FIT3158 Note - W3 Sensitivity analysis

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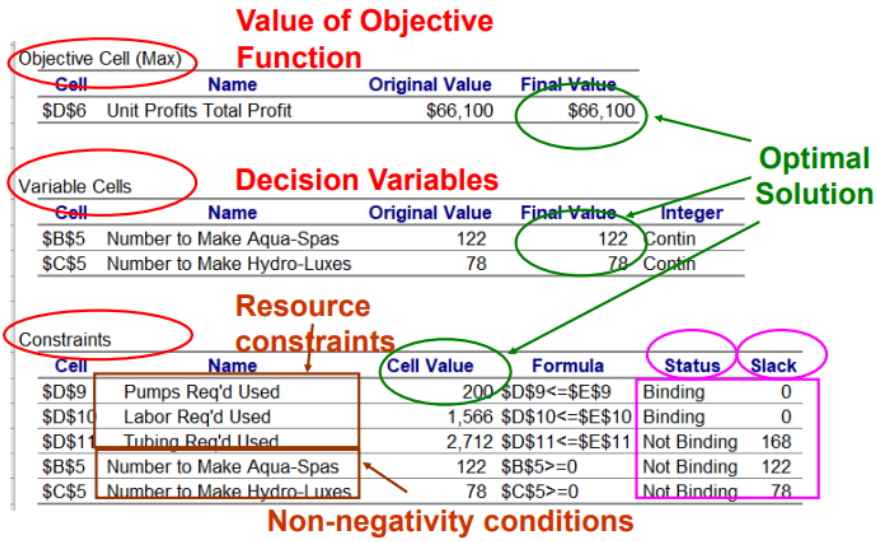
▼ What sorta questions are we answering from the sensitivity report?

- Amounts by which objective function coefficients can change without changing the optimal solution.
- The impact on the optimal objective function value of changes in constrained resources.
- The impact on the optimal objective function value of forced changes in decision variables.
- The impact changes in constraint coefficients will have on the optimal solution.

▼ Excel notation (Answer report) — optimal solution, final value of decision variables, resource usage



Section 1 and 2 together tell us the optimal values for the decision variables and the optimal value of the objective function (e.g. profit or return).

Section 3 gives info. on the constraints:

- Resource constraints:
- Which are binding i.e. which resources are all used up
- Which are non-binding i.e. some of the resource is not used
- Non-negativity constraints
- Which decision variables have values greater than their lower bound of zero

- Section 1 and 2 together tell us the optimal values for the decision variables and the optimal value of the objective function (e.g. profit or return).
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  - Resource constraints:
  - Which are binding – i.e. which resources are all used up
  - Which are non-binding – i.e. some of the resource is not used
  - Non-negativity constraints
  - Which decision variables have values greater than their lower bound of zero

- Sections 1 and 2 tell us the optimal values for the decision variables
  - X_1= Aqua-spas = 122
  - X_2 = Hydro-Luxe = 78
  - The optimal value of the objective function = $66,100
- Section 3 gives info on the constraints:
  - Resource constraints- Which are binding/non-binding (i.e. which resources are all used up and which are not.)
  - Pumps and labour are binding.
  - Tubing is non-binding.
  - Non-negativity constraints: Both decision variables (Aqua-spas and Hydro-Luxes) are greater than their lower bound of zero.

▼ Excel notation (Sensitivity report) — 1) Adjustable cells – final value, reduced cost, allowable increase/decrease; 2) Constraints – final value, shadow price, allowable increase/decrease;

The sensitivity report consists of 2 sections:

- The variable cells section
  - What happens when we change the values of the coefficients in the objective function
  - What happens if we include variables which are not part of the optimal solution
- The constraints section
  - What happens to the value of the objective function if we increase (or decrease) the amount of available resources

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Lecture 3.xlsm]Production Report (Solution)
Report Created: 4/08/2018 12:24:15 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number to Make Aqua-Spas	122	0	350	100	50
C5	Number to Make Hydro-Luxes	78	0	300	50	66.6667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	Pumps Req'd Used	200	200	200	7	26
D10	Labor Req'd Used	1566	16.6667	1566	234	126
D11	Tubing Req'd Used	2712	0	2880	1E+30	168

▼ Variable section

若然RC 係-VE,
即係蝕錢,
無理由繼續produce

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number to Make Aqua-Spas	122	0	350	100	50
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▼ Further explanation of Reduced Costs

Def.:

$$\textit{The reduced cost for each product} = (\textit{its per-unit marginal profit}) - (\textit{the per-unit value of the resources it consumes (priced at their shadow prices)}).$$

- Products whose marginal profits are less than the marginal value of the goods required for their production will not be produced in an optimal solution (unless a constraint forces the product to be produced)
- Reduced costs tell us by how much we would have to increase the profitability of an item before it would be included in the optimal solution (for a maximisation problem). And vice-versa for a minimisation problem.

- Tells the amount by which each objective function coefficient can change without affecting the optimal solution – assuming all other coefficients remain constant.
- Max. increase in profit for Aqua-Spas is $100 Max. decrease in profit for Aqua-Spas is $50

值多有多$100 Increase
or
$50 decrease (without changing optimal sol.)

- In the absence of degeneracy, any zero value in Allowable Increase or Decrease for any objective function coefficient indicates alternate optimal solutions exist.

如果其中一項有0,
即代表有alternative sol.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number to Make Aqua-Spas	122	0	350	100	50
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若然RC 係-VE,
即係蝕錢,
無理由繼續produce

如果呢兩個其中一個係0,
亦即表示有alternate solution

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number to Make Aqua-Spas	122	0	350	100	50
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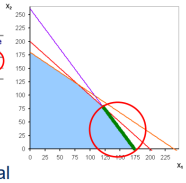
Alternate Optimal Solutions

Note: Values of zero (0) in "Allowable Increase" or "Allowable Decrease" columns for the Changing Cells indicate that an alternate optimal solution exists

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number to make Aqua-Spas	174	0	450	1E+30	0
C5	Number to make Hydro-Luxes	0	0	300	0	1E+30



e.g. (122,78) and (174,0) are both optimal

Alternate Optimal Solutions

▼ Constraint section

▼ Why is the shadow price always 0 when `constraint = Non-binding` ?

(because adding more of resource that has not been used up is no help!

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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Constraints

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D11	Tubing Req'd Used	2712	0	2880	1E+30	

If we go up one more pump,
our total profit is gonna increase by 200.
So, the objective function value will
change by 200 for a unit increase in the constraint right hand side

你爭係允許（加/減）甘多unit

E.g 7 Allowable Incr. in Pump
So, our total profit will go up
for each added pump
only up to an increase of
seven.

Beyond seven we cannot tell

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number to Make Aqua-Spas	122	0	350	100	50
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Constraints

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If we go up one more pump,
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So, the objective function value will
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Shadow Price

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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C5	Number to Make Hydro-Luxes	78	0	300	50	66.6667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
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D11	Tubing Req'd Used	2712	0	2880	1E+30	

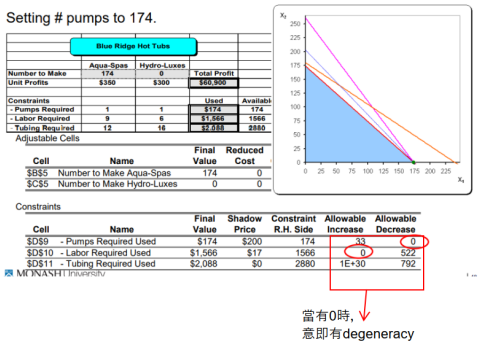
- Shadow prices for non-binding constraints are always zero.

If we go up one more pump,
our total profit is gonna increase by 200.
So, the objective function value will
change by 200 for a unit increase in the constraint right hand side

Def. : the amount by which the objective function value changes given a unit increase in the RHS value of the constraint, assuming all other coefficients remain constant.

▼ Degenerate

- Setting # pumps to 174.



當有0時,
意即有degeneracy

When the solution is degenerate:

1. The methods mentioned earlier for detecting alternate optimal solutions cannot be relied upon.
(你吾可以好似之前甘interpret 成有alternate sol from variable sections, in which when Allow. Incr. or decr ge value 係 0.)

2. The reduced costs for the changing cells may not be unique. Also, the objective function coefficients for changing cells must change by at least as much as (and possibly more than) their respective reduced costs before the optimal solution would change.

3. The allowable increases and decreases for the objective function coefficients still hold and, in fact, the coefficients may have to be changed beyond the allowable increase and decrease limits before the optimal solution changes.

4. The given shadow prices and their ranges may still be interpreted in the usual way but they may not be unique.
That is, a different set of shadow prices and ranges may also apply to the problem (even if the optimal solution is unique!). (可能有第二種scenario with diff. Shadow price.)

▼ The 100% Rule determines if the current solution remains optimal, when more than one objective function coefficient changes.

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I11	1: Def OzMade LHS	0	-210	0	0.4	0.6
I12	2: Demand for drug LHS	7	90	7	0.4	0.6
I13	3: Senior JobTime limit LHS	0	30	0	1.2	1E+30
I14	4: Junior JobTime limit LHS	-1.2	0	0	1E+30	1.2
I15	5: Tech < Junior LHS	0	1	0	6	4
I16	6: Senior regular hours LHS	8	25	8	1.2	0.8
I17	7: Junior regular hours LHS	8	13	8	0.75	1.333333333
I18	8: Senior overtime limit LHS	1.2	0			
I19	9: Junior overtime limit LHS	0	0			

If we decrease 1 unit,
the sp will be decreeased by 13
Profit decreases by $13

is this combination going to preserve
shadow presses the individual

Sensitivity Report 1 | Sheet1

Now, we can examine if the allowable dec/inc is changed *per resource*, how much will it affect our obj. function (e.g., profit).

Problem: But what if I wanna examine the if the current solution remains optimal WHEN there is more than one objective function coefficient changes?

Solution: Then we need The 100% rule.

Two situations could arise when applying this rule:

- Case 1. All variables whose objective function coefficients change have nonzero reduced costs. (全部RC 唔係0)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C5	Qty Doors	20	0	500	300	233.3333333
D5	Qty Windows	40	0	400	350	150

Case 2 : 最少一個0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E9	Cutting Used	40	350	40	40	13.33333333
E10	Sanding Used	40	300	40	6.666666667	20
E11	Finishing Usec	50	0	60	1E+30	10

Case 2. At least one variable whose objective function coefficient changes has a reduced cost of zero.

How to perform:

Let's say we wanna see if the sol. remains optimal, if we
1. decrease 1 unit in constraint 9
2. increase 2 units in constraint 5

19			Final	Shadow	Constraint	Allowable	Allowable
20	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
21	I11	1: Def OzMade LHS	0	-210	0	0.4	0.6
22	I12	2: Demand for drug LHS	7	90	7	0.4	0.6
23	I13	3: Senior JobTime limit LHS	0	30	0	1.2	1E+30
24	I14	4: Junior JobTime limit LHS	-1.2	0	0	1E+30	1.2
25	I15	5: Tech < Junior LHS	0	1	0	6	4
26	I16	6: Senior regular hours LHS	8	25	8	1.2	0.8
27	I17	7: Junior regular hours LHS	8	13	8	0.75	1.333333333
28	I18	8: Senior overtime limit LHS	1.2	0	2	1E+30	0.8
29	I19	9: Junior overtime limit LHS	0	0	2	1E+30	2
30	所以當我地sum these two (.333 + .5) <1,			0.33333			
31	sol. 係optimal, vice versa			=1/2			

changes is 2 (+2) = 2/allow. inc (6) = .3333
changes is 1 (-1) = 2/allow. inc (2) = .5

Sensitivity Report 1 | Sheet1

and the allowable decrease on the right
hand side for constraint nine is two

▼ Excel notation (Limits report) — how the objective function varies as each variable ranges between its limits;

Cell	Target Name	Value
D6	Unit Profits Total Profit	$66,100

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
B5	Number to Make Aqua-Spas	122	0	23400	122	66100
C5	Number to Make Hydro-Luxes	78	0	42700	78	66100

Summarises the value of the objective function as each variable cell ranges through its limits (and the others remain constant).

▼ To be studied further:

- Comments About Changes in Constraint RHS Values (p.25 - p. 27)
- Further explanation of Reduced Costs

$$The\ reduced\ cost\ for\ each\ product\ = \\ (its\ per-unit\ marginal\ profit)\ -\ (the\ per-unit\ value\ of\ the\ resources\ it\ consumes\ (priced\ at\ their\ shadow\ prices).$$

10 M + 15O + 10T ≤ 60