[§2.3] 2. (a) $\boxed{\text{No.}}$ For all nonzero $n \in \mathbb{Z}$, f attempts to assign more than one element of $\mathbb{R}$.

(b) $\boxed{\text{Yes.}}$ For all $n \in \mathbb{Z}$, f assigns exactly one element of $\mathbb{R}$.

(c) $\boxed{\text{No.}}$ Although $2 \in \mathbb{Z}$, it cannot be in the domain of f (to avoid division by zero).

6. (a) Domain: $\mathbb{Z}^+ \times \mathbb{Z}^+$
Range: $\mathbb{Z}^+$

(b) Domain: $\mathbb{Z}^+$
Range: $\{x \in \mathbb{Z} \mid 1 \leq x \leq 9\}$

8. (f) $\boxed{-2}$

(g) $\lfloor \frac{1}{2} + 1 \rfloor = \boxed{1}$

(h) $\lceil 0 + 1 + \frac{1}{2} \rceil = \boxed{2}$

23. (a) $\boxed{\text{Yes.}}$
f is one-to-one: $2x + 1 \neq 2y + 1$ when $x \neq y$.
f is onto: for all $y \in \mathbb{R}$ there is an $x \in \mathbb{R}$ such that $2x + 1 = y$. Namely, $x = \frac{y-1}{2}$.

(b) $\boxed{\text{No.}}$ For example, $f(1) = 2 = f(-1)$.

28. $f : \mathbb{R} \to \mathbb{R}$ is not invertible because there is no $x \in \mathbb{R}$ such that $e^x \leq 0$.
$f : \mathbb{R} \to \mathbb{R}^+$ is invertible because it is:

- one-to-one because e^x is strictly increasing, and
- onto because for all $y \in \mathbb{R}^+$ there is an $x \in \mathbb{R}$ such that $e^x = y$. Namely, $x = \log_e(y)$.

31. (a) $\boxed{f(S) = \{1, 0, 3\}}$

x	$f(x)$
± 2	$\lfloor 4/3 \rfloor = 1$
± 1	$\lfloor 1/3 \rfloor = 0$
0	$\lfloor 0/3 \rfloor = 0$
3	$\lfloor 9/3 \rfloor = 3$

(b) $\boxed{f(S) = \{1, 0, 3, 5, 8\}}$

x	$f(x)$
4	$\lfloor 16/3 \rfloor = 5$
5	$\lfloor 25/3 \rfloor = 8$

36. $f \circ g$ is $f(g(x)) = f(x + 2) = (x + 2)^2 + 1$
$g \circ f$ is $g(f(x)) = g(x^2 + 1) = x^2 + 3$

38. $f(g(x)) = a(cx + d) + b$ and $g(f(x)) = c(ax + b) + d$. Rearrangement gives

$$f(g(x)) = acx + ad + b \text{ and}$$
$$g(f(x)) = acx + cb + d,$$

where the acx term is common, so $\boxed{ad + b = cb + d \text{ is necessary and sufficient for } f \circ g = g \circ f.}$

39. f is invertible because it is:

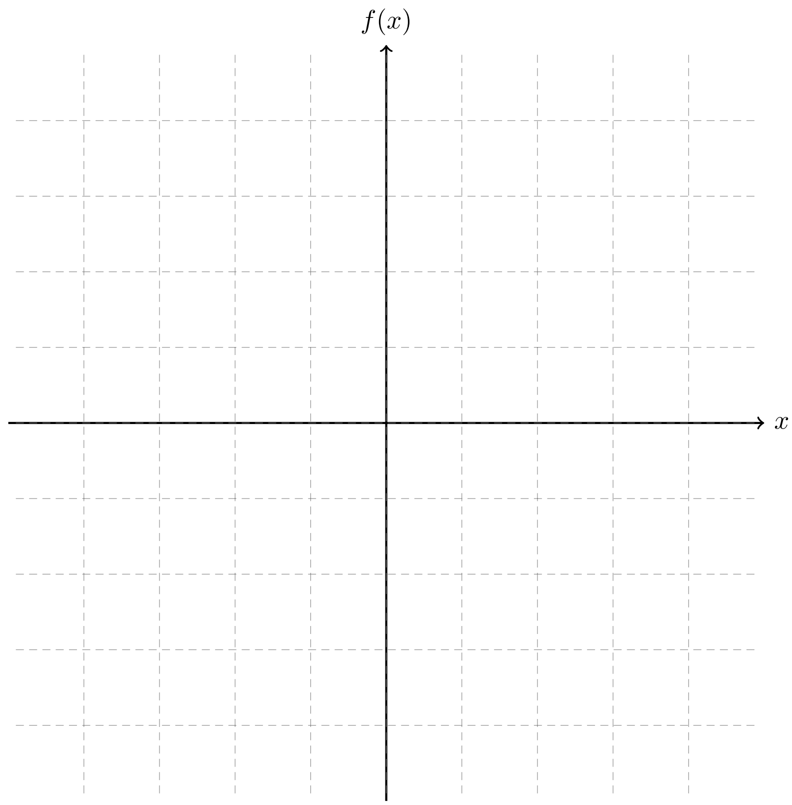
   - one-to-one because $ax + b \neq ay + b$ when $a \neq 0$ and $x \neq y$, and
   - onto because, given $a \neq 0$, for all $y \in \mathbb{R}$ there is an $x \in \mathbb{R}$ such that $ax + b = y$. Namely, $x = \frac{y-b}{a}$ and $a \neq 0$.

   Therefore the inverse of f is

$$f^{-1}(y) = \frac{y-b}{a} \quad (a \neq 0).$$

64.

x	$\cdots$	-4	$\cdots$	-2	$\cdots$	0	$\cdots$	2	$\cdots$	4	$\cdots$
$\lfloor x/2 \rfloor$	$\cdots$	-2	-2	-1	-1	0	0	1	1	2	$\cdots$



(plotted over gridlines by hand)