

[§6.3] 28.  $\binom{40}{17} = 88,732,378,800$  keys

33.  $\binom{10}{3}\binom{15}{3} = 54,600$  committees

37. Length-10 bit strings of at least three of both ① and ② must be comprised of either

- $3 \times \textcircled{1}$  and  $7 \times \textcircled{2}$ ,
- $4 \times \textcircled{1}$  and  $6 \times \textcircled{2}$ ,
- $5 \times \textcircled{1}$  and  $5 \times \textcircled{2}$ ,
- $6 \times \textcircled{1}$  and  $4 \times \textcircled{2}$ , or
- $7 \times \textcircled{1}$  and  $3 \times \textcircled{2}$ ;

thus there are  $\sum_{3 \leq k \leq 7} \binom{10}{k} = \boxed{912 \text{ strings}}$ .

[§6.4] 2. (a)  $(x+y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y)$  gives rise to six terms:

- i.  $x^5$ , obtainable  $\binom{5}{5} = 1$  way (viz.  $xxxxx$ )
- ii.  $x^4y$ , obtainable  $\binom{5}{4} = 5$  ways ( $xxxxy, xxxyx, \dots$ )
- iii.  $x^3y^2$ , obtainable  $\binom{5}{3} = 10$  ways
- iv.  $x^2y^3$ , obtainable  $\binom{5}{2} = 10$  ways
- v.  $xy^4$ , obtainable  $\binom{5}{1} = 5$  ways
- vi.  $y^5$ , obtainable  $\binom{5}{0} = 1$  way

Therefore,  $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$ .

(b) Using the binomial theorem,

$$\begin{aligned} (x+y)^5 &= \sum_{0 \leq j \leq 5} \binom{5}{j} x^{5-j} y^j \\ &= \binom{5}{0} x^5 + \binom{5}{1} x^4 y + \binom{5}{2} x^3 y^2 + \binom{5}{3} x^2 y^3 + \binom{5}{4} x y^4 + \binom{5}{5} y^5 \\ &= x^5 + 5x^4 y + 10x^3 y^2 + 10x^2 y^3 + 5x y^4 + y^5. \end{aligned}$$

4. The coefficient of  $x^5 y^8$  is  $\binom{13}{8} = \boxed{1287}$ .

7. The coefficient of  $x^9$  is  $(-1) \binom{19}{9} 2^{(19-9)} = \boxed{-94,595,072}$ .

12. Row 11 is  $\boxed{1 \quad 11 \quad 55 \quad 165 \quad 330 \quad 462 \quad 462 \quad 330 \quad 165 \quad 55 \quad 11 \quad 1}$ .

15. Using the binomial theorem with  $x = y = 1$ ,  $2^n$  can be expressed as

$$(1+1)^n = \sum_{0 \leq k \leq n} \binom{n}{k} 1^k 1^{n-k} = \sum_{0 \leq k \leq n} \binom{n}{k},$$

and since  $(\sum \binom{n}{k}) \geq \binom{n}{k}$ , it follows that  $\binom{n}{k} \leq 2^n$ .  $\square$

19. Pascal's Identity is

$$\begin{aligned}
 \binom{n}{r-1} + \binom{n}{r} &= \frac{n!}{(r-1)!(n-(r-1))!} + \frac{n!}{r!(n-r)!} \\
 &= \frac{n!(r + (n-r+1))}{r!(n-r+1)!} \\
 &= \frac{(n+1)!}{r!(n+1-r)!} \\
 &= \binom{n+1}{r}.
 \end{aligned}$$

□

[§6.5] 1.  $3^5 = 243$

3.  $26^6 = 308,915,776$

6.  $\binom{3+5-1}{5} = 21$

8.  $\binom{21+12-1}{12} = 225,792,840$

11. The number of pennies and nickels is irrelevant as long as there are at least eight of each.

There are  $\binom{2+8-1}{8} = \boxed{9 \text{ ways to choose eight coins}}$ .

17.  $\frac{10!}{2!3!5!} = 2,520$  strings