# FIT3158 Note - W11 Queuing theory

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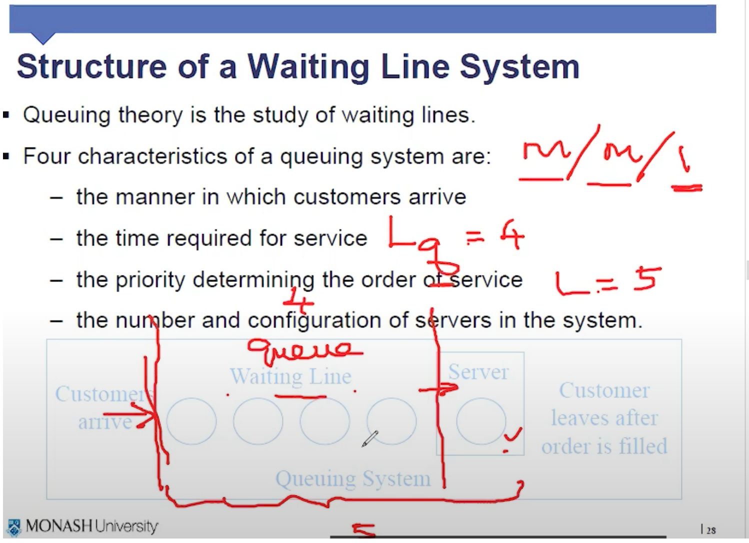
## What is the In-The-System ($L$) and Queue ($L_q$) ?

L refers to number of unit / People in the system.




**L = in the system**
**4 個人等 + 1個人serving = 5**
**所以L= 5**
**-----------------------------**
**4 個人等**
**L_queue = 4**

## What is W?

Same sense as above, but $W = time\ spent\ waiting\ in\ the\ queue + time\ being\ served\ in\ the\ system.$

Whereas $W_q = time\ spent\ waiting\ in\ the\ queue.$

### What are $\mu$ ?

$\mu = Service\ rate$ ; so $Service\ time = 1/\mu$ ?

The difference between service rate and service hour is for example $\mu = 25$ (Customers/hour)

- if it is $1/25 * 60$, this is your service time.

Queuing, Probability and Simulation

Service and waiting times for a single server queue, Poisson arrivals, Exponential service:

$\lambda$ = the average number of arrivals per time period (arrival rate)

$\frac{1}{\lambda}$ = the average time between arrivals

$\mu$ = the average number of services per time period (service rate)

$\frac{1}{\mu}$ = the average time taken for each service

$P_0 = 1 - \frac{\lambda}{\mu}$ the probability that no units are in the system

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ the average number of units in the waiting line

$L = L_q + \frac{\lambda}{\mu}$ the average number of units in the system

$W_q = \frac{L_q}{\lambda}$ the average time a unit spends in the waiting line

$W = W_q + \frac{1}{\mu}$ the average time a unit spends in the system

$P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service

$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ the probability of $n$ units in the system

Probability distributions:

The Poisson distribution

$f(x) = \frac{\theta^x e^{-\theta}}{x!}$ for a distribution having mean $\theta, (e = 2.71828\ldots)$

The exponential distribution

$f(x) = \frac{1}{\theta} e^{-x/\theta}$ for a distribution having mean $\theta, (e = 2.71828\ldots)$

$P(x \le x_0) = 1 - e^{-x_0/\theta}$

$P(x \ge x_0) = e^{-x_0/\theta}$ for a given value of $x_0$

Linear congruential generation of uniform random variables

Let $X_0$ be an integer chosen at random (the random seed) then uniformly distributed integers are generated as $X_{n+1} = AX_n \bmod B$ where $A$ and $B$ are large co prime integers. Random numbers between 0 and 1 are calculated as $r_n = \frac{X_n - 1}{B - 2}$.

Generation of Exponentially distributed random variables

Exponential variates with mean $b$ are generated from uniform [0,1] random numbers, $r_n$, by the transformation $t_n = -b \log_e(r_n)$.

Service and waiting times for an M/M/S queue:

$$P_0 = 1 \Big/ \left[ \sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!} \left( \frac{1}{1 - \lambda/S\mu} \right) \right] \qquad L = L_q + \frac{\lambda}{\mu}$$

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & \text{if } 0 \le n \le S \\ \frac{(\lambda/\mu)^n}{S!\,S^{n-S}} P_0 & \text{if } n \ge S \end{cases} \qquad W_q = \frac{L_q}{\lambda} \qquad W = W_q + \frac{1}{\mu}$$

$$L_q = \frac{(\lambda/\mu)^S (\lambda/S\mu)}{S!(1 - \lambda/S\mu)^2} P_0 \qquad \rho = \frac{\lambda}{S\mu}$$

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$P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service **/ Service utilisation**

$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ the probability of $n$ units in the system