Read mapping and the bwt

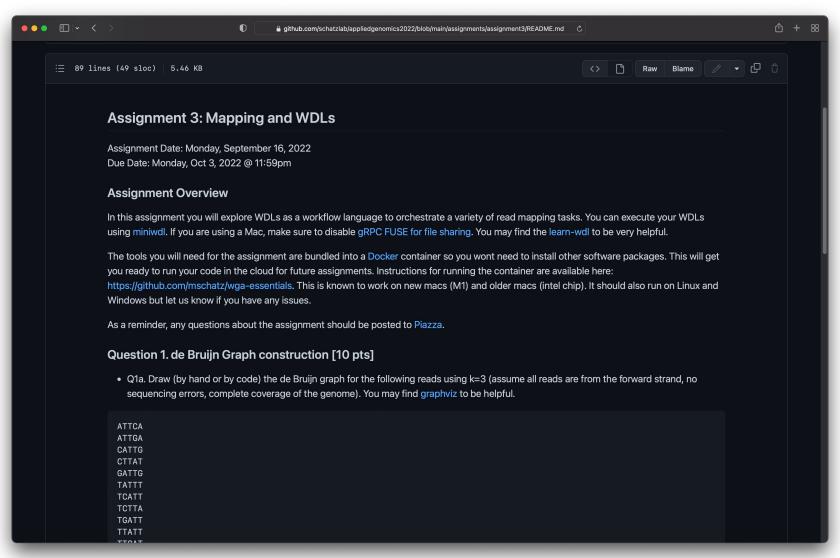
Michael Schatz

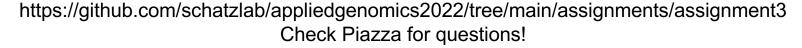
Sept 26, 2022

Lecture 8: Applied Comparative Genomics



Assignment 3: Mapping and WDL Due Monday Oct 3 by 11:59pm





Parallel Algorithm Spectrum

Embarrassingly Parallel



Each item is Independent

Loosely Coupled



Independent-Sync-Independent

Tightly Coupled



Constant Sync

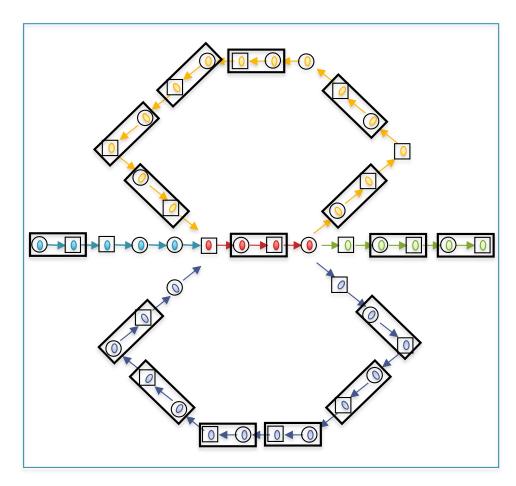
Fast Path Compression

Challenges

- Nodes stored on different computers
- Nodes can only access direct neighbors

Randomized List Ranking

- Randomly assign (H)/ T to each compressible node
- Compress (H)→T links



Initial Graph: 42 nodes

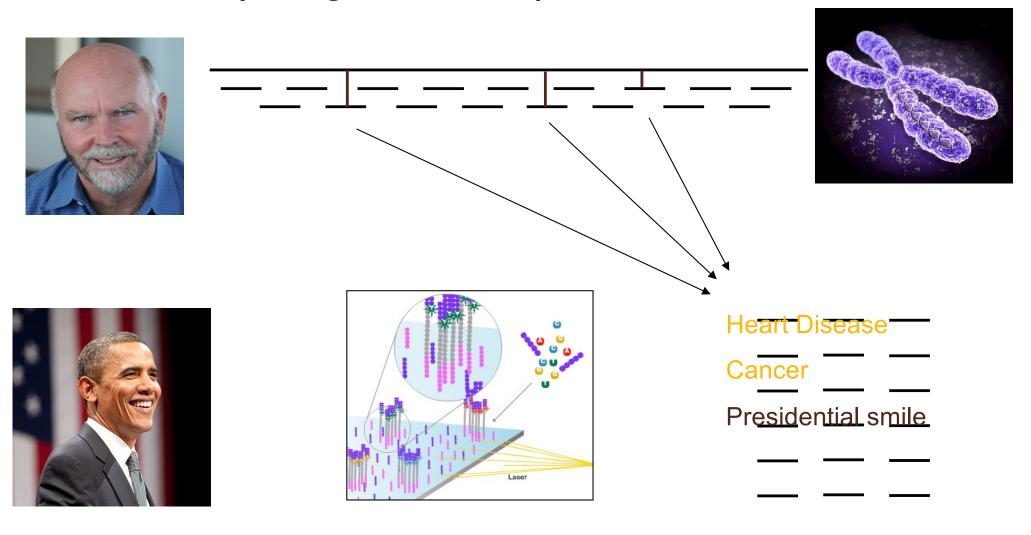
Randomized Speed-ups in Parallel Computation.

Vishkin U. (1984) ACM Symposium on Theory of Computation. 230-239.

Read Mapping

Personal Genomics

How does your genome compare to the reference?



Brute Force Analysis



- Brute Force:
 - At every possible offset in the genome:
 - Do all of the characters of the query match?
- Analysis
 - Simple, easy to understand

— Genome length = n	[3B]
– Query length = m	[7]
Comparisons: (n-m+1) * m	[21B]

Overall runtime: O(nm)

[How long would it take if we double the genome size, read length?] [How long would it take if we double both?]

Brute Force Reflections

Why check every position?

- GATTACA can't possibly start at position 15

[WHY?]

I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	•••
Т	G	Α	Т	Т	Α	С	Α	G	Α	Т	Т	Α	С	С	• • •
								G	Α	Т	Т	Α	С	Α	

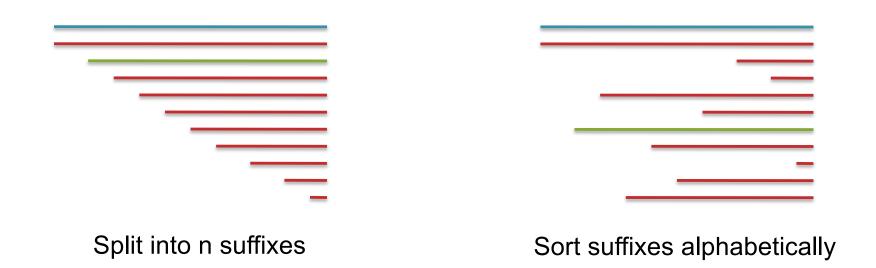
- Improve runtime to O(n + m)

[3B + 7]

- If we double both, it just takes twice as long
- Knuth-Morris-Pratt, 1977
- Boyer-Moyer, 1977, 1991
- For one-off scans, this is the best we can do (optimal performance)
 - We have to read every character of the genome, and every character of the query
 - For short queries, runtime is dominated by the length of the genome

Suffix Arrays: Searching the Phone Book

- What if we need to check many queries?
 - We don't need to check every page of the phone book to find 'Schatz'
 - Sorting alphabetically lets us immediately skip 96% (25/26) of the book without any loss in accuracy
- Sorting the genome: Suffix Array (Manber & Myers, 1991)
 - Sort every suffix of the genome



[Challenge Question: How else could we split the genome?]

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15;



#	Sequence	Pos
I	ACAGATTACC	6
2	ACC	13
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4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
7	CAGATTACC	7
8	CC	14
9	GATTACAGATTACC	2
10	GATTACC	9
Ш	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	I
14	TTACAGATTACC	4
15	TTACC	П



- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
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 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
 - Middle = Suffix[8] = CC



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5	ATTACC
6	C
7	CAGATTACC
8	CC
9	GATTACAGATTACC
10	GATTACC

TACAGATTACC...

TTACAGATTACC...

TGATTACAGATTACC...

TACC...

15 TTACC...



П

12

#

Sequence

ACC...

ACAGATTACC...

Pos

13

8

10

15

14

9

5

12

4

П



- Strategy 2: Binary search
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 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
 - Middle = Suffix[8] = CC=> Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC

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 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 11;





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 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC

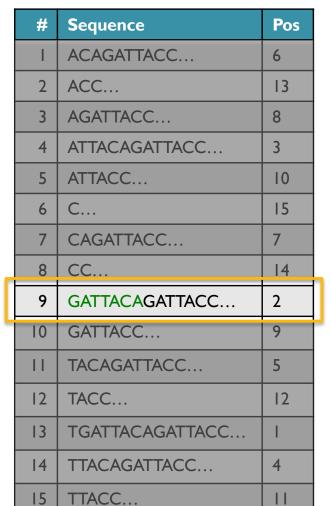
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 - Middle = Suffix[10] = GATTACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 9;



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 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 9; Mid = (9+9)/2 = 9
 - Middle = Suffix[9] = GATTACA...=> Match at position 2!





Binary Search Analysis

Binary Search

```
Initialize search range to entire list

mid = (hi+lo)/2; middle = suffix[mid]

if query matches middle: done

else if query < middle: pick low range

else if query > middle: pick hi range

Repeat until done or empty range
```

[WHEN?]

- Analysis
 - More complicated method
 - How many times do we repeat?
 - How many times can it cut the range in half?
 - Find smallest x such that: $n/(2^x) \le 1$; $x = \lg_2(n)$

[32]

- Total Runtime: O(m lg n)
 - More complicated, but much faster!
 - Looking up a query loops 32 times instead of 3B

[How long does it take to search 6B or 24B nucleotides?]



Binary Search Analysis

Binary Search

```
Initialize search range to entire list

mid = (hi+lo)/2; middle = suffix[mid]

if query matches middle: done

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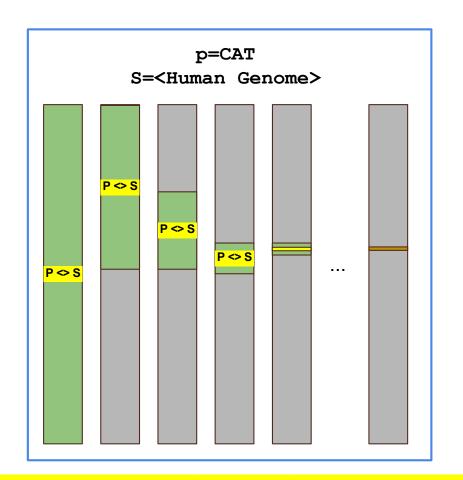
[32]

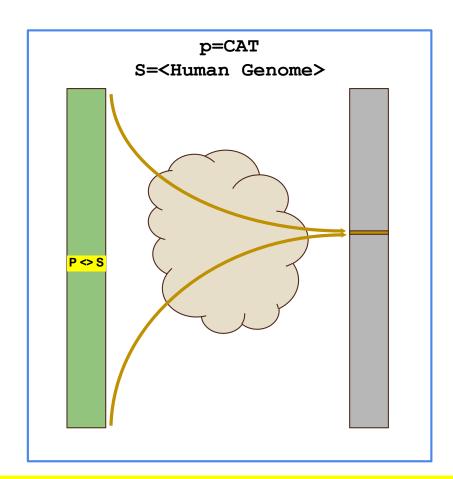
- Total Runtime: O(m lg n)
 - More complicated, but much faster!
 - Looking up a query loops 32 times instead of 3B



Can be reduced to O(m + lg n) using an auxiliary data structure called the LCP array

Sapling: Accelerating Suffix Array Queries with Learned Data Models





What if instead of a slow algorithmic approach to find the correct rows, we could somehow quickly guess/predict the correct rows?

Kirsche, M, Das, A, Schatz, MC (2020) Bioinformatics doi: https://doi.org/10.1093/bioinformatics/btaa911

Part 2: Burrows Wheeler Transform

Algorithmic challenge

How can we combine the speed of a suffix array O(m + lg(n)) (or even O(m)) with the size of a brute force analysis (n bytes)?

What would such an index look like?



Bowtie: Ultrafast and memory efficient alignment of short DNA sequences to the human genome

Slides Courtesy of Ben Langmead