EE 131	HW #2	Jason Chapman

(1) a.)

$$S: \left\{ \begin{matrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,2) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (1,3) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \\ (1,4) & (2,4) & (3,4) & (4,4) & (5,4) & (6,4) \\ (1,5) & (2,5) & (3,5) & (4,5) & (5,5) & (6,5) \\ (1,6) & (2,6) & (3,6) & (4,6) & (5,6) & (6,6) \end{matrix} \right\}$$

b.)

$$P(\text{different \#'s}) = \frac{30}{36} = \frac{5}{6}$$

c.)

$$\text{differences: } \left\{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{matrix} \right\} < \text{second \#: } \left\{ \begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 \end{matrix} \right\}$$

A: absolute value of the difference between two #'s < 2^{nd} #

$$P(A) = \frac{27}{36} = \frac{3}{4}$$

(2) Two Independent Tosses of Coin, Show A, B, C are Pairwise independent, & that they're not independent

$S: \{HH, HT, TH, TT\}$

$A: \{HH, HT\}$ $\quad P(A) = \frac{1}{2}$

$B: \{HH, TH\}$ $\quad P(B) = \frac{1}{2}$

$C: \{HH, TT\}$ $\quad P(C) = \frac{1}{2}$

$A \cap B : \{HH\}$ $\quad P(A \cap B) = \frac{1}{4}$ $\quad P(A \cap B) = P(A)P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ ✓

$A \cap C : \{HH\}$ $\quad P(A \cap C) = \frac{1}{4}$ $\quad P(A \cap C) = P(A)P(C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ ✓

$B \cap C : \{HH\}$ $\quad P(B \cap C) = \frac{1}{4}$ $\quad P(B \cap C) = P(B)P(C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ ✓

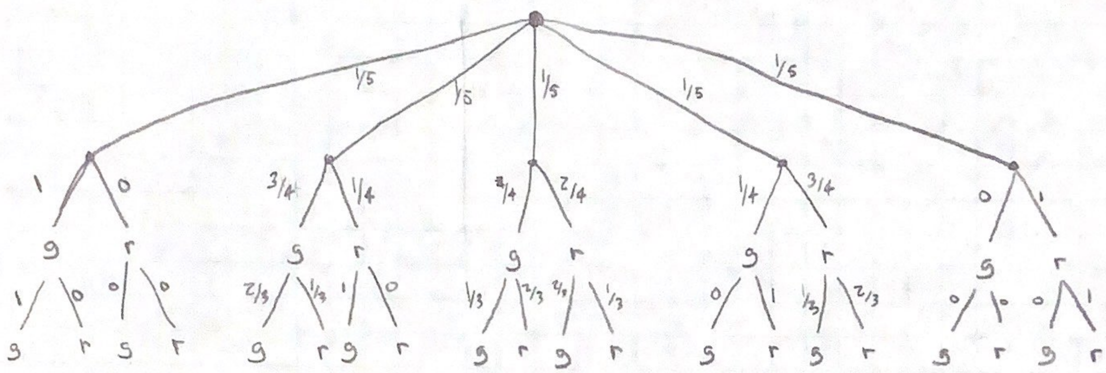
$\Rightarrow$ The events are pairwise independent

$A \cap B \cap C : \{HH\}$ $\quad P(A \cap B \cap C) = \frac{1}{4}$ $\quad P(A \cap B \cap C) = P(A)P(B)P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \neq \frac{1}{4}$ ×

$\Rightarrow$ The events are <u>NOT</u> independent

③ 5 urns which i^{th} urn contains $i-1$ Red balls & $5-i$ green balls

Urn 1: 4g Urn 2: 1r 3g Urn 3: 2r 2g Urn 4: 3r 1g Urn 5: 4r



a) A: 2^{nd} Ball is green

$$P(A) = \sum_{k=1}^{n} P(A|B_k)P(B_k)$$ when B_k represent either green or red for 1^{st} ball from the i^{th} urn

$$P(A) = \left(1\cdot 1\cdot \tfrac{1}{5}\right) + \left(0\cdot 0\cdot \tfrac{1}{5}\right) + \left(\tfrac{2}{3}\cdot \tfrac{3}{4}\cdot \tfrac{1}{5}\right) + \left(1\cdot \tfrac{1}{4}\cdot \tfrac{1}{5}\right) + \left(\tfrac{1}{3}\cdot \tfrac{2}{4}\cdot \tfrac{1}{5}\right) + \left(\tfrac{2}{3}\cdot \tfrac{2}{4}\cdot \tfrac{1}{5}\right)$$
$$+ \left(0\cdot \tfrac{1}{4}\cdot \tfrac{1}{5}\right) + \left(\tfrac{1}{3}\cdot \tfrac{3}{4}\cdot \tfrac{1}{5}\right) + \left(0\cdot 0\cdot \tfrac{1}{5}\right) + \left(0\cdot 1\cdot \tfrac{1}{5}\right) = \tfrac{1}{2}$$

$P(2^{nd}$ Ball Green$) = \frac{1}{2}$

b) C: 1^{st} Ball is green

$$P(A|C) = \left(1\cdot 1\cdot \tfrac{1}{5}\right) + \left(\tfrac{2}{3}\cdot \tfrac{3}{4}\cdot \tfrac{1}{5}\right) + \left(\tfrac{1}{3}\cdot \tfrac{2}{4}\cdot \tfrac{1}{5}\right) + \left(0\cdot \tfrac{1}{4}\cdot \tfrac{1}{5}\right) + \left(0\cdot 0\cdot \tfrac{1}{5}\right) = \tfrac{1}{3}$$

$P(2^{nd}$ Ball Green given 1^{st} Ball is Green$) = \frac{1}{3}$

④ Throw 2 6-sided Dice: X_1 = # dots on 1^{st} die, X_2 = # dots on 2^{nd} die, $Z = 2X_1 + 2X_2$

a)

z	4	6	8	10	12	14	16	18	20	22	24
$P(Z=z)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$P(Z=z) = \begin{cases} \dfrac{6 - \left|\frac{14-z}{2}\right|}{36} & \text{for } z = 4,6,8,10,12,14,16,18,20,22,24 \\ 0 & \text{for all other } z \end{cases}$$

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b) $\mathbb{E}[Z] = \sum_{z \in S} z_i p(z_i) = 4 \cdot \frac{1}{36} + 6 \cdot \frac{2}{36} + 8 \cdot \frac{3}{36} + 10 \cdot \frac{4}{36} + 12 \cdot \frac{5}{36} + 14 \cdot \frac{6}{36} + 16 \cdot \frac{5}{36} + 18 \cdot \frac{4}{36}$

$+ 20 \cdot \frac{3}{36} + 22 \cdot \frac{2}{36} + 24 \cdot \frac{1}{36} = 14$

$\mathbb{E}[Z] = 14$

$Var(Z) = \mathbb{E}(Z^2) - m_x^2 = \mathbb{E}[Z^2] - (\mathbb{E}[Z])^2 = \Big[4^2 \cdot \frac{1}{36} + 6^2 \cdot \frac{2}{36} + 8^2 \cdot \frac{3}{36} + 10^2 \cdot \frac{4}{36}$

$+ 12^2 \cdot \frac{5}{36} + 14^2 \cdot \frac{6}{36} + 16^2 \cdot \frac{5}{36} + 18^2 \cdot \frac{4}{36} + 20^2 \cdot \frac{3}{36} + 22^2 \cdot \frac{2}{36} + 24^2 \cdot \frac{1}{36}\Big] - 14^2 = 23.\bar{3}$

$Var(Z) = 23.\bar{3}$

c) $Z = 20$

$Z = 20$ iff $X_1 = 6, \; X_2 = 4$
$X_1 = 4, \; X_2 = 6$
$X_1 = 5, \; X_2 = 5$

$P(X_1 = 1) = 0$

$P(X_1 = 2) = 0$

$P(X_1 = 3) = 0$

$P(X_1 = 4) = \frac{1}{3}$

$P(X_1 = 5) = \frac{1}{3}$

$P(X_1 = 6) = \frac{1}{3}$

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