EE 131A | HW #3 | Jason Chapman

①

x_i	-2	-1	0	1	2
$P(X=x_i)$	0.3	0.2	0.2	0.2	0.1

$Y = X^2$

a.)

y_i	4	1	0	1	4
$P(Y=y_i)$	0.3	0.2	0.2	0.2	0.1

$\Rightarrow$ $P(Y=4) = 0.4$
$P(Y=1) = 0.4$
$P(Y=0) = 0.2$

$$P_Y(Y) = \begin{cases} 0.4 & Y=4 \\ 0.4 & Y=1 \\ 0.2 & Y=0 \\ 0 & Y \neq 0,1,4 \end{cases}$$

b.) $\mathbb{E}[Y] = 4(0.4) + 1(0.4) + 0(0.2) = 2 \Rightarrow \boxed{\mathbb{E}[Y] = 2}$

c.) $\mathbb{E}[Y^2] = 4^2(0.4) + 1^2(0.4) + 0^2(0.2) = 6.8$

$\mathrm{VAR}[Y] = \mathbb{E}[Y^2] - (E[Y])^2 = 6.8 - (2)^2 = 2.8 \Rightarrow \boxed{\mathrm{VAR}[Y] = 2.8}$

② X is binomial RV w/ $n=4$, $p=p$

a.) $\mathbb{E}\left[\sin\left(\frac{\pi x}{2}\right)\right] = \sum_{x=0}^{n} \sin\frac{\pi x}{2} \cdot P(x) = \cancelto{0}{\sin\frac{\pi 0}{2} P(X=0)} + \cancelto{1}{\sin\frac{\pi}{2}} \cdot P(X=1) + \cancelto{0}{\sin\frac{2\pi}{2} \cdot P(X=2)} + \cancelto{-1}{\sin\frac{3\pi}{2}} \cdot P(X=3) + \cancelto{0}{\sin\frac{4\pi}{2} \cdot P(X=4)}$

$= P(X=1) - P(X=3)$

$= \left[\binom{4}{1} p^1 (1-p)^{4-1}\right] - \left[\binom{4}{3} p^3 (1-p)^{4-3}\right]$

$= 4p(1-p)^3 - 4p^3(1-p) = 4p(1-p)\left[(1-p)^2 - p^2\right]$

$= 4p(1-p)\left[1 - 2p + \cancel{p^2} - \cancel{p^2}\right] = 4p(1-p)(1-2p)$

$\Rightarrow \boxed{\mathbb{E}\left[\sin\left(\frac{\pi x}{2}\right)\right] = 4p(1-p)(1-2p)}$

b.) $\mathbb{E}\left[\cos\left(\frac{\pi x}{2}\right)\right] = \sum_{x=0}^{n} \cos\frac{\pi x}{2} \cdot P(x) = \cancelto{1}{\cos(0)} P(X=0) + \cancelto{0}{\cos\frac{\pi}{2} P(X=1)} + \cancelto{-1}{\cos\pi} P(X=2) + \cancelto{0}{\cos\frac{3\pi}{2} P(X=3)} + \cancelto{1}{\cos 2\pi} P(X=4)$

$= P(X=0) - P(X=2) + P(X=4)$

$= \left[\binom{4}{0} p^0 (1-p)^{4-0}\right] - \left[\binom{4}{2} p^2 (1-p)^{4-2}\right] + \left[\binom{4}{4} p^4 (1-p)^{4-4}\right]$

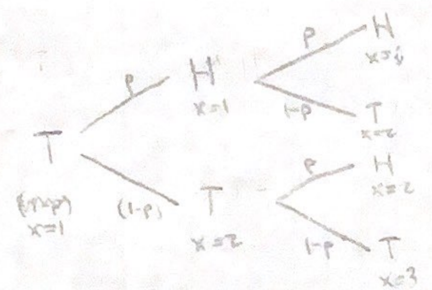
$= (1-p)^4 - 6p^2(1-p)^2 + p^4 = (1-2p+p^2)(1-2p+p^2) - 6p^2(1-2p+p^2) + p^4$

$= 1 - \cancel{2p} + \cancel{p^2} - \cancel{2p} + \cancel{4p^2} - \cancel{2p^3} + \cancel{p^2} - \cancel{2p^3} + p^4 - \cancel{6p^2} + \cancel{12p^3} - 6p^4 + p^4$

$= 1 - 4p + 8p^3 - 4p^4$

$\Rightarrow \boxed{\mathbb{E}\left[\cos\left(\frac{\pi x}{2}\right)\right] = 1 - 4p + 8p^3 - 4p^4}$

③ Flip n biased coins w/ P(heads) = p, X = # tails.



On 3rd flip: $P(x=1 \mid \text{First Tail}) = p^2$

$P(x=2 \mid \text{first Tail}) = p(1-p) + p(1-p) = 2p(1-p)$

$P(x=3 \mid \text{first Tail}) = (1-p)^2$

On 4th flip: $P(x=1 \mid \text{First Tail}) = p^3$

$P(x=2 \mid \text{First Tail}) = 2p^2(1-p) + p^2(1-p) = 3p^2(1-p)$

$P(x=3 \mid \text{First Tail}) = p(1-p)^2 + p(1-p)^2 + \dots = 3p(1-p)^2$

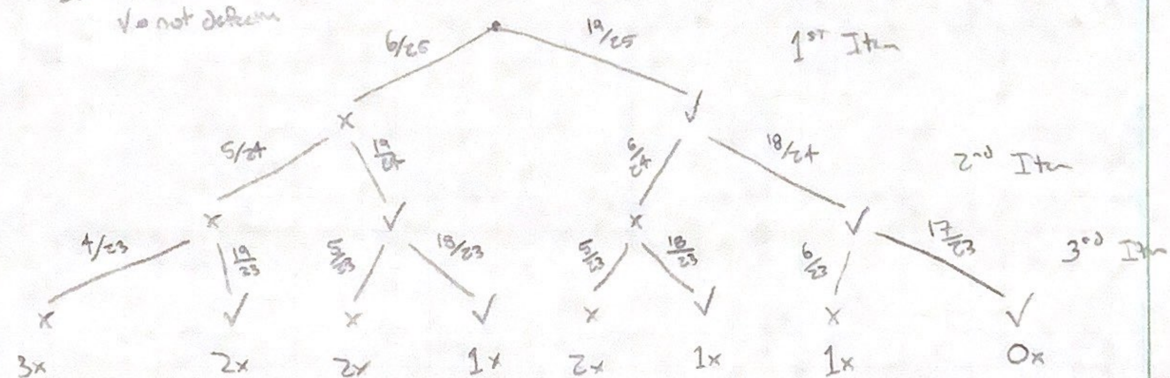
$P(x=4 \mid \text{First Tail}) = (1-p)^3$

$$\Rightarrow \boxed{P(X=x \mid \text{First Coin Heads}) = \binom{n-1}{x-1} p^{n-x} (1-p)^{x-1}}$$

④ Sample 3 items out of 25 w/ 6 defective.

Let x ≡ defective

✓ ≡ not defective



$P(3 \text{ defective}) = \frac{6}{25} \cdot \frac{5}{24} \cdot \frac{4}{23} = \frac{1}{115}$

$P(2 \text{ defective}) = \left(\frac{6}{25} \cdot \frac{5}{24} \cdot \frac{19}{23}\right) + \left(\frac{6}{25} \cdot \frac{19}{24} \cdot \frac{5}{23}\right) + \left(\frac{19}{25} \cdot \frac{6}{24} \cdot \frac{5}{23}\right) = \frac{19}{460} + \frac{19}{460} + \frac{19}{460} = \frac{57}{460}$

$P(1 \text{ defective}) = \left(\frac{6}{25} \cdot \frac{19}{24} \cdot \frac{18}{23}\right) + \left(\frac{19}{25} \cdot \frac{6}{24} \cdot \frac{18}{23}\right) + \left(\frac{19}{25} \cdot \frac{18}{24} \cdot \frac{6}{23}\right) = 0.446086957$

$P(0 \text{ defective}) = \frac{19}{25} \cdot \frac{18}{24} \cdot \frac{17}{23} = 0.421304348$

$\mathbb{E}[\# \text{ of defective items}] = 3\left(\frac{1}{115}\right) + 2\left(\frac{57}{460}\right) + 1(0.446086957) + 0(0.421304348)$

$= 0.72$

$$\Rightarrow \boxed{\mathbb{E}[\# \text{ of defective items}] = 0.72}$$

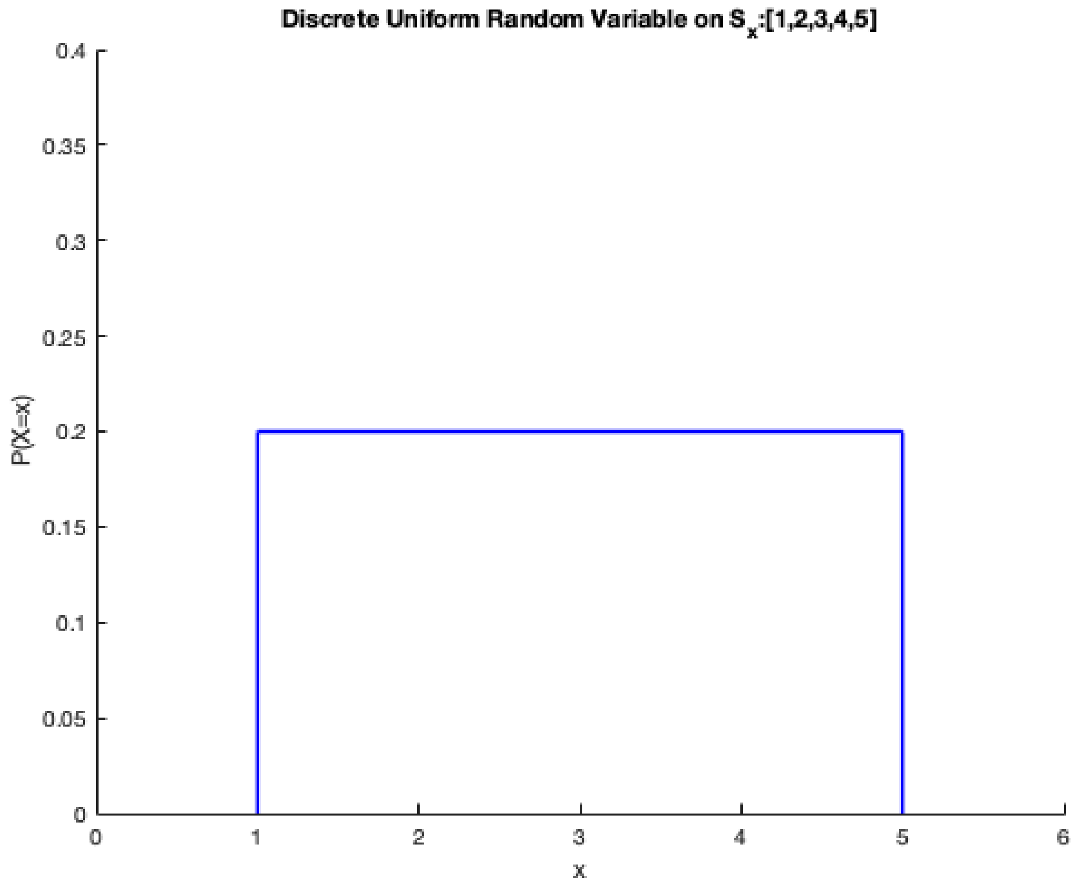
⑤ See Attached

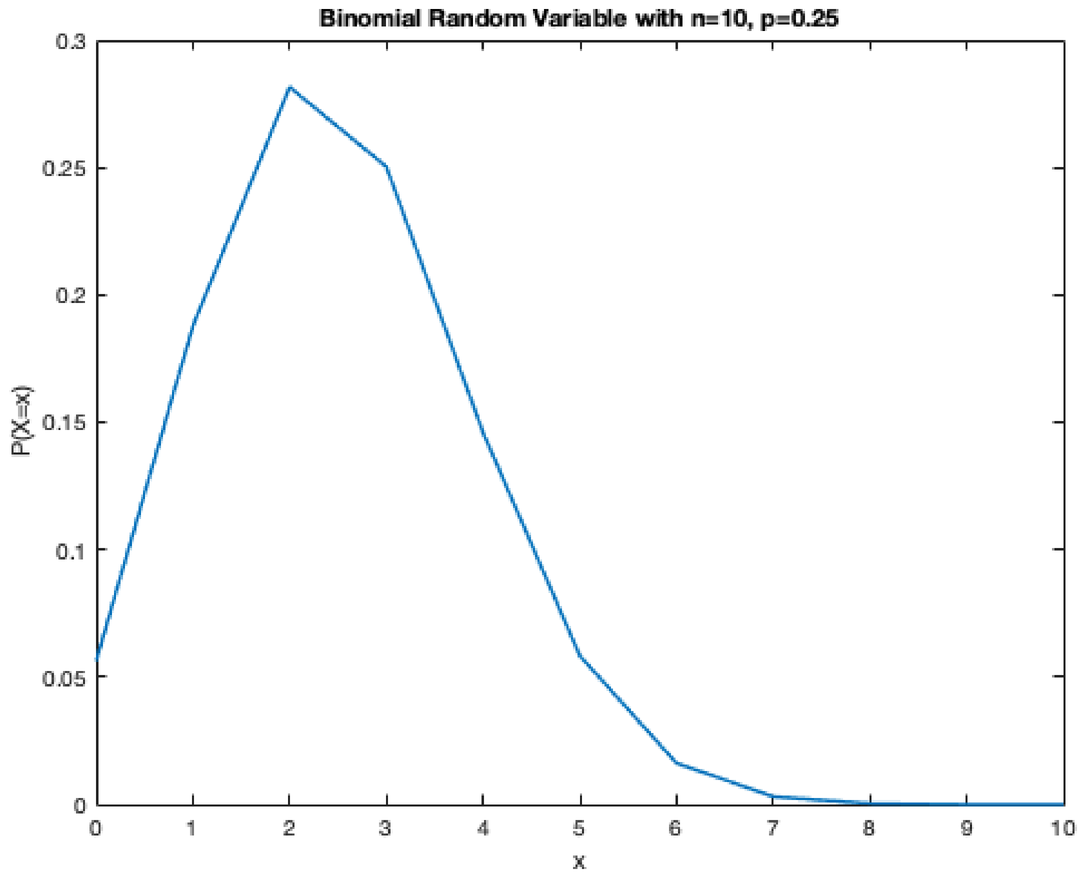
TOPS 35500

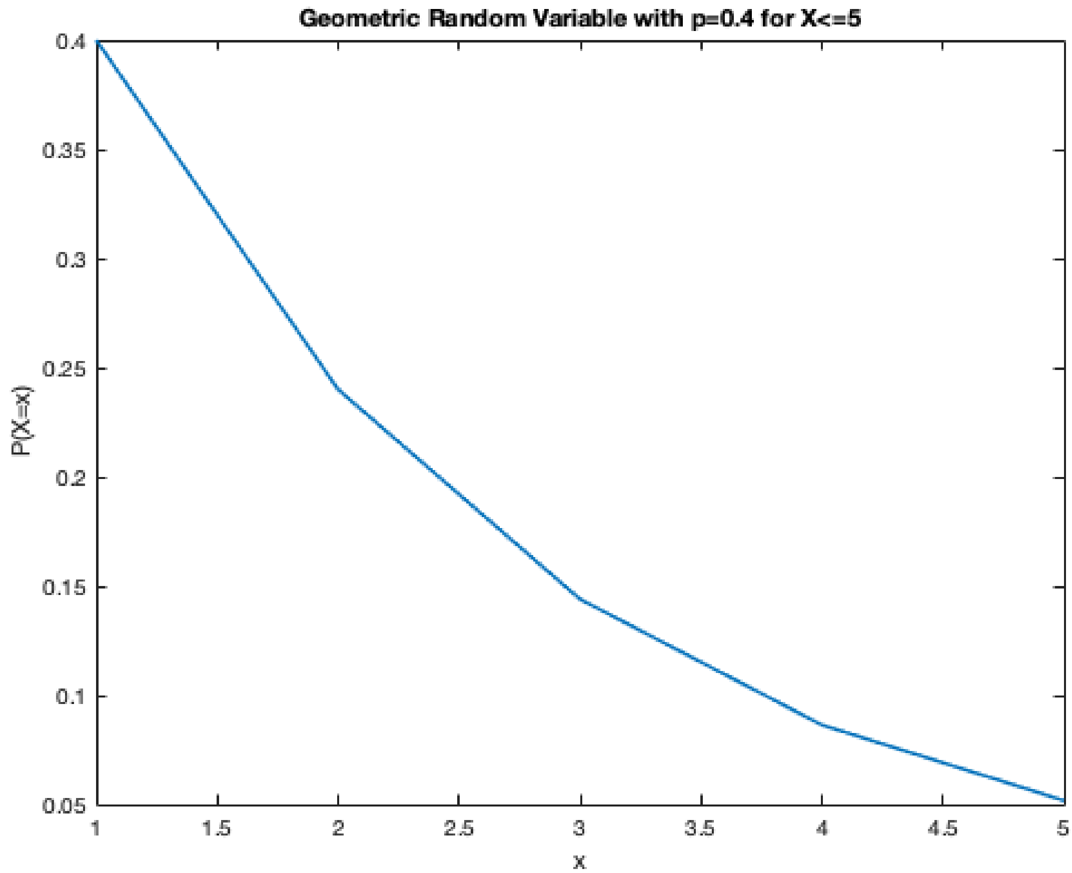
EE 131A - HW #1

Jason Chapman

```
close all; clear;

% 5a - Discrete Uniform RV on set{1,2,3,4,5}
Sx = 1:5;
L = length(Sx);
Px_uni = 1/L;

figure
hold on
plot(Sx,linspace(Px_uni,Px_uni,L),'b','Linewidth',1.5)
plot([1 1],[0 Px_uni],'b','Linewidth',1.5)
plot([5 5],[0 Px_uni],'b','Linewidth',1.5)
xlabel('x')
ylabel('P(X=x)')
title('Discrete Uniform Random Variable on S_{x}:[1,2,3,4,5]')
xlim([0 6])
ylim([0 2*Px_uni])

% 5b - Binomial RV with n=10 and p=0.25
n = 10;
p = 0.25;
for i = 0:n
    Px_bi(i+1) = (factorial(n)/(factorial(n-i))*(1/factorial(i)))*(p^i)*(1-
p)^(n-i);
    k(i+1) = i;
end

figure
plot(k,Px_bi,'Linewidth',1.5)
xlabel('x')
ylabel('P(X=x)')
title('Binomial Random Variable with n=10, p=0.25')

% 5c - Geometric RV with p=0.4 for values <=5
p = 0.4;
k = 1;
while k < 6
    Px_geo(k) = (1-p)^(k-1)*p;
    iter(k) = k;
    k = k+1;
end

figure
plot(iter,Px_geo,'Linewidth',1.5)
xlabel('x')
ylabel('P(X=x)')
title('Geometric Random Variable with p=0.4 for X<=5')
```


Discrete Uniform Random Variable on S_x:[1,2,3,4,5]
P(X=x)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
x


Binomial Random Variable with n=10, p=0.25
P(X=x)
x
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8
9
10



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