The hot hand fallacy

Notes on Behavioural Economics

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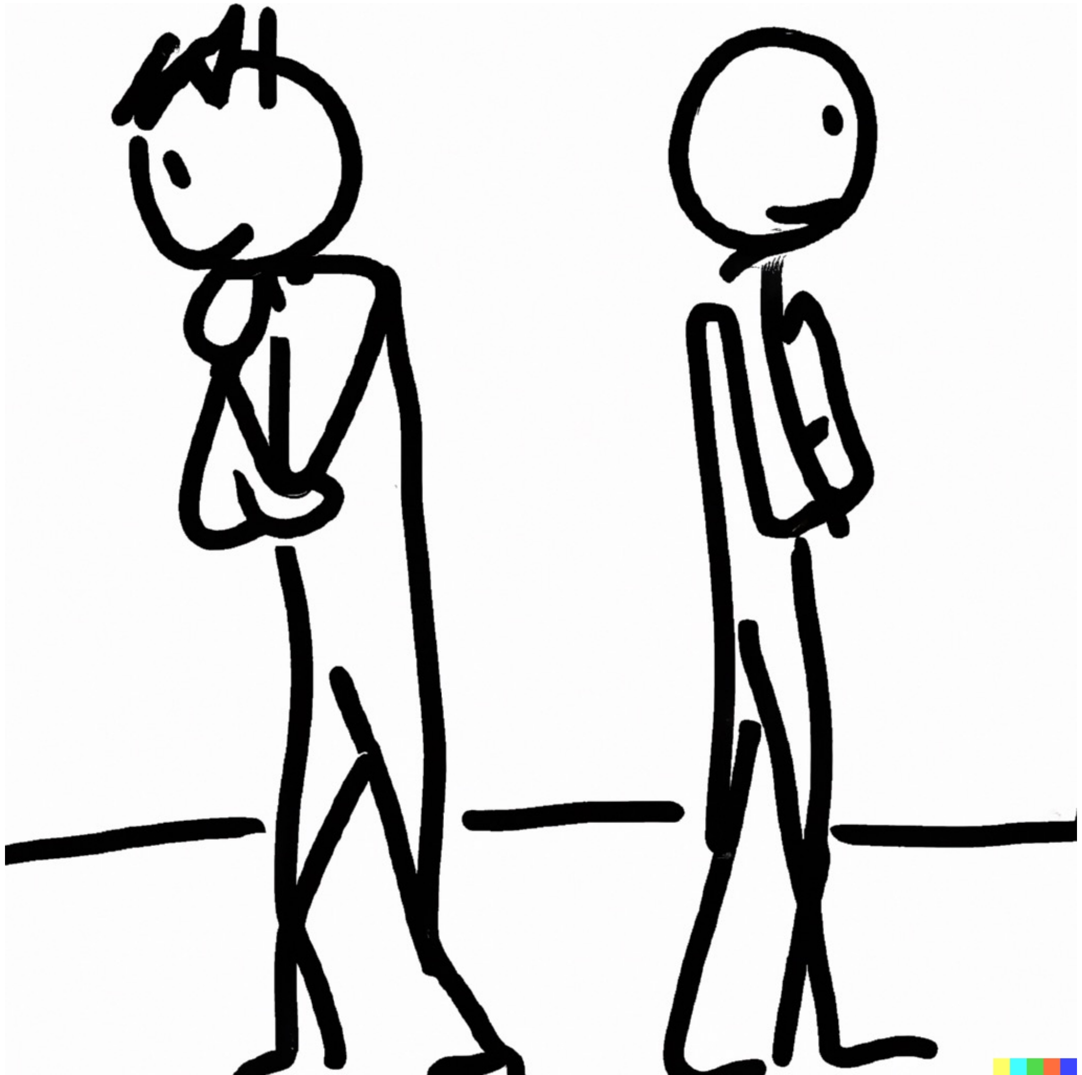
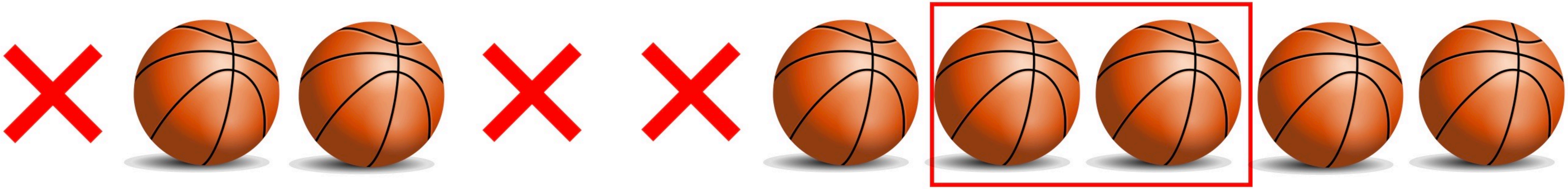




Sequence
H H H
H H T
H T H
H T T
T H H
T H T
T T H
T T T

Sequence	Proportion heads after heads
H H H	100%
H H T	50%
H T H	0%
H T T	0%
T H H	100%
T H T	0%
T T H	-
T T T	-

Sequence	Proportion heads after heads
H H H	100%
H H T	50%
H T H	0%
H T T	0%
T H H	100%
T H T	0%
T T H	-
T T T	-
Expected value	**41.7%**

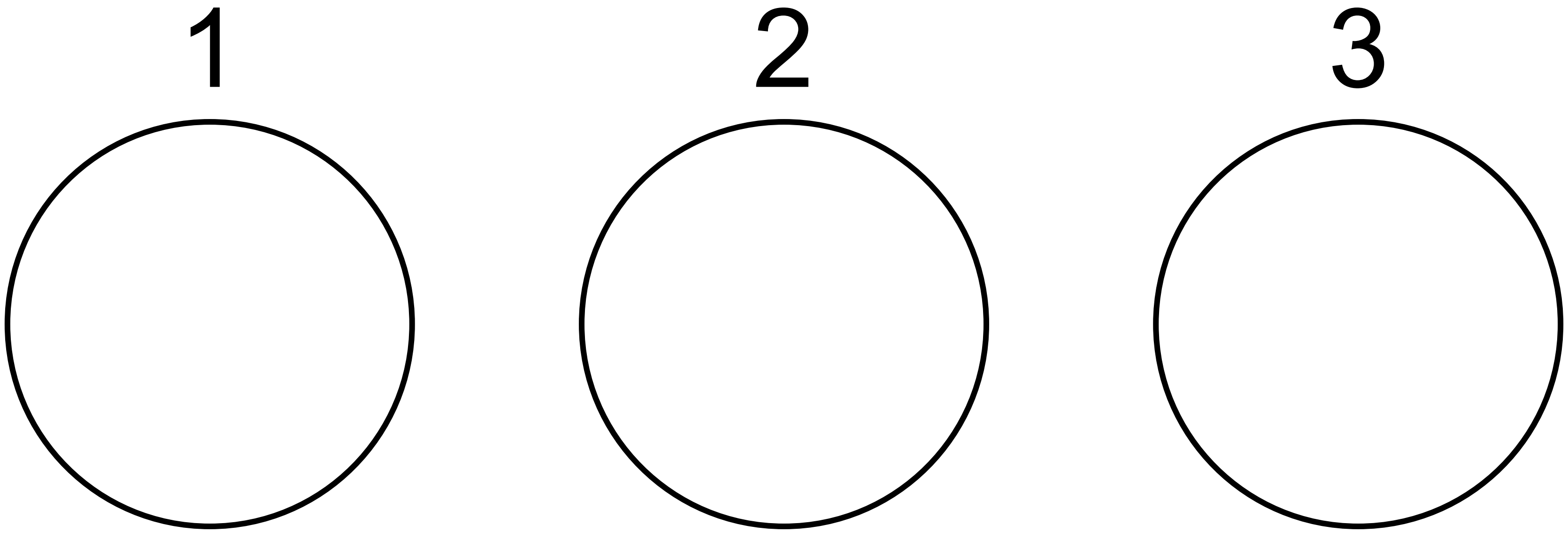
A B C

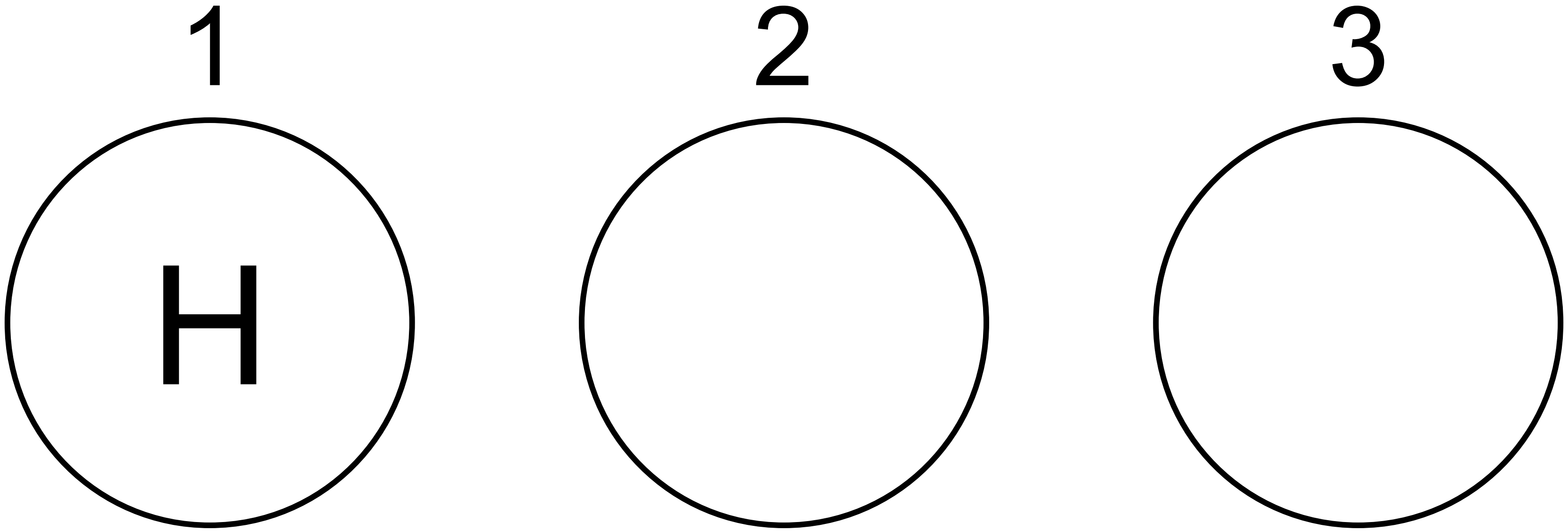
$$P(\mathrm{A} \cap B \cap C) = P(A \cap B|C)P(C)$$
$$= P(A|B \cap C)P(B \cap C)$$
$$= P(B|A \cap C)P(A \cap C)$$
$$= P(C|A \cap B)P(A \cap B)$$

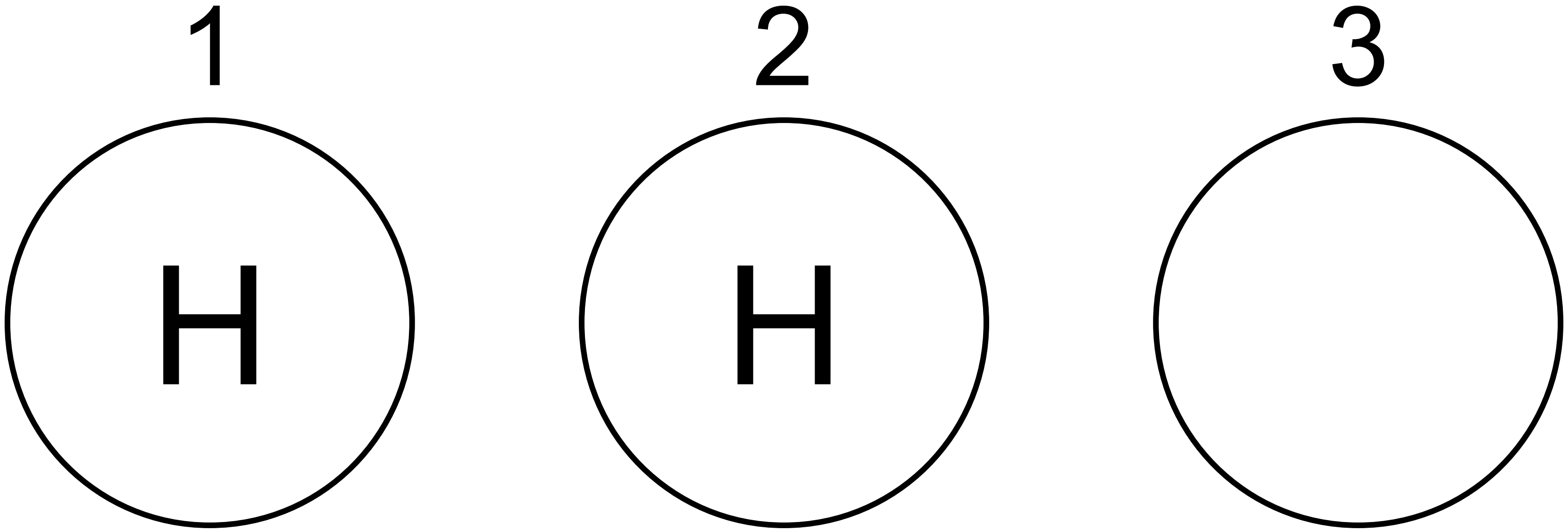
And so on.

$$P(A|B \cap C)P(B \cap C) = P(B|A \cap C)P(A \cap C)$$

$$P(A|B \cap C) = \frac{P(B|A \cap C)P(A \cap C)}{P(B \cap C)}$$


1
2
3


1
2
3
H


1
2
3
H
H

$$P(T_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(H_1 \cap X_2)}$$

$$P(T_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(H_1 \cap X_2)}$$

$$= \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2) + P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}$$

$$P(T_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(H_1 \cap X_2)}$$

$$= \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2) + P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}$$

$$= \frac{1 \times 0.25}{1 \times 0.25 + 0.5 \times 0.25}$$

$$
\begin{aligned}
P(T_2|H_1 \cap X_2) &= \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(H_1 \cap X_2)} \\
&= \frac{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2)}{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2) + P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)} \\
&= \frac{1 \times 0.25}{1 \times 0.25 + 0.5 \times 0.25} \\
&= \frac{2}{3}
\end{aligned}
$$

$$P(H_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(H_1 \cap X_2)}$$

$$P(H_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(H_1 \cap X_2)}$$

$$= \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2) + P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}$$

$$P(H_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(H_1 \cap X_2)}$$

$$= \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2) + P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}$$

$$= \frac{0.5 \times 0.25}{1 \times 0.25 + 0.5 \times 0.25}$$

$$P(H_2|H_1 \cap X_2) = \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(H_1 \cap X_2)}$$

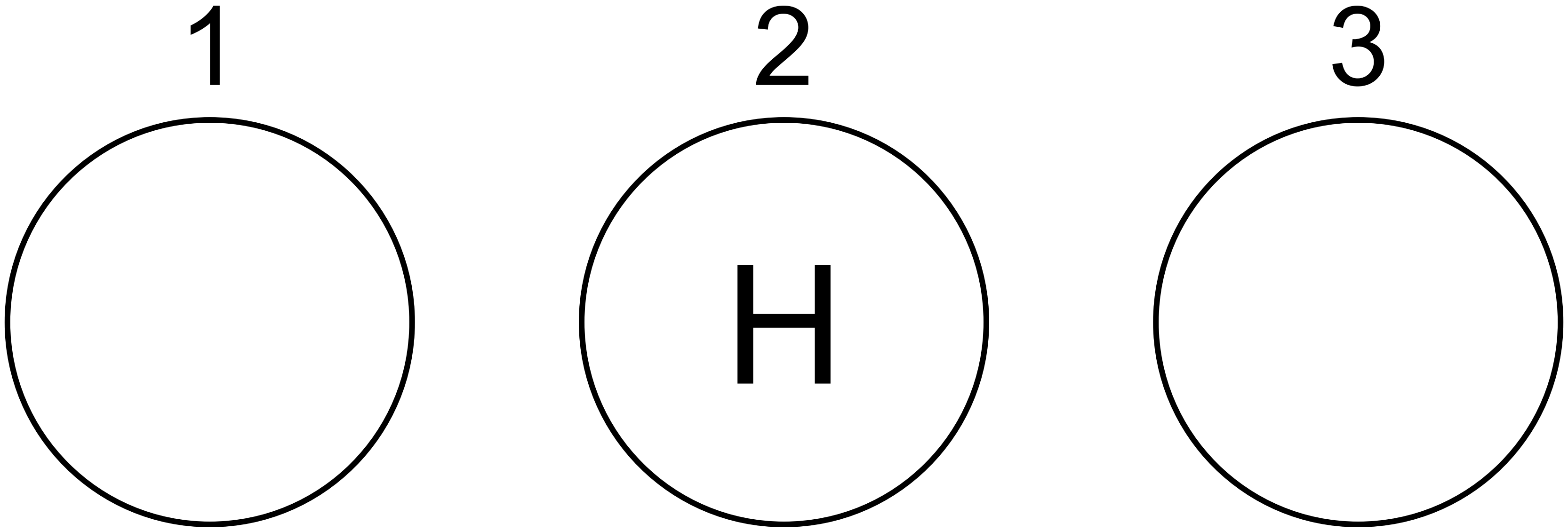
$$= \frac{P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}{P(X_2|H_1 \cap T_2)P(H_1 \cap T_2) + P(X_2|H_1 \cap H_2)P(H_1 \cap H_2)}$$

$$= \frac{0.5 \times 0.25}{1 \times 0.25 + 0.5 \times 0.25}$$

$$= \frac{1}{3}$$

$$P(T_2|H_1 \cap X_2) = P(T_2|X_2) = \frac{2}{3}$$

$$P(H_2|H_1 \cap X_2) = P(H_2|X_2) = \frac{1}{3}$$


1
2
3
H

$$P(T_3|H_2 \cap X_3) = P(T_3) = \frac{1}{2}$$

$$P(H_3|H_2 \cap X_3) = P(H_3) = \frac{1}{2}$$

$$
\begin{aligned}
P(H) &= P(X_2) \times P(H_2|X_2) + P(X_3) \times P(H_3|X_3) \\
&= 0.5 \times 0.33 + 0.5 \times 0.5 \\
&= 0.417
\end{aligned}
$$