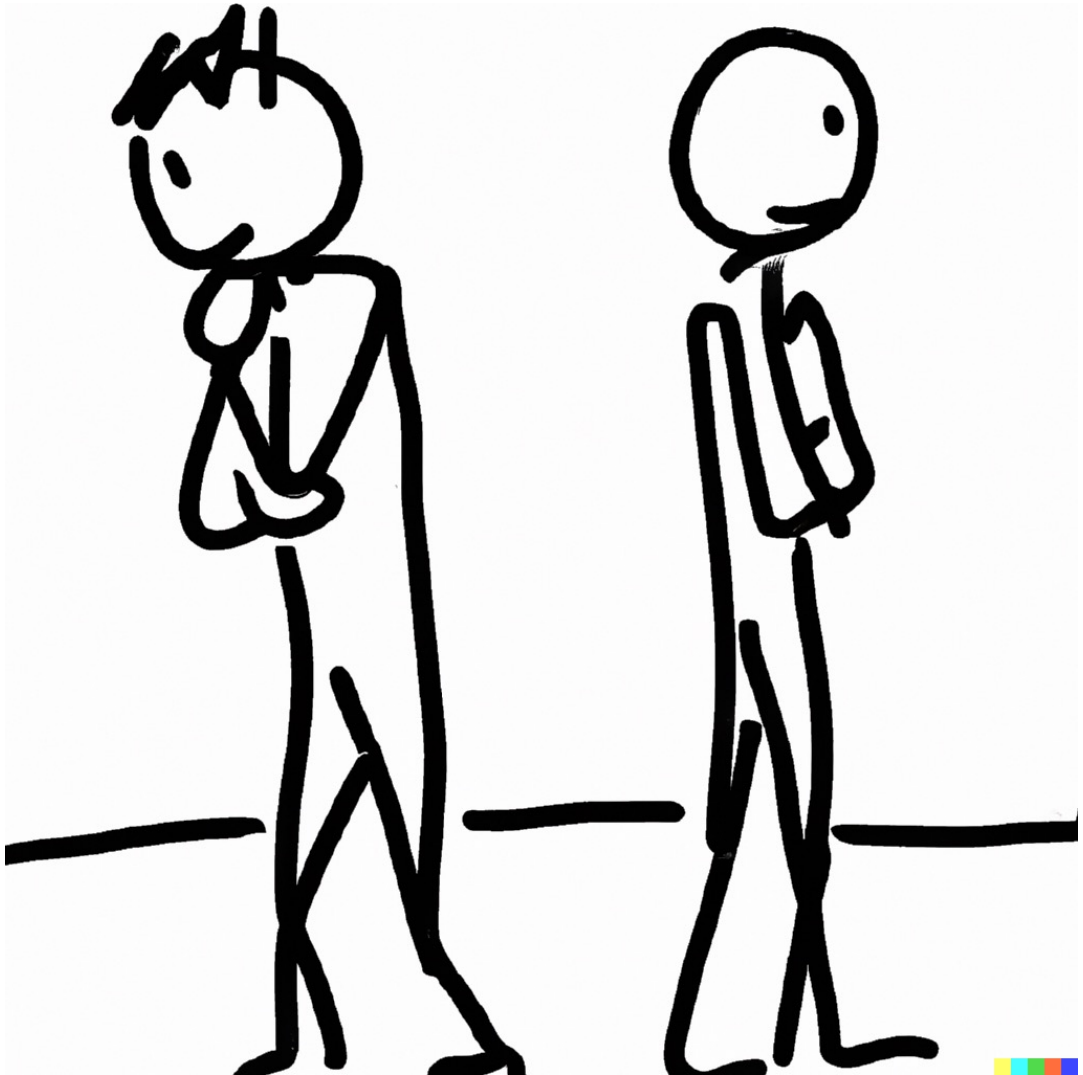
Level-k thinking

Notes on Behavioural Economics

Jason Collins



Level-k player $\Longrightarrow$ best-responds to level-(k-1) players

- Level-0 player do not engage in strategic thinking.
- Level-1 player $\Longrightarrow$ best-responds to level-0 players
- Level-2 player $\Longrightarrow$ best-responds to level-1 players
- And so on.

[P]rofessional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.

Keynes (1936)

The p-beauty contest

- Each of n players pick a number $y \in [0,100]$.
- The winner is the player whose chosen number is closest to the mean of all the chosen numbers ($\bar{y}$) multiplied by a parameter p.

The p-beauty contest

Suppose $p = 2/3$.

Level-0: Randomly select from the interval [0, ..., 100].

Level-1: The best response to level-0 players is:

$$y_1 = \frac{2}{3}\bar{y} = \frac{2}{3} \times 50 = 33.3$$

The p-beauty contest

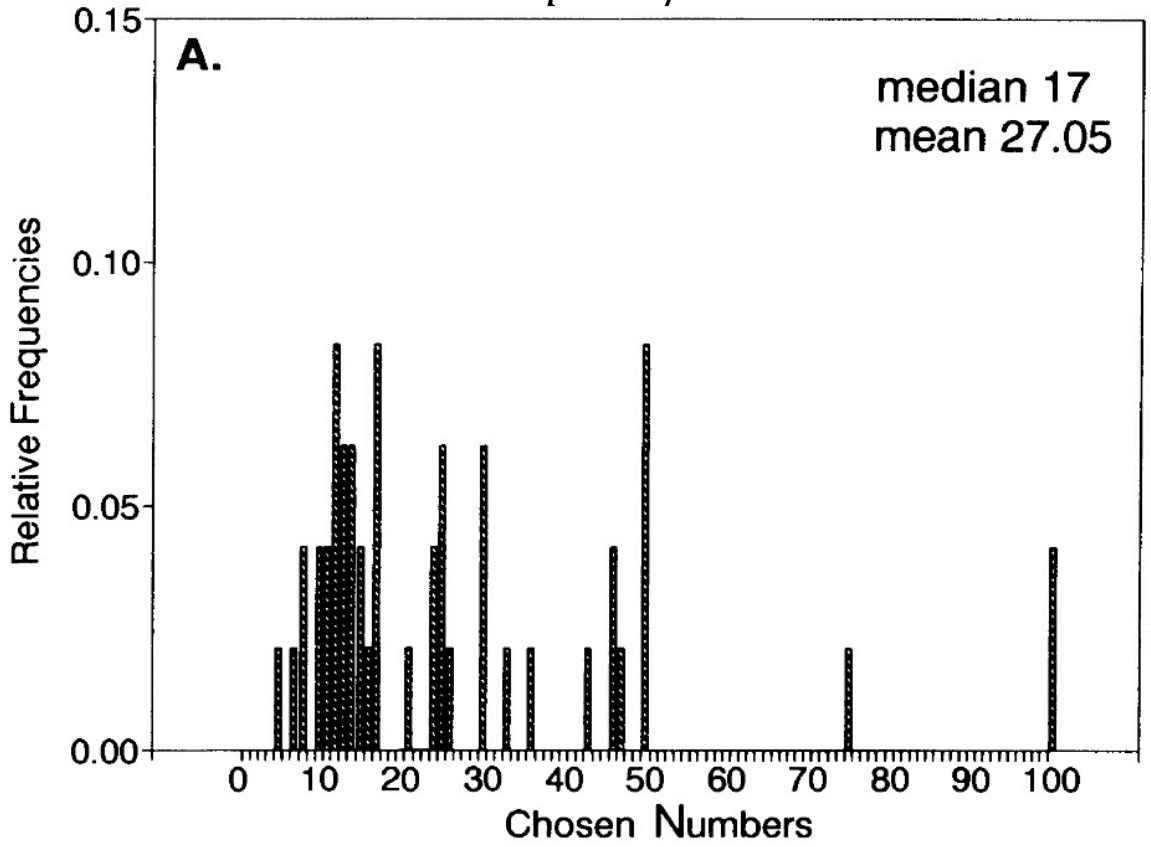
Level-2: The best response to level-1 players is:

$$y_2 = \frac{2}{3}\bar{y} = \frac{2}{3} \cdot 33.3 = 22.2$$

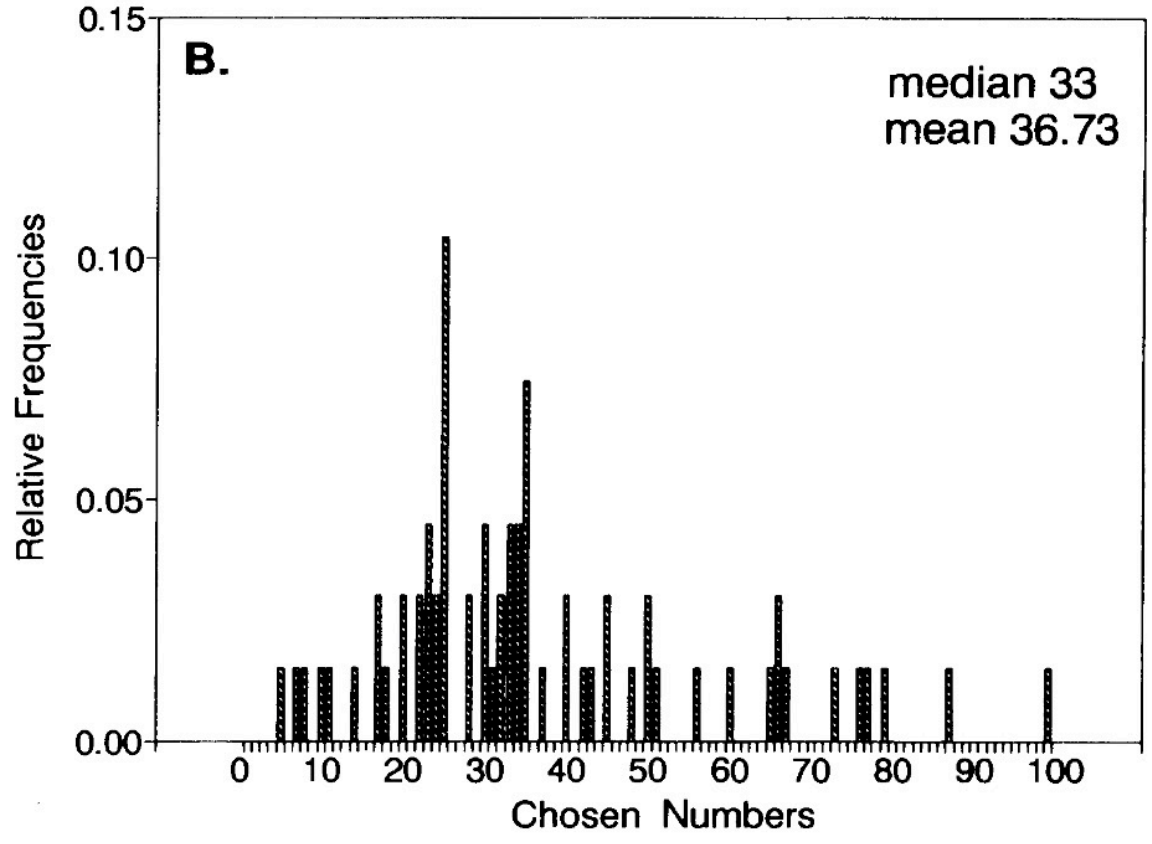
Level-3: The best response to level-2 players is:

$$y_3 = \frac{2}{3}\bar{y} = \frac{2}{3} \cdot 22.2 = 14.8$$

$p = 1/2$


A.
median 17
mean 27.05
Relative Frequencies
0.15
0.10
0.05
0.00
0 10 20 30 40 50 60 70 80 90 100
Chosen Numbers

$p = 2/3$


B.
median 33
mean 36.73
Relative Frequencies
0.15
0.10
0.05
0.00
0 10 20 30 40 50 60 70 80 90 100
Chosen Numbers

The assignment game

		Player B	
		Work	Shirk
Player A	Work	7,4	1,9
	Shirk	9,-1	0,0

The assignment game

		Player B: Work	Player B: Shirk
Player A	Work	7,4	1,9
	Shirk	9,-1	0,0

Level-1 player A

Expected payoff from playing work is:

$$\frac{1}{2} \times 7 + \frac{1}{2} \times 1 = 4$$

Expected payoff from playing shirk is:

$$\frac{1}{2} \times 9 + \frac{1}{2} \times 0 = 4.5$$

Level-1 player B

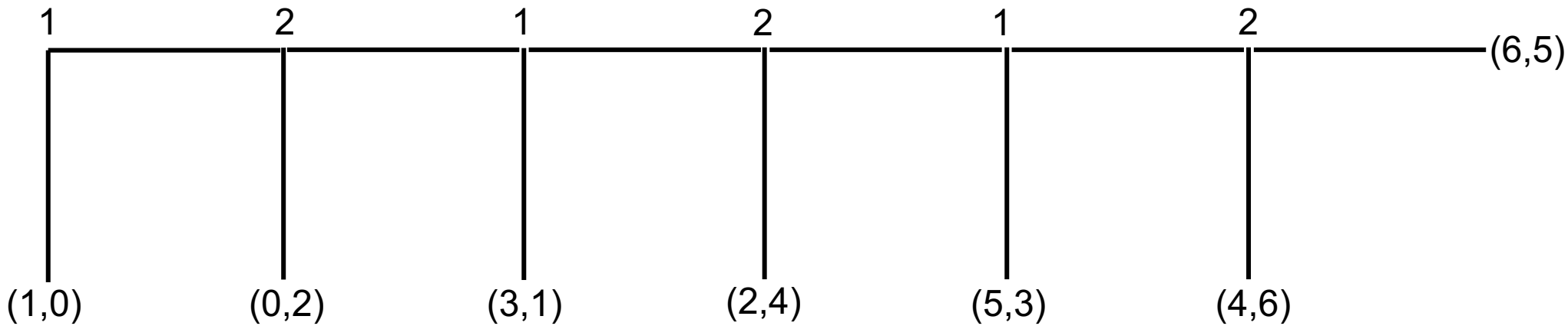
Expected payoff from playing work is:

$$\frac{1}{2} \times 4 + \frac{1}{2} \times -1 = 1.5$$

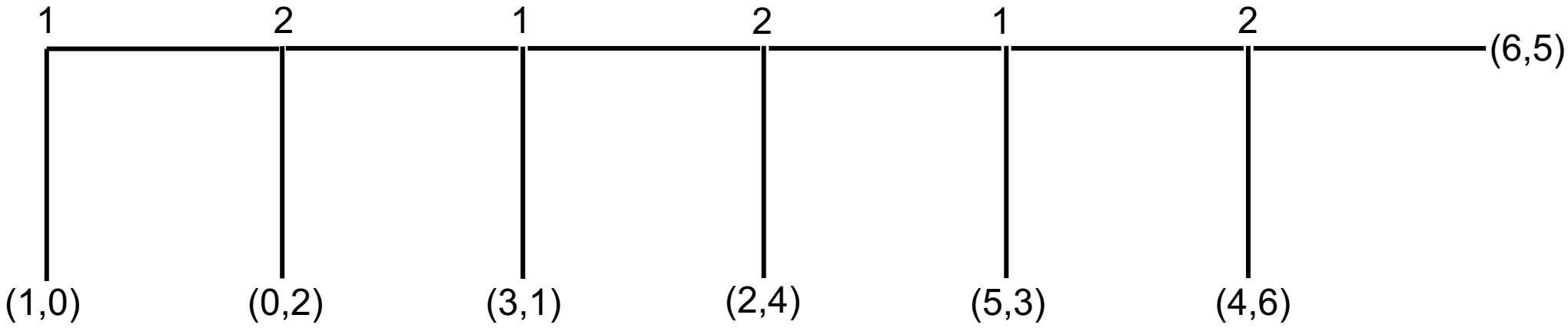
Expected payoff from playing shirk is:

$$\frac{1}{2} \times 9 + \frac{1}{2} \times 0 = 4.5$$

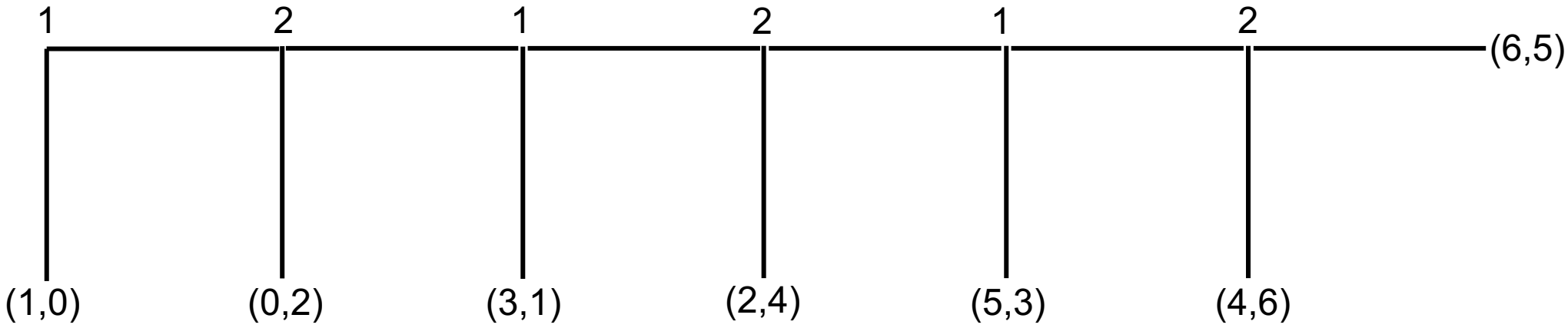
Level-k	Player A	Player B
k = 0	Random	Random
k = 1	Shirk	Shirk
k = 2	Work	Shirk
k = 3	Work	Shirk
k = 4	Work	Shirk


1
2
1
2
1
2
(6,5)
(1,0)
(0,2)
(3,1)
(2,4)
(5,3)
(4,6)

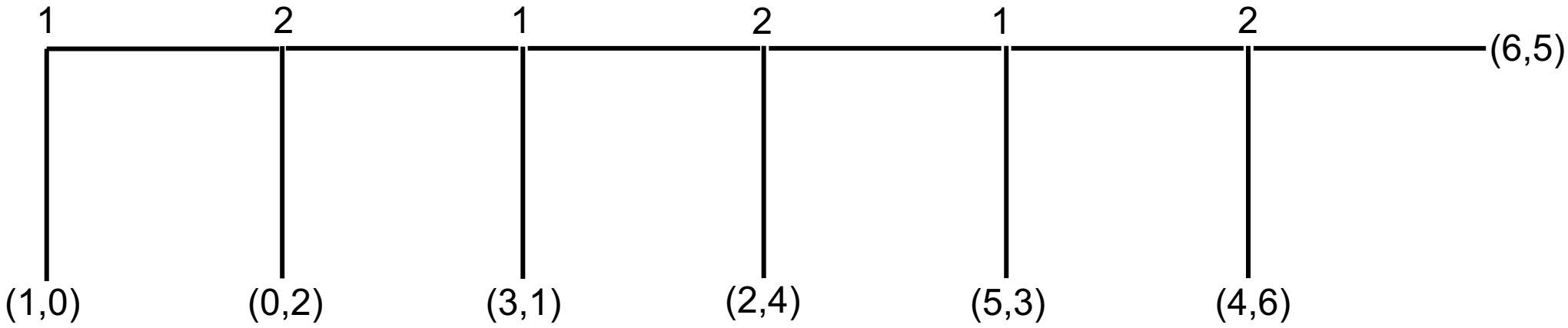
Assume a level-0 player passes until the end.



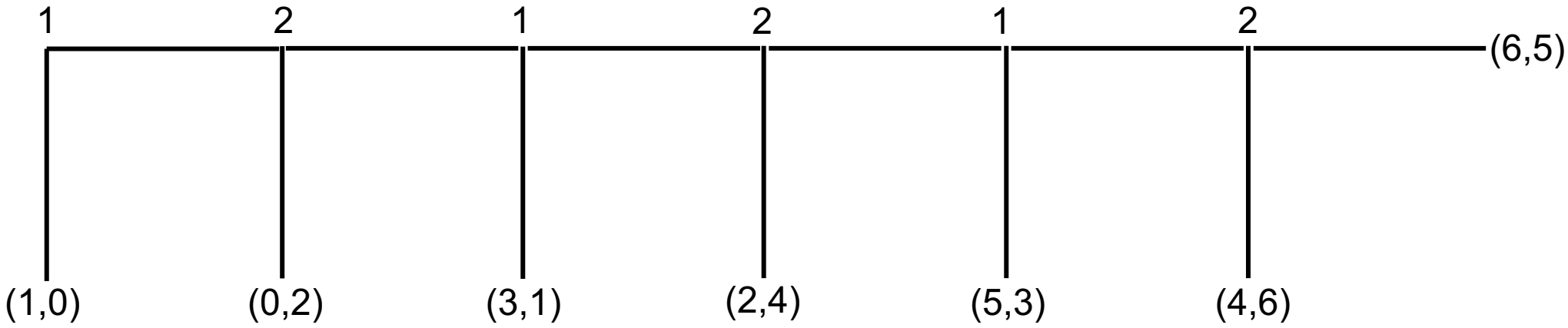
Level-1 player 2: take at (4,6) as the level-0 player 1 would pass until then.



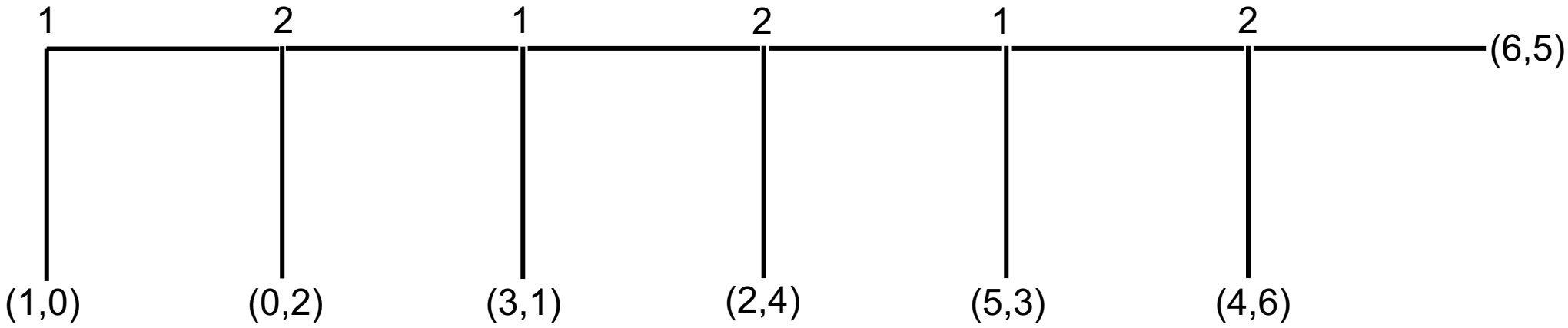
Level-1 player 1: take at (6,5) as the level-0 player 2 will keep passing.



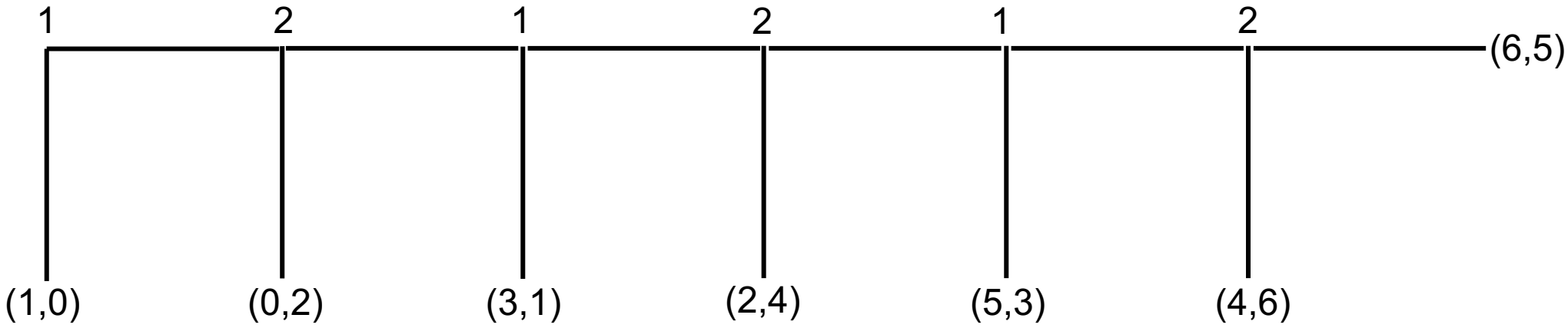
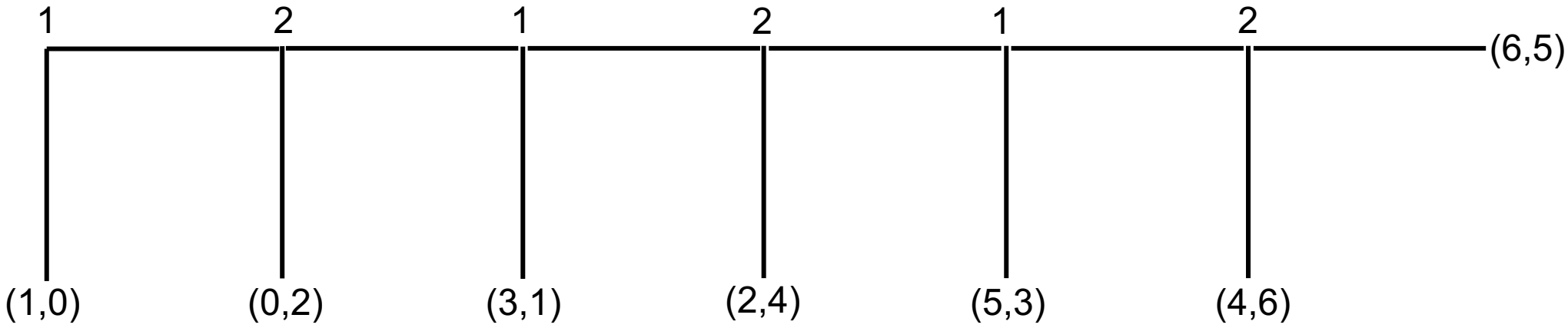
Level-2 player 2: take at (4,6) as the level-1 player 1 passes.



Level-2 player 1: take at (5,3) as a level-1 player 2 would take at (4,6).



Level-3 player 2: take at (2,4) as the level-1 player 1 will take at (5,3).



Level-3 player 1: take at (5,3) as a level-2 player 2 would take at (4,6).