Propositional Logic

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

logical equivalent

"for all cases, truth values are the same"

$$\neg p \to (q \to r) \equiv q \to (p \vee r)$$

* you can start wherever

$$\underbrace{\neg p}_{p} \to \underbrace{(q \to r)}_{q} \equiv \cancel{\neg p \to \neg p \wedge r}$$

#11a

$$\equiv \neg\neg p \vee (q \to r)$$

#7a

$$\equiv p \vee (q \to r)$$

#11a

$$\equiv p \vee (\neg q \vee r)$$

$$\equiv (p \vee \neg q) \vee r$$

#2a

$$\equiv (\neg q \vee p) \vee r$$

#3a

#2a $\equiv \neg q \vee (p \vee r)$

#11a $\equiv q \rightarrow (p \vee r)$

$$\boxed{\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)}$$

Predicate Logic

- define the U universe

universal $\forall x \; P(x)$

existential $\exists x \; P(x)$

Ex. 5 domain = set of students @ uni)

$E(x)$ = x is enrolled in class

$T(x)$ = x took the test

a) Someone who is enrolled in the class took the test

$$\exists x (E(x) \wedge T(x))$$

b) all students enrolled in the class took the test

$$\cancel{\forall x (E(x) \wedge T(x))}$$

$$\forall x (E(x) \rightarrow T(x))$$

not all students have to be in class + take test

"of all students, the ones in the class enrolled took the test"

c) Everyone who took the test is enrolled in the class.

$$\forall x \left(T(x) \rightarrow E(x) \right)$$

d) At least one Student who is enrolled in the class did not take the test.

$$\exists x \left(E(x) \wedge \neg T(x) \right)$$

<u>Rule of thumb</u>: $\exists x$ most likely has & $\wedge$

$\forall x$ most likely implies $\rightarrow$

<u>Ex. 6</u> - Domain = (set of real numbers)

True or False?

$\forall$ wants $\rightarrow$ false
CAME
$\exists$ wants $\rightarrow$ true

a) $\forall x \; \exists y \; (x + y = 0)$

~~false~~ $\boxed{\text{True}}$

↳ this is universal statement

* order of quantifiers matter

"for all x" "this needs to be true)

x $\quad$ $P(x)$

$x = 1$: $\exists y \; (1 + y = 0) \Rightarrow T$

$x = 7$: $\exists y \; (7 + y = 0) \Rightarrow T$

b) $\exists_x \underbrace{\forall_y (x + y = 0)}_{P(x)}$

"there exists one x ... thats all we need

$x = 1$ $\quad \forall_y (1 + y = 0) \Rightarrow F$

$x = 7$ $\quad \forall_y (7 + y = 0) \Rightarrow F$

$\boxed{\text{False}}$

c) $\exists_x \underbrace{\forall_y (xy = y)}_{P(x)}$

"There exists an x in which all y is true."

$x = 1$ $\quad \forall_y (1y = y) \Rightarrow T$ ✓

$x = 2$ $\quad \forall_y (2y = y) \Rightarrow F$

$\boxed{\text{True}}$