Lecture 78 Review Question - Solution

Business decision modelling (Monash University)

Problems

Deterministic Inventory

1. The high cost of ordering door knobs is presenting a problem to the Handy-Dandy Hardware Store. At $20 per order, use the following data to determine the economic order quantity. Amount sold per year: 400 Net cost of each knob: $4 Cost of holding inventory: $.10.

2. Lucky Charms sells 2,500 silver medallions yearly. The medals cost $5 each, and it costs $100 to place an order with the supplier. Annual holding costs, including taxes and insurance, total $.40 per dollar value of items in inventory.
   (a) In order to minimize total inventory cost, how many medallions should be ordered each time?
   (b) Orders should be placed once every what fraction of a year?

3. G.I. Joe's must decide how many one-pound cans of Spam to order. Annual demand is for 1,000 cans. The supplier will fill any order for a flat charge of $50 plus $1 per can. Freight is another $.10 per pound. G.I. Joe's arranges working capital financing at 1.5 % per month. On an annual basis, 10% of all Spam cans held in inventory spoil and must be disposed of.
   (a) Indicate the following: A k c h
   (b) Find the economic order quantity.

4. Gourmet Goodies carries a unique variety of imported cheese. It costs $20 to place each order of the cheese, which is obtained for $1 per pound. Annual demand is for 1,200 pounds. The annual holding cost, including refrigeration, is $.30 per pound. Gourmet's customers are willing to backorder out-ofstock items, for which the annual penalty has been estimated at $.10 per pound. Determine the optimal values for: (a) the order quantity; (b) the order level; and (c) the time (fraction of a year) between orders.

5. A baseball card dealer must determine how many 1955 reproduced Willie Mays cards to stock. He experiences an annual demand of 100 cards. Each card is acquired from a big dealer for $2. Each shipment must be sent by registered mail at a cost of $4-regardless of quantity. Inventory is financed through a 16 % bank loan.
   (a) Assuming that no shortages are allowed, what is the dealer's economic order quantity'?
   (b) Suppose a shortage penalty applies in the amount of $.04 per card short (on an annual basis).
      (1) Find the economic order quantity.
      (2) Find the optimal order level.
      (3) Determine the number of cards on backorder when a shipment arrives and the optimal policy is used.

6. The demand for Double-Whammies is 10,000 units per year. They can be produced at the rate of 50,000 per year. Each production run costs $2,000 to set up, and each item has a variable cost of $10. The annual holding cost is $.25 per dollar value of inventory. Calculate the following:
   (a) Optimal production quantity.
   (b) Optimal duration of a production run.
   (c) Duration of an inventory cycle under the optimal policy.
   (d) Total annual relevant cost of the optimal policy.

7. Bell Computer Corporation purchases all its laser printer cartridges through a national wholesaler, Cartridges Inc. Annual demand for one particular type of cartridge is 45,000 units. Each cartridge costs Bell $200. The company policy is to charge a holding cost of $.10 annually per dollar value for items in inventory. Their fixed cost of placing an order is $20. What is the best number of cartridges to order? What is the total annual relevant cost associated with the best order quantity?

8. Air Design Heating and Cooling Company specialize in air conditioning add-ons, replacements and new installations. It is concerned about its inventory of air conditioners. It experiences an annual demand of 1,000 for their deluxe model, which costs $500. They have a policy of using a holding cost of $.25 per dollar value of an item in inventory, a fixed order cost of $25, and a shortage cost of $5 per unit per year I (since shortages occur from time to time).
   (a) Find the economic order quantity for the air conditioners.
   (b) Find the optimal order level.
   (c) What is the maximum number of shortages?
   (d) What is the total annual relevant cost?

9. Allied Electronics Company manufactures electronic parts such as LEDs, switches, fuses, relays, test equipment, and so forth. It is concerned about one particular switch that has sales of 100,000 units per year. Its unit cost is $10. The set-up cost to produce the switch is $10,0000 per run and the company policy is to charge holding costs of $.10 per dollar value of each item stored in inventory. The company manufactures the switches at a rate of 500,000 annually.
   (a What is the economic production quantity for the switches?
   (b) Find the optimal duration of a production run.
   (c) Determine the duration of an inventory cycle under the optimal policy.
   (d) What is the total annual relevant cost of the optimal policy?

10. Allied Electronics Company manufactures electronic parts such as LEDs, switches, fuses, relays, test equipment, and so forth. One particular relay is of concern because it can either be purchased from an outside supplier or manufactured in-house. Annual demand for the relay is 4,800 units. The company's policy is to charge a holding cost of $.20 per dollar value of items in inventory.
   (a) If the product is purchased from the outside supplier it will cost $12 per unit and the company will incur a fixed order cost of $10. Find the economic order quantity for the relays. What is the associated total annual relevant cost?
   (b) If the product is manufactured in house it will cost $10 per unit and the company will incur a fixed set-up cost of $100 in order to produce the items at a rate of 7,200 per year. What is the economic production quantity for the relays? Find the associated total annual relevant cost.
   (c) Should the company purchase the product from the outsider supplier or manufacture it in-house? Why?

Lecture 8: Stochastic Inventory:

Source: Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 16 Questions 3, 4, 5, 6, 7, 8, 9

16-2 The Green Thumb roadside fruit and vegetable stand must order its cherries from a nearby orchard before they are picked. The following probability distribution for seasonal cherry demand applies:

Possible Demand	**Probability**
D = 100 boxes	.15
D= 150	.20
D= 200	.30
D= 250	.20
D= 300	.15

Green Thumb buys its cherries for $2 a box and sells them for $3 a box. Unsold, overripe cherries are picked up for disposal by a hog farmer, who charges $.10 for each box. Green Thumb must determine how many boxes to order so that expected profit is maximized.

(a) Construct the cumulative probability distribution for Green Thumb's cherry demand.

(b) Use the newsvendor problem approach to determine the optimal number of cherries to order to minimize total expected cost.

16-4 Personalized Printing Company must decide how many Christmas gift calendars should be made this season. All unsold calendars will be purchased by schools for $.10 each. Each calendar costs $.50 and ordinarily sells for $1. Loss of goodwill for shortages is $.05 per calendar. The following probability distribution for demand applies:

Demand	**Probability**
2,000	.05
3,000	.20
4,000	.25
5,000	.30
6,000	.20

(a) Compute the expected demand and determine the cumulative probabilities.
(b) How many calendars should be ordered?
(c) For the quantity in (b), determine (1) the expected shortage, (2) the expected surplus, and (3) total expected cost.
(d) For the quantity in (b), determine the probability that there will be a shortage.

16-5 The demand for Halloween pumpkins at the Black Cat's Patch is normally distributed with a mean of 1,000 and a standard deviation of 200. Each pumpkin costs $.50 and sells for $.90. Unsold pumpkins are disposed of at a cost of $.10 each.

(a) How many pumpkins should be ordered?
~~(b) For the quantity in (a), determine (1) the expected shortage, (2) the expected surplus, and (3) total expected cost.~~
~~(c) For the quantity in (a), determine the probability that there will be a shortage.~~

16-6 A newsvendor must decide how many copies of the *Berkeley Barb* he should leave in a sidewalk stand. Each paper sells for $.50 and costs him a quarter. Unsold papers are then converted into fireplace logs, giving them a nickel in value. The newsvendor believes that any quantity between 51 and 70 papers, inclusively, is equally likely to be demanded. There is no goodwill lost due to unfilled demands.

(a) How many *Berkeley Barbs* should be placed to minimize total expected daily cost?
(b) What is the total expected daily cost of the optimal inventory policy?
(c) What is the newsvendor's maximum expected daily profit from selling *Berkeley Barbs?*

16-7 A baker must decide how many dozen donuts to bake. Leftovers are ordinarily sold the next day, but she is closing her shop today to take a vacation. The following probability distribution for daily demand applies:

Donuts (Dozen)	**Probability**
5	.10
10	.15
15	.30
20	.20
25	.15
30	.10

Donuts cost $.50 per dozen to make, and they sell for $1 per dozen. Before closing the shop, the baker plans to throwaway any unsold donuts.

(a) Compute the expected demand and determine the cumulative probabilities.
(b) How many donuts should be baked?
(c) For the quantity in (b), determine (1) the expected shortage, (2) the expected surplus, and (3) total expected cost.
(d) For the quantity in (b), determine the probability that there will be a shortage.

16-8 The captain of a tramp steamer picks up a load of cocoa beans whenever he travels to West Africa. He always sells them to a candy-maker in Rotterdam for twice what he paid for them. The candy-maker buys only what she needs and the captain must dispose of any excess beans at less than cost to a cocoa dealer who also resides in Rotterdam. The present African price is 2 guilders per kilogram, and the Dutch cocoa dealer buys the beans for 1.50 guilders per kilogram. The following probability distribution for the candy-maker's demand applies:

Demand (kg)	**Probability**
100	.05
200	.12
300	.18
400	.25
500	.22
600	.09
700	.09

(a) How many kilograms of cocoa beans should the captain take on?
(b) For the quantity in (a), determine (1) the expected shortage, (2) the expected surplus and (3) total expected cost.

16-9 Horatio Dull is a college professor who supplements his paltry salary each year by selling Christmas trees - a venture that has been immensely successful in the past. He must order his trees in October for delivery in early December. His net cost for each tree is $5, and the trees sell for an average of $15. On those occasions when there have been unsold trees, Horatio has managed to dispose of every excess tree on Christmas Eve for a price of $0.50. Having built up a loyal clientele, Dull values any goodwill lost for each tree short at $20. Demand for his trees has been established as approximately normally distributed, with a mean of μ= 5,000 trees and a standard deviation of σ = 1,000.

(a) How many trees should Professor Dull order to minimize the total expected cost?
~~(b) For the quantity in (a), determine (1) the expected shortage, (2) the expected surplus, and (3) total expected cost.~~

Solutions:

Lecture 7: Deterministic Inventory

1. $Q^* = \sqrt{\dfrac{2(400)20}{4(.10)}} = 200$ door knobs

2. (a) $Q^* = \sqrt{\dfrac{2(100)2{,}500}{.4(5)}} = 500$ medallions

   (b) T* = 500/2,500 = .2 year

3. (a) A = 1,000   c = \$1.00 + .10 = \$1.10
   k = \$50   h = \$.18 + .10 = \$.28

   (b) $Q^* = \sqrt{\dfrac{2(1{,}000)50}{.28(1.10)}} = 569.8$ or 570

   (b) Q* = 2(1,000)(50) = 569.8 or 570

4. (a) $Q^* = \sqrt{\dfrac{2(1{,}200)20}{.3(1)}}\sqrt{\dfrac{.10+.30}{.10}} = 400(2) = 800$ pounds

   (b) $S^* = \sqrt{\dfrac{2(1{,}200)20}{.3(1)}}\sqrt{\dfrac{.10}{.10+.30}} = 400(1/2) = 200$ pounds

   (c) T* = 800/1,200 = 2/3 year

5. (a) $Q^* = \sqrt{\dfrac{2(100)4}{.16(2)}} = 50$

   (b) (1) $Q^* = \sqrt{\dfrac{2(100)4}{.16(2)}}\sqrt{\dfrac{.04+.16}{.04}} = 111.80$

   (2) $S^* = \sqrt{\dfrac{2(100)4}{.16(2)}}\sqrt{\dfrac{.04}{.04+.16}} = 50/3 = 16\ 2/3$

   (3) Q* – S* = 150 – 16 2/3 = 133 1/3

6. (a) $Q^* = \sqrt{\dfrac{2(10{,}000)2{,}000}{10(.25)}}\sqrt{\dfrac{50{,}000}{50{,}000-10{,}000}} = 4{,}000(1.118) = 4{,}472$

   (b) T_1^* = 4,472/50,000 = .089 year
   (c) T* = 4,472/10,000 = .4472 year

   (d) $TC(4{,}472) = \left(\dfrac{10{,}000}{4{,}472}\right)2{,}000 + .25(10)\left(\dfrac{4{,}472}{2}\right)\left(\dfrac{50{,}000-10{,}000}{}50{,}000\right)$

   = \$4,472 + \$4,472 = \$8,944

7. $Q^* = \sqrt{\dfrac{2(45{,}000)20}{.10(200)}} = 300$

$$TC(300) = \left(\frac{45{,}000}{300}\right)20 + .10(200)\left(\frac{300}{2}\right) = \$3{,}000 + \$3{,}000 = \$6{,}000$$

8. (a) $Q^* = \sqrt{\frac{2(1{,}000)25}{.25(500)}}\sqrt{\frac{5+125}{5}} = 101.98$

(b) $S^* = \sqrt{\frac{2(1{,}000)25}{.25(500)}}\sqrt{\frac{5}{5+125}} = 3.92$

(c) Q* − S* = 101.98 − 3.92 = 98.06

(d) $TC(101.98, 3.92) = \left(\frac{1{,}000}{101.98}\right)25 + \frac{.25(500)(3.92)^2}{2(101.98)} + \frac{5(101.98-3.92)^2}{2(101.98)}$

=\$245.15 + \$9.43 + \$235.72 = \$490.29

9. (a) $Q^* = \sqrt{\frac{2(100{,}000)10{,}000}{.10(100)}}\sqrt{\frac{500{,}000}{500{,}000-100{,}000}} = 44{,}721.36(1.118) = 50{,}000$

(b) T_1^* = 50,000/500,000 = .1 year

(c) T^* = 50,000/100,000 = .5 year

(d) $TC(50{,}000) = \left(\frac{100{,}000}{50{,}000}\right)10{,}000 + .10(10)\left(\frac{50{,}000}{2}\right)\left(\frac{500{,}000-100{,}000}{500{,}000}\right)$

= \$20,000 + \$20,000 = \$40,000

10. (a) $Q^* = \sqrt{\frac{2(4{,}800)10}{.20(12)}} = 200$

$$TC(200) = \left(\frac{4{,}800}{200}\right)10 + .20(12)\left(\frac{200}{2}\right) = \$240 + \$240 = \$480$$

(b) $Q^* = \sqrt{\frac{2(4{,}800)100}{.20(10)}}\sqrt{\frac{7{,}200}{7{,}200-4{,}800}} = 692.82(1.732) = 1{,}200$

$$TC(1{,}200) = \left(\frac{4{,}800}{1{,}200}\right)100 + .20(10)\left(\frac{1{,}200}{2}\right)\left(\frac{7{,}200-4{,}800}{7{,}200}\right)$$

= \$400 + \$400 = \$800

(a) Purchase in-house because the total annual relevant cost is less.

Lecture 8: Stochastic Inventory:

16-3 (a)

Demand d	Probability $Pr[D=d]$	Cumulative Probability $Pr[D \le d]$
100	.15	.15
150	.20	.35
200	.30	.65
250	.20	.85
300	.15	1.00

(b) $c = \$2$ $h_E = \$.10$ $P_S = 0$ $P_R = \$3$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + 3 - 2}{(0 + 3 - 2) + (.1 + 2)} = .32$$

The smallest level for Q having a cumulative probability at least as large is Q* = 150.

16-4 (a)

Demand d	$Pr[D = d]$	$Pr[D \le d]$	$d \times Pr[D = d]$
2,000	.05	.05	100
3,000	.20	.25	600
4,000	.25	.50	1,000
5,000	.30	.80	1,500
6,000	.20	1.00	1,200
			$\mu = 4{,}400$

$c_v = h_E + c = 0.40$
$c_u = P_S + P_S - c = 0.55$

(b) $c = \$.50$ $p_S = .05$ $p_R = \$1.00$ $h_E = -\$.10$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{.05 + 1.00 - .50}{(.05 + 1.00 - .50) + (-.10 + .50)} = .58$$

The smallest cumulative probability greater than or equal to the above occurs for a demand of 5,000 calendars. Thus, Q* = 5,000 calendars.

(c)

		(1) Shortage		(2) Surplus	
d	$Pr[D = d]$	$d - Q^*$	$(d - Q^*)Pr[D = d]$	$Q^* - d$	$(Q^* - d)Pr[D = d)$
2,000	.05	0	0	3,000	150
3,000	.20	0	0	2,000	400
4,000	.25	0	0	1,000	250
5,000	.30	0	0	0	0
6,000	.20	1,000	200	0	0
			B(Q) = 200 expected shortage		800 expected surplus

(3) $TEC(Q) = c\mu + (h_E + c)[Q - \mu + B(Q)] + (p_S + p_R - c)B(Q)$

$$TEC(5{,}000) = \$.50(4{,}400) + (-.10 + .50)(800) + (.05 + 1.00 - .50)(200)$$
$$= \$2{,}200 + 320 + 110 = \$2{,}630$$

Or simply:
$TEC(Q) = c\mu + [c_v \text{ x Expected Surplus}] + [c_u \text{ x Expected Shortage}]$
$= \$0.50(4{,}400) + (0.4 \text{ x } 800) + (0.55 \text{ x } 200) = \$2{,}630$

(d) Pr[shortage] = Pr[D > 5,000] = .20

16-5 (a) $c = \$.50 \quad p_S = 0 \quad p_R = \$.90 \quad h_E = \$.10$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + .90 - .50}{(0 + .90 - .50) + (-.10 + .50)} = .40$$

Not examinable

$\Pr[D \le Q^*] = .40 \qquad z = -.25$
$Q^* = \mu + z\sigma = 1{,}000 - .25(200) = 950$ pumpkins

(b) (1) $B(Q) = \mu - Q + \sigma L[(\mu - Q)/\sigma]$

$$B(950) = 1{,}000 - 950 + 200L([1{,}000 - 950]/200)$$
$$= 50 + 200L(.25)$$
$$= 50 + 200(.2863)$$
$$= 107.26$$

(2) Expected surplus $= Q - \mu + B(Q)$
$= 950 - 1{,}000 + 107.26$
$= 57.26$

(3) $TEC(950) = \$.50(1{,}000) + (.10 + .50)(950 - 1{,}000 + 107.26) + (0 + .90 - .50)(107.26)$
$= \$500 + 34.36 + 42.90 = \$577.26$

(c) Pr[shortage] = Pr[D > 950]
$= 1 - .40 = .60$

16-6 (a) $c = \$.25 \quad p_S = 0 \quad p_R = \$.50 \quad h_E = -\$.05$ (negative, since revenue is received)

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + .50 - .25}{(0 + .50 - .25) + (-.05 + .25)} = .56$$

$Q^* = 62$ papers, since this quantity involves a cumulative demand probability of .60, the smallest one exceeding the above ratio.

(b)

Demand d	Probability	Demand × Prob. dPr[D = d]	Cumulative Probability	Holding Cost .20(62 – d)	Cost × Prob.	Shortage Cost .25(d – 62)	Cost × Prob.
51	.05	2.55	.05	2.2	.11	-	-
52	.05	2.60	.10	2.0	.10	-	-
53	.05	2.65	.15	1.8	.09	-	-
54	.05	2.70	.20	1.6	.08	-	-
55	.05	2.75	.25	1.4	.07	-	-
56	.05	2.80	.30	1.2	.06	-	-
57	.05	2.85	.35	1.0	.05	-	-
58	.05	2.90	.40	.8	.04	-	-
59	.05	2.95	.45	.6	.03	-	-
60	.05	3.00	.50	.4	.02	-	-
61	.05	3.05	.55	.2	.01	-	-
62	.05	3.10	.60	0.0	.00	-	-
63	.05	3.15	.65	-	-	.25	.0125
64	.05	3.20	.70	-	-	.50	.0250
65	.05	3.25	.75	-	-	.75	.0375
66	.05	3.30	.80	-	-	1.00	.0500
67	.05	3.35	.85	-	-	1.25	.0625
68	.05	3.40	.90	-	-	1.50	.0750
69	.05	3.45	.95	-	-	1.75	.0875
70	.05	3.50	1.00	-	-	2.00	.1000
		$\mu = 60.50$		66			\$.4500

$c\mu = \$.25(60.5) = \$15.125$

TEC(62) = \$15.125 + .66 + .4500 = \$16.235

(c) This newsvendor's maximum expected daily profit is achieved by stocking Q* = 62 Berkeley Barbs This may be obtained in the same manner as with the Fortune problem. Or, more directly, we may first multiply expected demand of 60.5 copies (the median value may be used to quickly find this, since the demand distribution is symmetrical) by the revenue per copy:

\$.50(60.5) = \$30.25

Then subtracting TEC(62) from the above, we have
Maximum expected profit = \$30.25 – 16.235
= \$14.015

16-7 (a)

Demand d	Probability Pr[D = d]	Cumulative Probability Pr[D ≤ d]	d × Pr[D = d]
5	.10	.10	.50
10	.15	.25	1.50
15	.30	.55	4.50
20	.20	.75	4.00
25	.15	.90	3.75
30	.10	1.00	3.00
			$\mu = 17.25$

(b) $c = .50$ $\quad p_S = 0$ $\quad p_R = 1.00$ $\quad h_E = 0$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + 1.00 - .50}{(0 + 1.00 - .50) + (0 + .50)} = .50$$

The smallest level for demand having a cumulative probability at least as large is for 15 dozen donuts, so Q* = 15.

(c)

		(1) Shortage		(2) Surplus	
d	Pr[D = d]	d – Q*	(d – Q*)Pr[D = d]	Q* – d	(Q* – d)Pr(D = d]
5	.10	0	0	10	1.00
10	.15	0	0	5	.75
15	.30	0	0	0	0
20	.20	5	1.00	0	0
25	.15	10	1.50	0	0
30	.10	15	1.50	0	0
			B(Q*) = 4.00 Expected Shortage		1.75 Expected Surplus

(3) TEC(15) = \$.50(17.25) + (0 + .50)[1.75] + (0 + 1.00 -.50)(4.00)
=\$8.625 + .875 + 2.00 = \$11.50

(d) Pr[shortage] = Pr[D > 15] = .20 + .15 + .10 = .45

16-8 (a) $c = \$2$ $\quad p_S = 0$ $\quad p_R = \$4$ $\quad h_E = -\$1.50$

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{0 + 4 - 2}{(0 + 4 - 2) + (-1.50 + 2)} = .80$$

Q* = 500 kg

(b)

			(1) Shortage		(2) Surplus	
d	Pr[D = d]	d × Pr[D = d]	d – Q*	(d – Q*)Pr[D = d]	Q* – d	(Q* – d)Pr(D = d]
100	.05	5.00	0	0	400	20.00
200	.12	24.00	0	0	300	36.00
300	.18	54.00	0	0	200	36.00
400	.25	100.00	0	0	100	25.00
500	.22	110.00	0	0	0	0
600	.09	54.00	100	9.00	0	0
700	.09	63.00	200	18.00	0	0
		410.00		B(Q*) = 27.00 Expected Shortage		117.00 Expected Surplus

(3) TEC(500) = \$2(410) + (–1.50 + 2)(117) + (0 + 4 – 2)(7)

$= \$820 + 58.50 + 54 = \$932.50$

16-9 (a) $c = \$5$ $\mu = 5{,}000$
$p_S = \$20$ $\sigma = 1{,}000$
$p_R = \$15$
$h_E = -\$.50$ (negative, since revenue is received)

$$\frac{p_S + p_R - c}{(p_S + p_R - c) + (h_E + c)} = \frac{20 + 15 - 5}{(20 + 15 - 5) + (-.50 + 5)} = .8696$$

Area = .8696 – .5000 = .3696
$z = 1.12$
$Q^* = \mu + z\sigma = 5{,}000 + 1.12(1{,}000)$
$= 6{,}120$ trees

Not examinable

Professor Dull should order 6,120 trees.

(b) (1)

$$B(6{,}120) = 1{,}000L\left(\frac{6{,}120 - 5{,}000}{1{,}000}\right) = 1{,}000L(1.12)$$
$$= 1{,}000(.06595) = 65.95 \text{ or } 66$$

(2) Expected surplus = 6,120 – 5,000 + 65.95 = 1,185.95 or 1,186

(3) $TEC(6{,}120) = \$5(5{,}000) + (-.50 + 5)(6{,}120 - 5{,}000 + 65.95) + (20 + 15 - 5)(65.95)$
$= \$25{,}000 + 5{,}336.78 + 1{,}978.50 = \$32{,}315.28$