



Inventory Models

- The study of <u>inventory models</u> is concerned with two basic questions:
 - How much should be ordered each time
 - When should the reordering occur (How Often?)
- The objective is to <u>minimize total variable cost</u> over a specified time period (assumed to be annual in the following review).

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Inventory Costs

- Ordering cost -- salaries and expenses of processing an order, regardless of the order quantity
- <u>Holding cost</u> -- usually a percentage of the value of the item assessed for keeping an item in inventory (including finance costs, insurance, security costs, taxes, warehouse overhead, and other related variable expenses)
- <u>Backorder cost</u> -- costs associated with being out of stock when an item is demanded (including lost goodwill or lost sales)
- Purchase cost -- the actual price of the items

Economic Order Quantity (EOQ): Introduction

- The simplest inventory models assume demand and the other parameters of the problem to be <u>deterministic</u> and constant.
- The most basic of the deterministic inventory models is the Economic Order Quantity (EOQ).
- The variable costs in this model are annual holding cost and annual ordering cost.
- For the EOQ, the annual holding and ordering costs are equal.

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The Economic Order Quantity (EOQ) Problem

- Involves determining the optimal quantity to purchase when orders are placed.
- Small orders result in:
 - low inventory levels & carrying/holding costs
 - frequent orders & higher ordering costs
- Large orders result in:
 - higher inventory levels & carrying/holding costs
 - infrequent orders & lower ordering costs

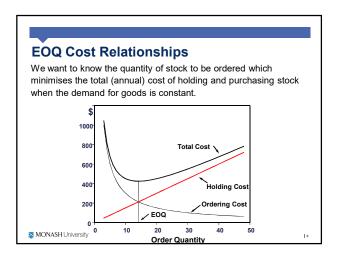
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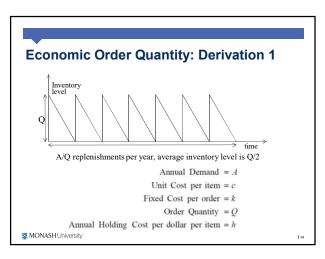
Sample Inventory Profiles

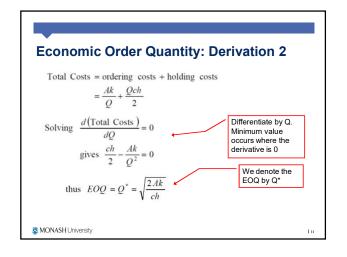
Economic Order Quantity: Assumptions

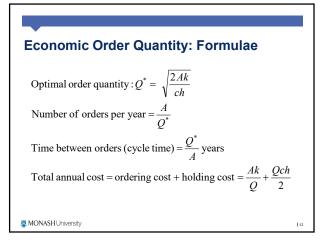
- Demand is constant throughout the year.
- Purchase cost per unit is constant (no quantity discount).
- Delivery time (lead time) is constant (We initially assume that delivery time is 0, that is, that delivery is instantaneous)
- Planned shortages are not permitted.
- Note that even when all of the assumptions of the economic order quantity (EOQ) do not hold, the model may still be used as a good guide to ordering.
- We assume that all values are determined over the same time period, taken to be a year in these notes.

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An EOQ Example: Ordering Paper For MetroBank

- Alan Wang purchases paper for copy machines and laser printers at MetroBank.
 - Annual demand (A) is for 24,000 boxes
 - Each box costs \$35 (c)
 - Each order costs \$50 (k)
 - Inventory carrying costs are 18% (h)
- What is the optimal order quantity Q*?

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MetroBank Example:

Optimal order quantity:

$$Q^{*} = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2 \times 24,000 \times 50}{35 * .18}} = 617.21 \approx 617$$

■ Costs

ordering costs =
$$\frac{Ak}{Q}$$
 = (24,000 * 50)/617.21 = 1944

holding costs =
$$\frac{Qch}{2}$$
 = 617.21 * 35 * .18 / 2 = 1944

Total costs

Total annual cost = ordering cost + holding cost = $\frac{Ak}{Q} + \frac{Q^*ch}{2} = 3889$

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MetroBank Example:

• In this case the number of orders per year is:

$$\frac{A}{Q}$$
 = 24,000 / 617 = 38.9 (approx)

• i.e. the time between orders is:

cycle time =
$$\frac{Q^{*}}{A}$$
 = 617/24,000 years = 9.38 days

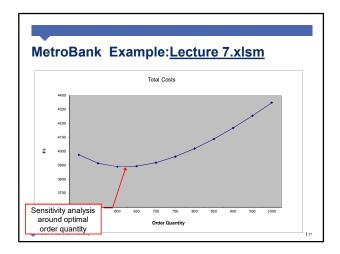
So the solution is to order 617 every 9.4 days

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MetroBank Example:

Question:

- What happens if we change the order quantity to 600?
- \bullet Very little change in Total Costs, but perhaps more convenient \ldots .
- The EOQ model is a very robust model i.e., small variations in the inputs do not change the output (i.e., total costs) much
- Lecture 7.xlsm



Example: Bart's Barometer Business

Economic Order Quantity Model

- Bart's Barometer Business (BBB) is a retail outlet which deals exclusively with weather equipment. Currently BBB is trying to decide on an inventory and reorder policy for home barometers.

 Unit cost per item, c

 Annual demand, A
- Barometers cost BBB \$50 each and demand is about 500 per year distributed fairly evenly throughout the year. Reordering costs are \$80 per order and holding costs are figured at 20% of the cost of the item.

 BBB is open 300 days a year (6 days a week and closed the year less in the part of 2000 days.)
- two weeks in August). Explore lead times of 20/60 days. % Holding cost per \$ per item, h

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Example: Bart's Barometer Business

■ Total Variable Cost Model

Total Costs =
$$\frac{Ak}{Q} + \frac{Qch}{2}$$

= $\frac{500 \times 80}{Q} + \frac{Q(0.2 \times 50)}{2}$
= $\frac{40000}{Q} + 5Q$

Optimal Reorder Quantity

$$Q^* = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2 \times 500 \times 80}{10}} = 89.44 \approx 90$$

■ Thus, if the lead time was 0, Bart should order 90 units when the inventory level is 0.

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Example: Bart's Barometer Business

In this case the number of orders per year is:

$$\frac{A}{Q}$$
 = 500/89.44 = 5.6 (approx)

• i.e. the time between orders is:

cycle time =
$$\frac{Q^*}{A}$$
 = 89.44/500 years = .178 years = approx. every 65 days

(for working days, see next slide)

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Example: Bart's Barometer Business

- Number of reorder times per year = (500/90) = 5.56 or once every (300/5.56) = 54 working days (about every 9 weeks).
- Total Annual Variable Cost:

Total Costs =
$$\frac{Ak}{Q} + \frac{Qch}{2}$$

• TC = (40,000/89.44) + 894.4/2 = 447 + 447 = \$894

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Example: Bart's Barometer Business

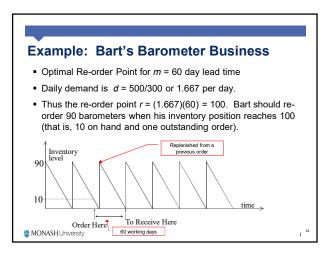
Variation:

- Suppose the lead time for delivery is no longer zero, but lead time is:
 - a. 20 working days
 - b. 60 working days
- We now need to know at what point we should place a new order – i.e., at what current level of inventory should we place a new order
- This is known as the re-order point or r

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Example: Bart's Barometer Business Optimal Re-order Point for Lead time m = 20 days Daily demand is d = 500/300 or 1.667 per day. Thus the re-order point r = (1.667)(20) = 33.34 (i.e., in 20 days, 33.34 barometers are sold) so Bart should re-order 90 barometers when his inventory position reaches 33 on hand. Inventory Order Here To Receive Here 20 working days Order Here To Receive Here 20 working days



Spreadsheet Model

- Spreadsheet showing summary calculations and the comparison of the EOQ with an alternative reorder quantity (in batches of 75).
- See <u>Lecture 7.xlsm</u>

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Annual Demand		500.00	
Ordering Cost	\$	80.00	
Annual Holding Rate%		20.00	
Cost Per Unit	S	50.00	
Working Days Per Year		300.00	
Lead Time (Days)		60.00	
Optimal Order Quantity		89.44	
Requested Order Quantity			75.00
% Change from EOQ			-16.15
Annual Holding Cost	\$	447.21	\$ 375.00
Annual Ordering Cost	\$	447.21	\$ 533.33
Total Annual Cost	\$	894.43	\$ 908.33
% Over Minimum TAC			1.55
Maximum Inventory Level		89.44	75.00
Average Inventory Level		44.72	37.50
Reorder Point		100.00	100.00
Number of Orders Per Year		5.59	6.67
Cycle Time		53.67	45.00

Example: Bart's Barometer Business

Summary of Spreadsheet Results

- A 16.15% negative deviation from the EOQ resulted in only a 1.55% increase in the Total Annual Cost.
- Annual Holding Cost and Annual Ordering Cost are no longer equal.
- The Re-order Point is not affected, in this model, by a change in the Order Quantity.

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EOQ with Planned Shortages

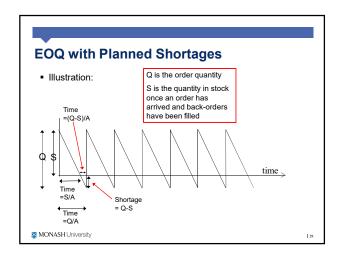
- With the <u>EOQ</u> with planned shortages model, a replenishment order does not arrive at or before the inventory position drops to zero.
- <u>Shortages</u> occur until a predetermined back-order quantity is reached, at which time the replenishment order arrives.
- The variable costs in this model are annual holding, backorder, and ordering.
- For the optimal order and back-order quantity combination, the sum of the annual holding and back-ordering costs equals the annual ordering cost.

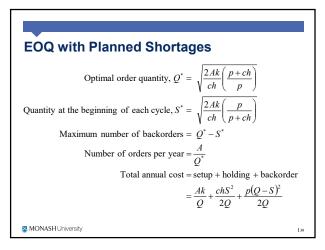
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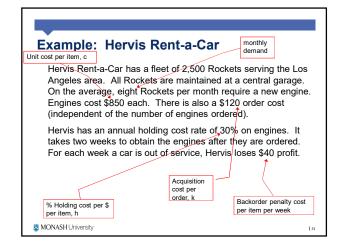
EOQ with Planned Shortages: Assumptions

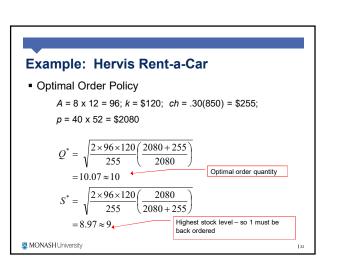
- Demand occurs at a constant rate of A items per year.
- Ordering cost: \$k per order.
- Holding cost: \$ch per item in inventory per year.
- Backorder penalty cost: \$p per item back-ordered per year.
- Purchase cost per unit is constant (no quantity discount).
- Set-up time (lead time) is constant.
- Planned shortages are permitted (back-ordered units are withdrawn from a replenishment order when it is delivered).











Example: Hervis Rent-a-Car

- Demand is 8 per month or 2 per week. Since lead time is 2 weeks, demand through lead time is 4.
- Thus, since the optimal policy is to order 10 to arrive when there is one back-order, the order should be placed when there are 3 engines remaining in inventory.

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Example: Hervis Rent-a-Car

How many days after receiving an order does Hervis run out of engines? How long is Hervis without any engines per cycle?

Solution:

ightharpoonup Inventory exists for p/(p+ch) = 2080/(255+2080) = .8908 of the order cycle.

(Note, S^*/Q^* = .8908 also before Q^* and S^* are rounded.)

- \triangleright An order cycle is $Q^*/A = .1049$ years = 38.3 days. Thus, Hervis runs out of engines .8908(38.3) = 34 days after receiving an order
- ➤ Hervis is out of stock for approximately 38 34 = 4 days

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EOQ with Quantity Discounts

- The EOQ with quantity discounts model is applicable where a supplier offers a lower purchase cost when an item is ordered in larger quantities.
- This model's variable costs are annual holding, ordering and purchase costs.
- For the optimal order quantity, the annual holding and ordering costs are not necessarily equal.

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EOQ with Quantity Discounts: **Asumptions**

- Demand occurs at a constant rate of A items per year.
- Ordering Cost is \$k per order.
- Holding Cost is h. This is equivalent to \$ch per item in inventory per year as per previous models.
- Purchase Cost is

 c_1 per item if the quantity ordered is between 0 and x_1 , c_2 if the order quantity is between x_1 and x_2 , etc.

- Delivery time (lead time) is constant.
- Planned shortages are not permitted.

EOQ with Quantity Discounts

Formulae

- · Optimal order quantity:
 - Calculate the smallest feasible Q* under each pricing structure. Choose the Q* which results in the smallest annual total cost.
- Number of orders per year: A/Q*
- Time between orders (cycle time): Q */A years
- Total annual cost: [(1/2)Q *ch] + [Ak/Q *] + Ac

(holding + ordering + purchase)

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Example: Nick's Camera Shop

- Nick's Camera Shop carries Zodiac instant print film. The film normally costs Nick \$3.20 per roll, and he sells it for \$5.25. Zodiac film has a shelf life of 18 months. Nick's average sales are 21 rolls per week. His annual inventory holding cost rate is 25% and it costs Nick \$20 to place an order with Zodiac.
- If Zodiac offers a 7% discount on orders of 400 rolls or more, a 10% discount for 900 rolls or more, and a 15% discount for 2000 rolls or more, determine Nick's optimal order quantity.

$$A = 21(52) = 1092$$
; $ch = 0.25(c)$; $k = 20$

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Example: Nick's Camera Shop

- Unit-Prices' Economical, Feasible Order Quantities
 - For c_4 = .85(3.20) = \$2.72
- To receive a 15% discount Nick must order at least 2,000 rolls.
 Unfortunately, the film's shelf life is 18 months. The demand in 18 months (78 weeks) is 78 x 21 = 1638 rolls of film.
- If he ordered 2,000 rolls he would have to scrap 362 of them. This would cost more than the 15% discount would save.

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Example: Nick's Camera Shop

- Unit-Prices' Economical, Feasible Order Quantities
 - For c_3 = .90(3.20) = \$2.88.

$$Q_3^* = \sqrt{\frac{2Ak}{c_3h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 2.88}} = 246.31 \text{ (not feasible)}$$

- $-\,$ The most economical, feasible quantity for ${\rm c_3}$ is 900
- For c_2 = .93(3.20) = \$2.976.

$$Q_2^* = \sqrt{\frac{2Ak}{c_2h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 2.976}} = 242.30 \text{ (not feasible)}$$

- The most economical, feasible quantity for c₂ is 400.

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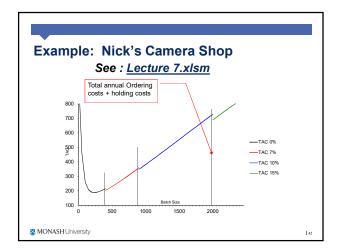
Example: Nick's Camera Shop

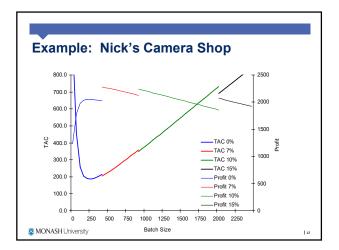
- Unit-Prices' Economical, Feasible Order Quantities
 - For $c_1 = 1.00(3.20) = 3.20 .

$$Q_1^* = \sqrt{\frac{2Ak}{c_1h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 3.20}} = 233.67 \text{ (feasible)}$$

- The following graph shows holding and ordering costs as a function of Q.
- When we reach a <u>computed</u> Q that is feasible we stop computing Q's. (In this problem we have no more to compute, anyway.)

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Example: Nick's Camera Shop

Total Cost Comparison

 Compute the total cost for the most economical, feasible order quantity in each price category for which a Q * was computed.

 $TC_i = [(1/2)Q * ch] + [Ak/Q *] + Ac$

 $TC_3 = (1/2)(900)(.72) + ((1092)(20)/900) + (1092)(2.88) = 3493$

 $TC_2 = (1/2)(400)(.744) + ((1092)(20)/400) + (1092)(2.976) = 3453$

 $TC_1 = (1/2)(234)(.80) + ((1092)(20)/234) + (1092)(3.20) = 3681$

 Comparing the total costs for 234, 400 and 900, the lowest total annual cost is \$3453. Nick should order 400 rolls at a time.

Economic Production Lot Size

- The economic production lot size model is a variation of the basic EOQ model.
- A replenishment order is not received in one lump sum as it is in the basic EOQ model.
- Inventory is replenished gradually as the order is produced (which requires the production rate to be greater than the demand rate)
- This model's variable costs are annual holding cost and annual set-up cost (equivalent to ordering cost).
- For the optimal lot size, annual holding and set-up costs are equal.

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Economic Production Lot Size: Assumptions

- Demand occurs at a constant rate of A items per year.
- Production rate is B items per year (and B > A).
- Set-up cost: \$k per run.
- Holding cost: \$ch per item in inventory per year.
- Manufacturing cost per unit is constant (no quantity discount).
- Set-up time (lead time) is constant.
- Planned shortages are not permitted.



Economic Production Lot Size: Formulae

Optimal production lot size: $Q^* = \sqrt{\frac{2Ak}{ch}} \sqrt{\frac{B}{B-A}}$

Number of production runs per year = $\frac{A}{Q^*}$

Time between setups (cycle time) = $\frac{Q^*}{A}$ years

Total annual cost = setup cost + holding cost = $\frac{Ak}{Q} + \frac{chQ}{2} \left(\frac{B-A}{B} \right)$

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Example: Non-Slip Tile Co.

Economic Production Lot Size Model

Non-Slip Tile (NST) Company has been using production runs of 100,000 tiles, 10 times per year to meet the demand of 1,000,000 tiles annually.

The set-up cost is \$5,000 per run and holding cost is estimated at 10% of the manufacturing cost of \$1 per tile.

The production capacity of the machine is 500,000 tiles per month.

The factory is open 365 days per year.

Example: Non-Slip Tile Co.

Total Annual Variable Cost Model

This is an economic production lot size problem with :

$$A = 1,000,000, B = 6,000,000, ch = .10, k = 5,000$$

Total annual cost = setup cost + holding cost

$$= \frac{Ak}{Q} + \frac{chQ}{2} \left(\frac{B-A}{B} \right)$$

= 5,000,000,000,000/Q + 0.04167Q

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Example: Non-Slip Tile Co.

Optimal production lot size
$$Q^* = \sqrt{\frac{2Ak}{ch}} \sqrt{\frac{B}{B-A}}$$

$$= \sqrt{100,000,000,000} \sqrt{\frac{6}{5}}$$

$$= 346,410$$

Number of Production Runs Per Year = A/Q* = 1000000 / 346410 = 2.89 times per year.

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Example: Non-Slip Tile Co.

Total Annual Variable Cost

- How much is NST losing annually by using their present production schedule? (Substitute Q into the Total Cost equation)
- Optimal *TC* = .04167(346,410) + 5,000,000,000 / 346,410
- = \$28,868
- Current TC = .04167(100,000) + 5,000,000,000 / 100,000
- = \$54,167
- Difference = 54,167 28,868 = \$25,299

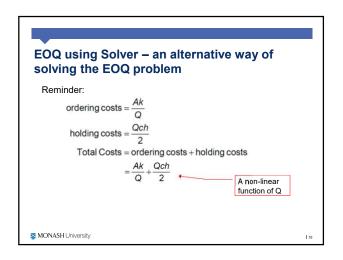
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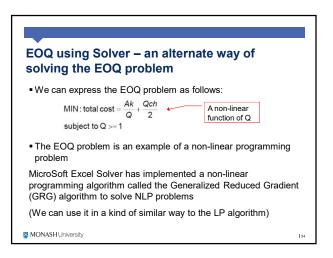
Example: Non-Slip Tile Co.

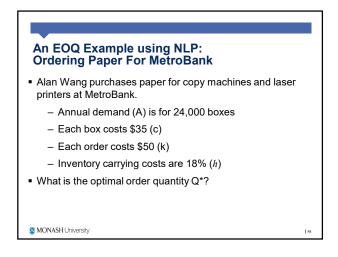
Idle Time Between Production Runs

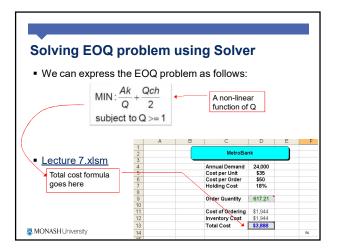
- There are 2.89 cycles per year. Thus, each cycle lasts (365/2.89) = 126.3 days.
- The time to produce 346,410 per run = (346,410/6,000,000)365
 = 21.1 days. Thus, the machine is idle for 126.3 21.1 = 105.2 days between runs.
- Maximum Inventory:
 - Current Policy: = ((B-A)/B)Q * = (5/6)100,000 \approx 83,333.
 - Optimal Policy: = (5/6)346,410 = 288,675.
- Machine Utilization: The machine is producing tiles A/B = 1/6 of the time. (Intuitively, this should be so!)

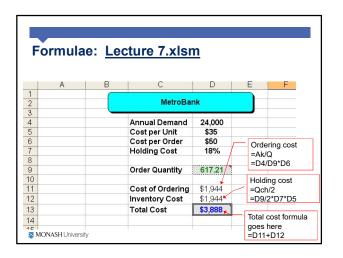
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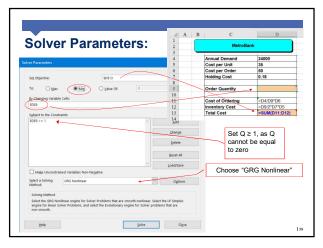


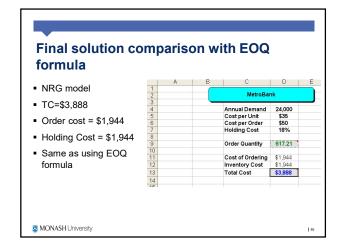


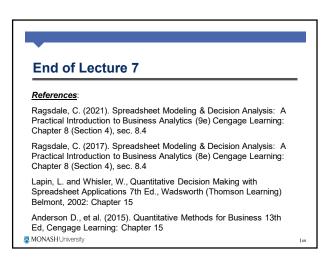












Homework

- ${\blacktriangleright}$ Go through today's lecture examples :
 - Familiarise yourself with the EOQ formulation and be able to
 - The economic order quantity (i.e., the quantity of stock to be ordered which minimises the total annual cost); *
 - How often should the order be placed;
 - Total annual relevant costs.
 - Optimal Inventory Policy with back-ordering (planned shortages)
 - Economic production quantity

Readings for next Lecture:

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 16
- Inventory Decisions with Uncertain Factors

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Tutorial 6 this week:

Network Modelling:

- Transportation Problem
- Assignment Problem
- Transhipment Problem
- > Various techniques will be explored:
 - North-west Corner Method;
 - Vogel's Approximation Method (VAM)
 - MODI (modified Dantzig Iteration) algorithm or the Closed-Loop Path