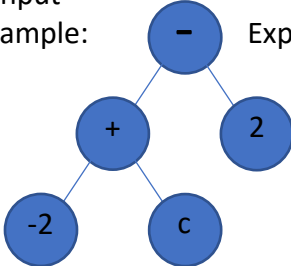


Description and justification of the tests

- testTree2prefix

Description: the input of this method is a correct structure of tree for Arithmetic Expression.

Input

Example:  Expected output: - + -2 c 2 which is a String

Purpose of the test: testing the function can return the correct prefix arithmetic expression when the tree contains negative integer nodes, letter nodes and non-negative nodes.

- testTree2prefixException

Description: the input of this method is an invalid structure of tree for Arithmetic Expression.

Input Example:

Expected output: throw an IllegalArgumentException

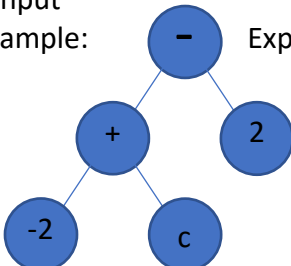


Purpose of the test: testing the function can detect the Illegal Arithmetic Expression in tree structure, which are two examples above, and throw an IllegalArgumentException to remind users.

- testTree2infix

Description: the input of the method is a correct structure of tree for Arithmetic Expression.

Input

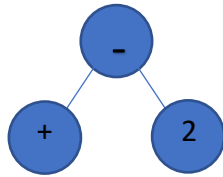
Example:  Expected output: ((-2+c)-2) which is a String

Purpose of the test: testing the function can return the correct infix arithmetic expression when the tree contains negative integer nodes, letter nodes and non-negative nodes.

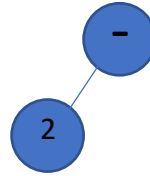
- testTree2infixException

Description: the input of this method is an invalid structure of tree for Arithmetic Expression.

Input Example:



Expected output: throw an IllegalArgumentException



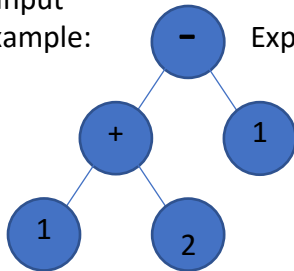
Purpose of the test: testing the function can detect the Illegal Arithmetic Expression in tree structure, which are two examples above, and throw an IllegalArgumentException to remind users.

- testSimplify1

Description: the input of the method is a simple LinkedBinaryTree, and the output is the simplify tree by following the arithmetic expression.

Input

Example:



Expected output: a simplify LinkedBinaryTree



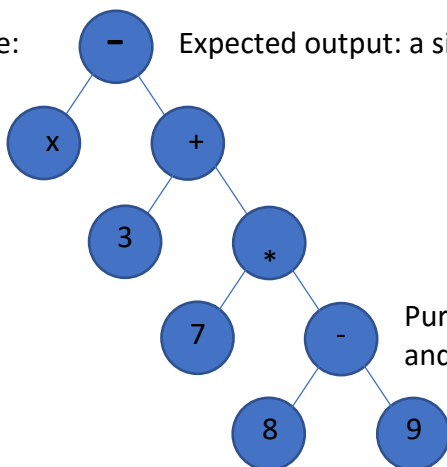
Purpose of the test: testing a simple tree to make sure the method work in basic situation.

- testSimplify2

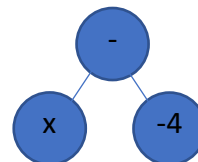
Description: the input of the method is a LinkedBinaryTree, and the output is the simplify tree by following the arithmetic expression.

Input

Example:



Expected output: a simplify LinkedBinaryTree



Purpose: testing every operators can work well and testing the rule that simplify cannot simplify a letter.

- testSimplify3 ---- testSimplify 8

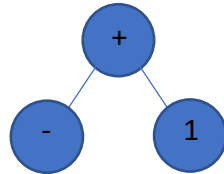
Description: the input of each method is a LinkedBinaryTree, and the output is the simplify tree by following the arithmetic expression.

Purpose: testing the method can handle more complicated trees such as tree with negative integer, positive integer, letter, three operators.

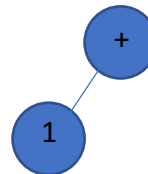
- testSimplifyException

Description: the inputs of the method are invalid LinkedBinaryTree(tree==null, and two trees below)

Input Example:



Expected output: throw an IllegalArgumentException



Purpose of the test: testing the function can detect the Illegal Arithmetic Expression in tree structure, null input tree, and throw an IllegalArgumentException.

- testSimplifyFancy1---testSimplifyFancy7

Description: * 1 x == x i.e. (1*x)==x input: left hand side(x can be letter or tree)
 * x 1 == x (x*1)==x output: right hand side
 * 0 x == 0 (0*x)==0 it is a general representation of my test input
 * x 0 == 0 (x*0)==0 in order to minimize the length of the report
 + 0 x == x (0+x)==x
 + x 0 == 0 (x+0)==x
 - x 0 == x (x-0)==x
 - x x == 0 (x-x)==0
 - 0 x == - 0 x cannot simplify

purpose: check the method can work perfectly follow by the simplifyFancy rules

- testSimplifyFancy8-9

Description: input: complicated tree containing letter, non-negative and negative integer

Output: really simple tree

Purpose: make sure that the method will work when simplify the tree with several different Fancy rules and with various kinds of nodes.

- testSubstitute

Description: input: linked binary tree with letters and Integer

Output: new tree with choosing letter substitute by Integer value

Purpose: testing the method can substitute choosing letter with non-negative or negative Integer

- test SubstituteException1—4

Description: input: an invalid tree expression, null tree, an invalid variable (e.g. operators, null, Integer), and a value

Output: throw IllegalArgumentException

Purpose: testing the method can correctly handle all the invalid input

- testSubstituteMap

Description: input: linked binary tree with several different letter and Integer, a HeapMap

Output: a substitute tree with every selected letters are substituted by given corresponding values

Purpose: testing method can substitute all variables correctly following the map

- testSubstituteMapException

Description: input: an invalid tree expression or null map

Output: throw IllegalArgumentException

Purpose: testing method can handle this kind of problem and throw exception correctly

- testisArithmeticException

Description: input: invalid Linked BinaryTree expression

Output: if the expression is right return true, else return false

Purpose: testing the method can return the right Boolean value for valid and invalid tree

- testMixed

Description: input: linked binary tree and map with variable and value

Output: simplify and substitute tree

Purpose: testing methods can use at the same time without any errors

Analysis run time cost

- **public static** String tree2prefix(LinkedBinaryTree<String> tree) throws IllegalArgumentException

Algorithm: I implemented the method recursively. The base case of the recursion is that If the node is a leaf, the recursion should stop and return the element in that node, since no more nodes exist after the leaf. I used a private helper method with one more parameter called Position<String> root to update the node that we choose for each recursion than the original function.

tree2prefix(LinkedBinaryTree<String> tree) throws IllegalArgumentException

if tree is null

throw IllegalArgumentException

return tree2prefix(tree, tree.root())

tree2prefix(LinkedBinaryTree<String> tree, Position<String> root) throws

IllegalArgumentException

Input: linked binary tree, root

Output: Arithmetic expression prefix in String

If tree.isExternal(root) is False

If node only has one child (tree.numChildren(root)=1)

Throw IllegalArgumentException

Return root.getelement()+" "+tree2prefix(tree, tree.left(root))+" "+tree2prefix(tree, tree.right(root))

if tree.isExternal(root) is True

if root is operator(+,-,*)

Throw IllegalArgumentException

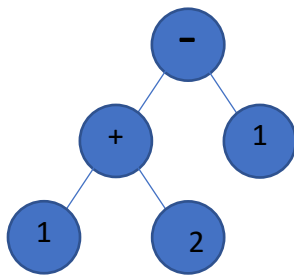
Return the element of the leaf

Big O worst case runtime and justification: $\rightarrow O(n)$

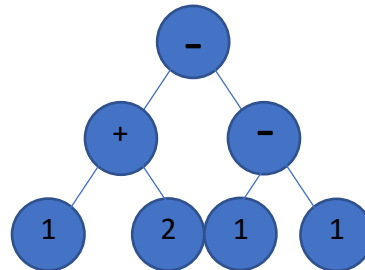
`root.getelement()+" "+tree2prefix(tree, tree.left(root))+" "+tree2prefix(tree, tree.right(root))`

$O(1)+T(n-1)+O(1)$ (first graph) $O(1)+T(n/2)+(T/2)$ (second graph)

Since Other lines are $O(1)$



$T(n-1)+O(1)=O(n)$



$2T(n/2)+O(1)=O(n)$

- `public static String tree2infix(LinkedBinaryTree<String> tree) throws
IllegalArgumentException`

Algorithm: The idea of the method is basically same as the previous method `tree2prefix`, the difference is that the output is in infix expression.

`tree2infix(LinkedBinaryTree<String> tree) throws IllegalArgumentException`

if tree is null

 throw `IllegalArgumentException`

 return `tree2infix(tree, tree.root())`

`tree2infix(LinkedBinaryTree<String> tree, Position<String> root) throws`

`IllegalArgumentException`

Input: linked binary tree, current node Output: Arithmetic expression infix in String

If `tree.isExternal(root)` is False

 If node only has one child (`tree.numChildren(root)=1`)

 Throw `IllegalArgumentException`

 Return `root.getelement()+" "+tree2infix(tree, tree.left(root))+" "+tree2infix(tree, tree.right(root))`

if `tree.isExternal(root)` is True

 if root is operator(+, -, *)

 Throw `IllegalArgumentException`

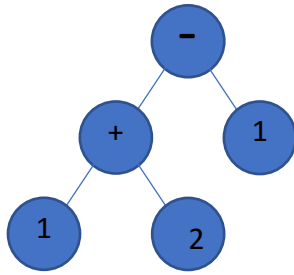
 Return the element of the leaf

Big O worst case runtime and justification: $\rightarrow O(n)$

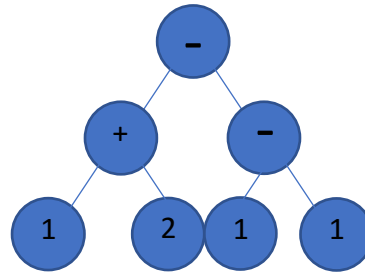
`root.getelement()+" "+tree2infix(tree, tree.left(root))+" "+tree2infix(tree, tree.right(root))`

$O(1)+T(n-1)+O(1)$ (first graph) $O(1)+T(n/2)+(T/2)$ (second graph)

Since Other lines are $O(1)$



$$T(n-1) + 2O(1) = T(n-1) + O(1) = O(n)$$



$$2T(n/2) + O(1) = O(n)$$

- `public static` `LinkedBinaryTree<String> simplify(LinkedBinaryTree<String> tree)`
`throws` `IllegalArgumentException`

Algorithm: I implement the method recursively. Traversing the the binary tree in preorder. I design a helper method that have one more input called `Position<String> root` than original method for recursion.

`simplify(LinkedBinaryTree<String> tree)` `throws` `IllegalArgumentException`

if tree is null

throw `IllegalArugmentException`

return `simplify(tree, tree.root())`

`simplify(LinkedBinaryTree<String> tree, Position<String> root)` `throws` `IllegalArumentException`

Input: linked binary tree and current node Output: a simplify tree

`LinkedBinaryTree<String> newtree=new LinkedBinaryTree<String>();`

If `tree.isExternal(root)` is False

`LinkedBinaryTree<String> lefttree= simplify(tree,tree.left(root));`

`LinkedBinaryTree<String> righttree=simplify(tree,tree.right(root));`

If root equals "+"

If left child (`simplify(tree, tree.left(root))`) of current root is a tree with only a root and the root is a letter

`newtree.addRoot("+");`

`newtree.attach(newtree.root(),lefttree,righttree)`

return newtree

If right child(`simplify(tree, tree.right(root))`) of current root is a tree with only a root and the root is a letter

`newtree.addRoot("+");`

`newtree.attach(newtree.root(),lefttree,righttree)`

return newtree

If left child of current root is a tree has number of nodes greater than 1

`newtree.addRoot("+");`

`newtree.attach(newtree.root(),lefttree,righttree)`

return newtree

If right child of current root is a tree has number of nodes greater than 1

`newtree.addRoot("+");`

`newtree.attach(newtree.root(),lefttree,righttree)`

return newtree

If left child(simplify(tree, tree.left(root))) of current root is a tree with only a root and is a negative Integer&&right child(simplify(tree, tree.right(root))) of current root is a tree with only a root and is a negative Integer

```
result=Integer.parseInt(lefttree.root().getElement().substring(1))+Integer.parseInt(righttree.root().getElement().substring(1));
newtree.addRoot("-"+result+"");
return newtree;
```

If left child(simplify(tree, tree.left(root))) of current root is a tree with only a root and is a negative Integer

```
int result=Integer.parseInt(righttree.root().getElement())
-Integer.parseInt(lefttree.root().getElement().substring(1));
newtree.addRoot(result+"");
return newtree;
```

If right child(simplify(tree, tree.right(root))) of current root is a tree with only a root and is a negative Integer

```
int result=Integer.parseInt(lefttree.root().getElement())
-Integer.parseInt(righttree.root().getElement().substring(1));
newtree.addRoot(result+"");
return newtree;
```

//if the left tree and the right tree of the current root does not satisfy above conditions, then go to the normal case

```
result=Integer.parseInt(lefttree.root().getElement())+Integer.parseInt(righttree.root().getElement());
newtree.addRoot(result+"");
return newtree;
```

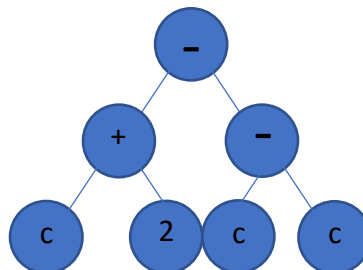
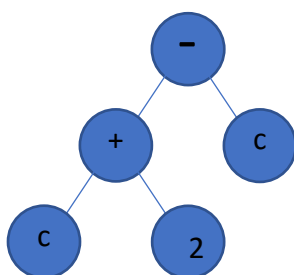
Notice: since there are three operators in the assignment, the method simplify should cover all conditions, which makes the method too long. In order to minimize the length of the report, I omit the “-” and “*” part of algorithm. The algorithm of the these two operator basically is same as “+” algorithm, except for some unique cases such as the 1 - -2 , -1 * 2 etc. I write comments for each line of my code to make my code readable.

if tree.isExternal(root) is True

```
newtree.addRoot(root.getElement());
return newtree
```

Big O worst case runtime and justification: $\rightarrow O(n)$

```
LinkedBinaryTree<String> lefttree= simplify(tree,tree.left(root));
LinkedBinaryTree<String> righttree=simplify(tree,tree.right(root));
T(n-1)+O(1) (graph one)          T(n/2)+T(n/2)(graph 2)
Since Other lines are O(1)
```



$$T(n-1)+O(1)=O(n)$$

$$2T(n/2)+O(1)=O(n)$$

- `public static` `LinkedBinaryTree<String> simplifyFancy(LinkedBinaryTree<String> tree)`
`throws` `IllegalArgumentException`

Algorithm: since the method is a update function of simplify, the algorithm of `simplifyFancy` is similar to `simplify`, except for some special rules that we should add in each operator if statements.

```
LinkedBinaryTree<String> lefttree= simplifyFancy(tree,tree.left(root));
LinkedBinaryTree<String> righttree=simplifyFancy(tree,tree.right(root));
```

Regarding to "+":

If left tree of current root only have one node is 0 or right tree of current root only have one node is 0

Return lefttree// if right tree equals 0

or Return righttree// if left tree equals 0

//in the code the algorithm above is implemented by using two if statement

Regarding to "-":

If left tree of current root only have one node is 0

```
newtree.addRoot("-");
```

```
newtree.attach(newtree.root(),lefttree,righttree);
```

```
return newtree;
```

If right tree of current root only have one node is 0

Return `simplifyFancy(tree,tree.left(root))`

If left tree and right tree of current root is completely equal

```
newtree.addRoot("0");
```

```
return newtree;
```

Regarding to "*":

If left tree of current root only have one node is 1 or right tree of current root only have one node is 1

Return lefttree// if right tree equals 1

or Return righttree// if left tree equals 1

//in the code the algorithm above is implemented by using two if statement

If left tree of current root only have one node is 0 or right tree of current root only have one node is 0

```
newtree.addRoot("0");
```

```
return newtree;
```

Big O worst case runtime and justification:

Except for operator "-", the worst case runtime **is O(n)**

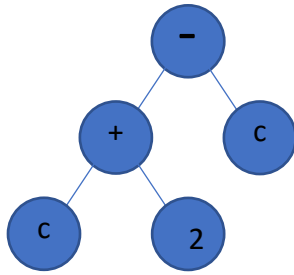
```
LinkedBinaryTree<String> lefttree= simplifyFancy(tree,tree.left(root));
```

```
LinkedBinaryTree<String> righttree=simplifyFancy(tree,tree.right(root));
```

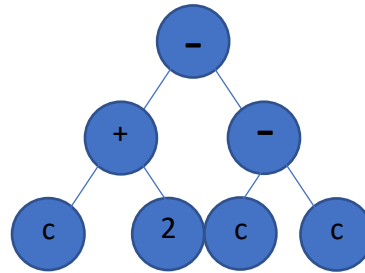
$T(n-1)+O(1)$ (graph one)

$T(n/2)+T(n/2)$ (graph 2)

Since Other lines are $O(1)$



$$T(n-1)+O(1)=O(n)$$

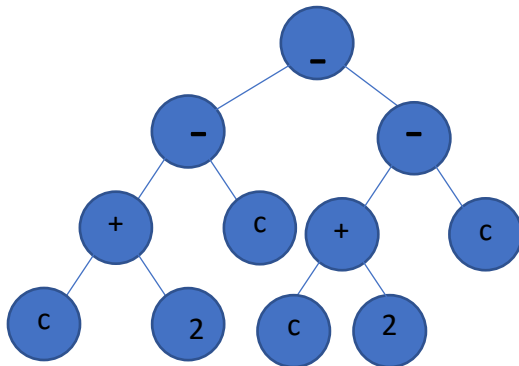


$$2T(n/2)+O(1)=O(n)$$

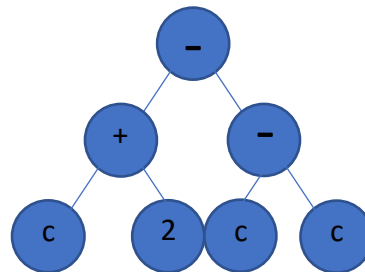
In the case of operator "-", the worst case runtime **is $O(n^2)$**

The special case $x-x$, the method use `equals(treeA,treeB)` to compare those two subtree whether completely same, which equals spend $O(n)$ to compare each node of the linked binary tree with height $O(n)$

Since other lines are $O(1)$,



$$T(n-1)+O(n)=O(n^2)$$



$$2T(n/2)+O(n)=O(n \log n)$$

- `public static LinkedBinaryTree<String> substitute(LinkedBinaryTree<String> tree, String variable, int value) throws IllegalArgumentException`

Algorithm: since we cannot substitute an operator and substitute leaf with null or operator, or Integer, the leaves are the nodes we can substitute Letter with Integer. Helper method same as above helper method add one more parameter `Position<String> root`. The base case is the root is a leaf.

```
substitute(LinkedBinaryTree<String> tree, String variable, int value) throws
IllegalArgumentException
```

```
    if tree is null or variable is null or variable is Integer or operator
```

```
        throw IllegalArugmentException
```

```
    return substitute(tree, tree.root(),variable,value)
```

```
substitute(LinkedBinaryTree<String> tree, Position<String> root, String variable, int
value) throws IllegalArumentException
```

```
Input: linked binary tree ,current node, variable, value  Output: a substitute tree
LinkedBinaryTree<String> newtree=new LinkedBinaryTree<String>();
```

```
If tree.isExternal(root) is False
```

```
    If node only has one child (tree.numChildren(root)=1)
```

```
        Throw IllegalArgumentException
```

```
    newtree.addRoot(root.getElement())
```

```

newtree.attach(newtree.root(),substitute(tree,tree.left(root),variable,value),
substitute(tree,tree.right(root),variable,value));
return newtree;
else
    if root is operator(+,-,*)
        Throw IllegalArgumentException
    If root.getElement equals variable
        newtree.addRoot(value+""");
        return newtree;
    else

        newtree.addRoot(root.getElement());
        return newtree;

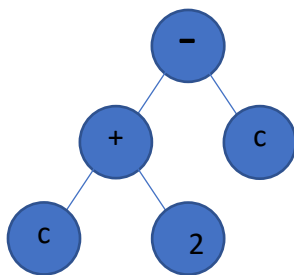
```

Big O worst case runtime and justification: $\rightarrow O(n)$

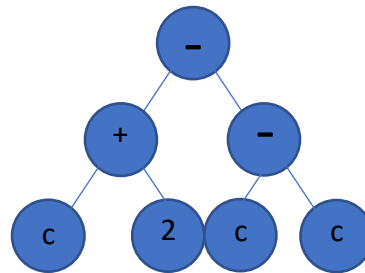
```

newtree.attach(newtree.root(), substitute(tree,tree.left(root),variable,value),
substitute(tree,tree.right(root),variable,value));
O(1)+O(1)+T(n-1)+O(1)( first graph)   O(1)+O(1)+T(n/2)+T(n/2)( second graph)
other lines are O(1)

```



$$T(n-1)+O(1)=O(n)$$



$$2T(n/2)+O(1)=O(n)$$

- `public static` `LinkBinaryTree<String> substitute(LinkBinaryTree<String> tree, HashMap<String, Integer> map) throws IllegalArgumentException`

Algorithm: The basic idea of the method is similar to the previous substitute method, except for the method can substitute several String with Integer instead of one pair in the previous substitute.

In the base case, adding a condition `Map.containsKey(root.getElement())` to check the current root whether contained in the map or not. If map does contain the root, replace it with provided value, if not changing nothing.

```

if(map.containsKey(root.getElement())){
    if(map.get(root.getElement())==null){
        throw new IllegalArgumentException();
    }
    newtree.addRoot(map.get(root.getElement())+""");
    return newtree;
}

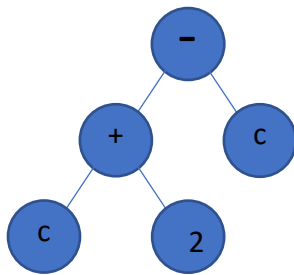
```

```
newtree.addRoot(root.getElement());
return newtree;
```

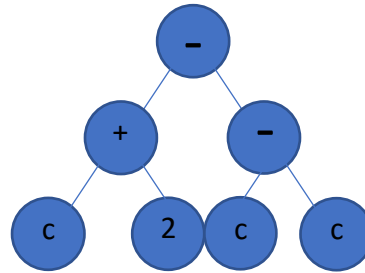
Big O worst case runtime and justification: $\rightarrow O(n)$

```
newtree.attach(newtree.root(), substitute(tree,tree.left(root),variable,value),
substitute(tree,tree.right(root),variable,value));
```

$O(1)+O(1)+T(n-1)+O(1)$ (first graph) $O(1)+O(1)+T(n/2)+T(n/2)$ (second graph)
since other lines are $O(1)$



$$T(n-1)+O(1)=O(n)$$



$$2T(n/2)+O(1)=O(n)$$

- **public static boolean** isArithmeticExpression(LinkedBinaryTree<String> tree)

Algorithm:

1. The tree provided is null
2. A node only have one child
3. Leaf is an operator

When the method reach one of the three conditions above, method should return false.

Implemented the method recursively. Base case is that the current node is a leaf, and if the lead is an operator, method return false, else return true

If tree.isExternal(root) is False

 If node only has one child (tree.numChildren(root)=1)

 return false;

 only if isArithmeticExpression(tree,tree.left(root)) and

 isArithmeticExpression(tree,tree.right(root)) all return true,

 method will return true

else

 return false

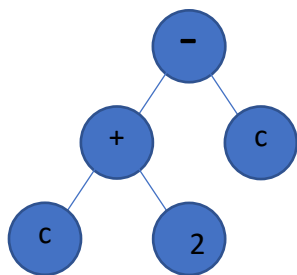
Big O worst case runtime and justification: $\rightarrow O(n)$

```
isArithmeticExpression(tree,tree.left(root))&&isArithmeticExpression(tree,tree.right(root))
```

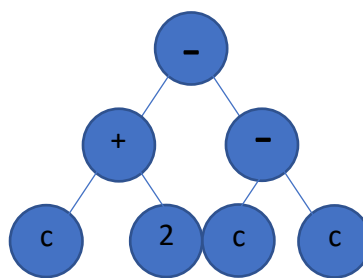
$T(n-1)+O(1)$ (first graph)

$T(n/2)+T(n/2)$ (second graph)

Since other lines are $O(1)$



$$T(n-1)+O(1)=O(n)$$



$$2T(n/2)+O(1)=O(n)$$