FIT3158
Business Decision Modelling

Lecture 6

- Network Modelling (Part 2)

Topics Covered:

- Solving transportation problems using North-West Corner method
- Solving transportation problems with Vogel Approximation Method (VAM)
- Solving transportation problems with MODI (closed loop) Method
- Modelling Assignment Problems

Introduction

Please recall …

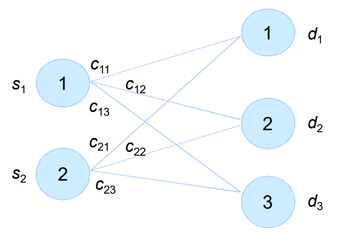
- A network model is one which can be represented by a set of nodes, a set of arcs, and functions (e.g., costs, supplies, demands, etc.) associated with the arcs (also called edges) and/or nodes (also called vertices).
- Each of these models can be formulated as a linear programming problem and solved by general purpose linear programming (LP) codes.

Introduction

- One of the most important applications of quantitative analysis in solving business problems is the physical distribution of products.
- Great cost savings can be achieved by more efficient routing, distribution and scheduling of goods and services from one node (*source*, where the supply is) to the required destination (*sink*, where the demand is).
- The transportation problem seeks to minimize the total shipping costs of transporting goods from m origins (each with a supply s_i) to n destinations (each with a demand d_j), when the unit shipping cost from an origin, i, to a destination, j, is c_{ij}.

Transportation Problem

- The following is a <u>network representation</u> of a transportation problem with two sources and three destinations



SOURCES DESTINATIONS

Transportation Problem - LP Formulation

The linear programming formulation in terms of the amounts shipped from the origins to the destinations, x_{ij}, can be written as:

$$\text{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \le s_i \quad for\ i = 1, 2, \ldots, m \quad \text{Supply}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \quad for\ i = 1, 2, \ldots, n \quad \text{Demand}$$

$x_{ij} \geq 0$ for all i and j

Transportation Problem

LP Formulation Special Cases

- Total supply exceeds total demand:

  No modification of LP formulation is necessary.

- Total demand exceeds total supply:

  Add a dummy origin with supply equal to the shortage amount. Assign a zero shipping cost per unit. The amount "shipped" from the dummy origin (in the solution) will not actually be shipped.

LP Formulation Special Cases

The following special-case modifications to the linear programming formulation can be made:

- ❖ Minimum shipping guarantee from i to j:

  $x_{ij} \geq L_{ij}$

- ❖ Maximum route capacity from i to j:

  $x_{ij} \leq L_{ij}$

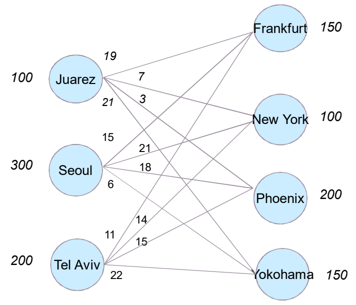
- ❖ Unacceptable route:

  Remove the corresponding decision variable.

Example: Ski Shipment Scheduling

From Plant	To Warehouse				
	Frankfurt	New York	Phoenix	Yokohama	Capacity
Juarez	19	7	3	21	100
Seoul	15	21	18	6	300
Tel Aviv	11	14	15	22	200
Demand	150	100	200	150	

Ski Shipment – Network Model



Plant (Source) Warehouse (Sink)

Implementing the Model & Solution

See file Transportation.xlsm (Method 1)

	A	B	C	D	E	F
1	Ski Shipment-Scheduling Illustration					
2						
3	From	To Warehouse				
4	Plant	Frankfurt	New York	Phoenix	Yokohama	Capacity
5	Juarez	19	7	3	21	100
6	Seoul	15	21	18	6	300
7	Tel Aviv	11	14	15	22	200
8	Demand	150	100	200	150	C(min)
9			Solution			$6,250
10	From	To Warehouse				
11	Plant	Frankfurt	New York	Phoenix	Yokohama	Total
12	Juarez	0	0	100	0	100
13	Seoul	50	0	100	150	300
14	Tel Aviv	100	100	0	0	200
15	Total	150	100	200	150	

=SUMPRODUCT(B5:E7,B12:E14)

=SUM(B12:B14)

=SUM(B12:E12)

Solution

Implementing the Model & the Solution

See file Transportation.xlsm (Method 2)

Ship	From		To		Unit Cost
0	1	Juarez	4	Frankfurt	$19
0	1	Juarez	5	New York	$7
100	1	Juarez	6	Phoenix	$3
0	1	Juarez	7	Yokohama	$21
50	2	Seoul	4	Frankfurt	$15
0	2	Seoul	5	New York	$21
100	2	Seoul	6	Phoenix	$18
150	2	Seoul	7	Yokohama	$6
100	3	Tel Aviv	4	Frankfurt	$11
100	3	Tel Aviv	5	New York	$14
0	3	Tel Aviv	6	Phoenix	$15
0	3	Tel Aviv	7	Yokohama	$22

Total Transportation Cost: $6,250

Transportation Models

We will now look at some of the techniques used to solve transportation problems.

- ❖ **Northwest Corner Method**
- ❖ **Vogel's Approximation Method (VAM)**
- ❖ **MODI (The Closed-Loop Method)**

Northwest Corner Method

Algorithm:

- Start in the top left hand (or Northwest) corner.
- Allocate the maximum supply possible to demand.
- Adjust the row and column entries.
- If demand is met, move to next column.
- If supply is exhausted, move to next row.
- Move from top left → bottom right

Northwest Corner Method – Example

Source	Destination				
	D1	D2	D3	D4	Capacity
S1	19 100	7	3	21	100
S2	15 50	21 100	18 150	6	~~250~~ 150 ~~300~~
S3	11	14	15 50	22 150	150 ~~200~~
Demand	50 ~~150~~	100	50 ~~200~~	~~150~~	600

Northwest Corner Method – Solution

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D1	100	19	1900
S2 → D1	50	15	750
S2 → D2	100	21	2100
S2 → D3	150	18	2700
S3 → D3	50	15	750
S3 → D4	150	22	3300
			$11,500

This is quite a 'simplistic' technique. It does not take the costs into consideration.
The solution generated is far from optimal – compare this with Slide 11 or 12 (where solution is far cheaper).

Vogel's Approximation Method (VAM)

This method was originally used for ammunition distribution.

The Basic Principle:

In choosing a route,

- Try to avoid high cost routes.
- Will be implicitly making decisions about alternative routes.
- Does not only consider direct costs but also the next best alternative.

Vogel's Approximation Method (VAM)

Algorithm:

1. Calculate the potential opportunity loss for rows. The opportunity loss is conservatively estimated as the difference between the lowest cost cell and the next lowest cost cell.
2. Do the same thing for columns.
3. Locate the highest potential opportunity loss. Break ties arbitrarily.
4. Allocate the maximum supply possible to the minimum cost cell in the row or column located in (3).
5. Adjust rows and columns.
6. Iterate

VAM – Example

Source	Destination					
	D1	D2	D3	D4	Capacity	
S1	19	7	3 100	21	100	7-3=4 0
S2	15 50	21	18 100	6 150	150 ~~300~~	15-6=9 18-15=3
S3	11 100	14 100	15	22	100 ~~200~~	14-11=3 15-11=4 0
Demand	50 ~~150~~	100	100 ~~200~~	150	600	
	15-11=4	14-7=7 21-14=7 0	15-3=12 18-15=3	21-6=15 0		

Vogel's Approximation Method – Solution

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D3	100	3	300
S2 → D1	50	15	750
S2 → D3	100	18	1800
S2 → D4	150	6	900
S3 → D1	100	11	1100
S3 → D2	100	14	1400
			$6,250

Clearly, this gives a better solution than the Northwest Corner Method. In fact, this is the optimal solution (approx. Slide 11).
But, VAM does not always guarantee an optimal solution. MODI does!

MODI (Closed-Loop Method)

Also known as:

- Modified Distribution Method ; or
- Modified Dantzig Iteration Algorithm

What we have noticed so far:

If there are 3 sources (N) and 4 destinations (M), the total number of allocations is 3 + 4 -1 = 6

So, for non-degenerate solutions, there will always be: N + M – 1 allocations.

MODI (Closed-Loop Method)

Algorithm:

(Recall that the C_{ij} are the edge costs.)

1. Generate a basic feasible solution (e.g., using Northwest Corner or Vogel's Approximation Method [VAM]).
2. Derive $R_i + K_j = C_{ij}$ for any cell with a shipment

   (where R_i = Row Indicators, K_j = Column Indicators).

   By convention, we always set $R_1 = 0$
3. Calculate the $C_{ij} – (R_i + K_j)$ values for cells with no shipment.

MODI (Closed-Loop Method)

4. Put a "+" sign in the most negative cell.
   - a) **If there is more than 1 negative, choose the biggest reduction. Break ties arbitrarily.**
   - b) **If there are no cells with a negative** $C_{ij} – (R_i + K_j)$ **values → *STOP* – the solution is optimal.**
5. Form a closed loop.
6. Determine maximum adjustment and modify solution.
7. Iterate

MODI (Closed-Loop Method) - Example

1. Start with a basic feasible solution – let's use the one from Northwest Corner Method (see above, approx. slide 15).

Source	Destination				
	D1	D2	D3	D4	Capacity
S1	19 / 100	7	3	21	100
S2	15 / 50	21 / 100	18 / 150	6	300
S3	11	14	15 / 50	22 / 150	200
Demand	150	100	200	150	600

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed

MODI (Closed-Loop Method) - Example

2. Derive $R_i + K_j = C_{ij}$ for any cell with a shipment

		K1 19	K2 25	K3 22	K4 29	K5	
	DEPOT	X1					
R1 0	D1	19 *100*	7				100
R2 -4	D2	15 *50*	21 *100*	18 *150*	6		300
R3 -7	D3	11	14	15 *50*	22 *150*		200
	DEMAND	150	100	200	150		

First, let $R_1 = 0$
Since $C_{11} = 19$, $R_1 + K_1 = 19 \rightarrow K_1 = 19$
And since $C_{21} = 15$, $R_2 + K_1 = 15 \rightarrow R_2 = 15-19 = -4$

So, $K_2 = 21 - (-4) = 25$, $K_3 = 18 - (-4) = 22$

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MODI (Closed-Loop Method) - Example

2. Derive $R_i + K_j = C_{ij}$ for any cell with a shipment

		K1 19	K2 25	K3 22	K4 29	K5	
	DEPOT	X1					
R1 0	D1	19 *100*	7				
R2 -4	D2	15 *50*	21 *100*	18 *150*	6		300
R3 -7	D3	11	14	15 *50*	22 *150*		200
	DEMAND	150	100	200	150		

First, let $R_1 = 0$
Since $C_{11} = 19$, $R_1 + K_1 = 19 \rightarrow K_1 = 19$
And since $C_{21} = 15$, $R_2 + K_1 = 15 \rightarrow R_2 = 15-19 = -4$

There are (M+N-1) allocations. And, setting $R_1 = 0$, there are (M+N)-1 values of R_i and K_j to find.

So, $K_1 = 19 - 0 = 19$, $R_2 = 15 - 19 = -4$, $K_2 = 21 - (-4) = 25$, $K_3 = 18 - (-4) = 22$, $R_3 = 15 - 22 = -7$, $K_4 = 22 - (-7) = 29$ (and this completes step 2). We now go to step 3 and calculate $C_{ij} - (R_i + K_j)$ values for cells with no shipment.

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MODI (Closed-Loop Method) - Example

3. Calculate the $C_{ij} - (R_i + K_j)$ values for cells with no shipment.

		K1 19	K2 25	K3 22	K4 29	K5	
		OUTLET					
	DEPOT	X1	X2	X3	X4	X5	SUPPLY
R1 0	D1	19 *100*	7 -18	3 -19	21 -8		100
R2 -4	D2	15 *50*	21 *100*				
R3 -7	D3	11 -1	14 -4	*50*	*150*		200
	DEMAND	150	100	200	150		

E.g.: Cell C_{12} & C_{13} – no shipment
So, $C_{12} - (R_1 + K_2) = 7 - (0 + 25) = -18$
And, $C_{13} - (R_1 + K_3) = 3 - (0 + 22) = -19$

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4. Put a "+" sign in the most negative cell.
5. Form a closed loop.
6. Determine maximum adjustment and modify solution

Entering cell
Put a "+" sign here

		K1 19	K2 25	K3 22	K4 29	K5	
		OUTLET					
	DEPOT	X1	X2	X3	X4	X5	SUPPLY
R1 0	D1	19 - ~~100~~ *0*	7 -18	3 + *+100* -19	21 -8		100
R2 -4	D2	15 + *50* *+100*	21 *100*	18 - ~~150~~ *50*	6 -19		300
R3 -7	D3	11 -1	14 -4	15 *50*	22		
	DEMAND	150	100	200			

Identify the minimum value in the cell with "-"; min{150, 100} = 100 in this case. Add or subtract '100' in opposite corners.

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MODI (Closed-Loop Method) - Example

After 2nd Iteration:

		K1 0	K2 6	K3 3	K4 10	K5	
		OUTLET					
	DEPOT	X1	X2	X3	X4	X5	SUPPLY
R1 0	D1	19 19	7 1	3 100	21 11		100
R2 15	D2	15 150	21 100	18 - ~~50~~ 0	6 + +50 -19		300
R3 12	D3	11 -1	14 -4	15 + 50 +50	22 - ~~150~~ 100		200
	DEMAND	150	100	200	150		

MODI (Closed-Loop Method) - Example

After 3rd Iteration:

		K1 19	K2 25	K3 3	K4 10	K5	
		OUTLET					
	DEPOT	X1	X2	X3	X4	X5	SUPPLY
R1 0	D1	19 0	7 -18	3 100	21 11		100
R2 -4	D2	15 150	21 - ~~100~~ 0	18 19	6 + 50 +100		300
R3 12	D3	11 -20	14 + +100 -23	15 100	22 - ~~100~~ 0		200
	DEMAND	150	100	200	150		

MODI (Closed-Loop Method) - Example

After 4th Iteration:

		K1 -4	K2 2	K3 3	K4 -13	K5	
		OUTLET					
	DEPOT	X1	X2	X3	X4	X5	SUPPLY
R1 0	D1	19 23	7 5	3 100	21 34		100
R2 19	D2	15 - ~~150~~ 50	21 0	18 + +100 -4	6 150		300
R3 12	D3	11 + +100 3	14 100	15 - ~~100~~ 0	22 23		200
	DEMAND	150	100	200	150		

After 5th Iteration:

There are no cells with a negative $C_{ij} – (R_i + K_j)$ values→ *STOP* – the solution is optimal.

		K1 0	K2 3	K3 3	K4 -9	K5	
		OUTLET					
	DEPOT	X1	X2	X3	X4	X5	SUPPLY
R1 0	D1	19 19	7 4	3 100	21 30		100
R2 15	D2	15 50	21 3	18 100	6 150		300
R3 11	D3	11 100	14 100	15 1	22 20		200
	DEMAND	150	100	200	150		

MODI (Closed-Loop Method) – Solution

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D3	100	3	300
S2 → D1	50	15	750
S2 → D3	100	18	1800
S2 → D4	150	6	900
S3 → D1	100	11	1100
S3 → D2	100	14	1400
			$6,250

MODI always gives the optimal solution.

Some issues to take note of:

1. When supply does not equal demand
   – When demand < supply, add a dummy column for demand to make up for the difference.

	X1	X2	Dummy	Supply
D1	8	9	0	200
D2	12	7	0	200
Demand	200	100	100	

   – Similarly, when demand > supply, add a dummy row for supply

In fact, the first step in doing any allocations is to check whether the demand and supply are equal.

Some issues to take note:

2. When there's a case of degeneracy

E.g.:

	X1	X2
D1	8 +	9 - 10
D2	12 - 10	7 + 10

→

	X1	X2
D1	8 10	9
D2	12 0	7 20

We now ended up having 2 allocations, thus breaking the (M + N – 1) rule.

To handle such a situation, we put a zero (0) in one of the cells.

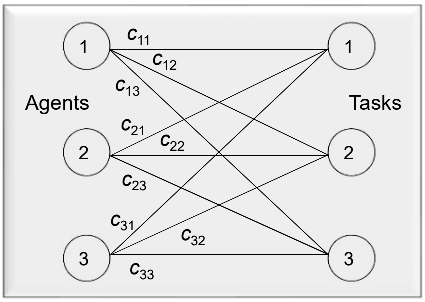
Assignment Problem

- An assignment problem seeks to minimize the total cost assignment of m workers to m jobs, given that the cost of worker i performing job j is c_{ij}.
- It assumes all workers are assigned and each job is performed.
- An assignment problem is a special case of a transportation problem in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.
- The network representation of an assignment problem with three workers and three jobs is shown on the next slide.

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning

Assignment Problem

- Network Representation



Assignment Problem

- Linear Programming Formulation

Using the notation:

$$x_{ij} = \begin{cases} 1 & \text{if agent } i \text{ is assigned to task } j \\ 0 & \text{otherwise} \end{cases}$$

c_{ij} = cost of assigning agent i to task j

continued →

Assignment Problem

- Linear Programming Formulation (continued)

$$\text{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad for\ j = 1, 2, \ldots, n \text{ Tasks}$$

$$\sum_{j=1}^{n} x_{ij} \leq 1 \quad for\ i = 1, 2, \ldots, m \text{ Agents}$$

$x_{ij} \geq 0$ for all i and j

Assignment Problem

- LP Formulation Special Cases
  - Number of agents exceeds the number of tasks:

    Extra agents simply remain unassigned.

  - Number of tasks exceeds the number of agents:

    Add enough dummy agents to equalize the number of agents and the number of tasks. The objective function coefficients for these new variables would be zero.

Assignment Problem

- LP Formulation Special Cases (continued)
  - The assignment alternatives are evaluated in terms of revenue or profit:

    Solve as a maximization problem.

  - An assignment is unacceptable:

    Remove the corresponding decision variable.

  - An agent is permitted to work t tasks:

$$\sum_{j=1}^{n} x_{ij} \leq t \quad \text{for } i = 1, 2, \ldots, m \text{ Agents}$$

Assignment Problem: Example

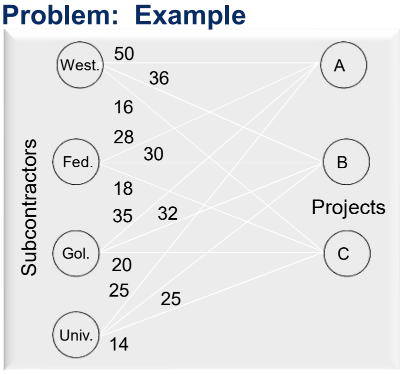
An electrical contractor pays her subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

	Projects		
Subcontractor	A	B	C
Westside	50	36	16
Federated	28	30	18
Goliath	35	32	20
Universal	25	25	14

How should the contractors be assigned so that total mileage is minimized?

Assignment Problem: Example

- Network Representation



Assignment Problem: Example

- Linear Programming Formulation

$$\begin{array}{ll} \text{Min} & 50x_{11}+36x_{12}+16x_{13}+28x_{21}+30x_{22}+18x_{23} \\ & +35x_{31}+32x_{32}+20x_{33}+25x_{41}+25x_{42}+14x_{43} \\ \text{s.t.} & \left.\begin{array}{l} x_{11}+x_{12}+x_{13} \leq 1 \\ x_{21}+x_{22}+x_{23} \leq 1 \\ x_{31}+x_{32}+x_{33} \leq 1 \\ x_{41}+x_{42}+x_{43} \leq 1 \end{array}\right\} \boxed{\text{Agents}} \\ & \left.\begin{array}{l} x_{11}+x_{21}+x_{31}+x_{41} = 1 \\ x_{12}+x_{22}+x_{32}+x_{42} = 1 \\ x_{13}+x_{23}+x_{33}+x_{43} = 1 \end{array}\right\} \boxed{\text{Tasks}} \\ & x_{ij} = 0 \text{ or } 1 \quad \text{for all } i \text{ and } j \end{array}$$

Implementation of Model & Solution:

See file Assignment Problems.xlsx (Contractor Assignment)

			Matrix of Indicators			
Matrix of Indicators	Projects		A	B	C	(Sum <= 1)
	Subcontractors	Westside	0	0	1	1
		Federated	1	0	0	1
		Goliath	0	0	0	0
		Universal	0	1	0	1
		(Sum = 1)	1	1	1	
Matrix of Weights	Projects		A	B	C	
	Subcontractors	Westside	50	36	16	
		Federated	28	30	18	
		Goliath	35	32	20	
		Universal	25	25	14	
Matrix of Products	Projects		A	B	C	
	Subcontractors	Westside	0	0	16	
		Federated	28	0	0	
		Goliath	0	0	0	
		Universal	0	25	0	
		Objective Function (total cost)				
		69				

Assignment Problem: Example

- The optimal assignment is:

Subcontractor	Project	Distance
Westside	C	16
Federated	A	28
Goliath	(unassigned)	
Universal	B	25
	Total Distance =	69 miles

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning

Job Assignment based on maximising preferences

- We have 10 people and we wish to assign each person one job to do. Each person makes a list of 3 preferences and we assign the jobs accordingly.
- This problem is probably getting quite close to the size that you could do with the solver in practice and it provides a good example of how the solver works.
- By observing partial solutions, we can see that the solver initially relaxes the constraint that indicators be integers and gradually enforces this condition as a solution is approached.

Setup: Input Data

- Input data, shown coded as a table of weights.

Preferences				Preferences coded 10 = first, 5 = second, 1 = thir								
Person	P1	P2	P3	1	2	3	4	5	6	7	8	9
a	2	7	8	0	10	0	0	0	0	5	1	0
b	2	4	5	0	10	0	5	1	0	0	0	0
c	9	8	6	0	0	0	0	0	1	0	5	10
d	6	9	1	1	0	0	0	0	10	0	0	5
e	6	4	7	0	0	0	5	0	10	1	0	0
f	6	7	9	0	0	0	0	0	10	5	0	1
g	6	8	7	0	0	0	0	0	10	1	5	0
h	1	7	5	10	0	0	0	1	0	5	0	0
I	3	7	4	0	0	10	1	0	0	5	0	0
j	1	9	8	10	0	0	0	0	0	0	1	9

See file Assignment Problems.xlsx (Job Choices)

Indicator Matrix

- Final settings showing indicator values and constraint values.

	1	2	3	4	5	6	7	8	9	10	Check
	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
Check	1	1	1	1	1	1	1	1	1	1	

The Travelling Salesperson Problem

A salesperson wants to find the least costly route for visiting clients in n different cities, visiting each city exactly once before returning home.

n	*(n-1)!*
3	*2*
5	*24*
9	*40,320*
13	*479,001,600*
17	*20,922,789,888,000*
20	*121,645,100,408,832,000*

Ragsdale, C.. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) , Cengage Learning Ragsdale (2021, 9e), chap. 8, sec. 8.14

Example: The Traveling Salesperson Problem

- Wolverine Manufacturing needs to determine the shortest distance for a drill bit to drill 9 holes in a fiberglass panel.

See file TSP.xlsm

Note: This is a Non-linear Programming (NLP) problem.

Ragsdale, C.. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) , Cengage Learning. Ragsdale (9e, 2021), chap. 8

End of Lecture 6

References:

Ragsdale, C. (2021). 9th edition, chapter 5,

Ragsdale, C. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) Cengage Learning: Chapter 8

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 12

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning: Chapter 10

Homework

- Go through today's lecture examples :
  - ✓ Familiarise yourself with the different algorithms used:
    - ❖ Northwest Corner Method
    - ❖ Vogel's Approximation Method
    - ❖ MODI (Closed-loop) Method
  - ✓ Understand how the spreadsheets are being modeled – for Assignment problems and Transportation problems

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- ***Readings for next Lecture:***
- C. T. Ragsdale (9th edn), chapter 8, secs. 8.4 – 8.5, Economic Order Quantity (EOQ)
- Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 15 - Inventory Decisions under Certainty

Tutorial 5 this week:

Network Modelling:

- The Shortest Route Problem
- Maximal Flow Problem
- Minimal Spanning Tree Problem