Expected utility theory examples

Notes on Behavioural Economics

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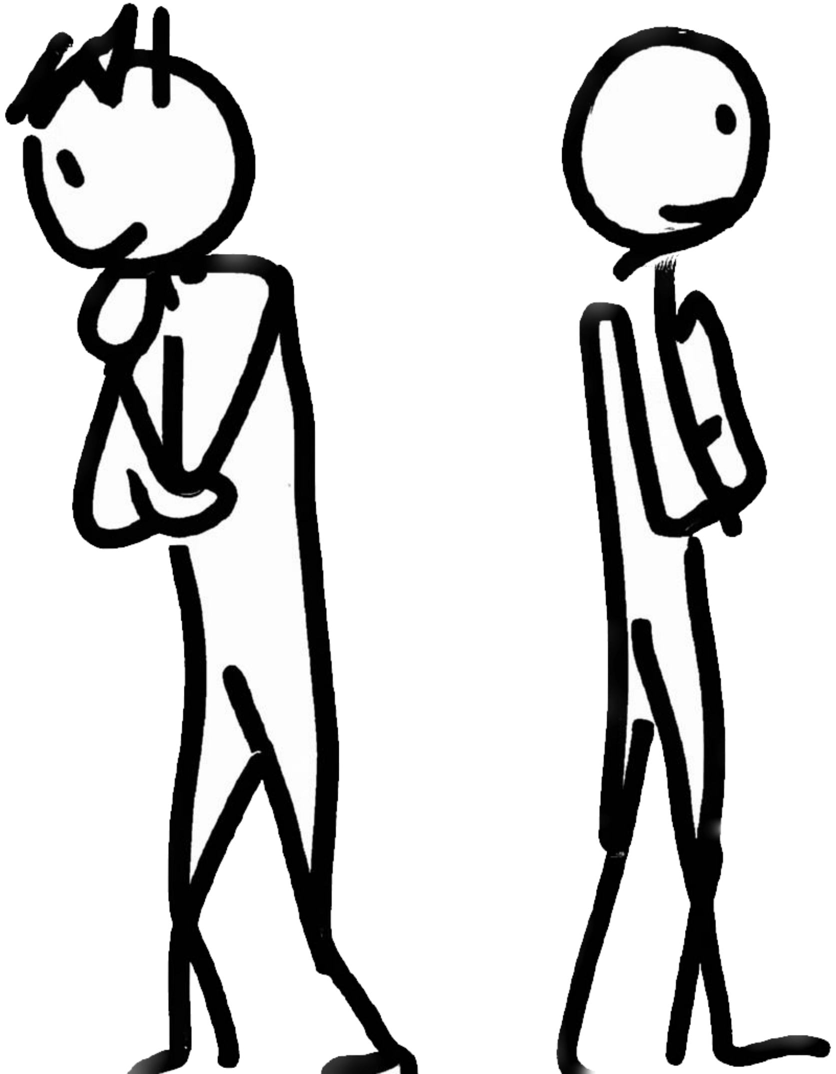

$$U(x) = \ln(x)$$

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$$X = (0.5, \$10; 0.5, -\$10) \qquad W = \$20$$

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$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$U(x) = \ln(x)$$

$$X = (0.5, \$10; 0.5, -\$10) \qquad W = \$20$$

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$= 0.5 \times \$10 + 0.5 \times (-\$10)$$

$$U(x) = \ln(x)$$

$$X = (0.5, \$10; 0.5, -\$10) \qquad W = \$20$$

$$\begin{aligned} E[X] &= \sum_{i=1}^{n} p_i x_i \\ &= 0.5 \times \$10 + 0.5 \times (-\$10) \\ &= 0 \end{aligned}$$

$$U(x) = \ln(x)$$

$$X = (0.5, \$10; 0.5, -\$10) \qquad W = \$20$$

$$E[U(W + X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$U(x) = \ln(x)$$

$$X = (0.5, \$10; 0.5, -\$10) \qquad W = \$20$$

$$E[U(W + X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$= 0.5 \times U(20 + 10) + 0.5 \times U(20 - 10)$$

$$U(x) = \ln(x)$$

$$X = (0.5, \$10; 0.5, -\$10) \qquad W = \$20$$

$$E[U(W + X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

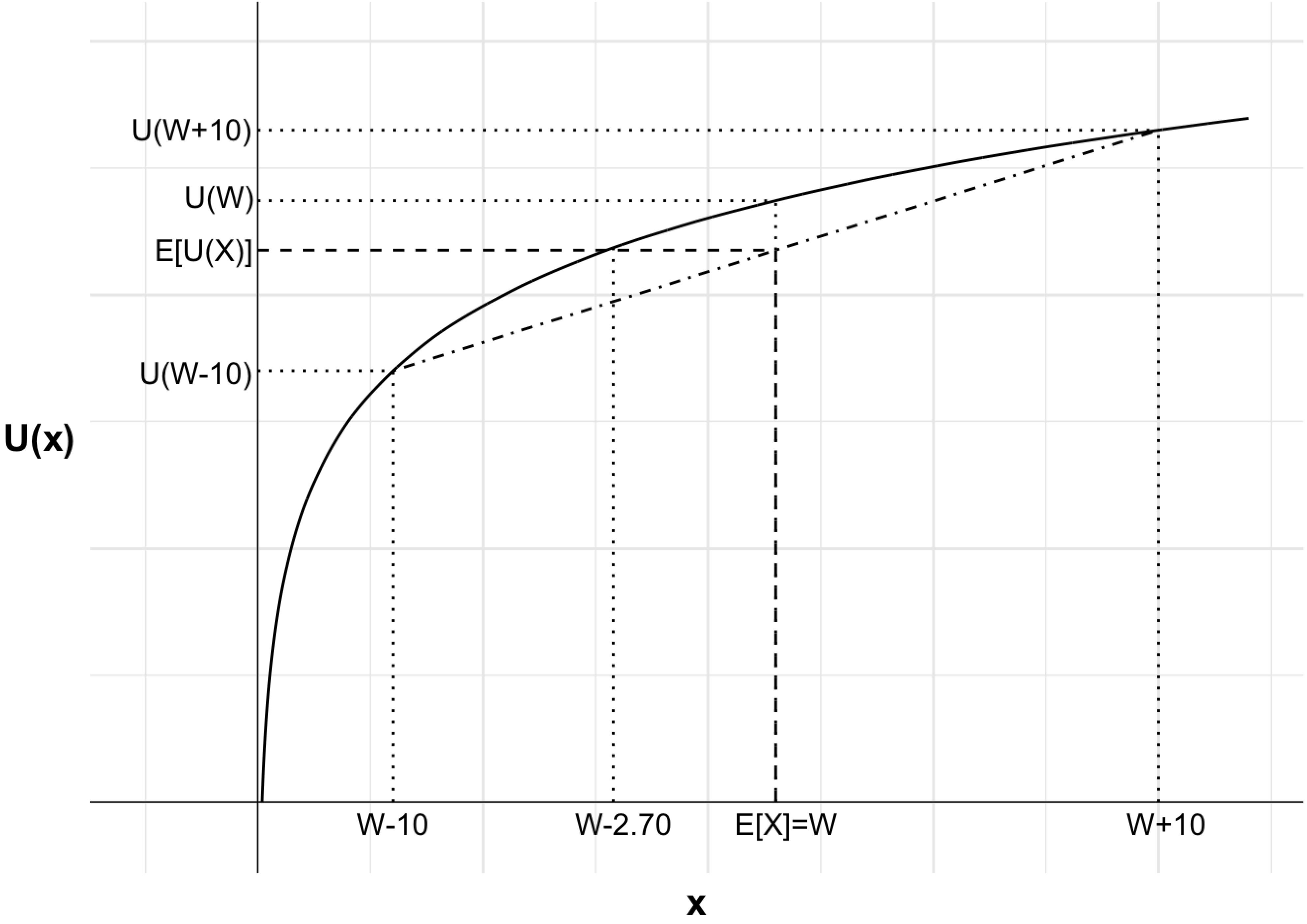
$$= 0.5 \times U(20 + 10) + 0.5 \times U(20 - 10)$$

$$= 0.5 \times \ln(30) + 0.5 \times \ln(10)$$

$$= 2.85$$

$$U(W) = \ln(W) = 2.85$$

$$W = e^{2.85} = \$17.30$$


U(W+10)
U(W)
E[U(X)]
U(W-10)
U(x)
W-10
W-2.70
E[X]=W
W+10
x

$$U(x) = \ln(x)$$

$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

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$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$U(x) = \ln(x)$$

$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$= 0.8 \times \$10 + 0.2 \times (-\$10)$$

$$U(x) = \ln(x)$$

$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$= 0.8 \times \$10 + 0.2 \times (-\$10)$$

$$= \$6$$

$$U(x) = \ln(x)$$

$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

$$E[U(W + x)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$U(x) = \ln(x)$$

$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

$$E[U(W + x)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$= 0.8 \times U(20 + 10) + 0.2 \times U(20 - 10)$$

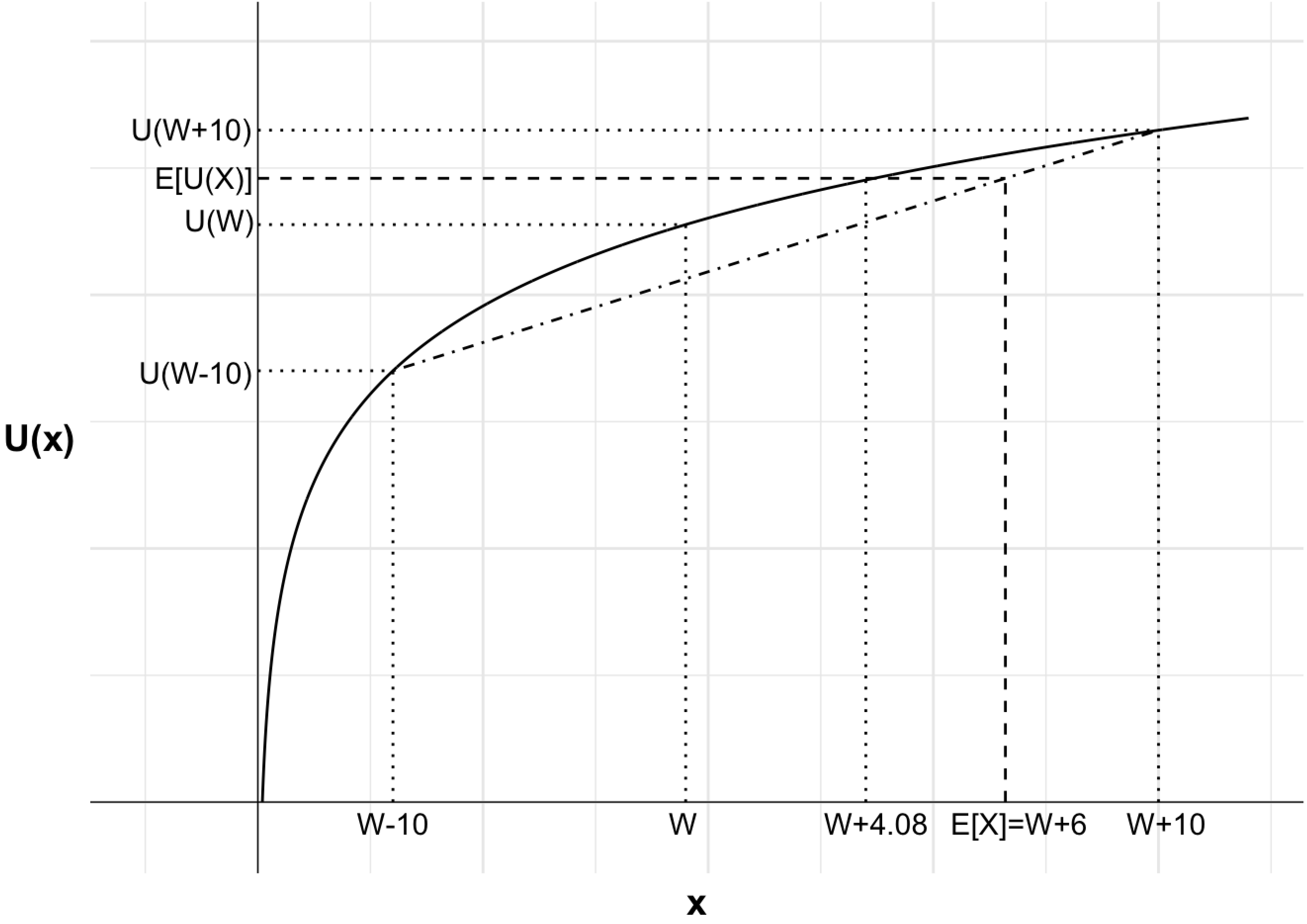
$$U(x) = \ln(x)$$

$$X = (0.8, \$10; 0.2, -\$10) \qquad W = \$20$$

$$\begin{aligned} E[U(W + x)] &= \sum_{i=1}^{n} p_i U(x_i + W) \\ &= 0.8 \times U(20 + 10) + 0.2 \times U(20 - 10) \\ &= 0.8 \times \ln(30) + 0.2 \times \ln(10) \\ &= 3.18 \end{aligned}$$

$$U(W) = \ln(W) = 3.18$$

$$W = e^{3.18} = \$24.08$$



U(W+10)
E[U(X)]
U(W)
U(W-10)
U(x)
W-10
W
W+4.08
E[X]=W+6
W+10
x

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$E[X] = \sum_{i=1}^{n} p_i x_i$$

$$= 0.5 \times 0.6W + 0.5 \times 1.5W$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$\begin{aligned}
E[X] &= \sum_{i=1}^{n} p_i x_i \\
&= 0.5 \times 0.6W + 0.5 \times 1.5W \\
&= 0.3W + 0.75W \\
&= 1.05W
\end{aligned}$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$E[U(W + X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$E[U(W + X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$= 0.5 \times U(0.6W) + 0.5 \times U(1.5W)$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

$$E[U(W+X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$= 0.5 \times U(0.6W) + 0.5 \times U(1.5W)$$

$$= 0.5 \times \ln(0.6) + 0.5 \times \ln(W) + 0.5 \times \ln(1.5) + 0.5 \times \ln(W)$$

$$U(x) = \ln(x)$$

$$X = (0.5, 0.5W; 0.5, -0.4W)$$

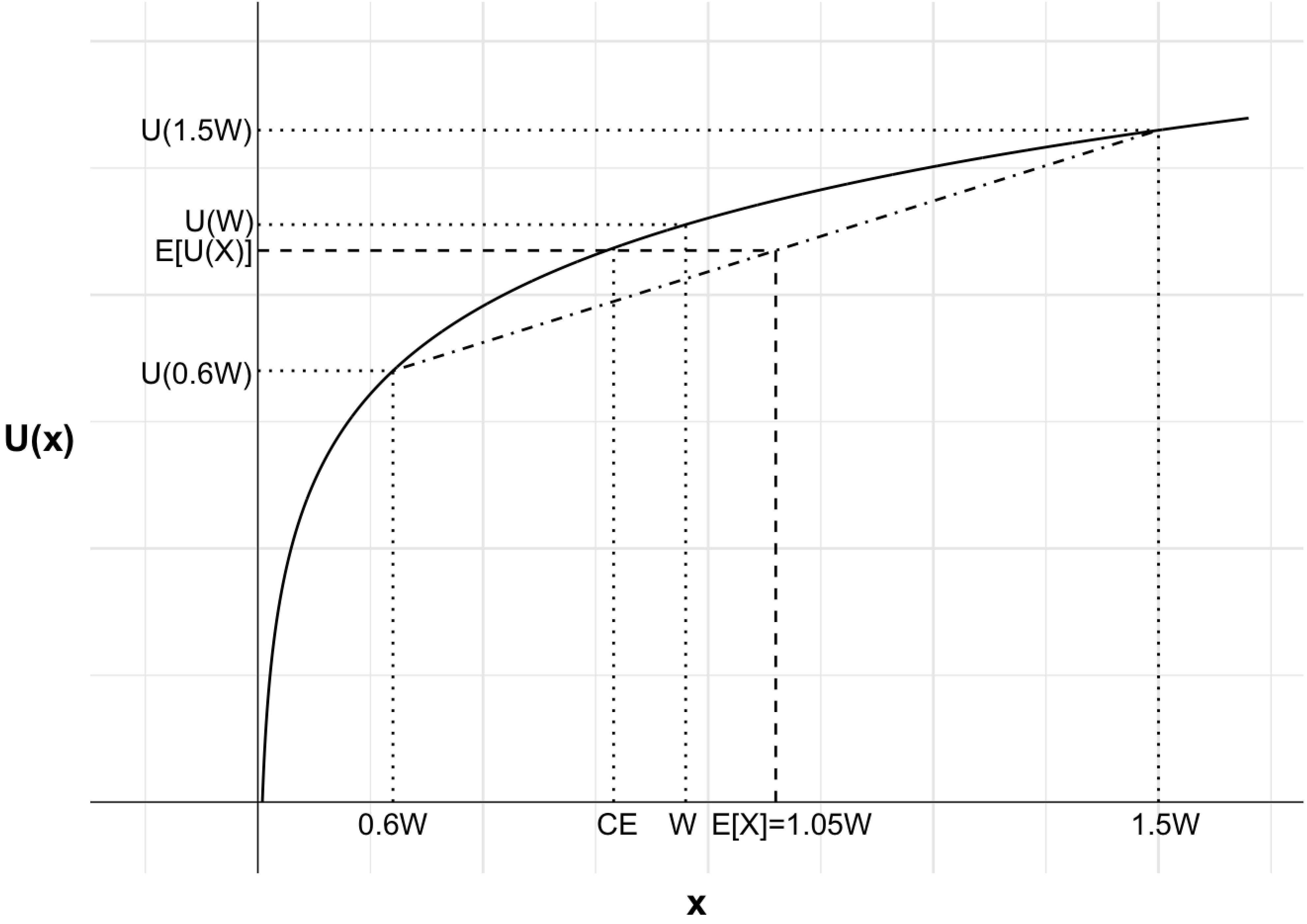
$$E[U(W+X)] = \sum_{i=1}^{n} p_i U(x_i + W)$$

$$= 0.5 \times U(0.6W) + 0.5 \times U(1.5W)$$

$$= 0.5 \times \ln(0.6) + 0.5 \times \ln(W) + 0.5 \times \ln(1.5) + 0.5 \times \ln(W)$$

$$= -0.255 + 0.203 + \ln(W)$$

$$= -0.053 + \ln(W)$$



U(1.5W)
U(W)
E[U(X)]
U(0.6W)
U(x)
0.6W
CE
W
E[X]=1.05W
1.5W
x

The St. Petersburg game

The St. Petersburg game

Tail on the 1st flip:	\$2
Tail on the 2nd flip:	\$4
Tail on the 3rd flip:	\$8
Tail on the 4th flip:	\$16

And so on.

The St. Petersburg game

The expected value of this game X is equal to:

$$E[X] = \underbrace{\frac{1}{2} \times 2}_{\textit{Tail first}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2}\right) \times}_{\textit{Tail second}} 4 + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)}_{\textit{Tail third}} \times 8 + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)}_{\textit{Tail fourth}} \times 16 + \cdots$$

The St. Petersburg game

The expected value of this game X is equal to:

$$E[X] = \underbrace{\frac{1}{2} \times 2}_{\textit{Tail first}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2}\right) \times 4}_{\textit{Tail second}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 8}_{\textit{Tail third}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 16}_{\textit{Tail fourth}} + \cdots$$

The St. Petersburg game

The expected value of this game X is equal to:

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The St. Petersburg game

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$$E[X] = \underbrace{\frac{1}{2} \times 2}_{\textit{Tail first}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2}\right) \times 4}_{\textit{Tail second}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 8}_{\textit{Tail third}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 16}_{\textit{Tail fourth}} + \cdots$$

The St. Petersburg game

The expected value of this game X is equal to:

$$E[X] = \underbrace{\frac{1}{2} \times\ 2}_{\textit{Tail first}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2}\right) \times\ 4}_{\textit{Tail second}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times\ 8}_{\textit{Tail third}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times\ 16}_{\textit{Tail fourth}} + \cdots$$

$$= 1 + 1 + 1 + 1 + \ldots$$

The St. Petersburg game

The expected value of this game X is equal to:

$$E[X] = \underbrace{\frac{1}{2} \times}_{\textit{Tail first}} \; 2 + \underbrace{\left(\frac{1}{2} \times \frac{1}{2}\right) \times}_{\textit{Tail second}} \; 4 + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times}_{\textit{Tail third}} \; 8 + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times}_{\textit{Tail fourth}} \; 16 + \cdots$$

$$= 1 + 1 + 1 + 1 + \ldots$$

$$= \sum_{k=1}^{\infty} 1$$

The St. Petersburg game

The expected value of this game X is equal to:

$$E[X] = \underbrace{\frac{1}{2} \times \ 2}_{\textit{Tail first}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2}\right) \times \ 4}_{\textit{Tail second}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times \ 8}_{\textit{Tail third}} + \underbrace{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times \ 16}_{\textit{Tail fourth}} + \cdots$$

$$= 1 + 1 + 1 + 1 + \ldots$$

$$= \sum_{k=1}^{\infty} 1$$

$$= \infty$$

The St. Petersburg game

The expected utility of this game X is equal to:

$$E[U(X)] = \underbrace{\frac{1}{2}U(W+2)}_{\textit{Tail first}} + \underbrace{\left(\frac{1}{2}*\frac{1}{2}\right)U(W+4)}_{\textit{Tail second}} + \underbrace{\left(\frac{1}{2}*\frac{1}{2}*\frac{1}{2}\right)U(W+8)}_{\textit{Tail third}}$$

$$+\underbrace{\left(\frac{1}{2}*\frac{1}{2}*\frac{1}{2}*\frac{1}{2}\right)U(W+16)}_{\textit{Tail fourth}} + \cdots$$

$$= \frac{1}{2}U(W+2) + \frac{1}{4}U(W+4) + \frac{1}{8}U(W+8) + \frac{1}{16}U(W+16) + \cdots$$

$$= \sum_{k=1}^{k=\infty} \frac{1}{2^k} U\left(W+2^k\right)$$

The St. Petersburg game

What is the maximum $\$c$ a risk neutral player with $U(x) = x$ would be willing to pay to play the game?

They will be indifferent when:

$$U(W) = E[U(X - c)]$$

The St. Petersburg game

$$U(W) = E[U(X - c)]$$

$$U(W) = \sum_{k=1}^{k=\infty} \frac{1}{2^k} U\left(W + \$2^k - c\right)$$

$$W = \sum_{k=1}^{k=\infty} \frac{1}{2^k} \left(W + 2^k - c\right)$$

The St. Petersburg game

$$W = \sum_{k=1}^{k=\infty} \frac{1}{2^k}\left(W + 2^k - c\right)$$

$$W = W - c + \sum_{k=1}^{k=\infty} 1 \qquad \left(\text{as } \sum_{k=1}^{k=\infty} \frac{1}{2^k} = 1\right)$$

$$c = \infty$$

The St. Petersburg game

What is the maximum $\$c$ a risk-averse player with $U(x) = ln(x)$ would be willing to pay to play the game? How does their wealth affect their willingness to pay?

They will be indifferent when:

$$U(W) = E[U(X - c)]$$

The St. Petersburg game

What is the maximum $\$c$ a risk-averse player with $U(x) = ln(x)$ would be willing to pay to play the game? How does their wealth affect their willingness to pay?

They will be indifferent when:

$$U(W) = E[U(X - c)]$$

$$U(W) = \sum_{k=1}^{k=\infty} \frac{1}{2^k} U\left(W + \$2^k - c\right)$$

$$ln(W) = \sum_{k=1}^{k=\infty} \frac{1}{2^k} ln\left(W + \$2^k - c\right)$$

The St. Petersburg game

What is the maximum $\$c$ a risk-averse player with $U(x) = ln(x)$ would be willing to pay to play the game? How does their wealth affect their willingness to pay?

Wealth	Willing to pay
$0.01	$2.01
$1000	$10.95
$1 million	$20.87

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{1}{2^k} U\left(\$2^k\right)$$

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{1}{2^k} U\left(\$2^k\right)$$

$$= \sum_{k=1}^{k=\infty} \frac{1}{2^k} ln\left(2^k\right)$$

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{1}{2^k} U(\$2^k)$$

$$= \sum_{k=1}^{k=\infty} \frac{1}{2^k} ln(2^k)$$

$$= \sum_{k=1}^{k=\infty} \frac{k}{2^k} ln(2)$$

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{k}{2^k} ln(2)$$

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{k}{2^k} ln(2)$$

$$= \frac{1}{2}\ln(2) + \frac{2}{4}\ln(2) + \frac{3}{8}\ln(2) + \frac{4}{16}\ln(2) + \frac{5}{32}\ln(2) + \cdots$$

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{k}{2^k} ln(2)$$

$$= \frac{1}{2}\ln(2) + \frac{2}{4}\ln(2) + \frac{3}{8}\ln(2) + \frac{4}{16}\ln(2) + \frac{5}{32}\ln(2) + \cdots$$

$$= \left(\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{5}{32} + \cdots\right)\ln(2)$$

The St. Petersburg game

What is the the utility of a risk-averse player whose only asset is the opportunity to play this game?

$$E[U(X)] = \sum_{k=1}^{k=\infty} \frac{k}{2^k} ln(2)$$

$$= \frac{1}{2}\ln(2) + \frac{2}{4}\ln(2) + \frac{3}{8}\ln(2) + \frac{4}{16}\ln(2) + \frac{5}{32}\ln(2) + \cdots$$

$$= \left(\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{5}{32} + \cdots\right)\ln(2)$$

$$= 2\ln(2)$$

Risk neutrality versus risk aversion

A: Lottery $A = (0.5, \$100; 0.5, \$20)$

B: $\$40$

Risk neutrality versus risk aversion

A: Lottery $A = (0.5, \$100; 0.5, \$20)$

B: $\$40$

Anika: Risk neutral

Risk neutrality versus risk aversion

A: Lottery $A = (0.5, \$100; 0.5, \$20)$

B: $\$40$

Anika: Risk neutral

$$
\begin{aligned}
\mathrm{E}[A] &= p_1 x_1 + p_2 x_2 \\
&= 0.5 \times \$100 + 0.5 \times \$20 \\
&= \$60
\end{aligned}
$$

Risk neutrality versus risk aversion

A: Lottery $A = (0.5, \$100; 0.5, \$20)$

B: $\$40$

Anika: Risk neutral

$$\begin{aligned} \mathrm{E}[A] &= p_1 x_1 + p_2 x_2 \\ &= 0.5 \times \$100 + 0.5 \times \$20 \\ &= \$60 \end{aligned}$$

$$\mathrm{E}[B] = \$40$$

Risk neutrality versus risk aversion

A: Lottery $A = (0.5, \$100; 0.5, \$20)$

B: $\$40$

Anthony: Risk averse with wealth \$100; $U(x) = \ln(x)$