Probability theory

Notes on Behavioural Economics

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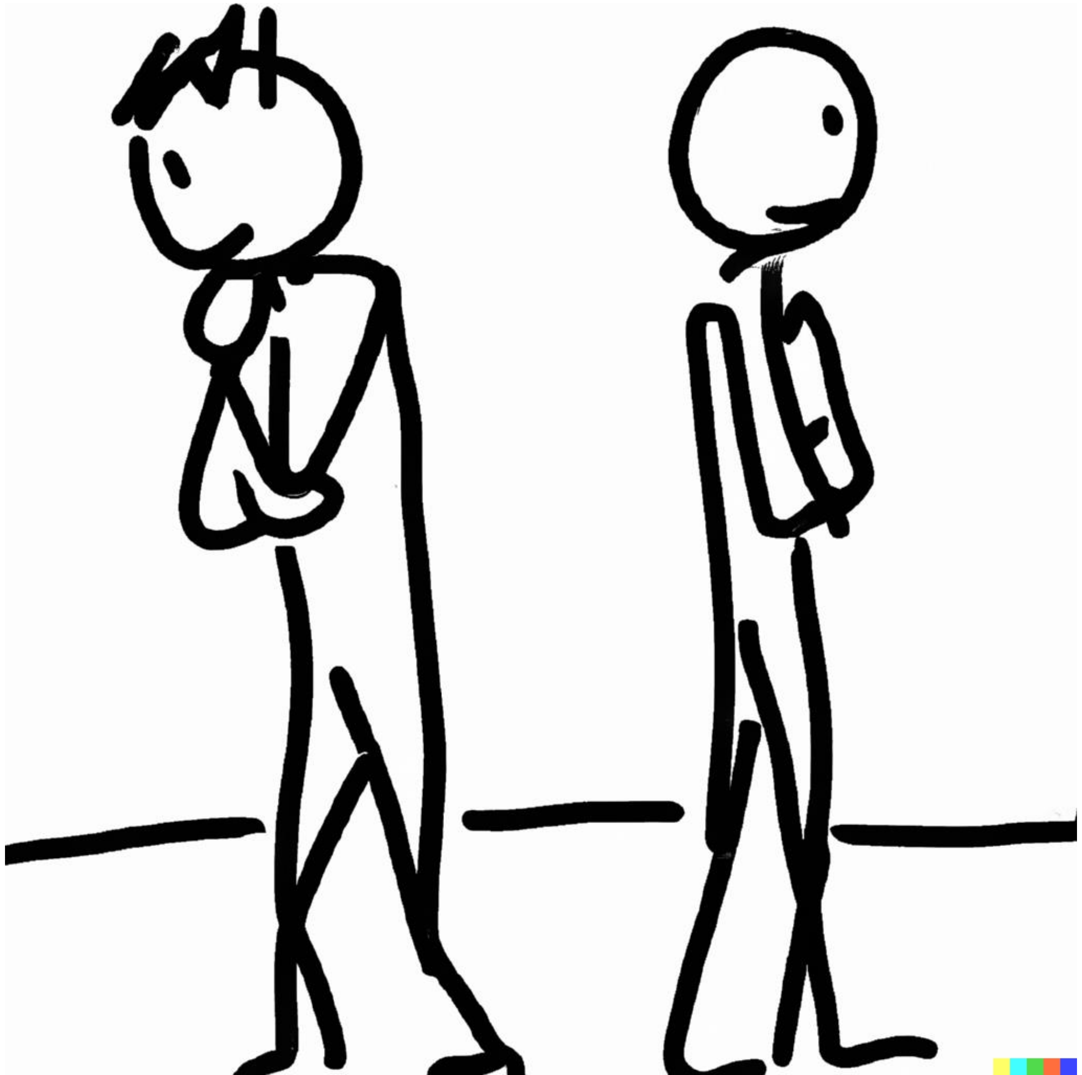
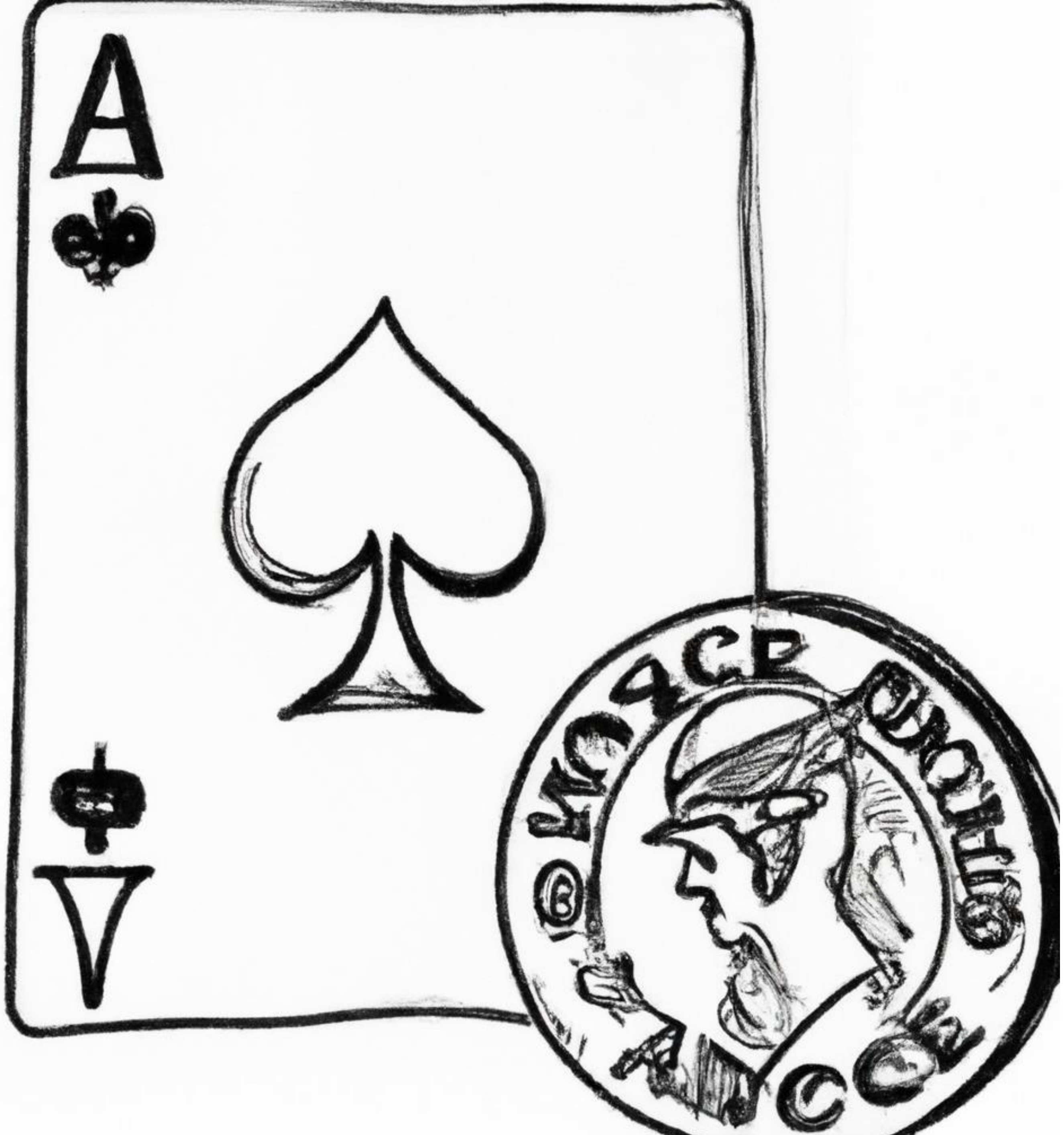


Probability of outcome A: $P(A)$

Probability function: $P(\cdot)$

The probability of outcome A lies between 0 and 1

$$0 \leq P(A) \leq 1$$


A

The probability of the entire outcome space equals 1

$$\sum_{n=1}^{n=52} {}^{1}/_{52} = 1$$

If outcomes A and B are mutually exclusive, the probability of A or B is the sum of the probability of A and the probability of B.

$$
\begin{aligned}
P(A \; or \, B) &= P(A \cup B) \\
&= P(A) + P(B)
\end{aligned}
$$

$$P(\mathrm{A}\spadesuit \cup \mathrm{A}\heartsuit \cup \mathrm{A}\diamondsuit \cup \mathrm{A}\clubsuit) = P(\mathrm{A}\spadesuit) + P(\mathrm{A}\heartsuit) + P(\mathrm{A}\diamondsuit) + P(\mathrm{A}\clubsuit)$$

$$= {}^{1}/_{52} + {}^{1}/_{52} + {}^{1}/_{52} + {}^{1}/_{52}$$

$$= {}^{4}/_{52}$$

If outcomes A and B are not mutually exclusive, the probability of A or B is the sum of the probability of A and the probability of B minus the probability of both occurring.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\mathrm{A} \cup \blacklozenge) = P(\mathrm{A}) + P(\blacklozenge) - P(A \cap \blacklozenge)$$

$$= {}^{4}/_{52} + {}^{1}/_{4} - {}^{1}/_{52}$$

$$= {}^{16}/_{52}$$

If outcomes A and B are independent, the conjunction of the two independent outcomes is the product of their probabilities.

$$P(A \cap B) = P(A) \cdot P(B)$$

$$
\begin{aligned}
P(A♠ \cap A♠) &= P(A♠) \cdot P(A♠) \\
&= {}^{1}/_{52} \times {}^{1}/_{52} \\
&= {}^{1}/_{2704}
\end{aligned}
$$

The probability of outcome A conditional on outcome B occurring.

$$P(A|B)$$

$$P(\text{Ace 1st} \cap \text{Ace 2nd}) = P(\text{Ace 1st}) \cdot P(\text{Ace 2nd} | \text{ Ace 1st})$$

$$= {}^{4}/_{52} \times {}^{3}/_{51}$$

$$= {}^{1}/_{221}$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

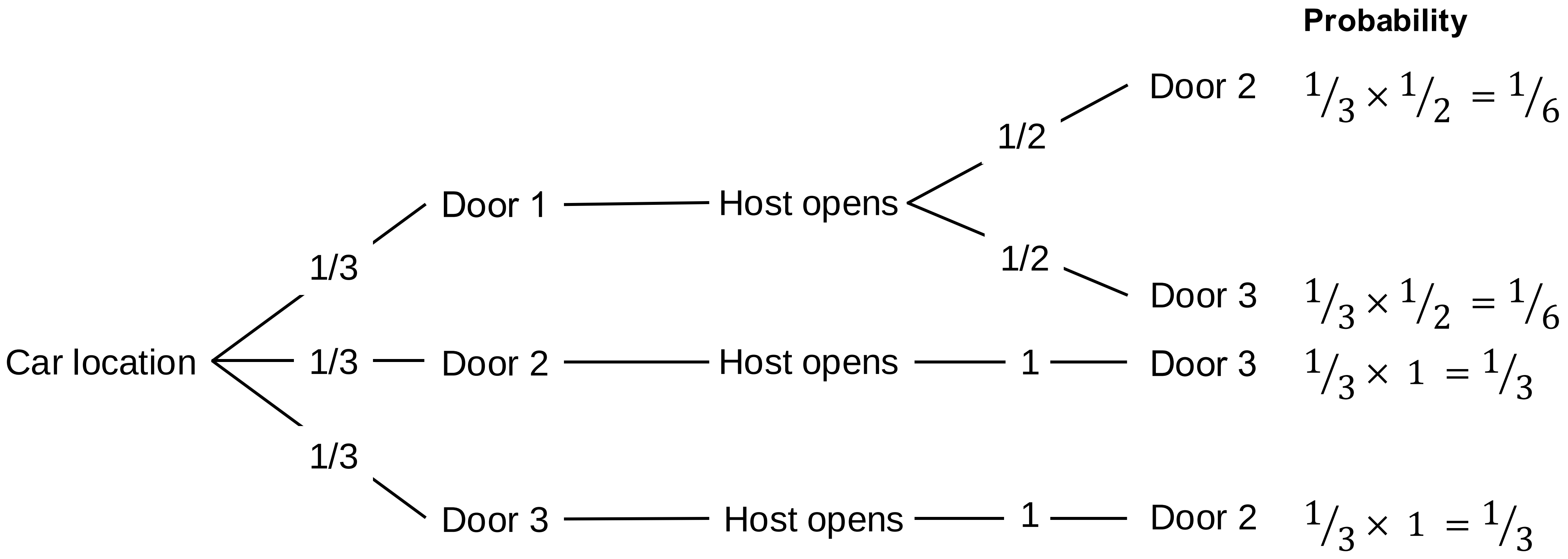
$$P(\text{Ace 2nd}|\ \text{Ace 1st}) = \frac{P(\text{Ace 1st} \cap \text{Ace 2nd})}{P(\text{Ace 1st})}$$

$$= \frac{{}^{1}/_{221}}{{}^{4}/_{52}}$$

$$= \frac{3}{51}$$

The Monty Hall problem

Suppose you're on a game show and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?



Probability
Car location
1/3
1/3
1/3
Door 1
Door 2
Door 3
Host opens
Host opens
Host opens
1/2
1/2
1
1
Door 2
Door 3
Door 3
Door 2
$^1/_3 \times {}^1/_2 = {}^1/_6$
$^1/_3 \times {}^1/_2 = {}^1/_6$
$^1/_3 \times 1 = {}^1/_3$
$^1/_3 \times 1 = {}^1/_3$

$$P(C2|D3) = \frac{P(C2 \cap D3)}{P(D3)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6}}$$

$$= \frac{2}{3}$$