

Variant Calling

Michael Schatz

October 3, 2022

Lecture 10: Applied Comparative Genomics



THE NOBEL PRIZE IN PHYSIOLOGY OR MEDICINE 2022

Illustration: Niklas Elmehed



Svante Pääbo

"for his discoveries concerning the genomes
of extinct hominins and human evolution"

THE NOBEL ASSEMBLY AT KAROLINSKA INSTITUTET



NovaSeq X Plus

Performance parameters*	1.5B flow cell*	10B flow cell*	25B flow cell*
Max output per run†	165 Gb–1 Tb	1–6 Tb	8–16 Tb
Single reads per run†	1.6–3.2 billion	10–20 billion	26–52 billion
Paired-end reads per run†	3.2–6.4 billion	20–40 billion	52–104 billion
Max read length	2 × 150 bp	2 × 150 bp	2 × 150 bp
Run time	~13–21 hr	~18–24 hr	~48 hr

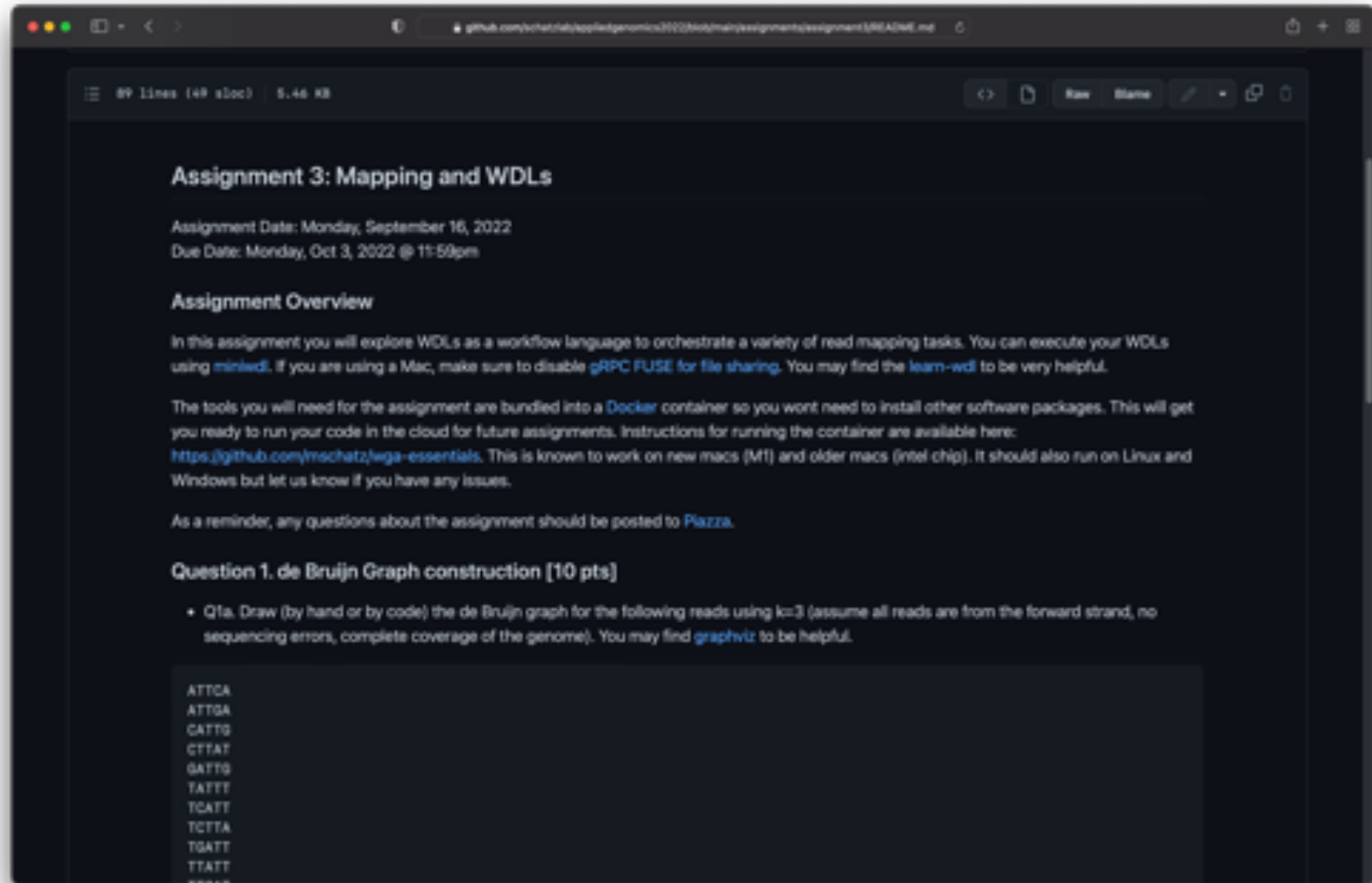
* NovaSeq X Plus system will be launched in Q1 2023. NovaSeq X system available later in 2023. 10B flow cell available Q1 2023. 1.5B and 25B flow cells available H2 2023. Performance metrics are subject to change.

† Highest output possible with dual flow cell runs on the NovaSeq X Plus system. The NovaSeq X Plus system is capable of single flow cell runs or dual flow cell runs. NovaSeq X system is capable of single flow cell runs.

Illumina says its NovaSeq X machine will get the price of sequencing down to \$200 per human genome.

Assignment 3: Mapping and WDL

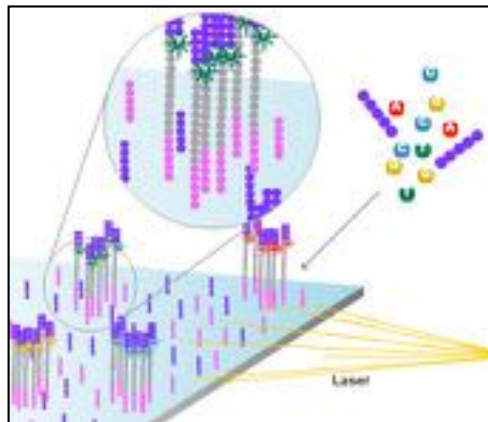
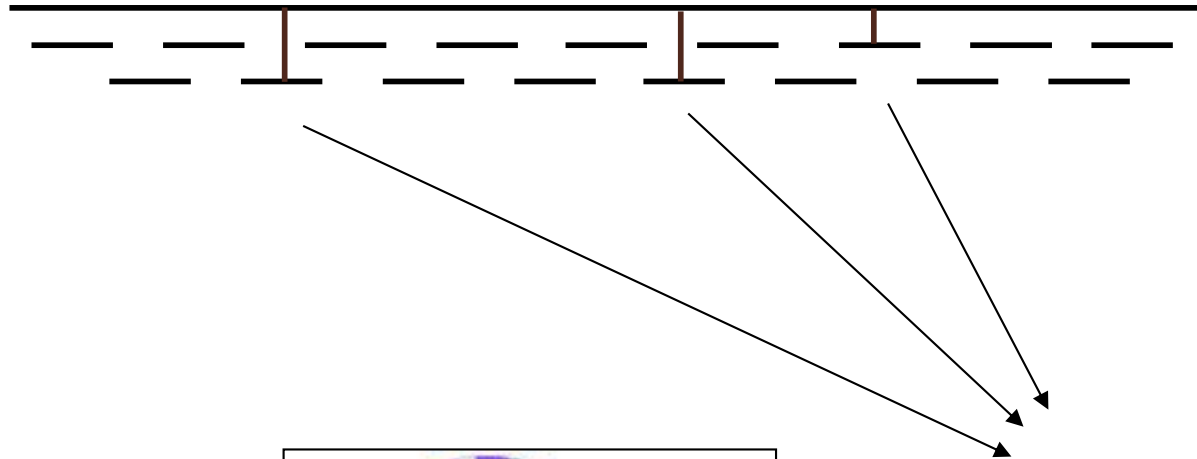
Due Monday Oct 3 by 11:59pm



<https://github.com/schatzlab/appliedgenomics2022/tree/main/assignments/assignment3>
Check Piazza for questions!

Personal Genomics

How does your genome compare to the reference?



Heart Disease

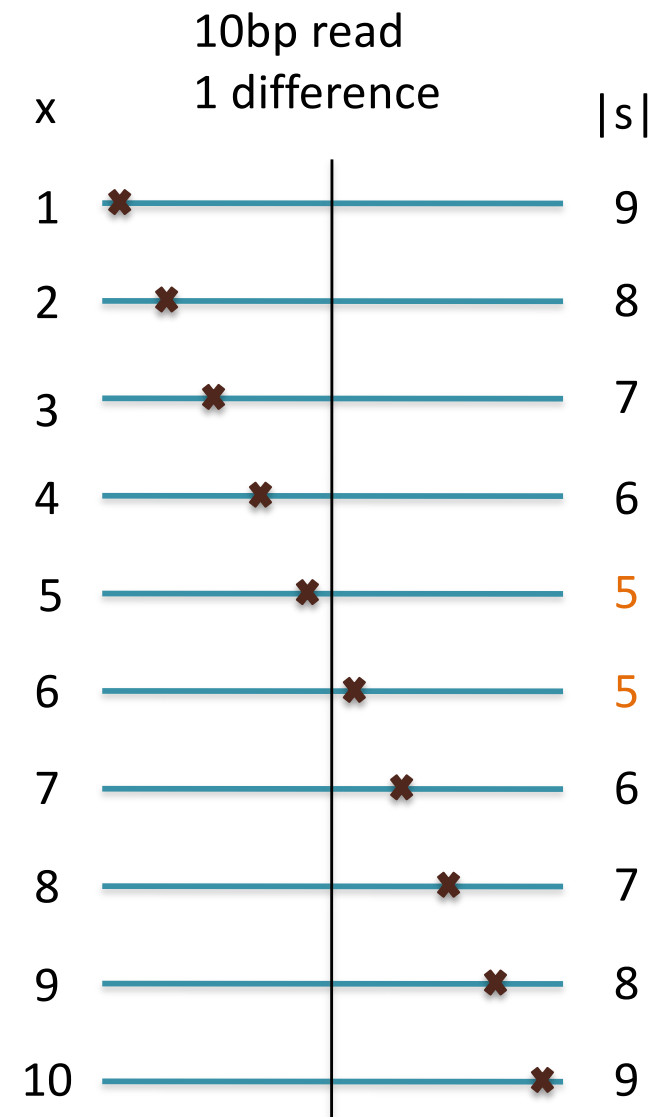
Cancer

Presidential smile

Seed-and-Extend Alignment

Theorem: An alignment of a sequence of length m with at most k differences **must** contain an exact match at least $s=m/(k+1)$ bp long
(Baeza-Yates and Perleberg, 1996)

- Proof: Pigeonhole principle
 - 1 pigeon can't fill 2 holes
- Seed-and-extend search
 - Use an index to rapidly find short exact alignments to seed longer in-exact alignments
 - BLAST, MUMmer, Bowtie, BWA, SOAP, ...
 - Specificity of the depends on seed length
 - Guaranteed sensitivity for k differences
 - Also finds some (but not all) lower quality alignments <- heuristic



Exact Matching Review & Overview

Where is GATTACA in the human genome?

Brute Force
(3 GB)

BANANA
BAN
ANA
NAN
ANA

$O(m * n)$

Slow & Easy

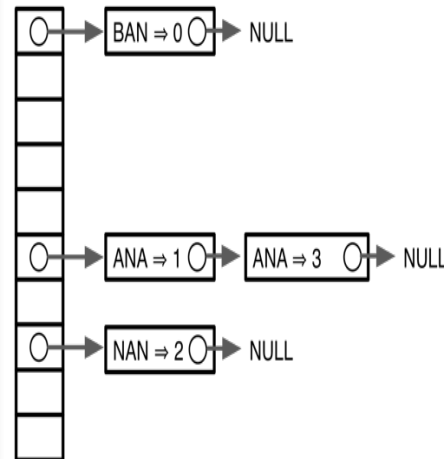
Suffix Array
(>15 GB)

6	\$
5	A\$
3	ANA\$
1	ANANA\$
0	BANANA\$
4	NA\$
2	NANA\$

$O(m + \lg n)$

Full-text index

Hash Table
(>15 GB)



$O(1)$

Fixed-length lookup

BWT
(3 GB)

BANANA\$
=>
\$BANANA**A**
A\$BANAN**N**
ANA\$BAN**N**
ANANA\$**B**
BANANA\$
NA\$BANA**A**
NANA\$BA**A**
=>
ANNB\$AA

$O(m)$

Full-text and concise

*** These are general techniques applicable to any text search problem ***

Burrows-Wheeler Transform

- Recreating T from BWT(T)
 - Start in the first row and apply **LF** repeatedly, accumulating predecessors along the way



[Decode this BWT string: ACTGA\$TTA]

Run Length Encoding

ref[614]:

It_was_the_best_of_times,_it_was_the_worst_of_times,_it_was_the_age_of_wisdom,_it_was_the_age_of_foolishness,_it_was_the_epoch_of_belief,_it_was_the_epoch_of_incredulity,_it_was_the_season_of_Light,_it_wa_s_the_season_of_Darkness,_it_was_the_spring_of_hope,_it_was_the_wint_er_of_despair,_we_had_everything_before_us,_we_had_nothing_before_us,_we_were_all_going_direct_to_Heaven,_we_were_all_going_direct_the_o ther_way_-_in_short,_the_period_was_so_far_like_the_present_period,_that_some_of_its_noisiest_authorities_insisted_on_its_being_received,_for_good_or_for_evil,_in_the_superlative_degree_of_comparison_only.\$

rle(bwt)[464]:

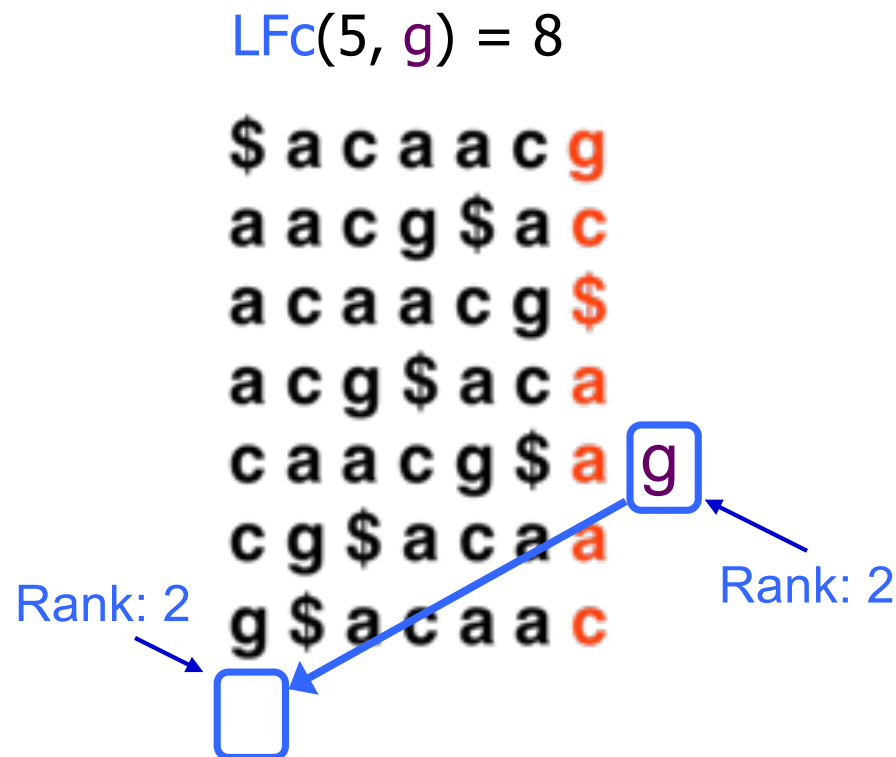
.dlms2ftysesdtrsns_y_2\$_yfofe4tg2sfefefg2e2drofr,l2re2f-,fs,9nfrsdn2 hereghet2edndete2ge2nste2,s5t,es3ns2f2te2dt10r,4e3feh2_2p_2fpDw11e2h l_ew_5eo2_ne3oa2eo2_4seph2r2hvh2w2egmgh7kr2w2h2s2Hr3vtr2ib2dbcbvs_2t hw2p3vm2irdn2ib_2eo12_4e2n6a2i_3ec2_2t18s_tsgltsLlvt2_3h2o2re_wr2ad2 wlors_9r_2lteiril2re_oua2no2i2oeo4i3hki6o_2ieitsp2ioi_12g2nodsc_s3_g fhf_f3hwh_nsmo_2ue2_sio3ae4o2_i2cgp2e2aoaeo2e2s2eu2tet11i_2ei_in_2a 2ie_e3rei

Saved 614-464 = 150 bytes (24%) with zero loss of information!

Common to save 50% to 90% on real world files with bzip2

BWT Exact Matching

- **LFc**(r, c) does the same thing as **LF**(r) but it ignores r's actual final character and “pretends” it's c:

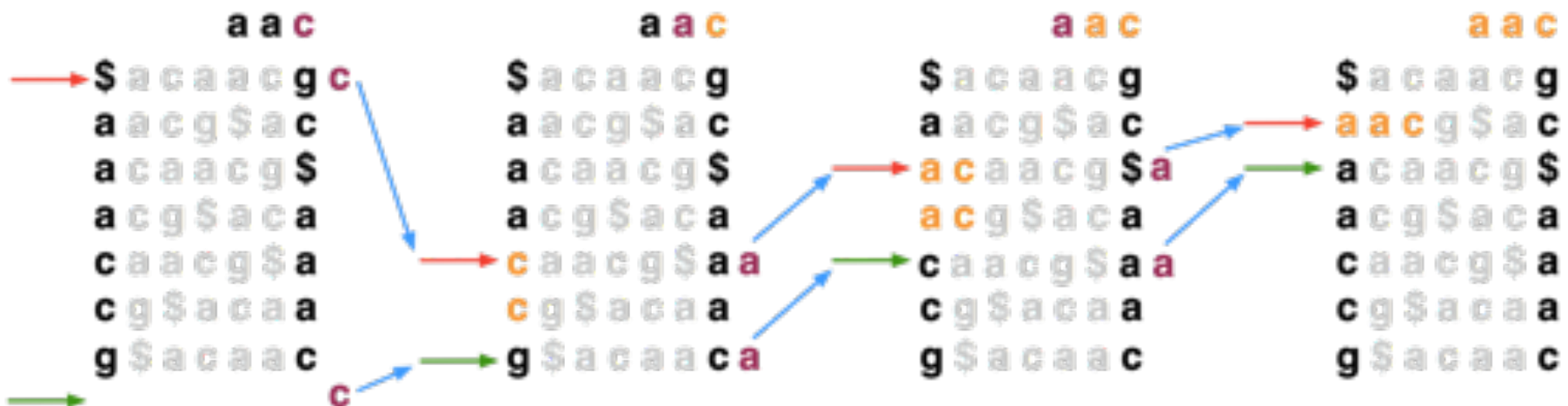


BWT Exact Matching

- Start with a range, (**top**, **bot**) encompassing all rows and repeatedly apply **LFc**:

top = **LFc**(**top**, **qc**); **bot** = **LFc**(**bot**, **qc**)

qc = the next character to the left in the query

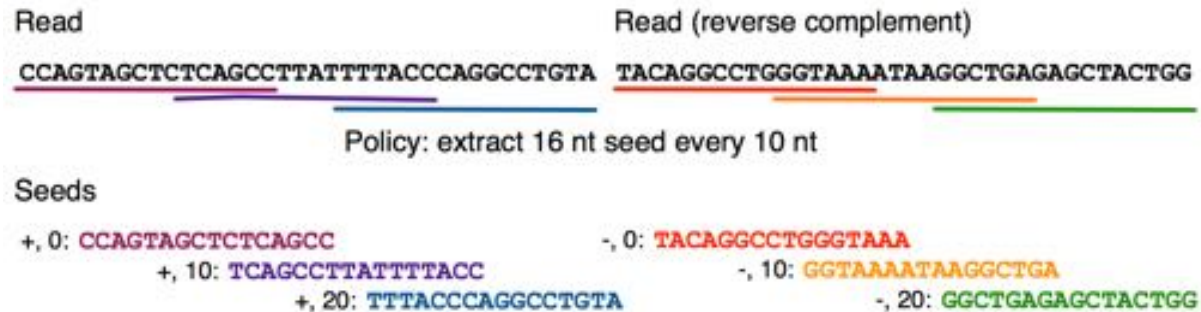


Ferragina P, Manzini G: Opportunistic data structures with applications. *FOCS. IEEE Computer Society; 2000.*

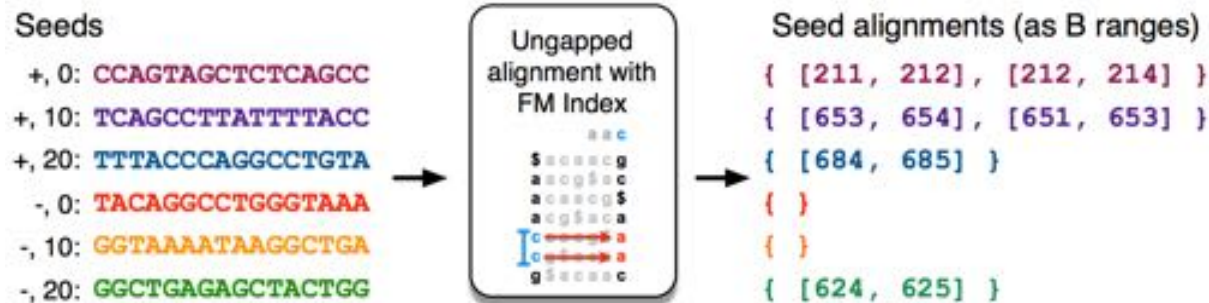
[Search for TTA this BWT string: ACTGA\$TTA]

Algorithm Overview

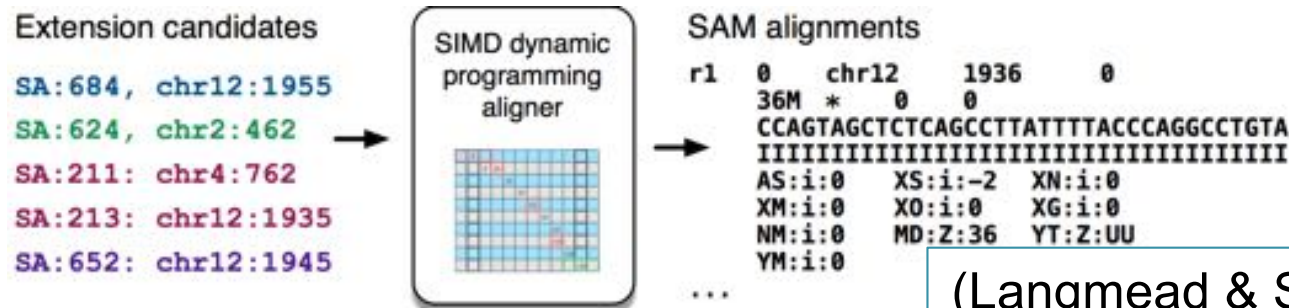
1. Split read into segments



2. Lookup each segment and prioritize



3. Evaluate end-to-end match



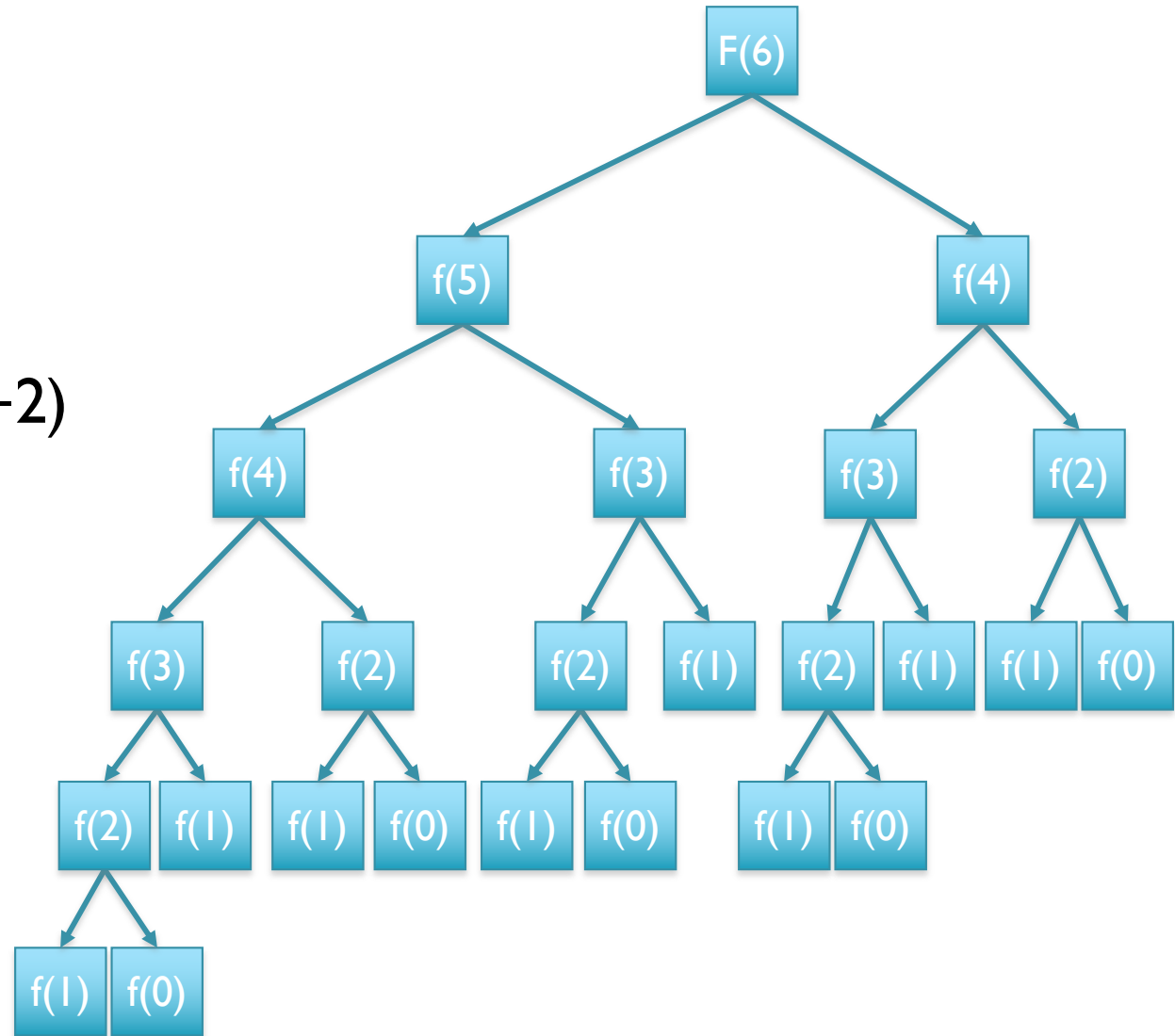
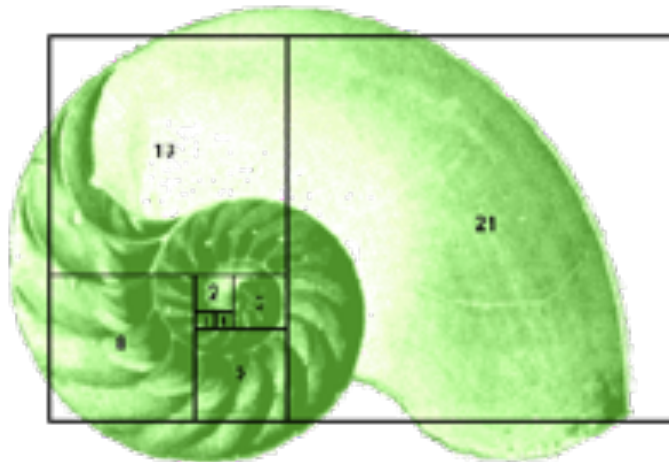
(Langmead & Salzberg, 2012)



Part 2: Dynamic Programming

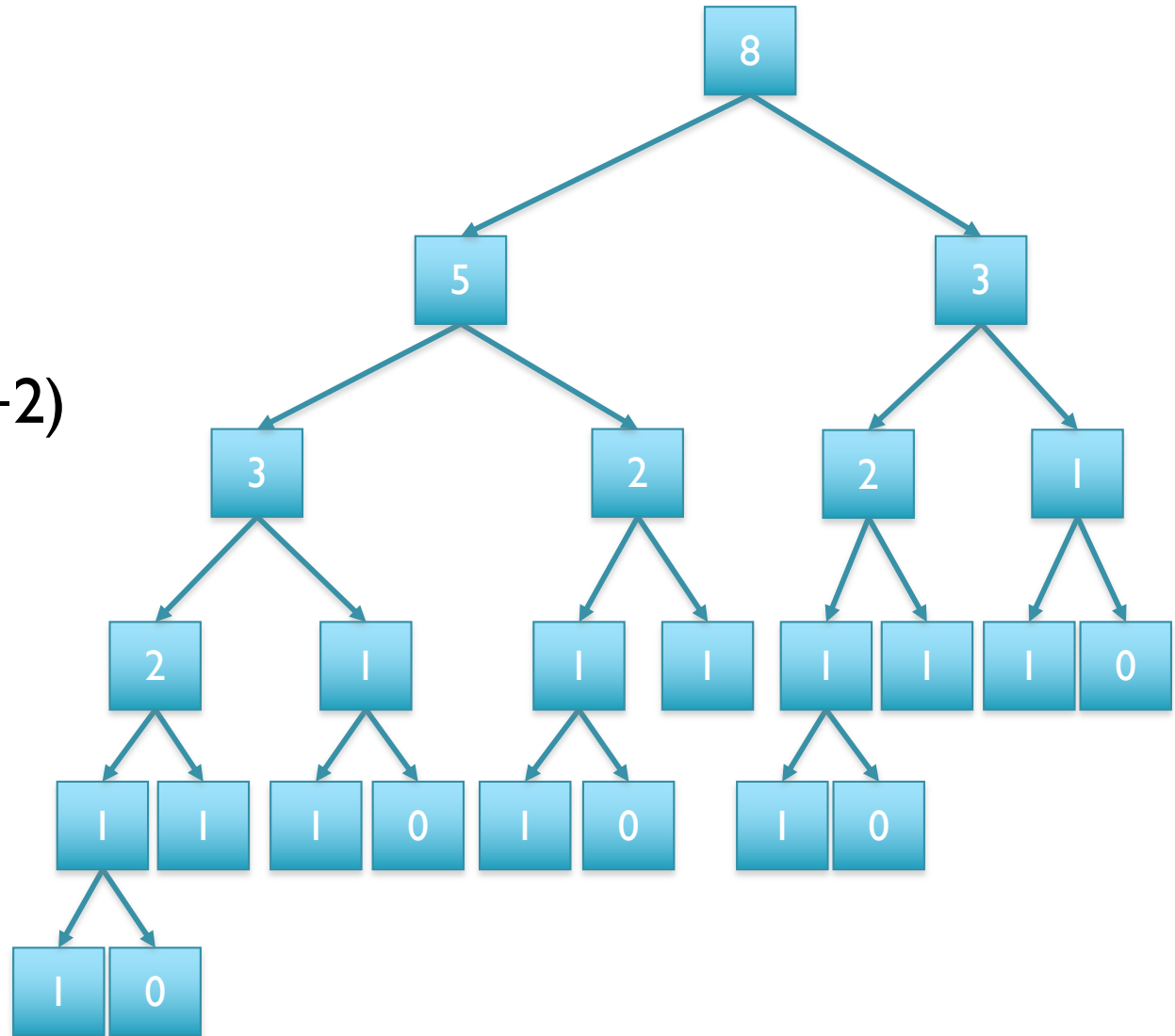
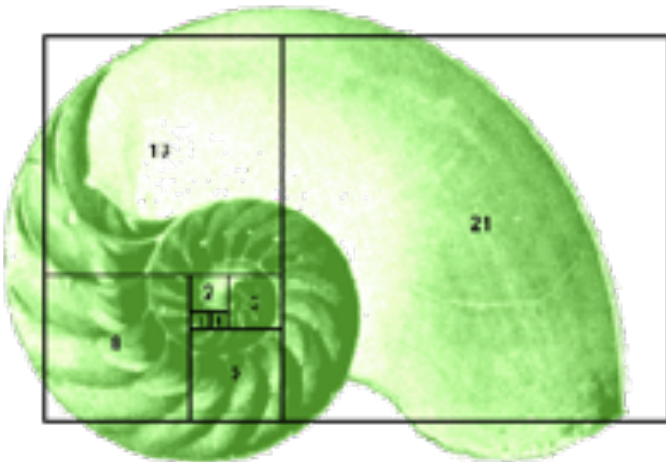
Fibonacci Sequence

```
def fib(n):  
    if n == 0 or n == 1:  
        return n  
    else:  
        return fib(n-1) + fib(n-2)
```



Fibonacci Sequence

```
def fib(n):  
    if n == 0 or n == 1:  
        return n  
    else:  
        return fib(n-1) + fib(n-2)
```



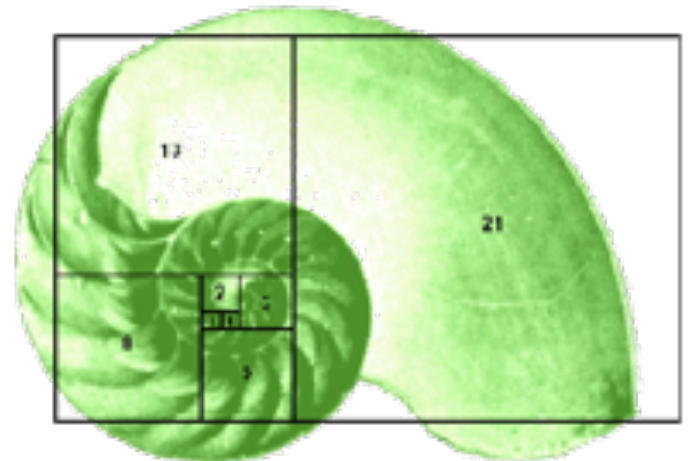
[What is the running time?]

Bottom-up Fibonacci Sequence

```
def fib(n):  
    table = [0] * (n+1)  
    table[0] = 0  
    table[1] = 1  
    for i in range(2,n+1):  
        table[i] = table[i-2] + table[i-1]  
    return table[n]
```

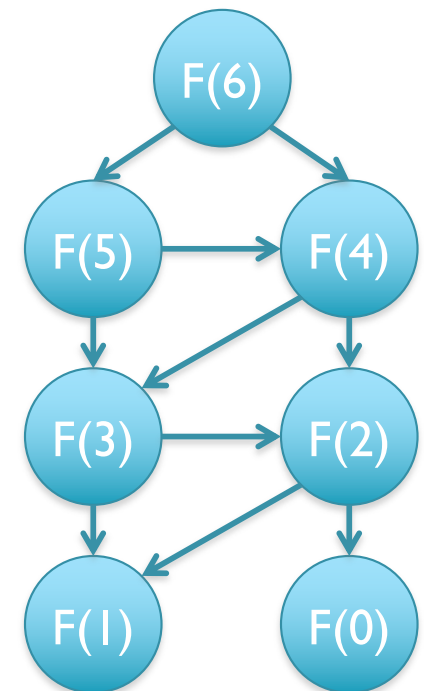
0	1	2	3	4	5	6
0	1	1	2	3	5	8

[What is the running time?]



Dynamic Programming

- General approach for solving (some) complex problems
 - When applicable, the method takes far less time than naive methods.
 - Polynomial time ($O(n)$ or $O(n^2)$) instead of exponential time ($O(2^n)$ or $O(3^n)$)
- Requirements:
 - **Overlapping subproblems**
 - **Optimal substructure**
- Applications:
 - Fibonacci
 - Longest Increasing Subsequence
 - Sequence alignment, Dynamic Time Warp, Viterbi
- Not applicable:
 - Traveling salesman problem, Clique finding, Subgraph isomorphism, ...
 - The cheapest flight from airport A to airport B involves a single connection through airport C, but the cheapest flight from airport A to airport C involves a connection through some other airport D.



In-exact alignment

- Where is GATTACA *approximately* in the human genome?
 - And how do we efficiently find them?
- It depends...
 - Define 'approximately'
 - Hamming Distance, Edit distance, or Sequence Similarity
 - Ungapped vs Gapped vs Affine Gaps
 - Global vs Local
 - All positions or the single 'best'?
 - Efficiency depends on the data characteristics & goals
 - Smith-Waterman: Exhaustive search for optimal alignments
 - BLAST: Hash-table based homology searches
 - Bowtie: BWT alignment for short read mapping

Similarity metrics

- Hamming distance

- Count the number of substitutions to transform one string into another

MIKESCHATZ

| | x | | x x x x |

MICESHATZZ

5

- Edit distance

- The minimum number of substitutions, insertions, or deletions to transform one string into another

MIKESCHAT-Z

| | x | | x | | | x |

MICES-HATZZ

3

Edit Distance Example

AGCACACA → ACACACTA in 4 steps

AGCACACA → (1. change G to C)

ACCACACA → (2. delete C)

ACACACA → (3. change A to T)

ACACACT → (4. insert A after T)

ACACACTA → done

[Is this the best we can do?]

Edit Distance Example

AGCACACA → ACACACTA in 3 steps

AGCACACA → (1. change G to C)

ACCACACA → (2. delete C)

ACACACA → (3. insert T after 3rd C)

ACACACTA → done

[Is this the best we can do?]

Reverse Engineering Edit Distance

$$D(\text{AGCACACA}, \text{ACACACTA}) = ?$$

Imagine we already have the optimal alignment of the strings, the last column can only be 1 of 3 options:

...M	...I	...D
...A	...-	...A
...A	...A	...-

The optimal alignment of last two columns is then 1 of 9 possibilities

...MM	...IM	...DM	...MI	...II	...DI	...MD	...ID	...DD
...CA	...-A	...CA	...A-	...--	...A-	...CA	...-A	...CA
...TA	...TA	...-A	...TA	...TA	...-A	...A-	...A-	...--

The optimal alignment of the last three columns is then 1 of 27 possibilities...

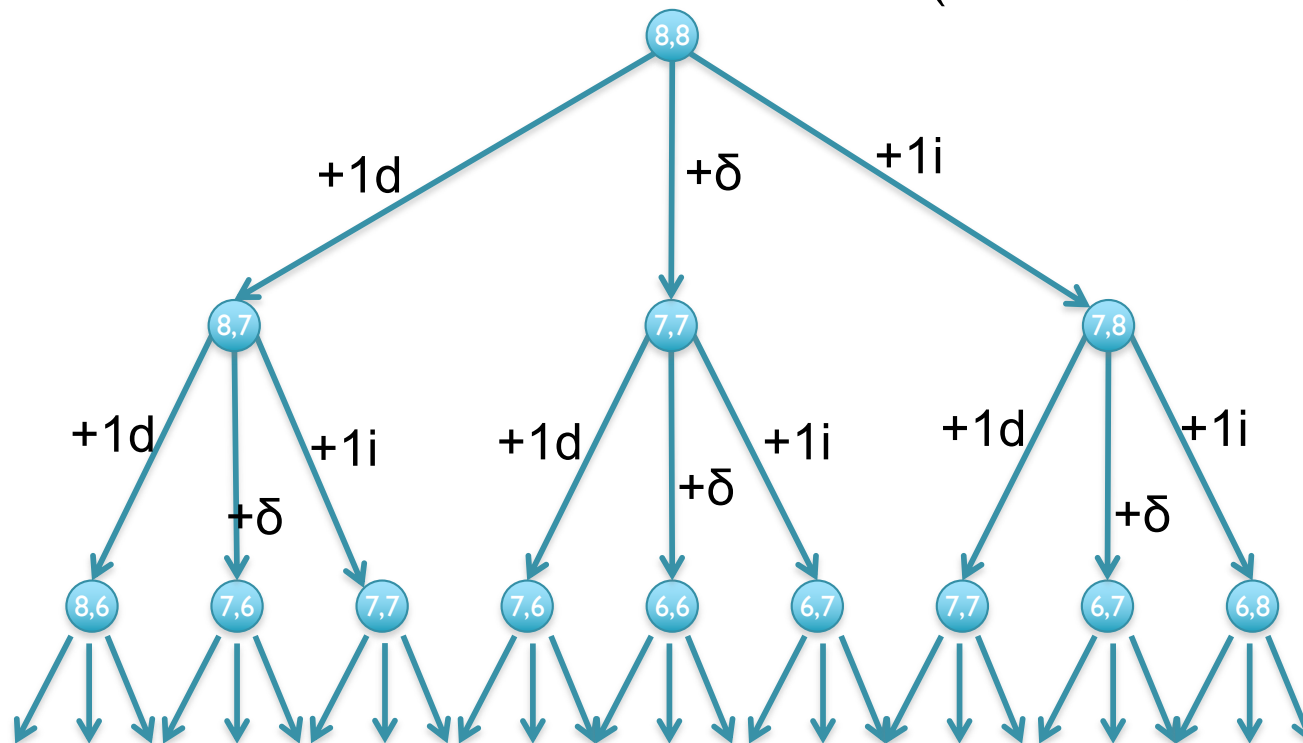
...M...	...I...	...D...
...X...	...-...	...X...
...Y...	...Y...	...-...

Eventually spell out every possible sequence of {I,M,D}

Recursive solution

- Computation of D is a recursive process.
 - At each step, we only allow matches, substitutions, and indels
 - $D(i,j)$ in terms of $D(i',j')$ for $i' \leq i$ and $j' \leq j$.

$$D(\text{AGCACACA}, \text{ACACACTA}) = \min\{D(\text{AGCACACA}, \text{ACACACT}) + 1, \\ D(\text{AGCACAC}, \text{ACACACTA}) + 1, \\ D(\text{AGCACAC}, \text{ACACACT}) + \delta(\text{A}, \text{A})\}$$



[What is the running time?]

Dynamic Programming

- We could code this as a recursive function call...
...with an exponential number of function evaluations
- There are only $(n+1) \times (m+1)$ pairs i and j
 - We are evaluating $D(i,j)$ multiple times
- Compute $D(i,j)$ bottom up.
 - Start with smallest $(i,j) = (1,1)$.
 - Store the intermediate results in a table.
 - Compute $D(i,j)$ *after* $D(i-1,j)$, $D(i,j-1)$, and $D(i-1,j-1)$

Recurrence Relation for D

Find the edit distance (minimum number of operations to convert one string into another) in $O(mn)$ time

- Base conditions:

- $D(i,0) = i$, for all $i = 0, \dots, n$
- $D(0,j) = j$, for all $j = 0, \dots, m$

- For $i > 0, j > 0$:

$$D(i,j) = \min \left\{ \begin{array}{ll} D(i-1,j) + 1, & // \text{align } 0 \text{ chars from } S, 1 \text{ from } T \\ D(i,j-1) + 1, & // \text{align } 1 \text{ chars from } S, 0 \text{ from } T \\ D(i-1,j-1) + \delta(S(i),T(j)) & // \text{align } 1+1 \text{ chars} \end{array} \right\}$$

[Why do we want the min?]

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	0	1	2	3	4	5	6	7	8
A	1								
G	2								
C	3								
A	4								
C	5								
A	6								
C	7								
A	8								

[What does the initialization mean?]

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	0	1	2	3	4	5	6	7	8
A	1	0							
G	2								
C	3								
A	4								
C	5								
A	6								
C	7								
A	8								

$$D[A,A] = \min\{D[A,]+1, D[,A]+1, D[,]+ \delta(A,A)\}$$

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	0	1	2	3	4	5	6	7	8
A	1	0	1						
G	2								
C	3								
A	4								
C	5								
A	6								
C	7								
A	8								

$$D[A,AC] = \min\{D[A,A]+1, D[,AC]+1, D[,A]+\delta(A,C)\}$$

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	0	1	2	3	4	5	6	7	8
A	1	0	1	2					
G	2								
C	3								
A	4								
C	5								
A	6								
C	7								
A	8								

$$D[A,ACA] = \min\{D[A,AC]+1, D[,ACA]+1, D[,AC]+\delta(A,A)\}$$

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
A	1	0	1	2	3	4	5	6	<u>7</u>
G	2								
C	3								
A	4								
C	5								
A	6								
C	7								
A	8								

$$D[A, ACACACTA] = 7$$

-----A
 ***** |
 ACACACTA

[What about the other A?]

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	5	6	7	8
A	1	0	1	2	3	<u>4</u>	5	6	7
G	2	1	1	2	3	4	<u>5</u>	<u>6</u>	<u>7</u>
C	3								
A	4								
C	5								
A	6								
C	7								
A	8								

$$D[AG, ACACACTA] = 7$$

-----AG--

***** | ***

ACACACTA

Dynamic Programming Matrix

		A	C	A	C	A	C	T	A
	<u>0</u>	1	2	3	4	5	6	7	8
A	1	<u>0</u>	1	2	3	4	5	6	7
G	2	<u>1</u>	1	2	3	4	5	6	7
C	3	2	<u>1</u>	2	2	3	4	5	6
A	4	3	2	<u>1</u>	2	2	3	4	5
C	5	4	3	2	<u>1</u>	2	2	3	4
A	6	5	4	3	2	<u>1</u>	2	3	3
C	7	6	5	4	3	2	<u>1</u>	<u>2</u>	3
A	8	7	6	5	4	3	2	2	<u>2</u>

$$D[\text{AGCACACA}, \text{ACACACTA}] = 2$$

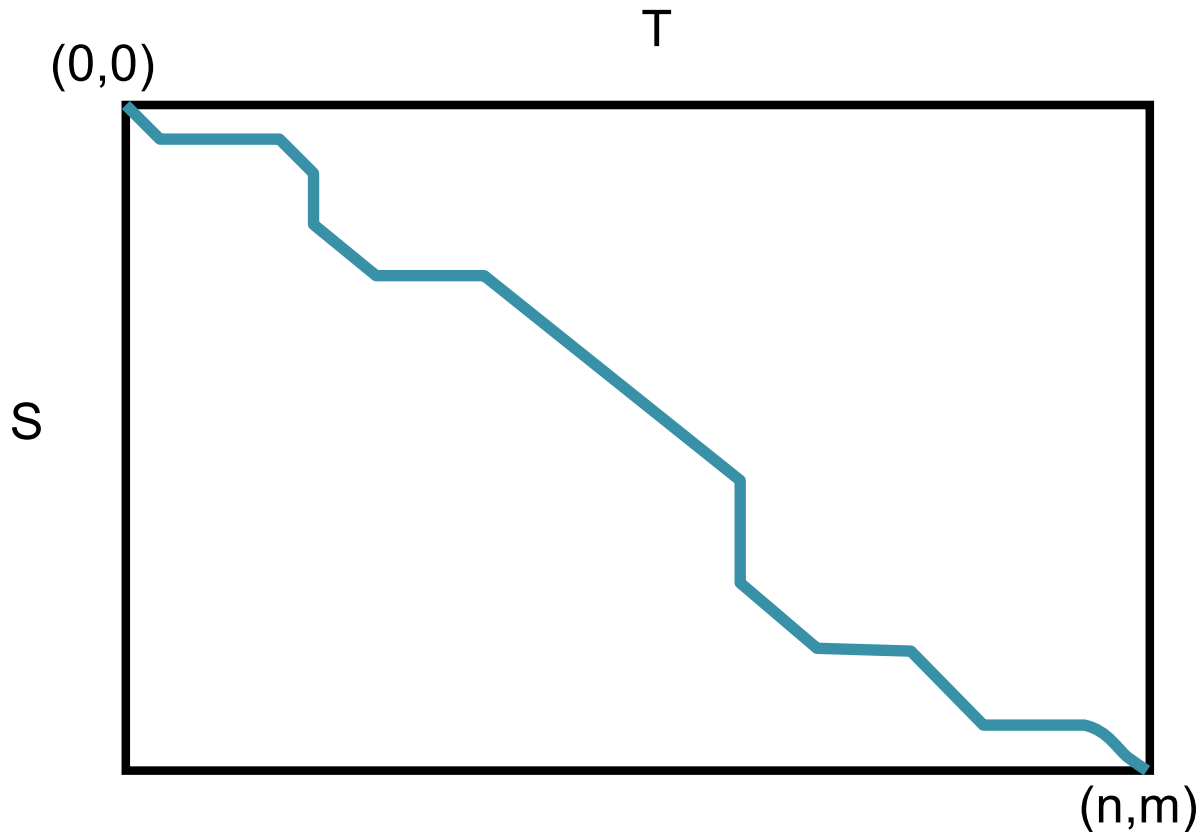
AGCACAC-A

| * | | | | * |

A-CACACTA

[Can we do it any better?]

Global Alignment Schematic

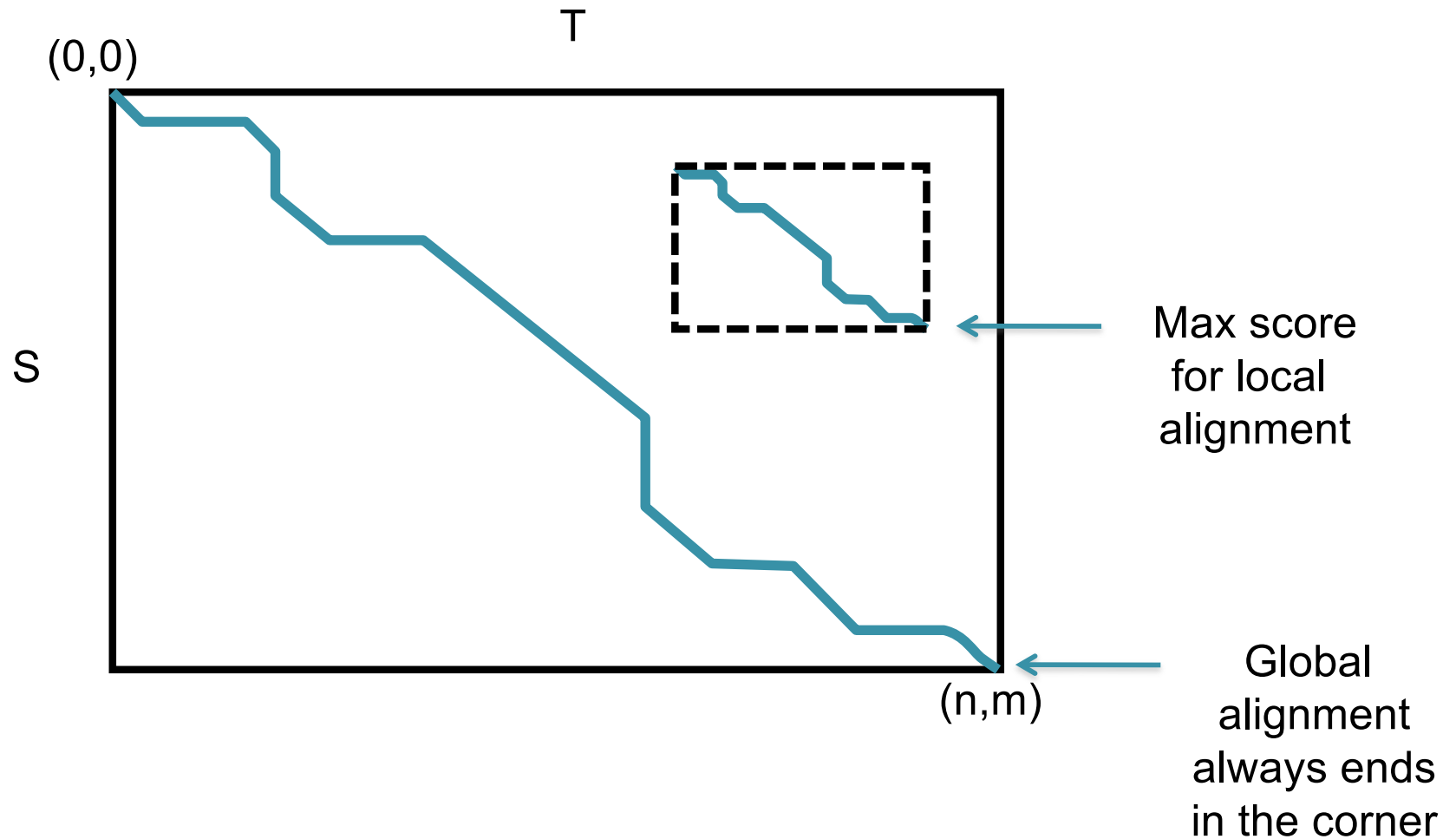


- A high quality alignment will stay close to the diagonal
 - If we are only interested in high quality alignments, we can skip filling in cells that can't possibly lead to a high quality alignment
 - Find the global alignment with at most edit distance d : $O(2dn)$

Local vs. Global Alignment

- The Global Alignment Problem tries to find the best end-to-end alignment between the two strings
 - Only applicable for very closely related sequences
- The Local Alignment Problem tries to find pairs of **substrings** with highest similarity.
 - Especially important if one string is substantially longer than the other
 - Especially important if there is only a distant evolutionary relationship

Global vs Local Alignment Schematic



Local vs. Global Alignment (cont' d)

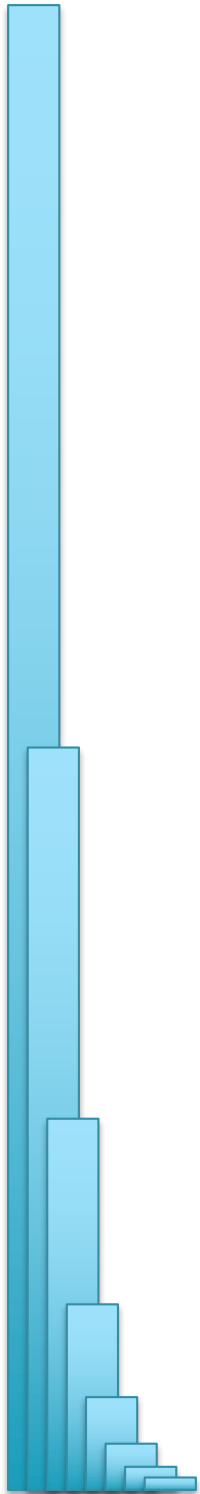
- Global Alignment

```
--T--CC-C-AGT--TATGT-CAGGGGACACG-A-GCATGCAGA-GAC
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
AATTGCCGCC-GTCGT-T-TTCAG-----CA-GTTATG-T-CAGAT--C
```

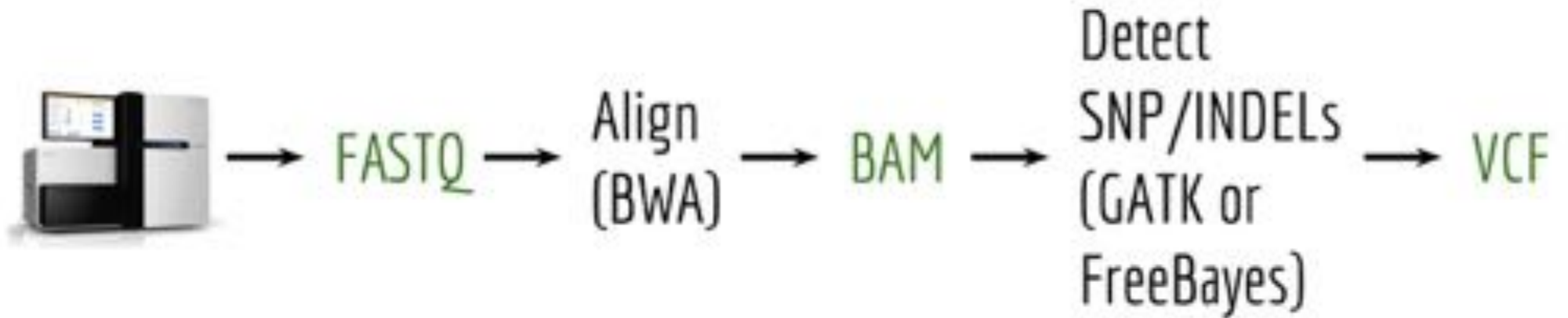
- Local Alignment—better alignment to find conserved segment

```
          tccCAGTTATGTCAGgggacacgagcatgcagagac
            |||||
aattgccgccgctcgtttttcagCAGTTATGTCAGatc
```

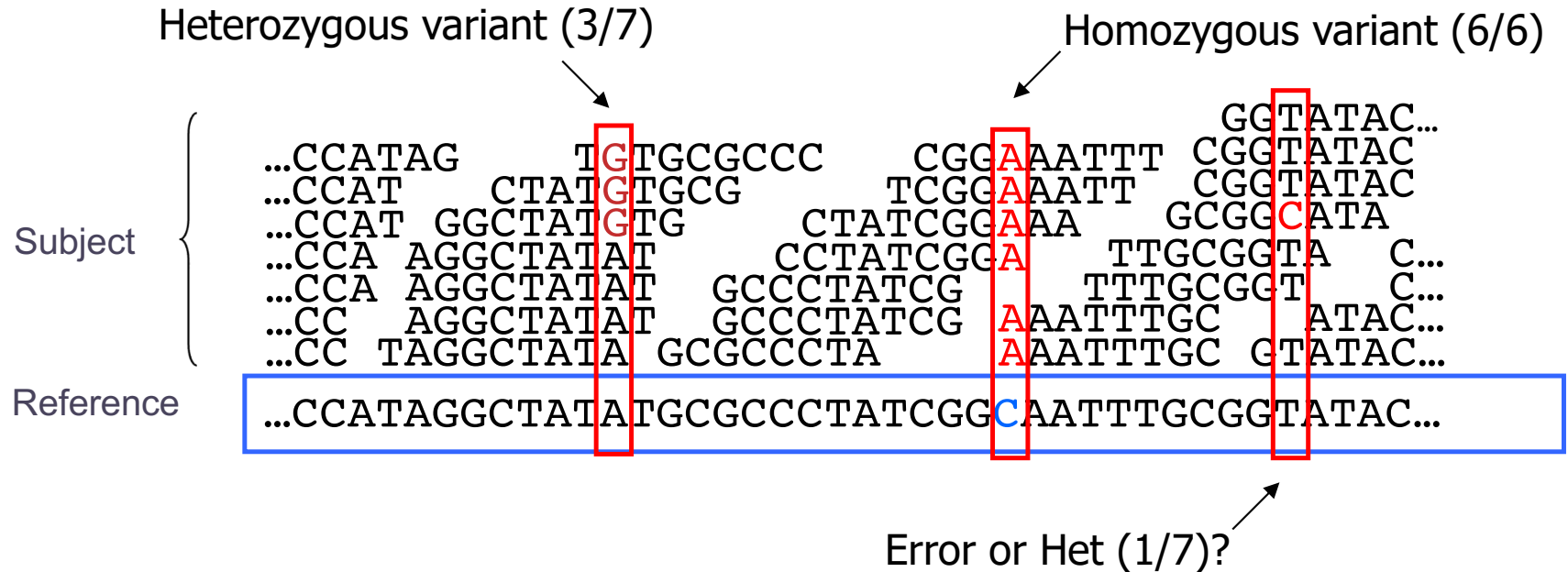

Part 3: Variant Calling



Variant Calling Overview



Genotyping Theory



- If there were no sequencing errors, identifying SNPs would be very easy: any time a read disagrees with the reference, it must be a variant!
- Sequencing instruments make mistakes
 - Quality of read decreases over the read length
- A single read differing from the reference is probably just an error, but it becomes more likely to be real as we see it multiple times

