Modelling and Solving LP Problems in a Spreadsheet

- The steps in implementing an LP Model in a spreadsheet
- Goals for good spreadsheet design
- Some examples in building spreadsheet models

Introduction

- Solving LP problems graphically is only possible when there are two decision variables
- Few real-world LP have only two decision variables
- Fortunately, we can now use spreadsheets to solve multi-variate LP problems
- Use spreadsheets to solve LP problems
  - The Solver in Excel.
  - The Simplex algorithm implemented in Excel's Solver searches all vertices to find the optimal solution.
- Main challenges are:
  - To formulate LP problem correctly.
  - Communicate formulation to computer accurately.
  - Use a recognised variable layout.

Software and LP Solvers

- Dozens of programs: some capable of solving problems with up to thousands variables and hundreds constraints
- Well developed interface and input-output facilities
- Special solvers callable from programming languages such as VB/Delphi etc..
- Spreadsheet solvers:
  - Analytic Solver Platform by Frontline Systems http://www.solver.com/student
  - Many Others!

The Steps in Implementing an LP Model in a Spreadsheet

1. Organize the data for the model on the spreadsheet.
2. Reserve separate cells in the spreadsheet for each decision variable in the model.
3. Create a formula in a cell in the spreadsheet that corresponds to the objective function.
4. For each constraint, create a formula in a separate cell in the spreadsheet that corresponds to the left-hand side (LHS) of the constraint.

Let's Implement a Model for the Blue Ridge Hot Tubs Example ...

MAX: $350X_1 + 300X_2$ } Maximise profit

S.T.: $1X_1 + 1X_2 <= 200$ } pumps

$9X_1 + 6X_2 <= 1566$ } labor

$12X_1 + 16X_2 <= 2880$ } tubing

$X_1, X_2 >= 0$ } non-negativity

Where:

- X_1 = # of Aqua-Spas to produce
- X_2 = # of Hydro-Luxes to produce

Implementing the Model

See file Lecture2-1.xls

- **Note the position of:**
  - Variables, Variable names,
  - Constraints (LHS), Constraint names, RHS,
  - Variable (Changing) Cells,
  - The Objective Function (Target cell)
- **Formulas make extensive use of Sumproduct function.**
- **Set X1 and X2 to 0 or 1 as place holders while model is set up.**

Implementing the model

The completed model looks like this. (Refer to Lecture 2-1.xlsm)

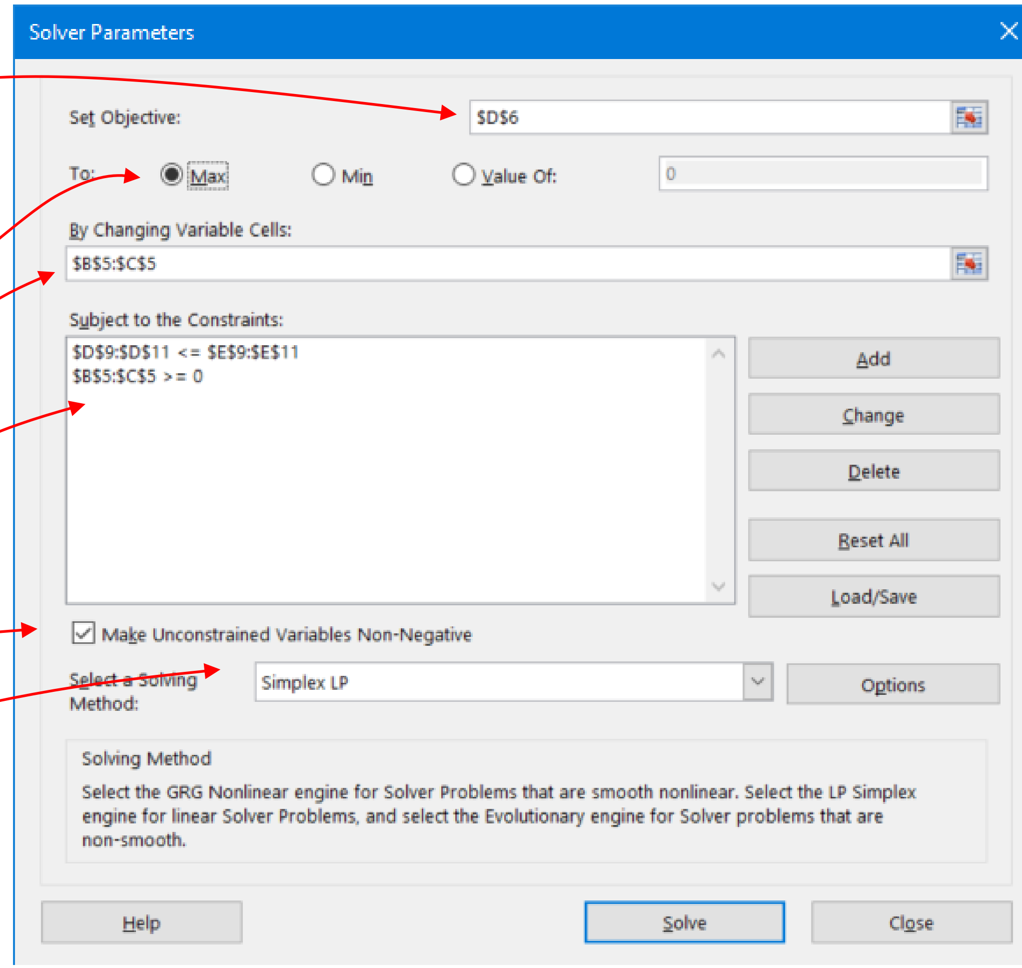
	A	B	C	D	E
1					
2		Blue Ridge Hot Tubs			
3					
4		Aqua-Spas	Hydro-Luxes		
5	Number to Make	0	0	Total Profit	
6	Unit Profits	$350	$300	$0	
7				LHS	RHS
8	Constraints			Used	Available
9	Pumps Req'd	1	1	0	200
10	Labor Req'd	9	6	0	1566
11	Tubing Req'd	12	16	0	2880

How Solver Views the Model

- Target cell - the cell in the spreadsheet that represents the *objective function*
- Changing cells - the cells in the spreadsheet representing the *decision variables*
- Constraint cells - the cells in the spreadsheet representing the *LHS formulas* on the constraints

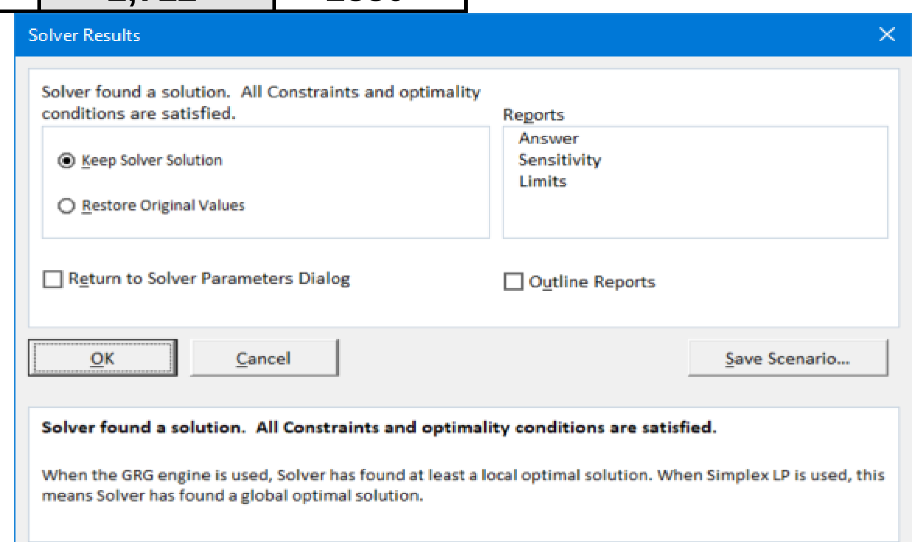
How the Solver views the model

- Data Tab (Analyze Group) → Solver
  - Set Objective (objective function)
  - Min or Max
  - Changing cells (decision variables),
  - Constraint cells: LHS and RHS,
  - Non-negativity
  - Assume linear model



The solution

	Blue Ridge Hot Tubs			
	Aqua-Spas	**Hydro-Luxes**		
Number to Make	**122**	**78**	**Total Profit**	
Unit Profits	**$350**	**$300**	**$66,100**	
Constraints			**Used**	**Available**
- Pumps Required	**1**	**1**	**200**	**200**
- Labor Required	**9**	**6**	**1,566**	**1566**
- Tubing Required	**12**	**16**	**2,712**	**2880**



Goals For Spreadsheet Design

- **Communication** - A spreadsheet's primary business purpose is communicating information to managers.
- **Reliability** - The output a spreadsheet generates should be correct and consistent.
- **Auditability** - A manager should be able to retrace the steps followed to generate the different outputs from the model in order to understand and verify results.
- **Modifiability** - A well-designed spreadsheet should be easy to change or enhance in order to meet dynamic user requirements.

Spreadsheet Design Guidelines - I

- Organize the data, then build the model around the data.
- Do not embed numeric constants in formulas.
- Things which are logically related should be physically related.
- Use formulae that can be copied.
- Column/rows totals should be close to the columns/rows being totaled.

Spreadsheet Design Guidelines - II

- The English-reading eye scans left to right, top to bottom.
- Use color, shading, borders and protection to distinguish changeable parameters from other model elements.
- Use text boxes and cell notes to document various elements of the model.

Make vs. Buy Decisions: The Electro-Poly Corporation

- Electro-Poly is a leading maker of slip-rings.
- A $750,000 order has just been received.

	Model 1	Model 2	Model 3
Number ordered	**3,000**	**2,000**	**900**
Hours of wiring/unit	**2**	**1.5**	**3**
Hours of harnessing/unit	**1**	**2**	**1**
Cost to Make	**$50**	**$83**	**$130**
Cost to Buy	**$61**	**$97**	**$145**

- The company has 10,000 hours of wiring capacity and 5,000 hours of harnessing capacity.

Defining the Decision Variables

M_1 = Number of model 1 slip rings to make in-house

M_2 = Number of model 2 slip rings to make in-house

M_3 = Number of model 3 slip rings to make in-house

B_1 = Number of model 1 slip rings to buy from competitor

B_2 = Number of model 2 slip rings to buy from competitor

B_3 = Number of model 3 slip rings to buy from competitor

Defining the Objective Function

Minimize the total cost of filling the order.

MIN: $50M_1 + 83M_2 + 130M_3 + 61B_1 + 97B_2 + 145B_3$

Defining the Constraints

- Demand Constraints

$M_1 + B_1 = 3{,}000$ } model 1

$M_2 + B_2 = 2{,}000$ } model 2

$M_3 + B_3 = 900$ } model 3

- Resource Constraints

$2M_1 + 1.5M_2 + 3M_3 <= 10{,}000$ } wiring

$1M_1 + 2.0M_2 + 1M_3 <= 5{,}000$ } harnessing

- Non-negativity Conditions

$M_1, M_2, M_3, B_1, B_2, B_3 >= 0$

Implementing the Model

See file [Lecture2-2.xlsm]

Solver Setup

Solver Parameters

Set Objective: $E$11

To: Max Min Value Of: 0

By Changing Variable Cells:

$B$6:$D$7

Subject to the Constraints:

$B$6:$D$7 >= 0
$B$13:$D$13 = $B$14:$D$14
$E$17:$E$18 <= $F$17:$F$18

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	A	B	C	D	E	F
1						
2		Electro-Poly Corporation				
3						
4		------- Slip Ring -------				
5	Number to	Model 1	Model 2	Model 3		
6	Make	0	0	0		
7	Buy	0	0	0		
8						
9	Cost to					
10	Make	$50	$83	$130	Total Cost	
11	Buy	$61	$97	$145	$0	
12						
13	# Available	0	0	0		
14	# Needed	3,000	2,000	900		
15						
16	Hours Required				Used	Available
17	Wiring	2	1.5	3	0	10,000
18	Harnessing	1	2	1	0	5,000

The Solution:

	A	B	C	D	E	F
1						
2		Electro-Poly Corporation				
3						
4		------- Slip Ring -------				
5	Number to	Model 1	Model 2	Model 3		
6	Make	3,000	550	900		
7	Buy	0	1,450	0		
8						
9	Cost to					
10	Make	$50	$83	$130	Total Cost	
11	Buy	$61	$97	$145	$453,300	
12						
13	# Available	3,000	2,000	900		
14	# Needed	3,000	2,000	900		
15						
16	Hours Required				Used	Available
17	Wiring	2	1.5	3	9,525	10,000
18	Harnessing	1	2	1	5,000	5,000

An Investment Problem: Retirement Planning Services, Inc.

- A client wishes to invest $750,000 in the following bonds.

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
Micro Modeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good

Investment Restrictions

- No more than 25% can be invested in any single company.
- At least 50% should be invested in long-term bonds (maturing in 10+ years).
- No more than 35% can be invested in DynaStar, Eagle Vision, and OptiPro as they are considered risky.

Defining the Decision Variables

X_1 = amount of money to invest in Acme Chemical

X_2 = amount of money to invest in DynaStar

X_3 = amount of money to invest in Eagle Vision

X_4 = amount of money to invest in MicroModeling

X_5 = amount of money to invest in OptiPro

X_6 = amount of money to invest in Sabre Systems

Defining the Objective Function

Maximize the total annual investment return:

$$\text{MAX: } .0865X_1 + .095X_2 + .10X_3 + .0875X_4 + .0925X_5 + .09X_6$$

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
Micro Modeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good

Defining the Constraints

- Total amount is invested

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 750{,}000$$

- No more than 25% in any one investment

$$X_i <= 187{,}500, \quad \text{for all } \boldsymbol{i}$$

- 50% long term investment restriction.

$$X_1 + X_2 + X_4 + X_6 >= 375{,}000$$

- 35% Restriction on DynaStar, Eagle Vision, and OptiPro.

$$X_2 + X_3 + X_5 <= 262{,}500$$

- Non-negativity conditions

$$X_i >= 0 \quad \text{for all } \boldsymbol{i}$$

Implementing the Model

	A	B	C	D	E	F	G	H	I
1				Retirement Planning Services, I					
2									
3									
4			Amount	Maximum		Years to	10+ years?		Risky?
5		Bond	Invested	25.0%	Return	Maturity	(1-yes, 0-no)	Rating	(1-yes, 0-no)
6		ACME Chemical	$0	$187,500	8.65%	11	1	1-Excellent	0
7		DynaStar	$0	$187,500	9.50%	10	1	3-Good	1
8		Eagle Vision	$0	$187,500	10.00%	6	0	4-Fair	1
9		MicroModeling	$0	$187,500	8.75%	10	1	1-Excellent	0
10		OptiPro	$0	$187,500	9.25%	7	0	3-Good	1
11		Sabre Systems	$0	$187,500	9.00%	13	1	2-Very Good	0
12		Total Invested:	$0	Total:	$0	Total:	$0	Total:	$0
13		Total Available:	$750,000			Required:	$375,000	Allowed:	$262,500

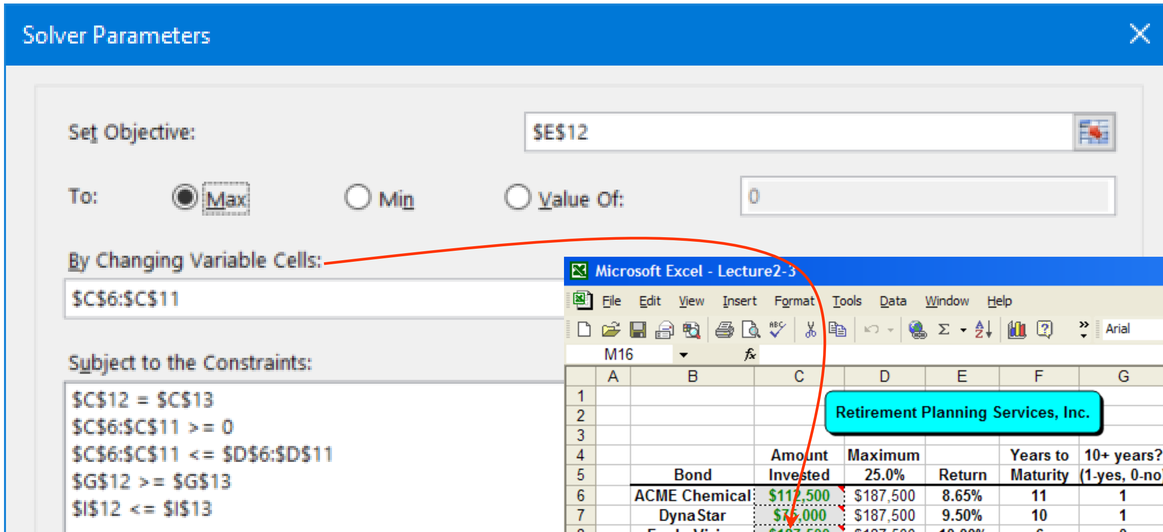
Objective function:
=SUMPRODUCT(E6:E11,$C$6:$C$11)

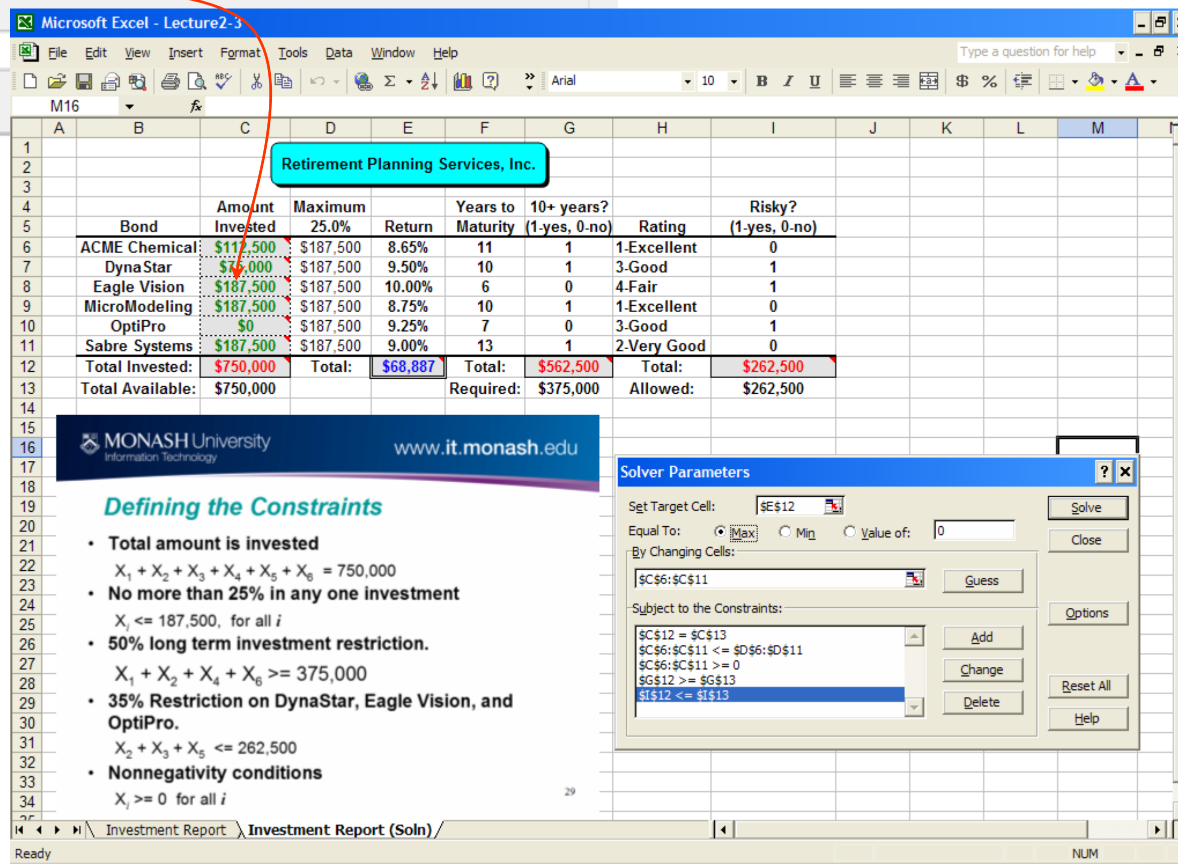
Total investment constraint:
=SUM(C6:C11)=C13

LT investment constraint:
=SUMPRODUCT(G6:G11,$C$6:$C$11)>=G13

Risky constraint:
=SUMPRODUCT(I6:I11,$C$6:$C$11)
<= I13

Solver settings…and the solution:





A Blending Problem: The Agri-Pro Company

- Agri-Pro has received an order for 8,000 pounds of chicken feed to be mixed from the following feeds.

	Percent of Nutrient in			
Nutrient	**Feed 1**	**Feed 2**	**Feed 3**	**Feed 4**
Corn	**30%**	**5%**	**20%**	**10%**
Grain	**10%**	**3%**	**15%**	**10%**
Minerals	**20%**	**20%**	**20%**	**30%**
Cost per pound	**$0.25**	**$0.30**	**$0.32**	**$0.15**

- The order must contain at least 20% corn, 15% grain, and 15% minerals.

Defining the Decision Variables

X_1 = pounds of feed 1 to use in the mix

X_2 = pounds of feed 2 to use in the mix

X_3 = pounds of feed 3 to use in the mix

X_4 = pounds of feed 4 to use in the mix

Defining the Objective Function

Minimize the total cost of filling the order.

$$\text{MIN:} \quad 0.25X_1 + 0.30X_2 + 0.32X_3 + 0.15X_4$$

Defining the Constraints

- Produce 8,000 pounds of feed

  $X_1 + X_2 + X_3 + X_4 = 8,000$

- Mix consists of at least 20% corn

  $(0.3X_1 + 0.05X_2 + 0.2X_3 + 0.1X_4)/8000 >= 0.2$

- Mix consists of at least 15% grain

  $(0.1X_1 + 0.03X_2 + 0.15X_3 + 0.1X_4)/8000 >= 0.15$

- Mix consists of at least 15% minerals

  $(0.2X_1 + 0.2X_2 + 0.2X_3 + 0.3X_4)/8000 >= 0.15$

- Non-negativity conditions

  $X_1, X_2, X_3, X_4 >= 0$

A Comment About Scaling

- Notice the coefficient for X_2 in the ‘corn’ constraint is 0.05/8000 = 0.00000625
- As Solver runs, intermediate calculations are made that make coefficients larger or smaller.
- Storage problems may force the computer to use approximations of the actual numbers.
- Such ‘scaling’ problems sometimes prevents Solver from being able to solve the problem accurately.
- Most problems can be formulated in a way to minimize scaling errors...

Re-Defining the Decision Variables

X_1 = ***thousands of pounds*** of feed 1 to use in the mix

X_2 = ***thousands of pounds*** of feed 2 to use in the mix

X_3 = ***thousands of pounds*** of feed 3 to use in the mix

X_4 = ***thousands of pounds*** of feed 4 to use in the mix

Re-Defining the Objective Function

Minimize the total cost of filling the order.

MIN: $250X_1 + 300X_2 + 320X_3 + 150X_4$

Re-Defining the Constraints

- Produce 8,000 pounds of feed

  $X_1 + X_2 + X_3 + X_4 = 8$
- Mix consists of at least 20% corn

  $(0.3X_1 + 0.05X_2 + 0.2X_3 + 0.1X_4)/8 >= 0.2$
- Mix consists of at least 15% grain

  $(0.1X_1 + 0.03X_2 + 0.15X_3 + 0.1X_4)/8 >= 0.15$
- Mix consists of at least 15% minerals

  $(0.2X_1 + 0.2X_2 + 0.2X_3 + 0.3X_4)/8 >= 0.15$
- Non-negativity conditions

  $X_1, X_2, X_3, X_4 >= 0$

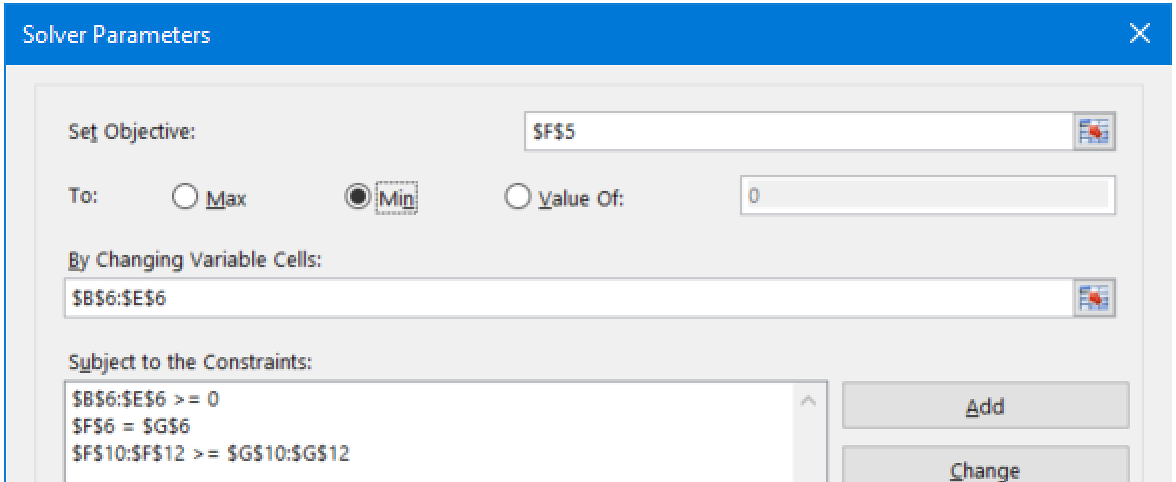
Scaling: Before and After

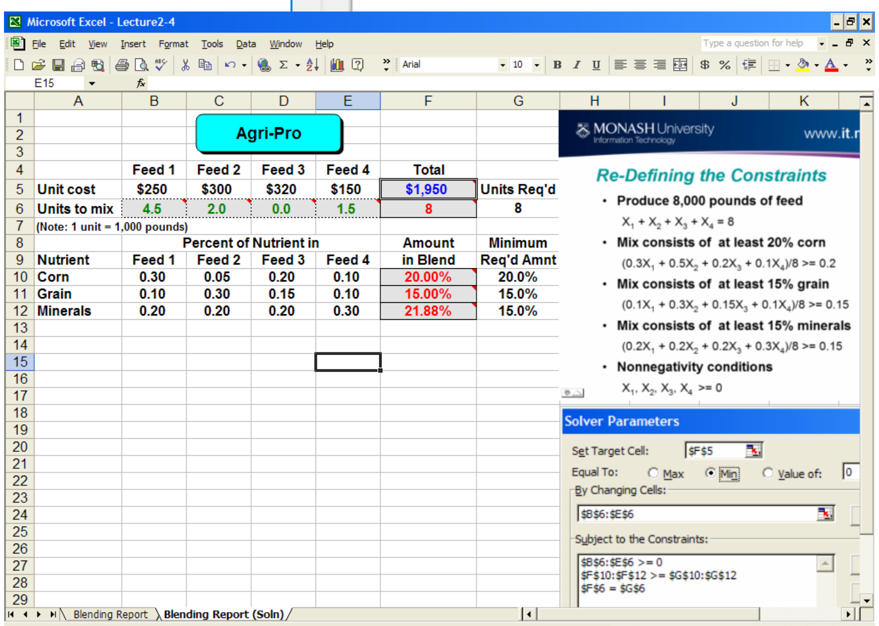
- Before:
  - Largest constraint coefficient was 8,000
  - Smallest constraint coefficient was

    0.05/8 = 0.00000625.
- After:
  - Largest constraint coefficient is 8
  - Smallest constraint coefficient is

    0.05/8 = 0.00625.
- The problem is now more evenly scaled!

Solver settings…and the solution:





A Multi-Period Cash Flow Problem: The Taco-Viva Sinking Fund - I

- Taco-Viva needs a sinking fund to pay $800,000 in building costs for a new restaurant in the next 6 months.
- Payments of $250,000 are due at the end of months 2 and 4, and a final payment of $300,000 is due at the end of month 6.
- The following investments may be used.

Investment	Available in Month	Months to Maturity	Yield at Maturity
A	1, 2, 3, 4, 5, 6	1	1.8%
B	1, 3, 5	2	3.5%
C	1, 4	3	5.8%
D	1	6	11.0%

Summary of Possible Cash Flows

	Cash Inflow/Outflow at the Beginning of Month						
Investment	**1**	**2**	**3**	**4**	**5**	**6**	**7**
A_1	**-1**	**1.018**					
B_1	**-1**	<——>	**1.035**				
C_1	**-1**	<——>	<——>	**1.058**			
D_1	**-1**	<——>	<——>	<——>	<——>	<——>	**1.11**
A_2		**-1**	**1.018**				
A_3			**-1**	**1.018**			
B_3			**-1**	<——>	**1.035**		
A_4				**-1**	**1.018**		
C_4				**-1**	<——>	<——>	**1.058**
A_5					**-1**	**1.018**	
B_5					**-1**	<——>	**1.035**
A_6						**-1**	**1.018**
Req'd Payments (in $1,000s)	**$0**	**$0**	**$250**	**$0**	**$250**	**$0**	**$300**

Defining the Decision Variables

A_i = amount (in \$1,000s) placed in investment A at the beginning of month i=1, 2, 3, 4, 5, 6

B_i = amount (in \$1,000s) placed in investment B at the beginning of month i=1, 3, 5

C_i = amount (in \$1,000s) placed in investment C at the beginning of month i=1, 4

D_i = amount (in \$1,000s) placed in investment D at the beginning of month i=1

Defining the Objective Function

Minimize the total cash invested in month 1.

$$\text{MIN: } A_1 + B_1 + C_1 + D_1$$

Defining the Constraints

- Cash Flow Constraints

$1.018A_1 - 1A_2 = 0$ } month 2

$1.035B_1 + 1.018A_2 - 1A_3 - 1B_3 = 250$ } month 3

$1.058C_1 + 1.018A_3 - 1A_4 - 1C_4 = 0$ } month 4

$1.035B_3 + 1.018A_4 - 1A_5 - 1B_5 = 250$ } month 5

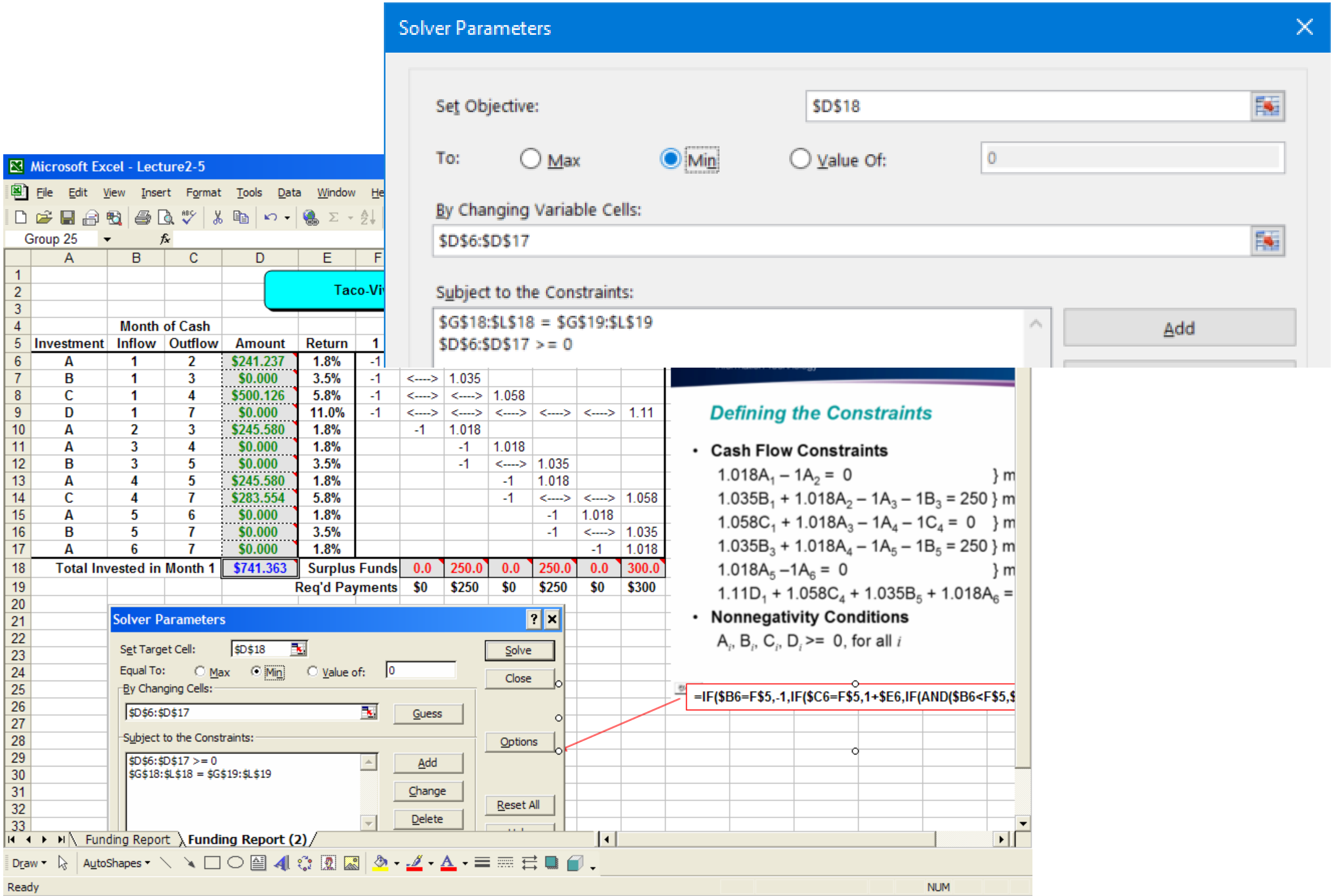
$1.018A_5 - 1A_6 = 0$ } month 6

$1.11D_1 + 1.058C_4 + 1.035B_5 + 1.018A_6 = 300$ } month 7

- Non-negativity Conditions

$A_i, B_i, C_i, D_i >= 0$, for all i

Solver settings…and the solution:



Tutorial 1 this week:

- LP: Graphical Method on two decision variables
  - Maximisation
  - Minimisation

  Multi-variable problem formulation