[§4.1] 2. (a) $1 = 1 \cdot a \Rightarrow 1 \mid a$ $\square$

(b) $0 = 0 \cdot a \Rightarrow a \mid 0$ $\square$

4. Since $a \mid b$ and $b \mid c$, we can write

$$b = as$$
$$c = bt$$

where s and t are integers. Then $c = ast \Rightarrow a \mid c$. $\square$

7. Since $ac \mid bc$, we can write $bc = acs$ for some integer s.
Since $c \neq 0$, the equation simplifies to $b = as \Rightarrow a \mid b$. $\square$

9. (b) quotient -11, remainder 10
(e) quotient 0, remainder 0
(g) quotient -1, remainder 2

10. (a) quotient 5, remainder 4
(d) quotient -1, remainder 22
(f) quotient 0, remainder 0

12. (a) $\boxed{\text{6:00.}}$ $(100 + 2) \bmod 24 = 6$.
(b) $\boxed{\text{15:00.}}$ $(12 - 45) \bmod 24 = 15$.

13. (a) $\boxed{c = 10.}$ $(9 \cdot 4) \bmod 13 = 10$.
(d) $\boxed{c = 9.}$ $(2 \cdot 4 + 3 \cdot 9) \bmod 13 = 9$.
(e) $\boxed{c = 6.}$ $(4^2 + 9^2) \bmod 13 = 6$.

15. Since $a \bmod m = b \bmod m$, there are integers s and t such that

$$a = sm + r$$
$$b = tm + r.$$

Subtracting the second equation from the first gives $a - b = (s - t)m$ which implies $m \mid (a - b)$, i.e., $a \equiv b \pmod{m}$. $\square$

23. (a) $228 \text{ div } 119 = 1$
$228 \bmod 119 = 109$
(c) $-10101 \text{ div } 333 = -31$
$-10101 \bmod 333 = 222$

24. (a) $\boxed{a = -3.}$ This is $43 + k \cdot 23$ for $k = -2$.
(c) $\boxed{a = 94.}$ This is $-11 + k \cdot 21$ for $k = 5$.

⚠ 31–32. *Consistent with the order in which questions were assigned, my answers to 31(a) and 32(a)(c) appear after 35 on the next page.*

35. $a \equiv b \pmod{m} \Rightarrow m \mid (a-b)$, and since $n \mid m$, it is also the case that $n \mid (a-b)$. Therefore, by definition, $a \equiv b \pmod{n}$. $\square$

31a. The expression simplifies to $(-133+261) \bmod 23 = \boxed{13}$

32. (a) $(19^2 \bmod 41) \bmod 9 = 33 \bmod 9 = \boxed{6}$

    (c) $(7^3 \bmod 23)^2 \bmod 31 = 441 \bmod 31 = \boxed{7}$

45.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$\cdot_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

[§4.3] 2. (d) $\boxed{\text{Yes.}}$ 101 has no positive factors other than 1 and 101

    (e) $\boxed{\text{Yes.}}$ 107 has no positive factors other than 1 and 107

4. (a) $39 = 3 \cdot 13$

    (e) $289 = 17 \cdot 17$

14. $\boxed{1, 5, 7, \text{ and } 11}$ are relatively prime to 12.

16a. $\boxed{\text{Yes, the set is pairwise relatively prime.}}$ $\gcd(21,34) = \gcd(21,55) = \gcd(34,55) = 1$.

32. (c) $\boxed{\gcd(277,123) = 1}$ because 1 is the last nonzero remainder.

$$
\begin{aligned}
277 &= 2 \cdot 123 + 31 \\
123 &= 3 \cdot 31 + 30 \\
31 &= 1 \cdot 30 + 1 \\
30 &= 30 \cdot 1
\end{aligned}
$$

    (d) $\boxed{\gcd(1529,14039) = 139}$ because 139 is the last nonzero remainder.

$$
\begin{aligned}
14039 &= 9 \cdot 1529 + 278 \\
1529 &= 5 \cdot 278 + 139 \\
278 &= 2 \cdot 139
\end{aligned}
$$

40. (b) By the Euclidean algorithm,

$$
\begin{aligned}
44 &= 1 \cdot 33 + 11 \\
33 &= 3 \cdot 11,
\end{aligned}
$$

so $\gcd(33,44) = 11$. By subtracting $1 \cdot 33$ from both sides of the first equation,

$$11 = -1 \cdot 33 + 1 \cdot 44.$$

(c) By the Euclidean algorithm,

$$\begin{aligned} 78 &= 2 \cdot 35 + 8 \\ 35 &= 4 \cdot 8 + 3 \\ 8 &= 2 \cdot 3 + 2 \\ 3 &= 1 \cdot 2 + 1 \\ 2 &= 2 \cdot 1, \end{aligned}$$

so $\gcd(35, 78) = 1$. Working upward to find substitute terms,

$$\begin{aligned} 1 &= 3 - 1 \cdot 2 \\ &= 3 - (8 - 2 \cdot 3) \\ &= (35 - 4 \cdot 8) - (8 - 2 \cdot (35 - 4 \cdot 8)) \\ &= 35 - 4 \cdot 8 - (8 - 2 \cdot 35 + 8 \cdot 8) \\ &= 35 - 4 \cdot (78 - 2 \cdot 35) - (9 \cdot (78 - 2 \cdot 35) - 2 \cdot 35) \\ &= 35 - 4 \cdot 78 + 8 \cdot 35 - (9 \cdot 78 - 18 \cdot 35 - 2 \cdot 35) \\ &= \underbrace{35 + 8 \cdot 35 + 20 \cdot 35}_{29 \cdot 35} - \underbrace{9 \cdot 78 - 4 \cdot 78}_{13 \cdot 78} \\ &= 29 \cdot 35 - 13 \cdot 78. \end{aligned}$$