Lecture 1 Review Question - Solution

Business decision modelling (Monash University)

Lecture 1: Introduction to Modelling and Decision Analysis
Introduction to Optimization and Linear Programming

Multiple Choice

1. The best models
   a. accurately reflect relevant characteristics of the real-world object or decision.
   b. are mathematical models.
   c. replicate all aspects of the real-world object or decision.
   d. replicate the characteristics of a component in isolation from the rest of the system.

ANSWER: A

2. A mathematical model is considered to be "valid" when
   a. it accurately represents the relevant characteristics of the object or decision.
   b. it has passed a validation test.
   c. it replicates all aspects of the object or decision.
   d. the left-hand and right-hand sides of expressions are equal.

ANSWER: A

3. All of the following are benefits of modeling except:
   a. Modeling delivers needed information on a more timely basis.
   b. Modeling finds the right answers to incorrect or flawed problem statements.
   c. Modeling is helpful in examining things that would be impossible to do in reality.
   d. Modeling is less expensive than implementing several alternative solutions.

ANSWER: B

4. The specification or description of the relationship between the dependent and independent variables is generally called
   a. a constraint.
   b. a declaration.
   c. a function.
   d. a mathematical model.

ANSWER: C

5. In the following expression, which is (are) the dependent variable(s)?
   PROFIT = REVENUE - EXPENSES
   a. Revenue
   b. Profit
   c. Expenses
   d. b and c

ANSWER: B

6. Which step of the problem-solving process is considered the most important?
   a. Identify problem.
   b. Analyze model.
   c. Test results.
   d. Implement solution.

ANSWER: A

7. Which of the following is true of "What if?" analysis?
   a. A well-designed spreadsheet facilitates "What if?" analysis.
   b. It is not very useful when working with non mathematical models.
   c. "What if?" analysis is an efficient optimization technique.
   d. "What if?" analysis is useful in creating a well-defined problem statement.

ANSWER: A

8. Which of the following fields of management science finds the optimal method of using resources to achieve the objectives of a business?
   a. Simulation
   b. Regression
   c. Mathematical programming
   d. Discriminant analysis

ANSWER: C

9. What are the three common elements of an optimization problem?
   a. objectives, resources, goals.
   b. decisions, constraints, an objective.
   c. decision variables, profit levels, costs.
   d. decisions, resource requirements, a profit function.

ANSWER: B

10. What is the goal in optimization?
    a. Find the best decision variable values that satisfy all constraints.
    b. Find the values of the decision variables that use all available resources.
    c. Find the values of the decision variables that satisfy all constraints.
    d. None of the above.

ANSWER: A

11. A common objective in the product mix problem is
    a. maximizing cost.
    b. maximizing profit.
    c. minimizing production time.
    d. maximizing production volume.

ANSWER: B

12. Limited resources are modeled in optimization problems as
    a. an objective function.
    b. constraints.
    c. decision variables.
    d. alternatives.

ANSWER: B

13. A manager has only 200 tons of plastic for his company. This is an example of a (an)
    a. decision.
    b. constraint.
    c. objective.
    d. parameter.

ANSWER: B

14. The symbols X1, Z1, are all examples of
    a. decision variables.
    b. constraints.
    c. objectives.
    d. parameters.

ANSWER: A

15. A production optimization problem has 4 decision variables and resource b1 limits how many of the 4 products can be produced. Which of the following constraints reflects this fact?
    a. $f(X_1, X_2, X_3, X_4) \leq b_1$
    b. $f(X_1, X_2, X_3, X_4) \geq b_1$
    c. $f(X_1, X_2, X_3, X_4) = b_1$
    d. $f(X_1, X_2, X_3, X_4) \neq b_1$

ANSWER: A

16. Which of the following is the general format of an objective function?
    a. $f(X_1, X_2, \ldots, X_n) \leq b$
    b. $f(X_1, X_2, \ldots, X_n) \geq b$
    c. $f(X_1, X_2, \ldots, X_n) = b$
    d. $f(X_1, X_2, \ldots, X_n)$

ANSWER: D

17. The first step in formulating a linear programming problem is
    a. Identify any upper or lower bounds on the decision variables.
    b. State the constraints as linear combinations of the decision variables.
    c. Understand the problem.
    d. Identify the decision variables.
    e. State the objective function as a linear combination of the decision variables.

ANSWER: C

18. The following linear programming problem has been written to plan the production of two products. The company wants to maximize its profits.

X_1 = number of product 1 produced in each batch
X_2 = number of product 2 produced in each batch

MAX: $150 X_1 + 250 X_2$
Subject to: $2 X_1 + 5 X_2 \leq 200$ - resource 1
$3 X_1 + 7 X_2 \leq 175$ - resource 2
$X_1, X_2 \geq 0$

How many units of resource 1 are consumed by each unit of product 1 produced?
a. 1
b. 2
c. 3
d. 5

ANSWER: B

19. A company uses 4 pounds of resource 1 to make each unit of X_1 and 3 pounds of resource 1 to make each unit of X_2. There are only 150 pounds of resource 1 available. Which of the following constraints reflects the relationship between X_1, X_2 and resource 1?
a. $4 X_1 + 3 X_2 \geq 150$
b. $4 X_1 + 3 X_2 \leq 150$
c. $4 X_1 + 3 X_2 = 150$
d. $4 X_1 \leq 150$

ANSWER: B

20. The constraint for resource 1 is $5 X_1 + 4 X_2 \leq 200$. If $X_1 = 20$, what it the maximum value for X_2?
a. 20
b. 25
c. 40
d. 50

ANSWER: B

21. The constraint for resource 1 is $5 X_1 + 4 X_2 \leq 200$. If $X_1 = 20$ and $X_2 = 5$, how much of resource 1 is unused?
a. 0
b. 80
c. 100
d. 200

ANSWER: B

22. The objective function for a LP model is 3 X_1 + 2 X_2. If X_1 = 20 and X_2 = 30, what is the value of the objective function?
    a. 0
    b. 50
    c. 60
    d. 120

ANSWER: D

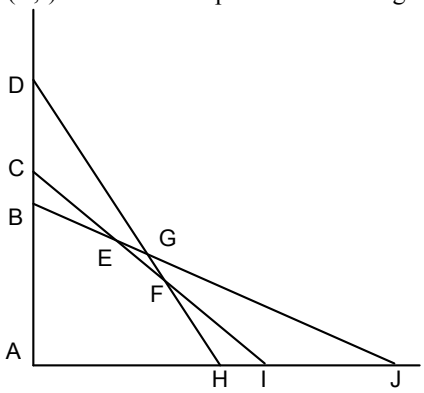
23. Why do we study the graphical method of solving LP problems?
    a. Lines are easy to draw on paper.
    b. To develop an understanding of the linear programming strategy,
    c. It is faster than computerized methods.
    d. It provides better solutions than computerized methods

ANSWER: B

24. Which of the following actions would expand the feasible region of an LP model?
    a. Loosening the constraints.
    b. Tightening the constraints.
    c. Multiplying each constraint by 2.
    d. Adding an additional constraint.

ANSWER: A

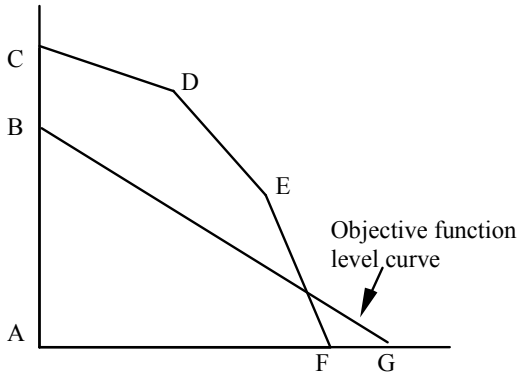
25. The following diagram shows the constraints for a LP model. Assume the point (0,0) satisfies constraint (B,J) but does not satisfy constraints (D,H) or (C,I). Which set of points on this diagram defines the feasible solution space?



    a. A, B, E, F, H
    b. A, D, G, J
    c. F, G, H, J
    d. F, G, I, J

ANSWER: D

26. This graph shows the feasible region (defined by points ACDEF) and objective function level curve (BG) for a maximization problem. Which point corresponds to the optimal solution to the problem?



a. A
b. B
c. C
d. D
e. E

ANSWER: D

27. When do alternate optimal solutions occur in LP models?
    a. When a constraint is parallel to a level curve.
    b. When a constraint is perpendicular to a level curve.
    c. When a constraint is parallel to another constraint.
    d. Alternate optimal solutions indicate an infeasible condition.

ANSWER: A [Note: level curve sits on feasible region edge, which implies parallel]

28. A redundant constraint is one which
    a. plays no role in determining the feasible region of the problem.
    b. is parallel to the level curve.
    c. is added after the problem is already formulated.
    d. can only increase the objective function value.

ANSWER: A

29. When the objective function can increase without ever contacting a constraint the LP model is said to be
    a. infeasible.
    b. open ended.
    c. multi-optimal.
    d. unbounded.

ANSWER: D

30. If there is no way to simultaneously satisfy all the constraints in an LP model the problem is said to be
    a. infeasible.
    b. open ended.
    c. multi-optimal.
    d. unbounded.

ANSWER: A

Problems

31. Solve the following LP problem graphically by enumerating the corner points.

MAX: $2 X_1 + 7 X_2$
Subject to: $5 X_1 + 9 X_2 \leq 90$
$9 X_1 + 8 X_2 \leq 144$
$X_2 \leq 8$
$X_1, X_2 \geq 0$

ANSWER: Obj = 63.20
$X_1 = 3.6$
$X_2 = 8$

32. Solve the following LP problem graphically using level curves.

MAX: $7 X_1 + 4 X_2$
Subject to: $2 X_1 + X_2 \leq 16$
$X_1 + X_2 \leq 10$
$2 X_1 + 5 X_2 \leq 40$
$X_1, X_2 \geq 0$

ANSWER: Obj = 58
$X_1 = 6$
$X_2 = 4$

33. Solve the following LP problem graphically by enumerating the corner points.

MIN: $8 X_1 + 3 X_2$
Subject to: $X_2 \geq 8$
$8 X_1 + 5 X_2 \geq 80$
$3 X_1 + 5 X_2 \geq 60$
$X_1, X_2 \geq 0$

ANSWER: Obj = 48
$X_1 = 0$
$X_2 = 16$

34. Solve the following LP problem graphically using level curves.

MAX: $5 X_1 + 3 X_2$
Subject to: $2 X_1 - 1 X_2 \leq 2$
$6 X_1 + 6 X_2 \geq 12$
$1 X_1 + 3 X_2 \leq 5$
$X_1, X_2 \geq 0$

ANSWER: Obj = 11.29
$X_1 = 1.57$
$X_2 = 1.14$

35. Solve the following LP problem graphically using level curves.

MIN: $5 X_1 + 7 X_2$
Subject to: $4 X_1 + 1 X_2 \geq 16$

$6 X_1 + 5 X_2 \geq 60$
$5 X_1 + 8 X_2 \geq 80$
$X_1, X_2 \geq 0$

ANSWER: Obj = 72.17
$X_1 = 3.48$
$X_2 = 7.83$

36. Jones Furniture Company produces beds and desks for college students. The production process requires carpentry and varnishing. Each bed requires 6 hours of carpentry and 4 hour of varnishing. Each desk requires 4 hours of carpentry and 8 hours of varnishing. There are 36 hours of carpentry time and 40 hours of varnishing time available. Beds generate $30 of profit and desks generate $40 of profit. Demand for desks is limited so at most 8 will be produced.

    Formulate the LP model for this problem.

ANSWER: Let X_1 = Number of Beds to produce
X_2 = Number of Desks to produce

MAX: $30 X_1 + 40 X_2$
Subject to: $6 X_1 + 4 X_2 \leq 36$ (carpentry)
$4 X_1 + 8 X_2 \leq 40$ (varnishing)
$X_2 \leq 8$ (demand for X_2)
$X_1, X_2 \geq 0$

    Solve the problem using the graphical method.

ANSWER: Obj = 240
$X_1 = 4$
$X_2 = 3$

37. The Big Bang explosives company produces customized blasting compounds for use in the mining industry. The two ingredients for these explosives are agent A and agent B. Big Bang just received an order for 1400 pounds of explosive. Agent A costs $5 per pound and agent B costs $6 per pound. The customer's mixture must contain at least 20% agent A and at least 50% agent B. The company wants to provide the least expensive mixture which will satisfy the customers requirements.

    Formulate the LP model for this problem.

ANSWER: Let X_1 = Pounds of agent A used
X_2 = Pounds of agent B used

MIN: $5 X_1 + 6 X_2$
Subject to: $X_1 \geq 280$ (Agent A requirement)
$X_2 \geq 700$ (Agent B requirement)
$X_1 + X_2 = 1400$ (Total pounds)
$X_1, X_2 \geq 0$

    Solve the problem using the graphical method.

ANSWER: Obj = 7700
$X_1 = 700$
$X_2 = 700$

38. Bob and Dora Sweet wish to start investing $1,000 each month. The Sweets are looking at five investment plans and wish to maximize their expected return each month. Assume interest rates remain fixed and once their investment plan is selected they do not change their mind. The investment plans offered are:

Fidelity	9.1% return per year
Optima	16.1% return per year
CaseWay	7.3% return per year
Safeway	5.6% return per year
National	12.3% return per year

Since Optima and National are riskier, the Sweets want a limit of 30% per month of their total investments placed in these two investments. Since Safeway and Fidelity are low risk, they want at least 40% of their investment total placed in these investments.

Formulate the LP model for this problem.

ANSWER:
MAX: $0.091X_1 + 0.161X_2 + 0.073X_3 + 0.056X_4 + 0.123X_5$
Subject to:

$X_1 + X_2 + X_3 + X_4 + X_5 = 1000$
$X_2 + X_5 \leq 300$
$X_1 + X_4 \geq 400$
$X_1, X_2, X_3, X_4, X_5 \geq 0$

Reference:
Ragsdale, C. T., Spreadsheet Modelling and Decision Analysis 5th Ed, Thomson, 2008: Chapter 1 & 2