[§2.1] 1. (a) $\{-1, 1\}$

(d) $\emptyset$

2b. $\{x \in \mathbb{Z} \mid |x| \leq 3\}$ where $\mathbb{Z}$ is the set of integers and $|x|$ is the absolute value of x

3. (a) $\boxed{\text{The second is a subset of the first.}}$ From New York to New Delhi, all nonstop airline flights are also airline flights, but some airline flights have stops, so the first is not a subset of the second.

(c) $\boxed{\text{The first is a subset of the second.}}$ All flying squirrels are also living creatures that can fly. Many living creatures that can fly are not flying squirrels, so the second is not a subset of the first.

7. (a) $\boxed{\text{Yes.}}$ 2 is in the domain (real numbers) and is an integer greater than 1.

(c) $\boxed{\text{Yes.}}$ The elements of this set are 2 and $\{2\}$.

(e) $\boxed{\text{No.}}$ The elements of this set are $\{2\}$ and $\{2, \{2\}\}$.

9. (a) $\boxed{\text{False.}}$ $\emptyset$ contains no elements, by definition.

(b) $\boxed{\text{False.}}$ The only element of $\{0\}$ is 0, which is not $\emptyset$.

(c) $\boxed{\text{False.}}$ There are no proper subsets of $\emptyset$.

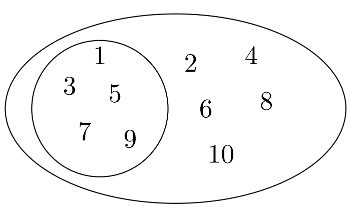
(d) $\boxed{\text{True.}}$ $\emptyset$ is a proper subset of any nonempty set.

(e) $\boxed{\text{False.}}$ The only element of $\{0\}$ is 0, which is not $\{0\}$.

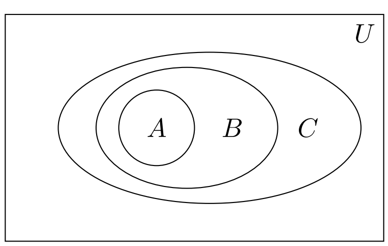
(f) $\boxed{\text{False.}}$ The sets are equal, so neither is a proper subset of the other.

(g) $\boxed{\text{True.}}$ Any set is a subset of itself.

12.



14.



where U is the universal set.

19. (a) $\boxed{1.}$ The only element is a.

(b) $\boxed{1.}$ The only element is $\{a\}$.

(c) $\boxed{2.}$ The two elements are a and $\{a\}$.

(d) $\boxed{3.}$ The three elements are a, $\{a\}$, and $\{a, \{a\}\}$.

23. (b) $\boxed{16.}$ $\{\emptyset, a, \{a\}, \{\{a\}\}\}$ has 4 elements, so its powerset has 2^4 elements.

(c) $\boxed{2.}$ $\mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$.

32. (a) $A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1),$
$(b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1),$
$(c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

(b) $C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c),$
$(1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$

(c) $C \times A \times B = \{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y),$
$(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)\}$

(d) $B \times B \times B = \{(x, x, x), (x, x, y), (x, y, x), (y, x, x),$
$(x, y, y), (y, x, y), (y, y, x), (y, y, y)\}$

36. $\boxed{mnp.}$ $A \times B \times C$ consists of all ordered triples (a, b, c) where $a \in A$, $b \in B$, and $c \in C$. Since there are m possibilities for a, n possibilities for b, and p possibilities for c, the total number of ordered triples (a, b, c) is the product of m, n, p.

41. (a) "The square of any real number is not -1." <u>True.</u>
The square of any real number is nonnegative.

(d) "There exists a real number equal to the square of itself." <u>True.</u>
For example, $0 = 0^2$.

[§2.2] 2. (a) $A \cap B$

(b) $A \cap \overline{B}$

(c) $A \cup B$

(d) $\overline{A \cap B}$ [this is the complement of (a)]

3. (a) $\{0, 1, 2, 3, 4, 5, 6\}$

(b) $\{3\}$

(c) $\{1, 2, 4, 5\}$

(d) $\{0, 6\}$

9. (a) $A \cup \overline{A} = \{x \mid \underbrace{(x \in A) \vee (x \notin A)}_{\text{always true}}\} = U$

(b) $A \cap \overline{A} = \{x \mid \underbrace{(x \in A) \wedge (x \notin A)}_{\text{impossible}}\} = \emptyset$

12. The other laws in Table 1 can be applied to the left hand side of the equation to show it equals the right hand side:

$$\begin{aligned} A \cup (A \cap B) &= (A \cup A) \cap (A \cup B) && \text{distributive law} \\ &= A \cap (A \cup B) && \text{idempotent law} \\ &= A && \text{second absorption law} \end{aligned}$$

18. (b) It suffices to show that if x belongs to $(A \cap B \cap C)$ then x also belongs to $(A \cap B)$. Suppose $x \in (A \cap B \cap C)$. Then, by definition,

$$(x \in A) \wedge (x \in B) \wedge (x \in C),$$

which implies, by simplification,

$$(x \in A) \wedge (x \in B).$$

Hence $x \in (A \cap B)$. $\square$

(c) It suffices to show that if x belongs to $(A - B) - C$ then x also belongs to $A - C$. Suppose $x \in (A - B) - C$. Then, by the definition of difference,

$$(x \in A - B) \wedge (x \notin C).$$

Equivalently, by the same definition,

$$(x \in A) \wedge (x \notin B) \wedge (x \notin C),$$

which implies, by simplification,

$$(x \in A) \wedge (x \notin C).$$

Hence $x \in A - C$. $\square$

29. (a) B is a subset of A, since this union implies B contains no elements not in A.

(b) A is a subset of B, since this intersection implies every common element is in A.

(c) A and B are disjoint, since there was no element in B that existed in A.

(d) A and B are any two sets. This is an identity (commutative law).

(e) A and B are equal. Since

$$A - B = \{x \in A \mid x \notin B\},$$
$$B - A = \{x \in B \mid x \notin A\},$$

and neither $(x \in A) \wedge (x \notin A)$ nor $(x \in B) \wedge (x \notin B)$ is ever possible, both differences result in the empty set;

$$A - B = B - A \quad \Longrightarrow \quad A - B = \emptyset = B - A,$$

which implies every element in A is in B, and vice versa.

50. (a) $A_1 = \{1, 2, 3, \ldots\}$, $A_2 = \{2, 3, 4, \ldots\}$, $A_3 = \{3, 4, 5, \ldots\}$, $\ldots$, so the generalized union is $A_1 = \mathbb{Z}^+$ and the generalized intersection is the empty set because no element is common to every set in the collection.

$$\boxed{\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}^+ \qquad \bigcap_{i=1}^{\infty} A_i = \emptyset}$$

(b) $A_1 = \{0, 1\}$, $A_2 = \{0, 2\}$, $\ldots$, so the generalized union is $\mathbb{N}$, the set of nonnegative integers, and the generalized intersection is $\{0\}$ because 0 is the only number in every set in the collection.

$$\boxed{\bigcup_{i=1}^{\infty} A_i = \mathbb{N} \qquad \bigcap_{i=1}^{\infty} A_i = \{0\}}$$

(c) $A_1 = (0, 1)$, $A_2 = (0, 2)$, $\ldots$, so the generalized union is $(0, \infty) = \mathbb{R}^+$ and the generalized intersection is $A_1 = (0, 1)$ which is the only set that is a subset of every set in the collection.

$$\boxed{\bigcup_{i=1}^{\infty} A_i = \mathbb{R}^+ \qquad \bigcap_{i=1}^{\infty} A_i = (0, 1)}$$

(d) $A_1 = (1, \infty)$, $A_2 = (2, \infty)$, $\ldots$, so the generalized union is $A_1 = (1, \infty)$ and the generalized intersection is the empty set because no element is common to every set in the collection.

$$\boxed{\bigcup_{i=1}^{\infty} A_i = (1, \infty) \qquad \bigcap_{i=1}^{\infty} A_i = \emptyset}$$