FIT3158 Business Decision Modelling

Tutorial 4 Solution

ILP, GP and MOLP

Exercise 1 (Ragsdale 6.10 8E/Ragsdale 6.11 9E)

Garden City Beach hires temporary lifeguards to ensure the safety of vacationing public. The lifeguards are assigned to work five consecutive days each week and then have two days off. However, the city insurance company requires that the city have at least the following numbers on duty each day of the week.

Minimum number of lifeguards required each day

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Lifeguards	18	17	16	16	16	14	19

The city manager would like to determine the minimum number of lifeguards that will have to be hired.

a. Formulate an ILP for this problem.

b. Implement it in a spreadsheet and solve it.

c. What is the optimal solution?

d. Several lifeguards have expressed preference to be off on Saturdays and Sundays. What is the maximum number of lifeguards that can be off on weekends without increasing the total number of lifeguards required?

Exercise 2 (Ragsdale 6.17 8E/Ragsdale 6.18 9E)

Radford Castings can produce brake shoes on six different machines. The following table summarizes the manufacturing costs associated with producing the brake shoes on each machine along with the available capacity on each machine. If the company has received an order for 1,800 brake shoes, how should it schedule these machines?

Machine	Fixed Cost ($)	Variable Cost ($)	Capacity
1	1,000	21	500
2	950	23	600
3	875	25	750
4	850	24	400
5	800	20	600
6	700	26	800

a) Formulate an ILP model for this problem.

b) Create a spreadsheet model for this problem and solve it.

c) What is the optimal solution?

(Hint: try using linking constraints to model the fixed costs).

Exercise 3 (Ragsdale 6.20 8E/Ragsdale 6.21 9E)

Tropicsun is a leading grower & distributor of fresh citrus products with 3 large citrus groves scattered around central Florida in the cities of Mt Dora, Eustis and Clermont. It currently has 275,000 bushels of citrus at the grove in Mt Dora, 400,000 bushels in Eustis and 300,000 bushels in Clermont. It has citrus processing plants in Ocala, Orlando and Leesburg with processing capacities to handle 200,000, 600,000 and 225,000 bushels respectively. Tropicsun contracts with a local trucking company to transport its fruit from the groves to processing plants. The trucking company charges a flat rate of $8 per kilometer regardless of how many bushels are transported. The table below summarizes the distances (in kilometers) between groves and processing plant.

Distances (in kilometers) between groves & plants:

Grove	**Ocala**	**Orlando**	**Leesburg**
Mt Dora	21	50	40
Eustis	35	30	22
Clermont	55	20	25

Tropicsun wants to determine how many bushels to ship from each grove to each processing plant to minimise the total transportation cost.

a. Formulate an ILP model for this problem
b. Create a spreadsheet model for this problem and solve it
c. What is the optimal solution?

(Hint: – try using 'big M').

<u>Goal Programming and Multiple Objective Optimisation</u>

Exercise 4 (Ragsdale 7.12 8E/Ragsdale 7.13 9E)

Back to Blue Ridge hot tubs! Howie Jones needs to decide how many of the two types Aqua-Spa and Hydro-Lux to manufacture for the next production cycle.

He buys the fibreglass hot tub shell from a local supplier and adds the pump and tubing to this shell to create his products. The supplier has the capacity to deliver as many shells required. Howie installs the same type of pump to both products. He will have only 200 pumps available during the next cycle. From a production standpoint the main difference between the two models of hot tubs is the amount of tubing and labour required.

Aqua Spa requires 9 hours labour and 12 feet tubing.

Hydro-Lux requires 6 hours labour and 16 feet of tubing.

Howie expects to have 1566 labour hours and 2880 feet of tubing. He earns a profit of $350 on each Aqua-Spa and $300 on each Hydro-Lux and is confident he can sell all tubs produced.

Production of each Aqua-Spa generates 15 pounds of toxin resin and each Hydro-Lux generates 10 pounds. Howie considers maximisation of profit half as important as minimisation of toxin resin when deciding how many of each to produce.

a. Formulate an MOLP for Howie's problem.
b. Implement it in a spreadsheet and solve it.
c. What is the optimal solution?

Exercise 5 (Ragsdale 7.1, 7.2 and 7.4)

a. What is the difference between an objective function and a goal?
b. Is there an optimal solution to a GP or MOLP problem? Explain.
c. In 2005, Hurricane Katrina decimated the gulf coast of the United States between Mobile, Alabama and New Orleans, Louisiana. The aftermath of the storm left the city flooded both with water and human victims. Responding to such a disaster is a logistical nightmare and presents governmental decision makers with an extremely difficult challenge.
    1. Identify several key objectives that the decision makers managing this problem must consider simultaneously.
    2. Identify the key resources that need to be allocated.
    3. Do the objectives identified conflict or compete with one another in terms of resource usage.
    4. How might techniques learned in last week's lecture help to allocate resources to achieve the objectives?

<u>Solution</u>

Exercise 1 (Ragsdale 6.10 8E/Ragsdale 6.11 9E)

a. MIN $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$

ST
$X_2 + X_3 + X_4 + X_5 + X_6 \geq 18$
$X_3 + X_4 + X_5 + X_6 + X_7 \geq 17$
$X_1 + X_4 + X_5 + X_6 + X_7 \geq 16$
$X_1 + X_2 + X_5 + X_6 + X_7 \geq 16$
$X_1 + X_2 + X_3 + X_6 + X_7 \geq 16$
$X_1 + X_2 + X_3 + X_4 + X_7 \geq 14$
$X_1 + X_2 + X_3 + X_4 + X_5 \geq 19$
$X_i \geq 0$ & integer

b. See file Prb6_10.xls
c. $X_1 = 5$, $X_2 = 2$, $X_3 = 6$, $X_4 = 2$, $X_5 = 6$, $X_6 = 2$, $X_7 = 1$ (alternate optimal solutions exist).
d. 3

Exercise 2 (Ragsdale 6.17 8E/Ragsdale 6.18 9E)

a. MIN $21X_1 + 23X_2 + 25X_3 + 24X_4 + 20X_5 + 26X_6$
$+ 1000Y_1 + 950Y_2 + 875Y_3 + 850Y_4 + 800Y_5 + 700Y_6$

ST
$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 1800$
$X_1 - 500\,Y_1 \leq 0$
$X_2 - 600\,Y_2 \leq 0$

$X_3 - 750\ Y_3 \le 0$
$X_4 - 400\ Y_4 \le 0$
$X_5 - 600\ Y_5 \le 0$
$X_6 - 800\ Y_6 \le 0$
$X_i \ge 0$
All Y_i are binary

b. See file: Prb6_17.xlsm
c. X_1=500, X_2=600, X_4=100, X_5=600
Total cost = $42,300

Exercise 3 (Ragsdale 6.20 8E/Ragsdale 6.21 9E)

a. Let X_{ij} = bushels (in 1000s) shipped from grove *i* to processing plant *j*
Y_{ij} = 1 if $X_{ij} \ge 0$, 0 otherwise

MIN $168\ Y_{14} + \$400\ Y_{15} + \$320\ Y_{16}$
$280\ Y_{24} + \$240\ Y_{25} + \$176\ Y_{26}$
$440\ Y_{34} + \$160\ Y_{35} + \$200\ Y_{36}$
ST $X_{14} + X_{15} + X_{16} = 275$
$X_{24} + X_{25} + X_{26} = 400$
$X_{34} + X_{35} + X_{36} = 300$
$X_{14} + X_{24} + X_{34} \le 200$
$X_{15} + X_{25} + X_{35} \le 600$
$X_{16} + X_{26} + X_{36} \le 225$
$X_{ij} - M_{ij}Y_{ij} \le 0$
$X_{ij} \ge 0$
Y_{ij} binary
Note: M_{ij} = MIN(supply of *i*, capacity of *j*)

b. See file: Prb6_20.xls
c. The solution is: X_{15}= 275, X_{24}= 200, X_{26}= 200, X_{35}= 300,
Y_{15}= 1, Y_{24}= 1, Y_{26}= 1, Y_{35}= 1.
Minimum trucking cost = $1,016.

Exercise 4 (Ragsdale 7.12 8E/Ragsdale 7.13 9E)

a. MIN Q
ST $1(66100 - 350X_1 - 300X_2) \le Q$
$2(15X_1 + 10X_2 - 0) \le Q$
$X_1 + X_2 \le 200$
$9X_1 + 6X_2 \le 1566$
$12X_1 + 16X_2 \le 2880$
$X_1, X_2 \ge 0$
b. See file: Prb7_12.xls
c. Aqua-Spas = 61, Hydro-Luxes = 134

Exercise 5 (Ragsdale 7.1, 7.2 and 7.4 8E/ Ragsdale 7.1, 7.2 and 7.5 9E)

1. Both are mathematical functions of the decision variables. However, there is no predetermined target level for an objective function -- we want to either maximize it or minimize it. A goal has some predetermined target level and we want to find a solution that comes as close as possible to achieving this target.

2. A GP or MOLP has no optimal solution in the sense of an LP problem -- where we can mechanically apply an algorithm to locate the optimal solution. Instead, both of these procedures require that we search for an optimal or "most satisfying" solution. This solution may differ from one decision maker to the next.

3. a. Objectives include: Maximizing lives saved, minimizing property damage, minimizing costs, minimizing political damage, etc.
   b. Resources: money, food, shelter, water, manpower, transportation services, medical assistance.
   c. Minimizing costs is at odds with the other objectives
   d. MOLP could be used to help decision makers analyze the trade-offs between the various objectives under different resource allocation scenarios.