*Universität Freiburg*
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# Machine Learning (SS2015)

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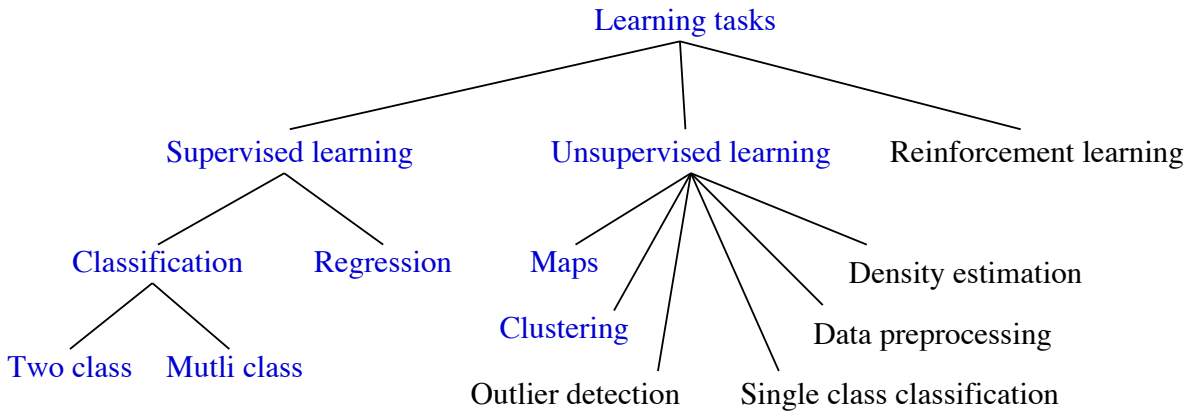
# Exercise Sheet 1

# Exercise 1.1: Introduction to Machine Learning

(a) Given an appropriate dataset, the problems given below can be solved by Machine Learning algorithms. Which of the problems would you apply supervised learning to?

(b) There are two types of supervised learning tasks, classification and regression. Decide for the supervised learning problems given below, whether it is a classification problem or not.

**Types of learning tasks**



| Task | Supervised Learning | Classification |
|---|---|---|
| Predict tomorrow's price of a particular stock. | ☒ | ☐ |
| Discover whether there are different types of spam mail and what categories there are. | ☐ | ☐ |
| Predict your life expectancy. | ☒ | ☐ |
| Predict if it is going to rain tomorrow. | ☒ | ☒ |
| Learn to grasp an object by trial and error. | ☐ | ☐ |

## Exercise 1.2: Version Spaces and Conjunctive Hypotheses

(a) What are the elements of the version space? How are they ordered? What can be said about the meaning and sizes of $S$ and $G$?

the elements of the version space

- hypotheses (descriptions of concepts)
- $VS_{H,D} \subseteq H$ with respect to the hypothesis space $H$ contains those hypotheses that are consistent with the training data $D$

how are they ordered?

- arranged in a general-to-specific ordering
- partial order: $\leq_g$, $<_g$

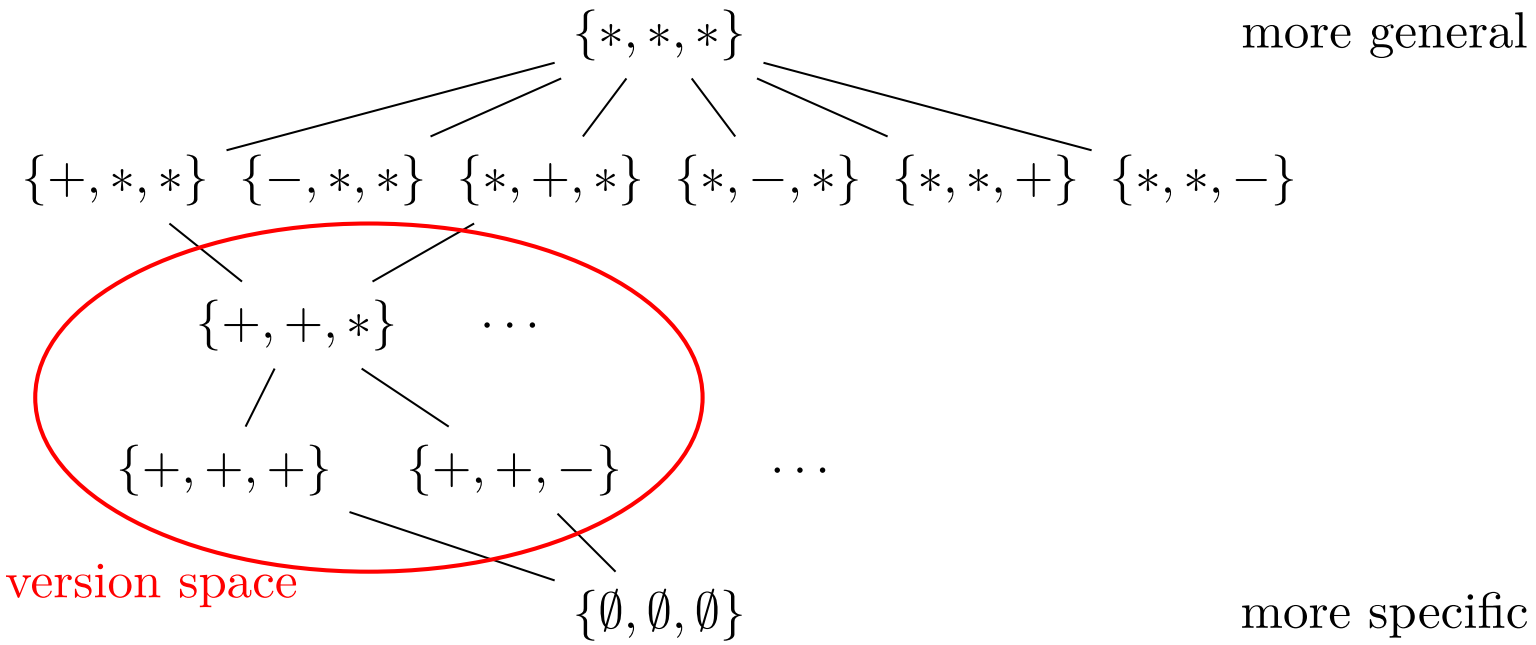
meaning and sizes of $G$ and $S$?

- They are sets containing the most general and most specific hypotheses consistent with the training data. Thus, they depict the general and specific boundary of the VS.
- For conjunctive hypotheses (which we consider here) it always holds $|S| = 1$, assuming consistent training data. $G$ attains its maximal size, if negative patterns with maximal hamming distance have been presented. Thus, in the case of binary constraints, it holds $|G| \leq n(n-1)$ where $n$ denotes the number of constraints per hypothesis.

(b) In the following, it is desired to describe whether a person is *ill*. We use a representation based on conjunctive constraints (three per subject) to describe individual person. These constraints are "running nose", "coughing", and "reddened skin", each of which can take the value true ('+') or false ('–'). We say that somebody is ill, if he is coughing and has a running nose. Each single symptom individually does not mean that the person is ill.

Specify the space of hypotheses that is being managed by the version space approach. To do so, arrange all hypotheses in a graph structure using the more-specific-than relation.

- hypotheses are vectors of constraints, denoted by $\langle N, C, R \rangle$
- with $N, C, R = \{-, +, \emptyset, *\}$



(c) Apply the candidate elimination (CE) algorithm to the sequence of training examples specified in Table 1 and name the contents of the sets $S$ and $G$ after each step.

| Training | running nose | coughing | reddened skin | **Classification** |
|---|---|---|---|---|
| $d_1$ | + | + | + | positive (ill) |
| $d_2$ | + | + | − | positive (ill) |
| $d_3$ | + | − | + | negative (healthy) |
| $d_4$ | − | + | + | negative (healthy) |
| $d_5$ | − | − | + | negative (healthy) |
| $d_6$ | − | − | − | negative (healthy) |

Table 1: List of training instances for the medical diagnosis task.

- Start (init): $G = \{\langle * * * \rangle\}$, $S = \{\langle \emptyset \emptyset \emptyset \rangle\}$
- **foreach** $d \in D$ **do**
  - $d_1 = [\langle + + + \rangle, pos] \Rightarrow G = \{\langle * * * \rangle\}$, $S = \{\langle + + + \rangle\}$
  - $d_2 = [\langle + + - \rangle, pos] \Rightarrow G = \{\langle * * * \rangle\}$, $S = \{\langle + + * \rangle\}$
  - $d_3 = [\langle + - + \rangle, neg]$
    - no change to $S$: $S = \{\langle + + * \rangle\}$
    - specializations of $G$: $G = \{\langle - * * \rangle, \langle * + * \rangle, \langle * * - \rangle\}$
    - there is no element in $S$ that is more specific than the first and third element of $G$
      $\rightarrow$ remove them from $G \Rightarrow G = \{\langle * + * \rangle\}$
  - $d_4 = [\langle - + + \rangle, neg]$
    - no change to $S$: $S = \{\langle + + * \rangle\}$
    - specializations of $G$: $G = \{\langle + + * \rangle, \langle * + - \rangle\}$
    - there is no element in $S$ that is more specific than the second element of $G$
      $\rightarrow$ remove it from $G \Rightarrow G = \{\langle + + * \rangle\}$

- Note:
  - At this point, the algorithm might be stopped, since $S = G$ and no further changes to $S$ and $G$ are to be expected.
  - However, by continuing we might detect inconsistencies in the training data.
- $d_5 = [\langle - - + \rangle, neg] \Rightarrow$ Both, $G = \{\langle + + * \rangle\}$ and $S = \{\langle + + * \rangle\}$ are consistent with $d_5$.
- $d_6 = [\langle - - - \rangle, neg] \Rightarrow$ Both, $G = \{\langle + + * \rangle\}$ and $S = \{\langle + + * \rangle\}$ are consistent with $d_6$.

(d) Does the order of presentation of the training examples (according to Table 1) to the learner affect the finally learned hypothesis?

- No, but it may influence the algorithm's running time.

(e) Assume a domain with two attributes, i.e. any instance is described by two constraints. How many positive and negative training examples are minimally required by the candidate elimination algorithm in order to learn an arbitrary concept?

- By learning an arbitrary concept, of course, we mean that the algorithm arrives at $S = G$.
- The algorithm is started with $S = \{\langle \emptyset, \emptyset \rangle\}$ and $G = \{\langle *, * \rangle\}$.
- We just consider the best cases, i.e. situations in where the training instances given to the CE algorithm allow for adapting $S$ or $G$.
- Negative Examples: Change $G$ from $\langle *, * \rangle$ to $\langle v, * \rangle$ or $\langle *, w \rangle$. Or they change $G$ from $\langle v, * \rangle$ or $\langle *, w \rangle$ to $\langle v, w \rangle$.
- Positive Examples: Change $S$ from $\langle \emptyset, \emptyset \rangle$, $\langle v, w \rangle$. Or they change $S$ from $\langle v, w \rangle$ to $\langle v, * \rangle$ or $\langle *, w \rangle$. Or from $\langle v, * \rangle$ or $\langle *, w \rangle$ to $\langle *, * \rangle$.
- At least one positive example is required (otherwise $S$ remains $\langle \emptyset, \emptyset \rangle$).
- Special case: Two positive patterns $\langle d_1, d_2 \rangle$, $\langle e_1, e_2 \rangle$ are sufficient, if it holds $d_1 \neq e_1$ and $d_2 \neq e_2$. $\Rightarrow S = \langle \emptyset, \emptyset \rangle \rightarrow \langle d_1, d_2 \rangle \rightarrow \langle *, * \rangle$

(f) We are now extending the number of constraints used for describing training instances by one additional constraint named "fever". We say that somebody is ill, if he has a running nose and is coughing (as we did before), or if he has fever.

How does the version space approach using the CE algorithm perform now, given the training examples specified in Table 2? What happens, if the order of presentation of the training examples is altered?

| Training | running nose | coughing | reddened skin | fever | **Classification** |
|---|---|---|---|---|---|
| $d_1$ | + | + | + | − | positive (ill) |
| $d_2$ | + | + | − | − | positive (ill) |
| $d_3$ | − | − | + | + | positive (ill) |
| $d_4$ | + | − | − | − | negative (healthy) |
| $d_5$ | − | − | − | − | negative (healthy) |
| $d_6$ | − | + | + | − | negative (healthy) |

Table 2: List of training instances using the extended representation.

- Initially: $S = \{\langle \emptyset\emptyset\emptyset\emptyset \rangle\}$, $G = \{\langle * * ** \rangle\}$
- $d_1 = [\langle + + + - \rangle, pos] \Rightarrow S = \{\langle + + + - \rangle\}$, $G = \{\langle * * ** \rangle\}$
- $d_2 = [\langle + + - - \rangle, pos] \Rightarrow S = \{\langle + + * - \rangle\}$, $G = \{\langle * * ** \rangle\}$
- $d_3 = [\langle - - + + \rangle, pos] \Rightarrow S = \{\langle * * ** \rangle\}$, $G = \{\langle * * ** \rangle\}$
  $\rightarrow$ We already arrive at $S = G$.
- $d_4 = [\langle + - - - \rangle, neg] \Rightarrow S = \{\langle * * ** \rangle\}$, $G = \{\langle * * ** \rangle\}$
  - Now, $S$ becomes empty since $\langle * * ** \rangle$ is inconsistent with $d_4$ and is removed from $S$.
  - $G$ would be specialized to $\{\langle - * ** \rangle, \langle * + ** \rangle, \langle * * +* \rangle, \langle * * *+ \rangle\}$. But it is required that at least one element from $S$ must be more specific than any element from $G$.
    $\rightarrow$ This requirement cannot be fulfilled

## Exercise 1.3: Decision Tree Learning with ID3

| Training | fever | vomiting | diarrhea | shivering | **Classification** |
|---|---|---|---|---|---|
| $d_1$ | no | no | no | no | healthy (H) |
| $d_2$ | average | no | no | no | influenza (I) |
| $d_3$ | high | no | no | yes | influenza (I) |
| $d_4$ | high | yes | yes | no | salmonella poisoning (S) |
| $d_5$ | average | no | yes | no | salmonella poisoning (S) |
| $d_6$ | no | yes | yes | no | bowel inflammation (B) |
| $d_7$ | average | yes | yes | no | bowel inflammation (B) |

(a) Apply the ID3 algorithm to the training data provided in the table above. Exemplary calculation for the first (root) node.

- entropy of the given data set $S$: $Entropy(S)$

$$= -\tfrac{1}{7}\log_2(\tfrac{1}{7}) - \tfrac{2}{7}\log_2(\tfrac{2}{7}) - \tfrac{2}{7}\log_2(\tfrac{2}{7}) - \tfrac{2}{7}\log_2(\tfrac{2}{7}) = 1.950$$

- consider attribute x="Fever"

| Values | H | I | S | B | $Entropy(S_i)$ |
|---|---|---|---|---|---|
| $S_1$ (no) | * | | | * | $[\frac{1}{2}, 0, 0, \frac{1}{2}] \rightarrow 1$ |
| $S_2$ (average) | | * | * | * | $[0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}] \rightarrow 1.585$ |
| $S_3$ (high) | | * | * | | $[0, \frac{1}{2}, \frac{1}{2}, 0] \rightarrow 1$ |

$\Rightarrow Entropy(S|Fever) = \frac{2}{7} \cdot 1 + \frac{3}{7} \cdot 1.585 + \frac{2}{7} \cdot 1 = 1.251$

- consider attribute x="Vomiting"

| Values | H | I | S | B | $Entropy(S_i)$ |
|---|---|---|---|---|---|
| $S_1$ (yes) | | | * | ** | $[0, 0, \frac{1}{3}, \frac{2}{3}] \rightarrow 0.918$ |
| $S_2$ (no) | * | ** | * | | $[\frac{1}{4}, \frac{2}{4}, \frac{1}{4}, 0] \rightarrow 1.5$ |

$\Rightarrow Entropy(S|Vomiting) = \frac{3}{7} \cdot 0.918 + \frac{4}{7} \cdot 1.5 = 1.251$

- consider attribute x="Diarrhea"

| Values | H | I | S | B | $Entropy(S_i)$ |
|---|---|---|---|---|---|
| $S_1$ (yes) | | | ** | ** | $[0, 0, \frac{2}{4}, \frac{2}{4}] \rightarrow 1$ |
| $S_2$ (no) | * | ** | | | $[\frac{1}{3}, \frac{2}{3}, 0, 0] \rightarrow 0.918$ |

$\Rightarrow Entropy(S|Diarrhea) = \frac{4}{7} \cdot 1 + \frac{3}{7} \cdot 0.918 = 0.965$

- consider attribute x="Shivering"

| Values | H | I | S | B | $Entropy(S_i)$ |
|---|---|---|---|---|---|
| $S_1$ (yes) | | * | | | $[0,0,1,0] \rightarrow 0$ |
| $S_2$ (no) | * | * | ** | ** | $[\frac{1}{6},\frac{1}{6},\frac{2}{6},\frac{2}{6}] \rightarrow 1.918$ |

$\Rightarrow Entropy(S|Shivering) = \frac{1}{7} \cdot 0 + \frac{6}{7} \cdot 1.918 = 1.644$

choose the attribute that maximizes the information gain

- Fever: $Gain(S) = Ent(S) - Ent(S|Fever) = 1.95 - 1.251 = 0.699$
- Vomiting: $Gain(S) = Ent(S) - Ent(S|Vomit) = 1.95 - 1.251 = 0.699$
- Diarrhea: $Gain(S) = Ent(S) - Ent(S|Diarrh) = 1.95 - 0.965 = 0.985$
- Shivering: $Gain(S) = Ent(S) - Ent(S|Shiver) = 1.95 - 1.644 = 0.306$

$\Rightarrow$ Attribute "Diarrhea" is the most effective one, maximizing the information gain.

- Now split the data using attribute "Diarrhea" and apply the algorithm recursively on subsets.

(b) Does the resulting decision tree provide a disjoint definition of the classes?

- Yes, the resulting decision tree provides disjoint class definitions.

(c) Consider the use of real-valued attributes, when learning decision trees, as described in the lecture. The table below shows the relationship between the body height and the gender of a group of persons (the records have been sorted with respect to the value of *height* in cm). Calculate the information gain for potential splitting thresholds and determine the best one.

| *Height* | 161 | 164 | 169 | 175 | 176 | 179 | 180 | 184 | 185 |
|---|---|---|---|---|---|---|---|---|---|
| *Gender* | F | F | M | M | F | F | M | M | F |

- Potential cut points must lie in the intervals $(164, 169)$, $(175, 176)$, $(179, 180)$, or $(184, 185)$.

- Calculate the information gain for the potential splitting thresholds
- $C_1 \in (164, 169)$
  - resulting class distribution: if $x < C_1$ then $2 - 0$ else $3 - 4$
  - conditional entropy: if $x < C_1$ then $E = 0$ else $E = -\frac{3}{7} \log_2 \frac{3}{7} - \frac{4}{7} \log_2 \frac{4}{7} = 0.985$
  - entropy: $E(C_1|S) = \frac{2}{9} \cdot 0 + \frac{7}{9} \cdot 0.985 = 0.766$
- $C_2 \in (175, 176)$
  - resulting class distribution: if $x < C_2$ then $2 - 2$ else $3 - 2$
  - entropy: $E(C_2|S) = \frac{4}{9} \cdot 1 + \frac{5}{9} \cdot 0.971 = 0.984$
- $C_3 \in (179, 180)$
  - resulting class distribution: if $x < C_3$ then $4 - 2$ else $1 - 2$
  - entropy: $E(C_3|S) = \frac{6}{9} \cdot 0.918 + \frac{3}{9} \cdot 0.918 = 0.918$
- $C_4 \in (184, 185)$
  - resulting class distribution: if $x < C_4$ then $4 - 4$ else $1 - 0$
  - entropy: $E(C_4|S) = \frac{8}{9} \cdot 1 + \frac{1}{9} \cdot 0 = 0.889$

- Prior entropy of $S$ is $-\frac{5}{9}\log_2\frac{5}{9} - \frac{4}{9}\log_2\frac{4}{9} = 0.991$.
- Information gain
  - $Gain(S, C_1) = 0.225$
  - $Gain(S, C_2) = 0.007$
  - $Gain(S, C_3) = 0.073$
  - $Gain(S, C_4) = 0.102$

  $\rightarrow$ First splitting point ($C_1$) is the best one.