



HONG KONG MOCK SECONDARY EDUCATION EXAMINATION
2021

2021 年香港中學模擬考試

MATHEMATICS Compulsory Part
數學 必修部分
PAPER 1 試卷一

Pre Mock

Avg	2020 - 2021	P1
5.4	92%	97
5*	86%	90
5	78%	82
4	67%	70
3	55%	58

Question and Answer Book 試題答題簿

(2 hour) 二小時完卷

INSTRUCTIONS 考生須知

- After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
宣布開考後，考生須首先在第1頁之適當位置填寫生編號，並在第1、3、5、7、9及11頁之適當位置貼上電腦條碼。
- This paper consists of THREE sections, A(1), A(2) and B.
本試卷分三部，即甲部(1)、甲部(2)和乙部。
- Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
本試卷各題均須作答，答案須寫在本試題答題簿中預留的空位內。不可在各頁邊界以外位置書寫。寫於邊界以外的答案，將不予評閱。
- Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.
如有需要，可要求派發方格紙及補充答題紙。每張紙均須寫下你的姓名及填畫試題編號，並用繩縛於簿內。
- Unless otherwise specified, all working must be clearly shown.
除特別指明外，須詳細列出所有算式。
- Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
除特別指明外，數值答案須用真確值，或準確至三位有效數字的近似值表示。
- The diagrams in this paper are not necessarily drawn to scale.
本試卷的附圖不一定依比例繪成。
- No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.
試場主任宣布停筆後，考生不會獲得額外時間貼上電腦條碼及填畫試題編號方格。

Section A(1) (30 marks)

甲部(1) (30 分)

1. Make x the subject of the formula $\frac{1}{x+y} = \frac{y}{y-x}$. (3 marks)

令 x 成為公式 $\frac{1}{x+y} = \frac{y}{y-x}$ 的主項。 (3 分)

$$y-x = y(x+y) \quad |M$$

$$y-x = yx + y^2$$

$$y - y^2 = yx + x \quad |M$$

$$x(y+1) = y - y^2$$

$$x = \frac{y - y^2}{y+1} \quad |A$$

2. Simplify $\frac{(x^{-1}y)^8}{x^{16}y^{-3}}$ and express the answer with positive indices. (3 marks)

化簡 $\frac{(x^{-1}y)^8}{x^{16}y^{-3}}$ ，並以正指數表示答案。 (3 分)

$$= \frac{x^{-8}y^8}{x^{16}y^{-3}} \quad |M$$

$$= \frac{y^8 \cdot y^3}{x^{16} \cdot x^{-8}} \quad |M$$

$$= \frac{y^{11}}{x^{24}} \quad |A$$

Answers written in the margins will not be marked.

寫於邊界以外的答案，將不予評閱。

3. Factorize 因式分解

- (a) $8x^3 + 27y^3$,
 (b) $8x^3 + 27y^3 - 4x - 6y$.

a)
$$\begin{aligned} & (2x)^3 + (3y)^3 \\ &= (2x+3y)[(2x)^2 - (2x)(3y) + (3y)^2] \\ &= (2x+3y)(4x^2 - 6xy + 9y^2) \quad |A \end{aligned}$$

(3 marks) (3 分)

b)
$$\begin{aligned} & (2x+3y)(4x^2 - 6xy + 9y^2) - 2(2x+3y) \quad |M \\ &= (2x+3y)(4x^2 - 6xy + 9y^2 - 2) \quad |A \end{aligned}$$

4. There are a number of cows and chickens in a farm. Nixon discovers the number of chickens is 40% of the number of cows and there are 72 cows more than chickens.

一農場有若干牛和雞。小力發現雞的數目是牛的40%，且牛比雞多出72隻。

- (a) Find the number of cows and chickens,

求該農場牛和雞的數目。

- (b) The total number of cows and chickens are 20% less than last year. Find the total number of cows and chickens.

該農場牛和雞的總數比去年少20%。求去年牛和雞的總數。

(5 marks) (5 分)

a) Let there are x cows, y chickens

$$\begin{cases} y = 0.4x & \text{-- (1)} \\ x = y + 72 & \text{-- (2)} \end{cases} \quad |M$$

put (1) into (2),

$$x = 0.4x + 72 \quad |M$$

$$0.6x = 72$$

$$x = 120$$

$$y = 0.4x = 48$$

\therefore there 120 cows, 48 chickens $|A$

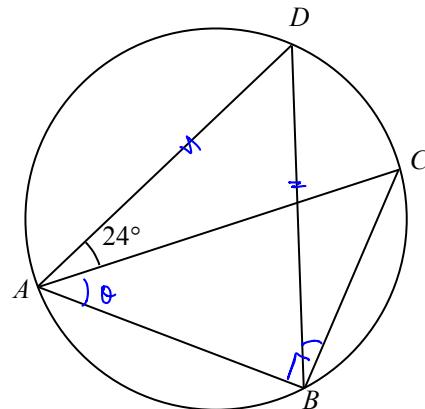
b) Total number = $(120+48) \div (1-20\%) \quad |M$
 $= 210, \quad |A$

Answers written in the margins will not be marked. 寫於邊界以外的答案，將不予評閱。

5. In the figure, AC is a diameter of the circle $ABCD$. If $AD = BD$ and $\angle CAD = 24^\circ$, find $\angle BAC$.

圖中， AC 為圓 $ABCD$ 的一直徑。若 $AD = BD$ 及 $\angle CAD = 24^\circ$ ，求 $\angle BAC$ 。

(4 marks) (4 分)



$$\angle ABC = 90^\circ \quad |M$$

$$\angle CBD = \angle CAD = 24^\circ \quad |M$$

$$\angle ABD = 90^\circ - 24^\circ = 66^\circ$$

$$\angle BAD = \angle ABD = 66^\circ \quad |M$$

$$\begin{aligned} \angle BAC &= 66^\circ - 24^\circ \\ &= 42^\circ, \quad |A \end{aligned}$$

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6. The cost of a calculator is \$ 160. If the calculator is sold at its marked price, then the percentage profit is 25%.

某計算機的成本為 \$160。若該計算機以其標價出售，則盈利百分率為25%，

- (a) Find the marked price of the calculator.

求該計算機的標價。

- (b) If the calculator is sold at a 10% discount on the marked price, find the percentage profit or percentage loss.

若該計算機以其標價九折出售，求盈利百分率或虧蝕百分率。

(4 marks) (4 分)

$$\text{a)} \quad \text{Marked Price} = 160 \times (1 + 25\%) \quad |M \\ = \$200 \quad |A$$

$$\text{b)} \quad \% \text{ profit} = \frac{200 \times (1 - 10\%) - 160}{160} \times 100\% \quad |M \\ = + 12.5\% \quad |A$$

7. (a) Solve the inequality 解不等式 $\frac{x+2}{2} > -3(x-4)$ 。

- (b) Find the least integer satisfying $3 \frac{x+2}{2} > -3(x-4)$ or $1 - 7x \leq 13$.

求滿足不等式 $3 \frac{x+2}{2} > -3(x-4)$ 或 $1 - 7x \leq 13$ 的最小整數。

(4 marks) (4 分)

$$\text{a)} \quad x+2 > -6x + 24$$

$$7x > 22$$

$$x > \frac{22}{7}$$

|A

$$\text{b)} \quad \frac{9}{2}x > 9 \quad \text{OR} \quad -7x \leq 12 \quad |M$$

$$x > 2 \quad \text{OR}$$

$$x \geq \frac{12}{7}$$

$$\therefore x \geq \frac{12}{7}$$

|A

$$\therefore \text{least integer} = 2 \quad |A$$

8. The polar coordinates of the points A is $(5, 13^\circ)$. A is rotated clockwise about the origin O through 90° to B . The polar coordinates of the points C are $(r, 193^\circ)$.

點 A 的極坐標為 $(5, 13^\circ)$ 。 B 為 A 沿原點 O 順時針旋轉 90° 的像。 C 點的極坐標為 $(r, 193^\circ)$ 。

- (a) Are O, B, C co-linear? Explain your answer.

O, B 與 C 是否共線？試解釋你的答案。

- (b) Express the area of ΔABC in terms of r .

以 r 表示 ΔABC 的面積。

(4 marks) (4 分)

a) $B = (5, 283^\circ)$

$$\angle BOC = 283^\circ - 193^\circ = 90^\circ \neq 180^\circ \text{ or } 0^\circ \quad 1M$$

\therefore No 1A

b) Area of $\Delta ABC = \frac{1}{2}(AC)(OB) = \frac{1}{2}(5+r)5 \quad 1M$

$$= \frac{5}{2}r + \frac{25}{2}, \quad 1A$$

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Section A(2) (30 marks)

甲部(2) (30 分)

9. (a) Find a such that $x - 3$ is a factor of $x^3 - 2x^2 - 5x + a$.

求 a 使得 $x - 3$ 為 $x^3 - 2x^2 - 5x + a$ 的因式。

- (b) Hence factorize $x^3 - 2x^2 - 5x + a$ completely.

由此，因式分解 $x^3 - 2x^2 - 5x + a$ 。

(c) Simplify 簡化 $\frac{x-4}{x^3 - 2x^2 - 5x + 6} \times \left(1 + \frac{6}{x-4}\right)$.

(8 marks) (8 分)

a) Let $f(x) = x^3 - 2x^2 - 5x + a$

$$f(3) = (3)^3 - 2(3)^2 - 5(3) + a = 0 \quad |M$$

$$a = 6 \quad |A$$

b) $f(x) = x^3 - 2x^2 - 5x + 6$

$$= (x-3)(x^2 + x - 2) \quad |M + |A$$

$$= (x-3)(x+2)(x-1), \quad |A$$

c) $\frac{x-4}{(x-3)(x+2)(x-1)} \times \left(\frac{x-4+6}{x-4} \right) \quad |M$

$$= \frac{(x-4)(x+2)}{(x-3)(x+2)(x-1)(x-4)} \quad |M$$

$$= \frac{1}{(x-3)(x-1)}, \quad |A$$

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10. A container $PQRSTUWV$. The base $PQRS$ is a square, and $PQVU$ and $SRWT$ are trapezium, where $PQ \parallel UV$ and $\angle PQV = 90^\circ$. Given $PQ = 24\text{ cm}$, $PU = 13\text{ cm}$ and $TW = 29\text{ cm}$.
 一個形狀為直立角柱體的容器 $PQRSTUWV$ 。該容器的底 $PQRS$ 是一個正方形，而 $PQVU$ 及 $SRWT$ 都是梯形，其中 $PQ \parallel UV$ 及 $\angle PQV = 90^\circ$ 。已知 $PQ = 24\text{ cm}$, $PU = 13\text{ cm}$ 及 $TW = 29\text{ cm}$ 。

- (a) Find the volume of the container.
 求該容器的容量。

(3 marks) (3 分)

- (b) The base $PQRS$ of the container is placed on the surface. Put a 16 cm tall cone to the container perpendicularly, the base of the cone is exactly put to the container base $PQRS$. Now the container is filled up with water.

該容器的底 $PQRS$ 置於一水平面上。把一個高 16 cm 的實心直立圓錐體鉛垂置於該容器內，圓錐體的底剛好放進該容器的底 $PQRS$ 內。現把該容器注滿水。

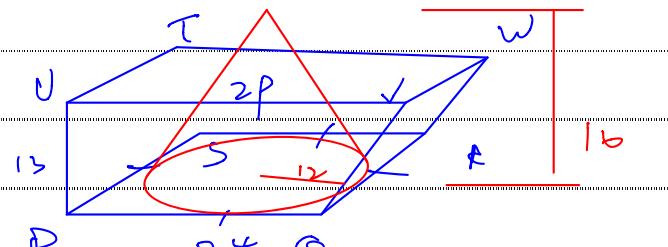
- (i) Find the wet surface area of the cone. Express the answer in terms of π .
 求圓錐體被浸濕的曲面面積，答案以 π 表示。

- (ii) Find the volume of the water in the container, correct to the nearest cm^3 .
 求該容器內水的體積，準確至最接近的 cm^3 。

(6 marks) (6 分)

a) $V_{\text{容器}} = \frac{(2P + 24)13 \times 24}{2} \quad (M)$

$= 8268 \text{ cm}^3 \quad (A)$



b):

Wet Area

$$= \pi \times 12 \sqrt{16^2 + 12^2} \left(1 - \left(\frac{16-13}{16} \right)^2 \right) \quad (M + M)$$

$$= 3705 \pi \text{ cm}^2 \quad (A)$$

ii)

$V_{\text{水}}$

$$= 8268 - \frac{1}{3} \pi \times 12^2 \times 16 \left(1 - \left(\frac{16-13}{16} \right)^3 \right) \quad (M + M)$$

$$= 5871.16 \dots$$

$$\approx 5870 \text{ cm}^3 \quad (R)$$

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11. The equation of the straight line L is $\sqrt{3}x - y = 3$. P is a moving point which maintains an equal distance from L and the x -axis.

直線 L 的方程是 $\sqrt{3}x - y = 3$ 。 P 是一動點使其與 L 和 x -軸保持等距。

- (a) Describe the geometric relationship between locus of P and angles formed by L and the x -axis.
描述 P 點的軌跡與 L 和 x -軸所成的角的幾何關係。
- (b) Find the equation of the locus of P .
求 P 點的軌跡方程。

a) angle bisector 角平分線 1A

(5 marks) (5 分)

b) slope of $L = \sqrt{3}$

angle of inclination of $L = \tan^{-1}(\sqrt{3}) = 60^\circ$
 L 的 仰角

angle of inclination of locus of $P = 30^\circ$
 P 的軌跡的 仰角

slope of locus of $P = \tan 30^\circ = \frac{\sqrt{3}}{3}$

$$\begin{cases} \sqrt{3}x - y = 3 \\ y = 0 \end{cases}$$

1M

1M

$x = \sqrt{3}, y = 0$

$$y - 0 = \tan\left(\frac{1}{2}\tan^{-1}\sqrt{3}\right)(x - \sqrt{3}) \quad 1M$$

T: $y = \frac{\sqrt{3}}{3}x - 1$ 1A

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12. Two hundred students took a test in Mathematics. The table shows the cumulative frequency and the frequency of the distribution of the test scores.

二百名學生參加一數學測驗，圖表顯示這測驗得分分佈的累積頻數及頻數。

Test scores 測驗得分 (x)	Cumulative frequency 累積頻數	Test scores 測驗得分 (x)	Frequency 頻數
$x \leq 50$	a	$40 < x \leq 50$	20
$x \leq 60$	77	$50 < x \leq 60$	29
$x \leq 70$	123	$60 < x \leq 70$	d
$x \leq 80$	b	$70 < x \leq 80$	e
$x \leq 90$	188	$80 < x \leq 90$	20
$x \leq 100$	c	$90 < x \leq 100$	f

< 28

- (a) Find求 d, e, f
 (b) Find the mean, mode, and median of the data.
 求數據的平均值、眾數及中位數。
 (c) If the passing score is 55, estimate the passing percentage of the students in the test.
 若該測驗的及格分數是55，估計這些學生的及格百分率。

(8 marks) (8 分)

a) $d = 123 - 77 = 46$
 $c = b - 123 = 188 - 77 = 111$
 $f = 200 - 188 = 12$

b) mean = $\frac{20 \cdot 28 + 45 \cdot 29 + 55 \cdot 20 + 65 \cdot 46 + 75 \cdot 45 + 85 \cdot 20 + 95 \cdot 12}{200}$ 1M + 1A
 $= 61.3$ 1A

mode = 65 or $(60 - 70)$ 1A

median = 1A

c) % passing = $\left(1 - \frac{28 + 29 + 20}{200}\right) \times 100\%$ 1M
 $= 68.75\%$ 1A

Section B (30 marks)

乙部 (30 分)

13. (a) Express the root(s) of $x^2 + x + 1 = 0$ in the form of $a + bi$, where a and b are real numbers.

把方程 $x^2 + x + 1 = 0$ 的根表成 $a + bi$ 的形式，其中 a 及 b 均為實數。

- (b) (i) Find the value of p and q such that the roots of equation $2x^2 + px + q = 0$ are

$$\frac{2}{a+bi} \text{ and } \frac{2}{a-bi}.$$

求 p 與 q 的值使二次方程 $2x^2 + px + q = 0$ 的根為 $\frac{2}{a+bi}$ 及 $\frac{2}{a-bi}$ 。

- (ii) By using the method of completing the square, find the coordinates of vertex of the graph of $y = 2x^2 + px + q$.

利用配方法，求 $y = 2x^2 + px + q$ 的圖像的頂點坐標。

(7 marks)(7 分)

a)
$$x = \frac{-1 \pm \sqrt{1^2 - 4(0)(1)}}{2(1)} \quad |M$$

$$= \frac{-1 \pm \sqrt{3}}{2}; \quad |A$$

b) (i) $a = \frac{1}{2}, b = \sqrt{3}$

$$\frac{2}{a+bi} = \frac{2}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{2(a-bi)}{a^2+b^2} = -1-\sqrt{3}i \quad |M$$

$$\frac{2}{a-bi} = -1+\sqrt{3}i$$

$$-P = (-1-\sqrt{3}i) + (-1+\sqrt{3}i) - P = 4$$

$$\frac{q-P}{2} = (-1-\sqrt{3}i)(-1+\sqrt{3}i), \quad q = 8 \quad |A$$

(ii) $y = 2x^2 + 4x + 8$

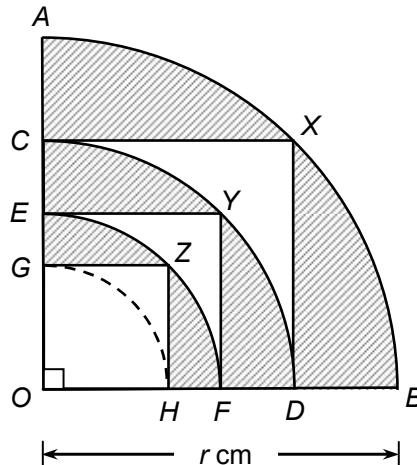
$$= 2(x^2 + 2x + 1) + 8 - 2 \quad |M$$

$$= 2(x+1)^2 + 7$$

$$\therefore \text{Vertex} = (-1, 7) \quad |A$$

14. In the figure, $OCXD$ is a square inscribed in the sector OAB with centre O and $\angle AOB = 90^\circ$. CD is then joined to form another sector OCD and another square $OEYF$ is inscribed in the sector, and this process is continued to form an infinite number of sectors and inscribed squares. Suppose the radius of sector OAB is r cm.

在圖中，正方形 $OCXD$ 內接於以 O 為圓心的扇形 OAB ，其中 $\angle AOB = 90^\circ$ 。連接 CD 後形成另一個扇形 OCD ，且另一個正方形 $OEYF$ 內接於該扇形。在不斷重複這個過程下，會形成無限個扇形和內接正方形。假設扇形 OAB 的半徑是 r cm。



- (a) (i) Find the radii of sectors OCD and OEF in terms of r .
試以 r 表示扇形 OCD 和 OEF 的半徑。
(ii) Hence, show that the radii of the sectors are in geometric sequence, and find the common ratio.
由此，證明扇形的半徑形成一個等比數列，並求其公比。
(b) If $r = 12$, find the sum of the areas of all the shaded regions in terms of π .
若 $r = 12$ ，求所有陰影部分的面積之和，答案以 π 表示。

(7 marks)

a) (i) $R_{OCD} = OD = OC$

$$OD^2 + OC^2 = DK^2 = r^2$$

$$R_{OCD} = \frac{1}{\sqrt{2}}r = \frac{\sqrt{2}}{2}r$$

$$R_{OEF} = OE = \frac{\sqrt{3}}{2}r = \frac{1}{2}\sqrt{3}r$$

$$\frac{OZ}{OY} = \frac{\frac{1}{2}r}{\frac{\sqrt{3}}{2}r} = \frac{1}{2} = \frac{\sqrt{2}}{2}$$

$$\frac{OY}{DX} = \frac{\frac{\sqrt{2}}{2}r}{\frac{1}{2}r} = \frac{\sqrt{2}}{2} = \frac{DZ}{OY}$$

$$\therefore \text{is G.S., Common rat. } = \frac{\sqrt{2}}{2}$$

b) Sum Area

$$= (\frac{1}{4}\pi r^2 - OD^2) / \left(-\left(\frac{\sqrt{2}}{2}\right)^2\right) \quad |A + M$$

$$= 72\pi - 144 \text{ cm}^2$$

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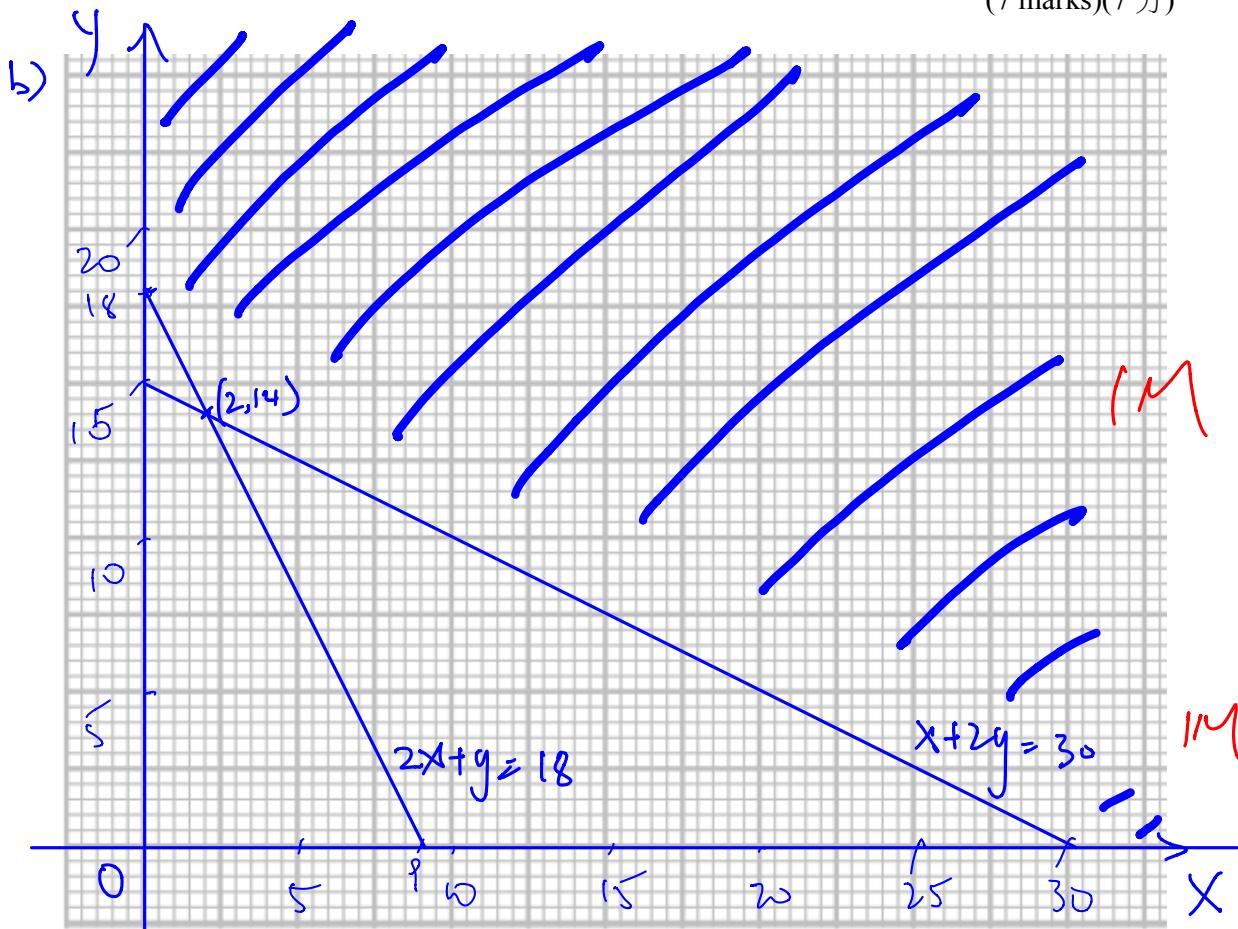
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15. A cafe serves set A and set B for desserts. Each unit of set A contains 20 g of chocolate and 100 g of cheese and costs \$25. Each unit of set B contains 40 g of chocolate and 50 g of cheese and costs \$30. It is found that at least 0.6 kg of chocolate and 0.9 kg of cheese are used per day. Let x and y be the numbers of units of set A and set B to be prepared each day.
- 一間咖啡店提供套餐 A 及套餐 B 的甜品。每個的套餐 A 有 20 g 的朱古力及 100 g 的芝士且其成本為 \$25。每個的套餐 B 有 40 g 的朱古力及 50 g 的芝士且其成本為 \$30。已知該咖啡店每天使用最少 0.6 kg 的朱古力及 0.9 kg 的芝士。

設 x 及 y 分別每天準備的套餐 A 及 B 的數目。

- Write down all the constraints for x and y . 以 x 及 y 寫出符合題設的約束條件。
- Draw and shade the region that satisfies all the constraints in (a). 繪畫表示 (a) 中約束條件的可行解區域，並塗上陰影。
- Find the minimum cost and the corresponding values of x and y . 求最低成本，並求出相關的 x 及 y 值。

(7 marks)(7 分)



a)

$$\left\{ \begin{array}{l} 20x + 40y \geq 600 \\ 100x + 50y \geq 900 \\ x \geq 0 \\ y \geq 0 \end{array} \right.$$

x, y are integers 整數 + 14

Vertices are A(6,18), B(2,14), C(30,0) 1A

Cost $P = 25x + 30y$

$$P_A = 25(6) + 30(18) = 540$$

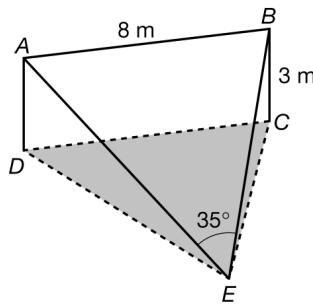
$$P_B = 25(2) + 30(14) = 470 \quad 1M$$

$$P_C = 25(30) + 30(0) = 750$$

$\therefore \min \text{ Cost} = \$470 \text{ when } x=2, y=14$ 1B

16. In the figure, a rectangular wall $ABCD$ with dimensions $8 \text{ m} \times 3 \text{ m}$ stands vertically on the horizontal ground. A piece of plate ABE is in the shape of an isosceles triangle, where $AE = BE$. The plate is placed with one side AB on the top of the wall and E is a point on the ground. The rays of sunlight shine from the sky vertically above. Find

圖中，一大小為 $8 \text{ m} \times 3 \text{ m}$ 的牆壁 $ABCD$ 直立於水平面上。 ABE 為一等腰三角形的膠板，其中 $AE = BE$ 。膠板的邊 AB 緊貼着牆壁的頂部，而 E 為地面上的一點。陽光從正上方照射下來。求



- (a) the length of AE ,
 AE 的長度；
- (b) the angle between the plate and the horizontal plane,
膠板與水平面的交角；
- (c) the area of the shadow.
影子的面積。

(9 marks) (9 分)

a) $\frac{AB}{2} / AE = \sin(\angle AEB / 2)$ 1M

$AB = 13.30203809$
 $\approx 13.3 \text{ m}$ 1A

b) let J on AB such that $EJ \perp AB$
 $EJ / AB = \frac{1}{2} AE^2 \sin 35^\circ$ 1M

$EJ = 12.68637921$

let K on DC , $GK \perp DC$

the angle requires is $\angle JEK$ 1M
 求的角為

$\sin \angle JEG = \frac{JK}{JE}$ 1M

$\angle JEG = 13.67853^\circ \approx 13.7^\circ$, 1A

c) Area of shadow

$$= \frac{1}{2} DC \cdot KG$$

1M

$$= \frac{1}{2} \cdot 8 \cdot \sqrt{JB^2 - JK^2}$$

1M

$$= 49.306 \dots$$

$$\approx 49.3 m^2,$$

1A