# FIT3158 Business Decision Modelling

#### Lecture 7

Inventory Modelling under Certainty

#### **Topics Covered:**





## **Inventory Models**

- The study of <u>inventory models</u> is concerned with two basic questions:
  - How much should be ordered each time
  - When should the reordering occur (How Often?)
- The objective is to minimize total variable cost over a specified time period (assumed to be annual in the following review).



#### **Inventory Costs**

- Ordering cost -- salaries and expenses of processing an order, regardless of the order quantity
- Holding cost -- usually a percentage of the value of the item assessed for keeping an item in inventory (including finance costs, insurance, security costs, taxes, warehouse overhead, and other related variable expenses)
- <u>Backorder cost</u> -- costs associated with being out of stock when an item is demanded (including lost goodwill or lost sales)
- Purchase cost -- the actual price of the items

# **Economic Order Quantity (EOQ):**Introduction

- The simplest inventory models assume demand and the other parameters of the problem to be <u>deterministic</u> and constant.
- The most basic of the deterministic inventory models is the <u>Economic Order Quantity (EOQ)</u>.
- The variable costs in this model are annual holding cost and annual ordering cost.
- For the EOQ, the annual holding and ordering costs are equal.

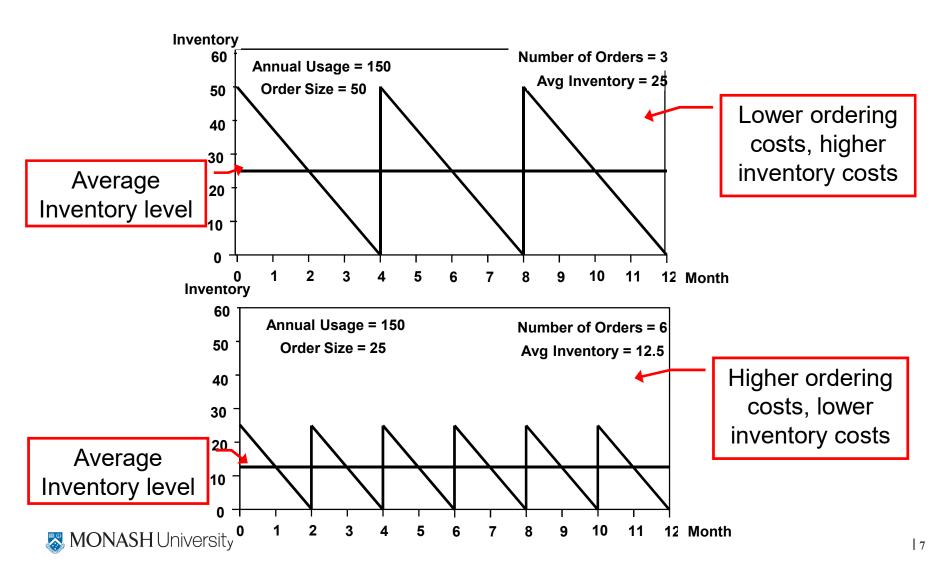


# The Economic Order Quantity (EOQ) Problem

- Involves determining the optimal quantity to purchase when orders are placed.
- Small orders result in:
  - low inventory levels & carrying/holding costs
  - frequent orders & higher ordering costs
- Large orders result in:
  - higher inventory levels & carrying/holding costs
  - infrequent orders & lower ordering costs



## Sample Inventory Profiles



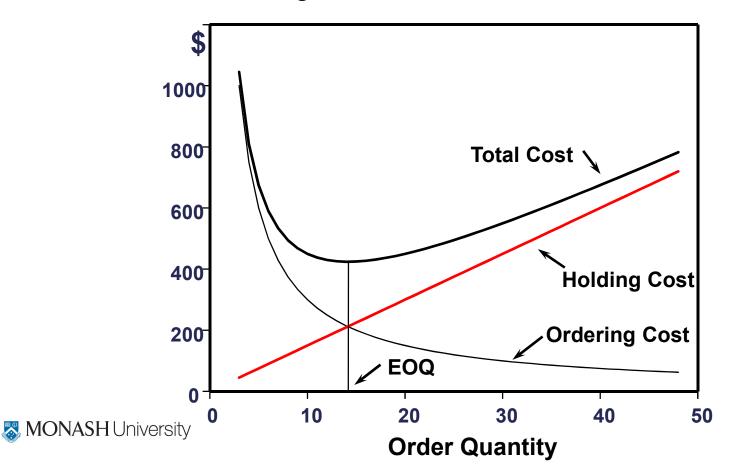
## **Economic Order Quantity: Assumptions**

- Demand is constant throughout the year.
- Purchase cost per unit is constant (no quantity discount).
- Delivery time (lead time) is constant (We initially assume that delivery time is 0, that is, that delivery is instantaneous)
- Planned shortages are not permitted.
- Note that even when all of the assumptions of the economic order quantity (EOQ) do not hold, the model may still be used as a good guide to ordering.
- We assume that all values are determined over the same time period, taken to be a year in these notes.

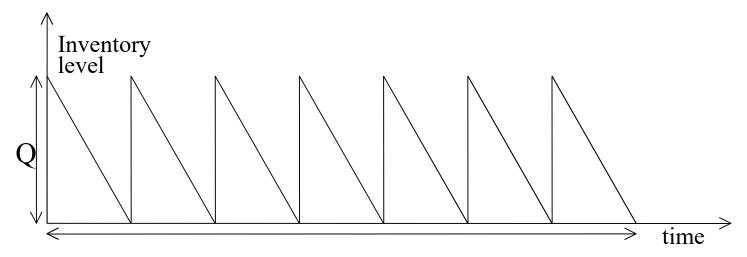


## **EOQ Cost Relationships**

We want to know the quantity of stock to be ordered which minimises the total (annual) cost of holding and purchasing stock when the demand for goods is constant.



#### **Economic Order Quantity: Derivation 1**



A/Q replenishments per year, average inventory level is Q/2

Annual Demand = A

Unit Cost per item = c

e.g., sending an email or clicking links Fixed Cost per order = k

Order Quantity = Q Find this

Annual Holding Cost per dollar per item = h



## **Economic Order Quantity: Derivation 2**

Total Costs = ordering costs + holding costs

$$= \frac{Ak}{Q} + \frac{Qch}{2}$$

Solving 
$$\frac{d(\text{Total Costs})}{dQ} = 0$$

gives  $\frac{ch}{2} - \frac{Ak}{Q^2} = 0$ 

thus 
$$EOQ = Q^* = \sqrt{\frac{2Ak}{ch}}$$

Differentiate by Q. Minimum value occurs where the derivative is 0

We denote the EOQ by Q\*

## **Economic Order Quantity: Formulae**

Optimal order quantity: 
$$Q^* = \sqrt{\frac{2Ak}{ch}}$$

Number of orders per year = 
$$\frac{A}{Q^*}$$

Time between orders (cycle time) = 
$$\frac{Q^*}{A}$$
 years

Total annual cost = ordering cost + holding cost = 
$$\frac{Ak}{Q} + \frac{Qch}{2}$$

## An EOQ Example: Ordering Paper For MetroBank

- Alan Wang purchases paper for copy machines and laser printers at MetroBank.
  - Annual demand (A) is for 24,000 boxes
  - Each box costs \$35 (c)
  - Each order costs \$50 (k)
  - Inventory carrying costs are 18% (h)
- What is the optimal order quantity Q\*?



#### MetroBank Example:

Optimal order quantity:

$$Q^* = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2 \times 24,000 \times 50}{35 * .18}} = 617.21 \approx 617$$

Costs:

ordering costs = 
$$\frac{Ak}{Q}$$
 = (24,000 \* 50)/617.21 = 1944

holding costs = 
$$\frac{Qch}{2}$$
 = 617.21 \* 35 \* .18 / 2 = 1944

Total costs

Total annual cost = ordering cost + holding cost =  $\frac{Ak}{Q^*} + \frac{Q^*ch}{2} = 3889$ 

#### MetroBank Example:

In this case the number of orders per year is:

$$\frac{A}{Q^*}$$
 = 24,000 / 617 = 38.9 (approx)

i.e. the time between orders is:

cycle time = 
$$\frac{Q^*}{A}$$
 = 617/24,000 years = 9.38 days

So the solution is to order 617 every 9.4 days

#### MetroBank Example:

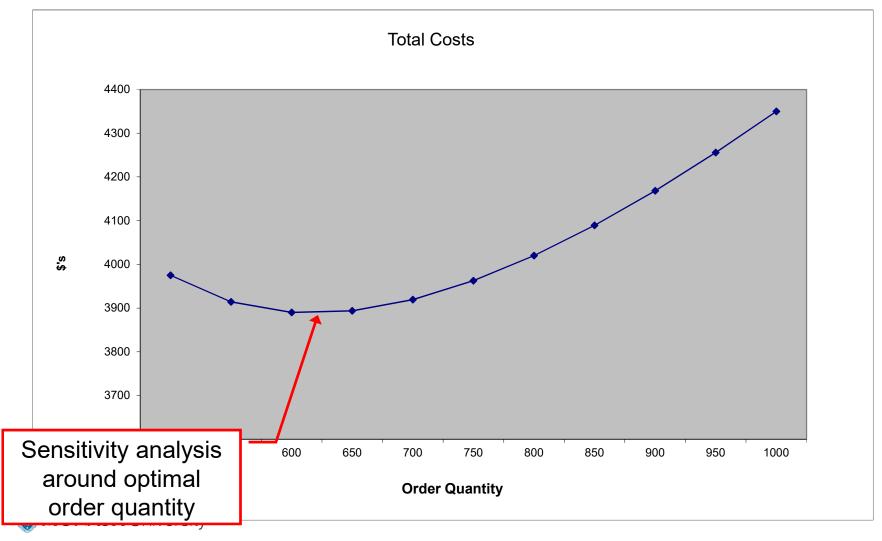
#### Question:

What happens if we change the order quantity to 600?

#### Answer:

- Very little change in Total Costs, but perhaps more convenient
   ....
- The EOQ model is a very robust model i.e., small variations in the inputs do not change the output (i.e., total costs) much
- Lecture 7.xlsm

## MetroBank Example: Lecture 7.xlsm



#### **Economic Order Quantity Model**

- Bart's Barometer Business (BBB) is a retail outlet which deals exclusively with weather equipment. Currently BBB is trying to decide on an inventory and reorder policy for home barometers.

  Unit cost per item, c
  Annual demand, A
- Barometers cost BBB \$50 each and demand is about 500 per year distributed fairly evenly throughout the year. Reordering costs are \$80 per order and holding costs are figured at 20% of the cost of the item.

  Acquisition cost per order (ordering cost), k
- BBB is open 300 days a year (6 days a week and closed two weeks in August). Explore lead times of 20/60 days.

  % Holding cost per \$ per item, h



Total Variable Cost Model

Total Costs = 
$$\frac{Ak}{Q} + \frac{Qch}{2}$$
$$= \frac{500 \times 80}{Q} + \frac{Q(0.2 \times 50)}{2}$$
$$= \frac{40000}{Q} + 5Q$$

Optimal Reorder Quantity

$$Q^* = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2 \times 500 \times 80}{10}} = 89.44 \approx 90$$

Thus, if the lead time was 0, Bart should order 90 units when the inventory level is 0.



In this case the number of orders per year is:

$$\frac{A}{Q^*} = 500/89.44 = 5.6 \text{ (approx)}$$

• i.e. the time between orders is:

cycle time = 
$$\frac{Q^*}{A}$$
 = 89.44/500 years = .178 years = approx. every 65 days

(for working days, see next slide)

■ Number of reorder times per year = (500/90) = 5.56 or once every (300/5.56) = 54 working days (about every 9 weeks).

Total Annual Variable Cost:

Total Costs = 
$$\frac{Ak}{Q} + \frac{Qch}{2}$$

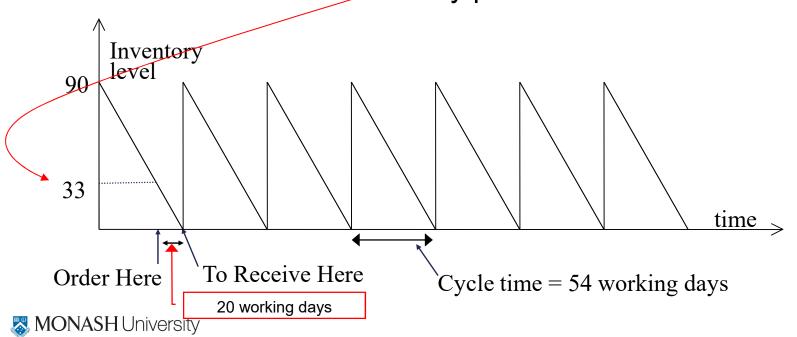
TC = (40,000/89.44) + 894.4/2 = 447 + 447 = \$894

#### Variation:

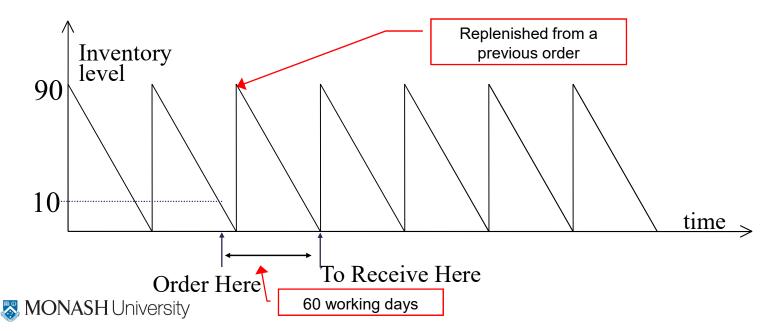
- Suppose the lead time for delivery is no longer zero, but lead time is:
  - a. 20 working days
  - b. 60 working days
- We now need to know at what point we should place a new order – i.e., at what current level of inventory should we place a new order
- This is known as the re-order point or r



- Optimal Re-order Point for Lead time *m* = 20 days
- Daily demand is *d* = 500/300 or 1.667 per day.
- Thus the re-order point r = (1.667)(20) = 33.34 (i.e., in 20 days, 33.34 barometers are sold) so Bart should re-order 90 barometers when his inventory position reaches 33 on hand.



- Optimal Re-order Point for m = 60 day lead time
- Daily demand is d = 500/300 or 1.667 per day.
- Thus the re-order point r = (1.667)(60) = 100. Bart should re-order 90 barometers when his inventory position reaches 100 (that is, 10 on hand and one outstanding order).



## **Spreadsheet Model**

- Spreadsheet showing summary calculations and the comparison of the EOQ with an alternative reorder quantity (in batches of 75).
- See <u>Lecture 7.xlsm</u>

500.00		
\$ 80.00		
20.00		
\$ 50.00		
300.00		
60.00		
89.44		
		75.00
		-16.15
\$ 447.21	\$	375.00
\$ 447.21	\$	533.33
\$ 894.43	\$	908.33
		1.55
89.44		75.00
44.72		37.50
100.00		100.00
5.59		6.67
53.67		45.00
\$ \$ \$ \$	\$ 80.00 20.00 \$ 50.00 300.00 60.00 89.44 \$ 447.21 \$ 447.21 \$ 894.43 89.44 44.72 100.00 5.59	\$ 80.00 20.00 \$ 50.00 300.00 60.00 89.44 \$ 447.21 \$ \$ 447.21 \$ \$ 894.43 \$ 89.44 44.72 100.00 5.59



#### Summary of Spreadsheet Results

- A 16.15% negative deviation from the EOQ resulted in only a 1.55% increase in the Total Annual Cost.
- Annual Holding Cost and Annual Ordering Cost are no longer equal.
- The Re-order Point is not affected, in this model, by a change in the Order Quantity.



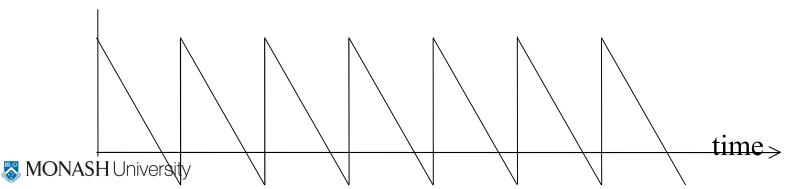
#### **EOQ** with Planned Shortages

- With the <u>EOQ with planned shortages model</u>, a replenishment order does not arrive at or before the inventory position drops to zero.
- Shortages occur until a predetermined back-order quantity is reached, at which time the replenishment order arrives.
- The variable costs in this model are annual holding, backorder, and ordering.
- For the optimal order and back-order quantity combination, the sum of the annual holding and back-ordering costs equals the annual ordering cost.



## **EOQ** with Planned Shortages: Assumptions

- Demand occurs at a constant rate of A items per year.
- Ordering cost: \$k per order.
- Holding cost: \$ch per item in inventory per year.
- Backorder penalty cost: \$p per item back-ordered per year.
- Purchase cost per unit is constant (no quantity discount).
- Set-up time (lead time) is constant.
- Planned shortages are permitted (back-ordered units are withdrawn from a replenishment order when it is delivered).

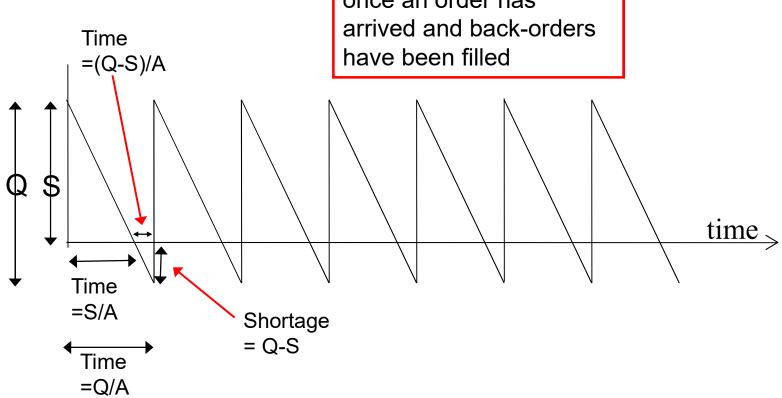


## **EOQ** with Planned Shortages

• Illustration:

Q is the order quantity

S is the quantity in stock once an order has





## **EOQ** with Planned Shortages

Optimal order quantity, 
$$Q^* = \sqrt{\frac{2Ak}{ch} \left(\frac{p+ch}{p}\right)}$$

Quantity at the beginning of each cycle, 
$$S^* = \sqrt{\frac{2Ak}{ch}} \left( \frac{p}{p+ch} \right)$$

Maximum number of backorders =  $Q^* - S^*$ 

Number of orders per year = 
$$\frac{A}{Q^*}$$

Total annual cost = setup + holding + backorder

$$= \frac{Ak}{Q} + \frac{chS^2}{2Q} + \frac{p(Q-S)^2}{2Q}$$

Unit cost per item, c

monthly demand

Hervis Rent-a-Car has a fleet of 2,500 Rockets serving the Los Angeles area. All Rockets are maintained at a central garage. On the average, eight Rockets per month require a new engine. Engines cost \$850 each. There is also a \$120 order cost (independent of the number of engines ordered).

Hervis has an annual holding cost rate of 30% on engines. It takes two weeks to obtain the engines after they are ordered. For each week a car is out of service, Hervis loses \$40 profit.

% Holding cost per \$ per item, h

Acquisition cost per order. k

Backorder penalty cost per item per week



Optimal Order Policy

$$A = 8 \times 12 = 96$$
;  $k = $120$ ;  $ch = .30(850) = $255$ ;  $p = 40 \times 52 = $2080$ 

$$Q^* = \sqrt{\frac{2 \times 96 \times 120}{255} \left(\frac{2080 + 255}{2080}\right)}$$
$$= 10.07 \approx 10$$

Optimal order quantity

$$S^* = \sqrt{\frac{2 \times 96 \times 120}{255}} \left( \frac{2080}{2080 + 255} \right)$$

$$=8.97 \approx 9$$

Highest stock level – so 1 must be back ordered

Demand is 8 per month or 2 per week. Since lead time is 2 weeks, demand through lead time is 4.

■ Thus, since the optimal policy is to order 10 to arrive when there is one back-order, the order should be placed when there are 3 engines remaining in inventory.



#### • Question:

How many days after receiving an order does Hervis run out of engines? How long is Hervis without any engines per cycle?

#### Solution:

 $\triangleright$  Inventory exists for p/(p+ch) = 2080/(255+2080) = .8908 of the order cycle.

(Note,  $S^*/Q^* = .8908$  also before  $Q^*$  and  $S^*$  are rounded.)

- $\triangleright$  An order cycle is  $Q^*/A = .1049$  years = 38.3 days. Thus, Hervis runs out of engines .8908(38.3) = 34 days after receiving an order
- ➤ Hervis is out of stock for approximately 38 34 = 4 days



#### **EOQ** with Quantity Discounts

- The <u>EOQ</u> with quantity discounts model is applicable where a supplier offers a lower purchase cost when an item is ordered in larger quantities.
- This model's variable costs are annual holding, ordering and purchase costs.
- For the optimal order quantity, the annual holding and ordering costs are not necessarily equal.



# **EOQ** with Quantity Discounts: Asumptions

- Demand occurs at a constant rate of A items per year.
- Ordering Cost is \$k per order.
- Holding Cost is h. This is equivalent to \$ch per item in inventory per year as per previous models.
- Purchase Cost is
  - $c_1$  per item if the quantity ordered is between 0 and  $x_1$ ,
  - $c_2$  if the order quantity is between  $c_1$  and  $c_2$ , etc.
- Delivery time (lead time) is constant.
- Planned shortages are not permitted.



## **EOQ** with Quantity Discounts

#### Formulae

- Optimal order quantity:
  - Calculate the smallest feasible Q\* under each pricing structure. Choose the Q\* which results in the smallest annual total cost.
- Number of orders per year: A/Q\*
- Time between orders (cycle time): Q \*/A years
- Total annual cost: [(1/2)Q \*ch] + [Ak/Q \*] + Ac

(holding + ordering + purchase)

- Nick's Camera Shop carries Zodiac instant print film. The film normally costs Nick \$3.20 per roll, and he sells it for \$5.25. Zodiac film has a shelf life of 18 months. Nick's average sales are 21 rolls per week. His annual inventory holding cost rate is 25% and it costs Nick \$20 to place an order with Zodiac.
- If Zodiac offers a 7% discount on orders of 400 rolls or more, a 10% discount for 900 rolls or more, and a 15% discount for 2000 rolls or more, determine Nick's optimal order quantity.

$$A = 21(52) = 1092$$
;  $ch = 0.25(c)$ ;  $k = 20$ 



Unit-Prices' Economical, Feasible Order Quantities

- For 
$$c_4$$
 = .85(3.20) = \$2.72

■ To receive a 15% discount Nick must order at least 2,000 rolls. Unfortunately, the film's shelf life is 18 months. The demand in 18 months (78 weeks) is 78 x 21 = 1638 rolls of film.

• If he ordered 2,000 rolls he would have to scrap 362 of them.
This would cost more than the 15% discount would save.



- Unit-Prices' Economical, Feasible Order Quantities
  - For  $c_3 = .90(3.20) = $2.88$ .

$$Q_3^* = \sqrt{\frac{2Ak}{c_3h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 2.88}} = 246.31 \text{ (not feasible)}$$

- The most economical, feasible quantity for c<sub>3</sub> is 900
- For  $c_2$  = .93(3.20) = \$2.976.

$$Q_2^* = \sqrt{\frac{2Ak}{c_2h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 2.976}} = 242.30 \text{ (not feasible)}$$

- The most economical, feasible quantity for  $c_2$  is 400.

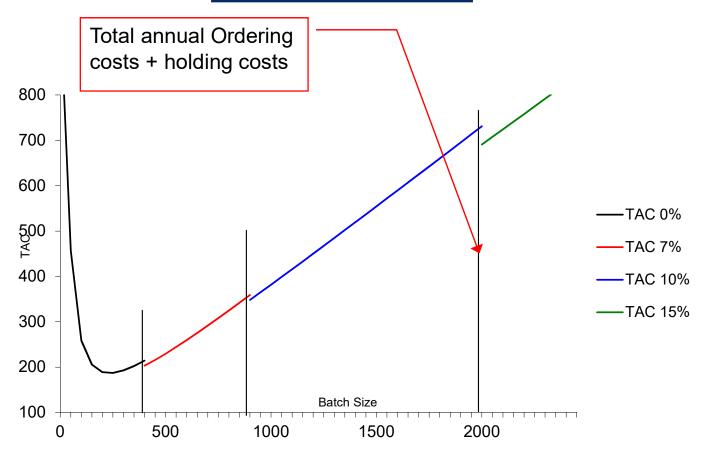
Unit-Prices' Economical, Feasible Order Quantities

- For 
$$c_1 = 1.00(3.20) = $3.20$$
.

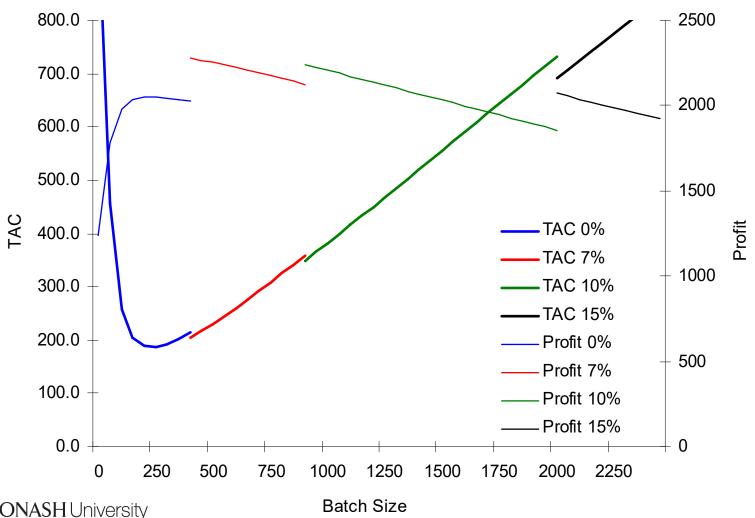
$$Q_1^* = \sqrt{\frac{2Ak}{c_1 h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 3.20}} = 233.67 \text{ (feasible)}$$

- The following graph shows holding and ordering costs as a function of Q.
- When we reach a <u>computed</u> Q that is feasible we stop computing Q's. (In this problem we have no more to compute, anyway.)

See: Lecture 7.xlsm







#### **Total Cost Comparison**

 Compute the total cost for the most economical, feasible order quantity in each price category for which a Q \* was computed.

$$TC_1 = [(1/2)Q *ch] + [Ak/Q *] + Ac$$
 $TC_3 = (1/2)(900)(.72) + ((1092)(20)/900) + (1092)(2.88) = 3493$ 
 $TC_2 = (1/2)(400)(.744) + ((1092)(20)/400) + (1092)(2.976) = 3453$ 
 $TC_1 = (1/2)(234)(.80) + ((1092)(20)/234) + (1092)(3.20) = 3681$ 

 Comparing the total costs for 234, 400 and 900, the lowest total annual cost is \$3453. Nick should order 400 rolls at a time.

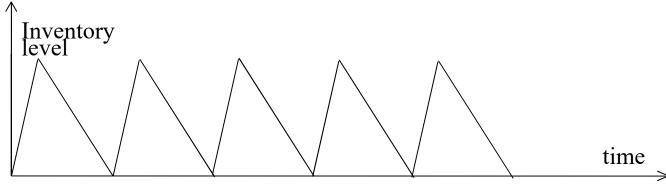
### **Economic Production Lot Size**

- The <u>economic production lot size model</u> is a variation of the basic EOQ model.
- A <u>replenishment order</u> is not received in one lump sum as it is in the basic EOQ model.
- Inventory is replenished gradually as the order is produced (which requires the production rate to be greater than the demand rate).
- This model's variable costs are annual holding cost and annual set-up cost (equivalent to ordering cost).
- For the optimal lot size, annual holding and set-up costs are equal.



## **Economic Production Lot Size: Assumptions**

- Demand occurs at a constant rate of A items per year.
- Production rate is B items per year (and B > A).
- Set-up cost: \$k per run.
- Holding cost: \$ch per item in inventory per year.
- Manufacturing cost per unit is constant (no quantity discount).
- Set-up time (lead time) is constant.
- Planned shortages are not permitted.



## **Economic Production Lot Size: Formulae**

Optimal production lot size: 
$$Q^* = \sqrt{\frac{2Ak}{ch}} \sqrt{\frac{B}{B-A}}$$

Number of production runs per year =  $\frac{A}{Q^*}$ 

Time between setups (cycle time) =  $\frac{Q^*}{A}$  years

Total annual cost = setup cost + holding cost =  $\frac{Ak}{Q} + \frac{chQ}{2} \left( \frac{B-A}{B} \right)$ 

#### Economic Production Lot Size Model

Non-Slip Tile (NST) Company has been using production runs of 100,000 tiles, 10 times per year to meet the demand of 1,000,000 tiles annually.

The set-up cost is \$5,000 per run and holding cost is estimated at 10% of the manufacturing cost of \$1 per tile.

The production capacity of the machine is 500,000 tiles per month.

The factory is open 365 days per year.



#### Total Annual Variable Cost Model

This is an economic production lot size problem with:

$$A = 1,000,000$$
,  $B = 6,000,000$ ,  $ch = .10$ ,  $k = 5,000$ 

Total annual cost = setup cost + holding cost

$$= \frac{Ak}{Q} + \frac{chQ}{2} \left( \frac{B - A}{B} \right)$$
  
= 5,000,000,000/Q + 0.04167 Q

Optimal production lot size 
$$Q^* = \sqrt{\frac{2Ak}{ch}} \sqrt{\frac{B}{B-A}}$$

$$= \sqrt{100,000,000,000} \sqrt{\frac{6}{5}}$$

$$= 346,410$$

Number of Production Runs Per Year = A/Q\*

= 1000000 / 346410 = 2.89times per year.

#### Total Annual Variable Cost

- How much is NST losing annually by using their present production schedule? (Substitute Q into the Total Cost equation)
- Optimal TC = .04167(346,410) + 5,000,000,000 / 346,410
- = \$28,868
- Current TC = .04167(100,000) + 5,000,000,000 / 100,000
- = \$54,167
- Difference = 54,167 28,868 = \$25,299



#### Idle Time Between Production Runs

- There are 2.89 cycles per year. Thus, each cycle lasts (365/2.89) = 126.3 days.
- The time to produce 346,410 per run = (346,410/6,000,000)365 = 21.1 days. Thus, the machine is idle for 126.3 21.1 = 105.2 days between runs.
- Maximum Inventory:
  - Current Policy: = ((B-A)/B)Q \* = (5/6)100,000 ≈ 83,333.
  - Optimal Policy: = (5/6)346,410 = 288,675.
- Machine Utilization: The machine is producing tiles A/B = 1/6 of the time. (Intuitively, this should be so!)



# **EOQ** using Solver – an alternative way of solving the EOQ problem

#### Reminder:

ordering costs = 
$$\frac{Ak}{Q}$$

holding costs = 
$$\frac{Qch}{2}$$

Total Costs = ordering costs + holding costs

$$=\frac{Ak}{Q}+\frac{Qch}{2}$$

A non-linear function of Q

# **EOQ** using Solver – an alternate way of solving the EOQ problem

We can express the EOQ problem as follows:

MIN: total cost = 
$$\frac{Ak}{Q} + \frac{Qch}{2}$$

Subject to Q >= 1

A non-linear function of Q

The EOQ problem is an example of a non-linear programming problem

MicroSoft Excel Solver has implemented a non-linear programming algorithm called the Generalized Reduced Gradient (GRG) algorithm to solve NLP problems

(We can use it in a kind of similar way to the LP algorithm)



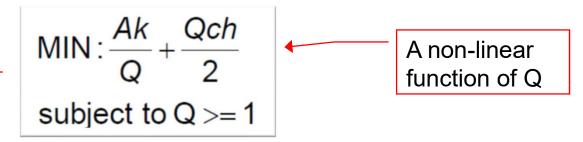
# An EOQ Example using NLP: Ordering Paper For MetroBank

- Alan Wang purchases paper for copy machines and laser printers at MetroBank.
  - Annual demand (A) is for 24,000 boxes
  - Each box costs \$35 (c)
  - Each order costs \$50 (k)
  - Inventory carrying costs are 18% (h)
- What is the optimal order quantity Q\*?



## Solving EOQ problem using Solver

We can express the EOQ problem as follows:



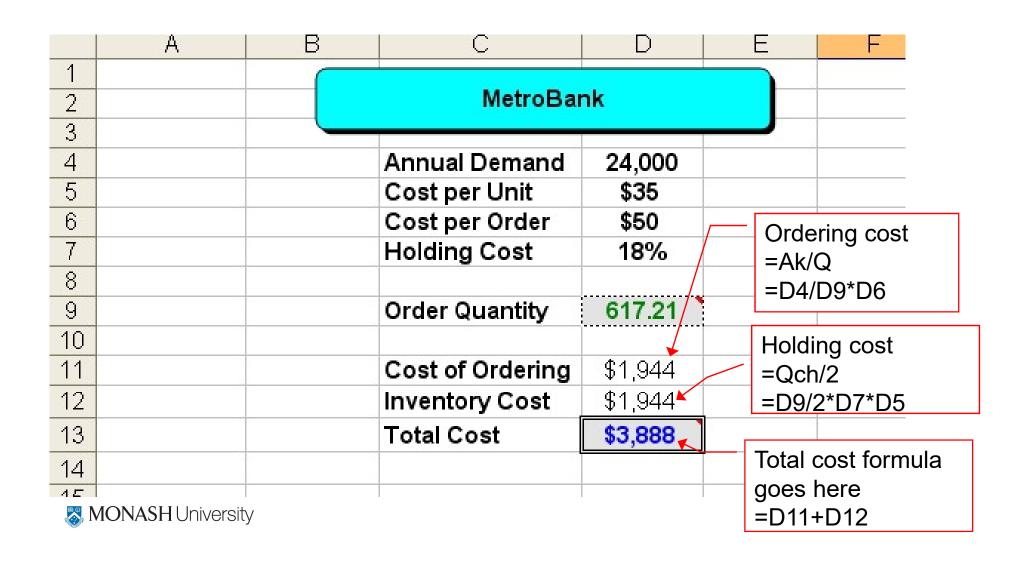
Lecture 7.xlsm

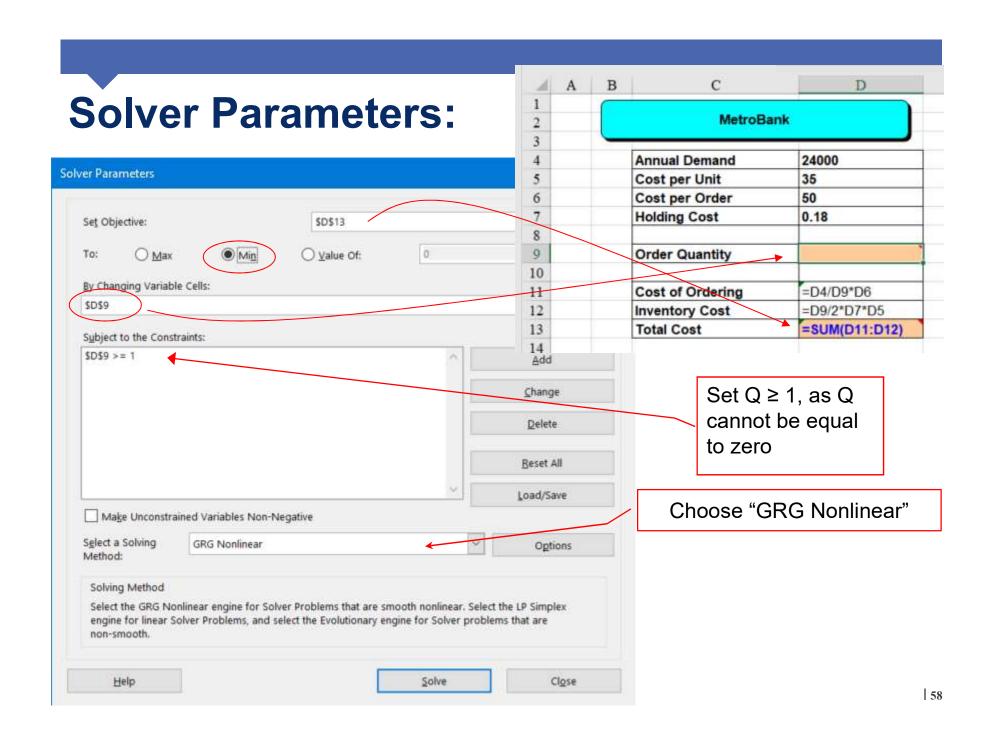
Total cost formula goes here

	А	В	C	D	E	F
1 2			MetroBank			
3 4			Annual Demand	24,000		
5 6 7			Cost per Unit Cost per Order	\$35 \$50		
8			Holding Cost Order Quantity	18% 617.21		
10			Cost of Ordering	\$1,944		
12			Inventory Cost Total Cost	\$1,944 <b>\$3,888</b>		
14			Total Cost	<b>\$3,688</b>		56



## Formulae: Lecture 7.xlsm





# Final solution comparison with EOQ formula

- NRG model
- TC=\$3,888
- Order cost = \$1,944
- Holding Cost = \$1,944
- Same as using EOQ formula

	А	В	C	D	Е		
1							
2			MetroBank				
3							
4			Annual Demand	24,000			
5			Cost per Unit	\$35	) () ()		
6			Cost per Order	\$50			
7			Holding Cost	18%	(d) (z)		
8							
9			Order Quantity	617.21			
10							
11			Cost of Ordering	\$1,944	04 02		
12			Inventory Cost	\$1,944			
13			Total Cost	\$3,888			
14			6.0		(C)		
15							



### **End of Lecture 7**

#### References:

Ragsdale, C. (2021). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (9e) Cengage Learning: Chapter 8 (Section 4), sec. 8.4

Ragsdale, C. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) Cengage Learning: Chapter 8 (Section 4), sec. 8.4

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 15

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning: Chapter 15



## Homework

- Go through today's lecture examples :
  - ✓ Familiarise yourself with the EOQ formulation and be able to determine:
    - The economic order quantity (i.e., the quantity of stock to be ordered which minimises the total annual cost);
    - How often should the order be placed;
    - Total annual relevant costs.
  - ✓ Optimal Inventory Policy with back-ordering (planned shortages)
  - ✓ Economic production quantity

#### Readings for next Lecture:

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 16

- Inventory Decisions with Uncertain Factors



### **Tutorial 6 this week:**

#### Network Modelling:

- Transportation Problem
- Assignment Problem
- Transhipment Problem
- ➤ Various techniques will be explored:
  - North-west Corner Method;
  - Vogel's Approximation Method (VAM)
  - MODI (modified Dantzig Iteration) algorithm or the Closed-Loop Path

