CS 264A | HW #3 | Jason Chapman

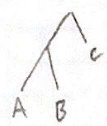
① $f = (\neg A \vee \neg B \vee C) \wedge (B \vee \neg C)$

a.) $X = \{A, B\}$, $Y = \{C\}$

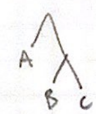
prime	sub
$A \wedge B$	C
$A \wedge \bar{B}$	$\bar{C}$
$\bar{A} \wedge B$	true
$\bar{A} \wedge \bar{B}$	$\bar{C}$

$\Rightarrow \quad f = (A \wedge B)(C) + (\bar{A} \wedge B)(\text{true}) + (\bar{B})(\bar{C})$

b.) Tree a



c) OBDDs are SDDs constructed using right-linear vtrees, so b should be used. Vtree c



will also lead to an OBDD, but with different variable ordering.

② $f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$

a.) $X = \{A, C\}$, $Y = \{B, D\}$

prime	sub
$A \wedge C$	$B \vee D$
$A \wedge \bar{C}$	B
$\bar{A} \wedge C$	$B \vee D$
$\bar{A} \wedge \bar{C}$	false

$\Rightarrow \quad f = (C)(B \vee D) + (A \wedge \bar{C})(B) + (\bar{A} \wedge \bar{C})(\text{false})$

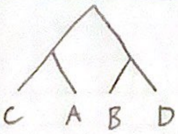
$\neg f = (\neg A \vee \neg B) \wedge (\neg B \vee \neg C) \wedge (\neg C \vee \neg D)$

prime	sub
$A \wedge C$	$\neg B \wedge \neg D$
$A \wedge \bar{C}$	$\neg B$
$\bar{A} \wedge C$	$\neg B \wedge \neg D$
$\bar{A} \wedge \bar{C}$	True

$\Rightarrow \quad \neg f = (C)(\neg B \wedge D) + (A \wedge \bar{C})(\neg B) + (\bar{A} + \bar{C})(\text{true})$

b.) In general to find an (X,Y)-partition for $\neg f$ from an (X,Y)-partition of f, simply <u>negate every sub</u> in the (X,Y)-partition of f.

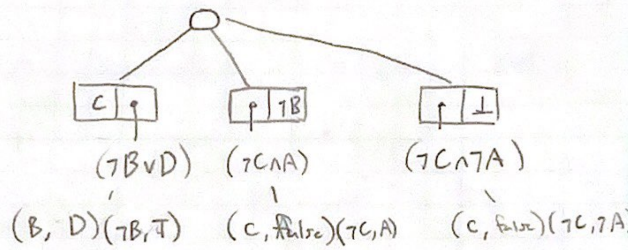
③ $f = (A \wedge \neg B) \vee (\neg B \wedge C) \vee (C \wedge D)$



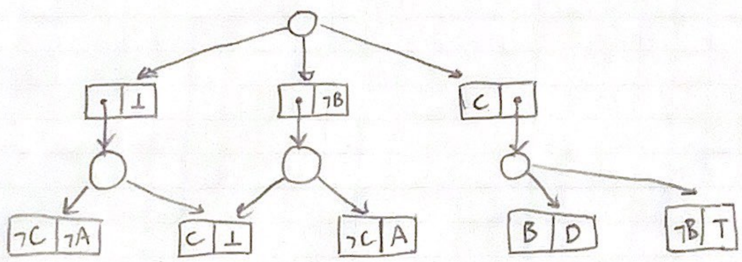
$X = \{C, A\}, \; Y = \{B, D\}$

prime	sub
$C \wedge A$	$\neg B \vee D$
$C \wedge \bar{A}$	$\neg B \vee D$
$\bar{C} \wedge A$	$\neg B$
$\bar{C} \wedge \bar{A}$	false

$\Rightarrow \; f = (C)(\neg B \vee D) + (\neg C \wedge A)(\neg B) + (\neg C \wedge \neg A)(\text{false})$



$\Rightarrow$



④ a.) The CNF ~~for~~ for this Boolean formula is $\bigwedge_{\substack{S \subseteq \{1, \ldots, n\} \\ |S| = k+1}} \left(\bigvee_{j \in S} X_j \right)$

In order to complete this you need to determine all combinations of k+1 objects in list of n items. Each value in the combination is disjoined & the results of each disjoined combination is conjoined. This ensures that every combination contains at least one high value

b.) The DNF for this Boolean formula is $\bigvee_{\substack{S \subseteq \{1, \ldots, n\} \\ |S| = k+1}} \left(\bigwedge_{j \in S} X_j \right)$

Similar to above. Each value in the combination is conjoined & the result of each conjoined combination is disjoined. This ensures that every combination contains at least one high value

c.) Yes, the structure space can be captured using an OBDD.

⑤ $\Delta = (\neg A \vee B \vee \neg C) \wedge (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C)$

a)
1,2 $(\neg A \vee B)$
2,5 $(B \vee C)$
4,5 $(A \vee C)$
3,4 $(A \vee \neg B)$

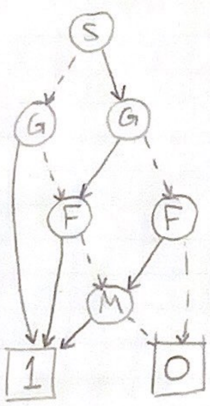
PI $\Rightarrow$ $(B \vee C) \wedge (A \vee C) \wedge (\neg A \vee B) \wedge (A \vee \neg B)$

b) $\neg \Delta = (A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge \neg C)$

1,2 $(A \wedge \neg B)$
2,5 $(\neg B \wedge \neg C)$
4,5 $(\neg A \wedge \neg C)$
3,4 $(\neg A \wedge B)$

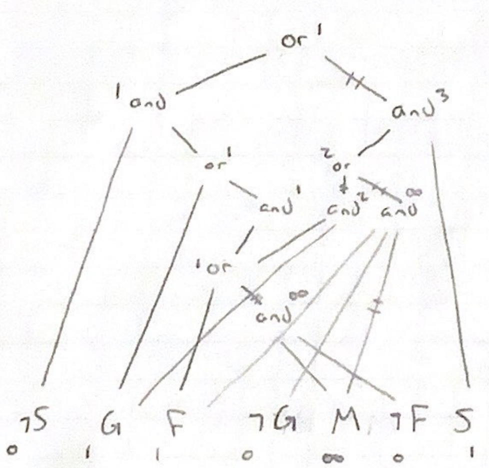
IP $\Rightarrow$ $(A \wedge \neg B) \vee (\neg B \wedge \neg C) \vee (\neg A \wedge \neg C) \vee (\neg A \wedge B)$

⑥

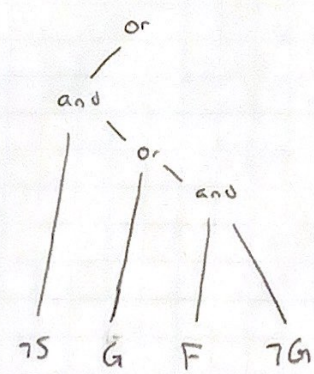


S - original Screenplay
G - great cinematrophy
F - famous cast
M - marketing

a) Instance: S=1, G=1, F=1, M=0 $\Rightarrow$ 1



$\Rightarrow$



The largest set is 3 without changing the decision on the current decision, with 2 instances: S=0, G=1, F=0, M=0

S=0, G=0, F=1, M=0

b) Instance: $S=1,\ G=0,\ F=1,\ M=1 \Rightarrow 1$

Reasons: $(S,F,M)(G,F,M)(F,M)$

Smallest set of feature $\alpha:\{F=1, M=1\}$ for this instance

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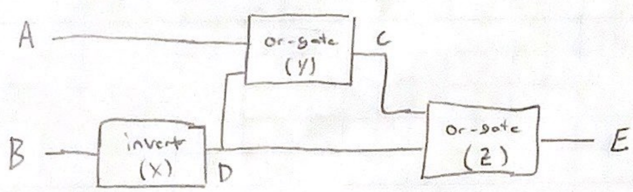

a)
$$\Delta = \begin{cases} okX \Rightarrow (B \Leftrightarrow \neg D) \\ okY \Rightarrow (A \vee D) \Leftrightarrow C \\ okZ \Rightarrow (D \vee C) \Leftrightarrow E \end{cases} \Rightarrow$$

CNF: $(\neg okX \vee \neg B \vee \neg D) \wedge (\neg okX \vee B \vee D) \wedge$
$(\neg okY \vee \neg A \vee C) \wedge (\neg okY \vee \neg D \vee C) \wedge$
$(\neg okY \vee A \vee \neg C) \wedge (\neg okY \vee D \vee \neg C) \wedge$
$(\neg okZ \vee \neg C \vee E) \wedge (\neg okZ \vee \neg D \vee E) \wedge$
$(\neg okZ \vee C \vee \neg E) \wedge (\neg okZ \vee D \vee \neg E)$

b.) System input: $A=1,\ B=0$; System output: $E=0$

Health(Δ, α) w/ $\alpha = A \wedge \neg B \wedge \neg E$

A	$\{\neg okY, \neg A, C\}, \{\neg okY, A, \neg C\}, \{A\}$	
B	$\{\neg okX, \neg B, \neg D\}, \{\neg okX, B, D\}, \{\neg B\}$	
C	$\{\neg okY, C, \neg D\}, \{\neg okY, \neg C, D\}, \{\neg okZ, \neg C, E\}, \{\neg okZ, C, \neg E\}$,	$\{\neg okY, C\}$
D	$\{\neg okZ, \neg D, E\}, \{\neg okZ, D, \neg E\}$	$\{\neg okX, D\}, \{\neg okY, D\}$
E	$\{\neg E\}$	$\{\neg okY, \neg okZ, E\}, \{\neg okX, \neg okZ, E\}, \{\neg okX, \neg okY, E\}$
okX	$\{\neg okX, \neg okZ\}, \{\neg okX, \neg okY\}$	
okY	$\{\neg okY, \neg okZ\}$	
okZ		

$\Rightarrow$ Health$(\Delta, \alpha) = (\neg okX \vee \neg okZ) \wedge (\neg okX \vee \neg okY) \wedge (\neg okY \vee \neg okZ)$

c) $(\neg okX \wedge \neg okY) \vee (\neg okX \wedge \neg okZ) \vee (\neg okY \wedge \neg okZ)$

d.) $(\neg okX \vee \neg okZ) \wedge (\neg okX \vee \neg okY) \wedge (\neg okY \vee \neg okZ)$