The Evaluation, Comparison, Selection, and Implementation of Derivative Pricing Methods into a Trading Algorithm

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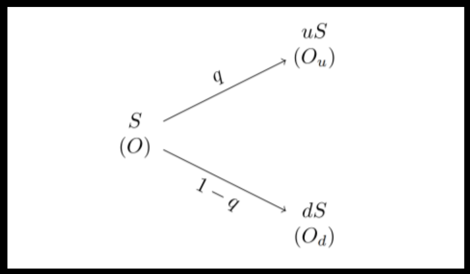
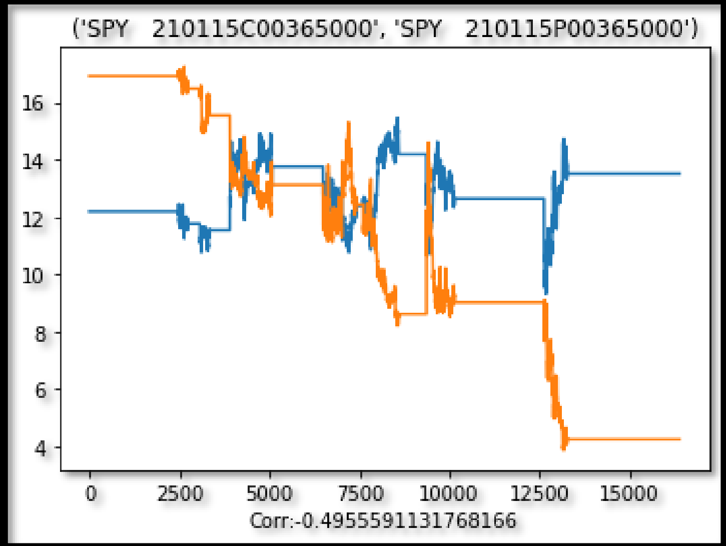


General Binomial Tree



Relative Error of Option Straddle Plotted against Time

Source: UIC

Option Contracts

- Classified as a derivative, other derivatives include futures, interest rate swaps
- Holder of the option has the right to buy or sell the underlying asset at the strike price, known as calls or puts respectively
- European options can only be exercised on expiration date, American options can be exercised anytime up until expiration date

- Common expirations include Weeklies, Monthlies, Quarterlies and LEAPs
- Each option contract has unique option symbol generated by Option Clearing Corporation (OCC)
- Expressed in the format shown below option symbol includes information relevant to the contract's underlying asset, expiration date, strike price and right.

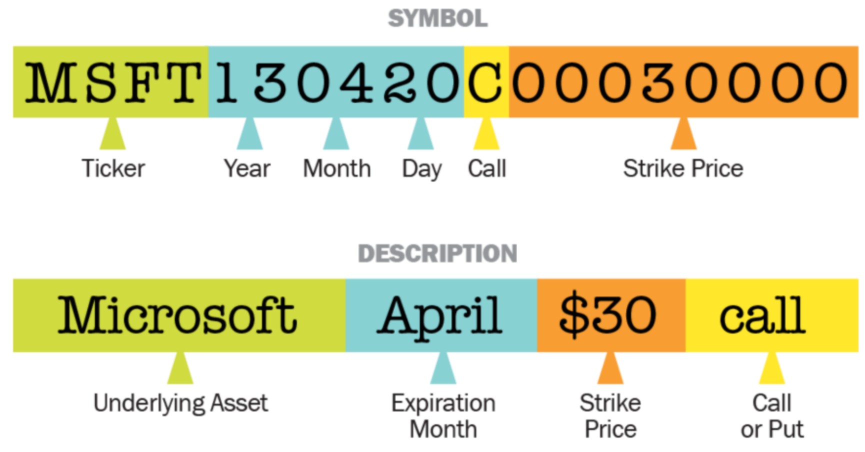


FIGURE 1.1: OCC Option Symbol Components

Source: Jeff Clark *Option Symbol Decomposition*

Pricing Methods

In the context of option pricing:

- Analytic Formulas give explicit formulas for the true value of the option, which are commonly European Style
- Analytic Approximations approximate the value of the option, commonly American Style, based on an underlying analytic formula
- Numerical Methods such as Binomial Trees and Finite Difference Methods approximate the true value with the use of a scheme across space and time.

Analytic Formulas

- Black Scholes Pricing Formulas, *1973* (BS-PF)

Analytic Approximations

- Barone-Adesi-Whaley Method, *1987* (B-A-W)
- Bjerksund-Stensland Method, *1993* (B-S)

Numerical Methods

- Binomial Trees
  - Cox-Ross-Rubinstein Binomial Tree, *1979* (C-R-R)
  - Jarrow-Rudd Binomial Tree, *1982* (J-R)
- Crank-Nicolson Finite Difference Method, *1985* (C-N)

Pricing Methods

- Most famous equation in option pricing is the Black-Scholes Pricing Formulas
- Formulas are the solutions to the Black-Scholes Partial Differential Equation (BS-PDE)
- Provide an explicit formula for value of European call or put (similar)
- American calls on stock with no dividend can be valued by (BS-PF)
- See Black's Approximation, 1975

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$$

Black Scholes Partial Differential Equation

$$C(S,t) = SN(d_1) - Ke^{-r(t)}N(d_2)$$

$$d_1 = \frac{\ln\frac{S}{K} + (r + \frac{\sigma^2}{2})(t)}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\frac{S}{K} + (r - \frac{\sigma^2}{2})(t)}{\sigma\sqrt{t}}$$

Black Scholes Pricing Formula for European Call, Put Similar Structure

Pricing Methods

- Barone-Adesi-Whaley (B-A-W) proposed the Quadratic Approximation of the American Call/Put
- Approximating the Analytic BS-PDE, the argument assumes any early exercise premium of American option follows BS-PDE
- Bjerksund-Stensland (B-S) proposed a class of general exercise strategies for American options specified by a trigger price
- One way to calculate the trigger price is with a time weighted average between infinite lived and an infinitesimal lived American Option

$$\begin{cases} C(S,T) = c(S,T) + A_2(\frac{S}{S^*})^{q_2} & S < S^* \\ C(S,T) = S - X & S \geq S^* \end{cases}$$

Quadratic Approximation of the American Call

$$\text{Such that } A_2(\tfrac{S}{S^*})^{q_2} \text{ is our approximation}$$

Approximation Term

$$c = \alpha(X)S^{\beta} - \alpha(X)\phi(S,T|\beta,X,X) + \phi(S,T|1,X,X) -$$
$$\phi(S,T|1,K,X) - K\phi(S,T|0,X,X) + K\phi(S,T|0,K,X)$$

Approximation of American Call given a General Exercise Strategy

$$X_T = B_0 + (B_\infty - B_0)(1 - e^h(T))$$

Time Weighted Average Trigger Price

Pricing Methods

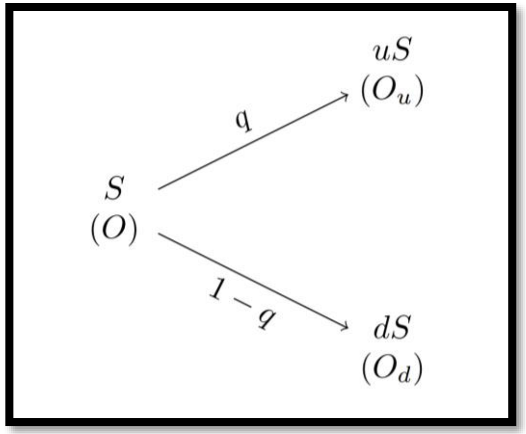
- Finite Difference Methods are often used in option pricing such as implicit, explicit, and Crank-Nicolson
- Performing central approximations in both time and space results in the following scheme
- Binomial Trees represent different moves the underlying asset can take
- Variations occur when restricting probability or magnitude of move
- Price of option is solved backward in time given payoffs at expiration

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$$

Black Scholes Partial Differential Equation

$$-a_j f_{i-1,j-1} + (1+2b_j) f_{i-1,j} + c_j f_{i-1,j+1} = a_j f_{i,j-1} + (1-2b_j) f_{i,j} + c_j f_{i,j+1}$$

Crank-Nicolson Finite Difference Scheme for Black-Scholes Partial Differential Equation



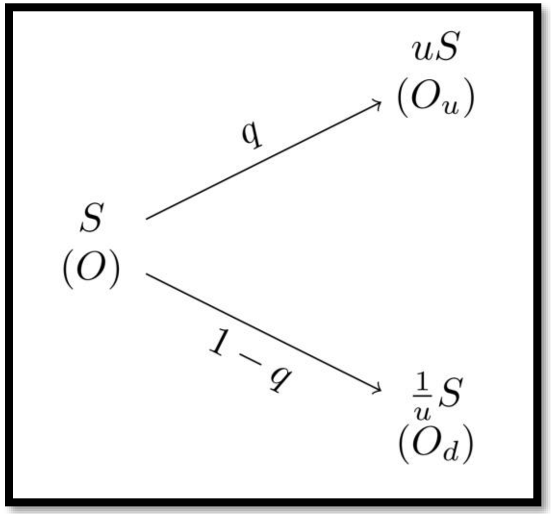
General Binomial Tree

$$Ee^{r\triangle t} = O_u q + O_d(1-q)$$

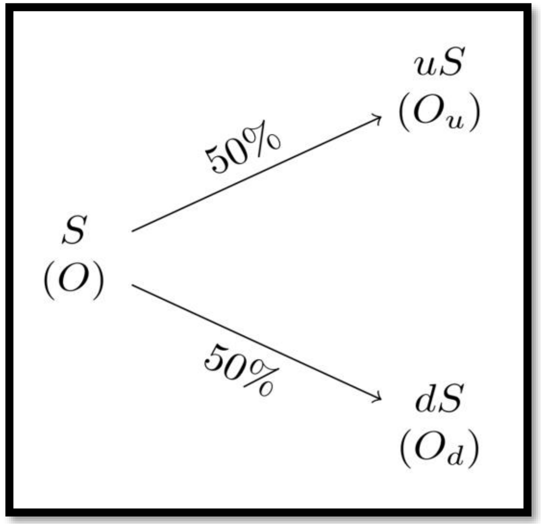
Expected Value of Option at Previous Node

Pricing Methods

- Cox-Ross-Rubinstein (C-R-R) proposed an Equal Jumps Binomial Tree where each node underlying asset can move up or down the same ratio with probabilities (q, 1-q)
- Jarrow-Rudd (J-R) proposed an Equal Probabilities Binomial Tree where each node underlying asset can move up or down with equal probabilities; the ratio of the moves can differ
- Both Trees converge to the Black Scholes Solution as number of steps goes to infinity.



General Equal Jumps Binomial Tree



General Equal Probabilities Binomial Tree

$$q = \frac{r-d}{u-d}$$

$$1-q = \frac{u-r}{u-d}$$

(C-R-R) Probability Formulas

$$u = e^{(r-\frac{\sigma^2}{2})\triangle t+\sigma\sqrt{\triangle t}}$$

$$d = e^{(r-\frac{\sigma^2}{2})\triangle t-\sigma\sqrt{\triangle t}}$$

(J-R) Step Size Formulas

Our Experiment

Goal:

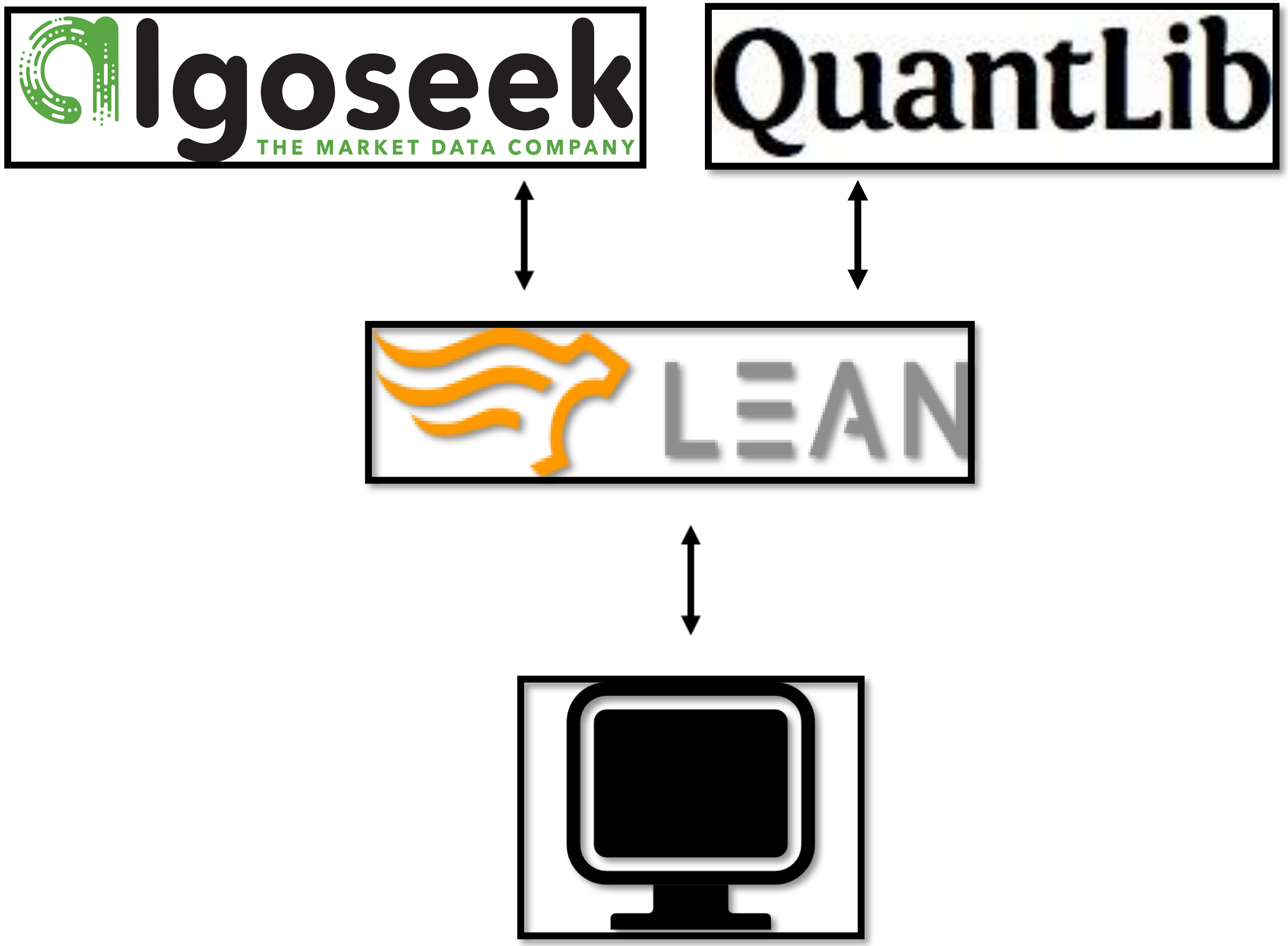
Evaluate the accuracy of chosen pricing methods across a variety of market condition to determine whether there is a significant difference between each methods results

Chosen Pricing Methods:

- Barone-Adesi-Whaley Analytic Approximation (B-A-W)
- Bjerkseud-Stensland Analytic Approximation (B-S)
- Crank-Nicolson Finite Difference Method (C-N)
- Cox-Ross-Rubinstein Binomial Tree (C-R-R)
- Jarrow-Rudd Binomial Tree (J-R)

Software Infrastructure

- LEAN is the Algorithmic Trading Engine used for the research and back-testing environment
- Algoseek Provides the Option Data
- QuantLib Provides Implementations of the Derivative Pricing Engines.



Sources: https://www.algoseek.com/company.html ; https://github.com/QuantConnect/Lean ; https://www.quantlib.org/

Universe

- Tests on accuracy will occur across three different time frames characterized by market conditions
  - Stable Market: 9/20/2019 - 10/18/2019
  - Bearish Market: 2/20/2020 - 3/20/2020
  - Bullish Market: 7/20/20 - 8/21/2020

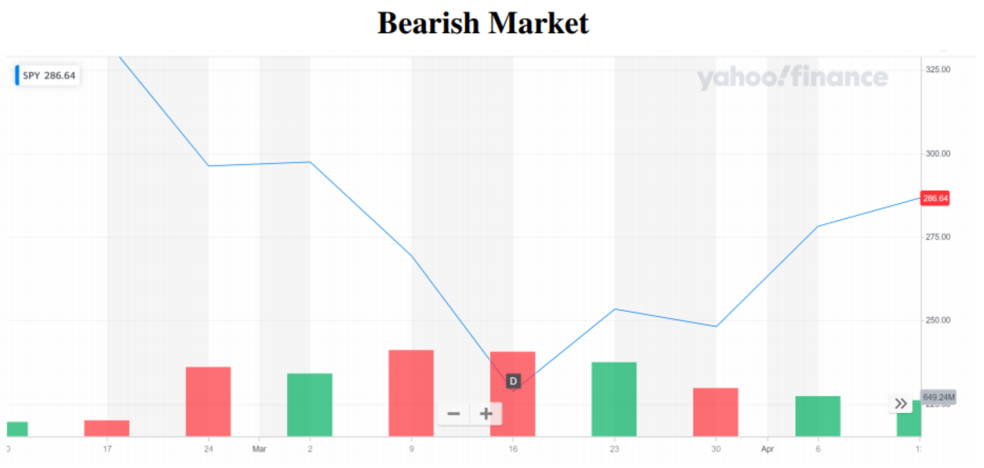


FIGURE 4.2: 2/20/2020 - 3/20/2020

Source: Yahoo Finance

For an option contract to be in our universe it must have the following:

- Underlying asset must be SPY
- American-style
- Expiration type is monthly
- Expiration date occurs in the last month of each chosen time frame
- Strike price within two standard deviations of the stock price at the start of the period

Pricing Methods

- Each pricing method requires the following estimators
  - Underlying Asset Volatility (60 Day Standard Deviations)
  - Dividend Yield (Constant=0)
  - Risk-Free Rate (Constant=0.01)

- Estimators are set in LEAN which are then passed to the pricing engines in QuantLib

```
if security.Type == SecurityType.Equity:
    #Perform history call for underlying volatility model
        security.VolatilityModel = StandardDeviationOfReturnsVolatilityModel(60)
        history = self.History(security.Symbol, 61, Resolution.Daily)
```

Above we Perform Necessary History Call to Populate our Underlying Volatility Estimator

```
Option.ConstantQLUnderlyingVolatilityEstimator.Estimate(...)=option.Underlying.VolatilityModel.
    Volatility;
```

Initialize our Underlying Volatility Estimator

```
Option.ConstantQLDividendYieldEstimator.Estimate(...)=0
```

Initialize our Dividend Yield Estimator

```
Option.ConstantQLRiskFreeRateEstimator.Estimate(...)=0.01
```

Initialize our Risk-Free Rate Estimator

Real Data Evaluation

- We run a backtest in LEAN over each time frame calculating the mid-spread (from quote data) and theoretical value (from pricing engine) each jth minute
- To standardize we calculate relative error for each contract in our universe
- Volume was also cached over the time frame

$$MS(C_{i(j)}) = \frac{Bid(C_{i(j)}) + Ask(C_{i(j)})}{2}$$

Mid-Spread Calculations from Bid and Ask of a given Option Contract

$$error_{C_i(j)} = \frac{MS(C_{i(j)}) - Th(C_{i(j)})}{MS(C_{i(j)})}$$

Relative Error Calculation between Mid-Spread and Theoretical Value

In Sample Results

- Average relative error of option contracts across Stable, Bullish, Bearish time frames

FIGURE 4.4: **Average Relative Error of Option Contracts**

Contract	Vol	B-A-W	B-S	C-N	C-R-R	J-R
Stable Market						
9/20/19 → 10/18/19						
191018C00302000	63376905	2.678623792	2.678623792	2.678623792	2.67437879	2.674454546
191018C00301000	59370390	2.012332278	2.012332278	2.012332278	2.012688603	2.012431568
191018C00300000	27230486	2.474583479	2.474583479	2.474583479	2.473374215	2.473545016
191018P00296000	49791381	4.312775181	4.312987549	4.312442347	4.311959884	4.311950649
191018P00295000	19021435	4.333976181	4.334243502	4.333624632	4.333363144	4.333300709
191018P00290000	17972131	4.219581335	4.219811531	4.219360764	4.218265365	4.218304149
Bearish Market						
2/20/20 → 3/20/20						
200320C00288000	151195022	18.95265	18.95265	18.95265	18.95318	18.95332
200320C00280000	41217042	15.14999	15.14999	15.14999	15.14947	15.14960
200320C00317000	40335972	8.08967	8.08967	8.08967	8.08864	8.08863
200320P00312000	121745412	12.17169	12.17186	12.17146	12.17068	12.17081
200320P00327000	68618846	13.73884	13.73908	13.73828	13.73820	13.73826
200320P00230000	36530004	14.09581	14.09581	14.09580	14.09590	14.09577
Bullish Market						
7/20/20 → 8/21/20						
200821C00341000	43059413	0.39444	0.39444	0.39444	0.39173	0.39185
200821C00340000	38731467	2.0782	2.0782	2.0782	2.0785	2.07909
200821C00339000	21222348	2.26745	2.26745	2.26745	2.26686	2.26741
200821P00320000	103406088	6.45580	6.45610	6.45557	6.45705	6.45693
200821P00335000	45367644	5.08538	5.08553	5.08508	5.08412	5.08437
200821P00320000	40093401	6.22273	6.22297	6.22241	6.2200	6.22015

Vol: Total Traded Volume
B-A-W: Barone-Adesi-Whaley Analytic Approximation [5]
B-S: Bjerksund-Stensland Analytic Approximation [7]
C-N : Crank-Nicolson Finite Difference Method [4][6]
C-R-R: Cox-Ross-Rubinstein Binomial Tree [2]
J-R: Jarrow-Rudd Binomial Tree [3]

In Sample Results

- One Way ANOVA Test where the Null Hypothesis is no true difference exists between the accuracy of the pricing methods
- Note the F-Statistic and Probability lead us to **not reject Null Hypothesis** in any of our time frames

FIGURE 4.5: **ANOVA Test for Stable, Bearish, and Bullish Markets**

Time Frame	F Statistic	Prob	Conclusion
Stable Market	$1.5 * 10^{-6}$	1	Do Not Reject H_0
Bearish Market	$1.7 * 10^{-7}$	1	Do Not Reject H_0
Bullish Market	$4.3 * 10^{-7}$	1	Do Not Reject H_0

Out of Sample Results

- Average relative error of option contracts across Out-of-Sample time frame
- Like before we do **not reject the Null Hypothesis** of the pricing methods having no true difference in accuracy
- Trading Algorithm would consistently buy options in universe due to downward bias

FIGURE 4.6: **Average Relative Error of Option Contracts**

Contract	Vol	B-A-W	B-S	C-N	C-R-R	J-R
Out-of-Sample data set						
$11/15/20 \rightarrow 1/15/21$						
210115C00380000	172887852	3.118747568	3.118747568	3.118747568	3.120401288	3.120199295
210115C00381000	105375385	2.953607075	2.953607075	2.953607075	2.954763296	2.954575036
210115C00382000	79773400	3.059143999	3.059143999	3.059143999	3.059199757	3.059308856
210115P00368000	52089251	10.63383405	10.6341782	10.63347105	10.63354459	10.63332738
210115P00375000	29945288	7.0013056371	7.001534286	7.000945899	6.999511168	6.999479611
210115P00365000	25779894	10.77798391	10.77834935	10.77764943	10.77656601	10.7765485

Vol: Total Traded Volume
B-A-W: Barone-Adesi-Whaley Analytic Approximation [5]
B-S: Bjerksund-Stensland Analytic Approximation [7]
C-N : Crank-Nicolson Finite Difference Method [4][6]
C-R-R: Cox-Ross-Rubinstein Binomial Tree [2]
J-R: Jarrow-Rudd Binomial Tree [3]

FIGURE 4.7: **ANOVA Test for Out-of-Sample data set**

Time Frame	F Statistic	Prob	Conclusion
Out-of-Sample data set	$5.8 * 10^{-7}$	1	Do Not Reject H_0

Conclusion

Presentation Topics:

- Introduced options
- Discussed pricing methods theoretically
- Performed an experiment to see the accuracy of pricing methods
- Concluded there is not a significant difference between the following pricing methods across the chosen contracts and time frames

Next Steps: Improved Estimators

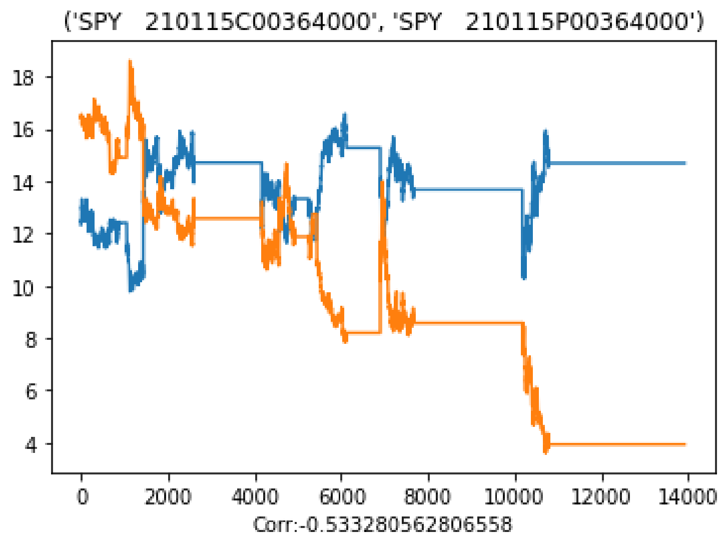
- Encountered a significant downward bias in error values across all the pricing methods and time frames
- Perhaps one way of decreasing bias is incorporating more realistic estimators
- Stochastic processes, GARCH models, EWMA models for underlying volatility estimator

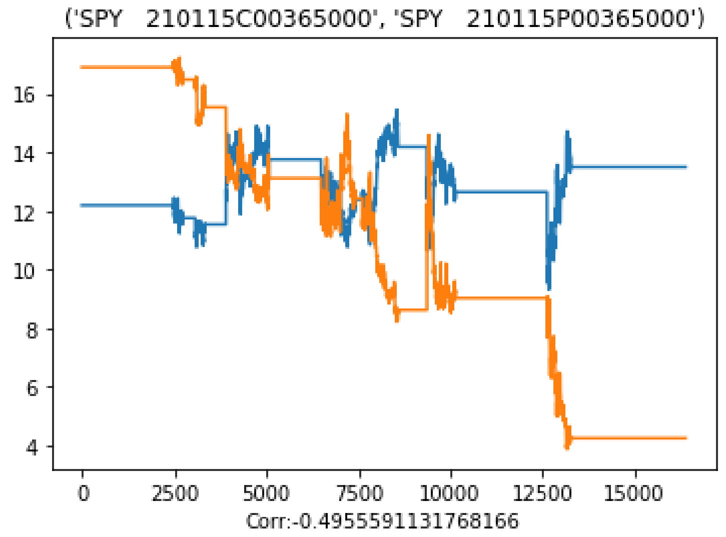
$$\boxed{\sigma_n^2 = \tau V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2}$$

GARCH (1,1) Model

Final Note

- Option straddles consist of a call and put contract on the same underlying with the same strike and expiration
- Surprisingly, we discovered inverse relationships in the relative error of option straddles
- Observation persisted across market conditions and contracts of varying liquidity
- We believe it relates to level of market volatility
  - Times of lower volatility prices of puts and calls are balanced
  - Times of higher volatility, more uncertainty, reflected in prices of puts and calls





Average Relative Error of Option Straddles

References

GitHub Repository

Software Licenses:

- LEAN Algorithmic Trading Engine
- QuantLib

Major sources listed, for a full extensive list please see the reference page in paper

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Q&A

Thank you for your time, please reach out to **jbohne3@uic.edu** with any remaining questions!

Source: UIC