Bayes' rule

Notes on Behavioural Economics

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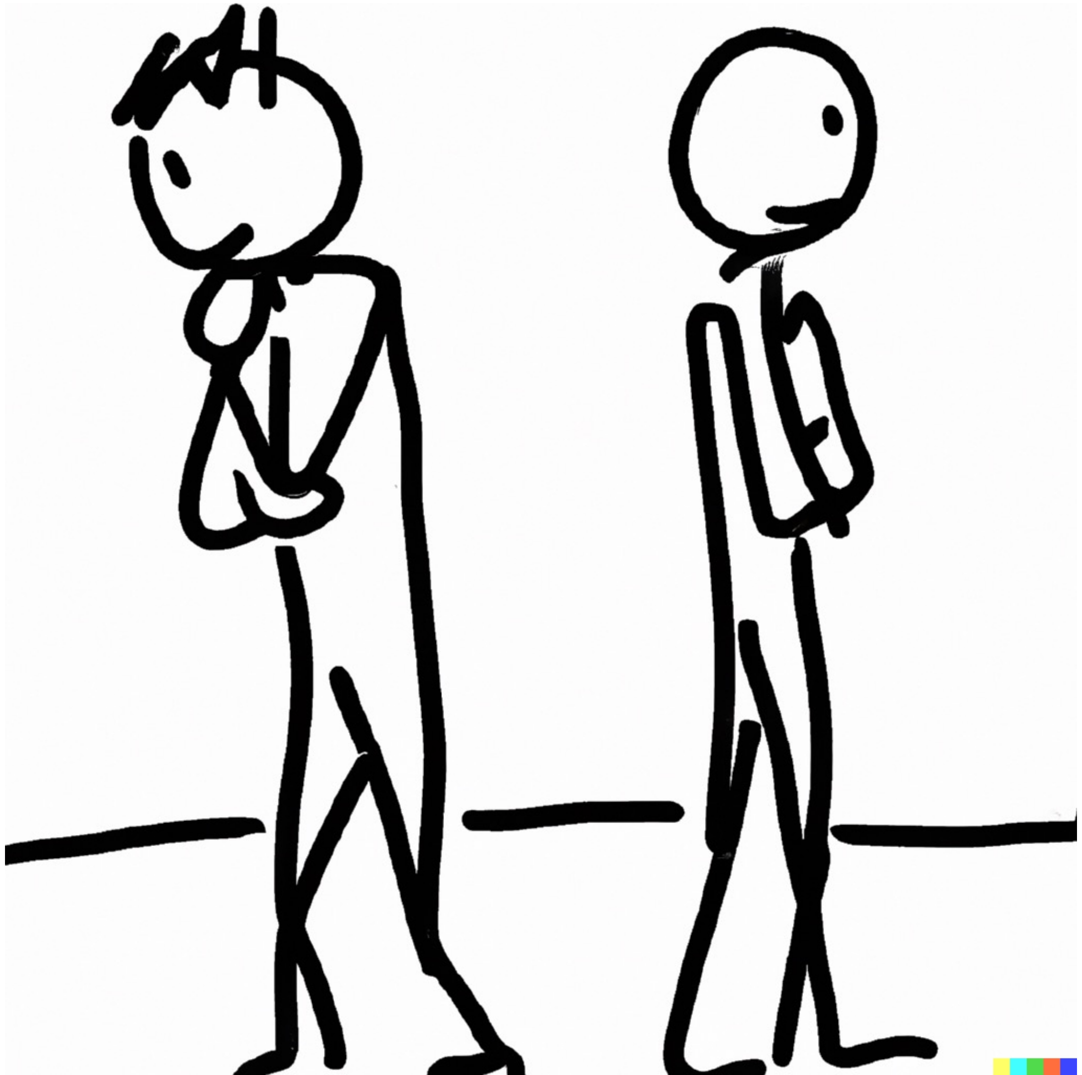


Estimate the conditional probability of outcome A given outcome B:

- The unconditional probability of outcome A.
- The probability of observing outcome B given outcome A.
- The total probability of outcome B

Bayes' rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B|A)P(A)}{P(B)}$$

Total probability of B

$$P(B) = P(B|A)P(A) + P(B|\neg A)P(\neg A)$$

Bayes' rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

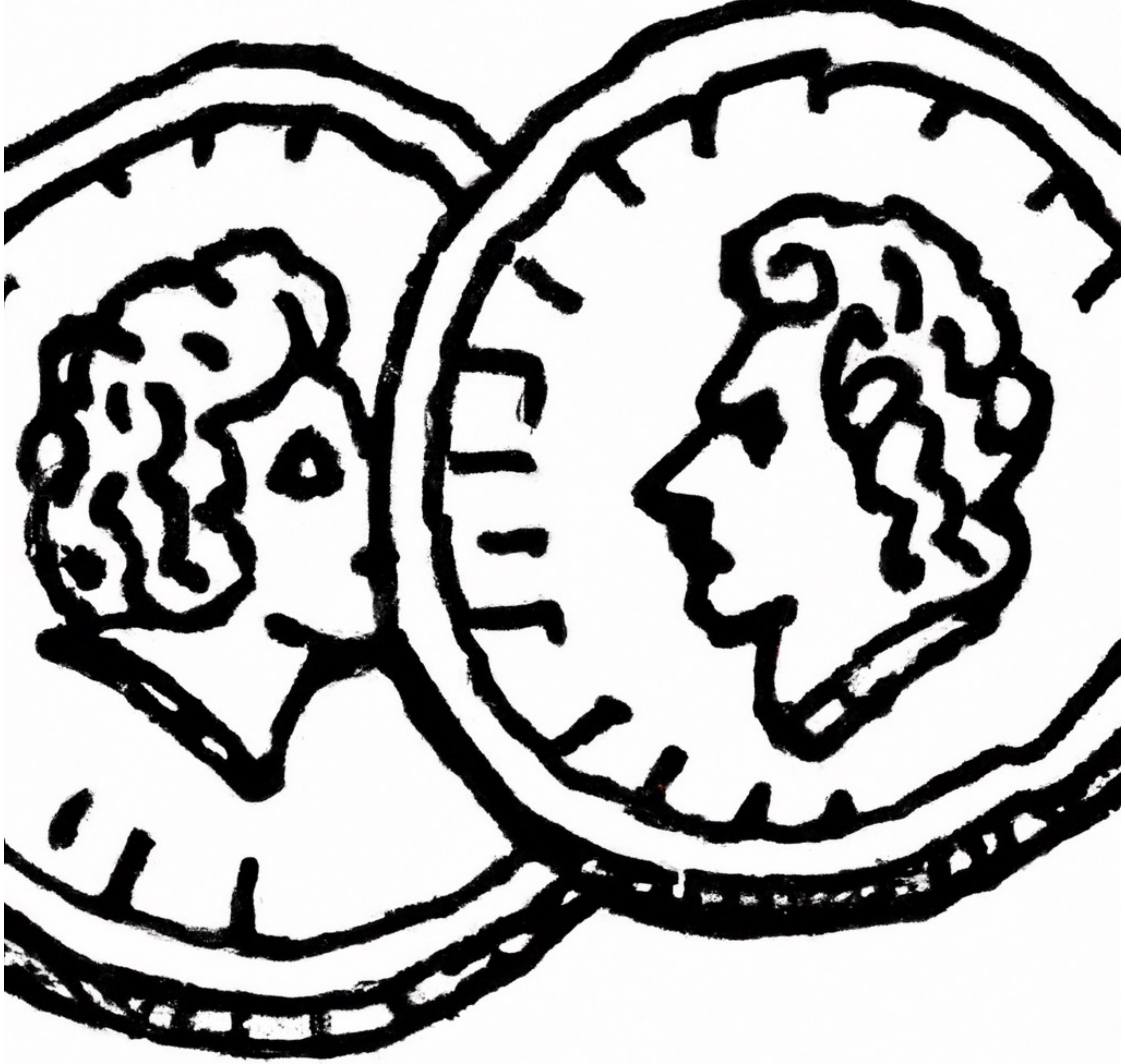
Updating a belief

- A hypothesis, H
- The prior probability of H being true, $P(H)$
- The probability of observing event E given a hypothesis H, $P(E|H)$
- The posterior probability of the hypothesis *H* given the event $E, P(H|E)$

Bayes' rule

$$\underbrace{P(H|E)}_{\text{Posterior belief}} = \frac{P(E|H)\overbrace{P(H)}^{\text{Prior belief}}}{P(E)}$$

$$= \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|\neg H)P(\neg H)}$$



Prior belief

$$P(\text{rigged}) = 0.5$$

Probability of observing event

$$P(\text{head} \mid \text{rigged}) = 1$$

Total probability of a head

$$P(\text{head}) = P(\text{head}|\text{rigged})P(\text{rigged}) + P(\text{head}|\text{fair})P(\text{fair})$$

$$= 1 \times 0.5 + 0.5 \times 0.5$$

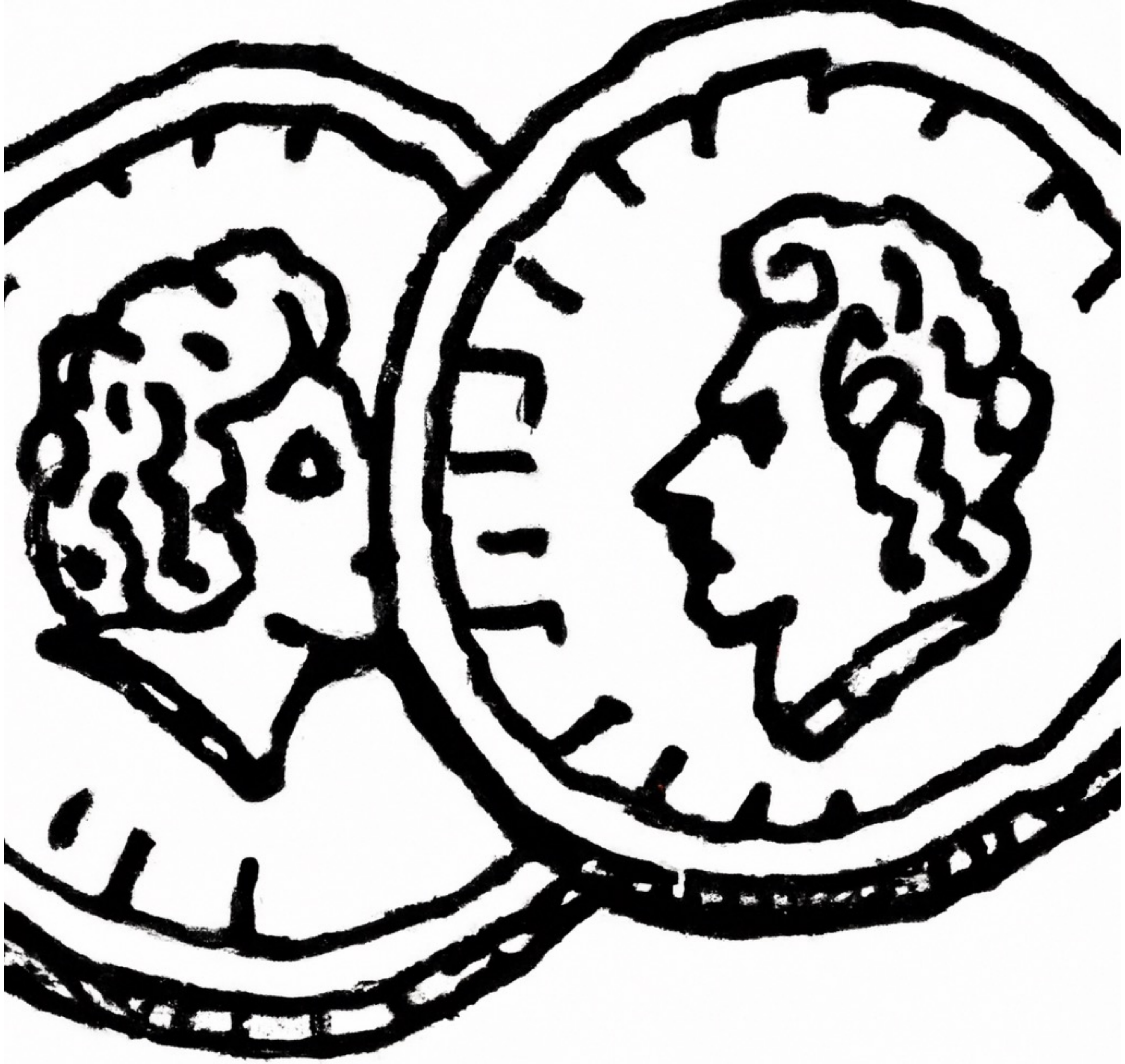
$$= 0.75$$

Bayes' rule

$$P(\text{rigged}|\text{head}) = \frac{P(\text{head}|\text{rigged})P(\text{rigged})}{P(\text{head})}$$

$$= \frac{1 \times 0.5}{0.75}$$

$$= \frac{2}{3}$$



Prior belief

$$P(\text{rigged}) = \frac{2}{3}$$

Total probability of a head

$$P(\text{head}) = P(\text{head}|\text{rigged})P(\text{rigged}) + P(\text{head}|\text{fair})P(\text{fair})$$

$$= 1 \times \frac{2}{3} + 0.5 \times \frac{1}{3}$$

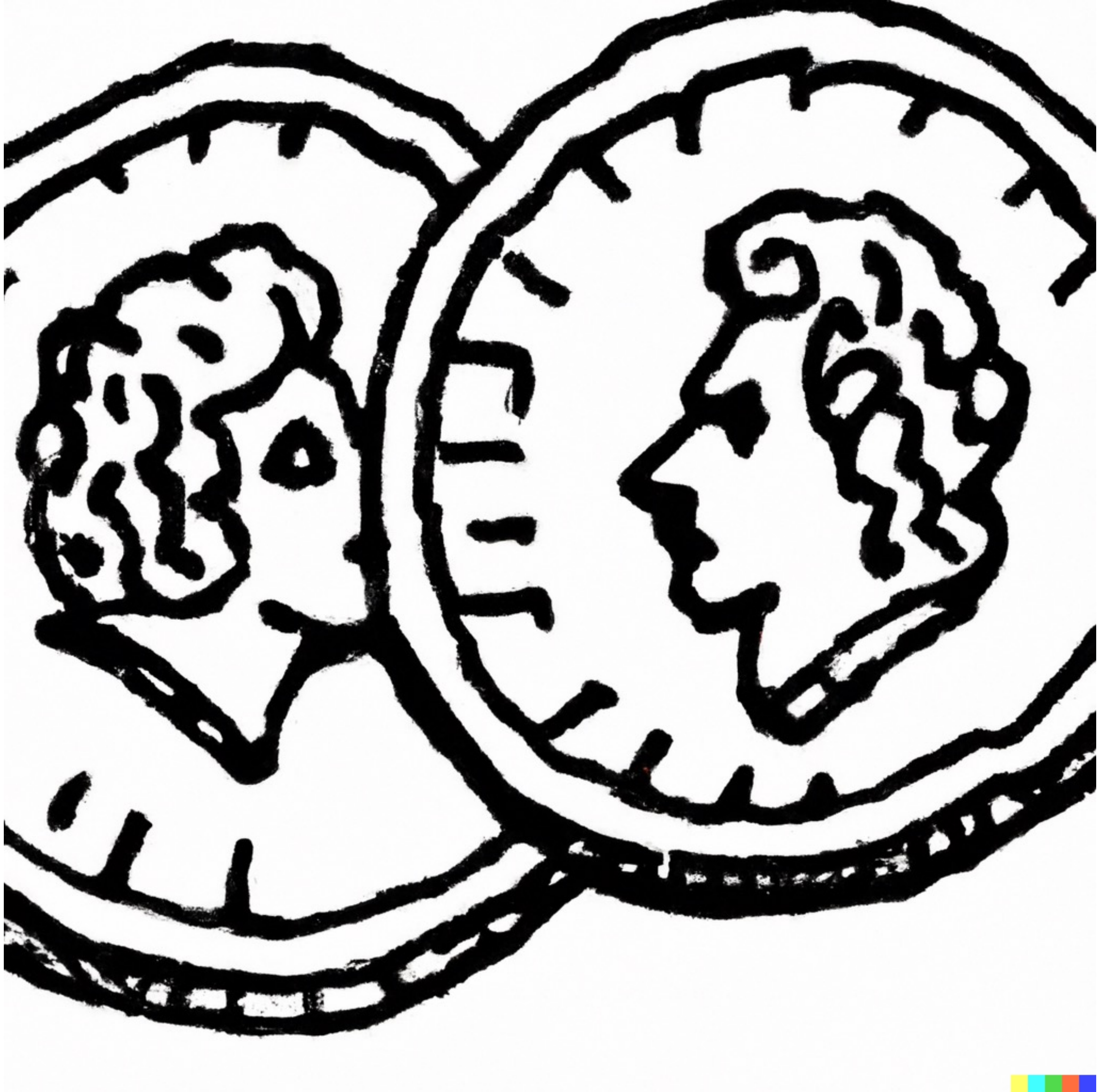
$$= \frac{5}{6}$$

Bayes' rule

$$P(\text{rigged}|\text{head}) = \frac{P(\text{head}|\text{rigged})P(\text{rigged})}{P(\text{head})}$$

$$= \frac{1 \times \frac{2}{3}}{\frac{5}{6}}$$

$$= \frac{4}{5}$$



Prior belief

$$P(\text{rigged}) = \frac{4}{5}$$

Total probability of 10 heads

$$P(10\ \text{heads}) = P(10\ \text{heads}|\text{rigged})P(\text{rigged}) + P(\text{head}|\text{fair})P(\text{fair})$$

$$= 1\times {}^{4}/_{5} + \left({}^{1}/_{2}\right)^{10} \times {}^{1}/_{5}$$

$$= 0.8001953$$

Bayes' rule

$$P(\text{rigged}|10\text{ heads}s) = \frac{P(10\text{ heads}|\text{rigged})P(\text{rigged})}{P(10\text{ heads})}$$

$$= \frac{1\times\frac{4}{5}}{0.8001953}$$

$$= 0.99976$$

Bayes' rule

$$P(\text{urn 1}|\text{yellow}) = \frac{P(\text{yellow} \mid \text{urn 1})P(\text{urn 1})}{P(\text{yellow})}$$

Prior belief

$$P(\text{urn } 1) = 0.5$$

Probability of observing event

$$P(\text{yellow} \mid \text{urn } 1) = 0.7$$

Total probability of yellow ball

$$
\begin{aligned}
P(\text{yellow}) &= P(\text{yellow} \mid \text{urn } 1)P(\text{urn } 1) + P(\text{yellow} \mid \text{urn } 2)P(\text{urn } 2) \\
&= 0.7 \times 0.5 + 0.3 \times 0.5 \\
&= 0.5
\end{aligned}
$$

Bayes' rule

$$P(\text{urn 1}|\text{yellow}) = \frac{P(\text{yellow} \mid \text{urn 1})P(\text{urn 1})}{P(\text{yellow})}$$

$$= \frac{0.7 \times 0.5}{0.5}$$

$$= 0.7$$

Prior belief

$$P(\text{urn } 1) = 0.7$$

Total probability of black ball

$$P(\text{black}) = P(\text{black} \mid \text{urn } 1)P(\text{urn } 1) + P(\text{black} \mid \text{urn } 2)P(\text{urn } 2)$$

$$= 0.3 \times 0.7 + 0.7 \times 0.3$$

$$= 0.42$$

Bayes' rule

$$P(\text{urn 1}|\text{black}) = \frac{P(\text{black}|\text{urn 1})P(\text{urn 1})}{P(\text{black})}$$

$$= \frac{0.3 \times 0.7}{0.42}$$

$$= 0.5$$

The Monty Hall problem

Suppose you’re on a game show and you’re given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what’s behind the doors, opens another door, say No. 3, which has a goat. He then says to you, “Do you want to pick door No. 2?” Is it to your advantage to switch your choice?

Bayes' rule

$$P(\mathrm{C2}|\mathrm{D3}) = \frac{P(\mathrm{D3}|\mathrm{C2})P(\mathrm{C2})}{P(\mathrm{D3})}$$

Total probability of D3

$$P(\mathrm{D3}) = P(\mathrm{D3}|\mathrm{C1})P(\mathrm{C1}) + P(\mathrm{D3}|C2)P(C2) + P(\mathrm{D3}|C3)P(C3)$$

Prior probability

$$P(\mathrm{C1}) = P(\mathrm{C2}) = P(\mathrm{C3}) = \frac{1}{3}$$

Probability of observing event

$$P(\mathrm{D3} \mid \mathrm{C1}) = \frac{1}{2}$$

$$P(\mathrm{D3} \mid \mathrm{C2}) = 1$$

$$P(\mathrm{D3} \mid \mathrm{C3}) = 0$$

Total probability of D3

$$P(\mathrm{D3}) = P(\mathrm{D3}|\mathrm{C1})P(\mathrm{C1}) + P(\mathrm{D3}|C2)P(C2) + P(\mathrm{D3}|C3)P(C3)$$

$$= \frac{1}{2} \times \frac{1}{3} + 1 \times \frac{1}{3} + 0 \times \frac{1}{3}$$

$$= \frac{1}{2}$$

Bayes' rule

$$P(\mathrm{C2}|\mathrm{D3}) = \frac{P(\mathrm{D3}|\mathrm{C2})P(\mathrm{C2})}{P(\mathrm{D3})}$$

$$= \frac{1 \times \frac{1}{3}}{\frac{1}{2}}$$

$$= \frac{2}{3}$$