Summary of formulas for Queues

Service and waiting times for an M/M/1 queue:

$\lambda =$ the average number of arrivals per time period (mean arrival rate)

$\frac{1}{\lambda} =$ the average time between arrivals

$\mu =$ the average number of services per time period (mean service rate)

$\frac{1}{\mu} =$ the average time taken for each service

$P_0 = 1 - \frac{\lambda}{\mu}$ the probability that no units are in the system

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ the average number of units in the waiting line

$L = L_q + \frac{\lambda}{\mu}$ the average number of units in the system $\left(= \frac{\lambda}{\mu - \lambda} \text{ for M/M/1 case} \right)$

$W_q = \frac{L_q}{\lambda}$ the average time a unit spends in the waiting line

$W = W_q + \frac{1}{\mu}$ the average time a unit spends in the system

$P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service

$P_n = \left(\frac{\lambda}{\mu} \right)^n P_0$ the probability of n units in the system

$\rho = \frac{\lambda}{\mu}$ the server utilisation factor

Service and waiting times for an M/G/1 queue:

$\sigma =$ the standard deviation of service time

$L_q = \frac{\lambda^2 \sigma^2 + (\lambda / \mu)^2}{2(1 - \lambda / \mu)}$, $L = L_q + \frac{\lambda}{\mu}$ other results as for M/M/1 case.

Service and waiting times for an M/M/S queue:

$$P_0 = 1 \Bigg/ \left[\sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!} \left(\frac{1}{1-\lambda/S\mu} \right) \right]$$

$$L = L_q + \frac{\lambda}{\mu}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & \text{if } 0 \le n \le S \\ \dfrac{(\lambda/\mu)^n}{S!\,S^{n-S}} P_0 & \text{if } n \ge S \end{cases}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1}{\mu}$$

$$L_q = \frac{(\lambda/\mu)^S (\lambda/S\mu)}{S!(1-\lambda/S\mu)^2} P_0$$

$$\rho = \frac{\lambda}{S\mu}$$

Probability distributions:

The Poisson distribution

$f(x) = \dfrac{\theta^x e^{-\theta}}{x!}$ for a distribution having mean θ, $x = 0,1,2,3\ldots$, $(e \approx 2.71828\ldots)$

The exponential distribution

$f(x) = \dfrac{1}{\theta} e^{-x/\theta}$ for a distribution having mean θ, $(e \approx 2.71828\ldots)$

$P(x \le x_0) = 1 - e^{-x_0/\theta}$

$P(x \ge x_0) = e^{-x_0/\theta}$ for a given value of x_0