

# STAT8020 Quantitative Strategies and Algorithmic Trading

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Lecture 2a – 20210128

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## TTR that trades only at market close

some misc TT  
rules slide 2-7

- Emmanuel Acar, Stephen Satchell. Chapters 4, 5 & 6, Advanced Trading Rules, Second Edition. Butterworth-Heinemann; 2nd edition. June 19, 2002.
- $P_t = \log\text{-price}$ ,  $X_t = P_t - P_{t-1}$  is the continuous compounded return
- $B_t = f(P_0, P_1, \dots, P_t) = \text{trading signal } (= +1/-1)$ , to be executed at the end of day  $t$
- Acar defines  $RR_t$  (rule return in day  $t$ ) as:  
 $RR_t = B_{t-1} * X_t$ ; If  $B_{t-1} = 1$ ,  $RR_t = X_t$ ;  
If  $B_{t-1} = -1$ ,  $RR_t = -X_t$ ;

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If the price process is known, we can calculate all sorts of operating characteristics of a TTR

$RR = \text{rule return}$

- Expected rule return  $= E(RR_1 + RR_2 + \dots + RR_t) = ?$
- Risk of rule return  $= \text{Std}(RR_1 + RR_2 + \dots + RR_t) = ?$
- Expected number of days for trading signal to change from short to long, or from long to short?
- Expected length of a long/short cycle
- Expected profit from a long/short cycle
- Expected profit per day
- All the above characteristics can be derived once you know the return generating mechanism

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## Some interesting return processes

- $R_t$  are iid normal  $N(\mu, \sigma^2)$ ,  $\mu > 0$  is the drift parameter  
Random walk model (RW), Bachelier (1900),
- $R_t$  are AR(p)
- $R_t$  are MA(q)
- $R_t$  are ARMA(p, q); in particular, Taylor investigated the model ARMA(1, 1) which he called the price trend model

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## Hidden Markov Chain

- $R_t$  are driven by a Markov process with two states, a bullish state and a bearish state.

Let  $Z_t$  be the Markov chain with value +1

(bullish) and 0 (bearish), with transition matrix

$$\text{where } \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$$

$p$  is the chance of remaining in a bullish state

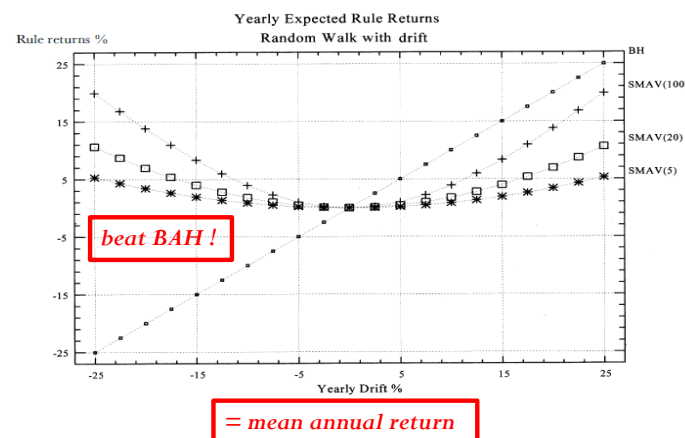
$q$  is the chance of remaining in a bearish state

- $R_t = \mu + Z_t * \pi_t - (1 - Z_t) * \delta_t$ ,  $\mu$  = long term mean return  
 $\pi_t$  are iid normal with positive mean residual return (bullish)  
 $\delta_t$  are iid normal with negative mean residual return (bearish)

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## Performance of EWMA under random walk assumption

- Although BAH is optimal under random walk assumption, it does not mean TT cannot make a profit under random walk



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## Revision of lecture 1

- Market efficiency (weak form and semi-strong form)
- Technical analysis to time the market in order to beat weak efficiency
- Fundamental analysis for stock selection to beat semi-strong efficiency
- Technical trading signal is given by  $B_t = f(P_0, P_1, \dots, P_t)$  ( $= +1/-1$ , to be executed at the end of day  $t$ ). We then discuss some price models on the return series: RW, AR(p), MA(q), ARMA(p,q), Hidden Markov chain
- Theoretically, if the price process is known, one can find a TTR that maximizes the expected profit. For example, if the price process is a random walk with positive drift, then BAH is the optimal trading rule
- What if the known price process is an AR(1)?

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## Optimal rule under AR(1) with no transaction cost

eg of TT given a time series price model AR(1)

- $X_t$  follows an AR(1) process:  
 $(X_t - \mu) = a(X_{t-1} - \mu) + \varepsilon_t$  where  $1 > a > 0$  is known,  $\varepsilon_t$  iid  $N(0, \tau^2)$ ,  $E(X_t) = \mu > 0$ .  $\text{corr}(X_t, X_{t-1}) = a$ ,  $X_t$  has a variance  $\sigma^2$
- Best forecast  $F_t$  for day  $t$ 's return is  
 $E(X_t | X_{t-1}) = a(X_{t-1} - \mu) + \mu$
- If  $X_{t-1} > (-1+a)\mu/a$  then  $B_t = +1$ ;  
 If  $X_{t-1} < (-1+a)\mu/a$  then  $B_t = -1$
- Reasonable values of  $a$ ,  $\mu$  and  $\sigma$ :  
 $a = 0.05$ ,  $\mu = 0.0004$ ,  $\sigma = 0.02$ ;
- $(1-a)\mu/a = 0.0076$ : the optimal trading rule is: when return  $< -0.76\%$ , sell short, otherwise take a long position

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## Expected profit of the optimal TTR

- Compute  $E(RR_t)$  assuming that  $\mu=0$ ,
- Let return in day  $t-1$  be  $w$ , i.e.  $X_{t-1}=w$   
 $X_t=aw + \varepsilon_t$ ,  $\varepsilon_t$  are iid  $N(0, \tau^2)$ . Construct TR  $B_t$  as follows:  
 When  $w>0$ ,  $B_{t-1}=1$ ,  $RR_t=aw + \varepsilon_t$   
 When  $w \leq 0$ ,  $B_{t-1}=-1$ ,  $RR_t=-aw + \varepsilon_t$
- $RR_t = a|w| + \varepsilon_t$ ,  
 $E(RR_t) = aE(|w|)$
- Since  $w/\sigma = z$  is a standard normal distribution,  
 $E(|w|) = \sigma E(|Z|) = \sigma \sqrt{\frac{2}{\pi}}$
- $E(RR_t) = a\sigma \sqrt{\frac{2}{\pi}}$
- The larger is  $a$ , the larger the profit

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## Expected profit of the optimal TTR (cont'd)

- Reasonable values of  $a$  ( $=0.05$ ) and  $\sigma$  ( $=0.02$ ) as before and  $\mu=0$ ,  $E(RR_t) = 0.000798$ , an annual return of 20%
- Optional exercise for those who would like to accept mathematical challenge:  
 Express  $E(RR_t)$  in terms of  $a$ ,  $\sigma$  and  $\mu$  when  $\mu \neq 0$
- Transaction cost has not been considered
- Other price process can be considered. Li and Lam (2000) fitted a ARMA(1,1) model and used the model to derive an optimal trading rule.

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## TT under a given price model

- “Optimal market timing strategies for ARMA(1,1) return processes”, W. Li and K. Lam, Advances in Investment Analysis and Portfolio Management, 2000
- First fit an ARMA(1,1) model to price data, compute an optimal strategy based on the model parameters and apply the strategy in a following period
- Another approach is to fit a Markov switching model to price data and compute the optimal filter rule under the model parameters and apply the strategy in a following period
- Xin, Yu and Lam (forthcoming) fitted a two regime Markov Switching model to six futures markets and derive the optimal filter size for each market.

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## Some market anomalies uncovered in early 1990's

study EMH using ACF, eg, sig short term ACF's violate EMH

Autocorrelations of Short-Term Returns 1896-1991 (Before publication)					
	1 Day	5 Days	10 Days	50 Days	125 Days
Correlation Coefficient	0.024	0.024	0.067	-0.01	0.067
p value	0.0002	0.097	0.001	0.923	0.356
Autocorrelations of Short-Term Returns 1992-2014 (After publication)					
	1 Day	5 Days	10 Days	50 Days	125 Days
Correlation Coefficient	-0.058	-0.103	-0.052	0.052	0.157
p value	<0.0001	0.0005	0.22	0.587	0.309
Autocorrelations of Long-Term Returns 1896-1991 (Before publication)					
	250 Days	500 Days	750 Days	1000 Days	
Correlation Coefficient	-0.128	-0.185	-0.279	-0.557	
p value	0.219	0.218	0.135	0.007	
Autocorrelations of Long-Term Returns 1992-2014 (After publication)					
	250 Days	500 Days	750 Days	1000 Days	
Correlation Coefficient	0.077	0.277	-0.05	-0.139	
p value	0.741	0.439	0.925	0.861	

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## Correlation coefficients (HSI) 1970-2014

Autocorrelations of Short-Term Returns						
	1 Day	5 Days	10 Days	30 Days	50 Days	125 Days
Correlation Coefficient	0.049	0.002	0.125	0.052	0.116	0.012
t-statistic	5.114	0.096	4.162	1.001	1.71	0.107
p value	<.0001	0.924	<.0001	0.318	0.089	0.915

Autocorrelations of Long-Term Returns				
	250 Days	500 Days	750 Days	1000 Days
Correlation Coefficient	-0.017	-0.085	-0.473	-0.46
t-statistic	-0.107	-0.363	-0.178	-1.37
p value	0.916	0.721	0.103	0.213

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## Evidence against weak EMH

- Many market anomalies disappeared after they were uncovered. They have no theoretical support.
- Empirical evidence: Positive autocorrelation of daily stock returns are observed but longer-term returns are negatively correlated.
- The idea of “short-term momentum trading and long-term contrarian trading” seems to work well and there are psychological reasons to support this anomaly
- D.M. Cutler, J. M. Poterba and L.H. Summers ( 1991 ) , 「 Speculative Dynamics 」 , The Review of Economic Studies, Vol. 58 , No.3. , pp. 529-546

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## Psychological explanation on under-and-overreaction

- Many successful trading results lack theoretical support, hence find it difficult to appear in academic journals in the 1980's
- However, short-term momentum trading and long-term contrarian trading find support from studies in human psychology.
- Strategies can either be contrarian ( against trend ) or momentum ( follow the trend ) depending on length of observation/investment periods
- Behavioral science emerges in the late 1980 and provide psychological evidence to justify the strategy
- Over-confidence and Conservatism to explain short- term momentum and long-term contrarian.
- Daniel, Hirshleifer and Subrahmanrum (1998), 'Investor psychology and security market under-and overreaction'

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*also called look-back method  
= here till last slide*

## Momentum strategy on a single asset

- Consider a single asset. look-back period=m days. holding period=n days following the look-back period
- Positive correlation of holding period return and look-back return
- Hence momentum trading works
- Why time series momentum exists?
  1. The slow diffusion, analysis, and acceptance of new information
  2. The forced sales or purchases of various type of funds
  3. Market manipulation by high frequency traders
  4. Persistence of roll returns in futures contracts
- Performance of momentum trading in two-year Treasury note future can be found in Chapter 6 of 'Algorithmic Trading' by Chan

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## Roll returns to explain interday momentum

- Futures return = spot return + roll return
- However, futures may have a discount or premium over spot
- If futures < spot: under backwardation
- If futures > spot: under contango
- At expiration: futures = spot
- If currently under backwardation/contango, the backwardation/contango will persist and gradually shrink to 0. This contributes to profitability of overnight momentum strategies

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## Return correlation reported by Chan two-year treasury note futures

look-back	holding	corr.coef	p-value
60	1	0.0313	0.169
60	5	0.0799	0.117
60	10	0.1718	0.017
60	25	0.2592	0.023
60	60	0.2162	0.234
60	120	-0.0331	0.860
60	250	0.3137	0.097

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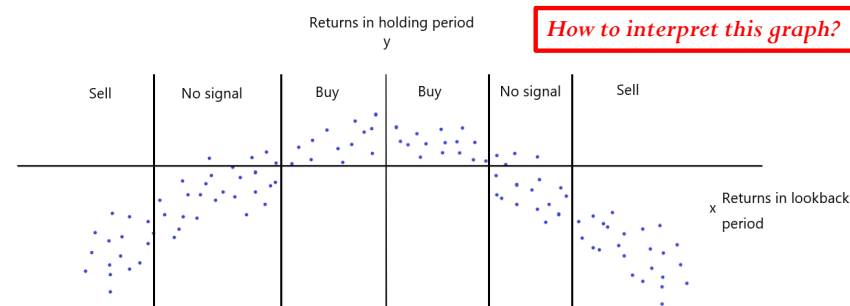
## Choice of algo parameters from back-testing results

- Largest mean profit, largest correlation or largest t?
- High correlation implies high predictability
- Needs a small p-value for significant evidence
- Compromise between correlation coefficient and p-value
- Chan chose (60,20), (60,25), (250,10), (250,60) (250,60) and (250,120)
- My comment: Choosing strategies based on correlation coefficient alone may not be the most effective method
- Correlation coefficient measures linear relationship. Curvilinear relationship may be present not detected by correlation but useful for trading purposes

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## An example of curvilinear relationship

- Curvilinear relationship can also contribute to profit



- Strategy profit is the most important

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## Equity curve for the momentum strategy

= cumulative  
wealth / return

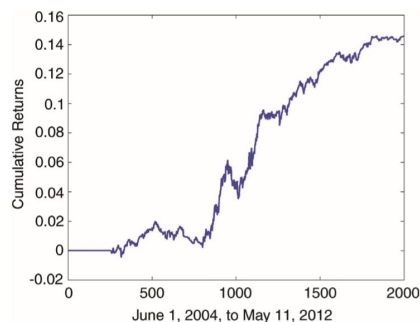


FIGURE 6.2 Equity Curve of TU Momentum Strategy

## Momentum strategy on HSI

- Daily closing index of HSI from 1980 onwards

look-back	holding	corr. Coeff	p-value
10	1	0.021	0.042
10	2	0.028	0.006
10	3	0.038	0.000
10	4	0.041	0.000
10	5	0.045	0.000
10	6	0.049	0.000
10	7	0.051	0.000
10	8	0.051	0.000
10	9	0.049	0.000
10	10	0.046	0.000

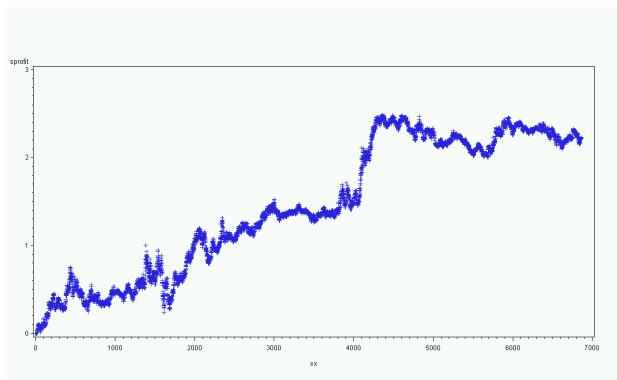
## SAS program to compute mean profit using 20 days look-back and 1 day holding period

- data hsi20\_1;
- set hsi\_ind;
- hold1ret=log(ind)-log(lag(ind));
- lookback20ret=log(ind)-log(lag20(ind));
- lookback20ret1=lag(lookback20ret);
- signal20\_1=sign(lookback20ret1);
- profit=signal20\_1\*hold1ret;
- sprofit+profit;
- year=int(date/10000);
- recent=1;
- if year<2015 then recent=0;
- xx=\_N\_;
- run;

## SAS program to plot equity curve

- proc corr data=hsi20\_1;
- var hold1ret lookback20ret1;
- by recent;
- run;
- proc means mean sum t std data=hsi20\_1;
- var profit;
- class recent;
- run;
- proc gplot;
- plot sprofit\*xx;
- run;
- .

## Equity curve for HSI momentum trading



## Dataset hsif was uploaded to moodle

- This is a very comprehensive daily dataset (hi1\_20170701\_20200609.csv in moodle under project) which I have collected and should be useful in devising overnight trading strategies for trading in various futures markets
- You can make use of it in doing your project

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## Short-term momentum and long-term contrarian

- If we change the investment horizon to a longer term, like 700 days ( roughly 3 years ) look-back and 250 days ( roughly one year ) holding, there is negative correlation between the returns
- Correlation coefficient becomes -0.366

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## Continuity in holding a futures contract

- Transaction cost much lower than trading stocks
- However, there is the complication in rolling over a futures contract to next month
- For look-back period, can use indices and roll over is not a problem *look-back=no position, roll over not needed, no problem !*
- Take more care to calculate the holding period return in which the roll over cost may surface

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