[§6.3] 28. $\binom{40}{17} = 88{,}732{,}378{,}800$ keys

33. $\binom{10}{3}\binom{15}{3} = 54{,}600$ committees

37. Length-10 bit strings of at least three of both ① and ⓪ must be comprised of either

- $3 \times$ ① and $7 \times$ ⓪,
- $4 \times$ ① and $6 \times$ ⓪,
- $5 \times$ ① and $5 \times$ ⓪,
- $6 \times$ ① and $4 \times$ ⓪, or
- $7 \times$ ① and $3 \times$ ⓪;

thus there are $\sum_{3 \le k \le 7} \binom{10}{k} = \boxed{\text{912 strings}}$.

[§6.4] 2. (a) $(x+y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y)$ gives rise to six terms:

i. x^5, obtainable $\binom{5}{5} = 1$ way (viz. $xxxxx$)
ii. x^4y, obtainable $\binom{5}{4} = 5$ ways $(xxxxy, xxxyx, \ldots)$
iii. x^3y^2, obtainable $\binom{5}{3} = 10$ ways
iv. x^2y^3, obtainable $\binom{5}{2} = 10$ ways
v. xy^4, obtainable $\binom{5}{1} = 5$ ways
vi. y^5, obtainable $\binom{5}{0} = 1$ way

Therefore, $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$.

(b) Using the binomial theorem,

$$\begin{aligned}(x+y)^5 &= \sum_{0 \le j \le 5} \binom{5}{j} x^{5-j} y^j \\ &= \binom{5}{0}x^5 + \binom{5}{1}x^4y + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}xy^4 + \binom{5}{5}y^5 \\ &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5.\end{aligned}$$

4. The coefficient of x^5y^8 is $\binom{13}{8} = \boxed{1287}$.

7. The coefficient of x^9 is $(-1)\binom{19}{9}2^{(19-9)} = \boxed{-94{,}595{,}072}$.

12. Row 11 is $\boxed{1 \quad 11 \quad 55 \quad 165 \quad 330 \quad 462 \quad 462 \quad 330 \quad 165 \quad 55 \quad 11 \quad 1}$.

15. Using the binomial theorem with $x = y = 1$, 2^n can be expressed as

$$(1+1)^n = \sum_{0 \le k \le n} \binom{n}{k} 1^k 1^{n-k} = \sum_{0 \le k \le n} \binom{n}{k},$$

and since $\left(\sum \binom{n}{k}\right) \ge \binom{n}{k}$, it follows that $\binom{n}{k} \le 2^n$. $\square$

19. Pascal's Identity is

$$\begin{aligned}\binom{n}{r-1}+\binom{n}{r} &= \frac{n!}{(r-1)!(n-(r-1))!}+\frac{n!}{r!(n-r)!}\\ &= \frac{n!(r+(n-r+1))}{r!(n-r+1)!}\\ &= \frac{(n+1)!}{r!(n+1-r)!}\\ &= \binom{n+1}{r}.\end{aligned}$$ $\square$

[§6.5] 1. $3^5 = 243$

3. $26^6 = 308{,}915{,}776$

6. $\binom{3+5-1}{5} = 21$

8. $\binom{21+12-1}{12} = 225{,}792{,}840$

11. The number of pennies and nickels is irrelevant as long as there are at least eight of each. There are $\binom{2+8-1}{8} = \boxed{\text{9 ways to choose eight coins}}$.

17. $\dfrac{10!}{2!3!5!} = 2{,}520$ strings