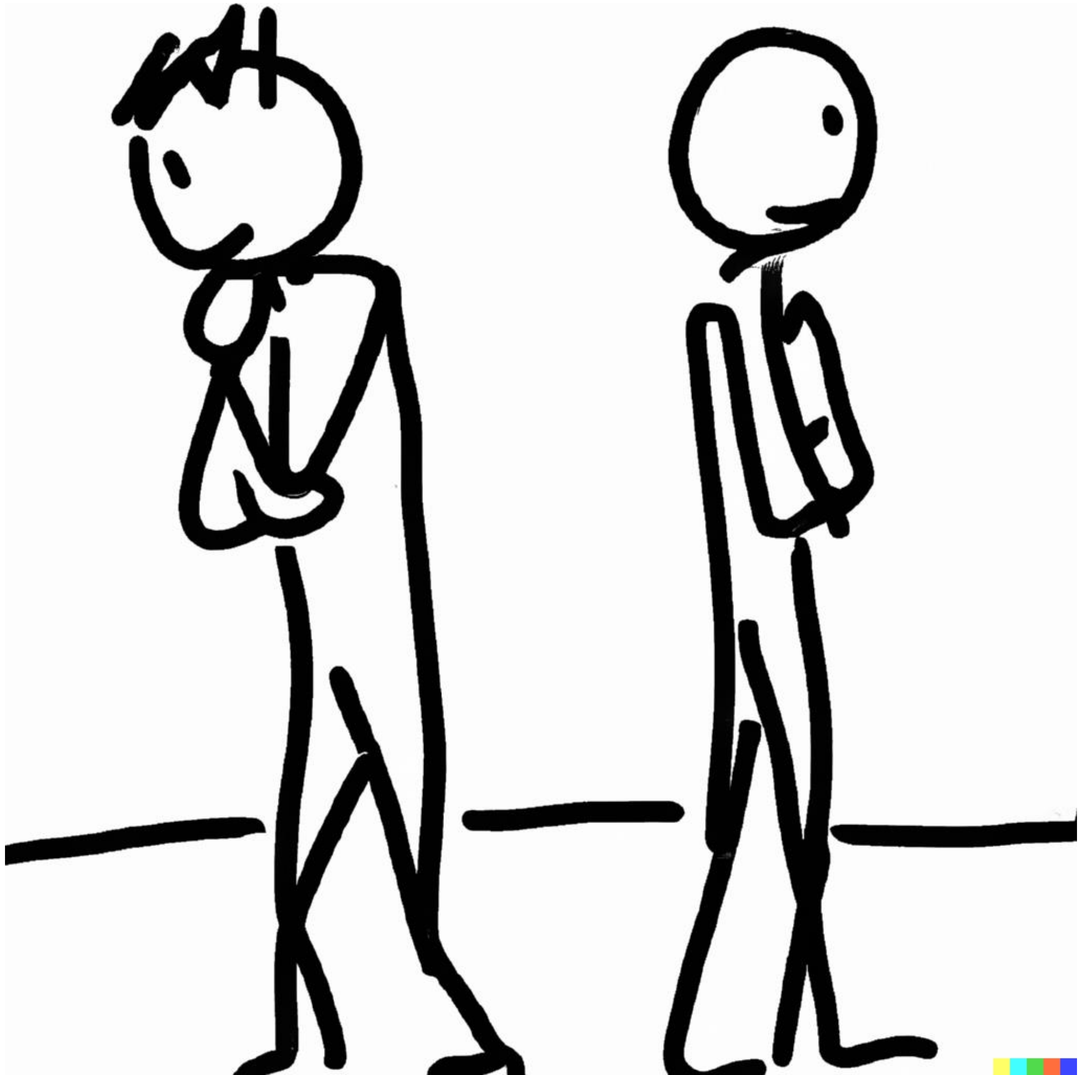
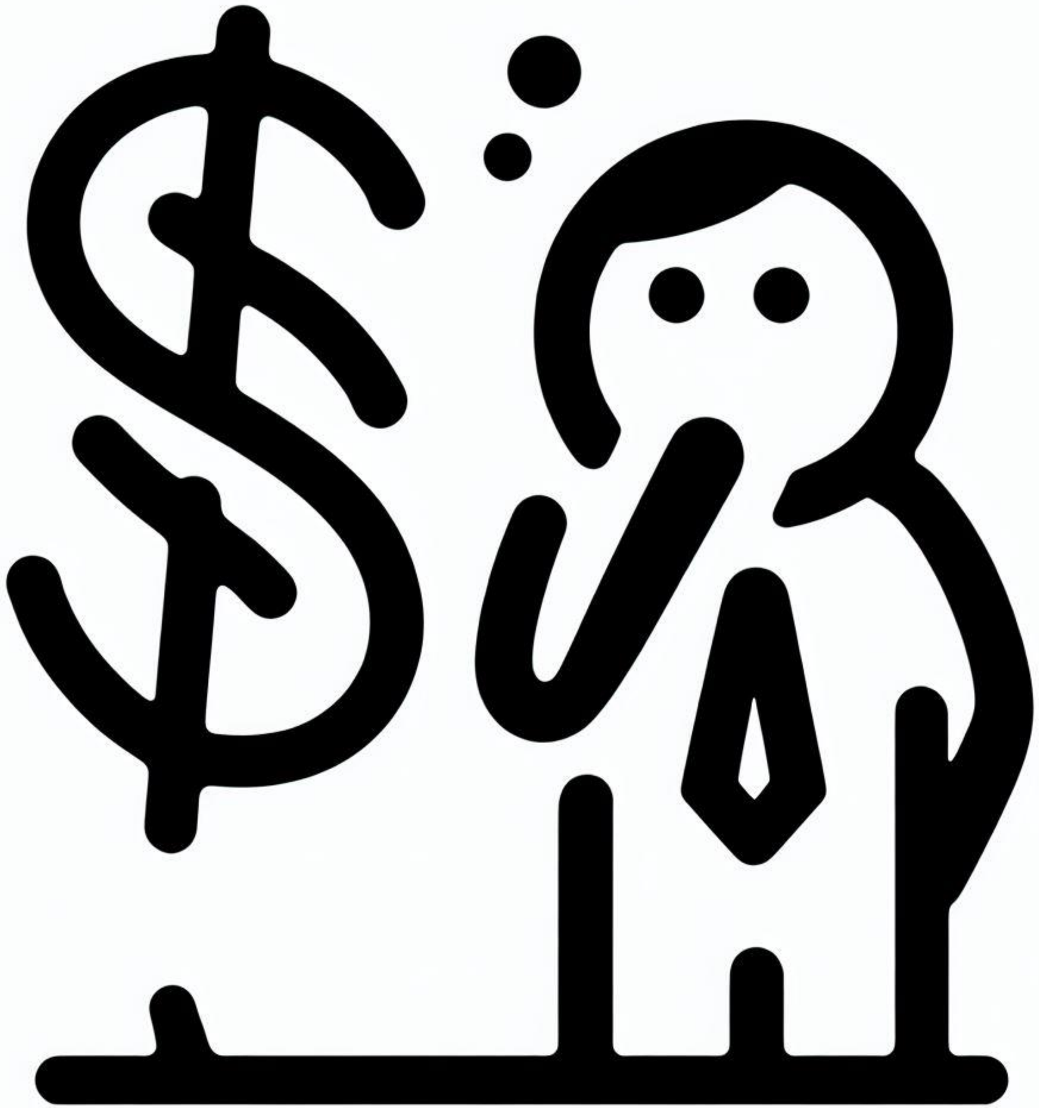
Subjective expected utility theory

Notes on Behavioural Economics

Jason Collins



Subjective expected utility theory



Subjective expected utility theory

Subjective expected utility: $\mathbb{E}[U(X)]$

Subjective probability: $\pi(x_i)$

Subjective expected utility theory

$$\mathbb{E}[U(X)] = \pi(x_1)u(x_1) + \pi(x_2)u(x_2) + \ ... + \pi(x_n)u(x_n)$$

$$= \sum_{i=1}^{n} \pi(x_i)u(x_i)$$

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2. Defining subjective probability $\pi(x_1)$ for each outcome.

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3. Weighting each outcome’s utility by its subjective probability.

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3. Weighting each outcomes utility by its subjective probability.
4. Summing the weighted utilities.

Axioms for subjective expected utility theory

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Sure-thing principle

Suppose there are two possible states of the world. If you prefer one option (A) over another (B) in one possible state, and you also prefer A over B under the alternative state, then you should prefer A over B even if you do not know which state will occur.

Coherent probabilities

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Non-negativity

$$0 \leq \pi(A) \leq 1$$

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Additivity

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Normalisation

$$\pi\ (\cdot) = 1$$

Baye's rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B|A)P(A)}{P(B)}$$

Why coherence matters

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Dutch book

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- Probability of rain tomorrow: 60%
- Probability of no rain tomorrow: 50%

Why coherence matters

Dutch book

- Probability of rain tomorrow: 60%
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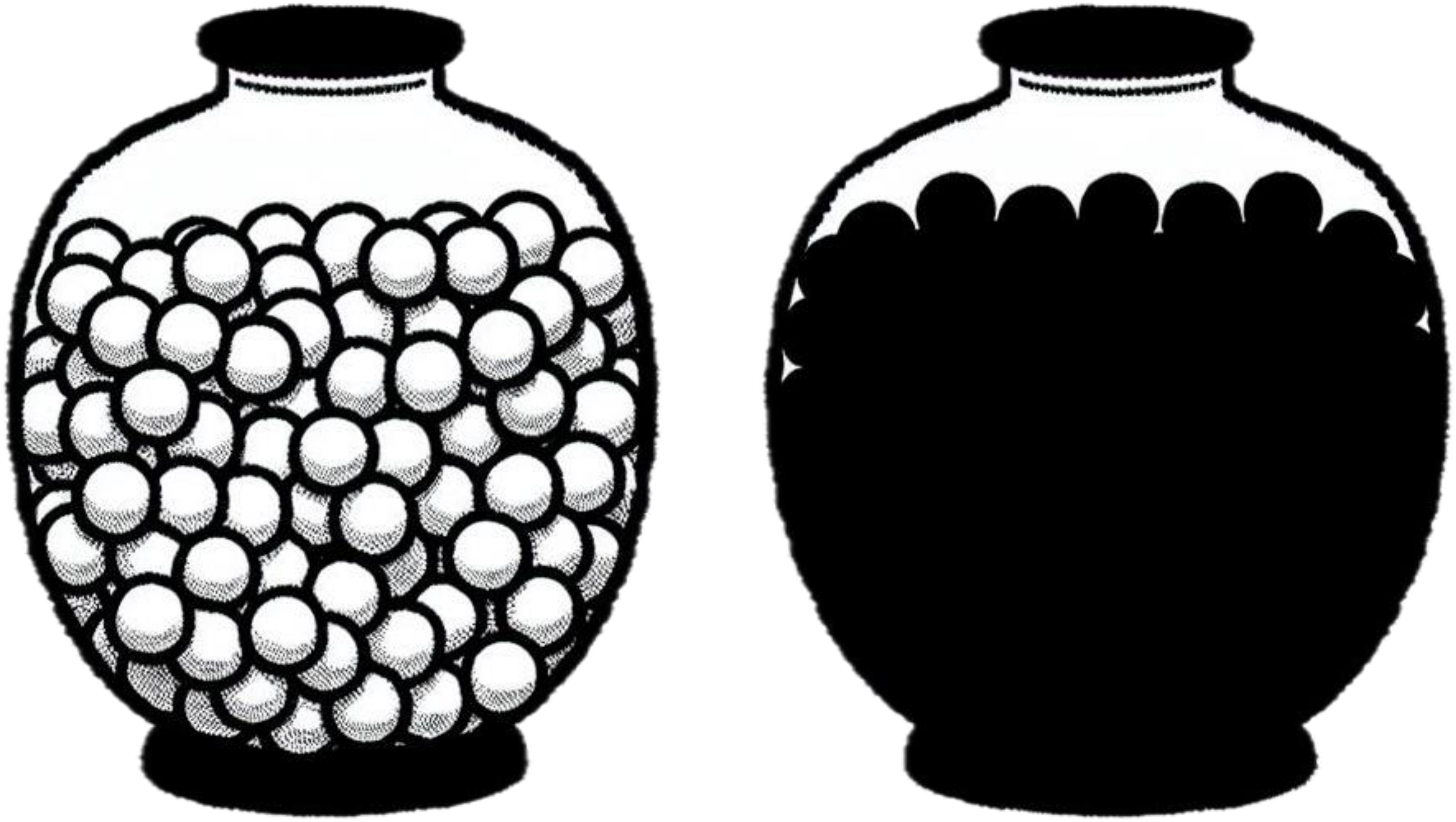
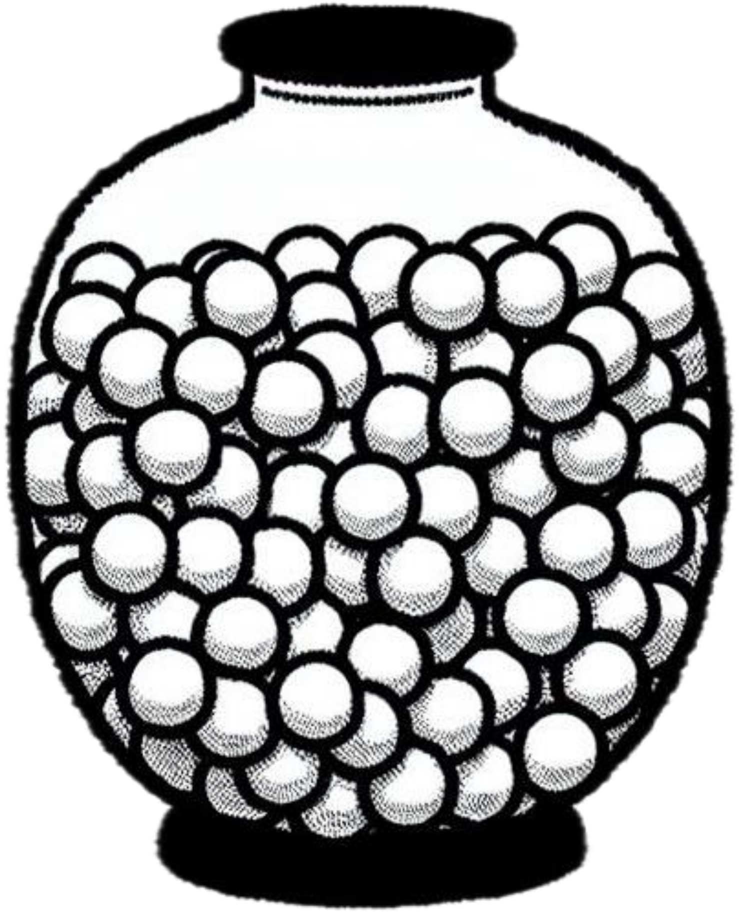
- Sell a $60 bet paying $100 if it rains
- Sell a $50 bet paying $100 if it doesn't rain

The Ellsberg paradox



The Ellsberg paradox



A: risky

B: ambiguous

The Ellsberg paradox

Gamble A

$$\mathbb{E}[U(A)] = 0.5u(\$100)$$

The Ellsberg paradox

Gamble A

$$\mathbb{E}[U(A)] = 0.5u(\$100)$$

Gamble B

Red: $\pi(r)$

Black: $\pi(b) = 1 - \pi(r)$

The Ellsberg paradox

Gamble A

$$\mathbb{E}[U(A)] = 0.5u(\$100)$$

Gamble B

Red: $\pi(r) \quad \Longrightarrow \quad \mathbb{E}[U(B_r)] = \pi(r)u(\$100)$

Black: $\pi(b) = 1 - \pi(r) \quad \Longrightarrow \quad \mathbb{E}[U(B_b)] = \big(1 - \pi(r)\big)u(\$100)$

The Ellsberg paradox

Gamble A

$$\mathbb{E}[U(A)] = 0.5u(\$100)$$

Gamble B

Red: $\pi(r) \quad \Longrightarrow \quad \mathbb{E}[U(B_r)] = \pi(r)u(\$100)$

Black: $\pi(b) = 1 - \pi(r) \quad \Longrightarrow \quad \mathbb{E}[U(B_b)] = \big(1 - \pi(r)\big)u(\$100)$

$$\mathbb{E}[U(B)] = \max\big\{\pi(r)u(\$100), \big(1 - \pi(r)\big)u(\$100)\big\}$$

$$= \max\big\{\pi(r), \big(1 - \pi(r)\big)\big\}\, u(\$100)$$

The Ellsberg paradox

Believe 0 red $\Longrightarrow$ Predict black $\mathbb{E}[U(B)] = u(\$100) > 0.5u(\$100) = \mathbb{E}[U(A)]$

Believe 1 red $\Longrightarrow$ Predict black $\mathbb{E}[U(B)] = 0.99u(\$100) > 0.5u(\$100) = \mathbb{E}[U(A)]$

$\dots$

Believe 50 red $\Longrightarrow$ Predict either $\mathbb{E}[U(B)] = 0.5u(\$100) = 0.5u(\$100)$
$= \mathbb{E}[U(A)]$

$\dots$

Believe 100 red $\Longrightarrow$ Predict red $\mathbb{E}[U(B)] = u(\$100) > 0.5u(\$100) = \mathbb{E}[U(A)]$