CS 246A | HW #1 | Jason Chapman

(1) • $(\neg A \Rightarrow B) \wedge (A \Rightarrow \neg B)$

W: A = true, B = false

$\therefore$ $(\neg A \Rightarrow B) = \neg\neg A \vee B = A \vee B = \text{True}$ ✓
$(A \Rightarrow \neg B) = \neg A \vee \neg B = \text{True}$

• $(A \wedge B) \Rightarrow (\neg A \vee \neg B)$

W: A = false or B = false

$\therefore$ $\neg A \vee \neg B \vee \neg A \vee \neg B = \text{True}$ if A = false or B = false ✓

(2) • $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$

	A	B	$(A \Rightarrow B)$	$(\neg B \Rightarrow \neg A)$	$(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$
w_1	T	T	T	T	T
w_2	T	F	F	F	T
w_3	F	T	T	T	T
w_4	F	F	T	T	T

✓

• $((A \vee B) \wedge (A \Rightarrow C)) \Rightarrow (B \vee C)$

	A	B	C	$(A \vee B) \wedge (A \Rightarrow C)$	$(B \vee C)$	$((A \vee B) \wedge (A \Rightarrow C)) \Rightarrow (B \vee C)$
w_1	T	T	T	T	T	T
w_2	T	T	F	F	T	T
w_3	T	F	T	T	T	T
w_4	F	T	T	T	T	T
w_5	T	F	F	F	F	T
w_6	F	T	F	T	T	T
w_7	F	F	T	F	T	T
w_8	F	F	F	F	F	T

✓

(3) a.) $\exists P(\Delta \vee \Gamma) = (\exists P \Delta) \vee (\exists P \Gamma)$

$\exists P(\Delta \vee \Gamma) = (\Delta \vee \Gamma)|P \vee (\Delta \vee \Gamma)|\neg P$

$= (\Delta|P \vee \Gamma|P) \vee (\Delta|\neg P \vee \Gamma|\neg P) = \Delta|P \vee \Gamma|P \vee \Delta|\neg P \vee \Gamma|\neg P$

$= (\Delta|P \vee \Delta|\neg P) \vee (\Gamma|P \vee \Gamma|\neg P)$

$= (\exists P \Delta) \vee (\exists P \Gamma)$ ✓

b) $\forall P(\Delta \wedge \Gamma) = (\forall P \Delta) \wedge (\forall P \Gamma)$

$$\forall P(\Delta \wedge \Gamma) = (\Delta \wedge \Gamma | P) \wedge (\Delta \wedge \Gamma | \neg P)$$
$$= (\Delta | P \wedge \Gamma | P) \wedge (\Delta | \neg P \wedge \Gamma | \neg P) = \Delta | P \wedge \Gamma | P \wedge \Delta | \neg P \wedge \Gamma | \neg P$$
$$= (\Delta | P \wedge \Delta | \neg P) \wedge (\Gamma | P \wedge \Gamma | \neg P)$$
$$= (\forall P \Delta) \wedge (\forall P \Gamma) \checkmark$$

4. $\Delta = A \Rightarrow B, \neg A \Rightarrow (\neg B \wedge C), (B \vee C) \Rightarrow D$

$$A \Rightarrow B = \neg A \vee B = \{\neg A, B\}$$
$$\neg A \Rightarrow (\neg B \wedge C) = \neg\neg A \vee (\neg B \wedge C) = (A \vee \neg B) \wedge (A \vee C) = \{A, \neg B\}, \{A, C\}$$
$$(B \vee C) \Rightarrow D = \neg(B \vee C) \vee D = (\neg B \wedge \neg C) \vee D = (\neg B \vee D) \wedge (\neg C \vee D) = \{\neg B, D\}, \{\neg C, D\}$$

$$\Rightarrow \Delta = \{\{\neg A, B\}, \{A, \neg B\}, \{A, C\}, \{\neg B, D\}, \{\neg C, D\}\}$$

5. Note that 2 sentences (Δ & Γ) are equivalent if $Mods(\Delta) = Mods(\Gamma)$. This test can be done by checking $Mods(\Gamma) \subseteq Mods(\Delta)$ and that $|Mods(\Gamma)| = |Mods(\Delta)|$.

6. $\Delta = \neg A \Rightarrow B, A \Rightarrow \neg C, \neg D \Rightarrow \neg B \wedge \neg C, A \Rightarrow E$ // show $D \vee E$ entailed by Δ

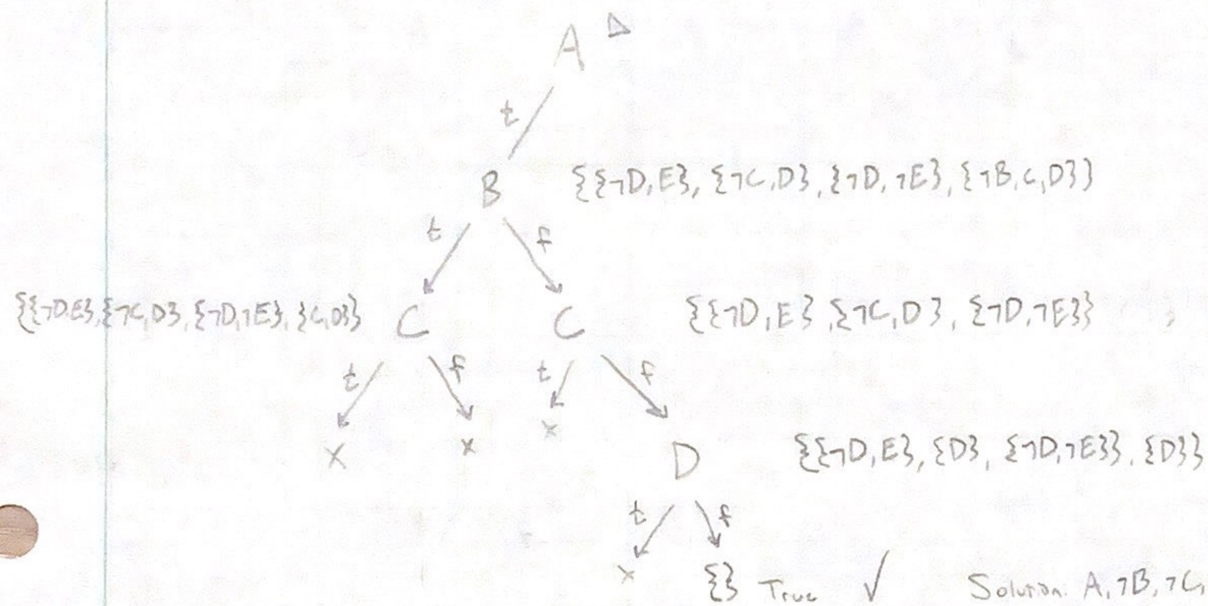
1.	$\{A, B\}$	
2.	$\{\neg A, \neg C\}$	
3.	$\{\neg B, D\}$	
4.	$\{\neg C, D\}$	
5.	$\{\neg A, E\}$	
6.	$\{\neg D\}$	
7.	$\{\neg E\}$	
8.	$\{\neg A\}$	5, 7
9.	$\{B\}$	1, 8
10.	$\{D\}$	3, 9
11.	$\{\}$	6, 10

$\Rightarrow$ Since we added $\neg D$ & $\neg E$ to Δ & showed that an empty clause could be derived, we have proven that $D \vee E$ is implied by the knowledge base

⑦ 1. $A \wedge D \Rightarrow E$ order A, B, C, D, E & true before false

$\Delta =$
2. $C \Rightarrow D$
3. $D \Rightarrow \neg E$
4. $B \wedge \neg C \Rightarrow D$

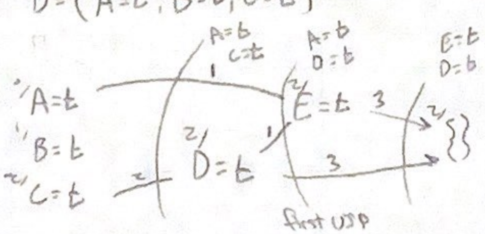
$\Rightarrow \Delta = \{\{\neg A, \neg D, E\}, \{\neg C, D\}, \{\neg D, \neg E\}, \{\neg B, C, D\}\}$



Solution: $A, \neg B, \neg C, \neg D$

⑧ $\Delta = \{\{\neg A, \neg D, E\}^{1}, \{\neg C, D\}^{2}, \{\neg D, \neg E\}^{3}, \{\neg B, C, D\}^{4}\}$, $D = ()$, $\Gamma = \{\}$

1^{st} iteration: $D = (A \overset{0}{=} t, B \overset{1}{=} t, C \overset{2}{=} t)$



conflict-driven clauses

$\{\neg \overset{0}{A}, \neg \overset{2}{C}\}$ al=0

$\{\neg \overset{0}{A}, \neg \overset{2}{D}\}$ al=0

$\{\neg \overset{2}{D}, \neg \overset{2}{E}\}$ not asserting

$\Rightarrow \alpha = \{\neg A, \neg D\}$

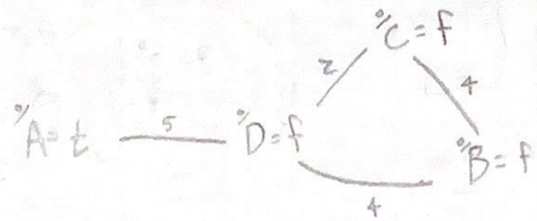
$m = 0$

$D \Leftarrow A = t$

$\Gamma \Leftarrow \{\{\neg A, \neg D\}\}$

2^{nd} Iteration: $\Delta, \Gamma = \{\{\neg A, \neg D\}\}$, $D = (A = t)$

$$\Delta = \begin{cases} 1.\ \{\neg A, \neg D, E\} \\ 2.\ \{\neg C, D\} \\ 3.\ \{\neg D, \neg E\} \\ 4.\ \{\neg B, C, D\} \\ 5.\ \{\neg A, \neg D\} \end{cases}$$

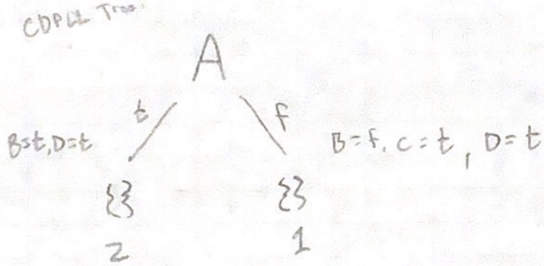


$\Rightarrow$ No conflict driven clause detected. We get: $\{A, \neg B, \neg C, \neg D\}$

④ $\Delta = A \Rightarrow B,\ \neg A \Rightarrow (\neg B \wedge C),\ (B \vee C) \Rightarrow D$

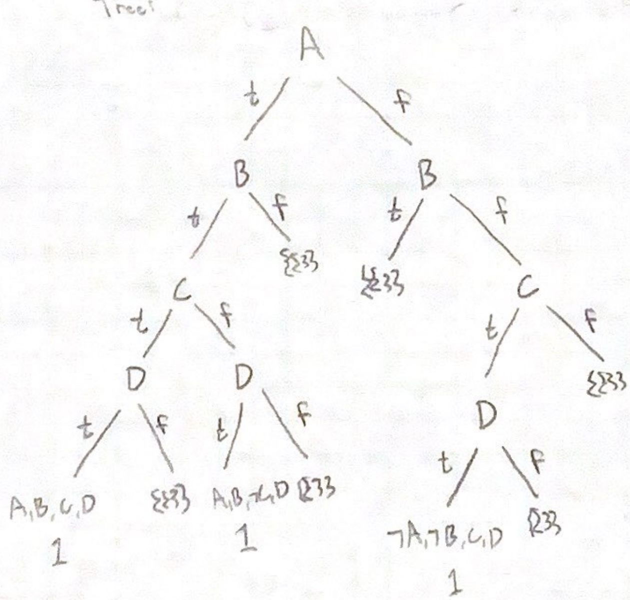
$\Delta = \{\{\neg A, B\}, \{A, \neg B\}, \{A, C\}, \{\neg B, D\}, \{\neg C, D\}\}$

CDPLL Tree:



$2 + 1 = 3$ Models

Termination Tree:



$= \surd$

$1 + 1 + 1 = 3$ Models

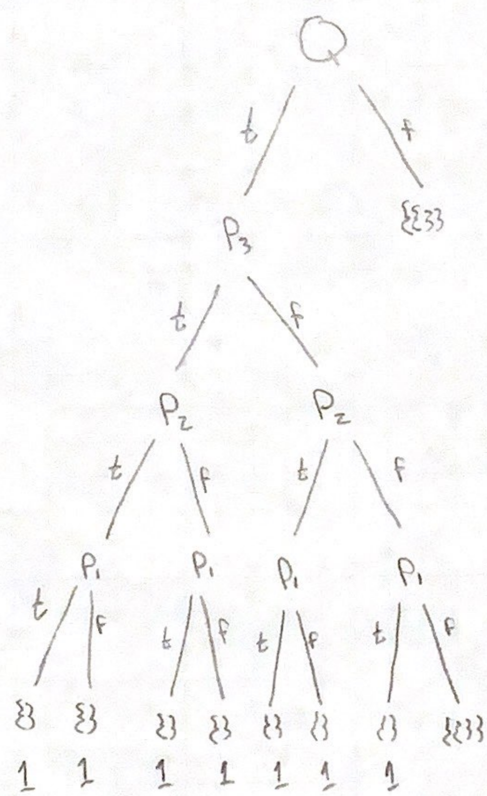
(10) $\Delta = P_1 \vee P_2 \vee P_3,\ P_1 \Rightarrow Q,\ P_2 \Rightarrow Q,\ P_3 \Rightarrow Q$

a) $\Delta = \{\{P_1, P_2, P_3\}, \{\neg P_1, Q\}, \{\neg P_2, Q\}, \{\neg P_3, Q\}\}$

b) Order: P_1, P_2, P_3, Q

		Directional Extension
P_1:	$\{P_1, P_2, P_3\}, \{\neg P_1, Q\}$	
P_2:	$\{\neg P_2, Q\}$	$\{P_2, P_3, Q\}$
P_3:	$\{\neg P_3, Q\}$	$\{P_3, Q\}$
Q:		$\{Q\}$

c)



7 Models