Chapter 14

Decision Analysis

14.0 Introduction

The previous chapters in this book describe a variety of modeling techniques that can help managers gain insight and understanding about the decision problems they face. But models do not make decisions—people do. Although the insight and understanding gained by modeling problems can be helpful, decision making often remains a difficult task. The two primary causes for this difficulty are uncertainty regarding the future and conflicting values or objectives.

For example, suppose that when you graduate from college you receive job offers from two companies. One company (company A) is in a relatively new industry that offers potential for spectacular growth—or rapid bankruptcy. The salary offered by this company is somewhat lower than you would like, but would increase rapidly if the company grows. This company is located in the city that is home to your favorite professional sports team and close to your friends and family.

The other job offer is from an established company (company B) that is known for its financial strength and long-term commitment to its employees. It has offered you a starting salary that is 10 percent more than you asked, but you suspect it would take longer for you to advance in this organization. Also, if you work for this company, you would have to move to a distant part of the country that offers few of the cultural and sporting activities that you enjoy.

Which offer would you accept? Or would you reject both offers and continue looking for employment with other companies? For many, this might be a difficult decision. If you accept the job with company A, you might be promoted twice within a year—or you could be unemployed in six months. With company B, you can be reasonably sure of having a secure job for the foreseeable future. But if you accept the job with company B and then company A grows rapidly, you might regret not accepting the position with company A. Thus, the uncertainty associated with the future of company A makes this decision difficult.

To further complicate the decision, company A offers a more desirable location than company B, but the starting salary with company A is lower. How can you assess the trade-offs between starting salary, location, job security, and potential for advancement in order to make a good decision? There is no easy answer to this question, but this chapter describes a number of techniques that can help you structure and analyze difficult decision problems in a logical manner.

14.1 Good Decisions vs. Good Outcomes

The goal of decision analysis is to help individuals make good decisions. But good decisions do not always result in good outcomes. For example, suppose that after carefully considering all the factors involved in the two job offers, you decide to accept the position with company B. After working for this company for nine months, it suddenly announces that, in an effort to cut costs, it is closing the office in which you work and eliminating your job. Did you make a bad decision? Probably not. Unforeseeable circumstances beyond your control caused you to experience a bad outcome, but it would be unfair to say that you made a bad decision. A good decision is one that is in harmony with what you know, what you want, what you can do, and to which you are committed. But good decisions sometimes result in bad outcomes.

The techniques for decision analysis presented in this chapter can help you make good decisions, but they cannot guarantee that good outcomes will always occur as a result of those decisions. Even when a good decision is made, luck often plays a role in determining whether a good or bad outcome occurs. However, using a structured approach to make decisions should give us enhanced insight and sharper intuition about the decision problems we face. As a result, it is reasonable to expect good outcomes to occur more frequently when using a structured approach to decision making than if we make decisions in a more haphazard manner.

14.2 Characteristics of Decision Problems

Although all decision problems are somewhat different, they share certain characteristics. For example, a decision must involve at least two alternatives for addressing or solving a problem. An **alternative** is a course of action intended to solve a problem. The job selection example described earlier involves three alternatives: you could accept the offer from company A, accept the offer from company B, or reject both offers and continue searching for a better one.

Alternatives are evaluated on the basis of the value they add to one or more decision criteria. The **criteria** in a decision problem represent various factors that are important to the decision maker and influenced by the alternatives. For example, the criteria used to evaluate the job offer alternatives might include starting salary, expected salary growth, desirability of job location, opportunity for promotion and career advancement, and so on. The impact of the alternatives on the criteria is of primary importance to the decision maker. Note that not all criteria can be expressed in terms of monetary value, making comparisons of the alternatives more difficult.

Finally, the values assumed by the various decision criteria under each alternative depend on the different states of nature that can occur. The **states of nature** in a decision problem correspond to future events that are not under the decision maker's control. For example, company A could experience spectacular growth, or it might go bankrupt. Each of these contingencies represents a possible state of nature for the problem. Many other states of nature are possible for the company; for example, it could grow slowly, or not grow at all. Thus, an infinite number of possible states of nature could exist in this, and many other, decision problems. However, in decision analysis, we often use a relatively small, discrete set of representative states of nature to summarize the future events that might occur.

14.3 An Example

The following example illustrates some of the issues and difficulties that arise in decision problems.

> Hartsfield-Jackson International Airport in Atlanta, Georgia is one of the busiest airports in the world. During the past 30 years, the airport has expanded again and again to accommodate the increasing number of flights being routed through Atlanta. Analysts project that this increase will continue well into the future.

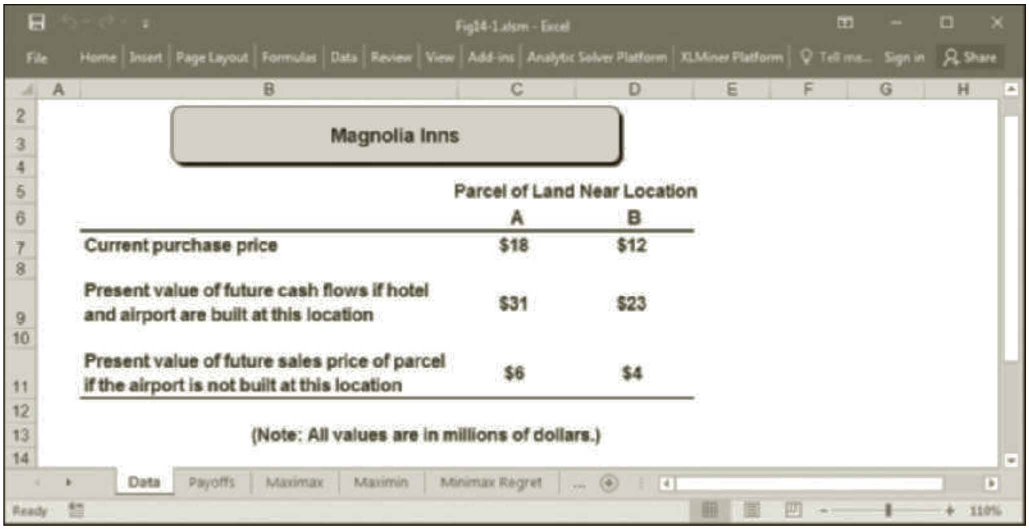
However, commercial development around the airport prevents it from building additional runways to handle the future air-traffic demands. As a solution to this problem, plans are being developed to build another airport outside the city limits. Two possible locations for the new airport have been identified, but a final decision on the new location is not expected to be made for another year.

The Magnolia Inns hotel chain intends to build a new facility near the new airport after its site is determined. Barbara Monroe is responsible for real estate acquisition for the company, and she faces a difficult decision about where to buy land. Currently, land values around the two possible sites for the new airport are increasing as investors speculate that property values will increase greatly in the vicinity of the new airport. The spreadsheet in Figure 14.1 (and in the file Fig14-1.xlsm that accompanies this book) summarizes the current price of each parcel of land, the estimated present value of the future cash flows that a hotel would generate at each site if the airport is ultimately located at the site, and the present value of the amount for which the company believes it can resell each parcel if the airport is not built at the site.

The company can buy either site, both sites, or neither site. Barbara must decide which sites, if any, the company should purchase.

FIGURE 14.1

Data for the Magnolia Inns decision problem



Magnolia Inns

	Parcel of Land Near Location	
	A	B
Current purchase price	$18	$12
Present value of future cash flows if hotel and airport are built at this location	$31	$23
Present value of future sales price of parcel if the airport is not built at this location	$6	$4

(Note: All values are in millions of dollars.)

14.4 The Payoff Matrix

A common way of analyzing this type of decision problem is to construct a payoff matrix. A **payoff matrix** is a table that summarizes the final outcome (or payoff) for each decision alternative under each possible state of nature. To construct a payoff matrix, we need to identify each decision alternative and each possible state of nature.

14.4.1 DECISION ALTERNATIVES

The following four decision alternatives are available to the decision maker in our example problem:

1. Buy the parcel at location A.
2. Buy the parcel at location B.
3. Buy the parcels at locations A and B.
4. Buy nothing.

14.4.2 STATES OF NATURE

Regardless of which parcel or parcels Magnolia Inns decides to purchase, two possible states of nature can occur. The two states of nature are as follows:

1. The new airport is built at location A.
2. The new airport is built at location B.

Figure 14.2 shows the payoff matrix for this problem. The rows in this spreadsheet represent the possible decision alternatives, and the columns correspond to the states of nature that might occur. Each value in this table indicates the financial payoff (in millions of dollars) expected for each possible decision under each state of nature.

Payoff Matrix

Land Purchased at Location(s)	Airport is Built at Location A	B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

FIGURE 14.2

Payoff matrix for the Magnolia Inns decision problem

14.4.3 THE PAYOFF VALUES

The value in cell B5 in Figure 14.2 indicates that if the company buys the parcel of land near location A, and the airport is built in this area, Magnolia Inns can expect to receive a payoff of $13 million. This figure of $13 million is computed from the data shown in Figure 14.1 as:

	Present value of future cash flows if hotel and airport are built at location A	$31,000,000
minus:		
	Current purchase price of hotel site at location A	−$18,000,000
		$13,000,000

The value in cell C5 in Figure 14.2 indicates that if Magnolia Inns buys the parcel of land at location A (for $18 million) and the airport is built at location B, the company would later resell the parcel at location A for only $6 million, incurring a loss of $12 million.

The calculations of the payoffs for the parcel near location B are computed using similar logic. The value in cell C6 in Figure 14.2 indicates that if the company buys the parcel of land near location B and the airport is built in this area, Magnolia Inns can expect to receive a payoff of $11 million. The value in cell B6 in Figure 14.2 indicates that if Magnolia Inns buys the parcel of land at location B (for $12 million) and the airport is built at location A, the company would later resell the parcel at location B for only $4 million, incurring a loss of $8 million.

Let's now consider the payoffs if the parcels at both locations A and B are purchased. The value in cell B7 in Figure 14.2 indicates that a payoff of $5 million will result if both parcels are purchased and the airport is built at location A. This payoff value is computed as:

	Present value of future cash flows if hotel and airport are built at location A	$31,000,000
plus:		
	Present value of future sales price for the unused parcel at location B	+$ 4,000,000
minus:		
	Current purchase price of hotel site at location A	−$18,000,000
minus:		
	Current purchase price of hotel site at location B	−$12,000,000
		$ 5,000,000

The value in cell C7 indicates that a loss of $1 million will occur if the parcels at both locations A and B are purchased, and the airport is built at location B.

The final alternative available to Magnolia Inns is not to buy either property at this point in time. This alternative guarantees that the company will neither gain nor lose anything, regardless of where the airport is located. Thus, cells B8 and C8 indicate that this alternative has a payoff of $0 regardless of which state of nature occurs.

14.5 Decision Rules

Now that the payoffs for each alternative under each state of nature have been determined, if Barbara knew with certainty where the airport was going to be built, it would be a simple matter for her to select the most desirable alternative. For example, if she knew the airport was going to be built at location A, a maximum payoff of $13 million could be obtained by purchasing the parcel of land at that location. Similarly, if she knew the airport was going to be built at location B, Magnolia Inns could achieve the maximum payoff of $11 million by purchasing the parcel at that location. The problem is that Barbara does not know where the airport is going to be built.

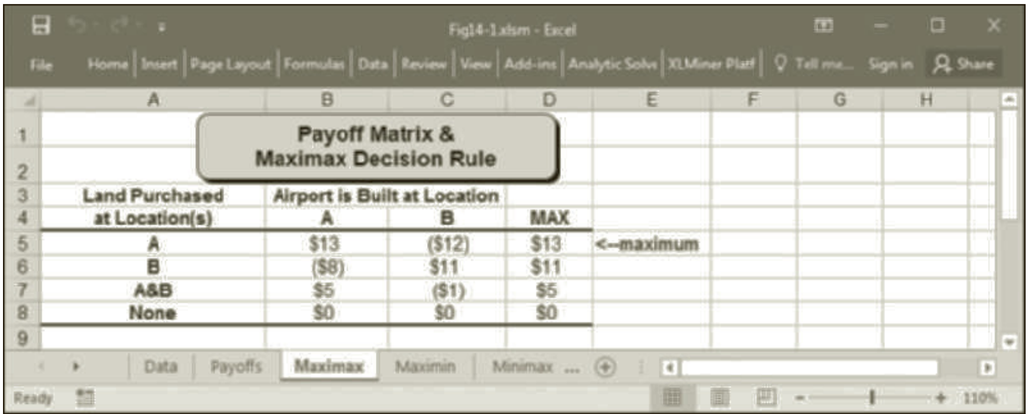
Several decision rules can be used to help a decision maker choose the best alternative. No one of these decision rules works best in all situations and, as you will see, each has some weaknesses. However, these rules help to enhance our insight and sharpen our intuition about decision problems so that we can make more informed decisions.

14.6 Nonprobabilistic Methods

The decision rules we will discuss can be divided into two categories: those that assume that probabilities of occurrence can be assigned to the states of nature in a decision problem (**probabilistic methods**), and those that do not (**nonprobabilistic methods**). We will discuss the nonprobabilistic methods first.

14.6.1 THE MAXIMAX DECISION RULE

As shown in Figure 14.2, the largest possible payoff will occur if Magnolia Inns buys the parcel at location A and the airport is built at this location. Thus, if the company optimistically believes that nature will always be "on its side" regardless of the decision it makes, the company should buy the parcel at location A because it leads to the largest possible payoff. This type of reasoning is reflected in the **maximax decision rule**, which determines the maximum payoff for each alternative and then selects the alternative associated with the largest payoff. Figure 14.3 illustrates the results of the maximax decision rule on our example problem.



Key Cell Formulas

Cell	Formula	Copied to
D5	=MAX(B5:C5)	D6:D8

FIGURE 14.3 *The maximax decision rule for the Magnolia Inns decision problem*

Although the alternative suggested by the maximax decision rule enables Magnolia Inns to realize the best possible payoff, it does not guarantee that this payoff will occur. The actual payoff depends on where the airport is ultimately located. If we follow the maximax decision rule and the airport is built at location A, the company would receive $13 million; but if the airport is built at location B, the company would lose $12 million.

In some situations, the maximax decision rule leads to poor decisions. For example, consider the following payoff matrix:

	State of Nature			
Decision	**1**	**2**	**MAX**	
A	30	−10,000	30	← maximum
B	29	29	29	

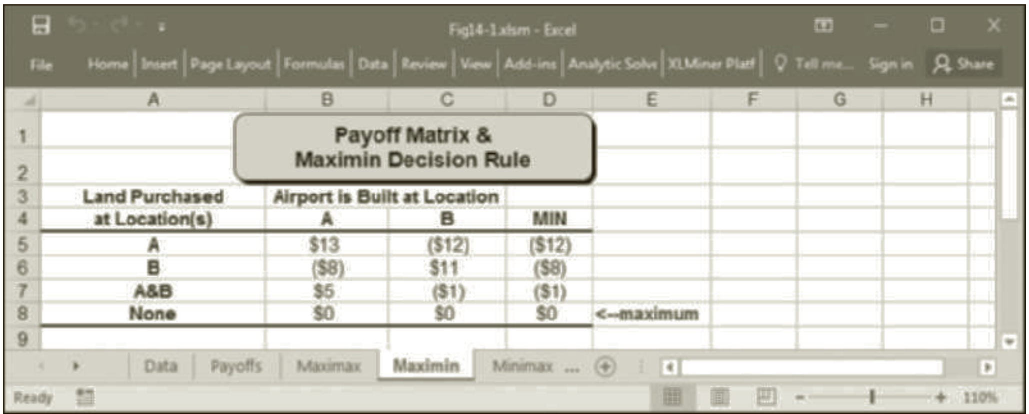
In this problem, alternative A would be selected using the maximax decision rule. However, many decision makers would prefer alternative B because its guaranteed payoff is only slightly less than the maximum possible payoff, and it avoids the potential large loss involved with alternative A if the second state of nature occurs.

14.6.2 THE MAXIMIN DECISION RULE

A more conservative approach to decision making is given by the **maximin decision rule**, which pessimistically assumes that nature will always be "against us" regardless of the decision we make. This decision rule can be used to hedge against the worst possible outcome of a decision. Figure 14.4 illustrates the effect of the maximin decision rule on our example problem.

FIGURE 14.4

The maximin decision rule for the Magnolia Inns decision problem



Land Purchased at Location(s)	Airport is Built at Location A	B	MIN	
A	$13	($12)	($12)	
B	($8)	$11	($8)	
A&B	$5	($1)	($1)	
None	$0	$0	$0	<--maximum

Key Cell Formulas

Cell	Formula	Copied to
D5	=MIN(B5:C5)	D6:D8

To apply the maximin decision rule, we first determine the minimum possible payoff for each alternative and then select the alternative with the largest minimum payoff (or the maximum of the minimum payoffs—hence the term "maximin"). Column D in Figure 14.4 lists the minimum payoff for each alternative. The largest (maximum) value in column D is the payoff of $0 associated with not buying any land. Thus, the maximin decision rule suggests that Magnolia Inns should not buy either parcel because, in the worst case, the other alternatives result in losses whereas this alternative does not.

The maximin decision rule can also lead to poor decision making. For example, consider the following payoff matrix:

	State of Nature			
Decision	**1**	**2**	**MIN**	
A	1,000	28	28	
B	29	29	29	← maximum

In this problem, alternative B would be selected using the maximin decision rule. However, many decision makers would prefer alternative A because its worst-case payoff is only slightly less than that of alternative B, and it provides the potential for a much larger payoff if the first state of nature occurs.

14.6.3 THE MINIMAX REGRET DECISION RULE

Another way of approaching decision problems involves the concept of **regret**, or opportunity loss. For example, suppose that Magnolia Inns decides to buy the parcel of land at location A as suggested by the maximax decision rule. If the airport is built at location A, the company will not regret this decision at all because it provides the largest possible payoff under the state of nature that occurred. However, what if the company buys the parcel at location A and the airport is built at location B? In this case, the company would experience a regret, or opportunity loss, of $23 million. If Magnolia Inns had bought the parcel at location B, it would have earned a payoff of $11 million, and the decision to buy the parcel at location A resulted in a loss of $12 million. Thus, there is a difference of $23 million in the payoffs between these two alternatives under this state of nature.

To use the minimax regret decision rule, we must first convert our payoff matrix into a regret matrix that summarizes the possible opportunity losses that could result from each decision alternative under each state of nature. Figure 14.5 shows the regret matrix for our example problem.

Regret Matrix & Minimax Regret Decision Rule

Land Purchased at Location(s)	Airport is Built at Location A	B	Max	
A	$0	$23	$23	
B	$21	$0	$21	
A&B	$8	$12	$12	<--minimum
None	$13	$11	$13	

Key Cell Formulas

Cell	Formula	Copied to
B5	=MAX(Payoffs!B$5:B$8)−Payoffs!B5	B5:C8
D5	=MAX(B5:C5)	D6:D8

FIGURE 14.5

The minimax regret decision rule for the Magnolia Inns decision problem

The entries in the regret matrix are generated from the payoff matrix as:

Formula for cell B5: =MAX(Payoffs!B$5:B$8) − Payoffs!B5
(Copy to B5 through C8.)

Each entry in the regret matrix shows the difference between the maximum payoff that can occur under a given state of nature and the payoff that would be realized from each alternative under the same state of nature. For example, if Magnolia Inns buys the parcel of land at location A and the airport is built at this location, cell B5 indicates that the company experiences 0 regret. However, if the company buys the parcel at location B and the airport is built at location A, the company experiences an opportunity loss (or regret) of $21 million $(13 - (-8) = 21)$.

Column D in Figure 14.5 summarizes the maximum regret that could be experienced with each decision alternative. The minimax regret decision corresponds to the alternative with the smallest (or minimum) maximum regret. As indicated in Figure 14.5, the minimax regret decision in our example problem is to buy the parcels at both sites. The maximum regret that could be experienced by implementing this decision is $12 million, whereas all other decisions could cause a larger regret.

The minimax regret decision rule can also lead to peculiar decision making. For example, consider the following payoff matrix:

	State of Nature	
Decision	**1**	**2**
A	9	2
B	4	6

The regret matrix and minimax regret decision for this problem are represented by:

	State of Nature			
Decision	**1**	**2**	**MAX**	
A	0	4	4	← minimum
B	5	0	5	

Thus, if the alternatives are given by A and B, the minimax regret decision rule would select alternative A. Now, suppose that we add a new alternative to this decision problem to obtain the following payoff matrix:

	State of Nature	
Decision	**1**	**2**
A	9	2
B	4	6
C	3	9

Notice that the payoffs for alternatives A and B have not changed—we simply added a new alternative (C). The regret matrix and minimax regret decision for the revised problem are represented by:

	State of Nature			
Decision	**1**	**2**	**MAX**	
A	0	7	7	
B	5	3	5	← minimum
C	6	0	6	

The minimax regret decision is now given by alternative B. Some decision makers are troubled that the addition of a new alternative, which is not selected as the final decision, can change the relative preferences of the original alternatives. For example, suppose that a person prefers apples to oranges, but would prefer oranges if given the options of apples, oranges, and bananas. This person's reasoning is somewhat inconsistent or incoherent. But such reversals in preferences are a natural consequence of the minimax regret decision rule.

14.7 Probabilistic Methods

Probabilistic decision rules can be used if the states of nature in a decision problem can be assigned probabilities that represent their likelihood of occurrence. For decision problems that occur more than once, it is often possible to estimate these probabilities from historical data. However, many decision problems (such as the Magnolia Inns problem) represent one-time decisions for which historical data for estimating probabilities are unlikely to exist. In these cases, probabilities are often assigned subjectively based on interviews with one or more domain experts. Highly structured interviewing techniques exist to solicit probability estimates that are reasonably accurate and free of the unconscious biases that may impact an expert's opinions. These interviewing techniques are described in several of the references at the end of this chapter. Here, we will focus on the techniques that can be used once appropriate probability estimates have been obtained either from historical data or expert interviews.

14.7.1 EXPECTED MONETARY VALUE

The **expected monetary value decision rule** selects the decision alternative with the largest expected monetary value (EMV). The EMV of alternative i in a decision problem is defined as:

$$\text{EMV}_i = \sum_j r_{ij} p_j$$

where

r_{ij} = the payoff for alternative i under the j^{th} state of nature

p_j = the probability of the j^{th} state of nature

Figure 14.6 illustrates the EMV decision rule for our example problem. In this case, Magnolia Inns estimates a 40% chance that the airport will be built at location A and a 60% chance that it will be built at location B.

The probabilities for each state of nature are computed in cells B10 and C10, respectively. Using these probabilities, the EMV for each decision alternative is calculated in column D as:

Formula for cell D5: =SUMPRODUCT(B5:C5,$B$10:$C$10)

(Copy to D6 through D8.)

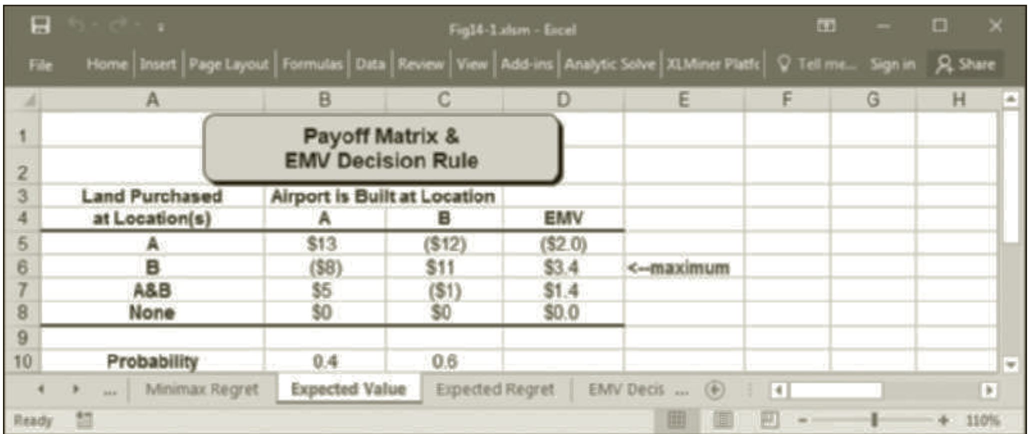


FIGURE 14.6

The expected monetary value decision rule for the Magnolia Inns decision problem

Key Cell Formulas

Cell	Formula	Copied to
D5	=SUMPRODUCT(B5:C5,$B$10:$C$10)	D6:D8
C10	=1−B10	--

The largest EMV is associated with the decision to purchase the parcel of land at location B. Thus, this is the decision suggested according to the EMV decision rule.

Let's consider the meaning of the figures in the EMV column in Figure 14.6. For example, the decision to purchase the parcel at location B has an EMV of $3.4 million. What does this figure represent? The payoff table indicates that Magnolia Inns will receive a payoff of $11 million if it buys this land and the airport is built there, or it will lose $8 million if it buys this land and the airport is built at the other location. So, there does not appear to be any way for the company to receive a payoff of $3.4 million if it buys the land at location B. However, imagine that Magnolia Inns faces this same decision not just once, but over and over again (perhaps on a weekly basis). If the company always decides to purchase the land at location B, we would expect it to receive a payoff of $11 million 60% of the time, and incur a loss of $8 million 40% of the time. Over the long run, then, the decision to purchase land at location B results in an average payoff of $3.4 million.

The EMV for a given decision alternative indicates the average payoff we would receive if we encounter the identical decision problem repeatedly and always select this alternative. Selecting the alternative with the highest EMV makes sense in situations where the identical decision problem will be faced repeatedly and we can "play the averages." However, this decision rule can be very risky in decision problems encountered only once (such as our example problem). For example, consider the following problem:

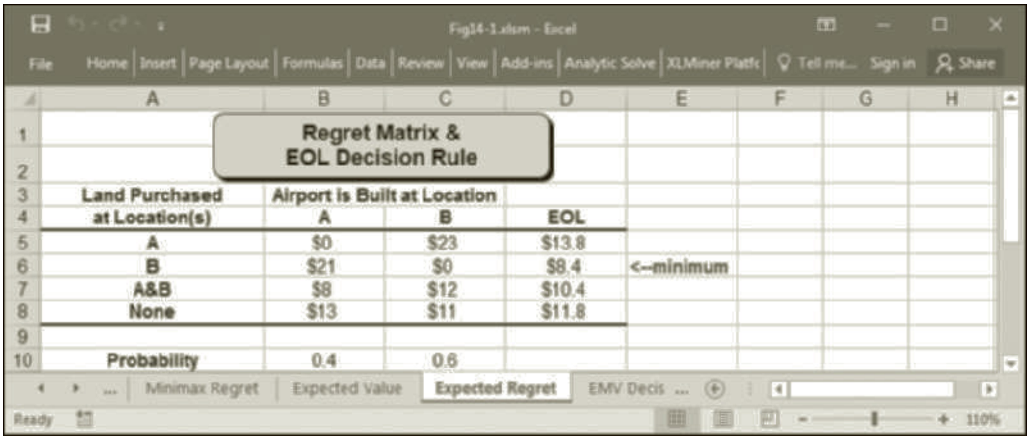
	State of Nature			
Decision	**1**	**2**	**EMV**	
A	15,000	−5,000	5,000	← maximum
B	5,000	4,000	4,500	
Probability	0.5	0.5		

If we face a decision with these payoffs and probabilities repeatedly and always select decision A, the payoff over the long run would average to $5,000. Because this is larger than decision B's average long-run payoff of $4,500, it would be best to always select decision A. But what if we face this decision problem only once? If we select decision A, we are equally likely to receive $15,000 or lose $5,000. If we select decision B, we are equally likely to receive payoffs of $5,000 or $4,000. In this case, decision A is more risky. Yet this type of risk is ignored completely by the EMV decision rule. Later, we will discuss a technique—known as the utility theory—that allows us to account for this type of risk in our decision making.

14.7.2 EXPECTED REGRET

We can also use the probability of the states of nature to compute the **expected regret**, or **expected opportunity loss** (EOL), for each alternative in a decision problem. Figure 14.7 illustrates this process for our example problem.

The calculations in Figure 14.7 are identical to those used in computing the EMVs, only here we substitute regret values (or opportunity losses) for the payoffs. As shown in Figure 14.7, the decision to purchase the parcel at location B results in the smallest EOL. It is not a coincidence that this same decision also resulted in the largest EMV in Figure 14.6. The decision with the smallest EOL will also have the largest EMV. Thus, the EMV and EOL decision rules always result in the selection of the same decision alternative.



	A	B	C	D	E
1		Regret Matrix & EOL Decision Rule			
3	Land Purchased	Airport is Built at Location			
4	at Location(s)	A	B	EOL	
5	A	$0	$23	$13.8	
6	B	$21	$0	$8.4	<--minimum
7	A&B	$8	$12	$10.4	
8	None	$13	$11	$11.8	
10	Probability	0.4	0.6		

Key Cell Formulas

Cell	Formula	Copied to
B5	=MAX(Payoffs!B$5:B$8)−Payoffs!B5	B5:C8
D5	=SUMPRODUCT(B5:C5,$B$10:$C$10)	D6:D8
C10	=1−B10	--

FIGURE 14.7

The expected regret decision rule for the Magnolia Inns decision problem

EMV and EOL

The expected monetary value (EMV) and expected opportunity loss (EOL) decision rules always result in the selection of the same decision alternative.

14.7.3 SENSITIVITY ANALYSIS

When using probabilistic decision rules, one should always consider how sensitive the recommended decision is to the estimated probabilities. For instance, the EMV decision rule shown in Figure 14.6 indicates that if there is a 60% probability of the new airport being built at location B, the best decision is to purchase the land at location B. However, what if this probability is 55%? Or 50%? Or 45%? Would it still be best to purchase the land at location B?

We can answer this by building a data table that summarizes the EMVs for each alternative as we vary the probabilities. Figure 14.8 shows how to set up a data table for this problem.

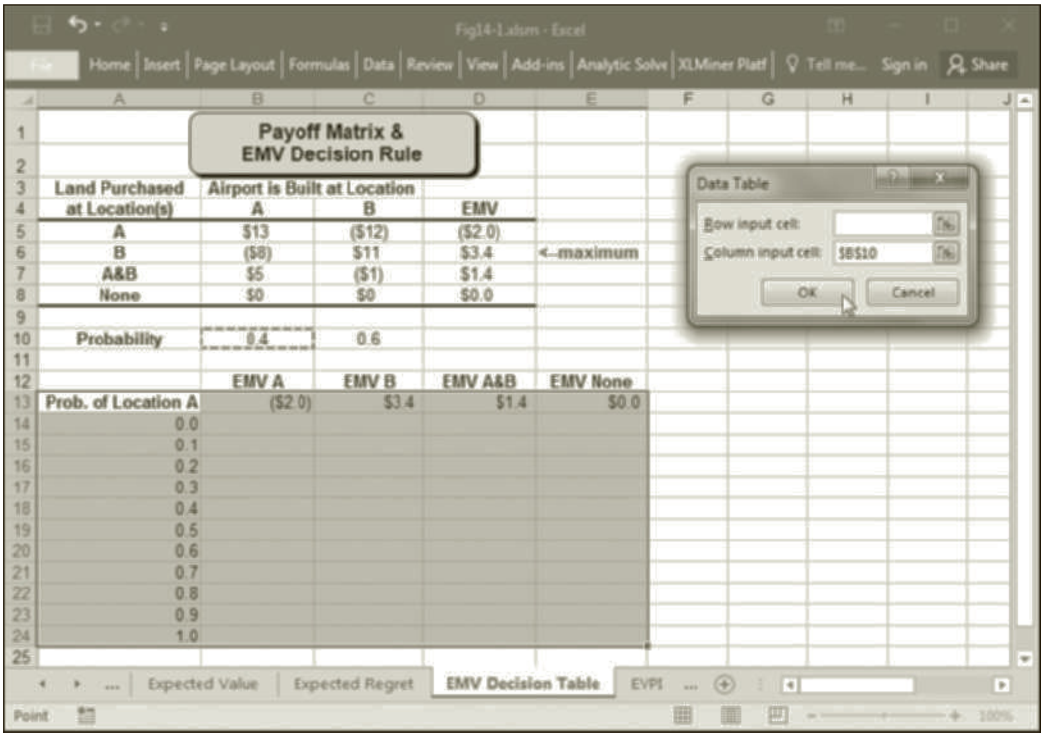
First, in cells A14 through A24, we entered the values from 0 to 1 representing different probabilities for the airport being built at location A. Next, in cells B13 through E13, we entered formulas that link back to the EMVs for each of the decision alternatives. To finish the data table, follow these steps:

1. Select cells A13 through E24.
2. Click Data, What-If Analysis, Data Table.
3. Specify a Column Input Cell of B10 (as shown in Figure 14.8).
4. Click OK.

This causes Excel to plug each of the values in cells A14 through A24 into cell B10, recalculate the spreadsheet, and then record the resulting EMVs for each decision

FIGURE 14.8

Creating a data table of EMVs for the various alternatives as the probabilities change



Key Cell Formulas

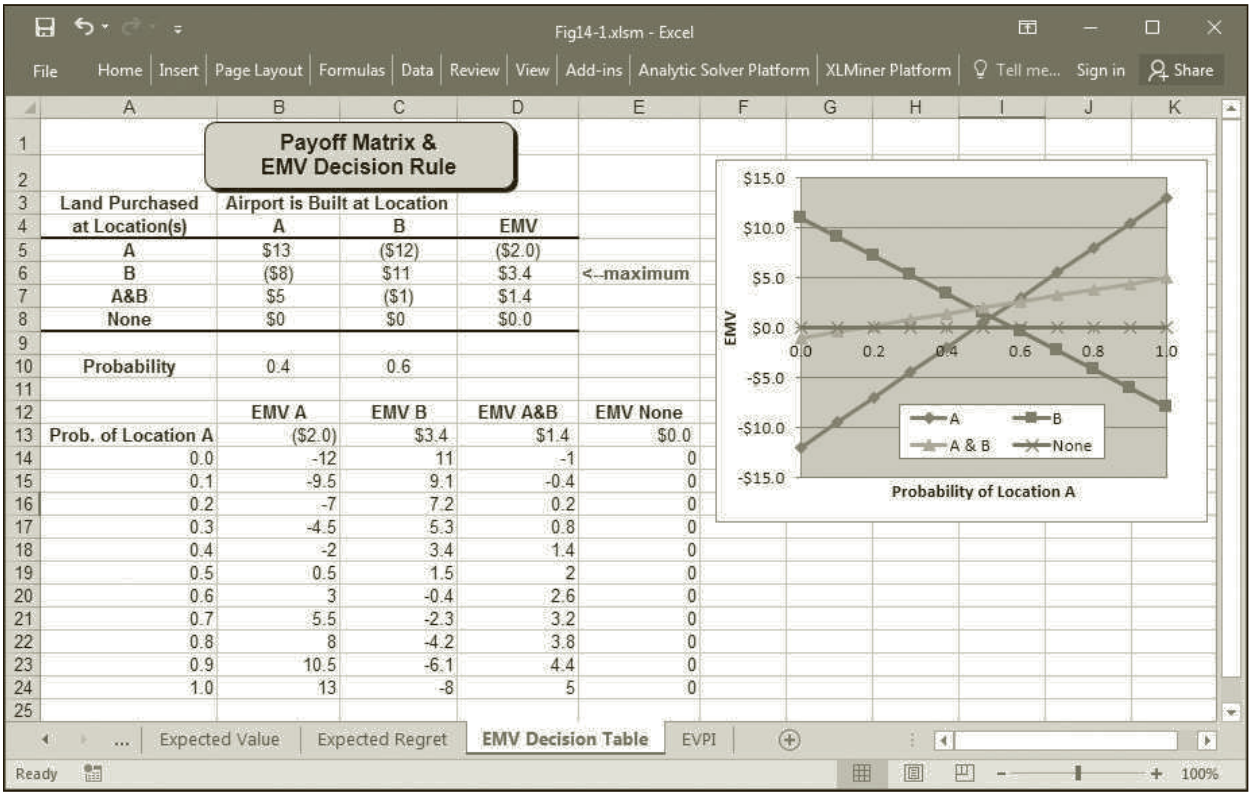
Cell	Formula	Copied to
D5	=SUMPRODUCT(B5:C5,$B$10:$C$10)	D6:D8
C10	=1−B10	--
A15	=A14+.1	A16:A24
B13	=D5	--
C13	=D6	--
D13	=D7	--
E13	=D8	--

alternative in our table. (Note that the formula in cell C10 makes the probability of the airport being built at location B dependent on the value in cell B10.) The resulting data table is shown in Figure 14.9.

The data table in Figure 14.9 indicates that if the probability of the airport being built at location A is 0.4 or less, then purchasing the land at location B has the highest EMV. However, if the airport is equally likely to be built at either location, then the decision to purchase land at both locations A and B has the highest EMV. If the airport is more likely to be built at location A, then purchasing the land at location A becomes the preferred decision.

The graph of the possible payoffs shown in Figure 14.9 makes it clear that buying the land at both locations A and B is a less risky alternative than buying either location individually. If there is much uncertainty in the probability estimates, the preferred alternative may well be to buy both pieces of property. For probability values between 0.4 and 0.6, the EMV of buying land at both locations A and B is always positive and varies from $1.4 million to $2.6 million. Within this same range of probabilities, a decision to buy at location A individually or location B individually poses a risk of a negative EMV.

FIGURE 14.9 *Data table for the Magnolia Inns decision problem*



	A	B	C	D	E
1		Payoff Matrix & EMV Decision Rule			
2					
3	Land Purchased	Airport is Built at Location			
4	at Location(s)	A	B	EMV	
5	A	$13	($12)	($2.0)	
6	B	($8)	$11	$3.4	<--maximum
7	A&B	$5	($1)	$1.4	
8	None	$0	$0	$0.0	
9					
10	Probability	0.4	0.6		
11					
12		EMV A	EMV B	EMV A&B	EMV None
13	Prob. of Location A	($2.0)	$3.4	$1.4	$0.0
14	0.0	-12	11	-1	0
15	0.1	-9.5	9.1	-0.4	0
16	0.2	-7	7.2	0.2	0
17	0.3	-4.5	5.3	0.8	0
18	0.4	-2	3.4	1.4	0
19	0.5	0.5	1.5	2	0
20	0.6	3	-0.4	2.6	0
21	0.7	5.5	-2.3	3.2	0
22	0.8	8	-4.2	3.8	0
23	0.9	10.5	-6.1	4.4	0
24	1.0	13	-8	5	0
25					

14.8 The Expected Value of Perfect Information

One of the primary difficulties in decision making is that we usually do not know which state of nature will occur. As we have seen, estimates of the probability of each state of nature can be used to calculate the EMV of various decision alternatives. However, probabilities do not tell us which state of nature will occur—they only indicate the likelihood of the various states of nature.

Suppose that we could hire a consultant who could tell us in advance and with 100% accuracy which state of nature will occur. If our example problem were a repeatable decision problem, 40% of the time the consultant would indicate that the airport will be built at location A, and the company would buy the parcel of land at location A and receive a payoff of $13 million. Similarly, 60% of the time the consultant would indicate that the airport will be built at location B, and the company would buy the parcel at location B and receive a payoff of $11 million. Thus, with advance perfect information about where the airport is going to be built, the average payoff would be the following:

$$\begin{aligned}\text{Expected value } \textit{with} \text{ perfect information} &= 0.40 \times \$13 + 0.60 \times \$11 \\ &= \$11.8 \text{ (in millions)}\end{aligned}$$

So, how much should Magnolia Inns be willing to pay this consultant for such information? From Figure 14.6, we know that *without* the services of this consultant, the best decision identified results in an EMV of \$3.4 million. Therefore, the information provided by the consultant would enable the company to make decisions that increase the EMV by \$8.4 million (\$11.8 − \$3.4 = \$8.4). Thus, the company should be willing to pay the consultant up to \$8.4 million for providing perfect information.

The **expected value of perfect information** (EVPI) is the expected value obtained with perfect information minus the expected value obtained without perfect information (which is given by the maximum EMV), that is:

$$\begin{array}{c}\text{Expected value of}\\ \text{perfect information}\end{array} = \begin{array}{c}\text{Expected value } \textit{with}\\ \text{perfect information}\end{array} - \text{maximum EMV}$$

Figure 14.10 summarizes the EVPI calculation for our example problem. Cell D6 in Figure 14.10 shows the calculation of the maximum EMV of \$3.4 million, which was described earlier in our discussion of the EMV decision rule. The payoffs of the decisions made under each state of nature with perfect information are calculated in cells B12 and C12 as:

Formula for cell B12: =MAX(B5:B8)
(Copy to C12.)

The expected value *with* perfect information is calculated in cell D12 as:

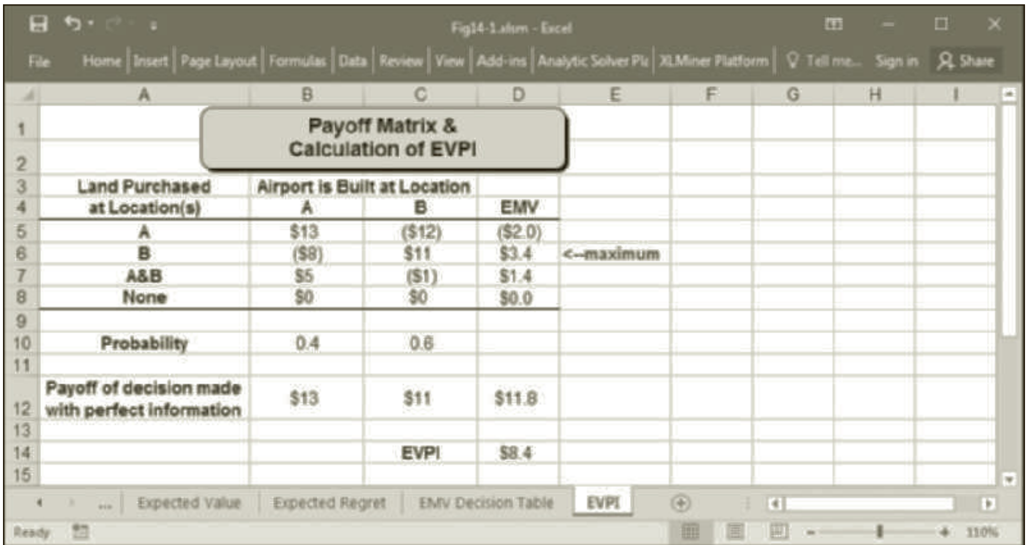
Formula for cell D12: =SUMPRODUCT(B12:C12,B10:C10)

Finally, the expected value *of* perfect information is computed in cell D14 as:

Formula for cell D14: =D12 − MAX(D5:D8)

FIGURE 14.10

The expected value of perfect information for the Magnolia Inns decision problem



Key Cell Formulas

Cell	Formula	Copied to
B12	=MAX(B5:B8)	C12
D5	=SUMPRODUCT(B5:C5,\$B\$10:\$C\$10)	D6:D8 & D12
D14	=D12−MAX(D5:D8)	--

Notice that the \$8.4 million EVPI figure in cell D14 is identical to the minimum EOL shown earlier in Figure 14.7. This is *not* just a coincidence. The minimum EOL in a decision problem will always equal the EVPI.

Key Point

The expected value of perfect information (EVPI) is equivalent to the minimum expected opportunity loss (EOL).

14.9 Decision Trees

Although some decision problems can be represented and analyzed effectively using payoff tables, we can also represent decision problems in a graphical form known as a **decision tree**. Figure 14.11 shows the decision problem for Magnolia Inns represented in this format.

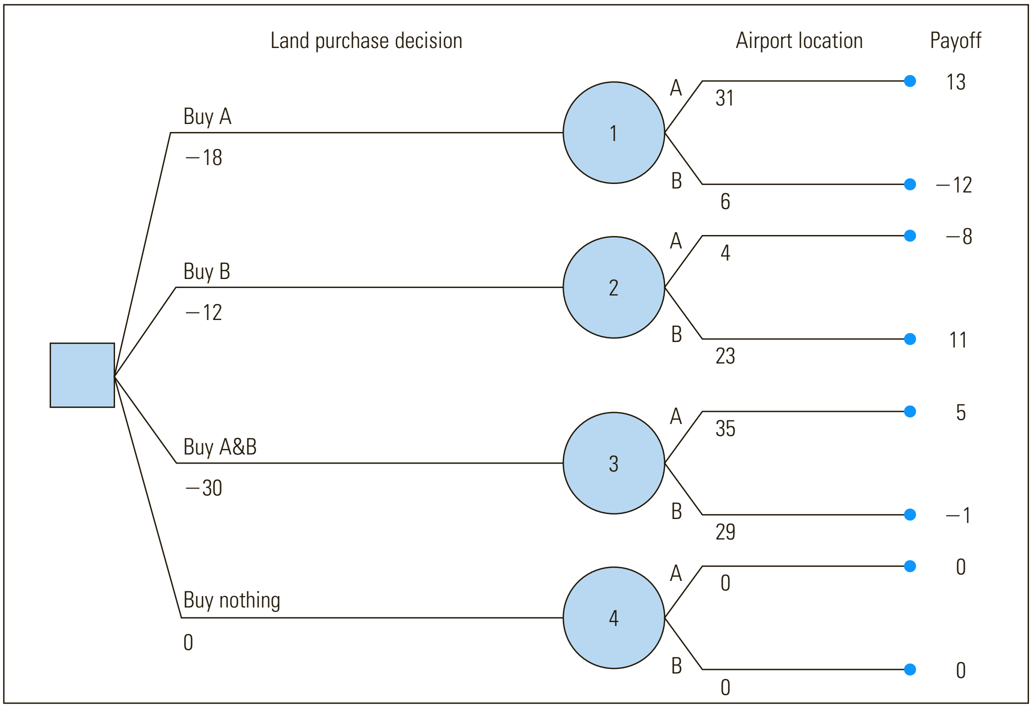


FIGURE 14.11

The decision tree representation of the Magnolia Inns problem

As shown in Figure 14.11, a decision tree is composed of a collection of nodes (represented by circles and squares) interconnected by branches (represented by lines). A square node is called a **decision node** because it represents a decision. Branches emanating from a decision node represent the different alternatives for a particular decision. In Figure 14.11, a single square decision node represents the decision Magnolia Inns faces about where to buy land. The four branches coming out of this decision node represent the four alternatives under consideration. The cash flow associated with each alternative is also listed. For example, the value –18 below the alternative labeled "Buy A" indicates that if the company purchases the parcel at location A, it must pay \$18 million.

The circular nodes in a decision tree are called **event nodes** because they represent uncertain events. The branches emanating from event nodes (called **event branches**) correspond to the possible states of nature or the possible outcomes of an uncertain event. Figure 14.11 shows that each decision alternative emanating from the decision node is followed by an uncertain event represented by the event nodes 1, 2, 3, and 4. The branches from each event node represent a possible location of the new airport. In each case, the airport can be built at location A or B. The value beneath each branch from the event nodes indicates the cash flow that will occur for that decision/event combination. For example, at node 1, the value 31 below the first event branch indicates that if the company buys the parcel at location A and the airport is built at this location, a cash flow of $31 million will occur.

The various branches in a decision tree end at objects called **leaves**. Because each leaf corresponds to one way in which the decision problem can terminate, leaves are also referred to as **terminal nodes**. Each terminal node in Figure 14.11 corresponds to an entry in the payoff table in Figure 14.2. The payoff occurring at each terminal node is computed by summing the cash flows along the set of branches leading to each leaf. For example, following the uppermost branches through the tree, a payoff of $13 million results if the decision to buy the parcel at location A is followed by the new airport being built at this location ($-18 + 31 = 13$). You should verify the cash-flow values on each branch and at each leaf before continuing.

14.9.1 ROLLING BACK A DECISION TREE

After computing the payoffs at each terminal node, we can apply any of the decision rules described earlier. For example, we could identify the maximum possible payoff for each decision and apply the maximax decision rule. However, decision trees are used most often to implement the EMV decision rule—that is, to identify the decision with the largest EMV.

We can apply a process known as **rolling back** to a decision tree to determine the decision with the largest EMV. Figure 14.12 illustrates this process for our example problem.

Because the EMV decision rule is a probabilistic method, Figure 14.12 indicates the probabilities associated with each event branch emanating from each event node (i.e., a 0.4 probability exists of the new airport being built at location A, and a 0.6 probability exists of it being built at location B). To roll back this decision tree, we start with the payoffs and work our way from right to left, back through the decision tree, computing the expected values for each node. For example, the event represented by node 1 has a 0.4 probability of resulting in a payoff of $13 million, and a 0.6 probability of resulting in a loss of $12 million. Thus, the EMV at node 1 is calculated as:

$$\text{EMV at node 1} = 0.4 \times 13 + 0.6 \times -12 = -2.0$$

The expected value calculations for the remaining event nodes in Figure 14.12 are summarized as:

$$\text{EMV at node 2} = 0.4 \times -8 + 0.6 \times 11 = 3.4$$
$$\text{EMV at node 3} = 0.4 \times 5 + 0.6 \times -1 = 1.4$$
$$\text{EMV at node 4} = 0.4 \times 0 + 0.6 \times 0 = 0.0$$

The EMV for a decision node is computed in a different way. For example, at the (square) decision node, we face a decision among four alternatives that lead to events

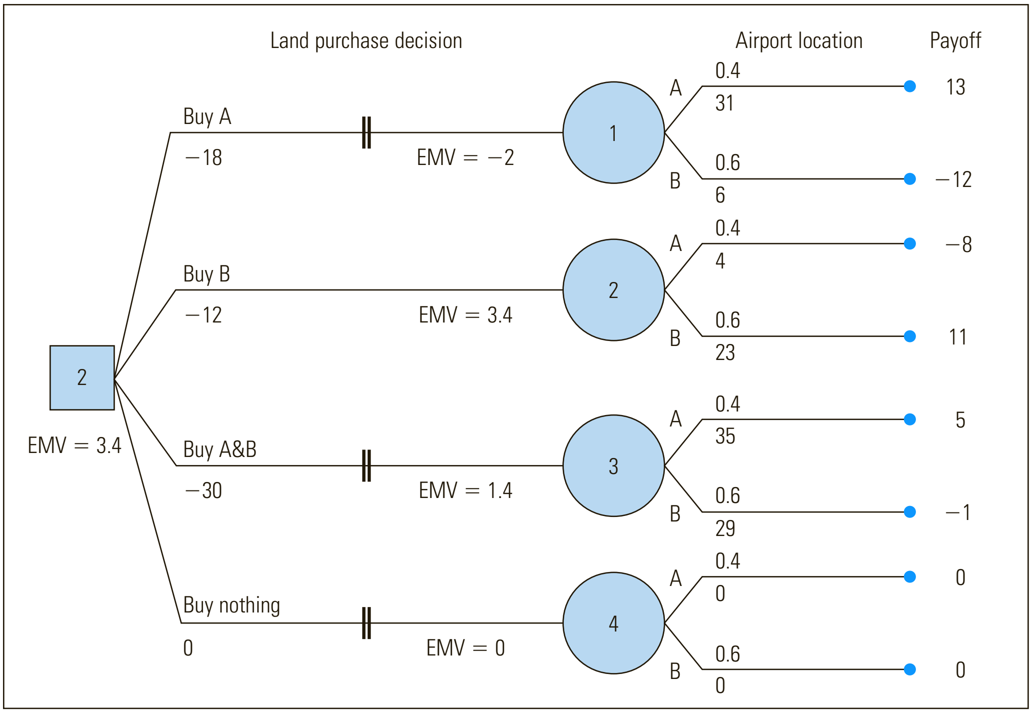


FIGURE 14.12

Rolling back the decision tree for the Magnolia Inns decision problem

with EMVs of −2, 3.4, 1.4, and 0, respectively. At a decision node, we always select the alternative that leads to the best EMV. Thus, the EMV at the decision node is 3.4, which corresponds to the EMV resulting from the decision to buy land at location B. (This decision is represented by the number 2 in the decision node because the decision to buy land at location B is the second decision alternative at this decision node.) The optimal alternative at a decision node is sometimes indicated by "pruning" the suboptimal branches. The pruned branches in Figure 14.12 are indicated by the double vertical lines (||) shown on the suboptimal alternatives emanating from the decision node.

The relationship between the decision tree in Figure 14.12 and the payoff table in Figure 14.2 should now be clear. However, you might wonder if it is necessary to include event node 4 in the tree shown in Figure 14.12. If Magnolia Inns decides not to buy either property, the payoff it receives does not depend on where the airport is ultimately built—regardless of where the airport is built, the company will receive a payoff of 0.

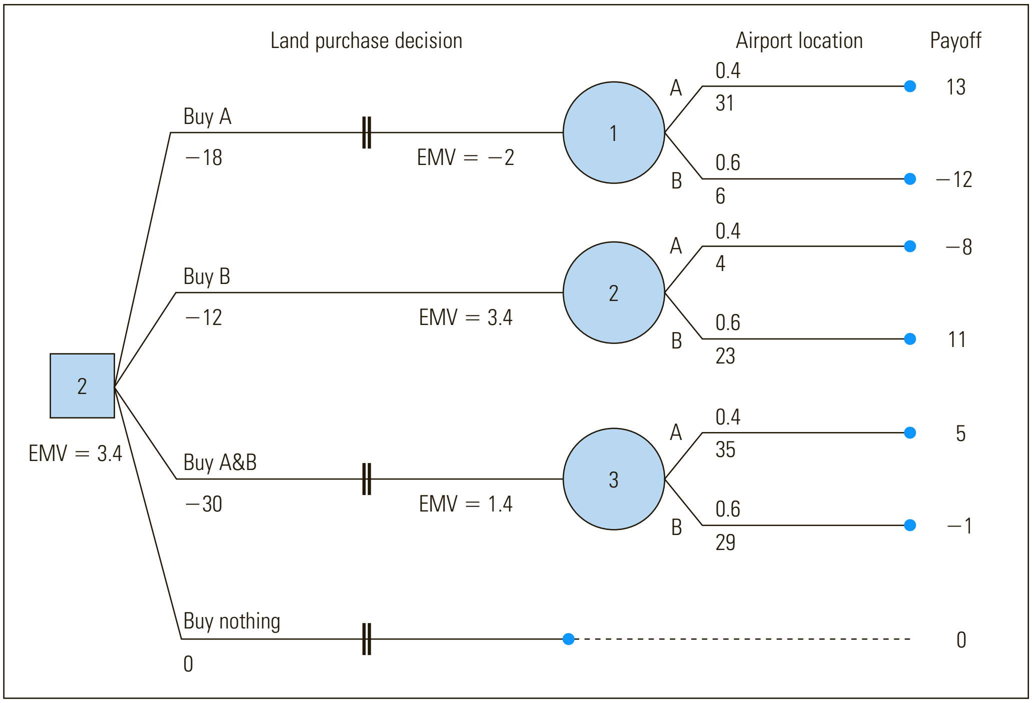
Figure 14.13 shows an alternative, and perhaps more efficient, way of representing this problem as a decision tree in which it is clear that the decision not to purchase either parcel leads to a definite payoff of 0.

14.10 Creating Decision Trees with Analytic Solver Platform

Analytic Solver Platform includes a tool that can help us create and analyze decision trees in Excel. We will illustrate how to use this tool to implement the decision tree shown in Figure 14.13 in Excel.

FIGURE 14.13

Alternative decision tree representation of the Magnolia Inns decision problem



To create a decision tree, open a new workbook and follow these steps:

1. Select cell A1.
2. Click the Analytic Solver Platform tab.
3. Click Decision Tree, Node, Add Node.

In response, Analytic Solver Platform displays the Decision Tree dialog box as shown in Figure 14.14. (Alternatively, you can also display the Decision Tree dialog box by selecting Decision Tree in the Analytic Solver task pane and clicking the green plus sign icon in the task pane.)

In Figure 14.14, we filled in the entries shown for the “Node Name” and supplied names and values (cash flows) for each of the four branches that should emanate from the initial decision node (as shown in Figure 14.13). When you click OK on the Decision Tree dialog box, an initial decision tree is created in Excel as shown in Figure 14.15.

In Figure 14.15, note that a representation of the decision tree is also created in the Decision Tree section of Analytic Solver task pane. If you click the different elements of the tree in the task pane, various properties associated with the selected element appear in the lower part of the task pane.

14.10.1 ADDING EVENT NODES

Each of the first three decision branches in Figure 14.13 leads to an event node with two event branches. Thus, we need to add similar event nodes to the decision tree shown in Figure 14.15. To add the first event node follow these steps:

1. Select the terminal node for the branch labeled Buy A (cell F3).
2. Click Decision Tree, Node, Change Node.

FIGURE 14.14 *Initial Decision Tree dialog box*

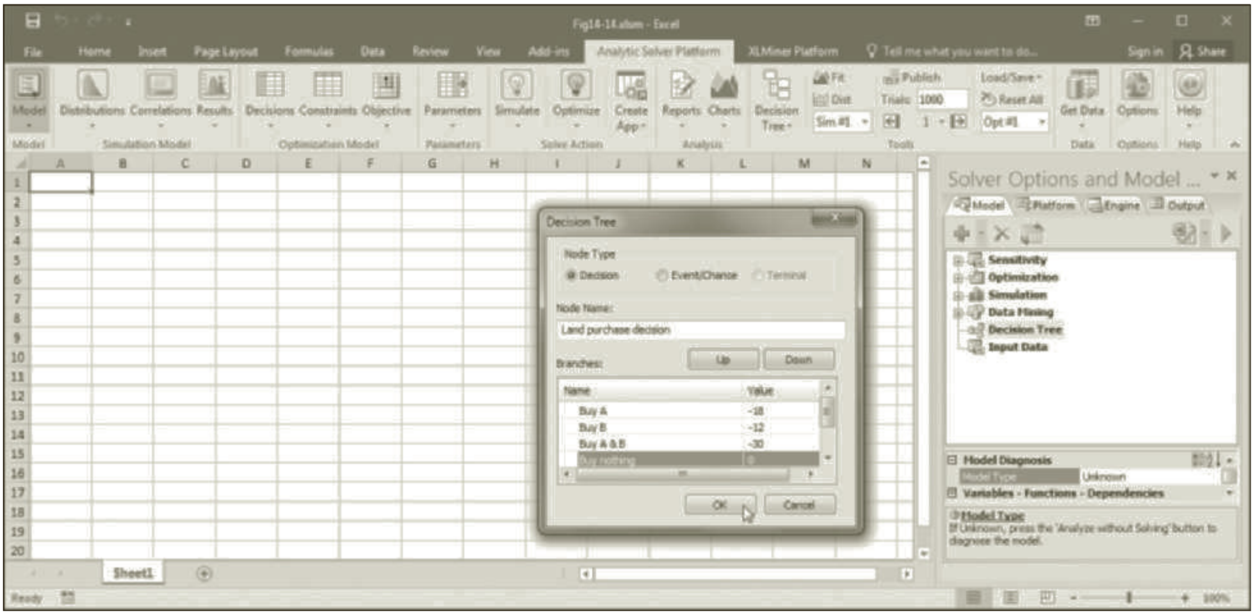
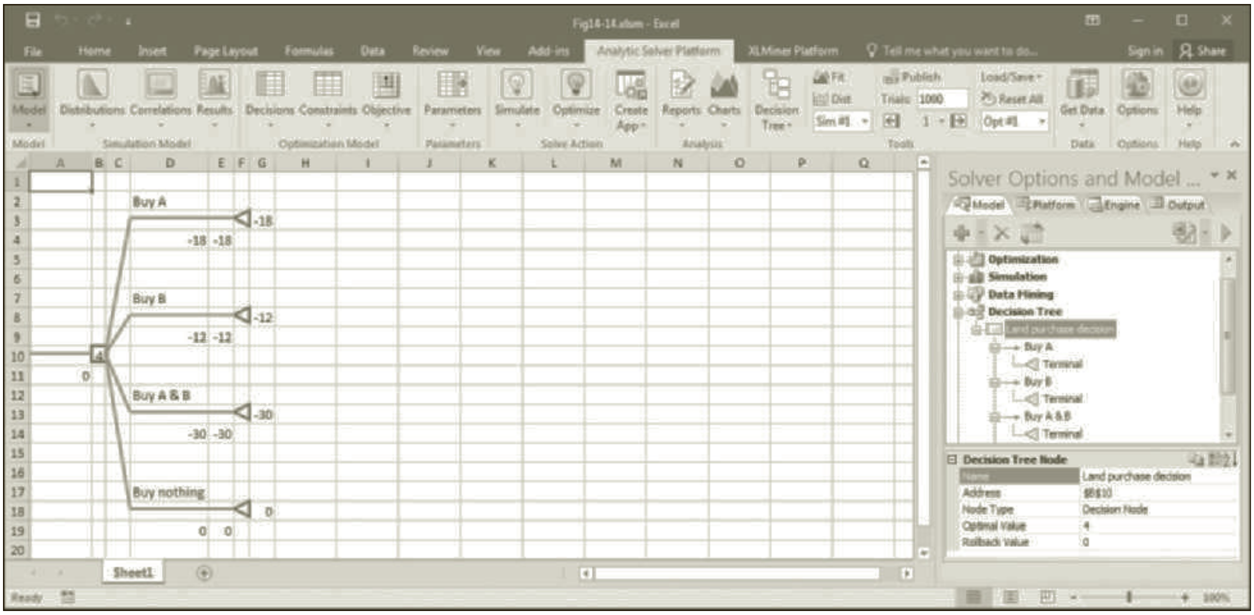


FIGURE 14.15 *Initial tree with four decision branches*



This causes the Decision Tree dialog box to appear again. However, as shown in Figure 14.16, this time we select the Event option and provide the name, value (or cash flows), and chance (probability) information associated with the event node we want to add to the tree. The resulting spreadsheet is shown in Figure 14.27.

FIGURE 14.16 *Adding an event node*

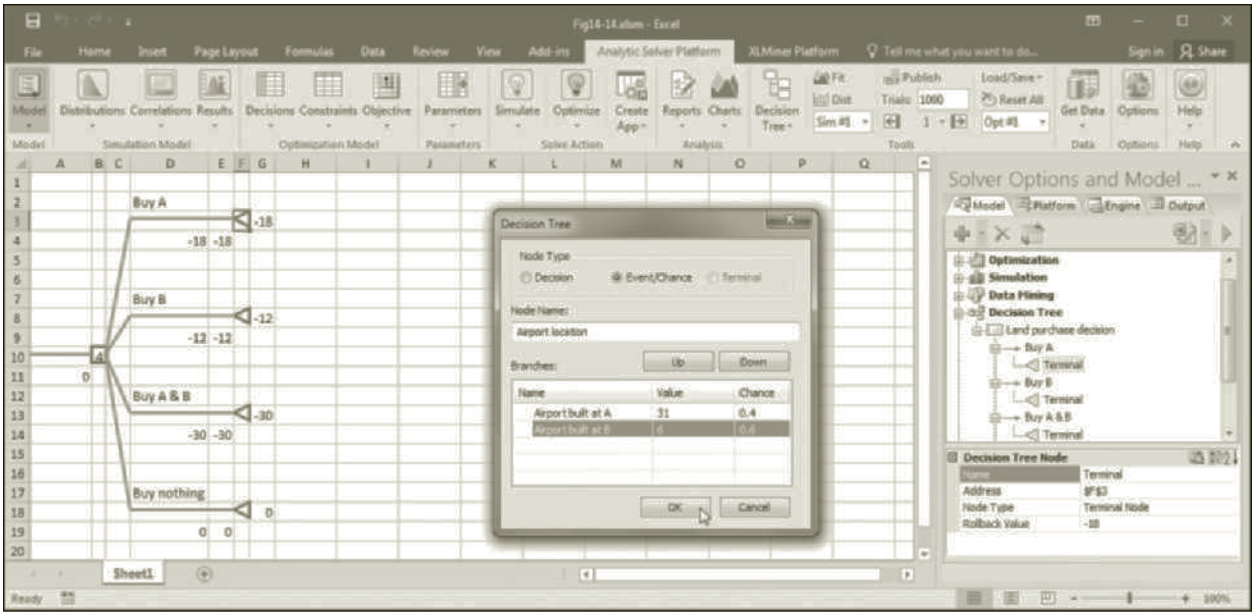
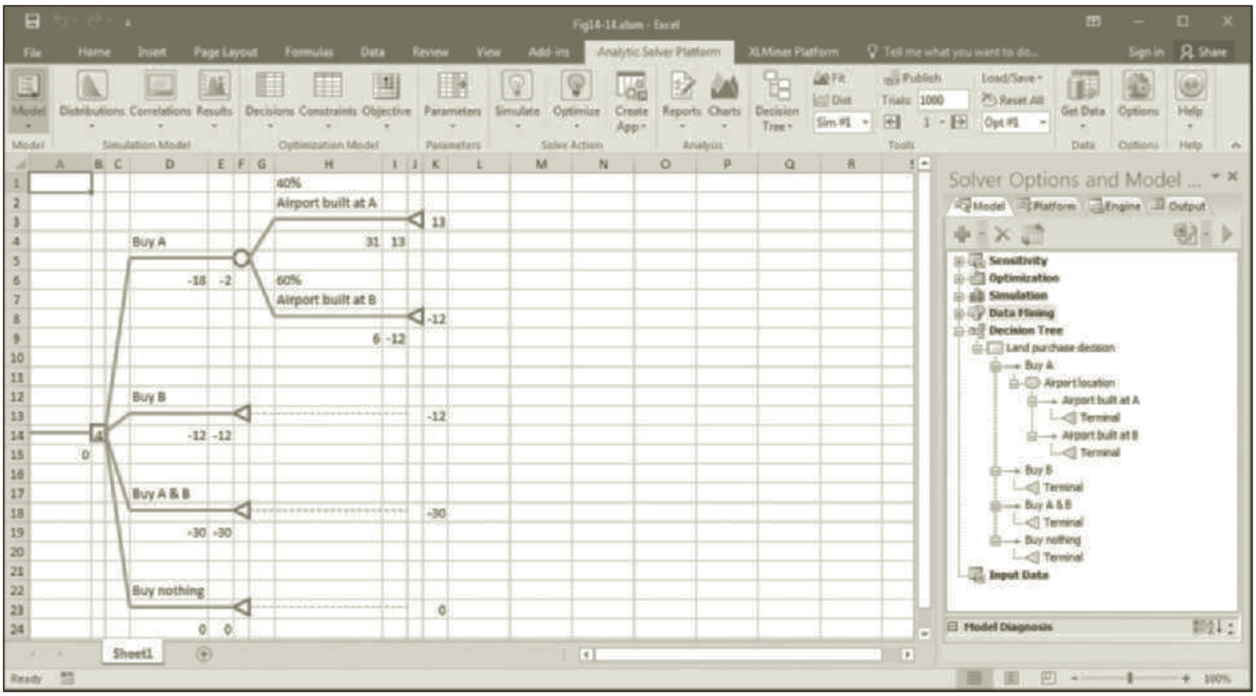


FIGURE 14.17 *Modified decision tree with an event node*



The procedure used to create the event node for the Buy A decision could be repeated to create event nodes for the decisions corresponding to Buy B and Buy A & B. However, because all of the event nodes are identical in this problem (except for the values on the branches), we could also simply copy the existing event node two times. You might be tempted to copy and paste the existing event node using the standard Excel commands—but if you do, Analytic Solver Platform cannot update the formulas in the tree properly. Thus, it is important to copy and paste portions of the decision tree using the Decision Tree tool's Copy Node and Paste Node commands. To create a copy of the event node:

1. Select the node you want to copy (cell F5).
2. Click Decision Tree, Node, Copy Node.

This creates a copy of the selected event node in your computer's memory. To paste a copy of this subtree onto the next branch in the decision tree:

1. Select the target cell location (cell F13).
2. Click Decision Tree, Node, Paste Node.

The resulting decision tree is shown in Figure 14.18. We can repeat this copy-and-paste procedure to create the third event node needed for the decision to buy the parcels at both locations A and B. Figure 14.19 shows the resulting spreadsheet after the cash flow values have been updated on the branches from the newly added event nodes.

14.10.2 DETERMINING THE PAYOFFS AND EMVs

Next to each terminal node, the Decision Tree tool automatically created a formula that sums the payoffs along the branches leading to that node. For example, cell K3 in

FIGURE 14.18 *Decision tree with three event nodes*

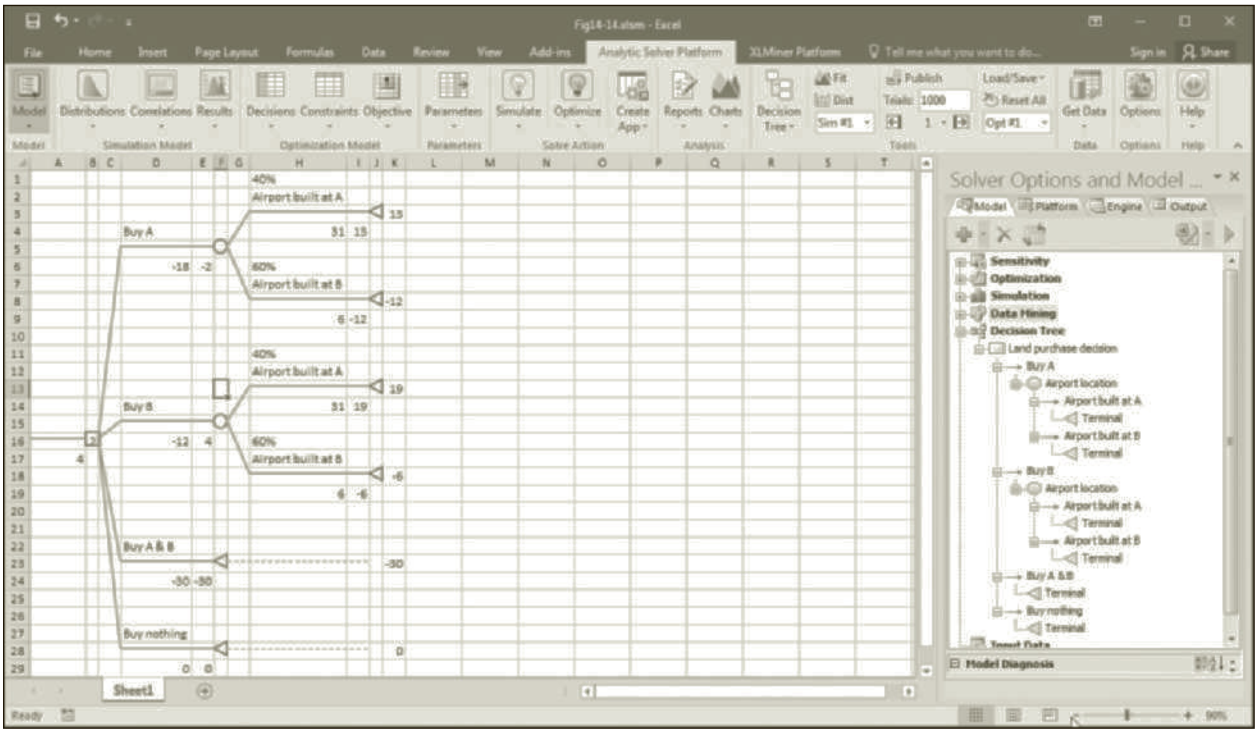


FIGURE 14.19 *Completed decision tree for the Magnolia Inns decision problem*

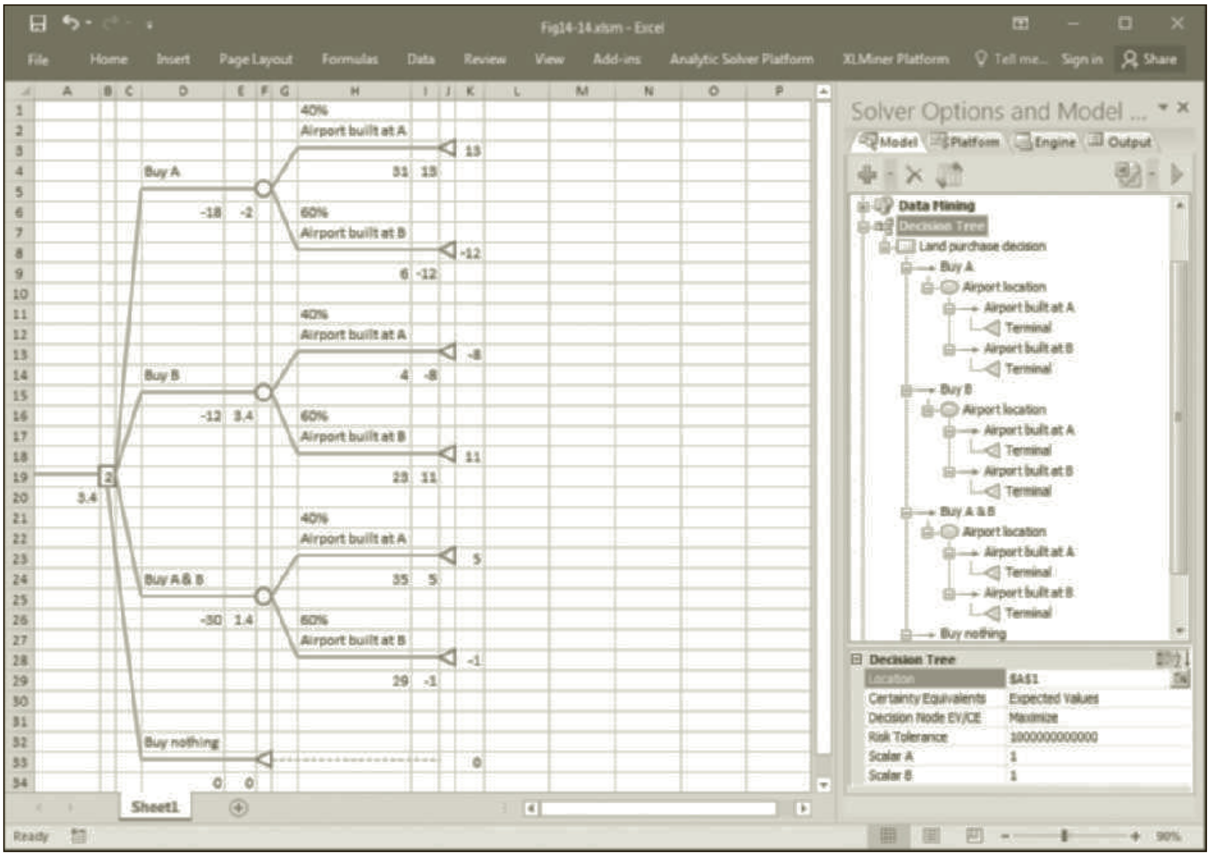


Figure 14.19 contains the formula =SUM(H4,D6). Thus, when we enter or change the cash flows for the branches in the decision tree, the payoffs are updated automatically.

Immediately below and to the left of each node, the Decision Tree tool created formulas that compute the EMV at each node in the same way as described earlier in our discussion of rolling back a decision tree. Thus, cell A20 in Figure 14.19 indicates that the largest EMV at the decision node is $3.4 million. The value 2 in the decision node (cell B19) indicates that this maximum EMV is obtained by selecting the second decision alternative (i.e., by purchasing the parcel at location B).

14.10.3 OTHER FEATURES

The preceding discussion was intended to give you an overview of how the Decision Tree tool operates and some of its capabilities and options. Many of its other capabilities are self-explanatory.

By default, the Decision Tree tool assumes that the EMVs it calculates represent profit values and that we want to identify the decision with the largest EMV. However, in some decision trees, the expected values could represent costs that we want to minimize. In Figure 14.19, note that when the Decision Tree component is selected on the Model tab in the Analytic Solver task pane, a number of properties are displayed in the lower portion of the task pane. The "Decision Node EV/CE" option allows you to specify whether you are using values that should be maximized or minimized.

Also by default, the Decision Tree tool assumes that we want to analyze the decision tree using expected values. However, another technique (described later) uses exponential utility functions in place of expected values. Thus, the Certainty Equivalents property controls whether the Decision Tree tool uses expected values or exponential utility functions while evaluating the alternatives in a decision tree.

14.11 Multistage Decision Problems

To this point, our discussion of decision analysis has considered only **single-stage** decision problems—that is, problems in which a single decision must be made. However, most decisions that we face lead to other decisions. As a simple example, consider the decision of whether to go out to dinner. If you decide to go out to dinner, you must then decide how much to spend, where to go, and how to get there. Thus, before you actually decide to go out to dinner, you'll probably consider the other issues and decisions that must be made if you choose that alternative. These types of problems are called **multistage** decision problems. The following example illustrates how a multistage decision problem can be modeled and analyzed using a decision tree.

The Occupational Safety and Health Administration (OSHA) has recently announced it will award an $85,000 research grant to the person or company submitting the best proposal for using wireless communications technology to enhance safety in the coal-mining industry. Steve Hinton, the owner of COM-TECH, a small communications research firm located just outside of Raleigh, North Carolina, is considering whether or not to apply for this grant. Steve estimates he would spend approximately $5,000 preparing his grant proposal and that he has about a 50–50 chance of actually receiving the grant. If he is awarded the grant, he would then need to decide whether to use microwave, cellular, or infrared communications technology. He has some experience in all three areas, but would need to acquire some new equipment depending on which technology is used. The cost of the equipment needed for each technology is summarized as:

Technology	Equipment Cost
Microwave	$4,000
Cellular	$5,000
Infrared	$4,000

In addition to the equipment costs, Steve knows he will spend money in research and development (R&D) to carry out the research proposal, but he does not know exactly what the R&D costs will be. For simplicity, Steve estimates the following best-case and worst-case R&D costs associated with using each technology, and he assigns probabilities to each outcome based on his degree of expertise in each area.

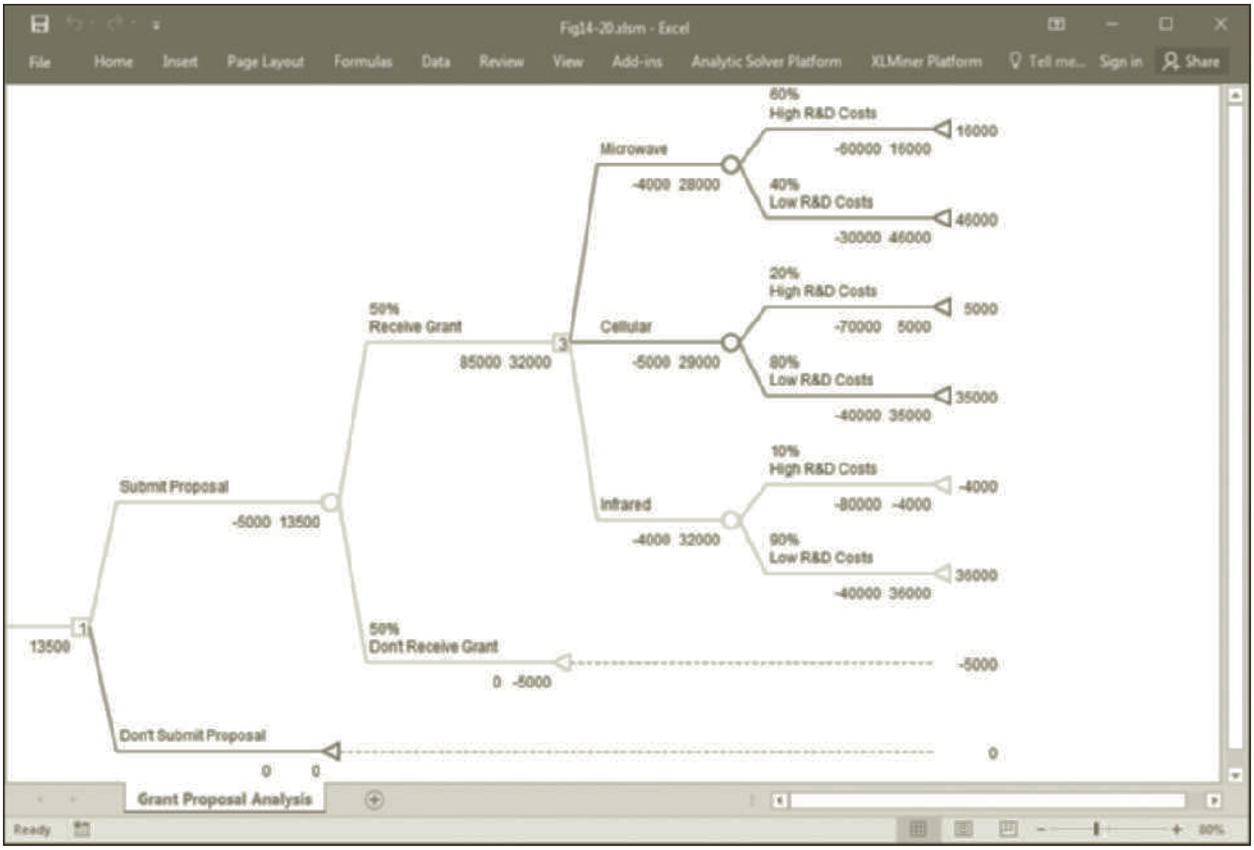
	Possible R&D Costs			
	Best Case		Worst Case	
	Cost	Prob.	Cost	Prob.
Microwave	$30,000	0.4	$60,000	0.6
Cellular	$40,000	0.8	$70,000	0.2
Infrared	$40,000	0.9	$80,000	0.1

Steve needs to synthesize all the factors in this problem to decide whether or not to submit a grant proposal to OSHA.

14.11.1 A MULTISTAGE DECISION TREE

The immediate decision in this example problem is whether or not to submit a grant proposal. To make this decision, Steve must also consider the technology selection decision that he will face if he receives the grant. So, this is a multistage decision problem. Figure 14.20 (and the file Fig14-20.xlsm that accompanies this book) shows the decision tree representation of this problem.

FIGURE 14.20 *Multistage decision tree for COM-TECH's grant proposal problem*



This decision tree clearly shows that the first decision Steve faces is whether or not to submit a proposal, and that submitting the proposal will cost $5,000. If a proposal is submitted, we then encounter an event node showing a 0.5 probability of receiving the grant (and a payoff of $85,000), and a 0.5 probability of not receiving the grant (leading to a net loss of $5,000). If the grant is received, we then encounter a decision about which technology to pursue. Each of the three technology options has an event node representing the best-case (lowest) and worst-case (highest) R&D costs that might be incurred. The final (terminal) payoffs associated with each set of decisions and outcomes are listed next to each terminal node. For example, if Steve submits a proposal, receives the grant, employs cellular technology, and encounters low R&D costs, he will receive a net payoff of $35,000.

According to this decision tree, Steve should submit a proposal because the expected value of this decision is $13,500 and the expected value of not submitting a proposal is $0. The decision tree also indicates that if Steve receives the grant, he should pursue the infrared communications technology because the expected value of this decision ($32,000) is larger than the expected values for the other technologies.

In Figure 14.20, note that the probabilities on the branches at any event node must always sum to 1 because these branches represent all the events that could occur. The R&D costs that would actually occur using a given technology could assume an infinite number of values. Some might argue that these costs could be modeled more accurately by some continuous random variable. However, our aim is to estimate the expected value of this random variable. Most decision makers probably would find it easier to assign subjective probabilities to a small, discrete set of representative outcomes for a variable such as R&D costs rather than try to identify an appropriate probability distribution for this variable.

14.11.2 DEVELOPING A RISK PROFILE

When using decision trees to analyze one-time decision problems, it is particularly helpful to develop a risk profile to make sure the decision maker understands all the possible outcomes that might occur. A **risk profile** is simply a graph or tree that shows the chances associated with possible outcomes. Figure 14.21 shows the risk profile associated with not submitting the proposal and that of the optimal EMV decision-making strategy (submitting the proposal and using infrared technology) identified from Figure 14.20.

FIGURE 14.21

A risk profile for the alternatives of submitting or not submitting the proposal

From Figure 14.21, it is clear that if the proposal is not submitted, the payoff will be $0. If the proposal is submitted, there is a 0.50 chance of not receiving the grant and incurring a loss of $5,000. If the proposal is submitted, there is a 0.05 chance ($0.5 \times 0.1 = 0.05$) of receiving the grant but incurring high R&D costs with the infrared technology and suffering a $4,000 loss. Finally, if the proposal is submitted, there is a 0.45 chance ($0.5 \times 0.9 = 0.45$) of enjoying low R&D costs with the infrared technology and making a $36,000 profit.

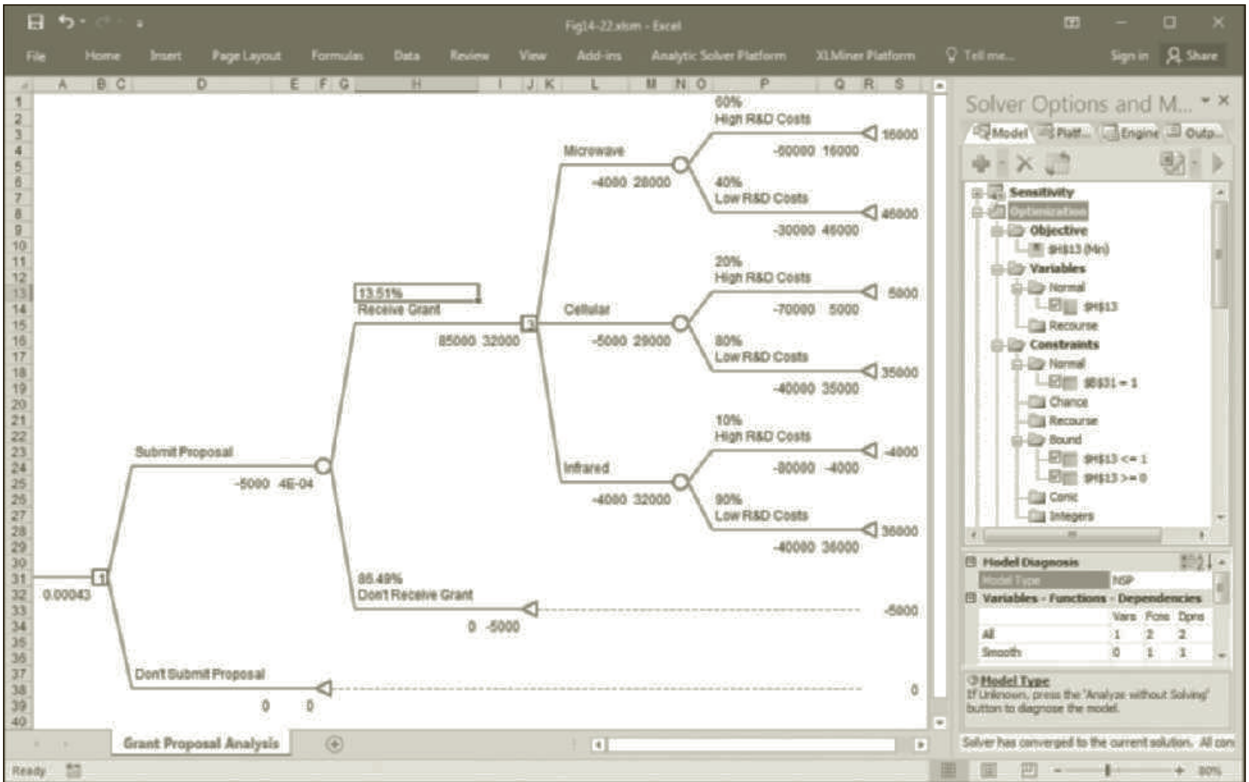
A risk profile is an effective tool for breaking an EMV into its component parts and communicating information about the actual outcomes that can occur as the result of various decisions. By looking at Figure 14.21, a decision maker could reasonably decide that the risks (or chances) of losing money if a proposal is submitted are not worth the potential benefit to be gained if the proposal is accepted and low R&D costs occur. These risks would not be apparent if the decision maker was provided only with information about the EMV of each decision.

14.12 Sensitivity Analysis

Before implementing the decision to submit a grant proposal as suggested by the previous analysis, Steve would be wise to consider how sensitive the recommended decision is to changes in values in the decision tree. For example, Steve estimated that a 50–50 chance exists that he will receive the grant if he submits a proposal. But what if that probability assessment is wrong? What if only a 30%, 20%, or 10% chance exists of receiving the grant? Should he still submit the proposal?

Using a decision tree implemented in a spreadsheet, it is fairly easy to determine how much any of the values in the decision tree can change before the indicated decision would change. For example, Figure 14.22 (and the file Fig14-22.xlsm that accompanies this book) shows how we can use optimization to determine how small the

FIGURE 14.22 *Using optimization to determine the sensitivity of a decision to changes in probabilities*



Key Cell Formulas

Cell	Formula	Copied to
H31	=1−H13	--

probability of receiving the grant would need to be before it would no longer be wise to submit the grant proposal (according to the EMV decision rule).

In this spreadsheet, we are using cell H13 (the probability of receiving the grant) as both our objective cell and our variable cell. In cell H31, we entered the following formula to compute the probability of not receiving the grant:

Formula for cell H31: =1 − H13

Minimizing the value in cell H13 (using Analytic Solver Platform's GRG nonlinear engine) while constraining the value of B31 to equal 1 determines the probability of receiving the grant that makes the EMV of submitting the grant equal to zero. The resulting probability (*i.e.*, approximately 0.1351) gives the decision maker some idea of how sensitive the decision is to changes in the value of cell H13.

If the EMV of submitting the grant is zero, most decision makers would probably not want to submit the grant proposal. Indeed, even with an EMV of $13,500 (as shown in Figure 14.20), some decision makers would still not want to submit the grant proposal because there is still a risk that the proposal would be turned down and a $5,000 loss incurred. As mentioned earlier, the EMV decision rule is most appropriately applied when we face a decision that will be made repeatedly and the results of bad outcomes can be balanced or averaged with good outcomes.

14.12.1 TORNADO CHARTS

As shown in the previous section, optimization can be used to determine the amount by which almost any value in a decision tree can be changed before a recommended decision (based on EMV) would change. However, given the number of probability and financial estimates used as inputs to a decision tree, it is often helpful to use tornado charts to identify the inputs that, if changed, have the greatest impact on the EMV. This helps to identify the areas where sensitivity analysis is most important and prioritize where time and resources should be applied in refining probability and financial estimates represented in the decision tree.

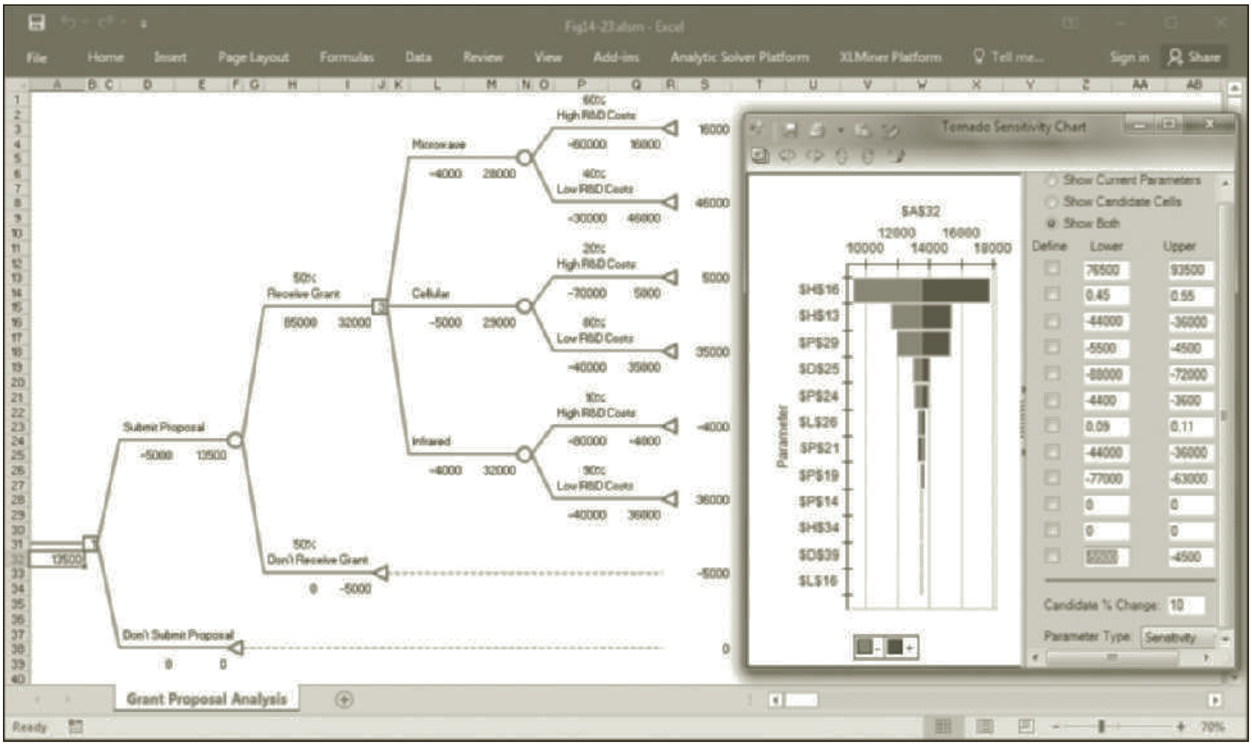
Analytic Solver Platform provides a simple way to create tornado charts. This tool allows you to specify an output cell of interest, and then it automatically identifies the input cells that have the greatest impact on the value of the output cell. (The tornado chart refers to the identified input cells as **candidate cells**.) The tornado chart tool incrementally changes the value of each input cell from its base case value (while holding the other input cells constant) within a specified percentage range (±10% by default) and records the effect of each change on the output cell's value.

As an example, suppose we are interested in identifying the input cells in COMTECH's decision tree shown in Figure 14.23 (and the file Fig14-23.xlsm that accompanies this book) that have the greatest influence on the EMV shown in cell A32. The tornado chart in Figure 14.23 summarizes the impact on the decision tree's EMV of each input cell being set at +10% and −10% of its original (base case) value. The input cell with the largest impact on the EMV's range is shown first, the input cell with the next largest impact is shown second, and so on, creating the tornado shaped appearance in the chart. To create the tornado chart shown in Figure 14.23, follow these steps:

1. Select the output cell of interest (cell A32).
2. On the Analytic Solver Platform tab, click Parameters, Identify.

At the top of the tornado chart, we readily see that cell H16 (the grant award amount of $85,000) has the largest impact on the EMV as it is adjusted from −10% to +10% of

FIGURE 14.23 *An automatic tornado chart for the COM-TECH decision problem*

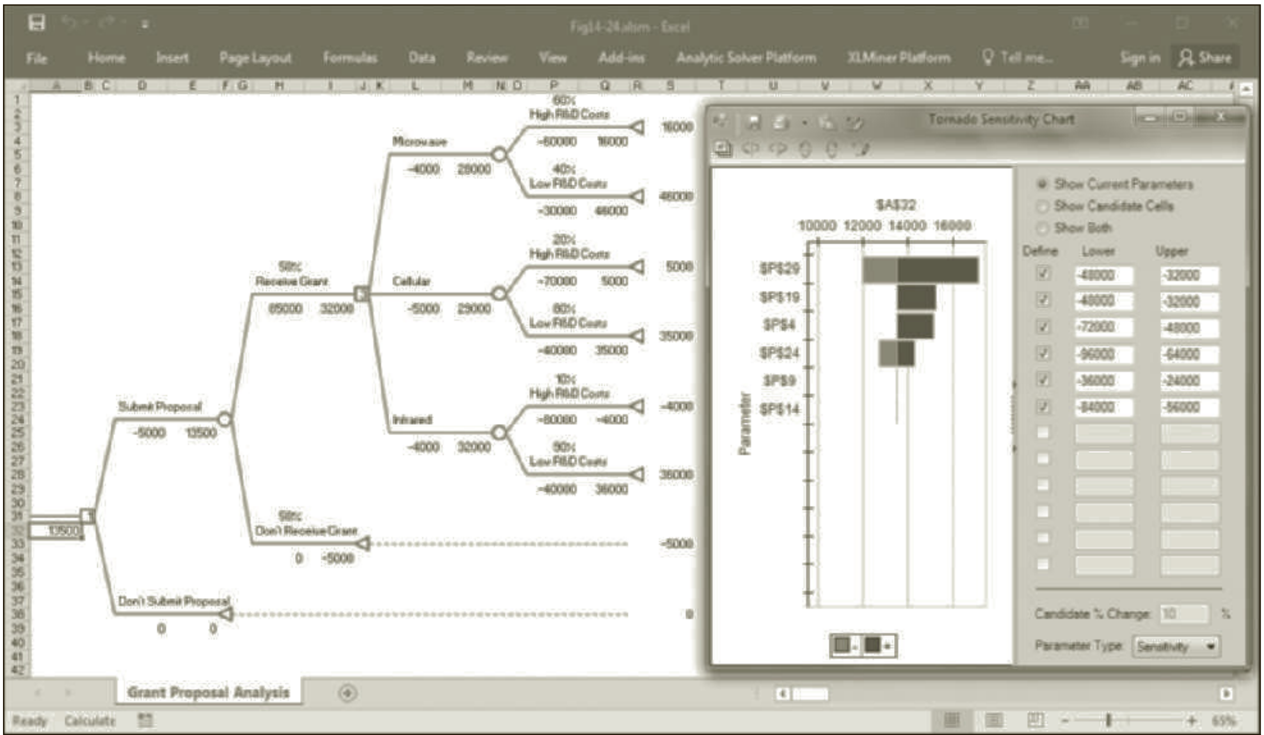


its original value. Cell H13 (the probability of receiving the grant) has the next largest influence, followed by cell P29 (the best case R&D costs for infrared technology), and so on. Thus, the tornado chart quickly gives us a good sense for which input cells have the most significant impact on the EMV and the associated recommended decision.

In Figure 14.23, note that Analytic Solver identified ***all*** the input cells that effect the output cell and summarized the most significant ones in the tornado chart. However, it is unlikely that the amount of the grant award (the "most significant" input cell) is actually going to differ from the stated value of $85,000. As a result, instead of having Analytic Solver automatically identify the input cells that, if changed, have the most significant impact on the output cell, we might want to only consider the input cells whose values are most uncertain and/or subject to change. Fortunately, it is possible to specify the input cells of interest and only include those in the tornado chart. This is done by defining those cells as sensitivity parameter cells using the PsiSenParam(L, U, B) function, where L and U represent, respectively, lower and upper limits on the range of possible values for each cell and B is the base case value for the cell.

For example, suppose we are only interested in considering the output cell's (cell A32) sensitivity to changes of up to 20% in the best case and worst case R&D costs for each of the possible technology choices (i.e., cells P4, P9, P14, P19, P24, and P29). We would first replace the values in each of the cells representing R&D costs with PsiSenParam( ) functions that allow those cells to be varied within plus or minus 20% of their original values as shown in Figure 14.24 (and the file Fig14-24 that accompanies this book). If you then create a new tornado chart and select the Show Current Parameters option, the chart shows the change in the output cell as each of the parameter cells (i.e., those cells containing PsiSenParam( ) functions) are varied from their lower limits to their upper limits.

FIGURE 14.24 *A customized tornado chart for the COM-TECH decision problem*



Key Cell Formulas

Cell	Formula	Copied to
P4	=PsiSenParam(–60000*1.2, –60000*0.8, –60000)	--
P9	=PsiSenParam(–30000*1.2, –30000*0.8, –30000)	--
P14	=PsiSenParam(–70000*1.2, –70000*0.8, –70000)	--
P19	=PsiSenParam(–40000*1.2, –40000*0.8, –40000)	--
P24	=PsiSenParam(–80000*1.2, –80000*0.8, –80000)	--
P29	=PsiSenParam(–40000*1.2, –40000*0.8, –40000)	--

Software Note

The results plotted in a tornado chart can consist of two different types of input cells: those that Analytic Solver identifies automatically (referred to as candidate cells), and those that we manually identify as sensitivity parameter (via PsiSenParam( ) functions). The option labeled "Candidate % Change" in the lower section of the tornado chart (see Figure 14.24) only applies to the candidate cells – it will ***not*** affect the cells containing PsiSenParam( ) functions. As a result, it is possible for a tornado chart to simultaneously display results where, for instance, candidate cells are varied from ±10% from their base case values and sensitivity parameters are varied a different amount, such as ±20% from their base case values.

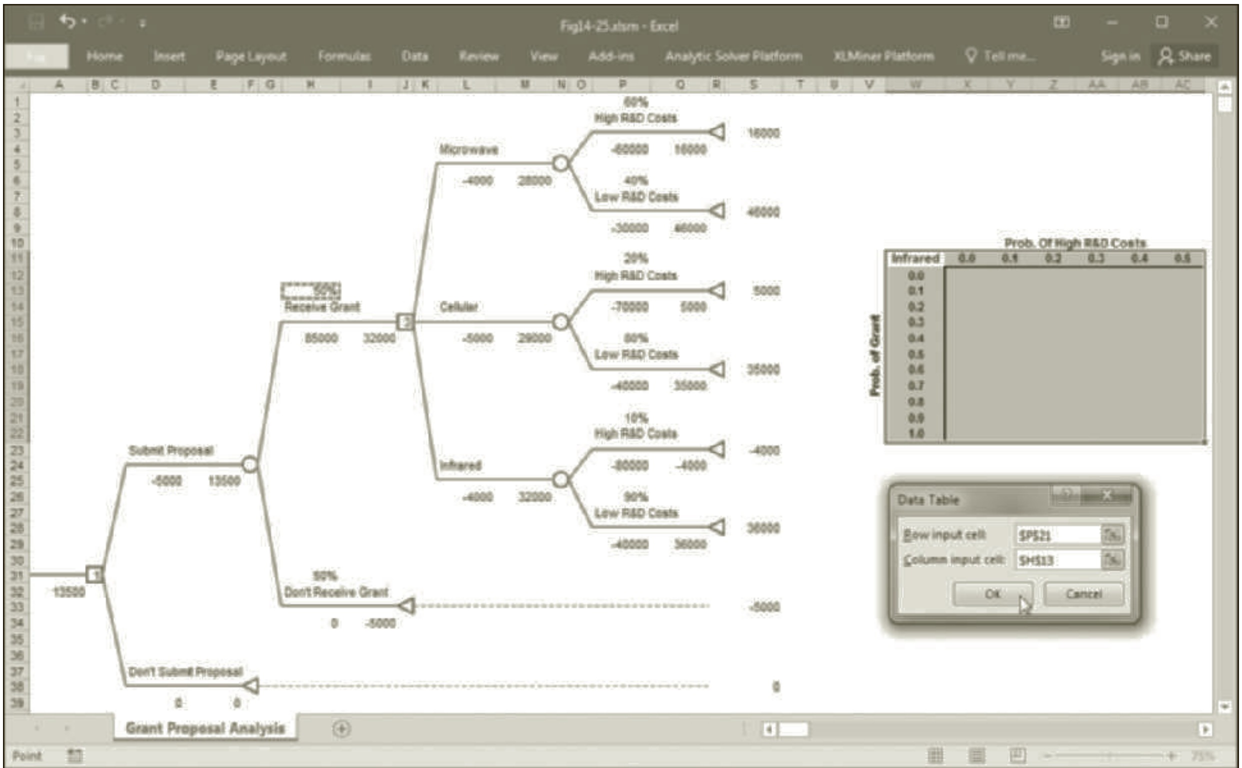
14.12.2 STRATEGY TABLES

A **strategy table** is another sensitivity analysis technique that allows a decision maker to analyze how the optimal decision strategy changes in response to two simultaneous changes in probability estimates. For example, the optimal strategy in Figure 14.20 is to submit the proposal and use infrared technology. However, suppose there is uncertainty about the probability of receiving the grant and the probability of encountering high R&D costs while carrying out the research proposal. Specifically, suppose the decision maker wants to see how the optimal strategy changes as the probability of receiving the grant varies from 0.0 to 1.0 and the probability of encountering high infrared R&D costs varies from 0.0 to 0.5. As shown in Figure 14.25 (and the file Fig14-25.xlsm that accompanies this book), a two-way data table can be used to analyze this situation.

In Figure 14.25, cells W12 through W22 represent different probabilities of receiving the grant. Using the Data Table command, we will instruct Excel to plug each of these values into cell H13, representing the probability of receiving the grant. The following formula was entered in cell H31 to calculate the complementary probability of not receiving the grant.

Formula for cell H31: $=1 - \text{H13}$

FIGURE 14.25 *Setting up a strategy table for the COM-TECH decision problem*



Key Cell Formulas

Cell	Formula	Copied to
H31	=1−H13	--
P26	=1−P21	--
W11	=IF(B31=1,CHOOSE(J15,"Microwave","Cellular","Infrared"),"Don't")	--

Cells X11 through AC11 represent different probabilities of encountering high R&D costs. Using the Data Table command, we will instruct Excel to plug each of these values into cell P21, representing the probability of receiving the grant. The following formula was entered in cell P26 to calculate the complementary probability of not receiving the grant.

Formula for cell P26: =1 − P21

As these different probabilities are changed, the spreadsheet will be recalculated and the value returned by the formula in cell W11 will be recorded in the appropriate cell in the data table.

Formula for cell W11: =IF(B31=1,CHOOSE (J15,"Microwave","Cellular","Infrared"),"Don't")

This formula first inspects the value of cell B31; which equals 1 if the EMV of submitting the proposal is positive. Thus, if B31 is equal to 1, the formula then returns (chooses) the label "Microwave", "Cellular", or "Infrared" depending on whether the value in cell J15 is one, two, or three, respectively. Otherwise, the previous formula returns the label "Don't" indicating that the proposal should not be submitted. The results of executing the Data Table command are shown in Figure 14.26.

FIGURE 14.26 *Completed strategy table for the COM-TECH decision problem*

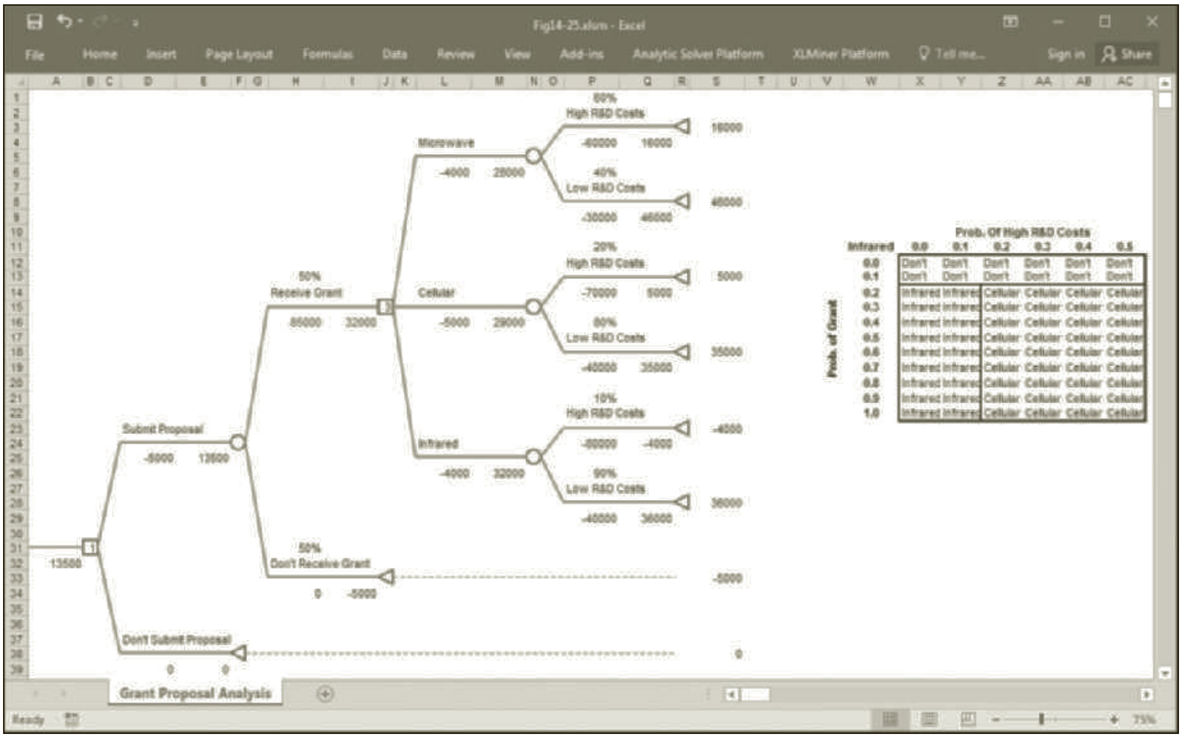
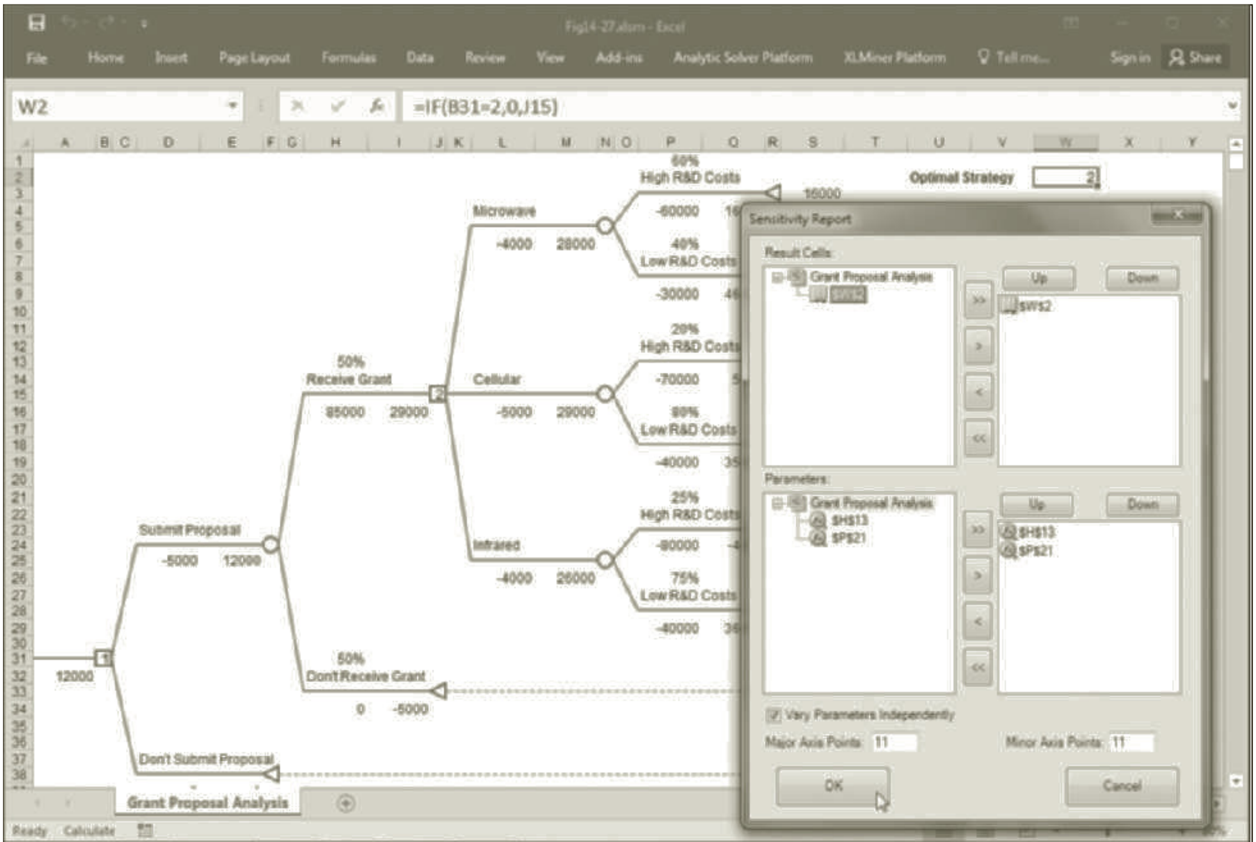


Figure 14.26 summarizes the optimal strategy for the various probability combinations. For instance, if the probability of receiving the grant is 0.10 or less, the company should not submit a proposal. Note that cell Y17 corresponds to the base case solution shown earlier in Figure 14.20. The strategy table makes it clear that this solution is relatively insensitive to changes in the probability of receiving the grant. However, if the probability of encountering high infrared R&D costs increases, the preferred strategy quickly switches to the cellular technology alternative. Thus, the decision maker might want to give closer attention to the risks of encountering high infrared R&D costs before implementing this strategy.

14.12.3 STRATEGY CHARTS

Similar to strategy tables, a **strategy chart** is a technique to graphically show how the optimal decision strategy changes in response to two simultaneous changes in probability estimates. Again, suppose there is uncertainty about the probability of receiving the grant and the probability of encountering high R&D costs while carrying out the research proposal. Specifically, assume the decision maker wants to see how the optimal strategy changes as the probability of receiving the grant varies from 0.0 to 1.0 and the probability of encountering high infrared R&D costs varies from 0.0 to 0.5. To create a strategy chart for this situation using Analytic Solver Platform, we first change cells P21 and H13 in Figure 14.27 (and

FIGURE 14.27 *Setting up a strategy chart for the COM-TECH decision problem*



Key Cell Formulas

Cell	Formula	Copied to
H13	=PsiSenParam(0,1)	--
H31	=1−H13	--
P21	=PsiSenParam(0,0.5)	--
P26	=1−P21	--
W2	=IF(B31=2,0,J15)	--

the file Fig14-27.xlsm that accompanies this book) to sensitivity parameter cells as follows,

Formula for cell P21: =PsiSenParam(0,0.5)
Formula for cell H13: =PsiSenParam(0,1)

We must also create an output cell that assigns a unique numeric value to each decision strategy. This is done in cell W2 as follows,

Formula for cell W2: =IF(B31=2,0,J15)

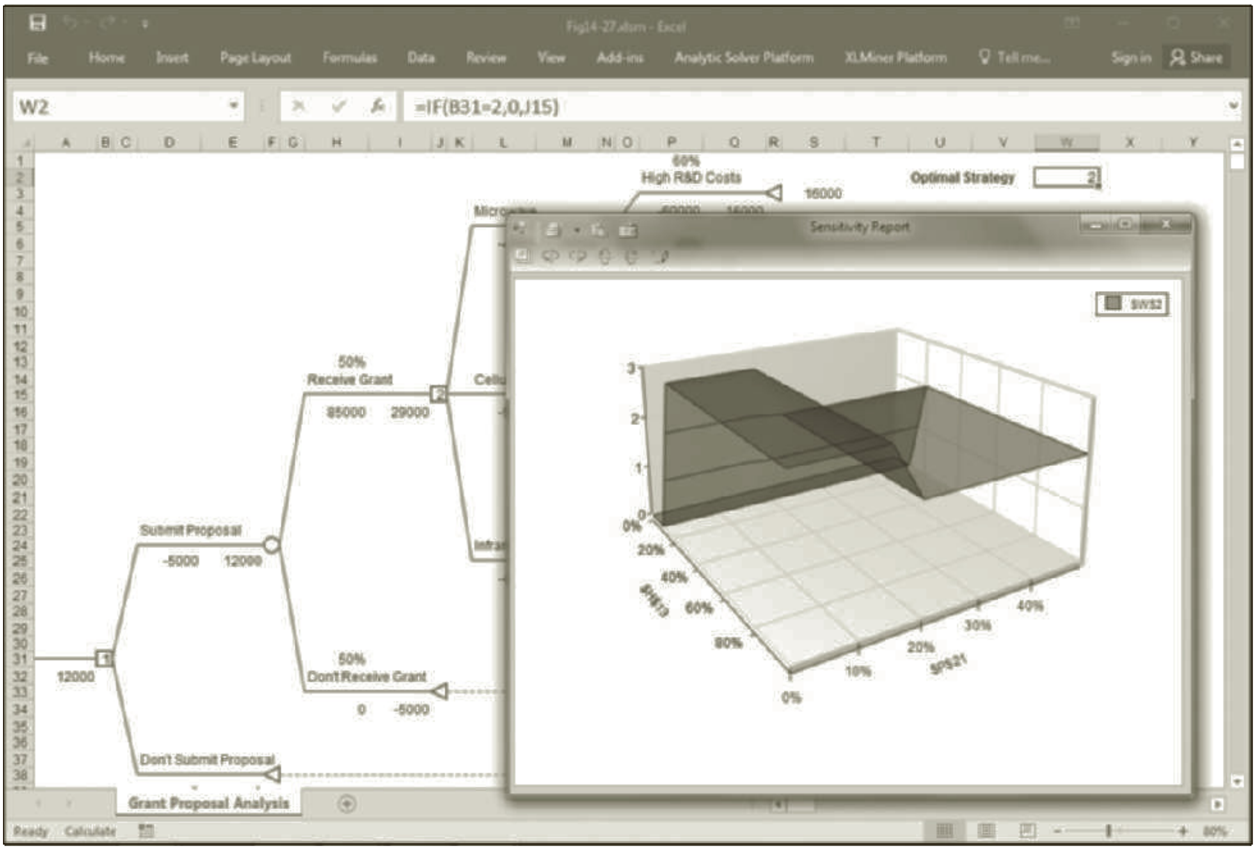
Note that the formula in cell W2 returns the value 0 if the optimal strategy is to not submit a proposal (i.e., if B31 = 2) and otherwise returns the value of 1, 2, or 3 (from cell J15) if the optimal decision is to submit a proposal and use microwave, cellular, or infrared technology, respectively.

We can then create a strategy chart for our problem as follows:

1. Select cell W2.
2. On the Analytic Solver ribbon, click Charts, Sensitivity Analysis, Parameter Analysis.
3. Complete the Sensitivity Report dialog box as shown Figure 14.27.
4. Click OK.

Analytic Solver Platform then creates the 3D area chart shown in Figure 14.28. This chart shows how the optimal decision strategy changes for different combinations of probability values in cells P21 and H13. Note that the decision chart provides a graphical summary of the results in the strategy table shown in Figure 14.26.

FIGURE 14.27 *Setting up a strategy chart for the COM-TECH decision problem*



14.13 Using Sample Information in Decision Making

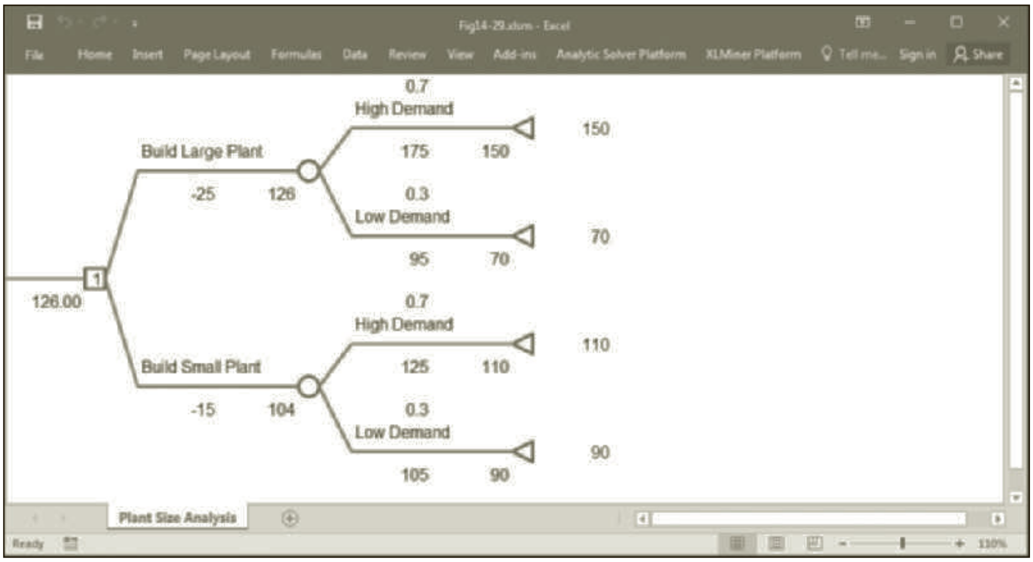
In many decision problems, we have the opportunity to obtain additional information about the decision before we actually make the decision. For example, in the Magnolia Inns decision problem, the company could have hired a consultant to study the economic, environmental, and political issues surrounding the site selection process and predict which site will be selected for the new airport by the planning council. This information might help Magnolia Inns make a better (or more informed) decision. The potential for using this type of additional sample information in decision making raises a number of interesting issues that are illustrated using the following example.

> Colonial Motors (CM) is trying to determine what size of manufacturing plant to build for a new car it is developing. Only two plant sizes are under consideration: large and small. The cost of constructing a large plant is $25 million and the cost of constructing a small plant is $15 million. CM believes there is a 70% chance that the demand for this new car will be high and a 30% chance that it will be low. The following table summarizes the payoffs (in millions of dollars) the company expects to receive for each factory size and demand combination (not counting the cost of the factory).

	Demand	
Factory Size	**High**	**Low**
Large	$175	$95
Small	$125	$105

A decision tree for this problem is shown in Figure 14.29 (and in the file Fig14-29.xlsm that accompanies this book). The decision tree indicates that the optimal decision is to build the large plant and that this alternative has an EMV of $126 million.

FIGURE 14.29 *Decision tree for the CM plant size problem*



Now suppose that before making the plant size decision, CM conducts a survey to assess consumer attitudes about the new car. For simplicity, we will assume that the results of this survey indicate either a favorable or unfavorable attitude about the new car. A revised decision tree for this problem is shown in Figure 14.30 (and in the file Fig14-30.xlsm that accompanies this book).

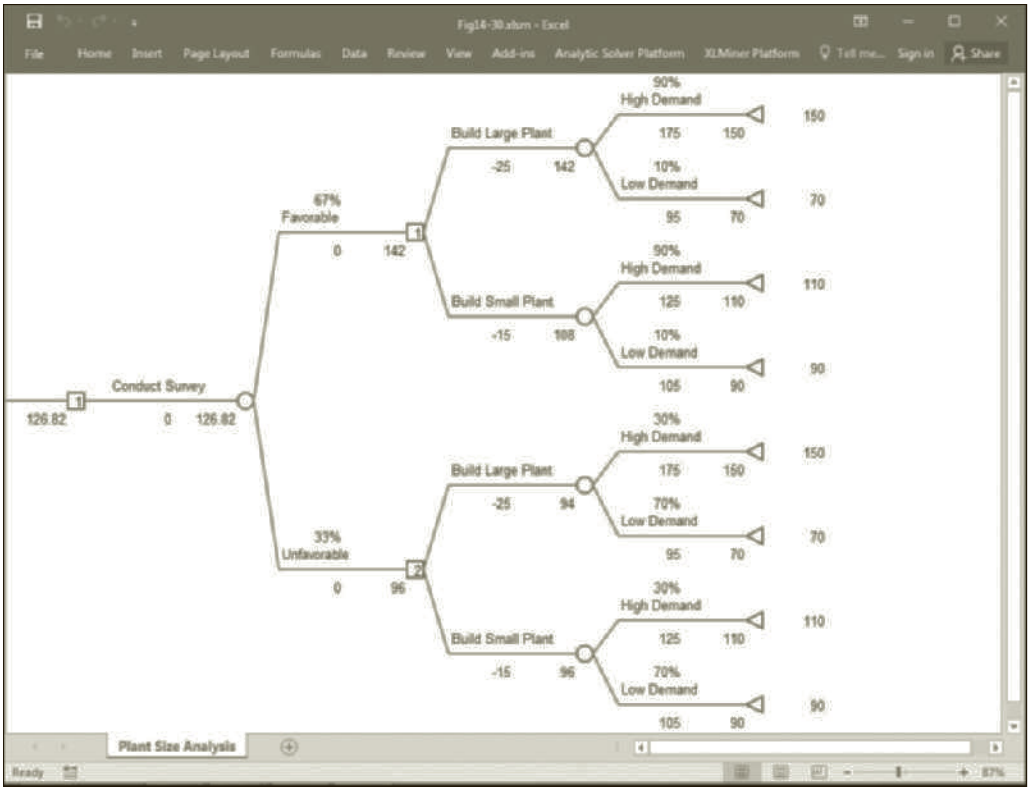


FIGURE 14.30

Decision tree if consumer survey is conducted before CM makes a plant size decision

The decision tree in Figure 14.30 begins with a decision node with a single branch representing the decision to conduct the market survey. For now, assume that this survey can be done at no cost. An event node follows, corresponding to the outcome of the market survey, which can indicate either favorable or unfavorable attitudes about the new car. We assume that CM believes that the probability of a favorable response is 0.67 and the probability of an unfavorable response is 0.33.

14.13.1 CONDITIONAL PROBABILITIES

After the survey results are known, the decision nodes in the tree indicate that a decision must be made about whether to build a large plant or a small plant. Following each decision branch, event nodes occur with branches representing the market demands for the car that could occur. Four event nodes represent the market demand that might occur for this car. However, the probabilities we assign to the branches of these nodes are likely to differ depending on the results of the market survey.

Earlier, we indicated that CM believed a 0.70 probability exists that demand for the new car will be high, expressed mathematically as:

$$P(\text{high demand}) = 0.7$$

In this formula $P(A) = X$ is read, "the probability of A is X." If the market survey indicates that consumers have a favorable impression of the new car, this will raise expectations that demand will be high for the car. Thus, given a favorable survey response, we might increase the probability assessment for a high-market demand to 0.90. This is expressed mathematically as the following *conditional* probability:

$$P(\text{high demand} \mid \text{favorable response}) = 0.90$$

In this formula $P(A \mid B) = X$ is read, "the probability of A given B is X."

As noted earlier, the probabilities on the branches at any event node must always sum to 1. If the favorable survey response increases the probability assessment of a high demand occurring, it must decrease the probability assessment of a low demand given this survey result. Thus, the probability of a low demand given a favorable response on the survey is:

$$\begin{aligned} P(\text{low demand} \mid \text{favorable response}) &= 1 - P(\text{high demand} \mid \text{favorable response}) \\ &= 1 - 0.90 = 0.10 \end{aligned}$$

These conditional probabilities are shown in Figure 14.30 on the first four event branches representing high and low demands given a favorable survey response.

If the market survey indicates consumers have an unfavorable response to the new car, this will lower expectations for high-market demand. Thus, given an unfavorable survey response, we might reduce the probability assessment of a high-market demand to 0.30:

$$P(\text{high demand} \mid \text{unfavorable response}) = 0.30$$

We must also revise the probability assessment for a low-market demand given an unfavorable market response as:

$$\begin{aligned} P(\text{low demand} \mid \text{unfavorable response}) &= 1 - P(\text{high demand} \mid \text{unfavorable response}) \\ &= 1 - 0.3 = 0.70 \end{aligned}$$

These conditional probabilities are shown on the last four demand branches in Figure 14.30. Later, we will discuss a more objective method for determining these types of conditional probabilities.

14.13.2 THE EXPECTED VALUE OF SAMPLE INFORMATION

The additional information made available by the market survey allows us to make more precise estimates of the probabilities associated with the uncertain market demand. This, in turn, allows us to make more precise decisions. For example, Figure 14.30 indicates that if the survey results are favorable, CM should build a large plant; and if the survey results are unfavorable, it should build a small plant. The expected value of this decision-making strategy is \$126.82 million, assuming that the survey can be done at no cost—which is unlikely. So, how much should CM be willing to pay to perform this survey? The answer to this question is provided by the expected value of sample information (EVSI), which is defined as:

$$EVSI = \begin{pmatrix} \text{Expected value of the best} \\ \text{decision with sample infor-} \\ \text{mation (obtained at no cost)} \end{pmatrix} - \begin{pmatrix} \text{Expected value of the best} \\ \text{decision without sample} \\ \text{information} \end{pmatrix}$$

The EVSI represents the *maximum* amount we should be willing to pay to obtain sample information. From Figure 14.30, we know that the expected value of the best decision *with* sample information for our example problem is \$126.82 million. From Figure 14.29, we know that the expected value of the best decision *without* sample information is \$126 million. So for our example problem, the EVSI is determined as:

$$\text{EVSI} = \$126.82 \text{ million} - \$126 \text{ million} = \$0.82 \text{ million}$$

Thus, CM should be willing to spend up to \$820,000 to perform the market survey.

14.14 Computing Conditional Probabilities

In our example problem, we assumed that the values of the conditional probabilities were assigned subjectively by the decision makers at CM. However, a company often has data available from which it can compute these probabilities. We will illustrate this process for the CM example. To simplify our notation, we will use the following abbreviations:

$$\begin{aligned} H &= \text{high demand} \\ L &= \text{low demand} \\ F &= \text{favorable response} \\ U &= \text{unfavorable response} \end{aligned}$$

To complete the decision tree in Figure 14.30, we determined values for the following six probabilities:

$$\begin{gathered} P(F) \\ P(U) \\ P(H|F) \\ P(L|F) \\ P(H|U) \\ P(L|U) \end{gathered}$$

Assuming that CM has been in the auto business for some time, it undoubtedly has performed other market surveys prior to introducing other new models. Some of these models probably achieved high consumer demand, whereas others achieved only low demand. Thus, CM can use historical data to construct the joint probability table shown at the top of Figure 14.31 (and in the file Fig14-31.xlsm that accompanies this book).

The value in cell B4 indicates that of all the new car models CM developed and performed market surveys on, 60% received a favorable survey response and subsequently enjoyed high demand. This is expressed mathematically as:

$$P(F \cap H) = 0.60$$

In this formula $P(A \cap B) = X$ is read, "the probability of A *and* B is X." Similarly, in the joint probability table we see that:

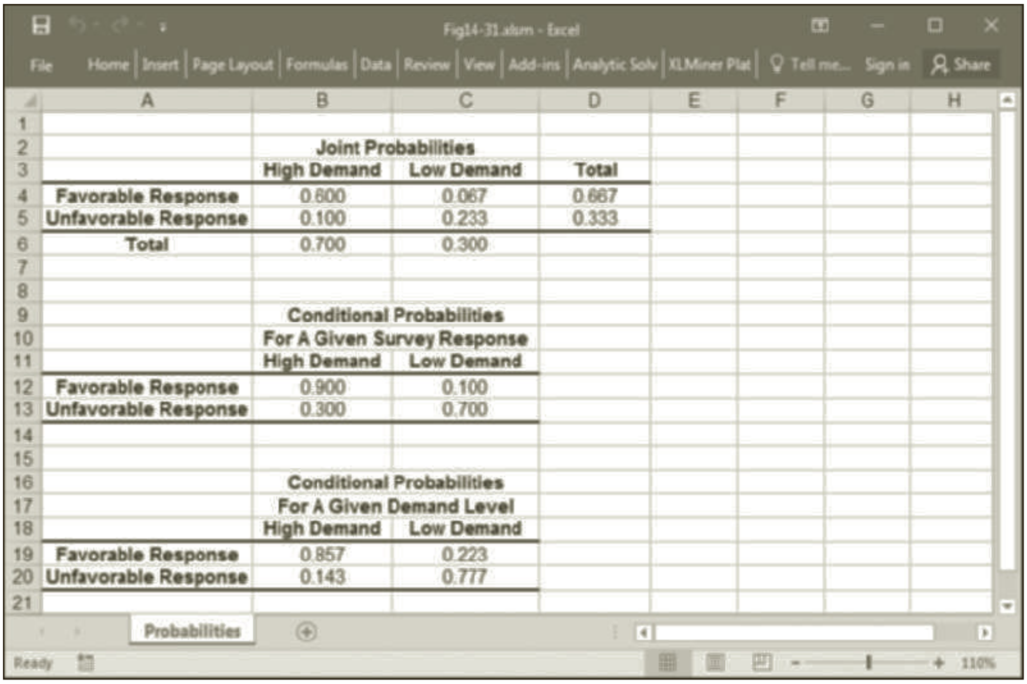
$$\begin{aligned} P(F \cap L) &= 0.067 \\ P(U \cap H) &= 0.10 \\ P(U \cap L) &= 0.233 \end{aligned}$$

The column totals in cells B6 and C6 represent, respectively, the estimated probabilities of high and low demands as:

$$\begin{aligned} P(H) &= 0.70 \\ P(L) &= 0.30 \end{aligned}$$

FIGURE 14.31

The calculation of conditional probabilities for the CM decision problem



	A	B	C	D
2		Joint Probabilities		
3		High Demand	Low Demand	Total
4	Favorable Response	0.600	0.067	0.667
5	Unfavorable Response	0.100	0.233	0.333
6	Total	0.700	0.300	
9		Conditional Probabilities		
10		For A Given Survey Response		
11		High Demand	Low Demand	
12	Favorable Response	0.900	0.100	
13	Unfavorable Response	0.300	0.700	
16		Conditional Probabilities		
17		For A Given Demand Level		
18		High Demand	Low Demand	
19	Favorable Response	0.857	0.223	
20	Unfavorable Response	0.143	0.777	

The row totals in cells D4 and D5 represent, respectively, the estimated probabilities of a favorable and unfavorable response. These values correspond to the first two of the six probability values listed earlier; that is:

$$P(F) = 0.667$$
$$P(U) = 0.333$$

With these values, we are now ready to compute the necessary conditional probabilities. One general definition of a conditional probability is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

We can use this definition, along with the values in the joint probability table, to compute the conditional probabilities required for Figure 14.30 as:

$$P(H|F) = \frac{P(H \cap F)}{P(F)} = \frac{0.60}{0.667} = 0.90$$

$$P(L|F) = \frac{P(L \cap F)}{P(F)} = \frac{0.067}{0.667} = 0.10$$

$$P(H|U) = \frac{P(H \cap U)}{P(U)} = \frac{0.10}{0.333} = 0.30$$

$$P(L|U) = \frac{P(L \cap U)}{P(U)} = \frac{0.233}{0.333} = 0.70$$

We can calculate these conditional probabilities of the demand levels for a given survey response in the spreadsheet. This is done in the second table in Figure 14.31 using the following formula:

Formula for cell B12: =B4/$D4

(Copy to B12 through C13.)

Although not required for Figure 14.30, we can also compute the conditional probabilities of the survey responses for a given level of demand as:

$$P(F|H) = \frac{P(H \cap F)}{P(H)} = \frac{0.60}{0.70} = 0.857$$

$$P(U|H) = \frac{P(H \cap U)}{P(H)} = \frac{0.10}{0.70} = 0.143$$

$$P(F|L) = \frac{P(L \cap F)}{P(L)} = \frac{0.067}{0.30} = 0.223$$

$$P(U|L) = \frac{P(L \cap U)}{P(L)} = \frac{0.233}{0.30} = 0.777$$

The third table in Figure 14.31 calculates conditional probabilities of the survey responses for a given level of demand using the following formula:

Formula for cell B19: =B4/B$6

(Copy to B20 through C20.)

14.14.1 BAYES's THEOREM

Bayes's Theorem provides another definition of conditional probability that is sometimes useful. This definition is:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})}$$

In this formula, A and B represent any two events, and $\overline{A}$ is the complement of A. To see how this formula might be used, suppose that we want to determine $P(H|F)$ but we do not have access to the joint probability table in Figure 14.31. According to Bayes's Theorem, we know that:

$$P(H|F) = \frac{P(F|H)P(H)}{P(F|H)P(H) + P(F|L)P(L)}$$

If we know the values for the various quantities on the RHS of this equation, we can compute $P(H|F)$ as in the following example:

$$P(H|F) = \frac{P(F|H)P(H)}{P(F|H)P(H) + P(F|L)P(L)} = \frac{(0.857)(0.70)}{(0.857)(0.70) + (0.223)(0.30)} = 0.90$$

This result is consistent with the value of $P(H|F)$ shown in cell B12 in Figure 14.31.

14.15 Utility Theory

Although the EMV decision rule is widely used, sometimes the decision alternative with the highest EMV is not the most desirable or most preferred alternative by the decision maker. For example, suppose that we could buy either of the two companies listed in the following payoff table for exactly the same price:

	State of Nature			
Company	**1**	**2**	**EMV**	
A	150,000	−30,000	60,000	← maximum
B	70,000	40,000	55,000	
Probability	0.5	0.5		

The payoff values listed in this table represent the annual profits expected from this business. Thus, in any year, a 50% chance exists that company A will generate a profit of \$150,000 and a 50% chance that it will generate a loss of \$30,000. On the other hand, in each year, a 50% chance exists that company B will generate a profit of \$70,000 and a 50% chance that it will generate a profit of \$40,000.

According to the EMV decision rule, we should buy company A because it has the highest EMV. However, company A represents a far more risky investment than company B. Although company A would generate the highest EMV over the long run, we might not have the financial resources to withstand the potential losses of \$30,000 per year that could occur in the short run with this alternative. With company B, we can be sure of making at least \$40,000 each year. Although company B's EMV over the long run might not be as great as that of company A, for many decision makers, this is more than offset by the increased peace of mind associated with company B's relatively stable profit level. However, other decision makers might be willing to accept the greater risk associated with company A in hopes of achieving the higher potential payoffs this alternative provides.

As this example illustrates, the EMVs of different decision alternatives do not necessarily reflect the relative attractiveness of the alternatives to a particular decision maker. **Utility theory** provides a way to incorporate the decision maker's attitudes and preferences toward risk and return in the decision-analysis process so that the most desirable decision alternative is identified.

14.15.1 UTILITY FUNCTIONS

Utility theory assumes that every decision maker uses a **utility function** that translates each of the possible payoffs in a decision problem into a nonmonetary measure known as a utility. The **utility** of a payoff represents the total worth, value, or desirability of the outcome of a decision alternative to the decision maker. For convenience, we will begin by representing utilities on a scale from 0 to 1, where 0 represents the least value and 1 represents the most.

Different decision makers have different attitudes and preferences toward risk and return. Those who are "risk neutral" tend to make decisions using the maximum EMV decision rule. However, some decision makers are risk avoiders (or "risk averse"), and others look for risk (or are "risk seekers"). The utility functions typically associated with these three types of decision makers are shown in Figure 14.32.

Figure 14.32 illustrates how the same monetary payoff might produce different levels of utility for three different decision makers. A "risk averse" decision maker assigns the largest relative utility to any payoff but has a diminishing marginal utility for increased

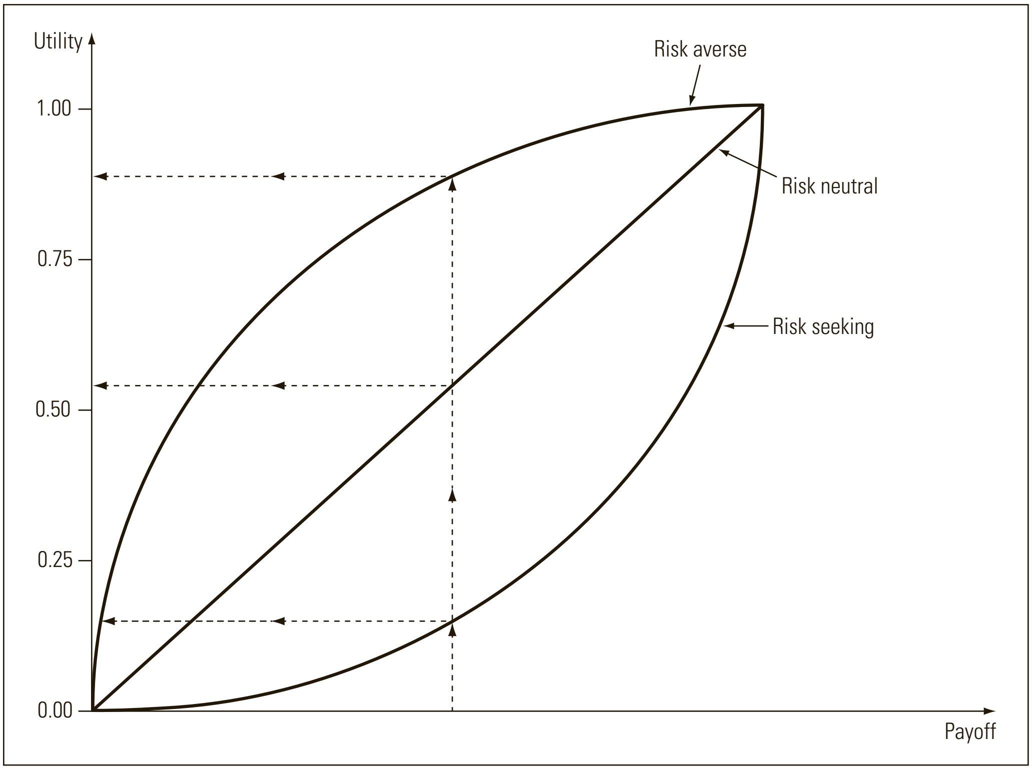


FIGURE 14.32

Three common types of utility functions

payoffs (i.e., every additional dollar in payoff results in smaller increases in utility). The "risk seeking" decision maker assigns the smallest utility to any payoff but has an increasing marginal utility for increased payoffs (i.e., every additional dollar in payoff results in larger increases in utility). The "risk neutral" decision maker (who follows the EMV decision rule) falls in between these two extremes and has a constant marginal utility for increased payoffs (i.e., every additional dollar in payoff results in the same amount of increase in utility). The utility curves in Figure 14.32 are not the only ones that can occur. In general, utility curves can assume virtually any form depending on the preferences of the decision maker.

14.15.2 CONSTRUCTING UTILITY FUNCTIONS

Assuming that decision makers use utility functions (perhaps at a subconscious level) to make decisions, how can we determine what a given decision maker's utility function looks like? One approach involves assigning a utility value of 0 to the worst outcome in a decision problem and a utility value of 1 to the best outcome. All other payoffs are assigned utility values between 0 and 1. (Although it is convenient to use endpoint values of 0 and 1, we can use any values provided that the utility value assigned to the worst payoff is less than the utility value assigned to the best payoff.)

We will let $U(x)$ represent the utility associated with a payoff of $\$x$. Thus, for the decision about whether to buy company A or B, described earlier, we have:

$$U(-30{,}000) = 0$$
$$U(150{,}000) = 1$$

Now suppose that we want to find the utility associated with the payoff of $70,000 in our example. To do this, we must identify the probability p at which the decision maker is indifferent between the following two alternatives:

Alternative 1. Receive $70,000 with certainty.

Alternative 2. Receive $150,000 with probability p and lose $30,000 with probability $(1 - p)$

If $p = 0$, most decision makers would choose alternative 1 because they would prefer to receive a payoff of $70,000 rather than lose $30,000. On the other hand, if $p = 1$, most decision makers would choose alternative 2 because they would prefer to receive a payoff of $150,000 rather than $70,000. So as p increases from 0 to 1, it reaches a point—p^*—at which the decision maker is indifferent between the two alternatives. That is, if $p < p^*$, the decision maker prefers alternative 1, and if $p > p^*$, the decision maker prefers alternative 2. The point of indifference, p^*, varies from one decision maker to another, depending on the decision maker's attitude toward risk and according to his ability to sustain a loss of $30,000.

In our example, suppose that the decision maker is indifferent between alternative 1 and 2 when $p = 0.8$ (so that $p^* = 0.8$). The utility of the $70,000 payoff for this decision maker is computed as:

$$U(70{,}000) = U(150{,}000)p^* + U(-30{,}000)(1 - p^*) = 1p^* + 0(1 - p^*) = p^* = 0.8$$

Notice that when $p = 0.8$, the expected value of alternative 2 is:

$$\$150{,}000 \times 0.8 - \$30{,}000 \times 0.2 = \$114{,}000$$

Because the decision maker is indifferent between a risky decision (alternative 2) that has an EMV of $114,000 and a nonrisky decision (alternative 1) that has a certain payoff of $70,000, this decision maker is "risk averse." That is, the decision maker is willing to accept only $70,000 to avoid the risk associated with a decision that has an EMV of $114,000.

The term **certainty equivalent** refers to the amount of money that is equivalent in a decision maker's mind to a situation that involves uncertainty. For example, $70,000 is the decision maker's certainty equivalent for the uncertain situation represented by alternative 2 when $p = 0.8$. A closely related term, **risk premium**, refers to the EMV that a decision maker is willing to give up (or pay) in order to avoid a risky decision. In our example, the risk premium is $114,000 − $70,000 = $44,000; that is:

$$\text{Risk premium} = \begin{pmatrix}\text{EMV of an} \\ \text{uncertain situation}\end{pmatrix} - \begin{pmatrix}\text{certainty equivalent of} \\ \text{the same uncertain situation}\end{pmatrix}$$

To find the utility associated with the $40,000 payoff in our example, we must identify the probability p at which the decision maker is indifferent between the following two alternatives:

Alternative 1: Receive $40,000 with certainty.

Alternative 2: Receive $150,000 with probability p and lose $30,000 with probability $(1 - p)$.

Because we reduced the payoff amount listed in alternative 1 from its earlier value of $70,000, we expect that the value of p at which the decision maker is indifferent would also be reduced. In this case, suppose that the decision maker is indifferent between the two alternatives when $p = 0.65$ (so that $p^* = 0.65$). The utility associated with a payoff of \$40,000 is:

$$U(40{,}000) = U(150{,}000)p^* + U(-30{,}000)(1 - p^*) = 1p^* + 0(1 - p^*) = p^* = 0.65$$

Again, the utility associated with the amount given in alternative 1 is equivalent to the decision maker's indifference point p^*. This is not a coincidence.

Key Point

When utilities are expressed on a scale from 0 to 1, the probability p^* at which the decision maker is indifferent between alternatives 1 and 2 always corresponds to the decision maker's utility for the amount listed in alternative 1.

Notice that when $p = 0.65$, the expected value of alternative 2 is:

$$\$150{,}000 \times 0.65 - \$30{,}000 \times 0.35 = \$87{,}000$$

Again, this is "risk averse" behavior because the decision maker is willing to accept only \$40,000 (or pay a risk premium of \$47,000) to avoid the risk associated with a decision that has an EMV of \$87,000.

For our example, the utilities associated with payoffs of −\$30,000, \$40,000, \$70,000, and \$150,000 are 0.0, 0.65, 0.80, and 1.0, respectively. If we plot these values on a graph and connect the points with straight lines, we can estimate the shape of the decision maker's utility function for this decision problem, as shown in Figure 14.38. Note that the shape of this utility function is consistent with the general shape of the utility function for a "risk averse" decision maker given in Figure 14.32.

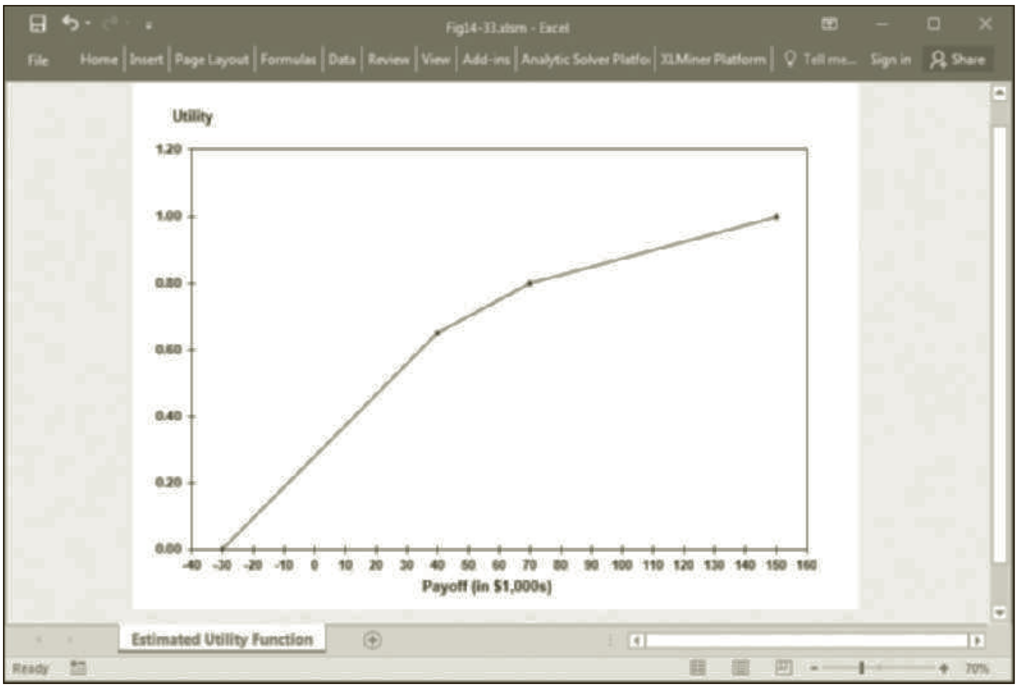


FIGURE 14.33

An estimated utility function for the example problem

14.15.3 USING UTILITIES TO MAKE DECISIONS

After determining the utility value of each possible monetary payoff, we can apply the standard tools of decision analysis to determine the alternative that provides the highest expected utility. We do so using utility values in place of monetary values in payoff tables or decision trees. For our current example, we substitute the appropriate utilities in the payoff table and compute the expected utility for each decision alternative as:

	State of Nature			
Company	**1**	**2**	**Expected Utility**	
A	1.00	0.00	0.500	
B	0.80	0.65	0.725	← maximum
Probability	0.5	0.5		

In this case, the decision to purchase company B provides the greatest expected level of utility to this decision maker—even though our earlier analysis indicated that its EMV of \$55,000 is less than company A's EMV of \$60,000. Thus, by using utilities, decision makers can identify the alternative that is most attractive given their personal attitudes about risk and return.

14.15.4 THE EXPONENTIAL UTILITY FUNCTION

In a complicated decision problem with numerous possible payoff values, it might be difficult and time-consuming for a decision maker to determine the different values for p^* that are required to determine the utility for each payoff. However, if the decision maker is "risk averse," the **exponential utility function** can be used as an approximation of the decision maker's actual utility function. The general form of the exponential utility function is:

$$\mathrm{U}(x) = 1 - e^{-x/R}$$

In this formula, e is the base of the natural logarithm $(e = 2.718281\ldots)$ and R is a parameter that controls the shape of the utility function according to a decision maker's risk tolerance. Figure 14.34 shows examples of the graph of this function for several values of R. Note that as R increases, the shape of the utility curve becomes flatter (or less "risk averse"). Also note that as x becomes large, $\mathrm{U}(x)$ approaches 1; when $x = 0$, then $\mathrm{U}(x) = 0$; and if x is less than 0, then $\mathrm{U}(x) < 0$.

To use the exponential utility function, we must determine a reasonable value for the risk tolerance parameter R. One method for doing so involves determining the maximum value of Y for which the decision maker is willing to participate in a game of chance with the following possible outcomes:

Win \$Y with probability 0.5
Lose \$Y/2 with probability 0.5

The maximum value of Y for which the decision maker would accept this gamble should give us a reasonable estimate of R. Note that a decision maker willing to accept this gamble only at very small values of Y is "risk averse," whereas a decision maker willing to play for larger values of Y is less "risk averse." This corresponds with the relationship between the utility curves and values of R shown in Figure 14.34. (As a rule of thumb, anecdotal evidence suggests that many firms exhibit risk tolerances of approximately one-sixth of equity or 125% of net yearly income.)

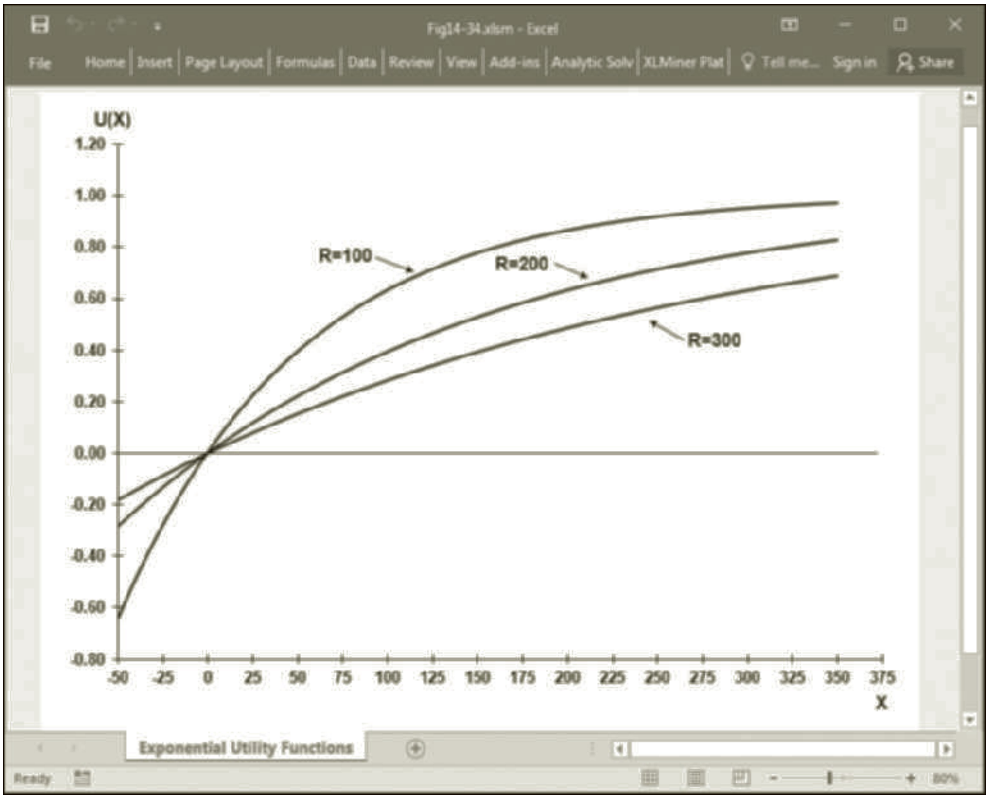


FIGURE 14.34

Examples of the exponential utility function

14.15.5 INCORPORATING UTILITIES IN DECISION TREES

Analytic Solver Platform's Decision Tree tool provides a simple way to use the exponential utility function to model "risk averse" decision preferences in a decision tree. We will illustrate this using the decision tree developed earlier for Magnolia Inns, where Barbara needs to decide which parcel of land to purchase. The decision tree developed for this problem is shown again in Figure 14.35 (and in the file Fig14-35.xlsm that accompanies this book).

To use the exponential utility function, we first construct a decision tree in the usual way. We then determine the risk tolerance value of R for the decision maker using the technique described earlier. Because Barbara is making this decision on behalf of Magnolia Inns, it is important that she provide an estimated value of R based on the acceptable risk levels of the corporation—not her own personal risk tolerance level.

In this case, let's assume that $4 million is the maximum value of Y for which Barbara believes Magnolia Inns is willing to gamble winning $Y with probability 0.5 and losing $Y/2 with probability 0.5. Therefore, $R = Y = 4$. (Note that the value of R should be expressed in the same units as the payoffs in the decision tree.)

We can now instruct the Decision Tree tool to use an exponential utility function to determine the optimal decision by following these steps:

1. Select the Decision Tree element in the Analytic Solver task pane.
2. Change the Risk Tolerance property to 4.
3. Change the Certainty Equivalents property to Exponential Utility Function.

The decision tree is then automatically converted so that the rollback operation is performed using expected utilities rather than EMVs. The resulting tree is shown in

FIGURE 14.35 *Decision tree for the Magnolia Inns land purchase problem*

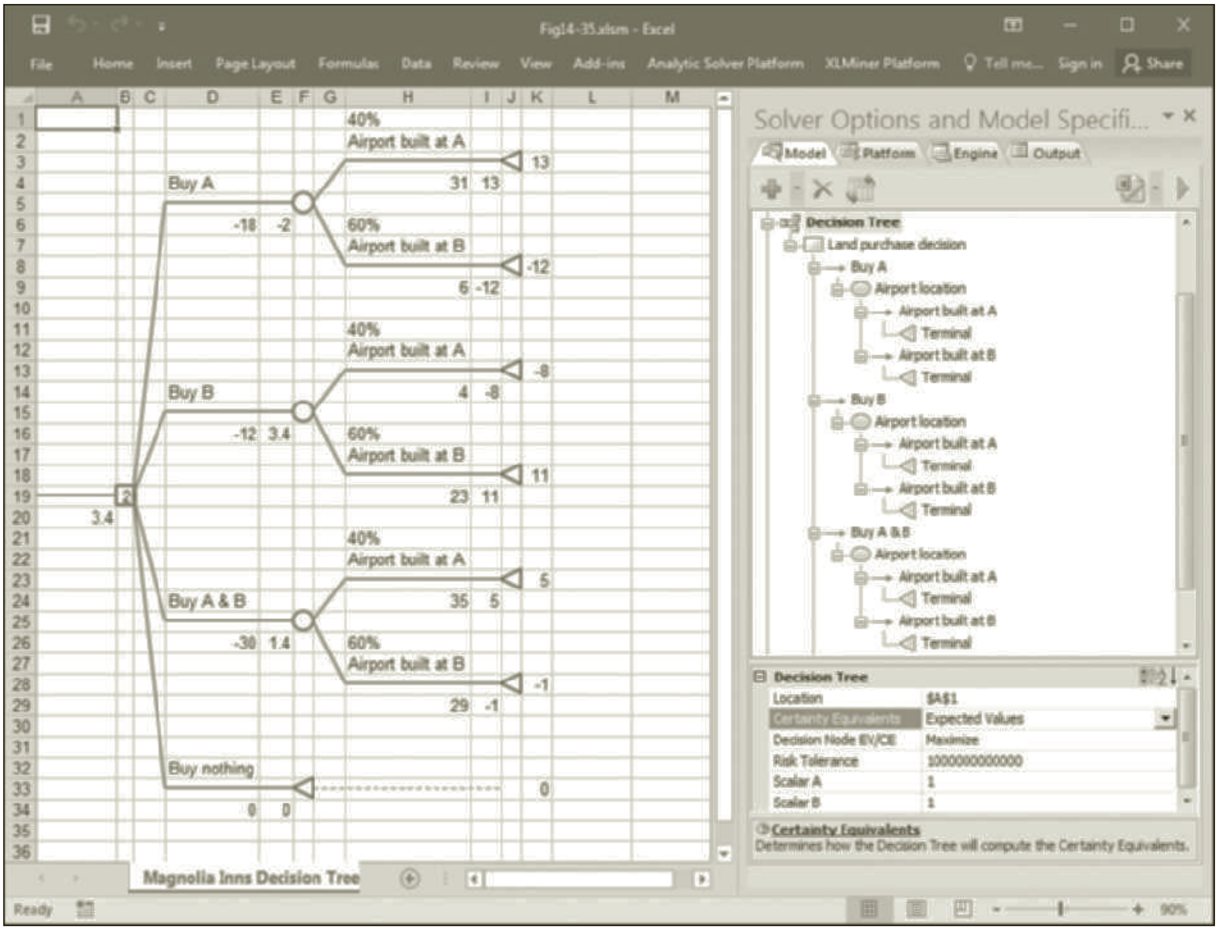
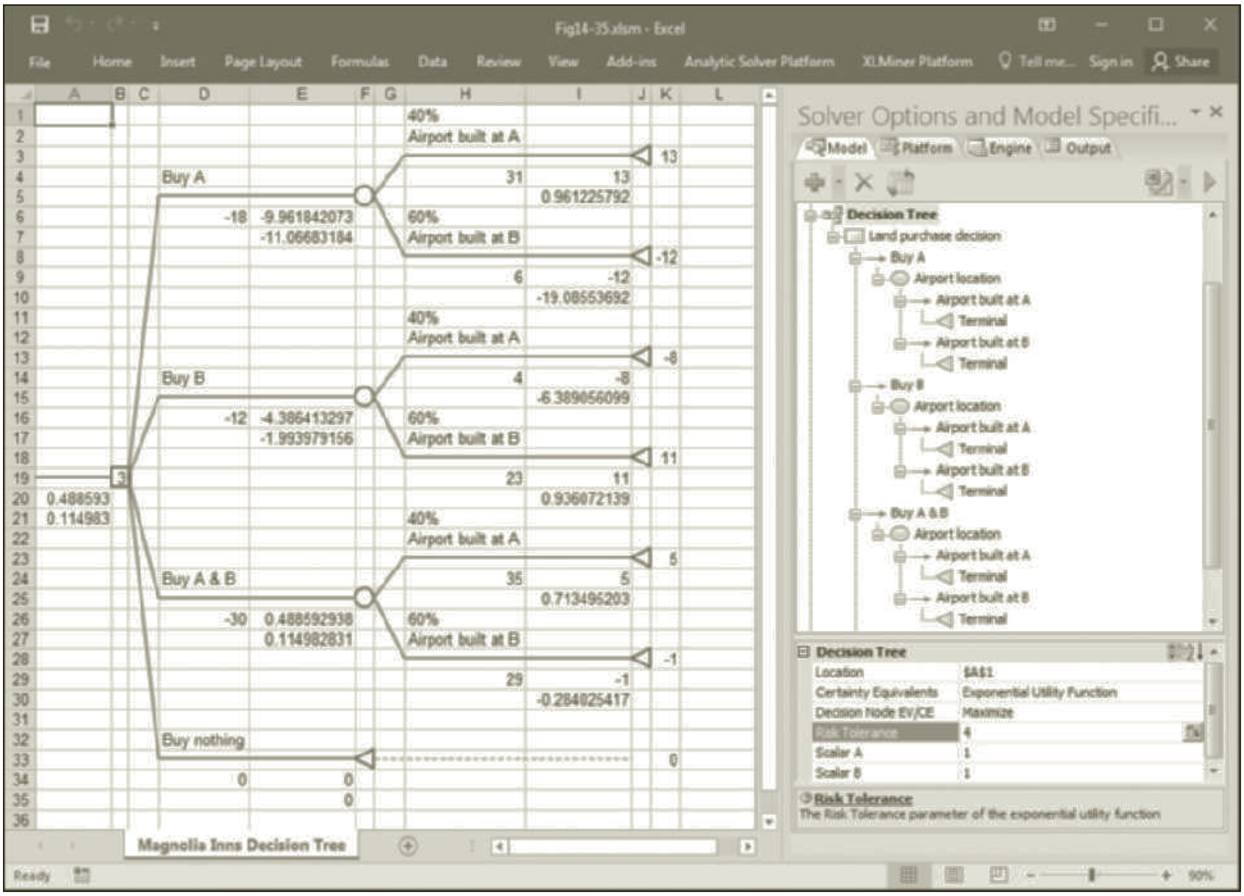


Figure 14.36. The certainty equivalent at each node appears in the cell directly below and to the left of each node (previously the location of the EMVs). The expected utility at each node appears immediately below the certainty equivalents. According to this tree, the decision to buy the parcels at locations A and B provides the highest expected utility for Magnolia Inns. Here again, it might be wise to investigate how the recommended decision might change if we had used a different risk tolerance value and/or different probabilities.

14.16 Multicriteria Decision Making

A decision maker often uses more than one criterion or objective to evaluate the alternatives in a decision problem. Sometimes, these criteria conflict with one another. For example, consider again the criteria of risk and return. Most decision makers desire high levels of return and low levels of risk. But high returns are usually accompanied by high risks, and low levels of return are associated with low risk levels. In making investment decisions, a decision maker must assess the trade-offs between risk and return to identify the decision that achieves the most satisfying balance of these two criteria. As we have seen, utility theory represents one approach to assessing the trade-offs between the criteria of risk and return.

FIGURE 14.36 *Analysis of the Magnolia Inns decision tree using an exponential utility function*



Many other types of decision problems involve multiple conflicting criteria. For example, in choosing between two or more different job offers, you must evaluate the alternatives on the basis of starting salary, opportunity for advancement, job security, location, and so on. If you purchase a video camcorder, you must evaluate a number of different models based on the manufacturer's reputation, price, warranty, size, weight, zoom capability, lighting requirements, and a host of other features. If you must decide whom to hire to fill a vacancy in your organization, you will likely have to evaluate a number of candidates on the basis of education, experience, references, and personality. This section presents two techniques that can be used in decision problems that involve multiple criteria.

14.17 The Multicriteria Scoring Model

The **multicriteria scoring model** is a simple procedure in which we score (or rate) each alternative in a decision problem based on each criterion. The score for alternative j on criterion i is denoted by s_{ij}. Weights (denoted by w_i) are assigned to each criterion indicating its relative importance to the decision maker. For each alternative, we then compute a weighted average score as:

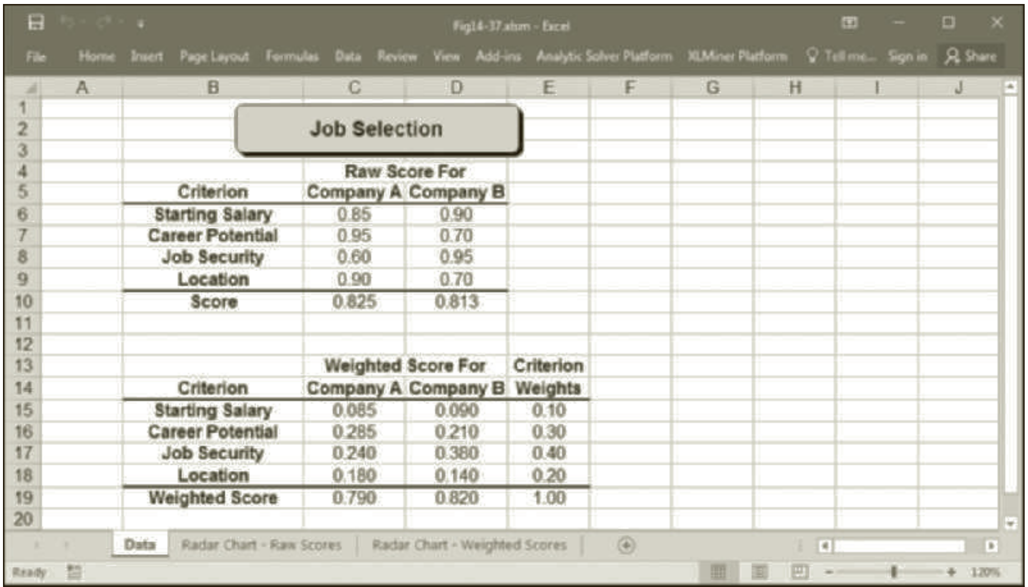
$$\text{Weighted average score for alternative } j = \sum_i w_i s_{ij}$$

We then select the alternative with the largest weighted average score.

The beginning of this chapter described a situation that many students face when they graduate from college—choosing between two job offers. The spreadsheet in Figure 14.37 (and in the file Fig14-37.xlsm that accompanies this book) illustrates how we might use a multicriteria scoring model to help in this problem.

FIGURE 14.37

A multicriteria scoring model



Job Selection

Criterion	Raw Score For Company A	Raw Score For Company B
Starting Salary	0.85	0.90
Career Potential	0.95	0.70
Job Security	0.60	0.95
Location	0.90	0.70
Score	0.825	0.813

Criterion	Weighted Score For Company A	Weighted Score For Company B	Criterion Weights
Starting Salary	0.085	0.090	0.10
Career Potential	0.285	0.210	0.30
Job Security	0.240	0.380	0.40
Location	0.180	0.140	0.20
Weighted Score	0.790	0.820	1.00

Key Cell Formulas

Cell	Formula	Copied to
C10	=AVERAGE(C6:C9)	D10
C15	=C6*$E15	C15:D18
C19	=SUM(C15:C18)	D19:E19

In choosing between two (or more) job offers, we would evaluate criteria for each alternative, such as the starting salary, potential for career development, job security, location of the job, and perhaps other factors as well. The idea in a scoring model is to assign a value from 0 to 1 to each decision alternative that reflects its relative worth on each criterion. These values can be thought of as subjective assessments of the utility that each alternative provides on the various criteria.

In Figure 14.37, scores for each criterion were entered in cells C6 through D9. These scores indicate the starting salary offered by company B provides the greatest value, but the salary offered by company A is not much worse. (Note that these scores do not necessarily mean that the starting salary offered by company B was the highest. These scores reflect the *value* of the salaries to the decision maker, taking into account such factors as the cost of living in the different locations.) The remaining scores in the table indicate that company A provides the greatest potential for career advancement and is in the most attractive location, but provides considerably less job security than that offered by company B. The average scores associated with each job offer are calculated in cells C10 and D10 as follows:

Formula for cell C10: =AVERAGE(C6:C9)
(Copy to D10.)

Notice that the offer from company A has a higher average score than that of company B. However, this implicitly assumes that all the criteria are of equal importance to the decision maker – which is not often the case.

Next, the decision maker specifies weights that indicate the relative importance of each criterion. Again, this is done subjectively. Hypothetical weights for each criterion in this example are shown in cells E15 through E18 in Figure 14.37. Note that these weights must sum to 1. The weighted scores for each criterion and alternative are calculated in cells C15 through D18 as:

Formula for cell C15: = C6*$E15
(Copy to C15 to D18.)

We can then sum these values to calculate the weighted average score for each alternative as:

Formula for cell C19: =SUM(C15:C18)
(Copy to E19.)

In this case, the total weighted average scores for company A and B are 0.79 and 0.82, respectively. Thus, when the importance of each criterion is accounted for via weights, the model indicates that the decision maker should accept the job with company B because it has the largest weighted average score.

Radar charts provide an effective way of graphically summarizing numerous alternatives in a multicriteria scoring model. Figure 14.38 shows the raw scores associated with each of the alternatives in our job selection example. A glance at this chart makes it clear that the offers from both companies offer very similar values in terms of salary,

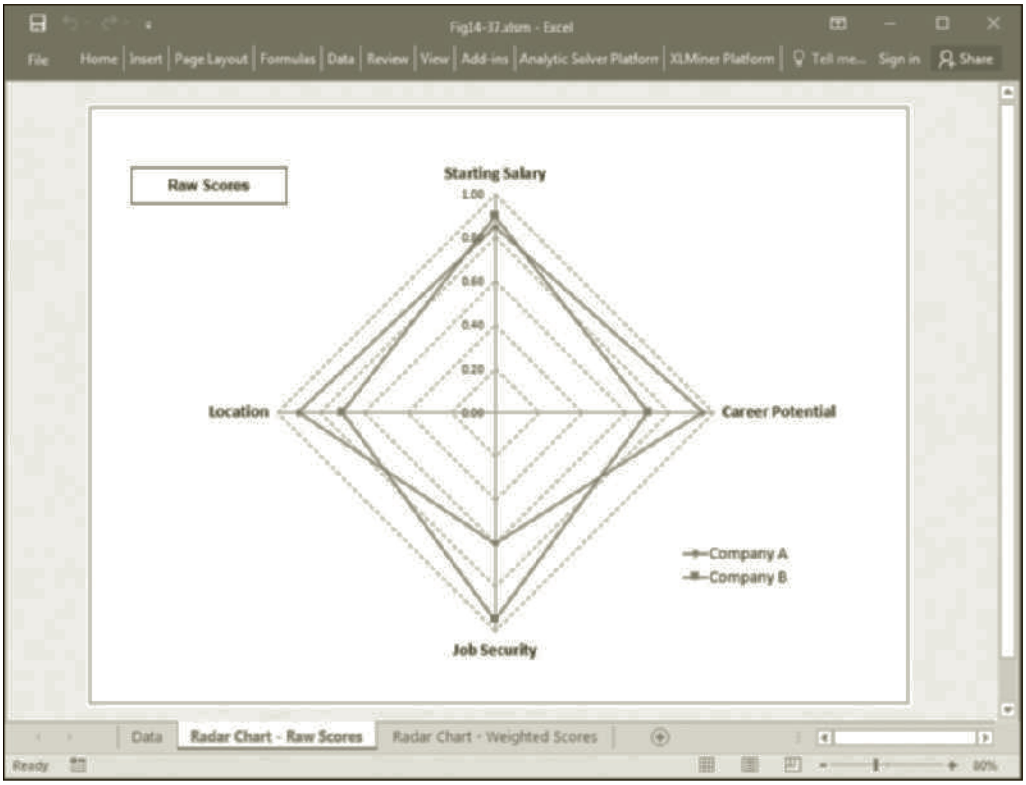


FIGURE 14.38

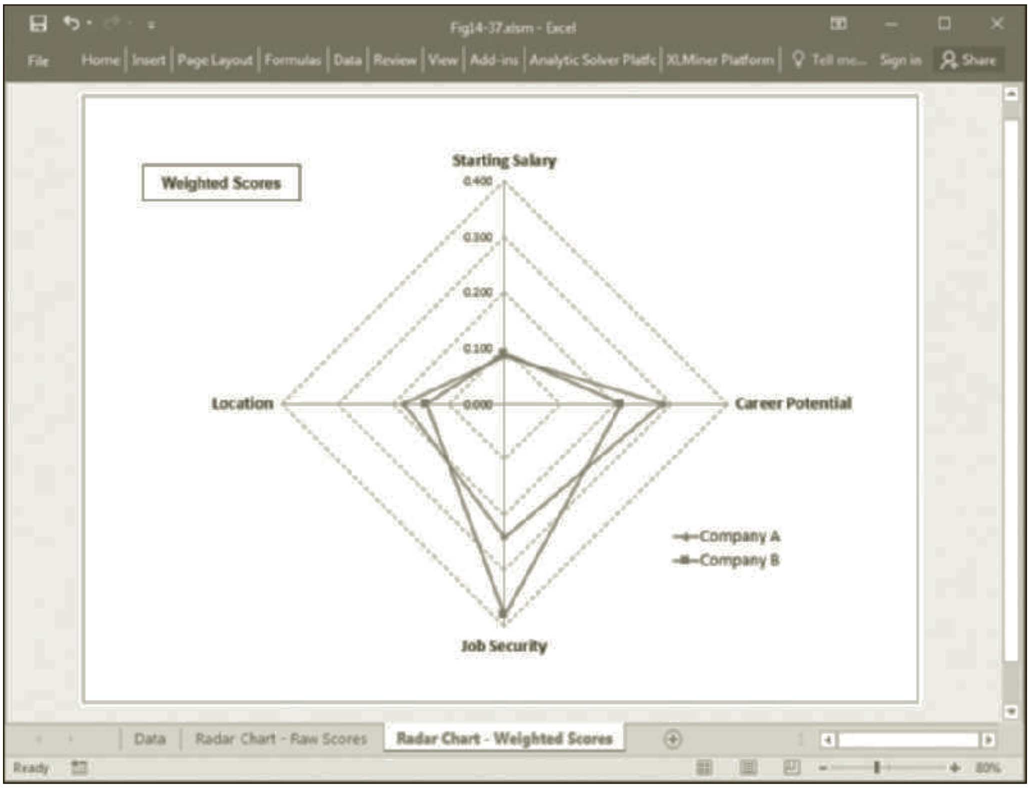
Radar chart of the raw scores

company A is somewhat more desirable in terms of career potential and location, and company B is quite a bit more desirable in terms of job security.

Figure 14.39 shows another radar chart of the weighted scores for each of the alternatives. Using the weighted scores, the radar chart tends to accentuate the differences on criteria that were heavily weighted. For instance, here the offers from the two companies are very similar in terms of salary and location and are most different with respect to career potential and job security. The radar chart's ability to graphically portray the differences in the alternatives can be quite helpful—particularly for decision makers that do not relate well to tables of numbers.

FIGURE 14.39

Radar chart of the weighted scores



Creating a Radar Chart

To create a radar chart like the one shown in Figure 14.39:

1. Select cells B14 through D18.
2. Click the Insert menu.
3. Click Other Charts.
4. Click Radar with Markers.

Excel then creates a basic chart that you can customize in many ways. Right-clicking a chart element displays a dialog box with options for modifying the appearance of the element.

14.18 The Analytic Hierarchy Process

Sometimes, a decision maker finds it difficult to subjectively determine the criterion scores and weights needed in the multicriteria scoring model. In this case, the analytic hierarchy process (AHP) can be helpful. AHP provides a more structured approach for determining the scores and weights for the multicriteria scoring model described earlier. This can be especially helpful in focusing attention and discussion on the important aspects of a problem in group decision-making environments. However, the validity of AHP is not universally accepted. As with any structured decision-making process, the recommendations of AHP should not be followed blindly but should be carefully considered and evaluated by the decision maker(s).

To illustrate AHP, suppose that a company wants to purchase a new payroll and personnel records information system and is considering three systems, identified as X, Y, and Z. The systems differ with respect to three key criteria: price, user support, and ease of use.

14.18.1 PAIRWISE COMPARISONS

The first step in AHP is to create a pairwise comparison matrix for each alternative on each criterion. We will illustrate the details of this process for the price criterion. The values shown in Figure 14.40 are used in AHP to describe the decision maker's preferences between two alternatives on a given criterion.

Value	Preference
1	Equally Preferred
2	Equally to Moderately Preferred
3	Moderately Preferred
4	Moderately to Strongly Preferred
5	Strongly Preferred
6	Strongly to Very Strongly Preferred
7	Very Strongly Preferred
8	Very Strongly to Extremely Preferred
9	Extremely Preferred

FIGURE 14.40

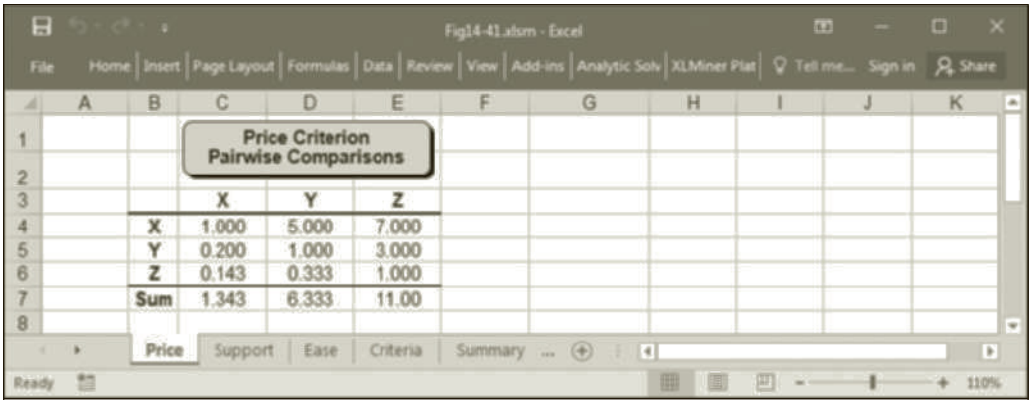
Scale for pairwise comparisons in AHP

To create a pairwise comparison matrix for the price criterion, we must perform pairwise comparisons of the prices of systems X, Y, and Z using the values shown in Figure 14.40. Let P_{ij} denote the extent to which we prefer alternative *i* to alternative *j* on a given criterion. For example, suppose that when comparing system X to Y, the decision maker strongly prefers the price of X. In this case, $P_{XY} = 5$. Similarly, suppose that when comparing system X to Z, the decision maker very strongly prefers the price of X, and when comparing Y to Z, the decision maker moderately prefers the price of Y. In this case, $P_{XZ} = 7$ and $P_{YZ} = 3$. We used the values of these pairwise comparisons to create the pairwise comparison matrix shown in Figure 14.41 (and the file Fig14-41.xlsm that accompanies this book).

The values of P_{XY}, P_{XZ}, and P_{YZ} are shown in cells D4, E4, and E5 in Figure 14.41. We entered the value 1 along the main diagonal in Figure 14.41 to indicate that if an alternative is compared against itself, the decision maker should equally prefer either alternative (because they are the same).

FIGURE 14.41

Pairwise comparisons of the price criterion for the three systems



Price Criterion Pairwise Comparisons

	X	Y	Z
X	1.000	5.000	7.000
Y	0.200	1.000	3.000
Z	0.143	0.333	1.000
Sum	1.343	6.333	11.00

Key Cell Formulas

Cell	Formula	Copied to
C5	=1/D4	--
C6	=1/E4	--
D6	=1/E5	--

The entries in cells C5, C6, and D6 correspond to P_{YX}, P_{ZX}, and P_{ZY}, respectively. To determine these values, we could obtain the decision maker's preferences between Y and X, Z and X, and Z and Y. However, if we already know the decision maker's preference between X and Y (P_{XY}), we can conclude that the decision maker's preference between Y and X (P_{YX}) is the reciprocal of the preference between X and Y; that is, $P_{YX} = 1/P_{XY}$. So, in general, we have:

$$P_{ji} = \frac{1}{P_{ij}}$$

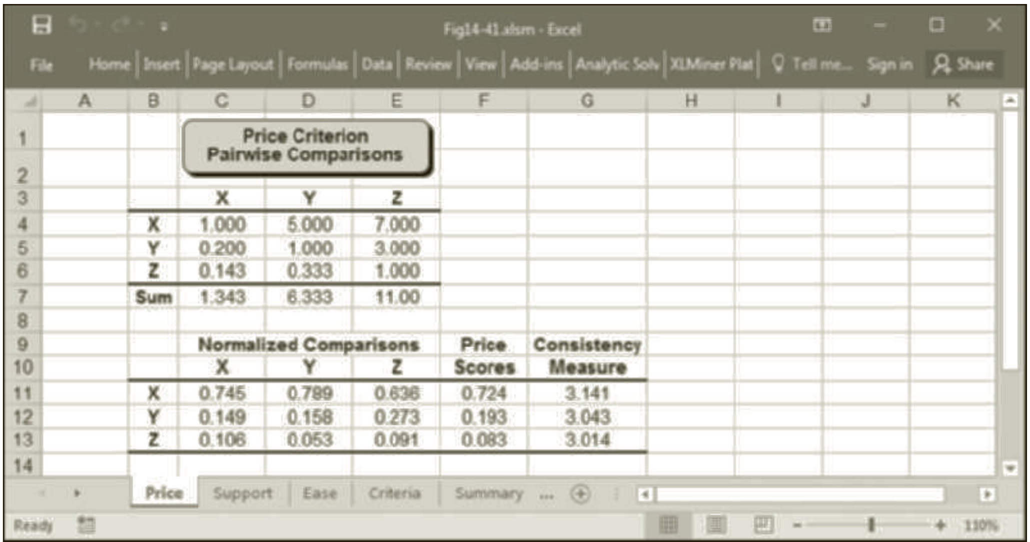
Thus, the values in cells C5, C6, and D6 are computed as:

Formula for cell C5:	= 1/D4
Formula for cell C6:	= 1/E4
Formula for cell D6:	= 1/E5

14.18.2 NORMALIZING THE COMPARISONS

The next step in AHP is to normalize the matrix of pairwise comparisons. To do this, we first calculate the sum of each column in the pairwise comparison matrix. We then divide each entry in the matrix by its column sum. Figure 14.42 shows the resulting normalized matrix.

We will use the average of each row in the normalized matrix as the score for each alternative on the criterion under consideration. For example, cells F11, F12, and F13 indicate that the average scores on the price criterion for X, Y, and Z are 0.724, 0.193, and 0.083, respectively. These scores indicate the relative desirability of the three alternatives to the decision maker with respect to price. The score for X indicates that this is by far the most attractive alternative with respect to price, and alternative Y is somewhat more attractive than Z. Note that these scores reflect the preferences expressed by the decision maker in the pairwise comparison matrix.



	C	D	E	F	G
	Price Criterion Pairwise Comparisons				
	X	Y	Z		
X	1.000	5.000	7.000		
Y	0.200	1.000	3.000		
Z	0.143	0.333	1.000		
Sum	1.343	6.333	11.00		
	Normalized Comparisons			Price	Consistency
	X	Y	Z	Scores	Measure
X	0.745	0.789	0.636	0.724	3.141
Y	0.149	0.158	0.273	0.193	3.043
Z	0.106	0.053	0.091	0.083	3.014

FIGURE 14.42

Price scores obtained from the normalized comparison matrix

Key Cell Formulas

Cell	Formula	Copied to
C7	=SUM(C4:C6)	D7:E7
C11	=C4/C$7	C11:E13
F11	=AVERAGE(C11:E11)	F12:F13

14.18.3 CONSISTENCY

In applying AHP, the decision maker should be consistent in the preference ratings given in the pairwise comparison matrix. For example, if the decision maker strongly prefers the price of X to that of Y, and strongly prefers the price of Y to that of Z, it would be inconsistent for the decision maker to indicate indifference (or equal preference) regarding the price of X and Z. Thus, before using the scores derived from the normalized comparison matrix, the preferences indicated in the original pairwise comparison matrix should be checked for consistency.

A consistency measure for each alternative is obtained as:

$$\text{Consistency measure for X} = \frac{0.724 \times 1 + 0.193 \times 5 + 0.083 \times 7}{0.724} = 3.141$$

$$\text{Consistency measure for Y} = \frac{0.724 \times 0.2 + 0.193 \times 1 + 0.083 \times 3}{0.193} = 3.043$$

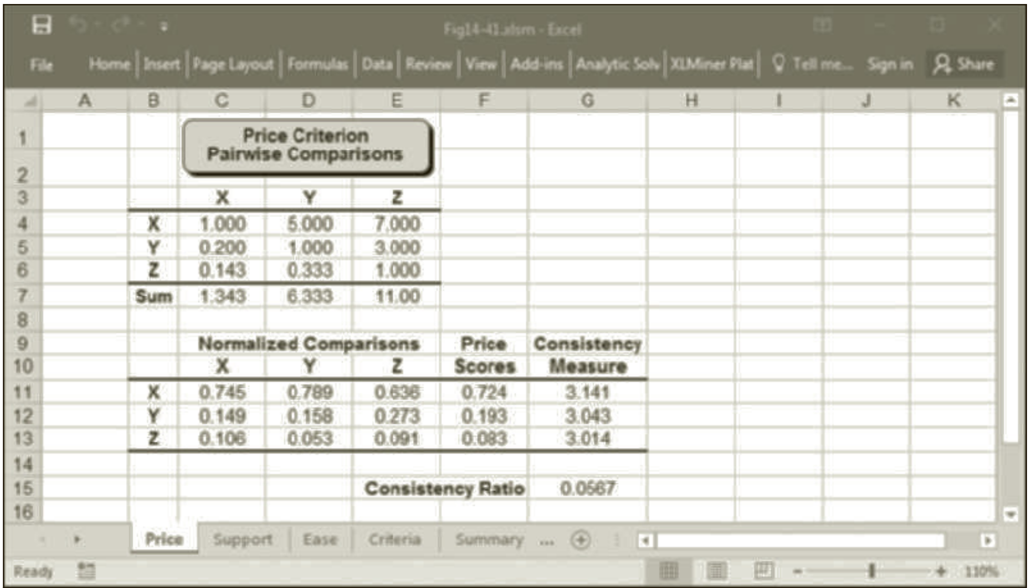
$$\text{Consistency measure for Z} = \frac{0.724 \times 0.143 + 0.193 \times 0.333 + 0.083 \times 1}{0.083} = 3.014$$

The numerator in each of these calculations multiplies the scores obtained from the normalized matrix by the preferences given in one of the rows of the original pairwise comparison matrix. The products are summed and then divided by the score for the alternative in question. These consistency measures are shown in Figure 14.43 in cells G11 through G13.

If the decision maker is perfectly consistent in stating preferences, each consistency measure will equal the number of alternatives in the problem (which, in this case, is three). So, there appears to be some amount of inconsistency in the preferences given in the pairwise comparison matrix. This is not unusual. It is difficult for a decision

FIGURE 14.43

Checking the consistency of the pairwise comparisons



Key Cell Formulas

Cell	Formula	Copied to
G11	=MMULT(C4:E4,$F$11:$F$13)/F11	G12:G13
G15	=(AVERAGE(G11:G13)−3)/(2*0.58)	--

maker to be perfectly consistent in stating preferences between a large number of pairwise comparisons. Provided that the amount of inconsistency is not excessive, the scores obtained from the normalized matrix will be reasonably accurate. To determine whether the inconsistency is excessive, we compute the following quantities:

$$\text{Consistency Index (CI)} = \frac{\lambda - n}{n - 1}$$

$$\text{Consistency Ratio (CR)} = \frac{\text{CI}}{\text{RI}}$$

where:

λ = the average consistency measure for all alternatives

n = the number of alternatives

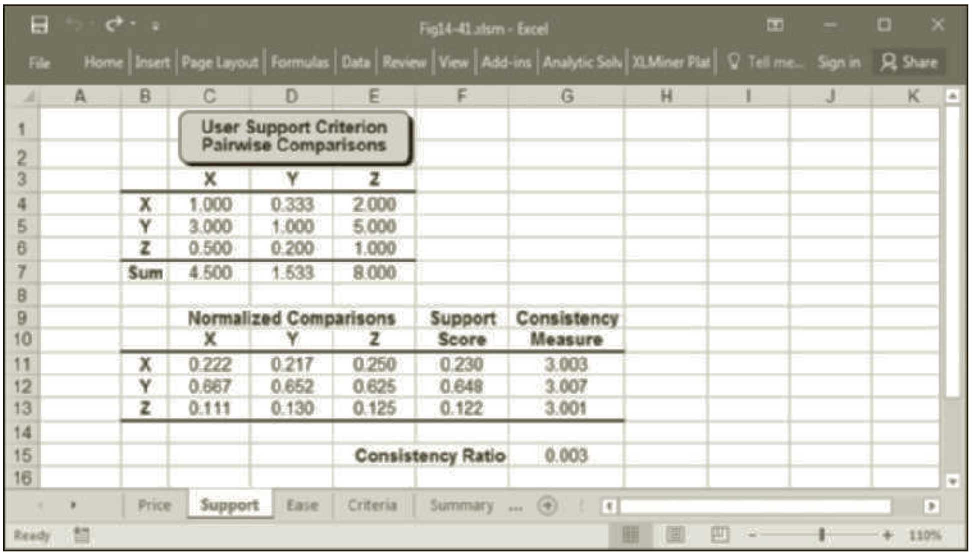
RI = the appropriate random index from Figure 14.44

FIGURE 14.44

Values of RI for use in AHP

n	RI
2	0.00
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41

If the pairwise comparison matrix is perfectly consistent, then $\lambda = n$ and the consistency ratio is 0. The values of RI in Figure 14.44 give the average value of CI if all the entries in the pairwise comparison matrix were chosen at random, given that all the diagonal entries equal 1 and $P_{ij} = 1/P_{ji}$. If CR $\leq$ 0.10, the degree of consistency in the pairwise comparison matrix is satisfactory. However, if CR $>$ 0.10, serious inconsistencies might exist and AHP might not yield meaningful results. The value for CR shown in cell G15 in Figure 14.43 indicates that the pairwise comparison matrix for the price criterion is reasonably consistent. Therefore, we can assume that the scores for the price criterion obtained from the normalized matrix are reasonably accurate.



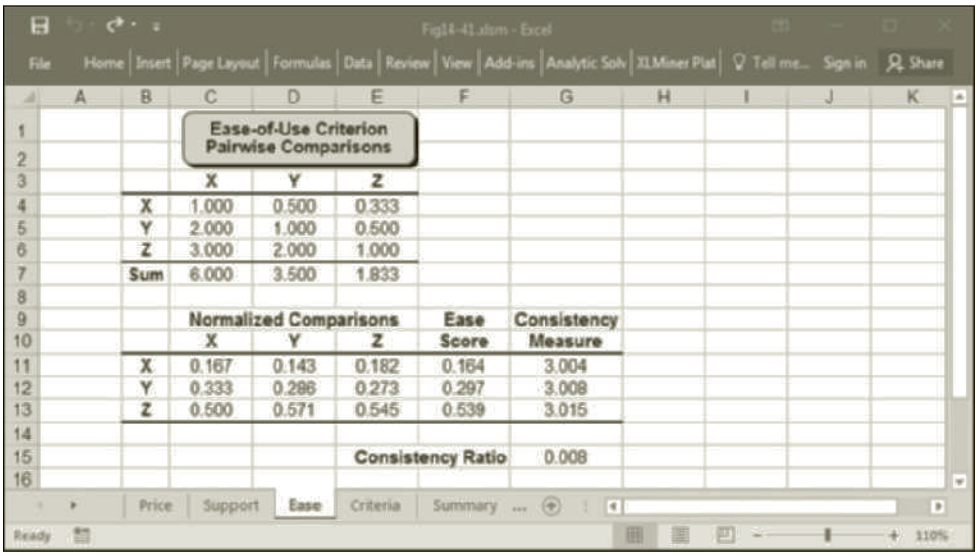
User Support Criterion Pairwise Comparisons

	X	Y	Z
X	1.000	0.333	2.000
Y	3.000	1.000	5.000
Z	0.500	0.200	1.000
Sum	4.500	1.533	8.000

	Normalized Comparisons X	Y	Z	Support Score	Consistency Measure
X	0.222	0.217	0.250	0.230	3.003
Y	0.667	0.652	0.625	0.648	3.007
Z	0.111	0.130	0.125	0.122	3.001
			Consistency Ratio		0.003

FIGURE 14.45

Spreadsheet used to calculate scores for the user support criterion



Ease-of-Use Criterion Pairwise Comparisons

	X	Y	Z
X	1.000	0.500	0.333
Y	2.000	1.000	0.500
Z	3.000	2.000	1.000
Sum	6.000	3.500	1.833

	Normalized Comparisons X	Y	Z	Ease Score	Consistency Measure
X	0.167	0.143	0.182	0.164	3.004
Y	0.333	0.286	0.273	0.297	3.008
Z	0.500	0.571	0.545	0.539	3.015
			Consistency Ratio		0.008

FIGURE 14.46

Spreadsheet used to calculate scores for the ease-of-use criterion

14.18.4 OBTAINING SCORES FOR THE REMAINING CRITERIA

We can repeat the process for obtaining the price criterion scores to obtain scores for the user support and ease-of-use criteria. Hypothetical results for these criteria are shown in Figures 14.45 and 14.46, respectively.

We can create these two spreadsheets easily by copying the spreadsheet for the price criterion (shown in Figure 14.43) and having the decision maker fill in the pairwise comparison matrices with preferences related to the user support and ease-of-use criteria. Notice that the preferences given in Figures 14.45 and 14.46 appear to be consistent.

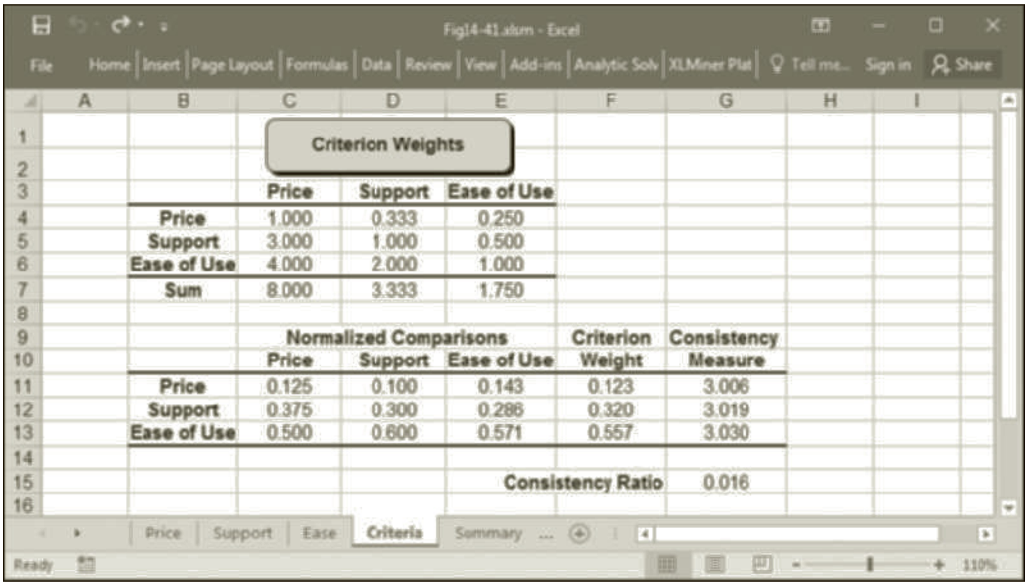
14.18.5 OBTAINING CRITERION WEIGHTS

The scores shown in Figures 14.43, 14.45, and 14.46 indicate how the alternatives compare with respect to the price, user support, and ease-of-use criteria. Before we can use these values in a scoring model, we must also determine weights that indicate the relative importance of the three criteria to the decision maker. The pairwise comparison process used earlier to generate scores for the alternatives on each criterion can also be used to generate criterion weights.

The pairwise comparison matrix in Figure 14.47 shows the decision maker's preferences for the three criteria. The values in cells C5 and C6 indicate that the decision maker finds user support and ease of use to be more important (or more preferred) than price, and cell D6 indicates that ease of use is somewhat more important than user support. These relative preferences are reflected in the criterion weights shown in cells F11 through F13.

FIGURE 14.47

Spreadsheet used to determine the criterion weights

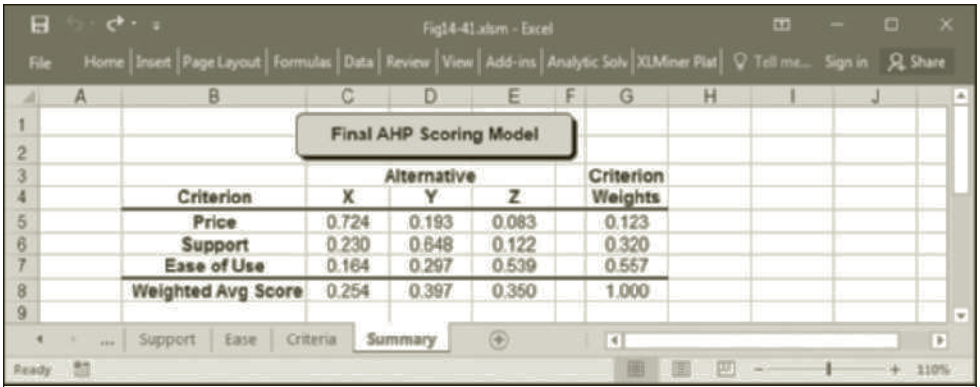


	Price	Support	Ease of Use
Price	1.000	0.333	0.250
Support	3.000	1.000	0.500
Ease of Use	4.000	2.000	1.000
Sum	8.000	3.333	1.750

	Normalized Comparisons			Criterion	Consistency
	Price	Support	Ease of Use	Weight	Measure
Price	0.125	0.100	0.143	0.123	3.006
Support	0.375	0.300	0.286	0.320	3.019
Ease of Use	0.500	0.600	0.571	0.557	3.030
				Consistency Ratio	0.016

14.18.6 IMPLEMENTING THE SCORING MODEL

We now have all the elements required to analyze this decision problem using a scoring model. Thus, the last step in AHP is to calculate the weighted average scores for each decision alternative. The weighted average scores are shown in cells C8 through E8 in Figure 14.48. According to these scores, alternative Y should be selected.



Criterion	Alternative X	Alternative Y	Alternative Z	Criterion Weights
Price	0.724	0.193	0.083	0.123
Support	0.230	0.648	0.122	0.320
Ease of Use	0.164	0.297	0.539	0.557
Weighted Avg Score	0.254	0.397	0.350	1.000

Key Cell Formulas

Cell	Formula	Copied to
C8	=SUMPRODUCT(C5:C7,$G$5:$G$7)	D8:E8

FIGURE 14.48

Final scoring model for selecting the information system

14.19 Summary

This chapter presented a number of techniques for analyzing a variety of decision problems. First, it discussed how a payoff table can be used to summarize the alternatives in a single-stage decision problem. Then, a number of nonprobabilistic and probabilistic decision rules were presented. No one decision rule works best in all situations, but together, the rules help to highlight different aspects of a problem and can help develop and sharpen a decision maker's insight and intuition about a problem so that better decisions can be made. When probabilities of occurrence can be estimated for the alternatives in a problem, the EMV decision rule is the most commonly used technique.

Decision trees are particularly helpful in expressing multistage decision problems in which a series of decisions must be considered. Each terminal node in a decision tree is associated with the net payoff that results from each possible sequence of decisions. A rollback technique determines the alternative that results in the highest EMV. Because different decision makers derive different levels of value from the same monetary payoff, the chapter also discussed how utility theory can be applied to decision problems to account for these differences.

Finally, the chapter discussed two procedures for dealing with decision problems that involve multiple conflicting decision criteria. The multicriteria scoring model requires the decision maker to assign a score for each alternative on each criterion. Weights are then assigned to represent the relative importance of the criteria, and a weighted average score is computed for each alternative. The alternative with the highest score is the recommended alternative. AHP provides a structured approach to determining the scores and weights used in a multicriteria scoring model if the decision maker has difficulty specifying these values.

14.20 References

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