CS 264A | HW #2 | Jason Chapman

① Δ

1	$(X \vee Y)$	
2	$(X \vee Z)$	
3	$(\neg Y \vee W)$	
4	$(\neg Z \vee \neg W)$	
5	$(X \vee \neg Y)$	
6	$(\neg X \vee W)$	
7	$(\neg X \vee \neg W \vee V)$	
8	$(\neg X \vee \neg W \vee \neg V)$	
9	$(Z \vee \neg X)$	
10	$(\neg Z \vee W)$	
11	$(\neg X \vee \neg W)$	7,8
12	$\neg X$	6,11
13	X	1,5
14	$\{\}$	12,13

⇒ "Minimal" Unsat core: $\Delta' = \{\{\neg X \vee \neg W \vee V\}, \{\neg X \vee \neg W \vee \neg V\}, \{\neg X \vee W\}, \{X \vee Y\}, \{X \vee \neg Y\}\}$

- For each clause, α, in Δ', $(\Delta' \setminus \alpha) \models \alpha$ is valid, & $\Delta' \setminus \alpha$ is satisfiable. Thus, Δ' is a min unsatisfiable core of Δ

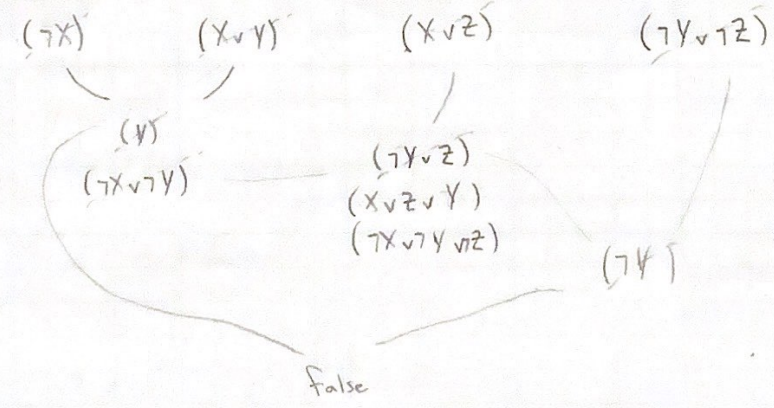
② $\Delta = (\neg X) \wedge (X \vee Y) \wedge (X \vee Z) \wedge (\neg Y \vee \neg Z)$

a)

X	Y	Z	Cost
T	T	T	1+0+0+1 = 2
T	T	F	1+0+0+0 = 1
T	F	T	1+0+0+0 = 1
F	T	T	0+0+0+1 = 1
T	F	F	1+0+0+0 = 1
F	F	T	0+1+0+0 = 1
F	T	F	0+0+1+0 = 1
F	F	F	0+1+1+0 = 2

- Any of these assignments are the optimal solutions because they have cost = 1
- Any solution except $\{X, Y, Z\}$ and $\{\neg X, \neg Y, \neg Z\}$

b)



$\Gamma = \{\{X \vee Y \vee Z\}, \{\neg X \vee \neg Y \vee \neg Z\}\}$, $K = 1$

③ $\Delta = (A \vee B \vee C) \wedge (\neg A \vee D \vee E)$

Resolvent: $B \vee C \vee D \vee E$

Computation Clauses: $A \vee B \vee C \vee \neg D$
$A \vee B \vee C \vee D \vee \neg E$
$\neg A \vee D \vee E \vee \neg B$
$\neg A \vee D \vee E \vee B \vee \neg C$

- No resolution left with resolvent

④ $\Delta = (\neg D \vee \neg E \vee B) \wedge (\neg B \vee E \vee \neg A) \wedge (\neg D \vee C \vee \neg B) \wedge (\neg B \vee C \vee E)$

a)

A	B	C	D	E	SAT
T	T	T	T	T	Yes
T	T	T	T	F	no
T	T	T	F	T	Yes
T	T	T	F	F	no
T	T	F	T	T	no
T	T	F	T	F	no
T	T	F	F	T	yes
T	T	F	F	F	no
T	F	T	T	T	no
T	F	T	T	F	Yes
T	F	T	F	T	Yes
T	F	T	F	F	Yes
F	T	T	T	T	Yes
F	T	T	T	F	Yes
F	T	T	F	T	Yes
F	T	T	F	F	Yes

A	B	C	D	E	SAT
F	F	F	F	T	no
F	F	F	T	F	Yes
F	F	F	F	T	Yes
F	F	F	F	F	Yes
F	F	T	T	T	no
F	F	T	T	F	Yes
F	F	T	F	T	Yes
F	F	T	F	F	Yes
F	T	F	T	T	no
F	T	F	T	F	no
F	T	F	F	T	Yes
F	T	F	F	F	no
T	F	F	T	T	no
T	F	F	T	F	Yes
T	F	F	F	T	Yes
T	F	F	F	F	Yes

$2^5 = 32$ $\quad \frac{20}{32} > \frac{1}{2} \Rightarrow$ MAJ-SAT? YES ✓

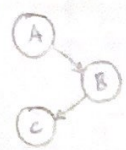
b) $X = \{A, B, C\}$, $Y = \{D, E\}$

- There are multiple X-instantiations under which the majority of Y-instantiations are satisfying. $\Rightarrow$ E-MAJ-SAT? YES ✓

$X = (A, \neg B, C)^1, (\neg A, B, C)^2, (\neg A, \neg B, \neg C)^3, (\neg A, \neg B, C)^4, (A, \neg B, \neg C)^5$

c) MAS-MAJ-SAT? YES ✓. As shown above 5/8 X-instantiations for which the majority of Y-instantiations are satisfying.

(5)



B

A	T	F
F	.1	.9
T	.8	.2

C

B	T	F
F	.3	.7
T	.25	.75

A

T	F
.6	.4

$\Theta_{B|\neg A}=0.1$, $\Theta_{\neg B|\neg A}=0.9$,
$\Theta_{B|A}=0.8$, $\Theta_{\neg B|A}=0.2$
$\Theta_{C|\neg B}=0.3$, $\Theta_{\neg C|\neg B}=0.7$
$\Theta_{C|B}=0.25$, $\Theta_{\neg C|B}=0.75$
$\Theta_A=0.6$, $\Theta_{\neg A}=0.4$

$\Delta = A \Leftrightarrow P_A,\ \neg A \Leftrightarrow P_{\neg A},\ A\wedge B \Leftrightarrow P_{B|A},\ A\wedge\neg B \Leftrightarrow P_{\neg B|A},\ \neg A\wedge B \Leftrightarrow P_{B|\neg A},$
$\neg A\wedge\neg B \Leftrightarrow P_{\neg B|\neg A},\ B\wedge C \Leftrightarrow P_{C|B},\ B\wedge\neg C \Leftrightarrow P_{\neg C|B},\ \neg B\wedge C \Leftrightarrow P_{C|\neg B},$
$\neg B\wedge\neg C \Leftrightarrow P_{\neg C|\neg B}$

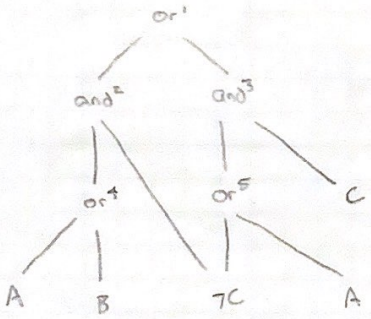
$W(A)=W(\neg A)=W(B)=W(\neg B)=W(C)=W(\neg C)=1$
$W(\neg P_{\alpha|\beta})=1$; $W(P_{\alpha|\beta})=\Theta_{\alpha|\beta}$

$M_1 = A, B, \neg C$ $\quad W(M_1)=\Theta_A\,\Theta_{B|A}\,\Theta_{\neg C|B}=(0.6)(0.8)(0.75)=0.36$

$M_2 = A, \neg B, \neg C$ $\quad W(M_2)=\Theta_A\,\Theta_{\neg B|A}\,\Theta_{\neg C|\neg B}=(0.6)(0.2)(0.7)=0.084$

$W(M)=W(M_1)+W(M_2)=0.36+0.084=$ **0.444**

(6)



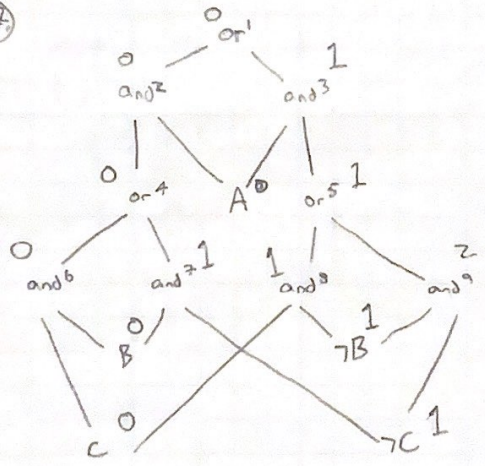
Decomposable: Disjoint variables over AND gates

- NOT DECOMPOSABLE because both inputs for and3 gate depend on C.

Deterministic: Mutual exclusive inputs for OR gates

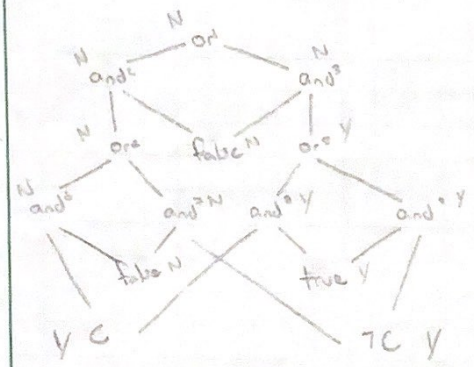
- NOT DETERMINISTIC because OR inputs have common model

(7)



a) Min Cardinality = 0

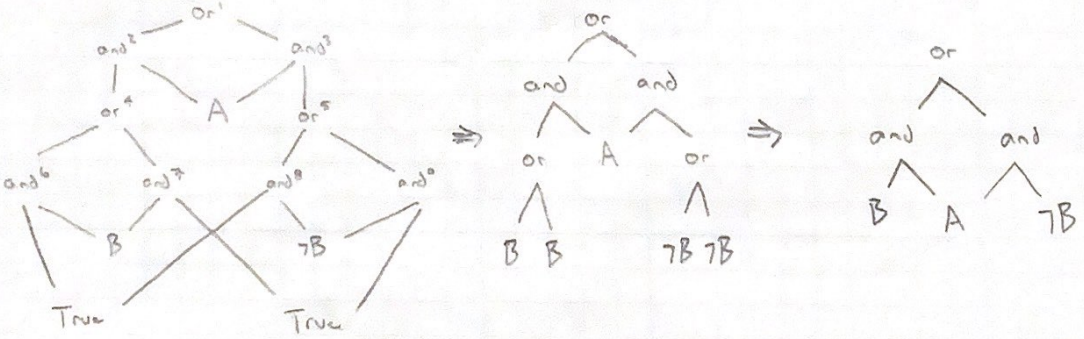
b) $\Delta \models A \vee B$ if $\Delta \cup \{\neg A, \neg B\}$ is inconsistent



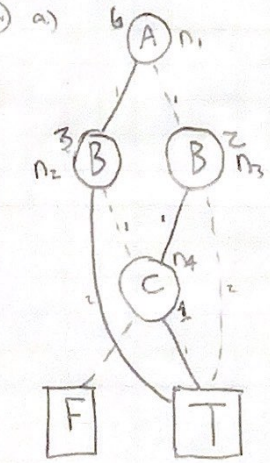
- Δ is not satisfiable, so $\Delta \cup \{\neg A, \neg B\}$ is inconsistent. Therefore,

$$\Delta \models A \vee B$$

c) $\exists C \Delta$. Circuit is decomposable, so set C to true



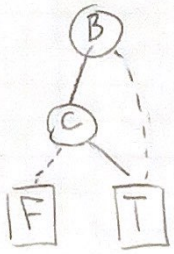
(B) a)



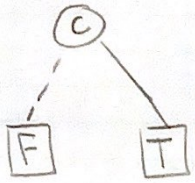
Total # satisfying assignments = 6

b) $\Delta \models A \vee \neg B$ if $\Delta \cup \{\neg A, B\}$ is inconsistent

$\Delta \cup \{\neg A\}$:



$\Delta \cup \{\neg A, B\}$:



- Since OBDD of $\Delta \cup \{\neg A, B\}$ is not $\boxed{F}$, $\Delta \cup \{\neg A, B\}$ is consistent

$$\Rightarrow \Delta \not\models A \vee \neg B$$

(9) a) $\Delta = \{X_1, X_2, \ldots X_N\}$

$\Delta' = \{X_1, X_2, \ldots X_{\lfloor N/2 \rfloor}\}$

If N is odd:

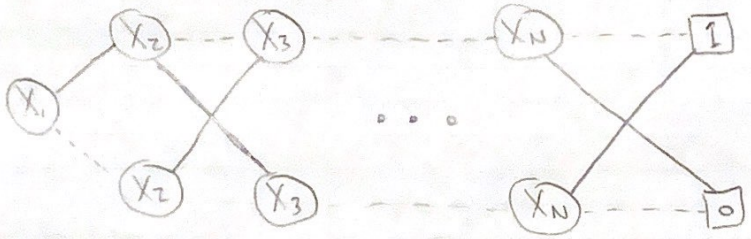
X_1	X_2	...	$X_{\lfloor N/2 \rfloor}$	$X_{\lfloor N/2 \rfloor+1}, X_{\lfloor N/2 \rfloor+2}, \ldots X_{N-1}, X_N$
0	0	...	0	
0	0	...	1	
⋮	⋮	...	⋮	
1	1	...	1	

If N is even

X_1	X_2	...	$X_{\lfloor N/2 \rfloor}$
0	0	...	0
0	0	...	1
⋮	⋮	...	⋮
1	1	...	1

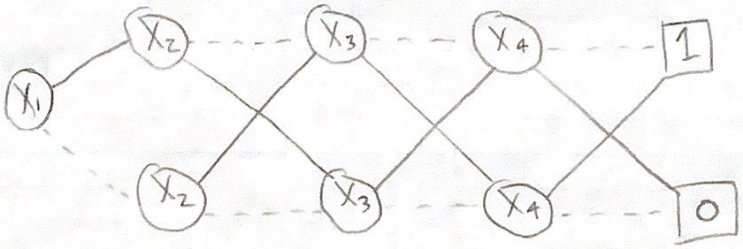
$$\text{Subfunctions} = \begin{cases} X_{\lfloor N/2 \rfloor+1}, X_{\lfloor N/2 \rfloor+2}, \ldots X_{N-1}, X_N & \text{if } \sum_{i=1}^{\lfloor N/2 \rfloor} \Delta'(X_i) \text{ is even} \\ \neg[X_{\lfloor N/2 \rfloor+1}, X_{\lfloor N/2 \rfloor+2}, \ldots X_{N-1}, X_N] & \text{if } \sum_{i=1}^{\lfloor N/2 \rfloor} \Delta'(X_i) \text{ is odd} \end{cases}$$

b)



Number of Nodes = $2N + 1$

c)



(10) Going over leaves of NNF circuit take linear time, So, one property of NNF is that all variable are in the leafs. If all of the literals are on the leafs, then we get the following:

$$\forall x(y) = y, \quad \forall x(\neg y) = \neg y, \quad \forall x(x) = F, \quad \forall x(\neg x) = F$$

Thus, if all the literals are on the leafs, $\forall x \Delta$ can be simply computed by replacing X & $\neg X$ with F & simplifying.