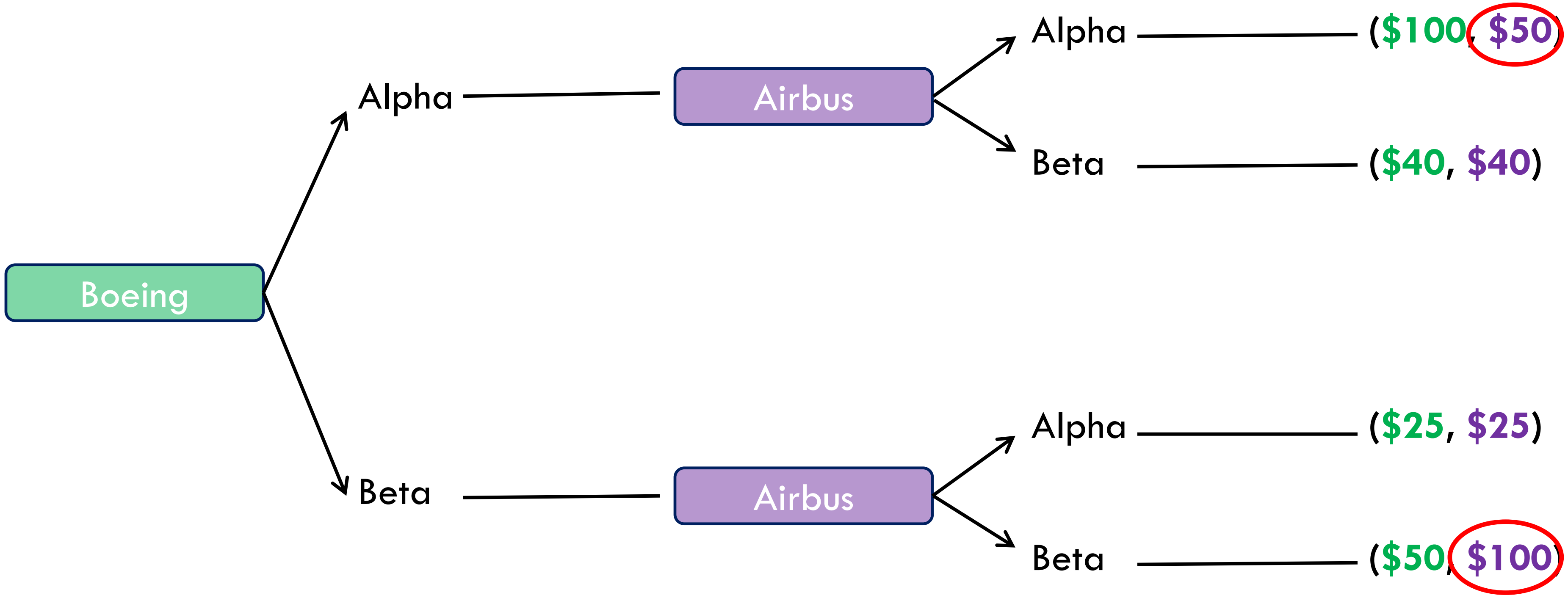
SINGLE PERIOD GAMES:

SEQUENTIAL MOVES

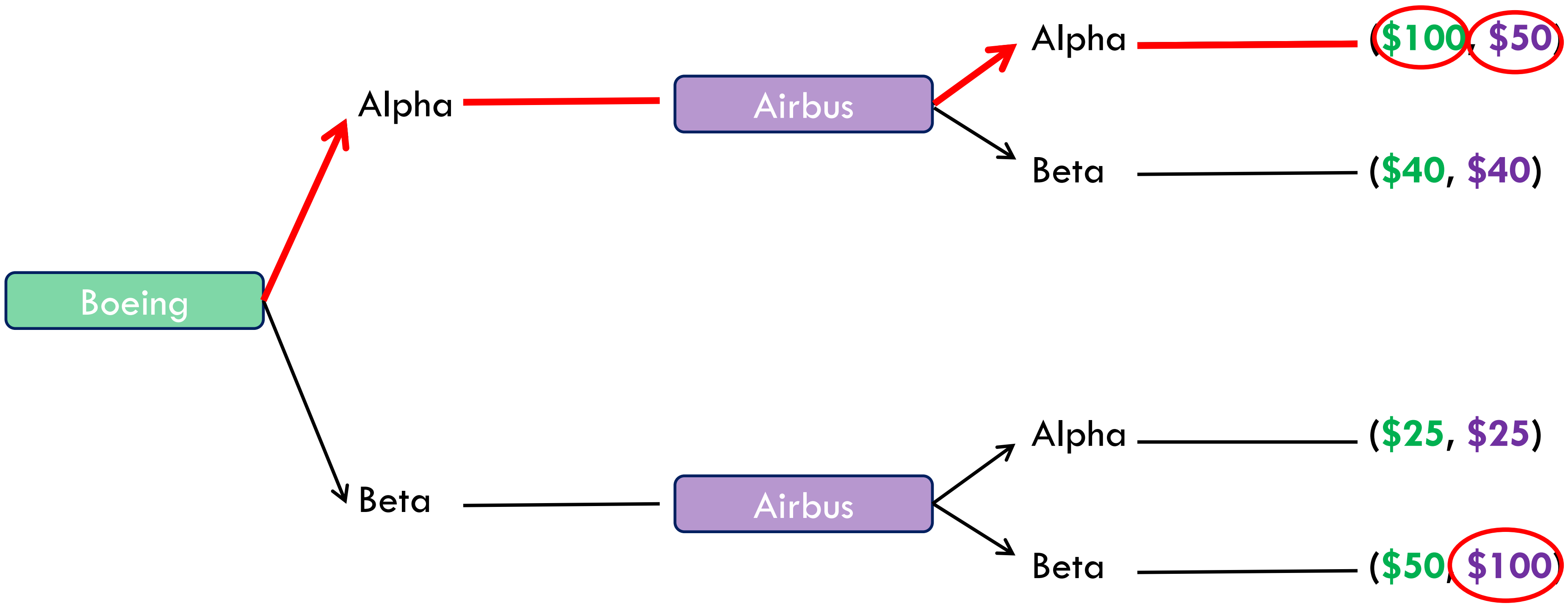
SEQUENTIAL DECISION MAKING



SEQUENTIAL DECISION MAKING



SEQUENTIAL DECISION MAKING



SEQUENTIAL DECISION MAKING

The extensive form representation explicitly shows the timing of play. The left number for each outcome is the payoff to the first player.

We can solve sequential games like this by backward induction using the concept of a Subgame perfect Nash equilibrium:

- solve for the decision nodes at the end of the game first
- work your way to the beginning of the game

A subgame is a part of a game that can be played as a game itself. A collection of nodes and branches that:

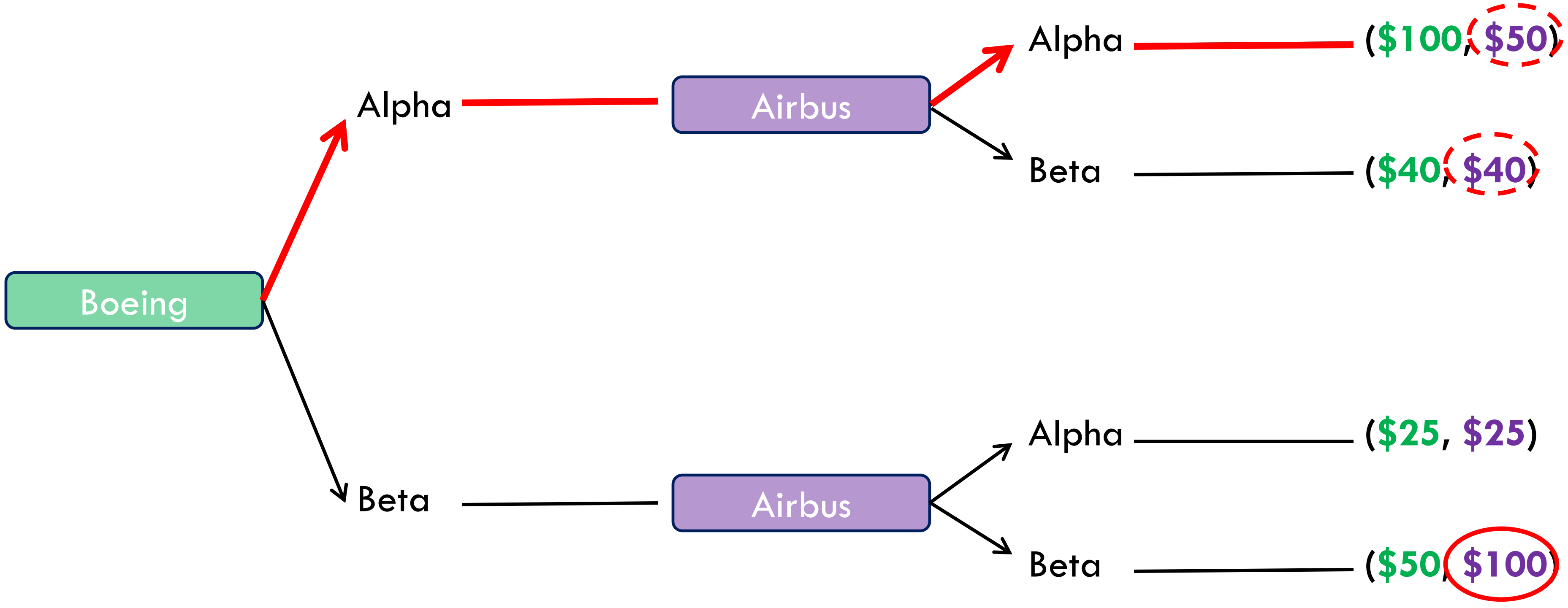
- Begins at a single node
- Contains every successor node.
- Contains all the relevant information

Why is the concept of subgame important? Threats exhibit a lack of credibility of at the time that they are to be carried out, the player does not maximise utility by carrying out the threat.

A Nash Equilibrium is subgame perfect if every player plays the Nash Equilibrium in every subgame

SEQUENTIAL DECISION MAKING

Here a threat by Airbus to always choose Beta is simply not credible as it is not subgame perfect



COMMITMENT

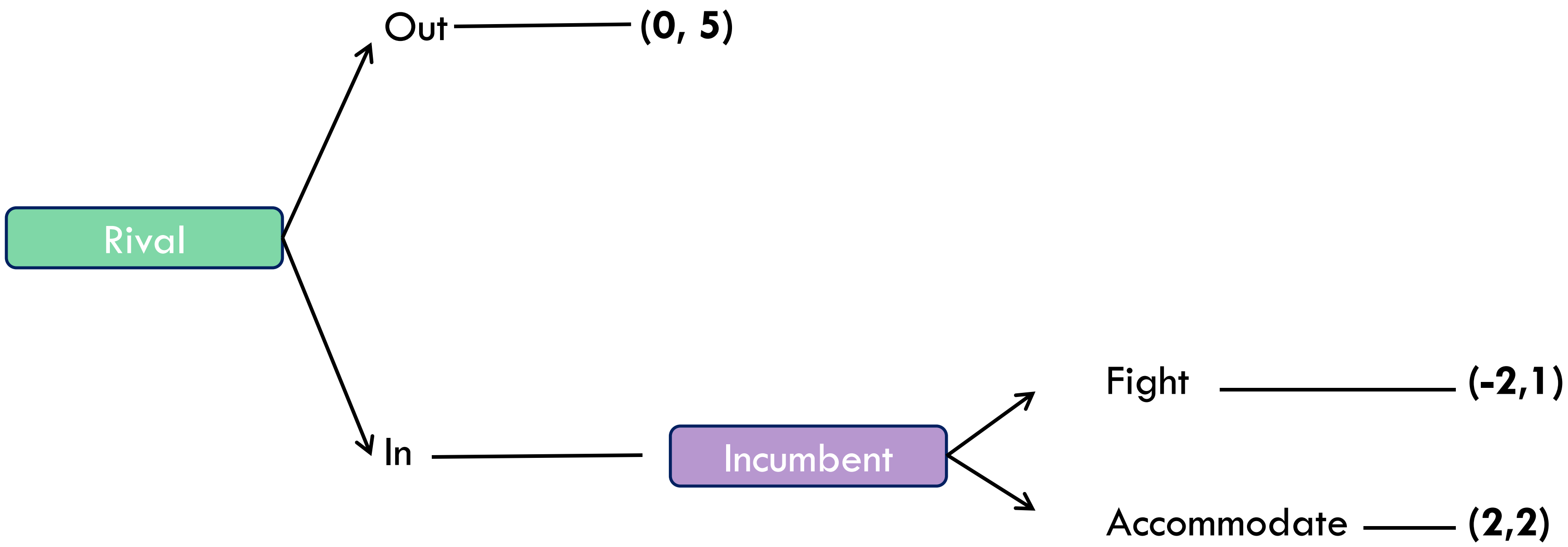
One way that a threat can be made credible is through some form of commitment.

For a commitment to be credible in general we would expect that it would

- Be visible
- Be understood by rivals
- Be credible – for example through some aspect of irreversibility such as capacity expansion in assets that are can not be redeployed or an agreement which makes a credible commitment not to compete on price.

COMMITMENT

In this game there are two Nash equilibrium, but only one Subgame perfect equilibrium as the incumbent's threat to fight is not credible.



COMMITMENT

If you can't see the Nash equilibrium in the extensive form, look at the game in normal form.

Rival \ Incumbent	Fight	Accommodate
Out	0, 5	0, 5
In	-2, 1	2, 2

COMMITMENT

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		Incumbent	
		Fight	Accommodate
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COMMITMENT

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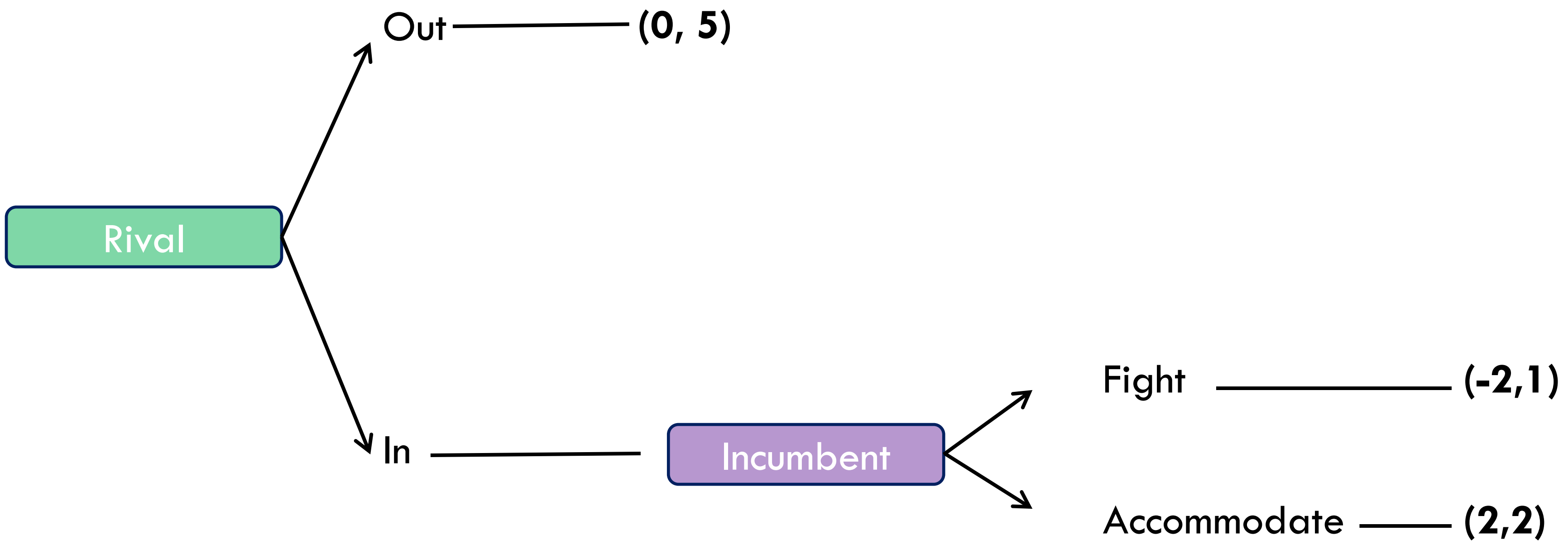


COMMITMENT

Consider a commitment such as building a new production facility that ensures the incumbent will fight. Assume that is such that ***rivals are aware*** of it and it ***cannot be reversed and is therefore credible.***

Consider the following timing:

- Stage 1 – the incumbent makes commitment to fight.
- Stage 2 – the rival decides whether to enter.

This games enables the other Nash equilibrium of the rival not entering to be reached.

LECTURE 2.4

REPEATED GAMES

REPEATED INTERACTION

With repeated interaction new equilibria can be supported.

Why?

Agents can cooperate and that cooperation can be sustained through punishment. That punishment and the cost of it might consist of the loss in long run profits from not cooperating as agreed.

Cooperation is more likely:

- The larger are the LR gains c.f. short run advantage
- When monitoring is less costly
- The expected length relationship is longer

SINGLE PERIOD SETTING

Consider two employees assigned to a team, Anna and Bert. Anna and Bert can work or shirk.

Payoffs reflecting the utility from exerting effort, along with the disutility of effort.

		Bert	
		Shirk	Work
Anna	Shirk	\$1000, \$1000	\$3000, \$0
	Work	\$0, \$3000	\$2000, \$2000

SINGLE PERIOD SETTING

Anna and Bert can work or shirk. Solution is that they both shirk. This is another version of the prisoner's dilemma.

		Bert	
		Shirk	Work
Anna	Shirk	\$1000, \$1000	\$3000, \$0
	Work	\$0, \$3000	\$2000, \$2000

REPEATED INTERACTION

Now suppose you expect to continue to work together into the future.

To formalise this, suppose you expect to work on the same team again with probability p, so probability working together for n periods is $p^{(n-1)}$.

To keep life easy we will consider that Anna and Bert have only two strategies available to them:

- Always shirk in which case the payoff is:

$$E(\textit{future earnings}) = \$1{,}000 + \$1{,}000p + \$1{,}000p^2 + \ldots = \frac{1000}{1-p}$$

- Work hard first period then if they ever shirk, punish them forever by always shirking in the future (grim trigger strategy).

REPEATED INTERACTION

Anna and Bert can work or shirk. What if they each think the other will play grim trigger?

		Bert: Always Shirk	Bert: Work then grim trigger
Anna	Always Shirk	\$1000/(1-p), \$1000/(1-p)	\$2000+\$1000/(1-p), -\$1000 +\$1000/(1-p)
	Work then grim trigger	-\$1000 +\$1000/(1-p), \$2000+\$1000/(1-p)	\$2000/(1-p), \$2000/(1-p)

REPEATED INTERACTION

But what if Anna thinks Bert will go grim trigger, may be in her interest to do so. In fact she will do so as long as p>0.5. That is:

$$\frac{2000}{1-p} > 2000 + \frac{1000}{1-p}$$

$$\frac{2000 - 2000 + 2000p}{1-p} > \frac{1000}{1-p}$$

$$p > 0.5$$

SINGLE PERIOD SETTING

If p=1/3, shirking.

		Bert	
		Shirk	Work then grim trigger
Anna	Shirk	\$1500, \$1500	\$3500, \$500
	Work then grim trigger	\$500, \$3500	\$3000, \$3000

SINGLE PERIOD SETTING

If p=3/4, two Nash equilibria. Initial expectations matter. What should a firm do …?

		Bert	
		Shirk	Work then grim trigger
Anna	Shirk	$4000, $4000	$6000, $3000
	Work then grim trigger	$3000, $6000	$8000, $8000

LECTURE 2.5

AUCTIONS

AUCTIONS

Auctions are just games. The design of auctions can be important for organisations given they are often used to allocate resources.

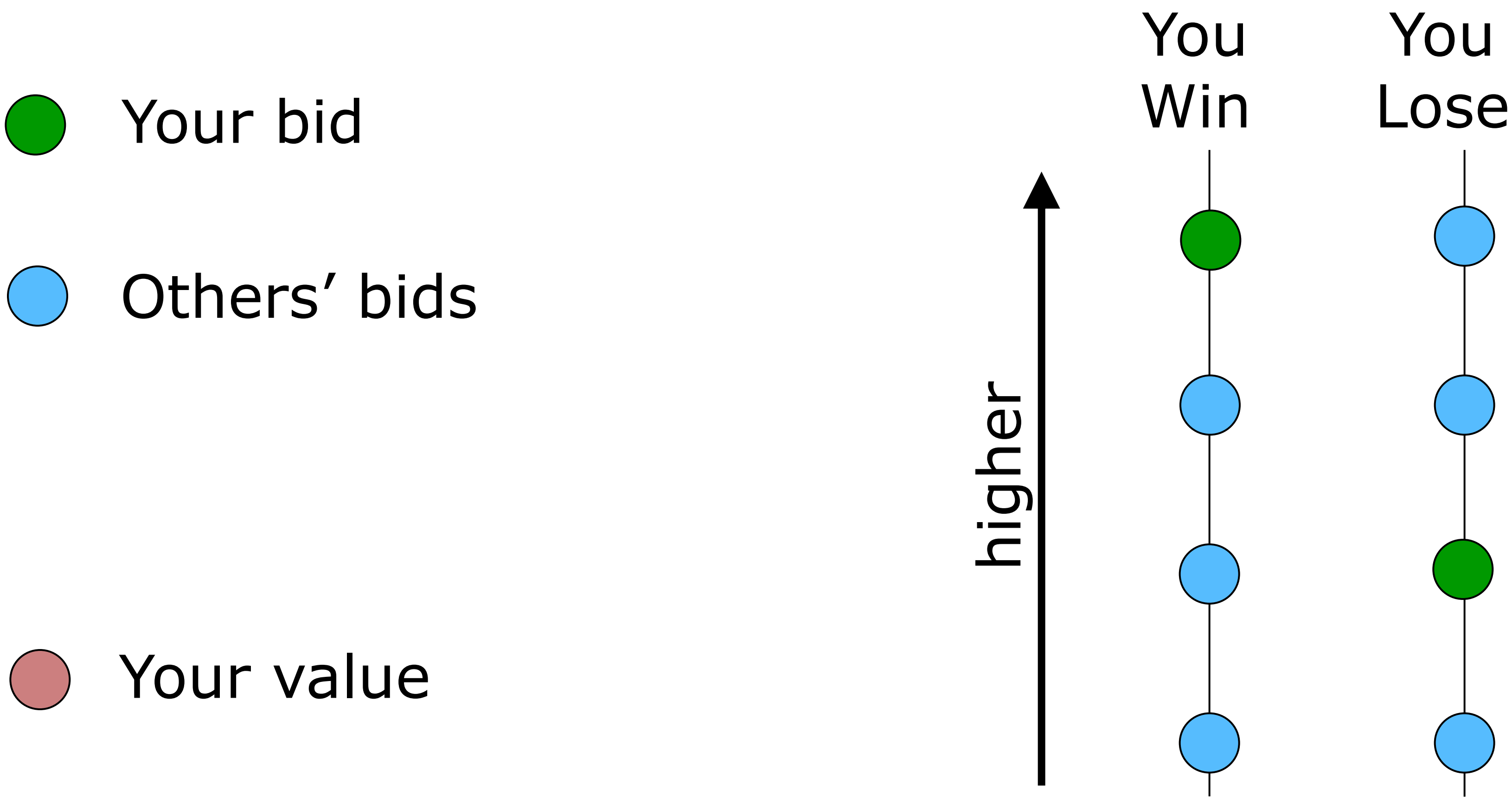
What is the best bidding strategy in a private value auction?

Let's consider a very simple example of a Vickery auction, a second price sealed bid auction, ala eBay.

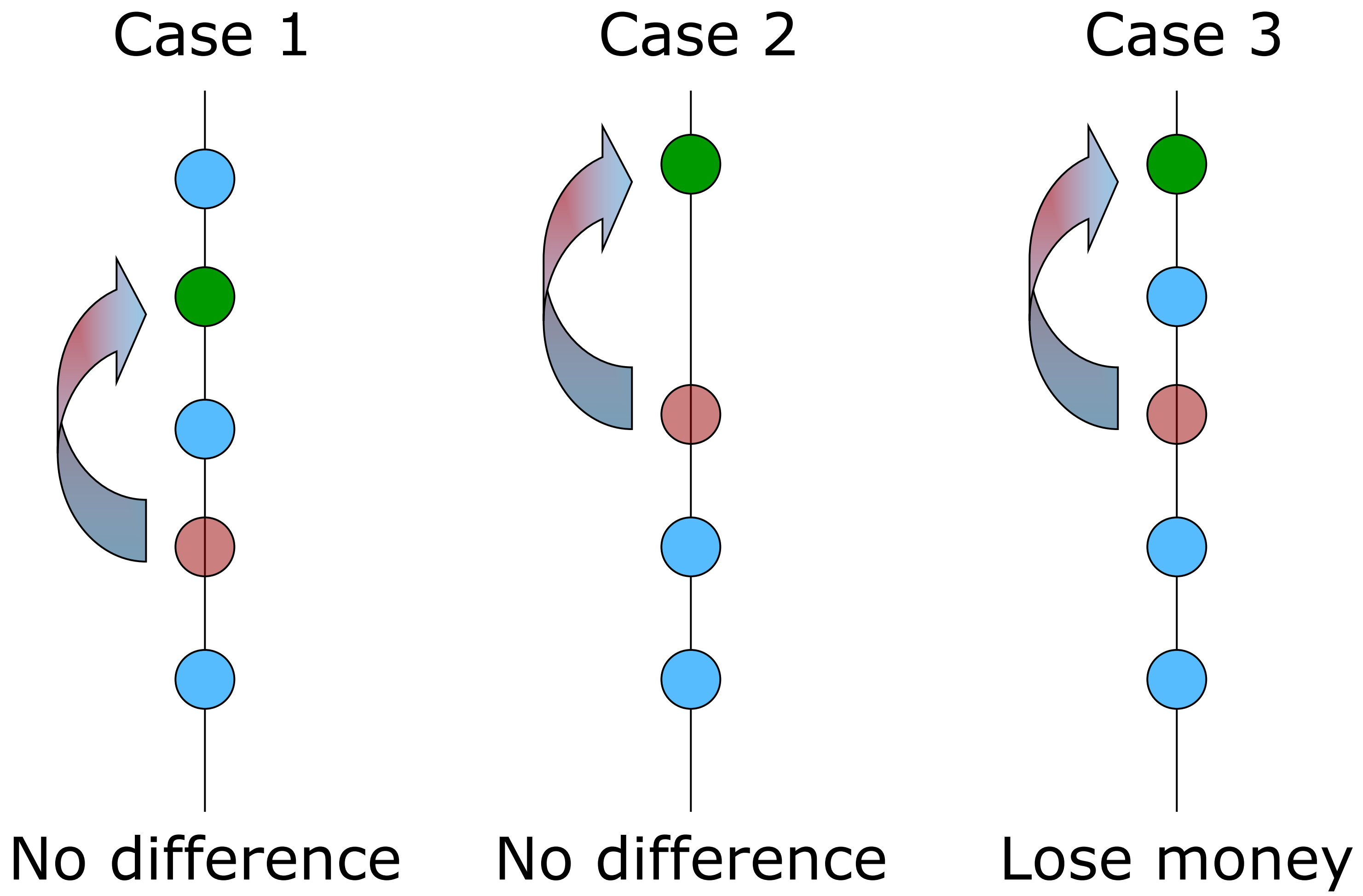
The question is, what is the optimal bidding strategy?

To tell the truth…!

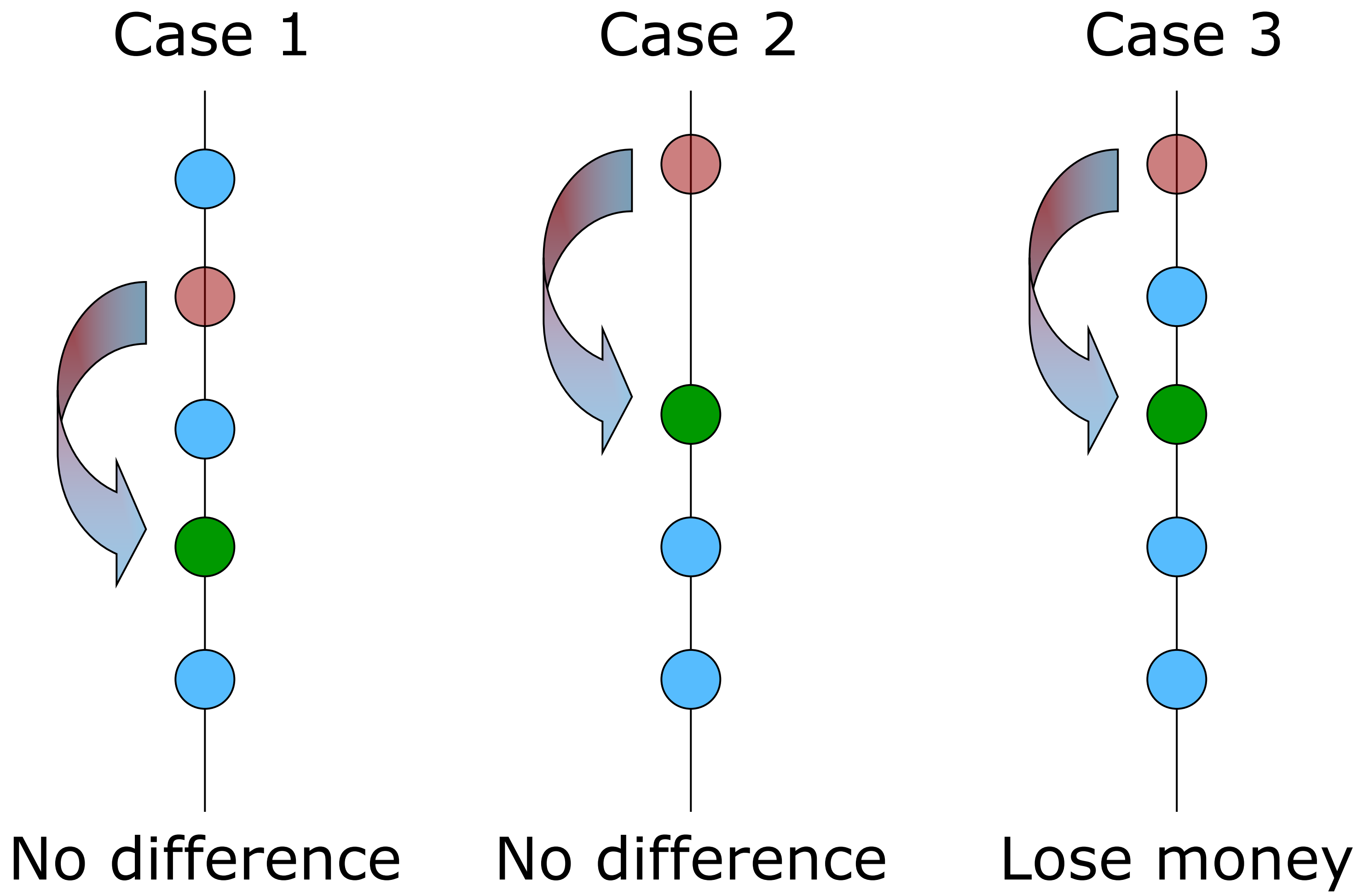
AUCTIONS – SECOND PRICE SEALED BID



AUCTIONS – BIDDING HIGHER THAN VALUATION



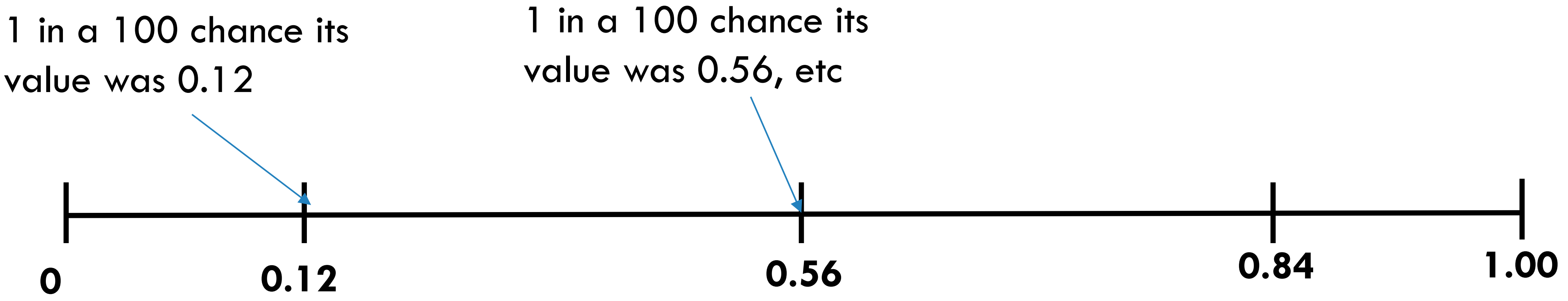
AUCTIONS – BIDDING LOWER THAN VALUATION



AUCTIONS

Now let's consider a common value auction.

Bidders for target firm are told that value of firm is uniformly distributed between 0 and 1.



AUCTIONS

Bidders also told that whatever value the firm has to the seller, it will be worth 1.5 times that to the buyer.

Think of this as you as the buyer can do better than the current owner at maximising value. That is, you purchased a firm that was worth 0.56 to the seller, then it was worth 0.84 (=0.56 ✖ 1.5) to the buyer.

In general, we expect the seller to sell the firm if the offer made by the buyer > value to the seller

The question is, what is the optimal bidding strategy?

AUCTIONS

Assume that the buyer chooses a bid of 0.48 (note this is chosen entirely randomly, and we could have chosen any figure between 0 and 1 and the argument we set out below will continue to hold)

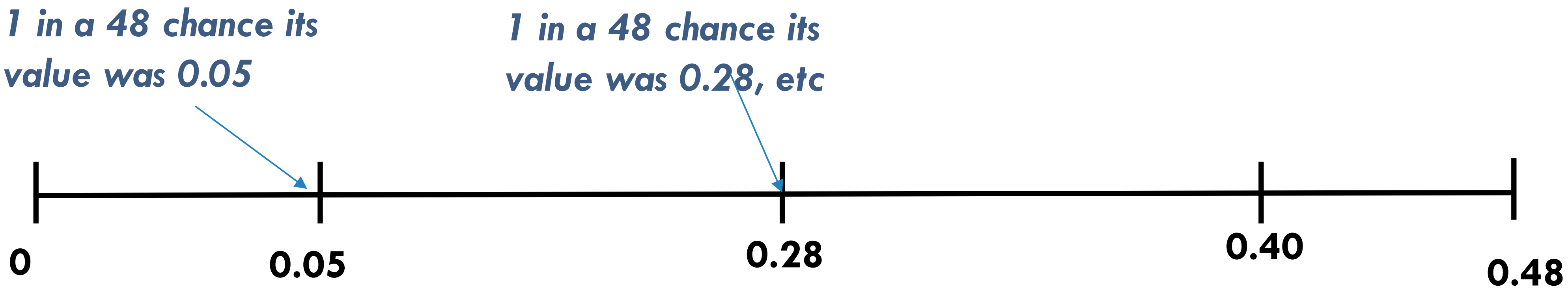
- If the bidder offers 0.48, the seller will only sell if the value to the seller is < 0.48. In this case, the average value of the firm (to the seller) will only be 0.24.
- Why? Consider how the value of the firm is distributed assuming its value is < 0.48.

AUCTIONS

Note although the average value of the firm (to the seller) will only be 0.24, the average value to the buyer will be 0.36 (=0.24 ✖ 1.5).

If the buyer pays 0.48 for something worth only 0.36, s/he will lose.

The winners curse



AUCTIONS

Intuition: As a buyer, you only get to buy the item if you bid more than it is worth (to the seller). Given uncertainty about its true value (asymmetric information), you will tend to bid too much and lose.

What is the best strategy for the buyer?