Jason Chapman
ECE 131A
December 9, 2022

# *Final Project*

1. *Tossing a Fair and Unfair Die*
   a. See Appendix for code.
   The estimated probabilities that X has odd value are shown below for each t, where t = [50, 100, 1000, 2000, 3000] respectively.

p_odds_a

1x5 double

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0.4000 | 0.5200 | 0.4940 | 0.5045 | 0.4903 |

   b. Mathematical analysis for fair die:

$$S_x = \{1,2,3,4,5,6,7,8\}$$

$$P(X=k) = \begin{cases} \frac{1}{8} & k \in S_x \\ 0 & \text{else} \end{cases}$$

A: X has odd value

$$P(A) = P(X=1)+P(X=3)+P(X=5)+P(X=7) = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow \boxed{P(X \text{ has odd value}) = \frac{1}{2}}$$

   c. t=50: No, the probability from simulation does not match the analytical result
   t=100: No
   t=1000: No
   t=2000: Yes
   t=3000: Yes

   As the number of tosses, t, increases, the estimated probability of an odd role from simulation gets closer to the analytical result, P(A)=0.5.

d. See Appendix for code.
The estimated probabilities that X has odd value are shown below for each t, where t = [50, 100, 1000, 2000, 3000] respectively.

p_odds_d

1x5 double

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0.4400 | 0.5200 | 0.4940 | 0.4945 | 0.5050 |

Mathematical analysis for unfair die:

$S_X = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$P(X=k) = \begin{cases} \frac{1}{5} & k = 1, 2 \\ \frac{1}{10} & k = 3, 4, 5, 6, 7, 8 \\ 0 & \text{else} \end{cases}$$

A: X has odd value

$$P(A) = P(X=1) + P(X=3) + P(X=5) + P(X=7) = \frac{1}{5} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \boxed{P(X \text{ has odd value}) = \frac{1}{2}}$$

t=50: No, the probability from simulation does not match the analytical result
t=100: No
t=1000: No
t=2000: Yes
t=3000: Yes

Like before, as the number of tosses, t, increases, the estimated probability of an odd role from simulation gets closer to the analytical result, P(A)=0.5.

2. *Maximum Likelihood Estimation*
    a. Analytical proof:

$X_1, X_2, X_3, \ldots X_{100000}$ where $X_i \sim N(\mu, \sigma^2)$ & $X$'s are iid.

- Since $X_i$'s are independent, the joint distribution is just the product of the individual pdfs

$$f_x(x_1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f_{x_1,x_2}(x_1, x_2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^2 e^{-\frac{(x_1-\mu)^2}{2\sigma^2} - \frac{(x_2-\mu)^2}{2\sigma^2}}$$

$$\vdots$$

$$f_{x_1,x_2,\ldots x_n}(x_1, x_2 \ldots x_n | \mu, \sigma) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\log\left(f_{x_1,x_2\ldots x_n}(x_1,x_2,\ldots,x_n|\mu,\sigma)\right) = \log\left[\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}}\right]$$

$$= \log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n + \log e^{-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}} \quad // \text{ log base: } e$$

$$= n\left[\cancelto{0}{\log(1)} - \log\left(\sqrt{2\pi\sigma^2}\right)\right] - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= -n\left[\log\left(2\pi\sigma^2\right)^{1/2}\right] - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= -\frac{n}{2}\log 2\pi\sigma^2 - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2} \quad \checkmark$$

b. Analytical expressions for $\mu_{MLE}, \sigma_{MLE}$:

$$\mu_{MLE} = \underset{\mu}{\text{argmax}} \left[ \log \left( f_{X_1, X_2, \ldots X_n}(X_1, X_2, \ldots X_n \mid \mu, \sigma) \right) \right] \qquad \text{// take derivative \$ set to zero}$$

$$\frac{d}{d\mu} \left[ -\frac{n}{2} \log 2\pi\sigma^2 - \sum_{i=1}^{n} \frac{(X_i - \mu)^2}{2\sigma^2} \right] = \sum_{i=1}^{n} \frac{2(X_i - \mu)}{2\sigma^2} = 0$$

$$\Rightarrow \sum_{i=1}^{n} (X_i - \mu) = 0 \quad \Rightarrow \quad \sum_{i=1}^{n} X_i - \sum_{i=1}^{n} \mu = 0$$

$$\Rightarrow \boxed{\mu_{MLE} = \frac{\sum_{i=1}^{n} X_i}{n}}$$

$$\sigma_{MLE} = \underset{\sigma}{\text{argmax}} \left[ \log \left( f_{X_1, X_2, \ldots X_n}(X_1, X_2, \ldots X_n \mid \mu_{MLE}, \sigma) \right) \right] \qquad \text{// take derivative \$ set to zero}$$

$$\frac{d}{d\sigma} \left[ -\frac{n}{2} \log 2\pi\sigma^2 - \sum_{i=1}^{n} (X_i - \mu)^2 (2\sigma^2)^{-1} \right] = -\frac{n}{2} \cdot \frac{1}{2\pi\sigma^2} \cdot 4\pi\sigma + \sum_{i=1}^{n} (X_i - \mu)^2 (2\sigma^2)^{-2} \cdot 4\sigma$$

$$\Rightarrow \frac{n}{\sigma} + \frac{\sum_{i=1}^{n} (X_i - \mu)^2}{\sigma^3} = 0$$

$$\Rightarrow \sum_{i=1}^{n} (X_i - \mu_{MLE})^2 = \frac{n}{\sigma} \cdot \sigma^3 \quad \Rightarrow \boxed{\sigma_{MLE} = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \mu_{MLE})^2}{n}}}$$

See Appendix for code.

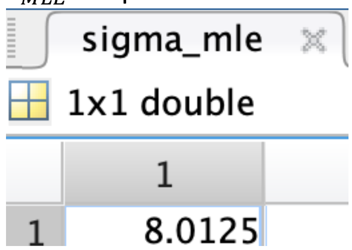
$\mu_{MLE}$ output:

mew_mle

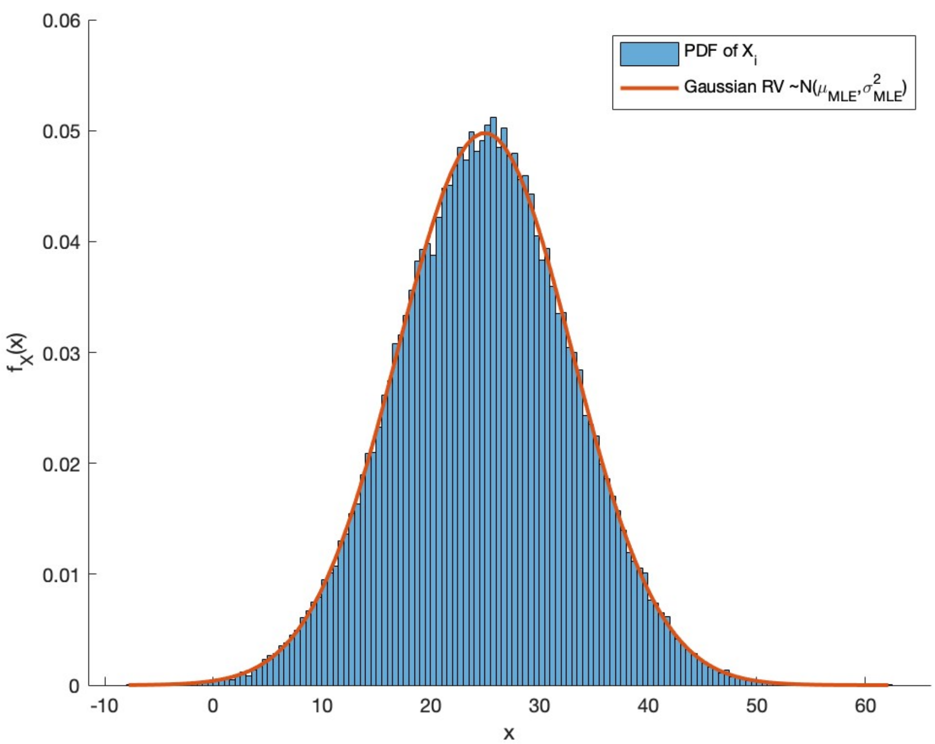
1x1 double

| | 1 |
|---|---|
| 1 | 24.9842 |

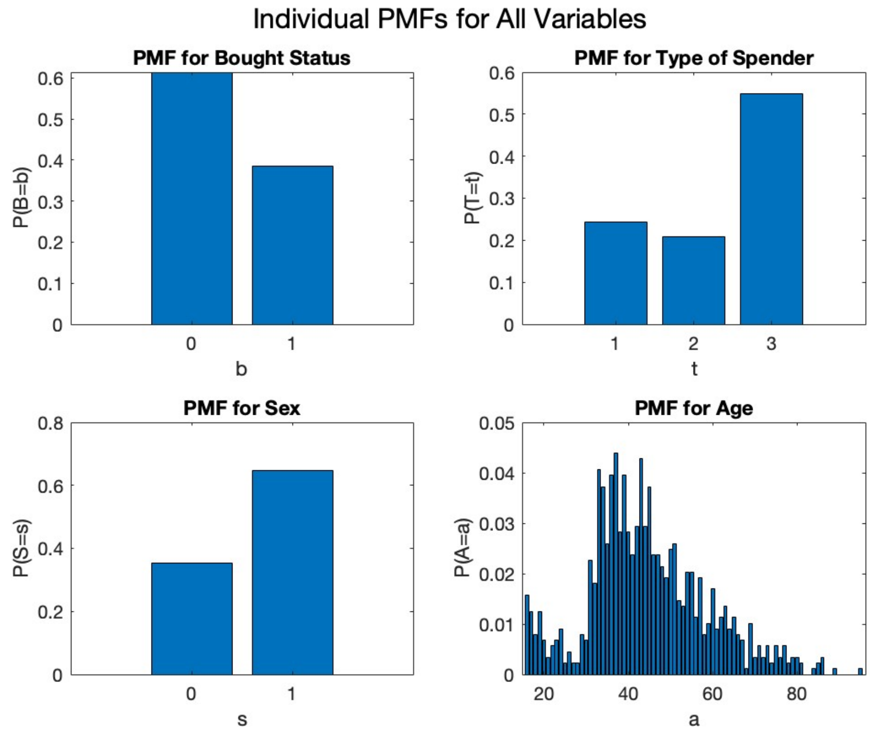
$\sigma_{MLE}$ output:



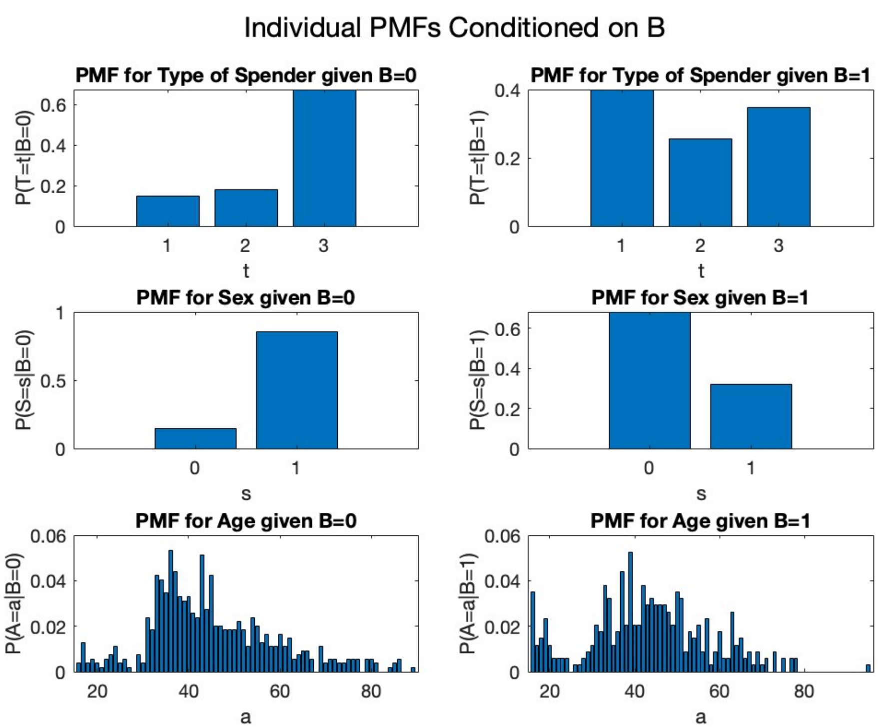

c. Plot of PDF of Xi superimposed with Gaussian RV:



3. *Naïve Bayes Classifier*
   a. Estimated individual PMFs:




   b. Conditional PMFs:



c. How to compute these probabilities:

$P(B=0, T=1, S=0, A \leq 55)$ // conditional independence $P(X \cap Y) = P(Y|X)P(X)$

$\Rightarrow P(T=1, S=0, A \leq 55 | B=0)P(B=0)$

$\Rightarrow P(T=1|B=0) \cdot P(S=0|B=0) \cdot P(A \leq 55|B=0) \cdot P(B=0)$

$$\Rightarrow \boxed{\begin{array}{l} P(B=0,T=1,S=0,A\leq 55) = P(T=1|B=0)P(S=0|B=0)P(A\leq 55|B=0)P(B=0) \\ \text{Similarly,} \\ P(B=1,T=1,S=0,A\leq 55) = P(T=1|B=1)P(S=0|B=1)P(A\leq 55|B=1)P(B=1) \end{array}}$$

$P(B=0, T=1, S=0, A \leq 55)$ output:

p_b0_t1_s0_a55

1x1 double

| | 1 | 2 |
|---|---|---|
| 1 | 0.0106 | |

$P(B=1, T=1, S=0, A \leq 55)$ output:

p_b1_t1_s0_a55

1x1 double

| | 1 | 2 |
|---|---|---|
| 1 | 0.0846 | |

d. How to compute these probabilities:

$$P(B=0 \mid T=1, S=0, A \le 55) = \frac{P(T=1, S=0, A \le 55 \mid B=0)\,P(B=0)}{P(T=1, S=0, A \le 55)} \quad \text{// Bayes Rule}$$

$$\text{//}\ P(T=1, S=0, A \le 55) = P(T=1, S=0, A \le 55 \mid B=0)\,P(B=0) + P(T=1, S=0, A \le 55 \mid B=1)\,P(B=1)$$

// Total Probability Law

$$\Rightarrow P(B=0 \mid T=1, S=0, A \le 55) = \frac{P(T=1, S=0, A \le 55 \mid B=0)\,P(B=0)}{P(T=1, S=0, A \le 55 \mid B=0)\,P(B=0) + P(T=1, S=0, A \le 55 \mid B=1)\,P(B=1)}$$

Similarly,

$$P(B=1 \mid T=1, S=0, A \le 55) = \frac{P(T=1, S=0, A \le 55 \mid B=1)\,P(B=1)}{P(T=1, S=0, A \le 55 \mid B=0)\,P(B=0) + P(T=1, S=0, A \le 55 \mid B=1)\,P(B=1)}$$

$P(B = 0|T = 1, S = 0, A \le 55)$ output:

pb0_conditioned

1x1 double

| | 1 | 2 |
|---|---|---|
| 1 | 0.1116 | |

$P(B = 1|T = 1, S = 0, A \le 55)$ output:

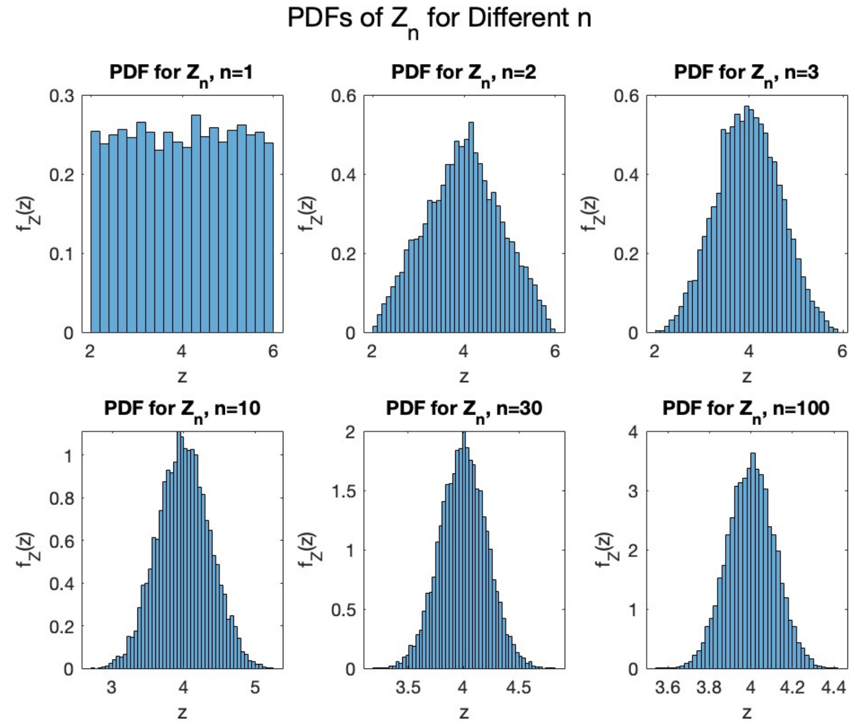
pb1_conditioned

1x1 double

| | 1 | 2 |
|---|---|---|
| 1 | 0.8884 | |

A female whose age is below 55 and who is a large spender WILL buy this product according to the Naïve Bayes Classifier. The probability that the individual will buy the product given the evidence is much higher than the probability that the individual will not buy the product given the evidence.

4. *Central Limit Theorem*
    a. See Appendix for code
    PDFs of Zn for different n:




As n increases, the distribution of Zn approaches a Gaussian distribution. This is evidence in the plots shown above.

b. Analytical derivation of mean and variance:

$X_i \sim \text{Uniform}(2,6)$

$$f_X(x) = \frac{1}{b-a} = \frac{1}{6-2} = \frac{1}{4} \qquad x \in (2,6)$$

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_2^6 x\left(\frac{1}{4}\right)dx = \frac{x^2}{8}\Big|_2^6 = \frac{36-4}{8} = 4$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_2^6 x^2\left(\frac{1}{4}\right)dx = \frac{x^3}{12}\Big|_2^6 = \frac{216-8}{12} = 17.33$$

$$VAR(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = 17.33 - 4^2 = \frac{4}{3}$$

$$Z_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\mathbb{E}[Z_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] = \frac{1}{n}\cdot n \cdot 4 = 4$$

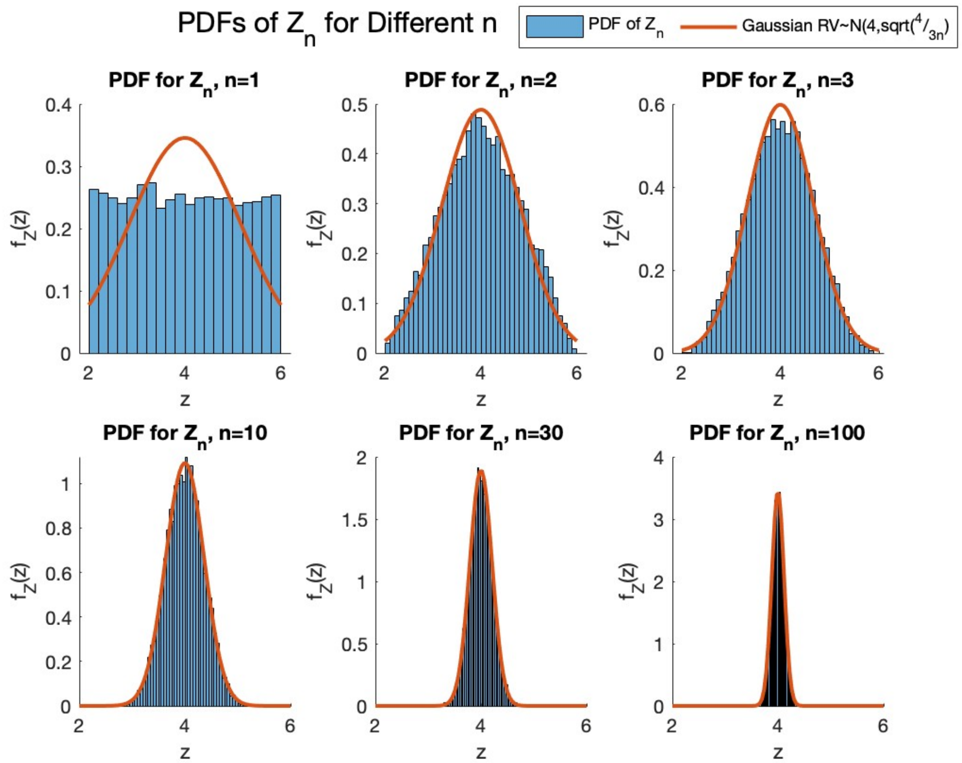
-Let $S_n = X_1 + X_2 + \dots + X_n$

$$VAR(S_n) = n \cdot VAR(X_i) = n \cdot \frac{4}{3} \qquad // \ X\text{'s iid}$$

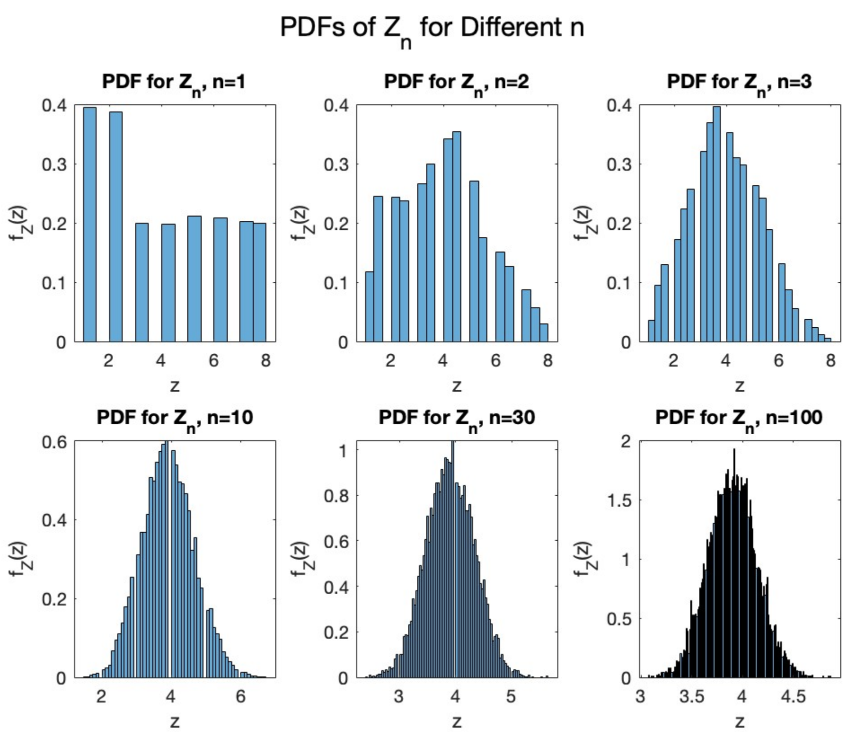
$$VAR(Z_n) = VAR\left(\frac{1}{n}S_n\right) = \frac{1}{n^2}VAR(S_n) = \frac{1}{n^2}\cdot n \cdot \frac{4}{3} = \frac{4}{3n}$$

$$\Rightarrow \boxed{\begin{array}{ll} \mathbb{E}[X] = 4 & VAR(X) = \frac{4}{3} \\ \mathbb{E}[Z_n] = 4 & VAR(Z_n) = \frac{4}{3n} \end{array}}$$

c. See Appendix for Code
PDFs of Zn for different n superimposed with Gaussian RV:




d. PDFs of Zn for different n:



Analytical derivation of mean and variance:

$$P(X=k) = \begin{cases} \frac{1}{5} & k=1,2 \\ \frac{1}{10} & k=3,4,5,6,7,8 \\ 0 & \text{else} \end{cases}$$

$$E[X] = \sum_{X_i \in S_X} k\,P(X=k) = 1\left(\tfrac{1}{5}\right)+2\left(\tfrac{1}{5}\right)+3\left(\tfrac{1}{10}\right)+4\left(\tfrac{1}{10}\right)+5\left(\tfrac{1}{10}\right)+6\left(\tfrac{1}{10}\right)+7\left(\tfrac{1}{10}\right)+8\left(\tfrac{1}{10}\right)$$

$$= 3.9$$

$$E[X^2] = \sum_{X_i \in S_X} k^2\,P(X=k) = 1^2\left(\tfrac{1}{5}\right)+2^2\left(\tfrac{1}{5}\right)+3^2\left(\tfrac{1}{10}\right)+4^2\left(\tfrac{1}{10}\right)+5^2\left(\tfrac{1}{10}\right)+6^2\left(\tfrac{1}{10}\right)+7^2\left(\tfrac{1}{10}\right)+8^2\left(\tfrac{1}{10}\right)$$

$$= 20.9$$

$$VAR(X) = E[X^2] + (E[X])^2 = 20.9 - (3.9)^2 = 5.69$$

$$Z_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$E[Z_n] = E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n}\cdot n \cdot 3.9 = 3.9$$

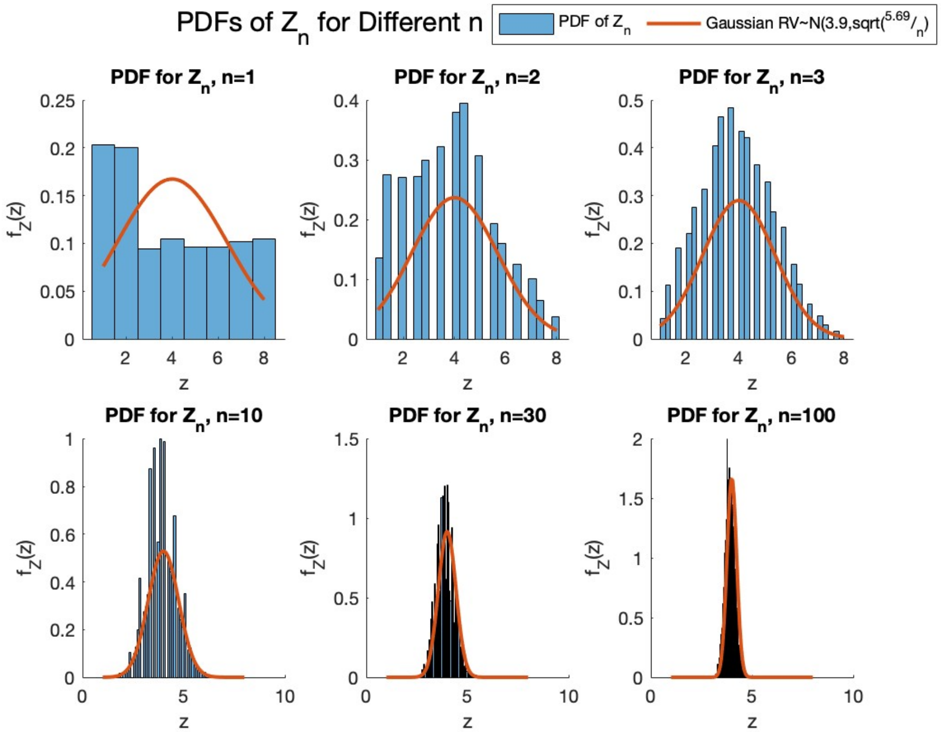
- Let $S_n = X_1 + X_2 + \ldots + X_n$

$$VAR(S_n) = n \cdot VAR(X_i) = n \cdot 5.69 \quad // \; X_i \text{ iid}$$

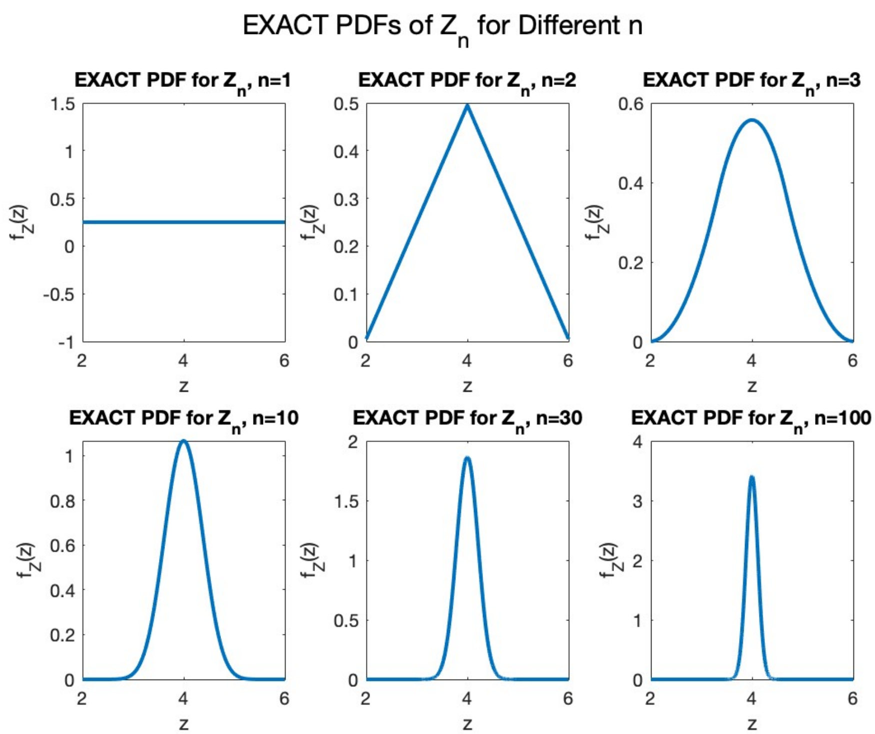
$$VAR(Z_n) = VAR\left(\frac{1}{n}S_n\right) = \frac{1}{n^2} VAR(S_n) = \frac{1}{n^2}\cdot n \cdot 5.69 = \frac{5.69}{n}$$

$$\Rightarrow \boxed{\begin{array}{ll} E[X] = 3.9 & VAR(X) = 5.69 \\ E[Z_n] = 3.9 & VAR(Z_n) = \frac{5.69}{n} \end{array}}$$

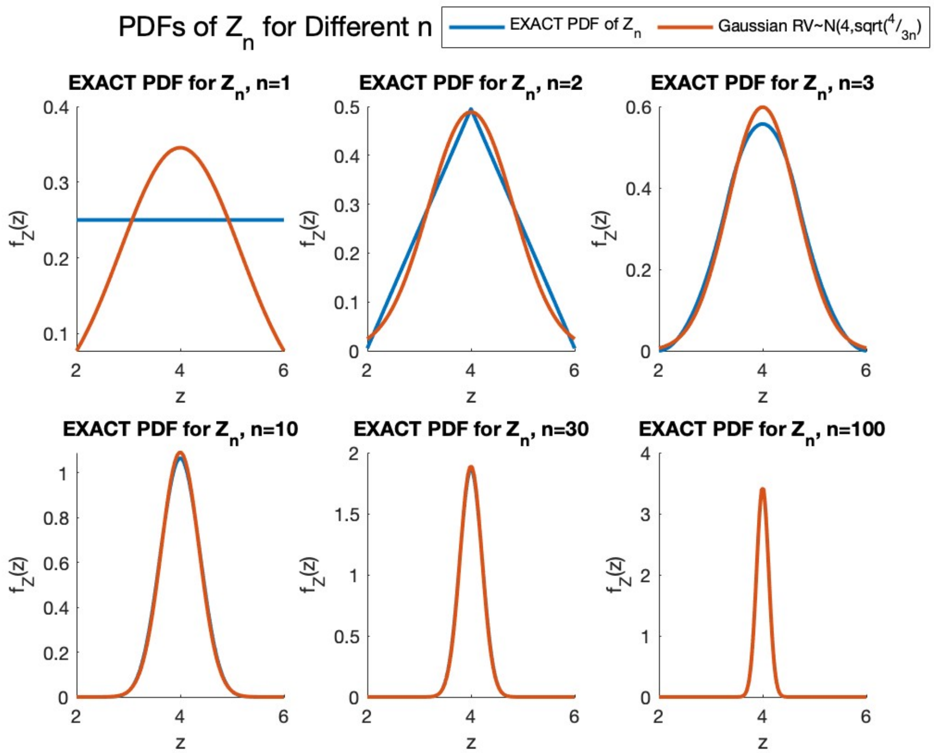
PDFs of Zn for different n superimposed with Gaussian RV:




e. Exact PDFs of Zn for different n:



Exact PDFs of Zn for different n superimposed with Gaussian RV:



How to compute PDF of Zn:

$X_i \sim \text{Uniform}(2,6)$

Let $S_2 = X_1 + X_2 \Rightarrow Z_2 = \frac{1}{2} S_2$

$$f_{X_1}(x_1) = f_{X_2}(x_2) = \begin{cases} \frac{1}{4} & x \in [2,6] \\ 0, & \text{else} \end{cases}$$

$$f_S(s) = \int_{-\infty}^{\infty} f_{X_1,X_2}(x_1, s - x_1)\, dx.$$

// We know $X_i$'s are independent

$$f_{X_1,X_2}(x_1, x_2) = f_{X_1}(x_1) \cdot f_{X_2}(x_2)$$

convolution

$$\Rightarrow f_S(s) = \int_{-\infty}^{\infty} f_{X_1}(x_1) \cdot f_{X_2}(s - x_1)\, dx = f_{X_1}(x_1) * f_{X_2}(x_2)$$

- So, to get pdf of $Z_n$, we find the convolution of $n$ uniform distributions & then divided by $n$.

Thus to find $Z_2$, it is the convolution of $f_{X_1}(x_1) * f_{X_2}(x_2)$ divided by 2. Then, for $Z_3$ it will be the convolution of the convolution of $f_{X_1}(x_1)$ & $f_{X_2}(x_2)$ & $f_{X_3}(x_3)$.

$$\Rightarrow \left[f_{X_1}(x_1) * f_{X_2}(x_2)\right] * \left[f_{X_3}(x_3)\right] \quad \text{\& so on}$$

for whatever $n$.

APPENDIX BELOW:

# Table of Contents



```
% ECE 131A
% Jason Chapman
% Project
close all; clearvars; clc;
```

# #1

1a

```
t = [50, 100, 1000, 2000, 3000];
for i = 1:length(t)
    data = eight_sided_dice(t(i));
    n_odds = sum(mod(data,2)==1);
    p_odds_a(i) = n_odds/t(i);
end

% 1d
for i = 1:length(t)
    data = eight_sided_dice_w_params(t(i));
    n_odds = sum(mod(data,2)==1);
    p_odds_d(i) = n_odds/t(i);
end
```

# #2

2b

```
f = fopen('data.txt');
data = textscan(f,'%s');
fclose(f);
x = str2double(data{1,1});

n = length(x);
mew_mle = sum(x)/n;
for i = 1:n
    sum_term(i) = (x(i)-mew_mle)^2;
end
sigma_mle = sqrt(sum(sum_term)/n);

% 2c
figure(1)
hold on
```

```
histogram(x,'Normalization','pdf')
x_gauss = linspace(min(x),max(x));
y_gauss = normpdf(x_gauss,mew_mle,sigma_mle);
plot(x_gauss,y_gauss,'Linewidth',2)
xlabel('x')
ylabel('f_{X}(x)')
legend('PDF of X_{i}','Gaussian RV ~N(\mu_{MLE},\sigma_{MLE}^{2})')
```

# 3

```
data = readmatrix('user_data.csv');

% 3a
str = ["Bought Status","Type of Spender","Sex","Age"];
strcap = ["B","T","S","A"];
strlo = ["b","t","s","a"];
for i = 1:4
    var = data(:,i);
    for j = 1:length(unique(var))
        unique_vals = unique(var);
        p(j) = numel(find(var==unique_vals(j)))/length(var);
        n(j) = unique_vals(j);
    end
    figure(2)
    subplot(2,2,i)
    bar(n,p)
    title(strcat("PMF for ",str(i)))
    xlabel(strlo(i))
    ylabel(strcat("P(",strcap(i),"=",strlo(i),")"))
    clear p n
end
sgtitle('Individual PMFs for All Variables')

% 3b
i_b0 = find(data(:,1)==0);
i_b1 = find(data(:,1)==1);
for i = 1:3
    var = data(:,i+1);
    var_b0 = var(i_b0);
    for j = 1:length(unique(var_b0))
        unique_vals_b0 = unique(var_b0);
        p_b0(j) = numel(find(var_b0==unique_vals_b0(j)))/length(var_b0);
        n_b0(j) = unique_vals_b0(j);
    end
    var_b1 = var(i_b1);
    for j = 1:length(unique(var_b1))
        unique_vals_b1 = unique(var_b1);
        p_b1(j) = numel(find(var_b1==unique_vals_b1(j)))/length(var_b1);
        n_b1(j) = unique_vals_b1(j);
    end
    figure(3)
    subplot(3,2,2*i-1)
    bar(n_b0,p_b0)
```

```
    title(strcat("PMF for ",str(i+1)," given B=0"))
    xlabel(strlo(i+1))
    ylabel(strcat("P(",strcap(i+1),"=",strlo(i+1),"|B=0)"))
    subplot(3,2,2*i)
    bar(n_b1,p_b1)
    title(strcat("PMF for ",str(i+1)," given B=1"))
    xlabel(strlo(i+1))
    ylabel(strcat("P(",strcap(i+1),"=",strlo(i+1),"|B=1)"))
    clear p_b0 n_b0 p_b1 n_b1
end
sgtitle('Individual PMFs Conditioned on B')

% 3c
data_b0 = data(i_b0,:);
data_b1 = data(i_b1,:);
pb0 = length(data_b0)/length(data);
pb1 = length(data_b1)/length(data);
pt1_b0 = numel(find(data_b0(:,2)==1))/length(data)/pb0;
pt1_b1 = numel(find(data_b1(:,2)==1))/length(data)/pb1;
ps0_b0 = numel(find(data_b0(:,3)==0))/length(data)/pb0;
ps0_b1 = numel(find(data_b1(:,3)==0))/length(data)/pb1;
pa55_b0 = numel(find(data_b0(:,4)<=55))/length(data)/pb0;
pa55_b1 = numel(find(data_b1(:,4)<=55))/length(data)/pb1;
p_b0_t1_s0_a55 = pt1_b0*ps0_b0*pa55_b0*pb0;
p_b1_t1_s0_a55 = pt1_b1*ps0_b1*pa55_b1*pb1;

% 3d
pb0_conditioned = p_b0_t1_s0_a55/(p_b0_t1_s0_a55+p_b1_t1_s0_a55);
pb1_conditioned = p_b1_t1_s0_a55/(p_b0_t1_s0_a55+p_b1_t1_s0_a55);
```

# 4

```
t = 10^4;
% 4a
n = [1,2,3,10,30,100];
for i = 1:length(n)
    for j = 1:t
        for k = 1:n(i)
            X(k) = unifrnd(2,6);
        end
        Zn(j) = sum(X)/n(i);
    end
    figure(4)
    subplot(2,3,i)
    histogram(Zn,'Normalization','pdf')
    ylabel('f_{Z}(z)')
    xlabel('z')
    title(strcat('PDF for Z_{n}, n=',num2str(n(i))))
    clear X
end
sgtitle('PDFs of Z_{n} for Different n')

% 4c
```

```
for i = 1:length(n)
    for j = 1:t
        for k = 1:n(i)
            X(k) = unifrnd(2,6);
        end
        Zn(j) = sum(X)/n(i);
    end
    x_gauss = linspace(2,6);
    variance = 4/(3*n(i));
    y_gauss = normpdf(x_gauss,4,sqrt(variance));
    figure(5)
    subplot(2,3,i)
    hold on
    histogram(Zn,'Normalization','pdf')
    plot(x_gauss,y_gauss,'Linewidth',2)
    ylabel('f_{Z}(z)')
    xlabel('z')
    title(strcat('PDF for Z_{n}, n=',num2str(n(i))))
    clear X
end
sgtitle('PDFs of Z_{n} for Different n                              ')
lg  = legend('PDF of Z_{n}','Gaussian RV~N(4,sqrt(^{4}/
_{3n})','Orientation','Horizontal');

% 4d
for i = 1:length(n)
    for j = 1:t
        for k = 1:n(i)
            X(k) = eight_sided_dice_w_params(1);
        end
        Zn(j) = sum(X)/n(i);
    end
    figure(6)
    subplot(2,3,i)
    histogram(Zn,'Normalization','pdf','BinWidth',1/(n(i)+1))
    ylabel('f_{Z}(z)')
    xlabel('z')
    title(strcat('PDF for Z_{n}, n=',num2str(n(i))))
    clear X
end
sgtitle('PDFs of Z_{n} for Different n')

for i = 1:length(n)
    for j = 1:t
        for k = 1:n(i)
            X(k) = eight_sided_dice_w_params(1);
        end
        Zn(j) = sum(X)/n(i);
    end
    x_gauss = linspace(1,8);
    variance = 5.69/n(i);
    y_gauss = normpdf(x_gauss,4,sqrt(variance));
    figure(7)
    subplot(2,3,i)
```

```
    hold on
    histogram(Zn,'Normalization','pdf')
    plot(x_gauss,y_gauss,'Linewidth',2)
    ylabel('f_{Z}(z)')
    xlabel('z')
    title(strcat('PDF for Z_{n}, n=',num2str(n(i))))
    clear X
end
sgtitle('PDFs of Z_{n} for Different n                              ')
lg  = legend('PDF of Z_{n}','Gaussian RV~N(3.9,sqrt(^{5.69}/
_{n})','Orientation','Horizontal');

% 4e

for i = 1:length(n)
    x = linspace(2,6);
    y = unifpdf(x,2,6);
    if i == 1
        conv_pdf = y;
    else
        for j = 1:n(i)-1
            if j == 1
                conv_pdf = y;
            end
            conv_pdf = conv(conv_pdf,y);
        end
    end
    x = linspace(2,6,length(conv_pdf));
    A = trapz(x,conv_pdf);
    conv_pdf = conv_pdf/A;
    figure(8)
    subplot(2,3,i)
    plot(x,conv_pdf,'Linewidth',2)
    ylabel('f_{Z}(z)')
    xlabel('z')
    title(strcat('EXACT PDF for Z_{n}, n=',num2str(n(i))))
end
sgtitle('EXACT PDFs of Z_{n} for Different n')

for i = 1:length(n)
    x = linspace(2,6);
    y = unifpdf(x,2,6);
    x_gauss = linspace(2,6);
    variance = 4/(3*n(i));
    y_gauss = normpdf(x_gauss,4,sqrt(variance));
    if i == 1
        conv_pdf = y;
    else
        for j = 1:n(i)-1
            if j == 1
                conv_pdf = y;
            end
            conv_pdf = conv(conv_pdf,y);
        end
```

```
    end
    x = linspace(2,6,length(conv_pdf));
    A = trapz(x,conv_pdf);
    conv_pdf = conv_pdf/A;
    figure(9)
    subplot(2,3,i)
    hold on
    plot(x,conv_pdf,'Linewidth',2)
    plot(x_gauss,y_gauss,'Linewidth',2)
    ylabel('f_{Z}(z)')
    xlabel('z')
    title(strcat('EXACT PDF for Z_{n}, n=',num2str(n(i))))
end
sgtitle('PDFs of Z_{n} for Different n
    ')
lg  = legend('EXACT PDF of Z_{n}','Gaussian RV~N(4,sqrt(^{4}/
_{3n})','Orientation','Horizontal');


function t = eight_sided_dice(sims)
    for i = 1:sims
        t(i) = randi(8);
    end
end
function t = eight_sided_dice_w_params(sims)
    for i = 1:sims
        num = randi(10);
        if num == 1 || num == 2
            t(i) = 1;
        elseif num == 3 || num == 4
            t(i) = 2;
        elseif num == 5
            t(i) = 3;
        elseif num == 6
            t(i) = 4;
        elseif num == 7
            t(i) = 5;
        elseif num == 8
            t(i) = 6;
        elseif num == 9
            t(i) = 7;
        elseif num == 10
            t(i) = 8;
        end
    end
end
```

*Published with MATLAB® R2022b*