is usually a good idea to try starting NLP algorithms from different points to determine if the problem has different local optimal solutions. This procedure often reveals the global optimal solution. (Two questions at the end of this chapter illustrate this process.)

A Note About "Optimal" Solutions

When solving an NLP problem, Solver normally stops when the first of three numerical tests is satisfied, causing one of the following three completion messages to appear:

1. "**Solver found a solution. All constraints and optimality conditions are satisfied.**" This means Solver found a local optimal solution, but does not guarantee that the solution is the global optimal solution. Unless you know that a problem has only one local optimal solution (which must also be the global optimal solution), you should run Solver from several different starting points to increase the chances that you find the global optimal solution to your problem. The easiest way to do this is to set the Engine tab Global Optimization group MultiStart option to True before you solve—this will automatically run the Solver from several randomly (but efficiently) chosen starting points.
2. "**Solver has converged to the current solution. All constraints are satisfied.**" This means the objective function value changed very slowly for the last few iterations. If you suspect the solution is not a local optimal solution, your problem may be poorly scaled. The convergence option in the Solver Options dialog box can be reduced to avoid convergence at suboptimal solutions.
3. "**Solver cannot improve the current solution. All constraints are satisfied.**" This rare message means that your model is degenerate and the Solver is cycling. Degeneracy can often be eliminated by removing redundant constraints in a model.

A Note About Starting Points

Solver sometimes has trouble solving an NLP problem if it starts at the null starting point, where all the decision variables are set equal to 0—even if this solution is feasible. Therefore, when solving an NLP problem, it is best to specify a non-null starting solution whenever possible.

We will now consider several examples of NLP problems. These examples illustrate some of the differences between LP and NLP problems and provide insight into the broad range of problems that cannot be modeled adequately using LP.

8.4 Economic Order Quantity Models

The economic order quantity (EOQ) problem is one of the most common business problems for which nonlinear optimization can be used. This problem is encountered when a manager must determine the optimal number of units of a product to purchase whenever an order is placed. The basic model for an EOQ problem makes the following assumptions:

1. Demand for (or use of) the product is fairly constant throughout the year.
2. Each new order is delivered in full when the inventory level reaches 0.

Figure 8.4 illustrates the type of inventory patterns observed for a product when the preceding conditions are met. In each graph, the inventory levels are depleted at a constant rate, representing constant demand. Also, the inventory levels are replenished instantly whenever the inventory levels reach 0.

The key issue in an EOQ problem is to determine the optimal quantity to order whenever an order is placed for an item. The trade-offs in this decision are evident in Figure 8.4. The graphs indicate two ways of obtaining 150 units of a product during the year. In the first graph, an order for 50 units is received whenever the inventory level drops to 0. This requires that three purchase orders be issued during the year

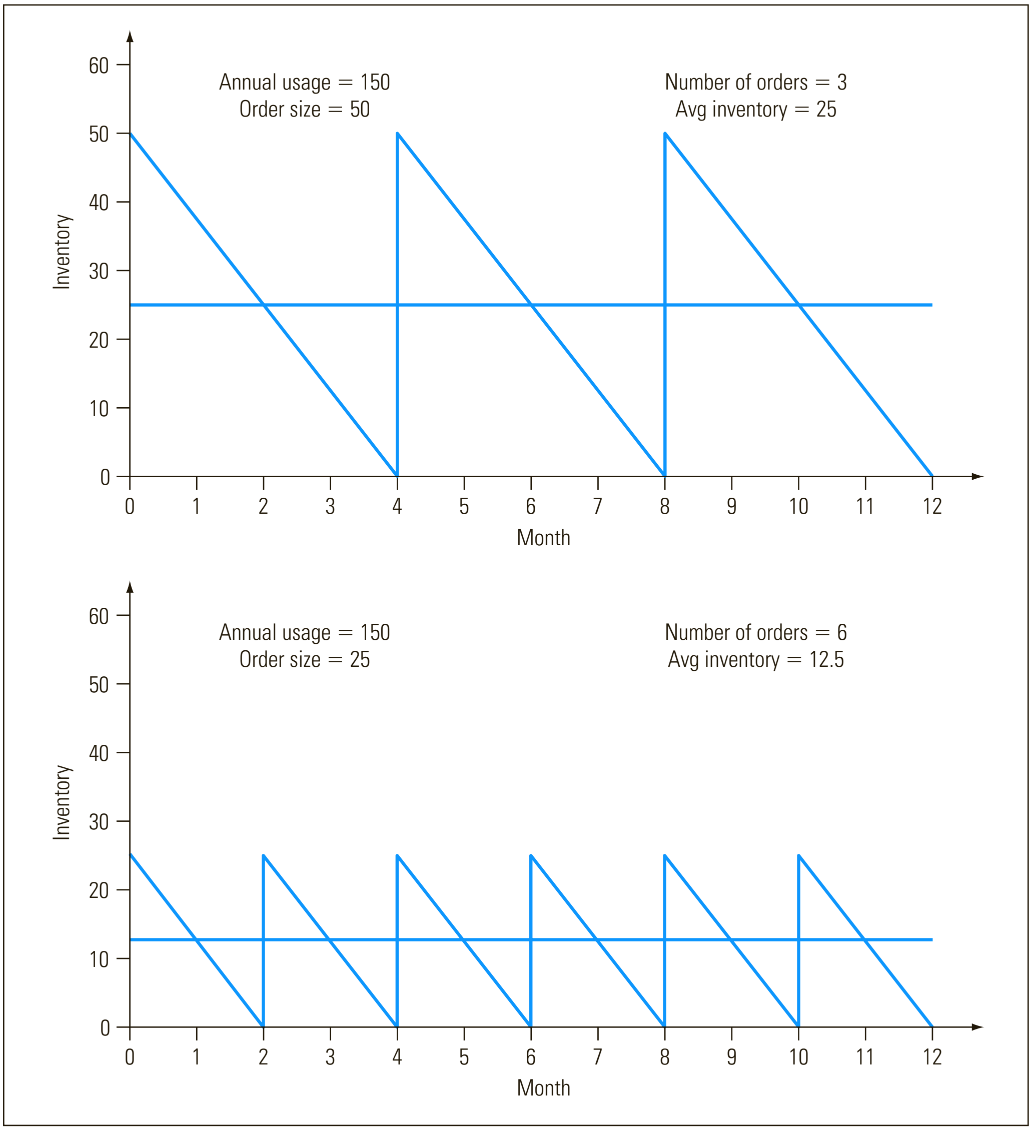


FIGURE 8.4

Inventory profiles of products for which the EOQ assumptions are met and results in an average inventory level of 25 units. In the second graph, an order for 25 units is received whenever the inventory level drops to 0. This requires that six purchase orders be issued throughout the year and results in an average inventory level of 12.5 units. Thus, the first ordering strategy results in fewer purchase orders (and lower ordering costs) but higher inventory levels (and higher carrying costs). The second ordering strategy results in more purchase orders (and higher ordering costs) but lower levels of inventory (and lower carrying costs).

In the basic EOQ model, the total annual cost of stocking a product is computed as the sum of the actual purchase cost of the product, plus the fixed cost of placing orders, plus the cost of holding (or carrying) the product in inventory. Figure 8.5 shows the relationships among order quantity, carrying cost, ordering cost, and total cost. Notice that as the order quantity increases, ordering costs decrease and carrying costs increase. The goal in this type of problem is to find the EOQ that minimizes the total cost.

The total annual cost of acquiring products that meet the stated assumptions is represented by:

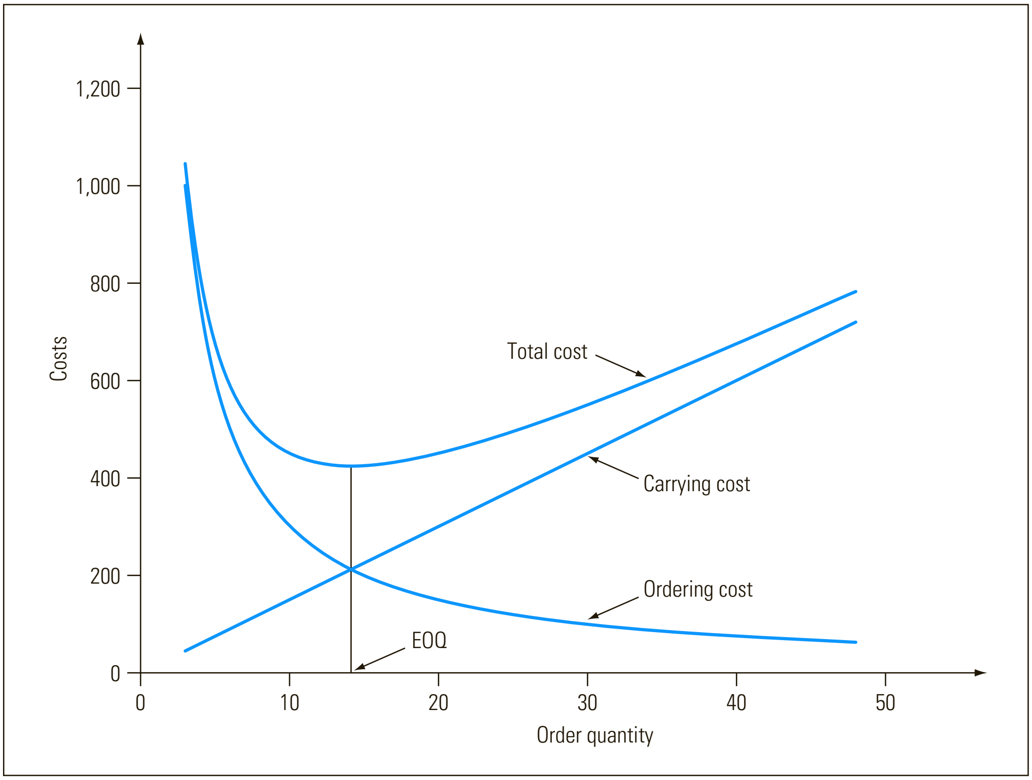
$$\text{Total annual cost} = DC + \frac{D}{Q}S + \frac{Q}{2}Ci$$

where:

D = annual demand for the item
C = unit purchase cost for the item
S = fixed cost of placing an order
i = cost of holding one unit in inventory for a year (expressed as a percentage of C)
Q = order quantity, or quantity ordered each time an order is placed

FIGURE 8.5

Relationship between order quantity, carrying cost, ordering cost, and total cost



The first term in this formula (DC) represents the cost of purchasing a year's worth of the product. The second term $\frac{D}{Q}S$ represents the annual ordering costs. Specifically, $\frac{D}{Q}$ represents the number of orders placed during a year. Multiplying this quantity by S represents the cost of placing these orders. The third term $\frac{Q}{2}Ci$ represents the annual cost of holding inventory. On average, $\frac{Q}{2}$ units are held in inventory throughout the year (refer to Figure 8.4). Multiplying this term by Ci represents the cost of holding these units. The following example illustrates the use of the EOQ model.

> Alan Wang is responsible for purchasing the paper used in all the copy machines and laser printers at the corporate headquarters of MetroBank. Alan projects that in the coming year he will need to purchase a total of 24,000 boxes of paper, which will be used at a fairly steady rate throughout the year. Each box of paper costs $35. Alan estimates that it costs $50 each time an order is placed (this includes the cost of placing the order plus the related costs in shipping and receiving). MetroBank assigns a cost of 18% to funds allocated to supplies and inventories because such funds are the lifeline of the bank and could be lent out to credit card customers who are willing to pay this rate on money borrowed from the bank. Alan has been placing paper orders once a quarter, but he wants to determine if another ordering pattern would be better. He wants to determine the most economical order quantity to use in purchasing the paper.

8.4.1 IMPLEMENTING THE MODEL

To solve this problem, we first need to create a spreadsheet model of the total cost formula described earlier, substituting the data for Alan's problem for the parameters D, C, S, and *i*. This spreadsheet implementation is shown in Figure 8.6 (and in the file Fig8-6.xlsm that accompanies this book).

In Figure 8.6, cell D4 represents the annual demand (D), cell D5 represents the per-unit cost (C), cell D6 represents the cost of placing an order (S), cell D7 represents the inventory holding cost (*i*) expressed as a percentage of an item's value, and cell D9 represents the order quantity (Q). The data corresponding to Alan's decision problem have been entered into the appropriate cells in this model. Because Alan places orders once a quarter (or four times a year), the order quantity in cell D9 is set at $24{,}000 \div 4 = 6{,}000$.

We calculate each of the three pieces of our total cost function in cells D11, D12, and D13. Cell D11 contains the cost of purchasing a year's worth of paper, cell D12 represents the cost associated with placing orders, and cell D13 is the inventory holding cost that would be incurred. The sum of these costs is calculated in cell D14.

Formula for cell D11: =D5*D4

Formula for cell D12: =D4/D9*D6

Formula for cell D13: =D9/2*D7*D5

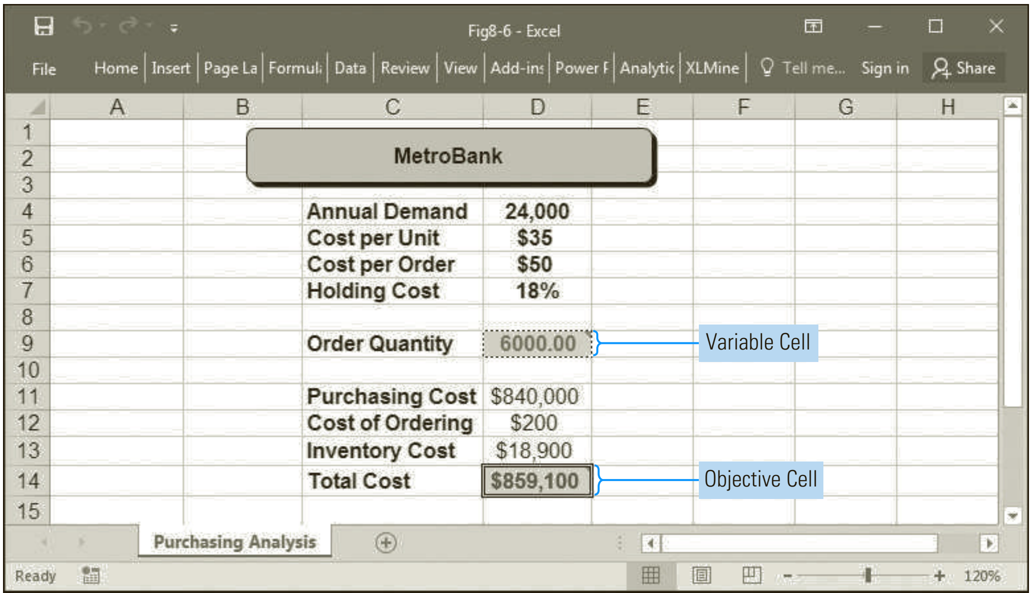
Formula for cell D14: =SUM(D11:D13)

8.4.2 SOLVING THE MODEL

The goal in this problem is to determine the order quantity (the value of Q) that minimizes the total cost. That is, we want Solver to determine the value for cell D9 that minimizes the value in cell D14. Figure 8.7 shows the Solver parameters and options required to solve this problem. Note that a constraint has been placed on cell D9 to prevent the order quantity from becoming 0 or negative. This constraint requires that at least one order must be placed during the year.

FIGURE 8.6

Spreadsheet implementation of MetroBank's paper purchasing problem



Key Cell Formulas

Cell	**Formula**	**Copied to**
D11	=D5*D4	--
D12	=D4/D9*D6	--
D13	=D9/2*D5*D7	--
D14	=SUM(D11:D13)	--

FIGURE 8.7

Solver parameters for MetroBank's paper purchasing problem

Solver Settings:

Objective: D14 (Min)
Variable cells: D9
Constraints:
B9:D9 >=1

Solver Options:
Standard LSGRG Nonlinear Engine

A Note About Engine Options

When solving an NLP problem, it is important *not* to select the Standard LP engine option. When this option is selected, Analytic Solver Platform conducts a number of internal tests to verify that the model is truly linear in the objective and constraints. If this option is selected and Solver's tests indicate that the model is *not* linear, a message appears indicating that the conditions for linearity are not satisfied.

8.4.3 ANALYZING THE SOLUTION

The optimal solution to this problem is shown in Figure 8.8. This solution indicates that the optimal number of boxes for Alan to order at any time is approximately 617. Because the total cost curve in the basic EOQ model has one minimum point, we can be sure that this local optimal solution is also the global optimal solution for this problem. Notice this solution occurs where the total ordering costs are in balance with the total holding costs. Using this order quantity, costs are reduced by approximately \$15,211 from the earlier level shown in Figure 8.6 when an order quantity of 6,000 was used.

If Alan orders 617 boxes, he needs to place approximately 39 orders during the year $(24{,}000 \div 617 = 38.89)$, or 1.333 orders per week $(52 \div 39 = 1.333)$. As a practical matter, it might be easier for Alan to arrange for weekly deliveries of approximately 461 boxes. This would increase the total cost by only \$167 to \$844,055 but probably would be easier to manage and still save the bank more than \$15,000 per year.

8.4.4 COMMENTS ON THE EOQ MODEL

There is another way to determine the optimal order quantity using the simple EOQ model. Using calculus, it can be shown that the optimal value of Q is represented by:

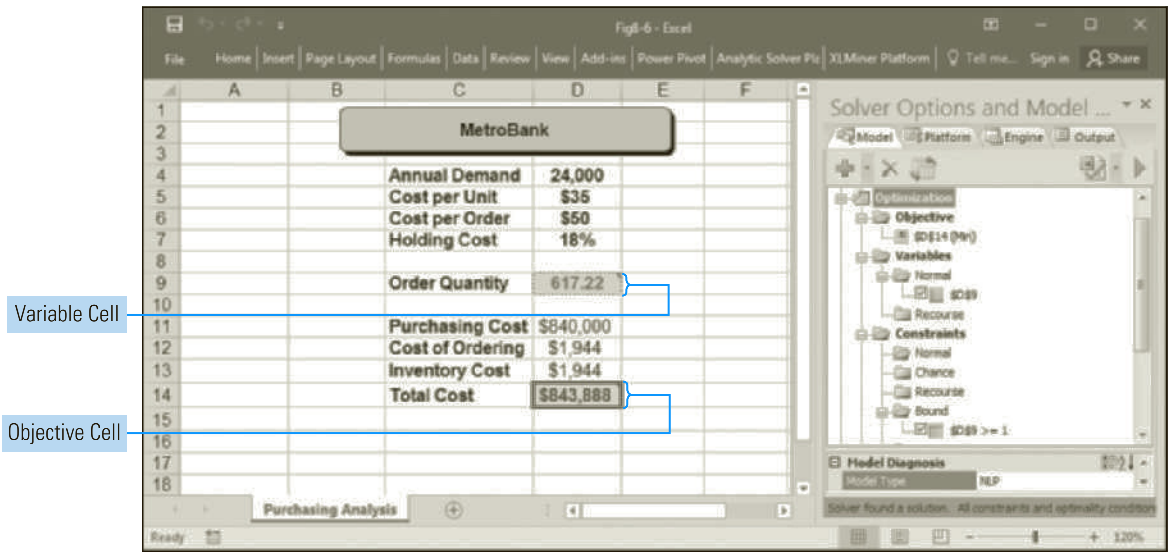
$$Q^* = \sqrt{\frac{2DS}{Ci}}$$

If we apply this formula to our example problem, we obtain:

$$Q^* = \sqrt{\frac{2 \times 24{,}000 \times 50}{35 \times 0.18}} = \sqrt{\frac{2{,}400{,}000}{6.3}} = 617.214$$

The value obtained using calculus is almost the same value obtained using Solver (refer to cell D9 in Figure 8.8). The slight difference in the results might be due to rounding, or to Solver stopping just short of converging on the exact solution.

FIGURE 8.8 *Optimal solution to MetroBank's paper purchasing problem*



Although the previous EOQ formula has its uses, we often must impose financial or storage space restrictions when determining optimal order quantities. The previous formula does not explicitly allow for such restrictions, but it is easy to impose these types of restrictions using Solver. In some of the problems at the end of this chapter, we will consider how the EOQ model can be adjusted to accommodate these types of restrictions, as well as quantity discounts. A complete discussion of the proper use and role of EOQ models in inventory control is beyond the scope of this text, but can be found in other texts devoted to production and operations management.

8.5 Location Problems

A number of decision problems involve determining the location of facilities or service centers. Examples might include determining the optimal location of manufacturing plants, warehouses, fire stations, or ambulance centers. The objective in these types of problems is often to determine a location that minimizes the distance between two or more service points. You might recall from basic algebra that the straight line (or Euclidean) distance between two points (X_1, Y_1) and (X_2, Y_2) on a standard X-Y graph is defined as:

$$\text{Distance} = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$$

This type of calculation is likely to be involved in any problem in which the decision variables represent possible locations. The distance measure might occur in the objective function (for example, we might want to minimize the distance between two or more points) or it might occur in a constraint (for example, we might want to ensure that some minimum distance exists between two or more locations). Problems involving this type of distance measure are nonlinear. The following example illustrates the use of distance measures in a location problem.

> The Rappaport Communications Company provides cellular telephone services in several mid-western states. The company is planning to expand its customer base by offering cellular service in northeastern Ohio to the cities of Cleveland, Akron, Canton, and Youngstown. The company will install the hardware necessary to service customers in each city on preexisting communications towers in each city. The locations of these towers are summarized in Figure 8.9.
>
> However, the company also needs to construct a new communications tower somewhere between these cities to handle intercity calls. This tower will also allow cellular calls to be routed onto the satellite system for worldwide calling service. The tower the company is planning to build can cover areas within a 40-mile radius. Thus, the tower needs to be located within 40 miles of each of these cities.

It is important to note that we could have overlaid the X- and Y-axes on the map in Figure 8.9 in more than one way. The origin in Figure 8.9 could be located anywhere on the map without affecting the analysis. To establish the X-Y coordinates, we need an absolute reference point for the origin, but virtually any point on the map could be selected as the origin. Also we can express the scaling of the X-axis and Y-axis in a number of ways: meters, miles, inches, feet, and so on. For our purposes, we will assume that each unit along the X- and Y-axes represents one mile.