

FIT3158 Note - W6 Degeneracy in Transportation Problem (MODI)

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▼ More exercises

[Transportation Problem | Set 7 \(Degeneracy in Transportation Problem \) - GeeksforGeeks](#)

▼ [Degeneracy: Transportation Problem \(universalteacherpublications.com\)](#)

Table 1

Factory	Dealer				Supply
	1	2	3	4	
A	2 ⁹⁰⁰	2 ¹⁰⁰	2	4	1000
B	4	6 ⁷⁰⁰	4	3	700
C	3	2	1 ⁵⁰⁰	0 ⁴⁰⁰	900
Requirement	900	800	500	400	

Lowest value and cannot form the loop

▼ Degeneracy in Transportation Problem (With Examples). | Operations Research (engineeringnotes.com)

Table 2 :

Destination Source	D ₁	D ₂	D ₃	D ₄	D ₅	Supply
S ₁	5	8	⑤ 6	6	③ 3	8
S ₂	④ 4	7	7	① 6	④ 5	5
S ₃	8	④ 4	6	6	⑤ 4	9
S _{success}	0	0	0	③ 0	0	3
Demand	4	4	5	4	8	25

This solution includes occupied cells but in a rule there will be $m + n - 1 = 5 + 4 - 1 = 8$ occupied cell.

lowest cost and cannot form loop

DEGENERACY IN TRANSPORTATION PROBLEMS in Quantitative Techniques for management Tutorial 27
(This example is weird!)

Example from (Here) YouTube illustrates how to solve degeneracy in transportation problem.

▼ Its explanation in pdf is here:

<https://s3-us-west-2.amazonaws.com/secure.notion-static.com/95414ef1-70b0-4509-a4f6-5d1e89095345/TP-Degeneracy-LeastCost-MODI.pdf>

		Destination (Retail Agency)			Availability
		A1	A2	A3	
Source (Factory)	F1	8	7	3	60
	F2	3	50	9	20
	F3	11	3	80	5
Demand		50	80	80	210

Suppose we used least cost / vogel to get this feasible solution. Now our task is to find the optimal one using MODI method.

When will we know the solution has degeneracy?

Step 1 of MODI: Check if there is a degeneracy.

		Destination (Retail Agency)			Availability
		A1	A2	A3	
Source (Factory)	F1	8	7	3	60
	F2	3	50	9	20
	F3	11	3	80	5
Demand		50	80	80	210

Using formula of $(m+n-1)$, 我地就知道有無degenerate.
e.g., 呢到 $M+N = 6$; $(m+n-1) = 5$
而呢到 #of allocated cell = 4
因為 $m+n-1 (5) \neq \text{#of allocated cell } (4)$, 所以degenerate

Now, we know there is a degeneracy. We want to find independent cells on which it can solve the problem.

What is independent cell?

Step 2 of MODI: Given there is a degeneracy, find independent cells on to solve the problem.

2 conditions:

1. Unallocated cells that cannot form a closed loop
2. Has the lowest value (e.g., cost)

Destination (Retail Agency)

Source (Factory)

	A1	A2	A3	Availability
F1	8	7	3	60
F2	3	8	9	70
F3	11	3	5	80
Demand	50	80	80	

不符合1, 2.
1. It can form a loop
2. C₃(F3, A3) has the lowest cost

Destination (Retail Agency)

Source (Factory)

	A1	A2	A3	Availability
F1	8	7	3	60
F2	3	8	9	70
F3	11	3	5	80
Demand	50	80	80	

符合1, 2.
1. 周圍form 吾到loop
2. 5 is the lowest cost of unallocated cells

Step 3 of MODI: After confirming the independent cell, the degenerate issues is solved.

$V_1 =$ $V_2 =$ $V_3 =$

$u_1 =$
 $u_2 =$
 $u_3 =$

	A1	A2	A3	Availability
F1	8	7	3	60
F2	3	8	9	70
F3	11	3	5	80
Demand	50	80	80	

$m+n-1 = 3+3-1 = 5$
呢到有5個allocated cells
所以無degeneracy

Step 4 we continue doing the MODI, until the optimality is reached.

Here is the solution:

$V_1 = -3$ $V_2 = 1$ $V_3 = 3$

$u_1 = 0$
 $u_2 = 6$
 $u_3 = 2$

	A1	A2	A3	Availability
F1	8	7	3	60
F2	3	8	9	70
F3	11	3	5	80
Demand	50	80	80	

Queuing, Probability and Simulation

Service and waiting times for a single server queue, Poisson arrivals, Exponential service:

λ = the average number of arrivals per time period (arrival rate)

$\frac{1}{\lambda}$ = the average time between arrivals

μ = the average number of services per time period (service rate)

$\frac{1}{\mu}$ = the average time taken for each service

$P_0 = 1 - \frac{\lambda}{\mu}$ the probability that no units are in the system

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ the average number of units in the waiting line

$L = L_q + \frac{\lambda}{\mu}$ the average number of units in the system

$W_q = \frac{L_q}{\lambda}$ the average time a unit spends in the waiting line

$W = W_q + \frac{1}{\mu}$ the average time a unit spends in the system

$P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service

$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ the probability of n units in the system

Probability distributions:

The Poisson distribution

$f(x) = \frac{\theta^x e^{-\theta}}{x!}$ for a distribution having mean θ , ($e = 2.71828...$)

The exponential distribution

$f(x) = \frac{1}{\theta} e^{-x/\theta}$ for a distribution having mean θ , ($e = 2.71828...$)

$P(x \leq x_0) = 1 - e^{-x_0/\theta}$

$P(x \geq x_0) = e^{-x_0/\theta}$ for a given value of x_0

Linear congruential generation of uniform random variables

Let X_0 be an integer chosen at random (the random seed) then uniformly distributed integers are generated as $X_{n+1} = AX_n \bmod B$ where A and B are large co prime integers. Random numbers between 0 and 1 are calculated as $r_n = \frac{X_n - 1}{B - 2}$.

Generation of Exponentially distributed random variables

Exponential variates with mean b are generated from uniform [0,1] random numbers, r_n , by the transformation $t_n = -b \log_e(r_n)$.

Service and waiting times for an M/M/S queue:

$$P_0 = 1 / \left[\sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!} \left(\frac{1}{1 - \lambda/S\mu} \right) \right]$$

$$L = L_q + \frac{\lambda}{\mu}$$

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & \text{if } 0 \leq n \leq S \\ \frac{(\lambda/\mu)^n}{S! S^{n-S}} P_0 & \text{if } n \geq S \end{cases}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1}{\mu}$$

$$L_q = \frac{(\lambda/\mu)^S (\lambda/S\mu)}{S! (1 - \lambda/S\mu)^2} P_0$$

$$\rho = \frac{\lambda}{S\mu}$$

Queuing, Probability and Simulation
Service and waiting times for a single server queue, Poisson arrivals, Exponential service:
 λ = the average number of arrivals per time period (arrival rate)
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 $P_0 = 1 - \frac{\lambda}{\mu}$ the probability that no units are in the system
 $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ the average number of units in the waiting line
 $L = L_q + \frac{\lambda}{\mu}$ the average number of units in the system
 $W_q = \frac{L_q}{\lambda}$ the average time a unit spends in the waiting line
 $W = W_q + \frac{1}{\mu}$ the average time a unit spends in the system
 $P_w = \frac{\lambda}{\mu}$ the probability that an arriving unit has to wait for service
 $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ the probability of n units in the system