[§9.5] 4. (1) Suppose two students are equivalent if they have the same graduation year. Then an equivalence class consists of the set of students graduating in any single year, assuming at least one student will graduate that year.

(2) Suppose two students are equivalent if they have the same academic advisor. Suppose advisors are Profs. A, B, and C; equivalence classes are the set of students assigend to Prof. A, the set of students assigned to Prof. B, and the set of students assigned to Prof. C, assuming each professor advises at least one student.

(3) Partition the set of students into two classes: those who will, and those who will not, pursue an advanced degree. Then every student who will (or will not) pursue an advanced degree is equivalent to every such student, including oneself.

9. (a) R is
   - ✓ reflexive: $f(x) = f(x)$
   - ✓ symmetric: $f(x) = f(y) \Leftrightarrow f(y) = f(x)$
   - ✓ transitive: $[f(x) = f(y)] \wedge [f(y) = f(z)] \Rightarrow f(x) = f(z)$

   (b) Equivalence classes are the sets $f^{-1}(b)$ for each b in the range of f

15. R is
   - ✓ reflexive: $a + b = b + a \Rightarrow ((a, b), (a, b)) \in R$
   - ✓ symmetric: $[a + d = b + c] \Leftrightarrow [c + b = d + a]$, thus

$$((a, b), (c, d)) \in R \Leftrightarrow ((c, d), (a, b)) \in R$$

   - ✓ transitive: $[a + d = b + c] \wedge [c + e = d + f] \Rightarrow a + e = b + f$, thus

$$[((a, b), (c, d)) \in R] \wedge [((c, d), (e, f)) \in R] \Rightarrow ((a, b), (e, f)) \in R$$

26. (a) $[0] = \{0\}, [1] = \{1\}, [2] = \{2\}, [3] = \{3\}$

   (c) $[0] = \{0\}, [1] = \{1, 2\}, [2] = \{1, 2\}, [3] = \{3\}$

   (e) (not an equivalence relation)

35. (a) $[2]_5 = \{i \mid i \equiv 2 \pmod 5\} = \{\ldots, -8, -3, 2, 7, 12, \ldots\}$

   (b) $[3]_5 = \{i \mid i \equiv 3 \pmod 5\} = \{\ldots, -7, -2, 3, 8, 13, \ldots\}$

   (c) $[6]_5 = \{i \mid i \equiv 6 \pmod 5\} = \{\ldots, -9, -4, 1, 6, 11, \ldots\}$

   (d) $[-3]_5 = \{i \mid i \equiv -3 \pmod 5\} = \{\ldots, -8, -3, 2, 7, 12, \ldots\}$

36. (a) $[4]_2 = \{i \mid i \equiv 4 \pmod 2\} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$

   (b) $[4]_3 = \{i \mid i \equiv 4 \pmod 3\} = \{\ldots, -5, -2, 1, 4, 7, \ldots\}$

   (c) $[4]_6 = \{i \mid i \equiv 4 \pmod 6\} = \{\ldots, -8, -2, 4, 10, 16, \ldots\}$

   (d) $[4]_8 = \{i \mid i \equiv 4 \pmod 8\} = \{\ldots, -12, -4, 4, 12, 20, \ldots\}$

37. For each $k = 0, 1, \ldots, 5$: $[k]_6 = \{i \mid i \equiv k \pmod 6\}$

41. (a) No — not disjoint

(b) Yes

(c) Yes

(d) No — missing the element 3

44. (a) Yes

(b) No — missing the element 0

(c) Yes

(d) Yes

(e) No — not disjoint

47. (b) $\{(0,0),(0,1),(1,0),(1,1),(2,2),(2,3),(3,2),(3,3),(4,4),(4,5),(5,4),(5,5)\}$

(c) $\{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2),(3,3),(3,4),(3,5),(4,3),(4,4),(4,5),(5,3),(5,4),(5,5)\}$

(d) $\{(0,0),(1,1),(2,2),(3,3),(4,4),(5,5)\}$

48c. $\{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d),(e,e),(e,f),(e,g),(f,e),(f,f),(f,g),(g,e),(g,f),(g,g)\}$

[§9.3] 1. (a)
$$\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

(b)
$$\begin{bmatrix}0 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

(c)
$$\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$$

4. (b) $\{(1,1),(1,2),(1,3),(2,2),(3,3),(3,4),(4,1),(4,4)\}$

(c) $\{(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)\}$

26. $\{(a,a),(a,b),(b,a),(b,b),(c,a),(c,c),(c,d),(d,d)\}$

27. $\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(d,d)\}$

28. [I will assume the unlabeled vertex is a.]

$\{(a,a),(a,b),(b,a),(b,b),(c,c),(c,d),(d,c),(d,d)\}$

32.

	reflexive	irreflexive	symmetric	antisymmetric	asymmetric	transitive
(26)	✓					
(27)			✓			
(28)	✓		✓			✓