FIT3158
Business Decision Modelling

Lecture 7

Inventory Modelling under Certainty

Topics Covered:

- Economic Order Quantity (EOQ) Model
- An Inventory Model with Planned Shortages
- Economic Production Lot Size Model
- Quantity Discounts for the EOQ Model
- EOQ being an NLP (Nonlinear Programming) Model

Inventory Models

- The study of inventory models is concerned with two basic questions:
  - How much should be ordered each time
  - When should the reordering occur (How Often?)

- The objective is to minimize total variable cost over a specified time period (assumed to be annual in the following review).

Inventory Costs

- Ordering cost -- salaries and expenses of processing an order, regardless of the order quantity
- Holding cost -- usually a percentage of the value of the item assessed for keeping an item in inventory (including finance costs, insurance, security costs, taxes, warehouse overhead, and other related variable expenses)
- Backorder cost -- costs associated with being out of stock when an item is demanded (including lost goodwill or lost sales)
- Purchase cost -- the actual price of the items

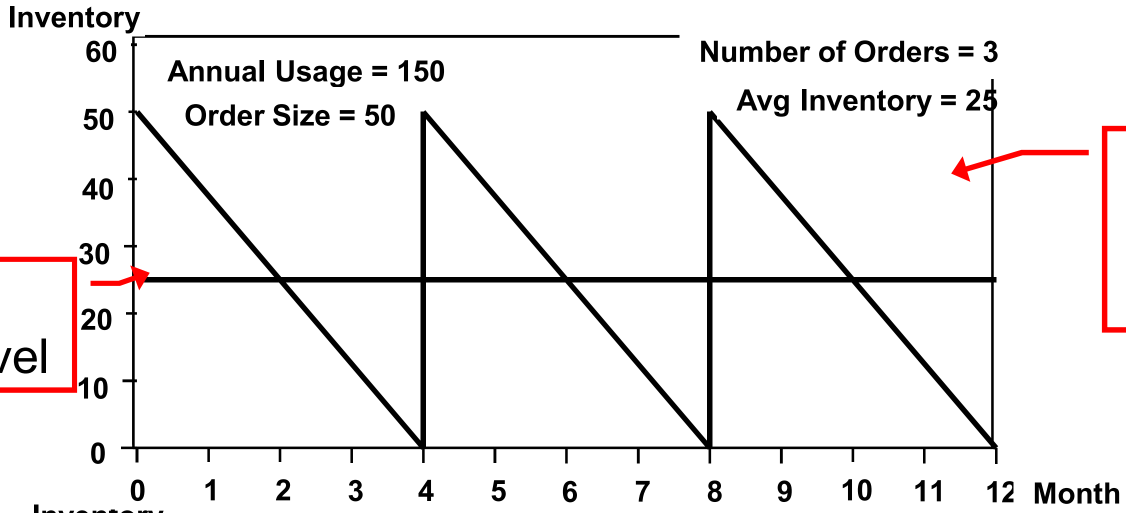
Economic Order Quantity (EOQ): Introduction

- The simplest inventory models assume demand and the other parameters of the problem to be deterministic and constant.
- The most basic of the deterministic inventory models is the Economic Order Quantity (EOQ).
- The variable costs in this model are annual holding cost and annual ordering cost.
- For the EOQ, the annual holding and ordering costs are equal.

The Economic Order Quantity (EOQ) Problem

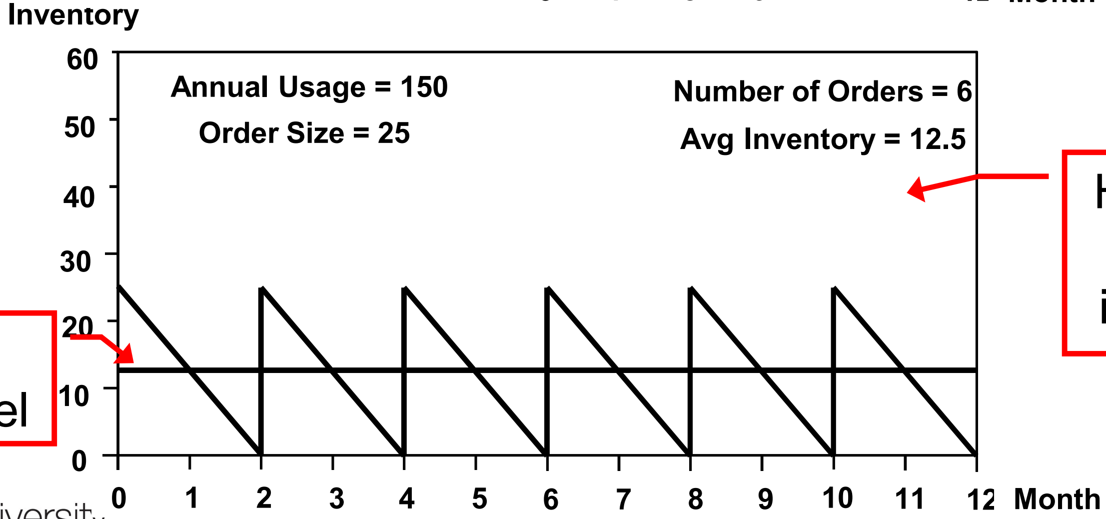
- Involves determining the optimal quantity to purchase when orders are placed.
- Small orders result in:
  - low inventory levels & carrying/holding costs
  - frequent orders & higher ordering costs
- Large orders result in:
  - higher inventory levels & carrying/holding costs
  - infrequent orders & lower ordering costs

Sample Inventory Profiles



Average Inventory level

Lower ordering costs, higher inventory costs



Average Inventory level

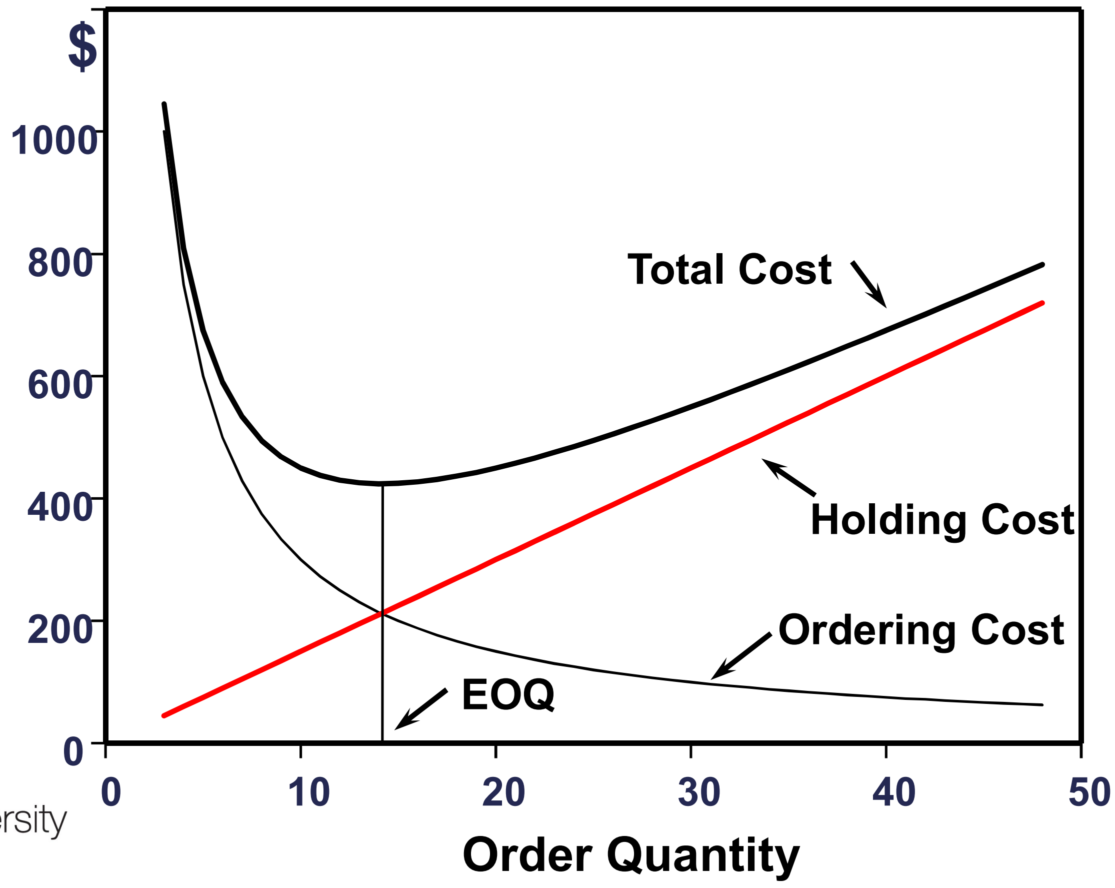
Higher ordering costs, lower inventory costs

Economic Order Quantity: Assumptions

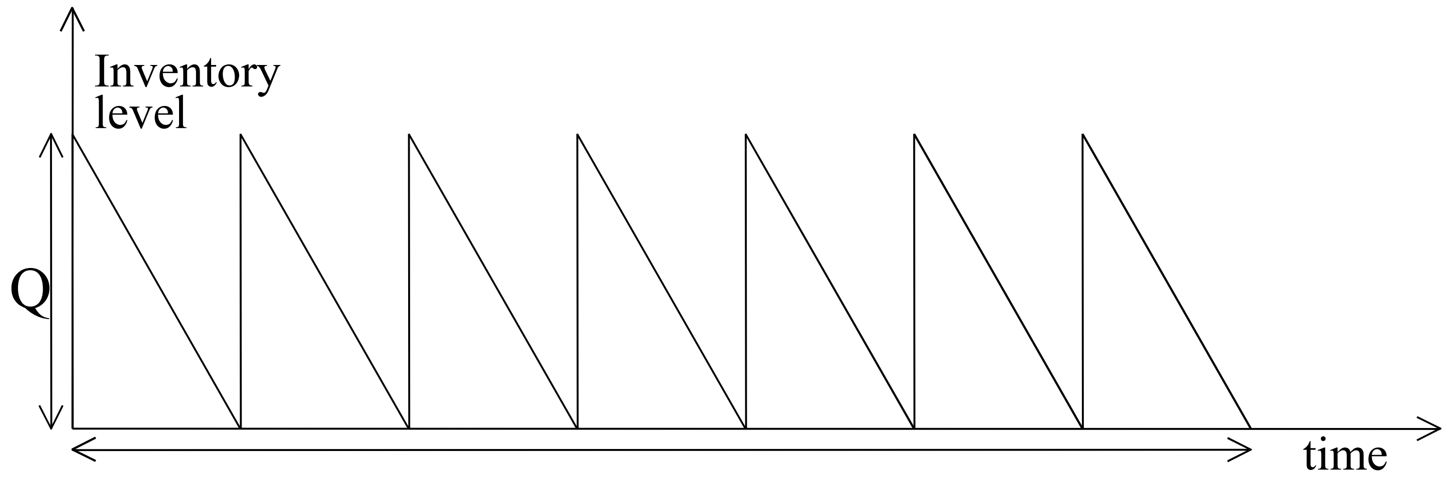
- Demand is constant throughout the year.
- Purchase cost per unit is constant (no quantity discount).
- Delivery time (lead time) is constant (We initially assume that delivery time is 0, that is, that delivery is instantaneous)
- Planned shortages are not permitted.
- Note that even when all of the assumptions of the economic order quantity (EOQ) do not hold, the model may still be used as a good guide to ordering.
- We assume that all values are determined over the same time period, taken to be a year in these notes.

EOQ Cost Relationships

We want to know the quantity of stock to be ordered which minimises the total (annual) cost of holding and purchasing stock when the demand for goods is constant.



Economic Order Quantity: Derivation 1



A/Q replenishments per year, average inventory level is Q/2

$$\text{Annual Demand} = A$$

$$\text{Unit Cost per item} = c$$

e.g., sending an email or clicking links $\text{Fixed Cost per order} = k$

$\text{Order Quantity} = Q$ 是 Find this

$$\text{Annual Holding Cost per dollar per item} = h$$

Economic Order Quantity: Derivation 2

$$\text{Total Costs} = \text{ordering costs} + \text{holding costs}$$

$$= \frac{Ak}{Q} + \frac{Qch}{2}$$

$$\text{Solving} \quad \frac{d\left(\text{Total Costs}\right)}{dQ} = 0$$

$$\text{gives} \quad \frac{ch}{2} - \frac{Ak}{Q^2} = 0$$

$$\text{thus} \quad EOQ = Q^* = \sqrt{\frac{2Ak}{ch}}$$

Differentiate by Q. Minimum value occurs where the derivative is 0

We denote the EOQ by Q*

Economic Order Quantity: Formulae

$$\text{Optimal order quantity}: Q^* = \sqrt{\frac{2Ak}{ch}}$$

$$\text{Number of orders per year} = \frac{A}{Q^*}$$

$$\text{Time between orders (cycle time)} = \frac{Q^*}{A} \text{ years}$$

$$\text{Total annual cost} = \text{ordering cost} + \text{holding cost} = \frac{Ak}{Q} + \frac{Qch}{2}$$

An EOQ Example: Ordering Paper For MetroBank

- Alan Wang purchases paper for copy machines and laser printers at MetroBank.
  - Annual demand (A) is for 24,000 boxes
  - Each box costs \$35 (c)
  - Each order costs \$50 (k)
  - Inventory carrying costs are 18% (*h*)
- What is the optimal order quantity Q*?

MetroBank Example:

- Optimal order quantity:

$$Q^* = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2 \times 24{,}000 \times 50}{35 * .18}} = 617.21 \approx 617$$

- Costs:

$$\text{ordering costs} = \frac{Ak}{Q} = (24{,}000 * 50)/617.21 = 1944$$

$$\text{holding costs} = \frac{Qch}{2} = 617.21 * 35 * .18 / 2 = 1944$$

- Total costs

$$\text{Total annual cost} = \text{ordering cost} + \text{holding cost} = \frac{Ak}{Q^*} + \frac{Q^* ch}{2} = 3889$$

MetroBank Example:

- In this case the number of orders per year is:

$$\frac{A}{Q^*} = 24{,}000/617 = 38.9 \text{ (approx)}$$

- i.e. the time between orders is:

$$\text{cycle time} = \frac{Q^*}{A} = 617/24{,}000 \text{ years} = 9.38 \text{ days}$$

- **So the solution is to order 617 every 9.4 days**

MetroBank Example:

Question:

- What happens if we change the order quantity to 600?

Answer:

- Very little change in Total Costs, but perhaps more convenient … .

- The EOQ model is a very robust model – i.e., small variations in the inputs do not change the output (i.e., total costs) much
- Lecture 7.xlsm

MetroBank Example:Lecture 7.xlsm

Sensitivity analysis around optimal order quantity

Example: Bart's Barometer Business

Economic Order Quantity Model

- Bart's Barometer Business (BBB) is a retail outlet which deals exclusively with weather equipment. Currently BBB is trying to decide on an inventory and reorder policy for home barometers.

Unit cost per item, c

Annual demand, A

- Barometers cost BBB $50 each and demand is about 500 per year distributed fairly evenly throughout the year. Re-ordering costs are $80 per order and holding costs are figured at 20% of the cost of the item.

Acquisition cost per order (ordering cost), k

- BBB is open 300 days a year (6 days a week and closed two weeks in August). Explore lead times of 20/60 days.

% Holding cost per $ per item, h

Example: Bart's Barometer Business

- *Total Variable Cost Model*

$$\text{Total Costs} = \frac{Ak}{Q} + \frac{Qch}{2}$$
$$= \frac{500 \times 80}{Q} + \frac{Q(0.2 \times 50)}{2}$$
$$= \frac{40000}{Q} + 5Q$$

- *Optimal Reorder Quantity*

$$Q^* = \sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2 \times 500 \times 80}{10}} = 89.44 \approx 90$$

- Thus, if the lead time was 0, Bart should order 90 units when the inventory level is 0.

Example: Bart's Barometer Business

- In this case the number of orders per year is:

$$\frac{A}{Q^*} = 500 / 89.44 = 5.6 \text{ (approx)}$$

- i.e. the time between orders is:

$$\text{cycle time} = \frac{Q^*}{A} = 89.44/500 \text{ years} = .178 \text{ years}$$
$$= \text{approx. every 65 days}$$

(for working days, see next slide)

Example: Bart's Barometer Business

- Number of reorder times per year = (500/90) = 5.56 or once every (300/5.56) = 54 working days (about every 9 weeks).

- Total Annual Variable Cost:

$$\text{Total Costs} = \frac{Ak}{Q} + \frac{Qch}{2}$$

- TC = (40,000/89.44) + 894.4/2 = 447 + 447 = $894

Example: Bart's Barometer Business

Variation:

- Suppose the lead time for delivery is no longer zero, but lead time is:

  a. 20 working days

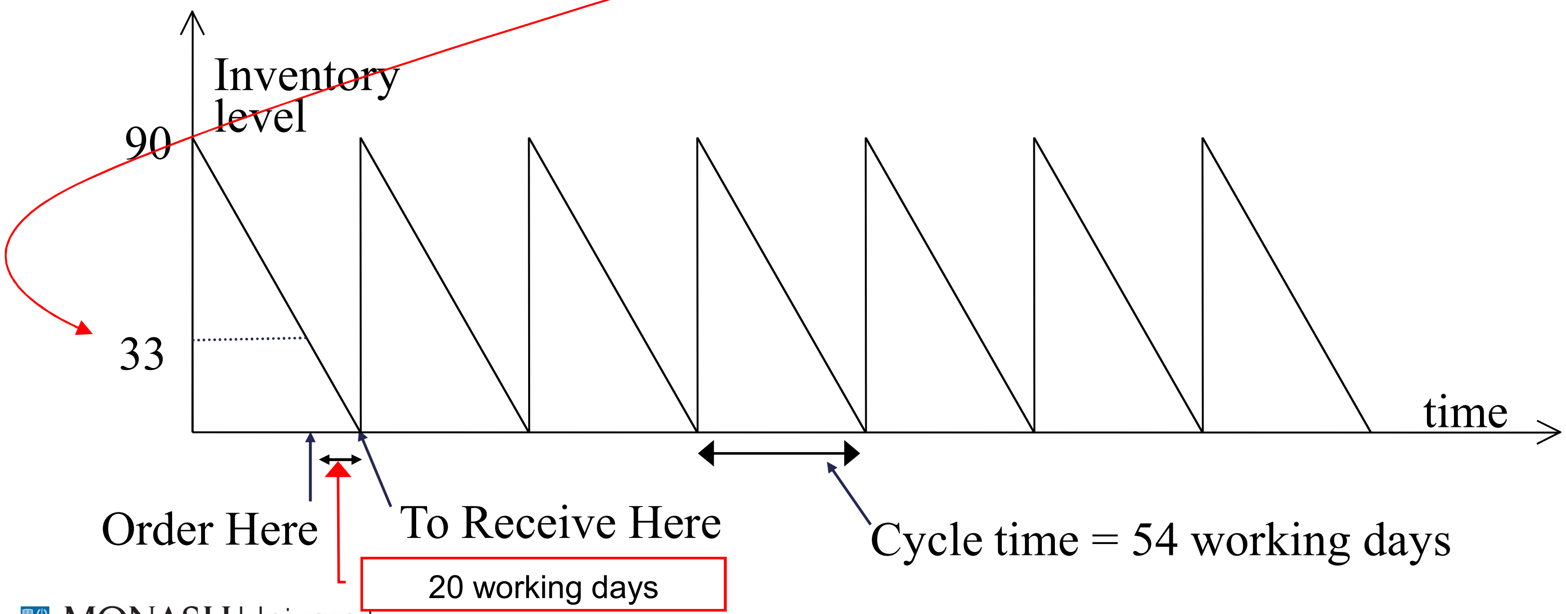
  b. 60 working days

- We now need to know at what point we should place a new order – i.e., at what current level of inventory should we place a new order

- This is known as the re-order point or r

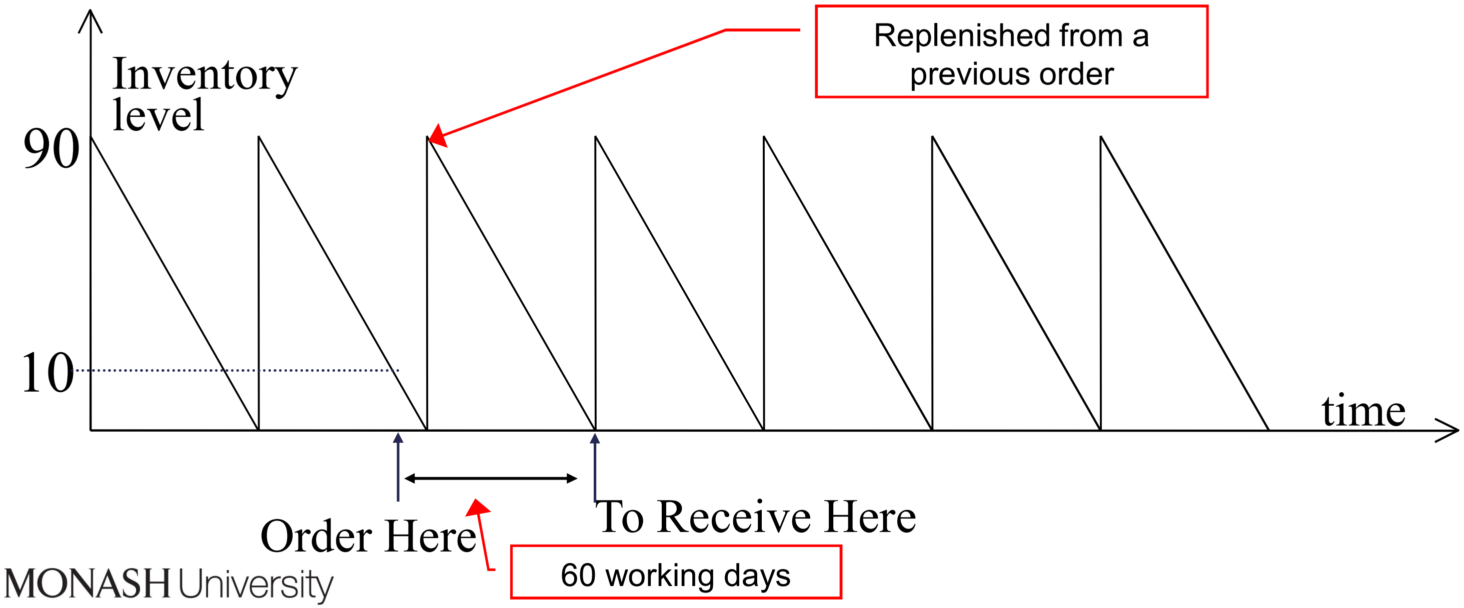
Example: Bart's Barometer Business

- Optimal Re-order Point for Lead time *m* = 20 days
- Daily demand is *d* = 500/300 or 1.667 per day.
- Thus the re-order point *r* = (1.667)(20) = 33.34 (i.e., in 20 days, 33.34 barometers are sold) so Bart should re-order 90 barometers when his inventory position reaches 33 on hand.



Example: Bart's Barometer Business

- Optimal Re-order Point for *m* = 60 day lead time
- Daily demand is *d* = 500/300 or 1.667 per day.
- Thus the re-order point *r* = (1.667)(60) = 100. Bart should re-order 90 barometers when his inventory position reaches 100 (that is, 10 on hand and one outstanding order).



Spreadsheet Model

- Spreadsheet showing summary calculations and the comparison of the EOQ with an alternative reorder quantity (in batches of 75).
- See [Lecture 7.xlsm]

Annual Demand		500.00	
Ordering Cost		$ 80.00	
Annual Holding Rate%		20.00	
Cost Per Unit		$ 50.00	
Working Days Per Year		300.00	
Lead Time (Days)		60.00	
Optimal Order Quantity		89.44	
Requested Order Quantity			75.00
% Change from EOQ			-16.15
Annual Holding Cost		$ 447.21	$ 375.00
Annual Ordering Cost		$ 447.21	$ 533.33
Total Annual Cost		$ 894.43	$ 908.33
% Over Minimum TAC			1.55
Maximum Inventory Level		89.44	75.00
Average Inventory Level		44.72	37.50
Reorder Point		100.00	100.00
Number of Orders Per Year		5.59	6.67
Cycle Time		53.67	45.00

Example: Bart's Barometer Business

Summary of Spreadsheet Results

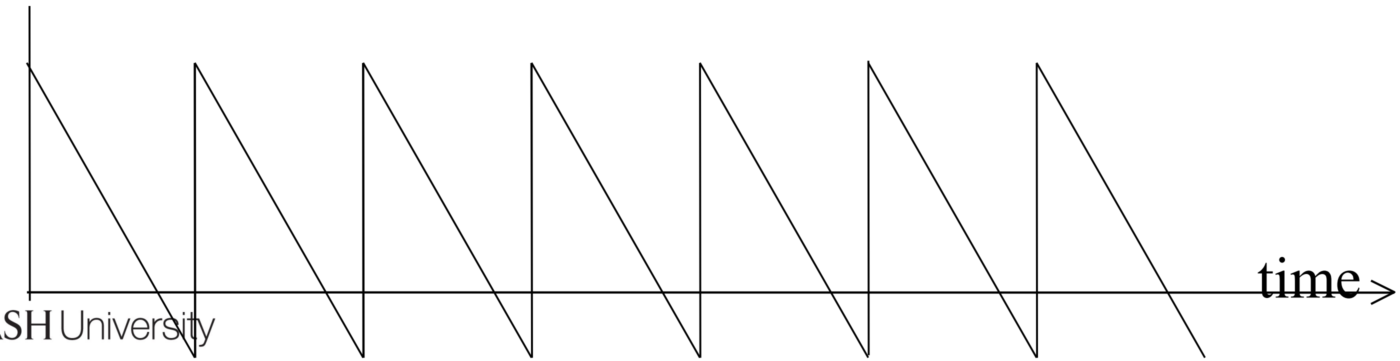
- A 16.15% negative deviation from the EOQ resulted in only a 1.55% increase in the Total Annual Cost.
- Annual Holding Cost and Annual Ordering Cost are no longer equal.
- The Re-order Point is not affected, in this model, by a change in the Order Quantity.

EOQ with Planned Shortages

- With the EOQ with planned shortages model, a replenishment order does not arrive at or before the inventory position drops to zero.
- Shortages occur until a predetermined back-order quantity is reached, at which time the replenishment order arrives.
- The variable costs in this model are annual holding, back-order, and ordering.
- For the optimal order and back-order quantity combination, the sum of the annual holding and back-ordering costs equals the annual ordering cost.

EOQ with Planned Shortages: Assumptions

- Demand occurs at a constant rate of *A* items per year.
- Ordering cost: $*k* per order.
- Holding cost: $*ch* per item in inventory per year.
- Backorder penalty cost: $*p* per item back-ordered per year.
- Purchase cost per unit is constant (no quantity discount).
- Set-up time (lead time) is constant.
- Planned shortages are permitted (back-ordered units are withdrawn from a replenishment order when it is delivered).

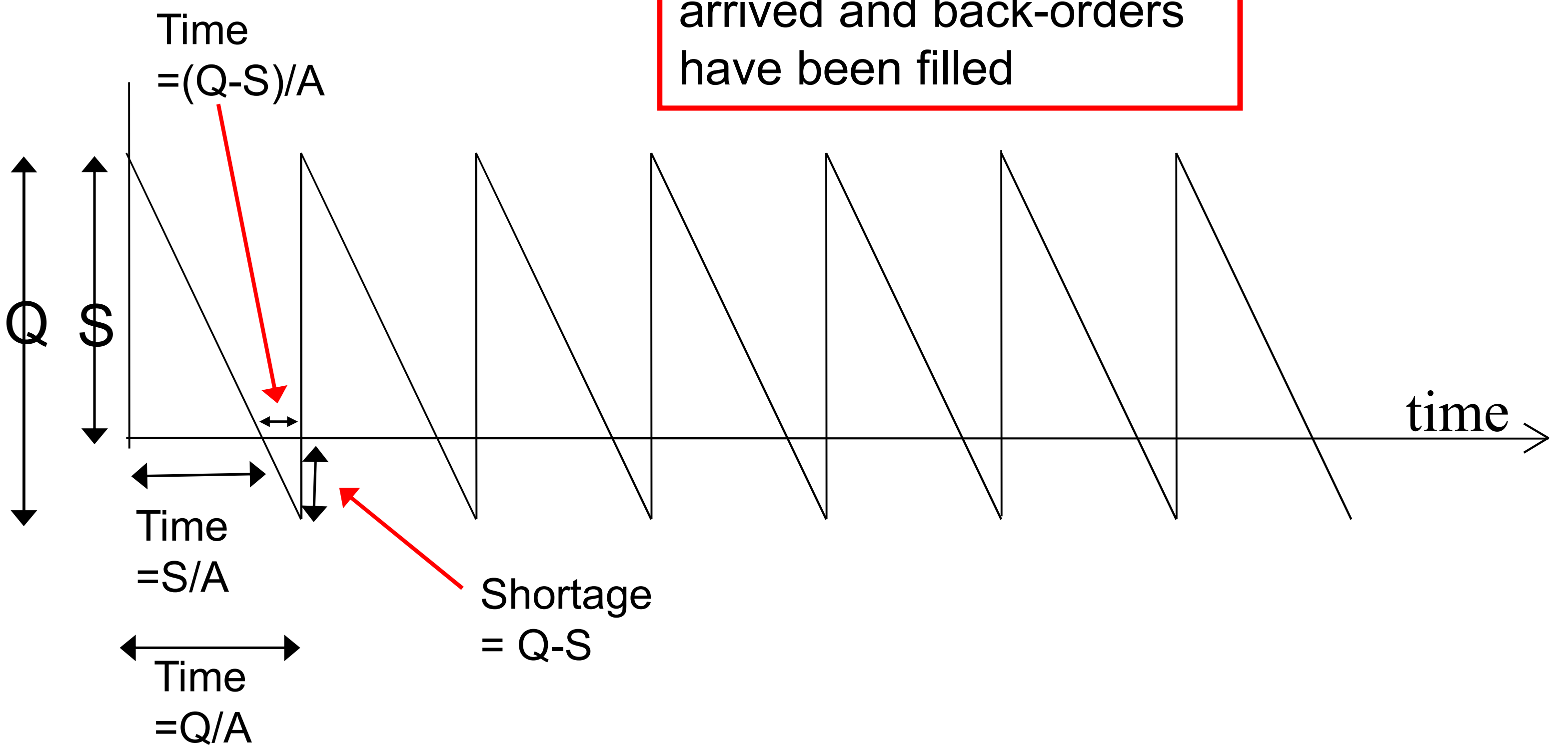


EOQ with Planned Shortages

- Illustration:

Q is the order quantity

S is the quantity in stock once an order has arrived and back-orders have been filled



EOQ with Planned Shortages

$$\text{Optimal order quantity, } Q^* = \sqrt{\frac{2Ak}{ch}\left(\frac{p+ch}{p}\right)}$$

$$\text{Quantity at the beginning of each cycle, } S^* = \sqrt{\frac{2Ak}{ch}\left(\frac{p}{p+ch}\right)}$$

$$\text{Maximum number of backorders} = Q^* - S^*$$

$$\text{Number of orders per year} = \frac{A}{Q^*}$$

$$\text{Total annual cost} = \text{setup} + \text{holding} + \text{backorder}$$

$$= \frac{Ak}{Q} + \frac{chS^2}{2Q} + \frac{p(Q-S)^2}{2Q}$$

Example: Hervis Rent-a-Car

Unit cost per item, c

monthly demand

Hervis Rent-a-Car has a fleet of 2,500 Rockets serving the Los Angeles area. All Rockets are maintained at a central garage. On the average, eight Rockets per month require a new engine. Engines cost $850 each. There is also a $120 order cost (independent of the number of engines ordered).

Hervis has an annual holding cost rate of 30% on engines. It takes two weeks to obtain the engines after they are ordered. For each week a car is out of service, Hervis loses $40 profit.

% Holding cost per $ per item, h

Acquisition cost per order, k

Backorder penalty cost per item per week

Example: Hervis Rent-a-Car

- Optimal Order Policy

  A = 8 x 12 = 96; k = \$120; ch = .30(850) = \$255;

  p = 40 x 52 = \$2080

$$Q^* = \sqrt{\frac{2 \times 96 \times 120}{255}\left(\frac{2080 + 255}{2080}\right)}$$

$$= 10.07 \approx 10$$

Optimal order quantity

$$S^* = \sqrt{\frac{2 \times 96 \times 120}{255}\left(\frac{2080}{2080 + 255}\right)}$$

$$= 8.97 \approx 9$$

Highest stock level – so 1 must be back ordered

Example: Hervis Rent-a-Car

- Demand is 8 per month or 2 per week. Since lead time is 2 weeks, demand through lead time is 4.

- Thus, since the optimal policy is to order 10 to arrive when there is one back-order, the order should be placed when there are 3 engines remaining in inventory.

Example: Hervis Rent-a-Car

- **Question:**

  How many days after receiving an order does Hervis run out of engines? How long is Hervis without any engines per cycle?

- **Solution:**

➢ Inventory exists for $p/(p+ch)$ = 2080/(255+2080) = .8908 of the order cycle.

  (Note, S^*/Q^* = .8908 also before Q^* and S^* are rounded.)

➢ An order cycle is Q^*/A = .1049 years = 38.3 days. Thus, Hervis runs out of engines .8908(38.3) = 34 days after receiving an order

➢ Hervis is out of stock for approximately 38 - 34 = 4 days

EOQ with Quantity Discounts

- The EOQ with quantity discounts model is applicable where a supplier offers a lower purchase cost when an item is ordered in larger quantities.
- This model's variable costs are annual holding, ordering and purchase costs.
- For the optimal order quantity, the annual holding and ordering costs are not necessarily equal.

EOQ with Quantity Discounts: Asumptions

- Demand occurs at a constant rate of A items per year.
- Ordering Cost is $\$k$ per order.
- Holding Cost is h. This is equivalent to $\$ch$ per item in inventory per year as per previous models.
- Purchase Cost is

  $\$c_1$ per item if the quantity ordered is between 0 and x_1,

  $\$c_2$ if the order quantity is between x_1 and x_2, etc.
- Delivery time (lead time) is constant.
- Planned shortages are not permitted.

EOQ with Quantity Discounts

Formulae

- Optimal order quantity:
  - Calculate the smallest feasible Q^* under each pricing structure. Choose the Q^* which results in the smallest annual total cost.
- Number of orders per year: A/Q^*
- Time between orders (cycle time): Q^*/A years
- Total annual cost: $[(1/2)Q^*ch] + [Ak/Q^*] + Ac$

(holding + ordering + purchase)

Example: Nick's Camera Shop

- Nick's Camera Shop carries Zodiac instant print film. The film normally costs Nick $3.20 per roll, and he sells it for $5.25. Zodiac film has a shelf life of 18 months. Nick's average sales are 21 rolls per week. His annual inventory holding cost rate is 25% and it costs Nick $20 to place an order with Zodiac.
- If Zodiac offers a 7% discount on orders of 400 rolls or more, a 10% discount for 900 rolls or more, and a 15% discount for 2000 rolls or more, determine Nick's optimal order quantity.

$$A = 21(52) = 1092; \quad ch = 0.25(c); \quad k = 20$$

Example: Nick's Camera Shop

- Unit-Prices' Economical, Feasible Order Quantities
  - **For c_4 = .85(3.20) = $2.72**

- To receive a 15% discount Nick must order at least 2,000 rolls. Unfortunately, the film's shelf life is 18 months. The demand in 18 months (78 weeks) is 78 x 21 = 1638 rolls of film.

- If he ordered 2,000 rolls he would have to scrap 362 of them. This would cost more than the 15% discount would save.

Example: Nick's Camera Shop

- Unit-Prices' Economical, Feasible Order Quantities
  - **For c_3 = .90(3.20) = $2.88.**

$$Q_3^* = \sqrt{\frac{2Ak}{c_3 h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 2.88}} = 246.31 \text{ (not feasible)}$$

  - **The most economical, feasible quantity for c_3 is 900**
  - **For c_2 = .93(3.20) = $2.976.**

$$Q_2^* = \sqrt{\frac{2Ak}{c_2 h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 2.976}} = 242.30 \text{ (not feasible)}$$

  - **The most economical, feasible quantity for c_2 is 400.**

Example: Nick's Camera Shop

- Unit-Prices' Economical, Feasible Order Quantities
  - **For c_1 = 1.00(3.20) = $3.20.**

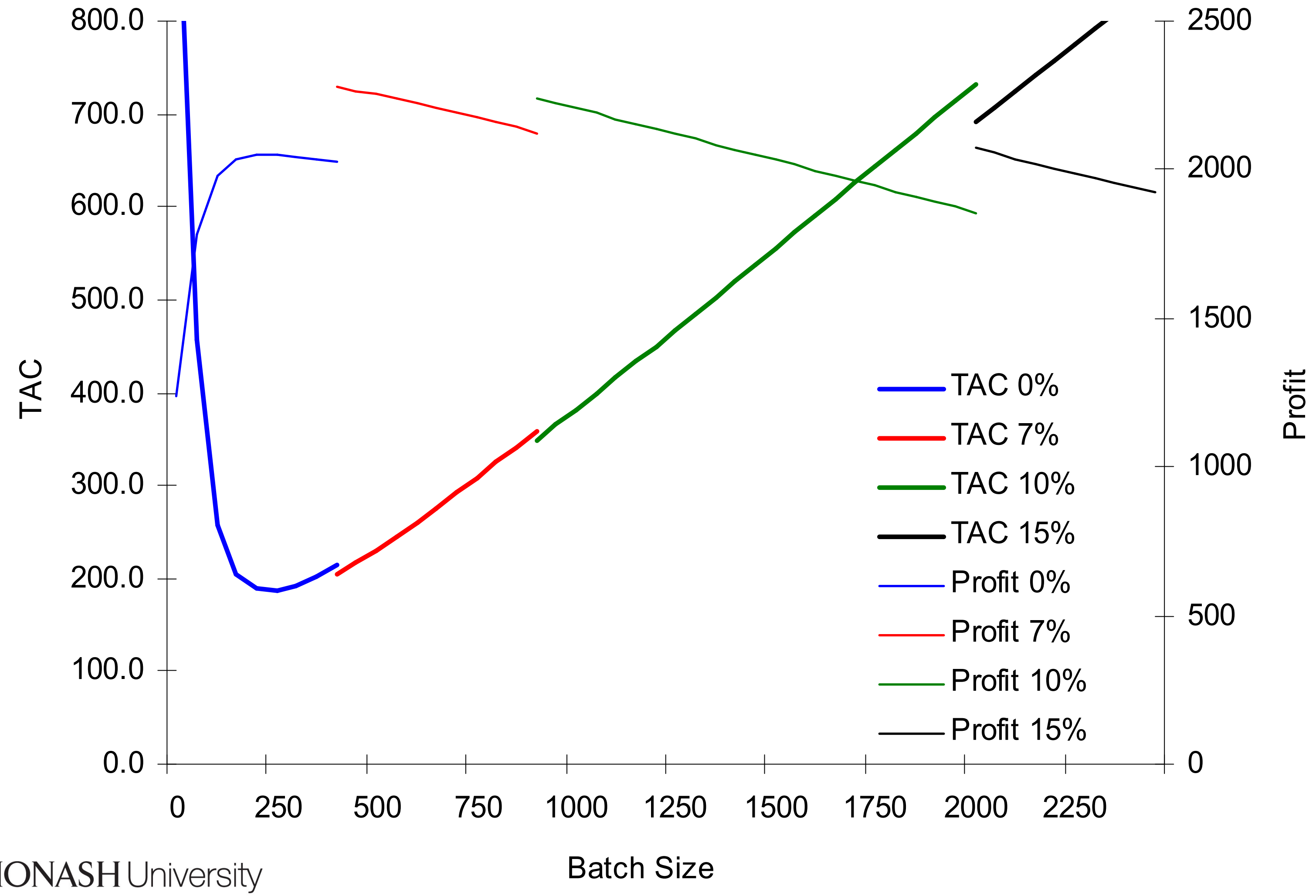
$$Q_1^* = \sqrt{\frac{2Ak}{c_1 h}} = \sqrt{\frac{2 \times 1092 \times 20}{0.25 \times 3.20}} = 233.67 \text{ (feasible)}$$

- The following graph shows holding and ordering costs as a function of *Q.*
- When we reach a computed *Q* that is feasible we stop computing *Q's*. (In this problem we have no more to compute, anyway.)

Example: Nick's Camera Shop

See : [Lecture 7.xlsm]

Example: Nick's Camera Shop



Example: Nick's Camera Shop

Total Cost Comparison

- Compute the total cost for the most economical, feasible order quantity in each price category for which a *Q* * was computed.

$$TC_i = [(1/2)Q^*ch] + [Ak/Q^*] + Ac$$

$$TC_3 = (1/2)(900)(.72) + ((1092)(20)/900) + (1092)(2.88) = 3493$$

$$TC_2 = (1/2)(400)(.744) + ((1092)(20)/400) + (1092)(2.976) = 3453$$

$$TC_1 = (1/2)(234)(.80) + ((1092)(20)/234) + (1092)(3.20) = 3681$$

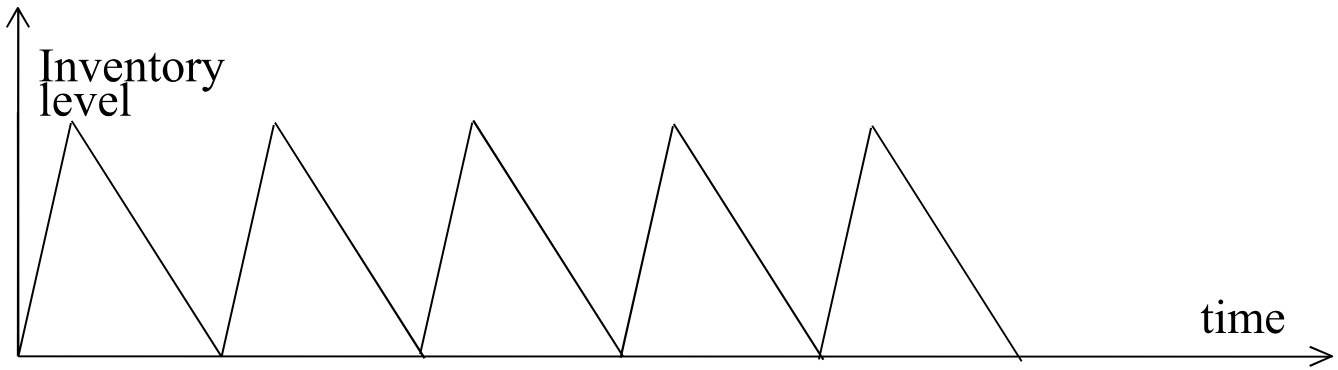
- Comparing the total costs for 234, 400 and 900, the lowest total annual cost is $3453. Nick should order 400 rolls at a time.

Economic Production Lot Size

- The economic production lot size model is a variation of the basic EOQ model.
- A replenishment order is not received in one lump sum as it is in the basic EOQ model.
- Inventory is replenished gradually as the order is produced (which requires the production rate to be greater than the demand rate).
- This model's variable costs are annual holding cost and annual set-up cost (equivalent to ordering cost).
- For the optimal lot size, annual holding and set-up costs are equal.

Economic Production Lot Size: Assumptions

- Demand occurs at a constant rate of A items per year.
- Production rate is B items per year (and $B > A$).
- Set-up cost: $\$k$ per run.
- Holding cost: $\$ch$ per item in inventory per year.
- Manufacturing cost per unit is constant (no quantity discount).
- Set-up time (lead time) is constant.
- Planned shortages are not permitted.



Economic Production Lot Size: Formulae

$$\text{Optimal production lot size}: Q^* = \sqrt{\frac{2Ak}{ch}}\sqrt{\frac{B}{B-A}}$$

$$\text{Number of production runs per year} = \frac{A}{Q^*}$$

$$\text{Time between setups (cycle time)} = \frac{Q^*}{A}\ \text{years}$$

$$\text{Total annual cost} = \text{setup cost} + \text{holding cost} = \frac{Ak}{Q} + \frac{chQ}{2}\left(\frac{B-A}{B}\right)$$

Example: Non-Slip Tile Co.

Economic Production Lot Size Model

Non-Slip Tile (NST) Company has been using production runs of 100,000 tiles, 10 times per year to meet the demand of 1,000,000 tiles annually.

The set-up cost is $5,000 per run and holding cost is estimated at 10% of the manufacturing cost of $1 per tile.

The production capacity of the machine is 500,000 tiles per month.

The factory is open 365 days per year.

Example: Non-Slip Tile Co.

Total Annual Variable Cost Model

This is an economic production lot size problem with :

A = 1,000,000, B = 6,000,000, ch = .10, k = 5,000

$$\begin{aligned}\text{Total annual cost} &= \text{setup cost} + \text{holding cost}\\ &= \frac{Ak}{Q} + \frac{chQ}{2}\left(\frac{B-A}{B}\right)\\ &= 5{,}000{,}000{,}000/Q \; + \; 0.04167\,Q\end{aligned}$$

Example: Non-Slip Tile Co.

$$\text{Optimal production lot size } Q^* = \sqrt{\frac{2Ak}{ch}}\sqrt{\frac{B}{B-A}}$$

$$= \sqrt{100{,}000{,}000{,}000}\sqrt{\frac{6}{5}}$$

$$= 346{,}410$$

Number of Production Runs Per Year = *A*/*Q* *

= 1000000 / 346410 = 2.89 times per year.

Example: Non-Slip Tile Co.

Total Annual Variable Cost

- How much is NST losing annually by using their present production schedule? (Substitute Q into the Total Cost equation)

- Optimal *TC* = .04167(346,410) + 5,000,000,000 / 346,410

= $28,868

- Current *TC* = .04167(100,000) + 5,000,000,000 / 100,000

= $54,167

- Difference = 54,167 - 28,868 = $25,299

Example: Non-Slip Tile Co.

Idle Time Between Production Runs

- There are 2.89 cycles per year. Thus, each cycle lasts (365/2.89) = 126.3 days.
- The time to produce 346,410 per run = (346,410/6,000,000)365 = 21.1 days. Thus, the machine is idle for 126.3 - 21.1 = 105.2 days between runs.
- Maximum Inventory:
  - Current Policy: = $((B\text{-}A)/B\ )Q^* = (5/6)100{,}000 \approx 83{,}333$.
  - Optimal Policy: = (5/6)346,410 = 288,675.
- Machine Utilization: The machine is producing tiles A/B = 1/6 of the time. (Intuitively, this should be so!)

EOQ using Solver – an alternative way of solving the EOQ problem

Reminder:

$$\text{ordering costs} = \frac{Ak}{Q}$$

$$\text{holding costs} = \frac{Qch}{2}$$

$$\begin{aligned}\text{Total Costs} &= \text{ordering costs} + \text{holding costs} \\ &= \frac{Ak}{Q} + \frac{Qch}{2}\end{aligned}$$

A non-linear function of Q

EOQ using Solver – an alternate way of solving the EOQ problem

- We can express the EOQ problem as follows:

$$\text{MIN : total cost} = \frac{Ak}{Q} + \frac{Qch}{2}$$

$$\text{subject to } Q >= 1$$

A non-linear function of Q

- The EOQ problem is an example of a non-linear programming problem

MicroSoft Excel Solver has implemented a non-linear programming algorithm called the Generalized Reduced Gradient (GRG) algorithm to solve NLP problems

(We can use it in a kind of similar way to the LP algorithm)

An EOQ Example using NLP: Ordering Paper For MetroBank

- Alan Wang purchases paper for copy machines and laser printers at MetroBank.
  - Annual demand (A) is for 24,000 boxes
  - Each box costs $35 (c)
  - Each order costs $50 (k)
  - Inventory carrying costs are 18% (h)
- What is the optimal order quantity Q*?

Solving EOQ problem using Solver

- We can express the EOQ problem as follows:

$$\text{MIN}: \frac{Ak}{Q} + \frac{Qch}{2}$$
$$\text{subject to } Q >= 1$$

A non-linear function of Q

- [Lecture 7.xlsm]

Total cost formula goes here

	A	B	C	D	E	F
1						
2		MetroBank				
3						
4			Annual Demand	24,000		
5			Cost per Unit	$35		
6			Cost per Order	$50		
7			Holding Cost	18%		
8						
9			Order Quantity	617.21		
10						
11			Cost of Ordering	$1,944		
12			Inventory Cost	$1,944		
13			Total Cost	$3,888		
14						

Formulae: Lecture 7.xlsm

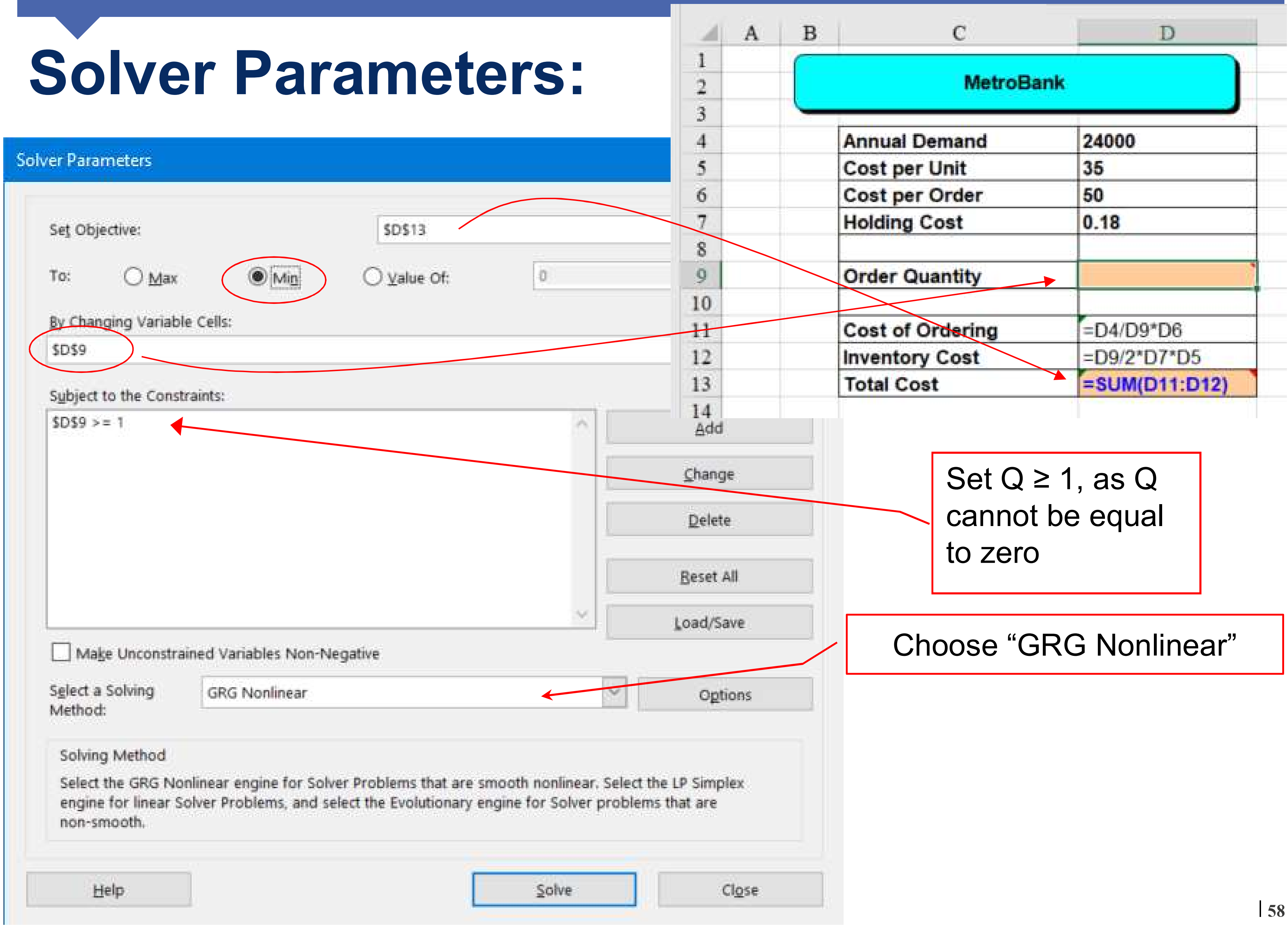
	A	B	C	D	E	F
1						
2			MetroBank			
3						
4			Annual Demand	24,000		
5			Cost per Unit	$35		
6			Cost per Order	$50		
7			Holding Cost	18%		
8						
9			Order Quantity	617.21		
10						
11			Cost of Ordering	$1,944		
12			Inventory Cost	$1,944		
13			Total Cost	$3,888		
14						

Ordering cost
=Ak/Q
=D4/D9*D6

Holding cost
=Qch/2
=D9/2*D7*D5

Total cost formula goes here
=D11+D12

Solver Parameters:



Set Q ≥ 1, as Q cannot be equal to zero

Choose “GRG Nonlinear”

Final solution comparison with EOQ formula

- NRG model
- TC=$3,888
- Order cost = $1,944
- Holding Cost = $1,944
- Same as using EOQ formula

MetroBank

Annual Demand	24,000
Cost per Unit	$35
Cost per Order	$50
Holding Cost	18%
Order Quantity	617.21
Cost of Ordering	$1,944
Inventory Cost	$1,944
Total Cost	$3,888

End of Lecture 7

References:

Ragsdale, C. (2021). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (9e) Cengage Learning: Chapter 8 (Section 4), sec. 8.4

Ragsdale, C. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) Cengage Learning: Chapter 8 (Section 4), sec. 8.4

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 15

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning: Chapter 15

Homework

- Go through today's lecture examples :
  - ✓ Familiarise yourself with the EOQ formulation and be able to determine:
    - ❖ The economic order quantity (i.e., the quantity of stock to be ordered which minimises the total annual cost);
    - ❖ How often should the order be placed;
    - ❖ Total annual relevant costs.
  - ✓ Optimal Inventory Policy with back-ordering (planned shortages)
  - ✓ Economic production quantity

☞ ***Readings for next Lecture*:**

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 16
- Inventory Decisions with Uncertain Factors

Tutorial 6 this week:

Network Modelling:

- **Transportation Problem**
- **Assignment Problem**
- **Transhipment Problem**

➢ Various techniques will be explored:

- North-west Corner Method;
- Vogel's Approximation Method (VAM)
- MODI (modified Dantzig Iteration) algorithm or the Closed-Loop Path

A/Q replenishments per year, average inventory level is Q/2

$$\text{Annual Demand} = A$$

$$\text{Unit Cost per item} = c$$

e.g., sending an email or clicking links $\text{Fixed Cost per order} = k$

$\text{Order Quantity} = Q$ 是 Find this

$$\text{Annual Holding Cost per dollar per item} = h$$

Economic Order Quantity: Formulae

$$\text{Optimal order quantity}: Q^* = \sqrt{\frac{2Ak}{ch}}$$

$$\text{Number of orders per year} = \frac{A}{Q^*}$$

$$\text{Time between orders (cycle time)} = \frac{Q^*}{A}\ \text{years}$$

$$\text{Total annual cost} = \text{ordering cost} + \text{holding cost} = \frac{Ak}{Q} + \frac{Qch}{2}$$

Economic Production Lot Size: Formulae

$$\text{Optimal production lot size}: Q^* = \sqrt{\frac{2Ak}{ch}}\sqrt{\frac{B}{B-A}}$$

$$\text{Number of production runs per year} = \frac{A}{Q^*}$$

$$\text{Time between setups (cycle time)} = \frac{Q^*}{A}\ \text{years}$$

$$\text{Total annual cost} = \text{setup cost} + \text{holding cost} = \frac{Ak}{Q} + \frac{chQ}{2}\left(\frac{B-A}{B}\right)$$

EOQ with Quantity Discounts

Formulae

- Optimal order quantity:
  - Calculate the smallest feasible Q* under each pricing structure. Choose the Q* which results in the smallest annual total cost.
- Number of orders per year: *A/Q**
- Time between orders (cycle time): *Q */A years*
- Total annual cost: [(1/2)*Q *ch*] + [*Ak/Q* *] + *Ac*

(holding + ordering + purchase)

EOQ using Solver – an alternative way of solving the EOQ problem

Reminder:

$$\text{ordering costs} = \frac{Ak}{Q}$$

$$\text{holding costs} = \frac{Qch}{2}$$

$$\text{Total Costs} = \text{ordering costs} + \text{holding costs}$$

$$= \frac{Ak}{Q} + \frac{Qch}{2}$$

A non-linear function of Q