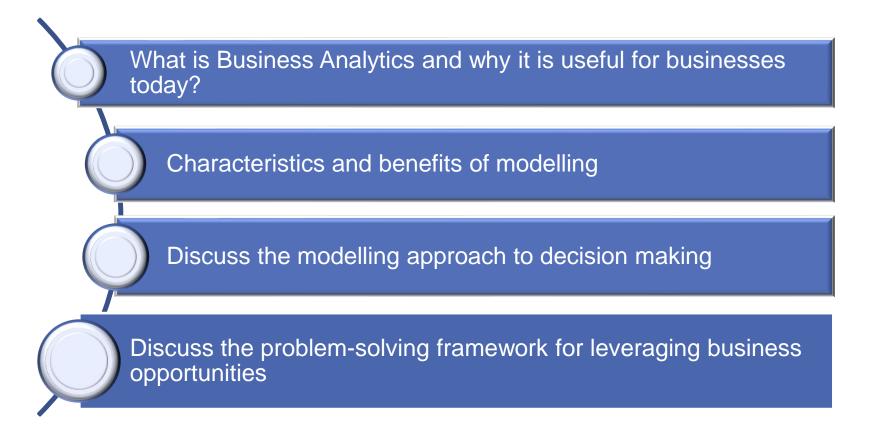
# Part 1 Introduction to Modelling and Decision Analysis



#### Introduction

- We face numerous decisions in life and business. Its not always easy to make good decision.
  - For any given situation there are often numerous possible courses of action.
- Decision analysis
  - A means of systematically identifying, evaluating & choosing best course of action.
- We can use computers to analyse the potential outcomes of decision alternatives.

# What is Business Analytics?

- A field of study that uses computers, statistics, and mathematics to solve business problems.
- Considerable overlap with:
  - Operations Research
  - Management Science
  - Decision Science

#### Chevron

- Developed optimization tool for
  - Operational & strategic planning
  - Mixing crude oils
  - Planning capital expenditures
- Benefits:
  - Annual savings of \$1.0 billion

#### Dell

- Built analytics models to
  - Optimize hardware configurations
  - Optimize its website's design
  - Optimize promotion design and timing
- Benefits:

Increased profit by \$142 million

#### Kroger

- Analytics team created models to determine reorder point
   & order up to quantities for items in 1,950 in-store
   pharmacies
- Benefits:
  - Reduced prescription stock outs by 1.6 million
  - Lowered inventory by over \$120 million
  - Increased revenue by \$80 million

- National Broadcast Network Company
  - Gov't owned broadband provider in Australia
  - Developed tool to optimize design of network servicing 8 million locations
  - Benefits:
    - Reduction in network design time
    - Savings of ~\$1.7 billion

- Alliance for Paired Donations (APD)
  - Transplant patients needing a kidney often have potential donors who are incompatible
  - APD finds exchanges with other patient-donor pairs to optimize paired matches
  - Benefits:
    - 220 lives saved since 2006
    - Value of savings: <u>priceless</u>

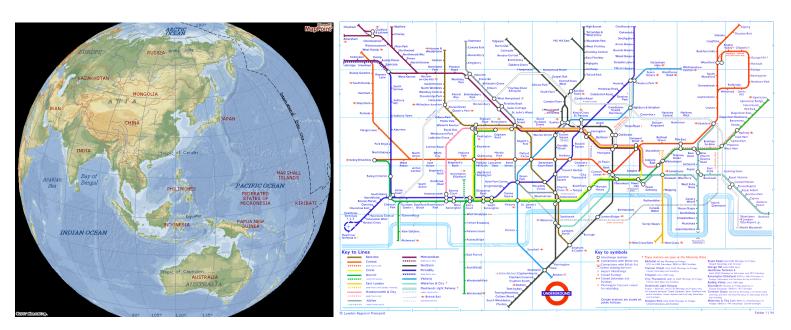
# A Modelling Approach to Decision Making

- A model is
  - A structure which has been built purposely to exhibit features and characteristics of some other objects
- Everyone uses models to make decisions
- Types of models:
  - Mental (arranging furniture)
  - Visual (blueprints, road maps)
  - Physical/Scale (aerodynamics, buildings)
  - Mathematical (what we'll be studying)



#### **Characteristics of Models**

- Usually <u>simplified</u> versions of things they represent
- A <u>valid</u> model accurately represents the <u>relevant</u> <u>characteristics</u> of the object or decision problem being studied





#### A more formal definition

- A model is a device which behaves approximately like part of the real world:
  - Train-sets, dolls, toy prams...
  - Small aircraft replicas in wind tunnels...
  - Sponge-rubber men used in car crash…
  - The solar system as a number of small balls moving in ellipses round a larger ball...



## **Models for Decision Making**

#### May include:

- Mathematical equations
  - Mathematical Models
- Structured set of rules or steps to follow
  - Algorithms and Discrete Event processes
- Simulation of real events
  - Queuing models, waiting lines
- Statistical techniques
  - Data Analysis, Equation Fitting



# Modelling: Why bother?

- Real Understanding:
  - The process may reveal hidden inner relationships within the system.
  - Formal Analysis (assumptions/relationships)
  - Experimentation can be used repeatedly
  - To large extent, independent of the data
     Relisten the explaination
  - Variety of models for the same problem



# **Benefits of Modelling**

#### Economy

Less costly to analyze decision problems.

#### Timeliness

 Deliver needed information more quickly than their realworld counterparts.

#### Feasibility

Used to do things that would be impossible in reality.

#### Insight & understanding

That improves decision making.

# **Advantages of Using Models**

- Less expensive and disruptive than experimenting with real world system
- Models enable us to ask 'what if' questions and test the sensitivity of particular values
- Models force a consistent, systematic approach to a problem
- Models require managers to be specific about constraints and goals
- Models can reduce the time taken to make decisions.



# **Disadvantages of Using Models**

- May be expensive to develop and test
- May be misused and misunderstood because of their complexity (e.g., there may be too much variables)
- Models may down play the value of non-quantifiable information
- May oversimplify the variables of the real world



### What is a "Computer Model"? = Spreadsheet model

- A set of mathematical relationships and logical assumptions implemented in a computer as an abstract representation of a real-world object or phenomenon.
- Spreadsheets provide the most convenient way for business people to build computer models
  - Spreadsheets are commonplace on most computers.
  - They are the tool of choice for today's managers.
  - They facilitate decision-making process by making it easier to play out various what-if scenarios.

## **Example of a Mathematical Model**

Profit = Revenue - Expenses or Profit = f(Revenue, Expenses)or  $Y = f(X_1, X_2)$ 

Dependent variable

Independent variables

#### **A Generic Mathematical Model**

$$Y = f(X_1, X_2, ..., X_n)$$

Where:

Y is DEPENDENT for the variable of X\_i

Y =dependent variable

(aka bottom-line performance measure)

 $X_i = independent variables$ 

(inputs having an impact on Y)

 $f(\cdot)$  = function defining the relationship between Y &  $X_i$ 



# **Mathematical Models & Spreadsheets**

 Most spreadsheet models are very similar to our generic mathematical model:

$$Y = f(X_1, X_2, ..., X_n)$$

• Most spreadsheets have input cells (representing  $X_i$ ) to which mathematical functions ( $f(\cdot)$ ) are applied to compute a bottom-line performance measure (or Y).

# **Categories of Mathematical Models**

| Model<br>Category | Relationship $\mathbf{Form} \ \mathbf{of} \ f(\cdot)$ | Independent<br>Variables | OR/MS<br>Techniques                                |
|-------------------|---|--------------------------|--|
| Prescriptive      | known,  | known or under           | LP, Networks, IP,                                  |
|                   | well-defined  | decision maker's         | CPM, EOQ, NLP,                                     |
|                   |   | control                  | GP, MOLP   |
| Predictive        | unknown,  | known or under           | Regression Analysis,                               |
|                   | ill-defined   | decision maker's         | Time Series Analysis,                              |
|                   |   | control                  | Discriminant Analysis                              |
| Descriptive       | known,<br>well-defined                                | unknown or<br>uncertain  | Simulation, PERT,<br>Queueing,<br>Inventory Models |

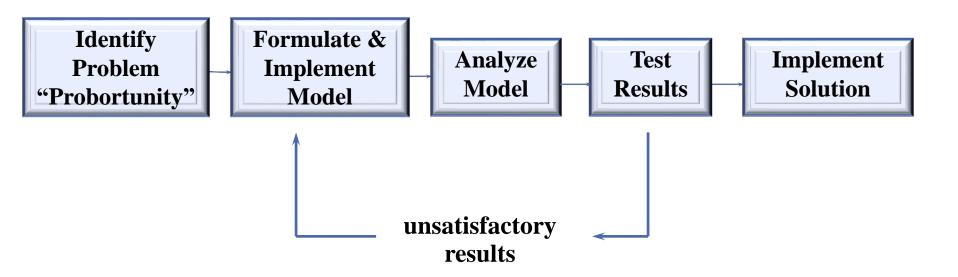
#### How to Use a Model

It wont make a decision for you, but it would help you make a decision

- Don't accept an answer without further analysis and questioning. Get better insight; question our assumption; get relationship between variables - Like BRAINSTORMING
- A mathematical model is only one of a number of tools for decision making.
- If the answer is unacceptable, the reasons for unacceptability should be spelled out and incorporated in a modified model.
- An acceptable answer is only an option.
- By successive questioning of the answers and altering the model, it is possible to clarify the options available and obtain a greater understanding of what is possible in a real situation.



# The Problem-Solving Framework for Leveraging Business Opportunities

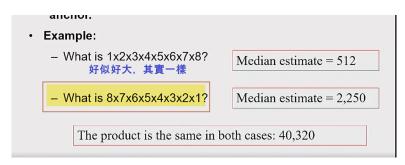


# **Psychology of Decision Making**

- Models can be used for structural aspects of decision problems.
- Other aspects cannot be structured easily, requiring intuition and judgment.
- Caution: Human judgment and intuition are not always rational!
- Errors in human judgment often arise because of anchoring and framing effects associated with decision problems.

#### **Anchoring Effects**

- Arise when trivial factors influence initial thinking about a problem.
- Decision-makers usually under-adjust from their initial "anchor."



# **Framing Effects**

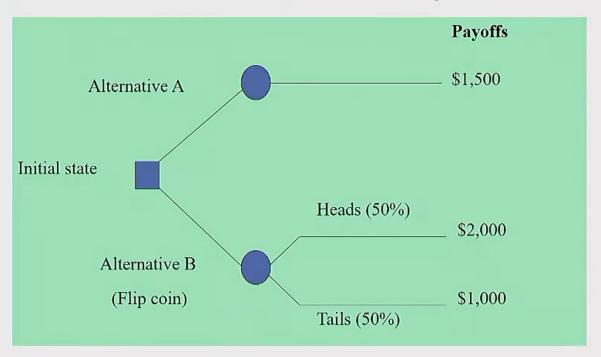
 Refers to how decision-makers view a problem from a win-loss perspective.

The way a problem is framed often influences choices in irrational ways...

#### Framing Effects 1

- Suppose you've been given \$1000 and must choose between:
  - A1. Receive \$500 more immediately |=> Most popular
  - B1. Flip a coin and receive \$1000 more if heads occurs or \$0 more if tails occurs.
- Now suppose you've been given \$2000 and must choose between:
  - A2. Give back \$500 immediately
  - B2. Flip a coin and give back \$0 if heads occurs or give back \$1000 if tails occurs. => Most popular

#### **A Decision Tree for Both Examples**



# Framing Effects 2 DARK PATTERNS!

Which option would you select?

This food is 95% fat free

VS

This food consists of 5% fat

You generally obtain different answers from the same person.

#### **Good Decisions vs. Good Outcomes**

Good decisions do not always lead to good outcomes...



 A structured, modelling approach to decision making helps us make good decisions, but can't guarantee good outcomes.

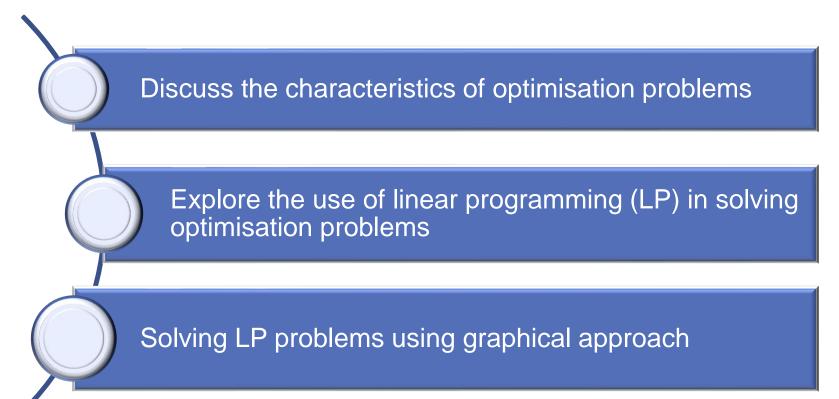
#### What is a good decision?

#### A balance among

- What we know
- What we can do
- What we want

|                     |      | Outcome Quality  |                |  |
|---------------------|------|------------------|----------------|--|
|                     |      | Good             | Bad            |  |
| Decision<br>Quality | Good | Deserved Success | Bad Luck       |  |
|                     | Bad  | Dumb Luck        | Poetic Justice |  |

# Part 2 Introduction to Optimisation and Linear Programming



#### Introduction

- We all face decision about how to use limited resources such as:
  - Oil in the earth
  - Land for dumps
  - Time
  - Money
  - Workers

# **Mathematical Programming...**

- A field of management science that finds the optimal, or most efficient, way of using limited resources to achieve the objectives of an individual of a business.
- Also referred to as ... Optimisation

# **Applications of Optimization**

#### Determining Product Mix

– How many of each product to produce to maximise profits or to satisfy demand at minimum cost?

#### Manufacturing

– Eg. For a circuit board, what is the drilling order that minimises total distance the drill bit must be moved?

#### Routing and Logistics

— What is the **least costly** method of transferring goods from warehouses to stores?

#### Financial Planning

– How much to save in superannuation to minimise tax liability?



# **Characteristics of Optimization Problems**

- Decisions
  - One or more decisions that must be made
- Constraints
  - Due to limited resources
- Objectives
  - Goal that decision maker considers when making decision
  - To maximise profit or minimise cost

## General Form of an Optimization Problem

MAX (or MIN): 
$$f_0(X_1, X_2, ..., X_n)$$

**Objective Function** 

Subject to: 
$$f_{I}(X_{1}, X_{2}, ..., X_{n}) <= b_{1}$$

$$\vdots$$

$$f_{k}(X_{1}, X_{2}, ..., X_{n}) >= b_{k}$$

$$\vdots$$
(Left Hand Side) 
$$f_{m}(X_{1}, X_{2}, ..., X_{n}) = b_{m}$$

Value of constraints

Note: If all the functions in an optimization are linear, the problem is a Linear Programming (LP) problem

# **Linear Programming (LP) Problems**

MAX (or MIN): 
$$c_1X_1 + c_2X_2 + ... + c_nX_n$$

Subject to: 
$$a_{11}X_1 + a_{12}X_2 + ... + a_{1n}X_n \le b_1$$
  

$$\vdots$$

$$a_{k1}X_1 + a_{k2}X_2 + ... + a_{kn}X_n \ge b_k$$

$$\vdots$$

$$a_{m1}X_1 + a_{m2}X_2 + ... + a_{mn}X_n = b_m$$

### **An Example LP Problem**

Blue Ridge Hot Tubs produces two types of hot tubs: Aqua-Spas & Hydro-Luxes.

|                    | Aqua-Spa | Hydro-Lux |
|--------------------|----------|-----------|
| Pumps              | 1        | 1         |
| Labor              | 9 hours  | 6 hours   |
| Tubing             | 12 feet  | 16 feet   |
| <b>Unit Profit</b> | \$350    | \$300     |

There are 200 pumps, 1566 hours of labor, and 2880 feet of tubing available. Decision: Hw many Aqua-Spa and Hydro-Lux can be produced



### 5 Steps In Formulating LP Models:

- 1. Understand the problem.
- 2. Identify the decision variables.

X₁=number of Aqua-Spas to produce

X<sub>2</sub>=number of Hydro-Luxes to produce

3. State the objective function as a linear combination of the decision variables.

MAX:  $350X_1 + 300X_2$ 

# 5 Steps In Formulating LP Models (continued)

4. State the constraints as linear combinations of the decision variables.

$$1X_1 + 1X_2 \le 200$$
 } pumps  
 $9X_1 + 6X_2 \le 1566$  } labor  
 $12X_1 + 16X_2 \le 2880$  } tubing

5. Identify any upper or lower bounds on the decision variables.

$$X_1 >= 0$$

$$X_2 >= 0$$

# LP Model for Blue Ridge Hot Tubs

MAX: 
$$350X_1 + 300X_2$$
  
S.T.:  $1X_1 + 1X_2 \le 200$   
 $9X_1 + 6X_2 \le 1566$   
 $12X_1 + 16X_2 \le 2880$   
 $X_1 >= 0$   
 $X_2 >= 0$ 

### Solving LP Problems: An Intuitive Approach

- Idea: Each Aqua-Spa (X<sub>1</sub>) generates the highest unit profit (\$350), so let's make as many of them as possible!
- How many would that be?

- Let 
$$X_2 = 0$$

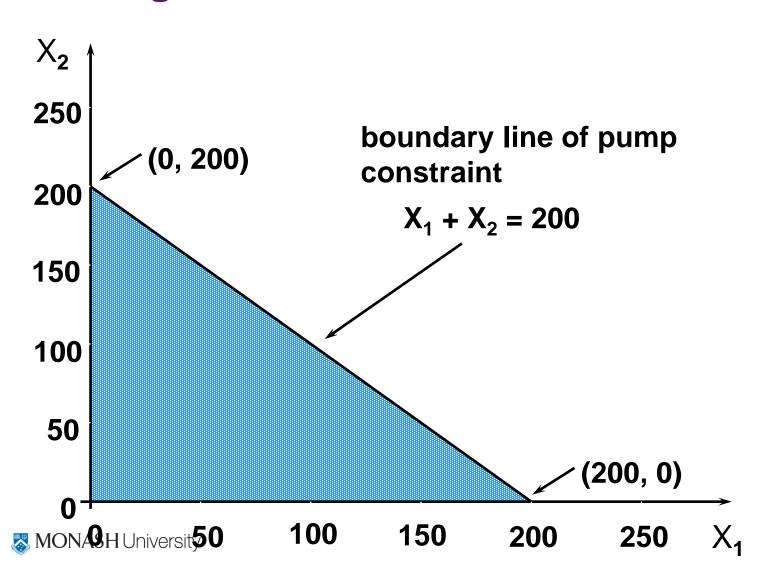
- 1st constraint:  $1X_1 \le 200$
- 2nd constraint:  $9X_1 <= 1566$  or  $X_1 <= 174$
- 3rd constraint:  $12X_1 \le 2880$  or  $X_1 \le 240$
- If  $X_2$ =0, the maximum amount of  $X_1$  you could make is 174 and the total profit is \$350\*174 + \$300\*0 = \$60,900
- This solution is *feasible*, but is it *optimal?*



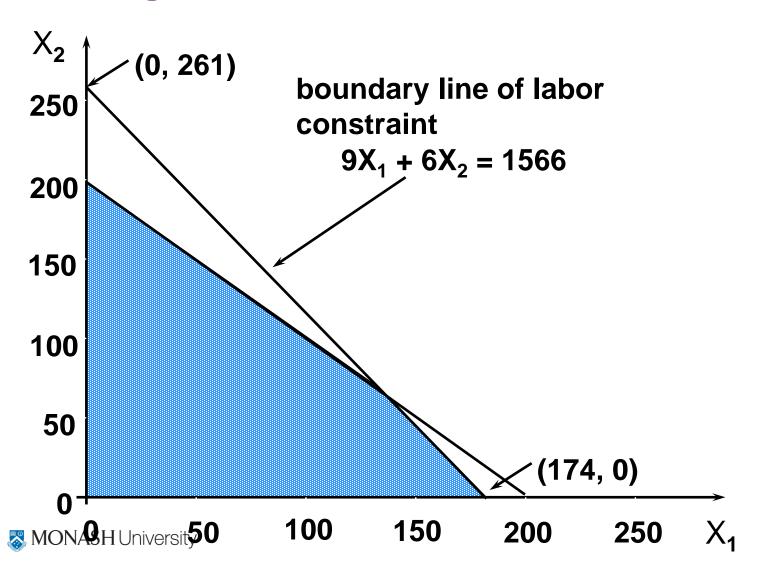
## Solving LP Problems: A Graphical Approach

- The constraints of an LP problem defines its feasible region.
- The best point in the feasible region is the optimal solution to the problem.
- For LP problems with 2 variables, it is easy to plot the feasible region and find the optimal solution.

### **Plotting the First Constraint**

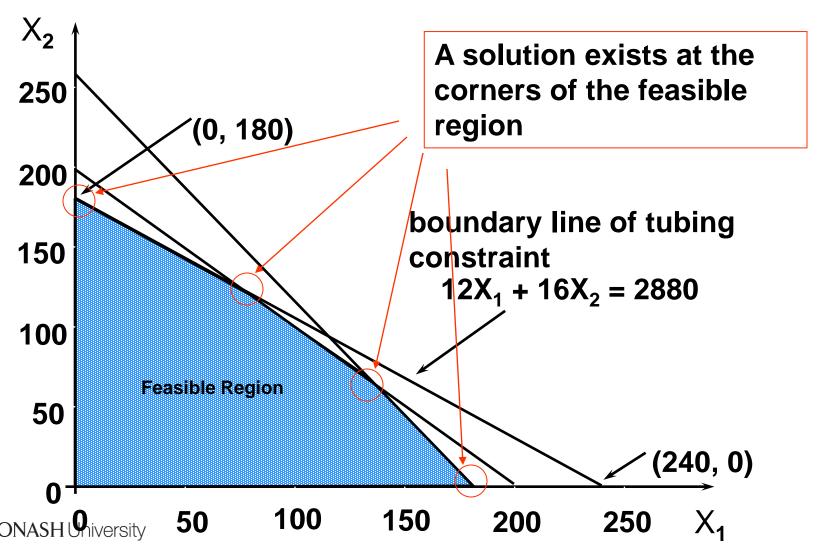


### **Plotting the Second Constraint**

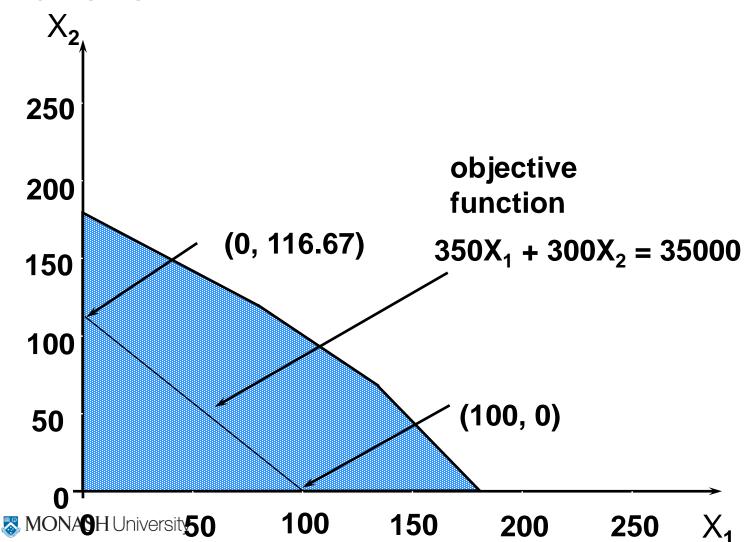


#### **Plotting the Third Constraint**

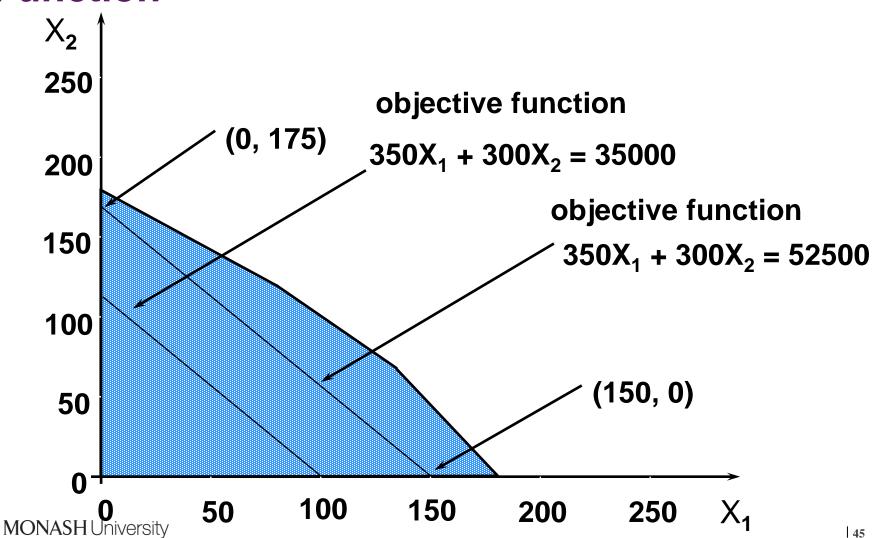
Feasible != Optimal



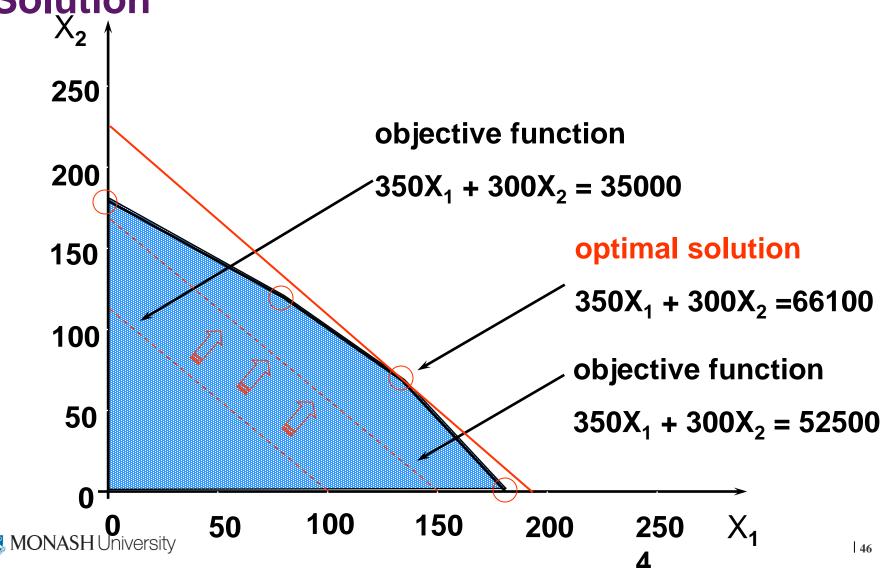
### Plotting A Level Curve of the Objective Function



### A Second Level Curve of the Objective Function



## Using A Level Curve to Locate the Optimal Solution



#### **Calculating the Optimal Solution**

- The optimal solution occurs where the "pumps" and "labor" constraints intersect.
- This occurs where:

$$X_1 + X_2 = 200 \tag{1}$$

and 
$$9X_1 + 6X_2 = 1566$$
 (2)

- From (1) we have,  $X_2 = 200 X_1$  (3)
- Substituting (3) for X<sub>2</sub> in (2) we have,

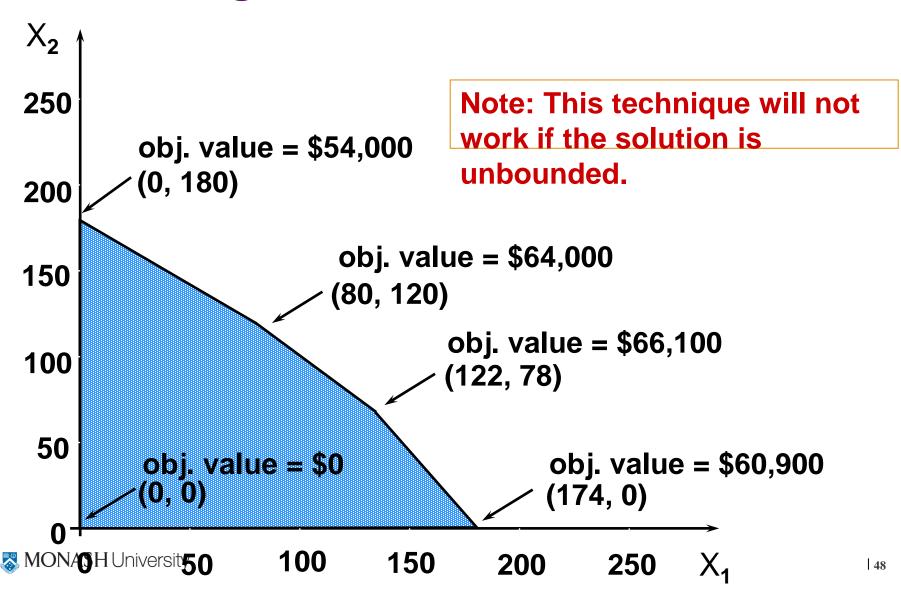
$$9X_1 + 6 (200 - X_1) = 1566$$
  
 $\rightarrow X_1 = 122$ 

So the optimal solution is:

Substituting 
$$X_1 = 122$$
 in  $X_2 = 200 - X_1 \rightarrow X_2 = 78$ 

Total Profit = 
$$$350*122 + $300*78 = $66,100$$

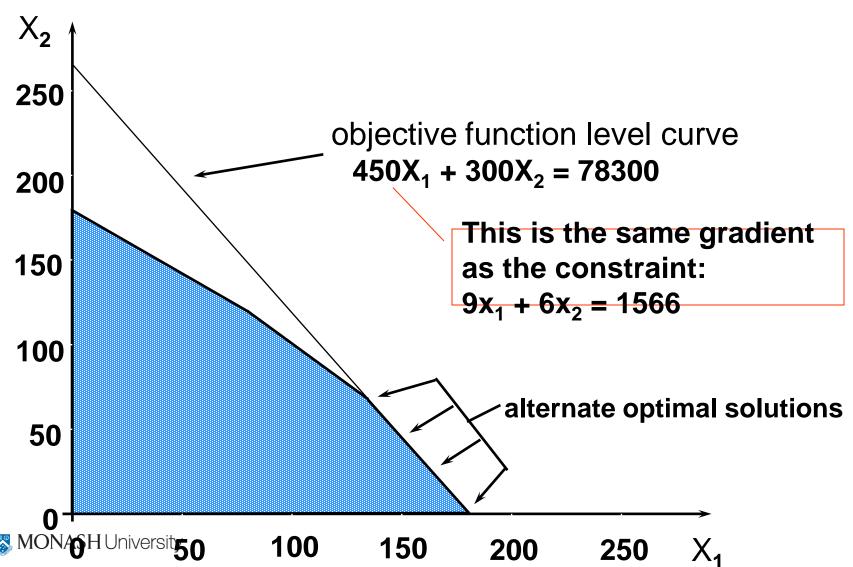
### **Enumerating The Corner Points**



### **Special Conditions in LP Models**

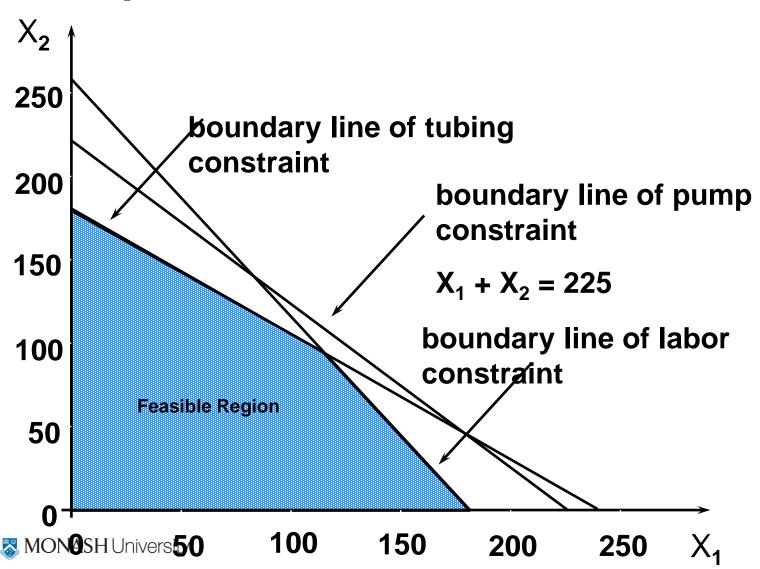
- A number of anomalies can occur in LP problems:
  - Alternate Optimal Solutions
  - Redundant Constraints
  - Unbounded Solutions
  - Infeasibility

### **Example of Alternate Optimal Solutions**

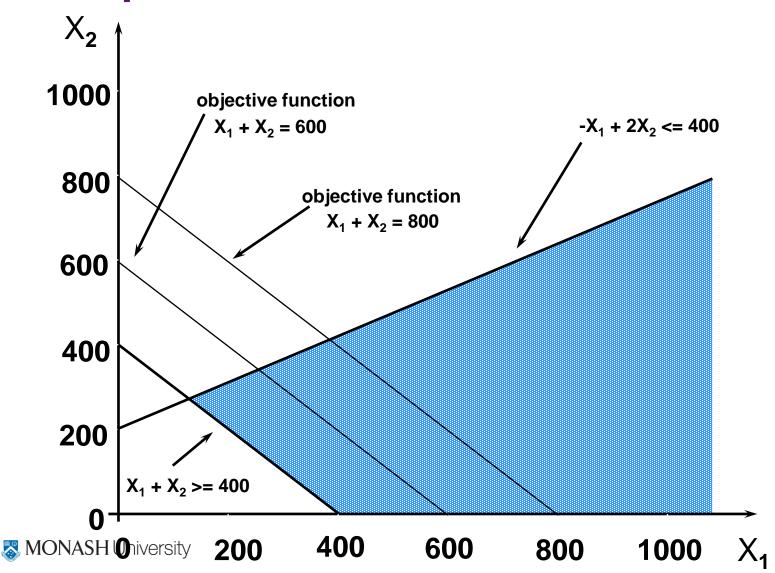


50

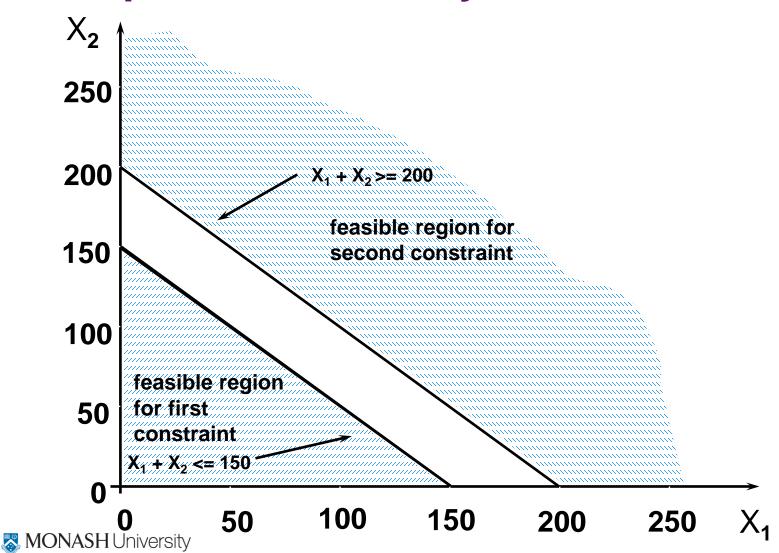
### **Example of a Redundant Constraint**



### **Example of an Unbounded Solution**



### **Example of Infeasibility**





#### There's no tutorial this week

Please go through the Excel exercises on Moodle on your own to brush up your Excel skills for subsequent exercises in the weeks to come.

#### Readings for next week's Lecture:

Ragsdale, C. T., Spreadsheet Modelling and Decision Analysis 8th Ed, Cengage Learning, 2017: Chapter 3

