Queuing Problems

1. A company has recorded the following list of service rates (customers/hour) for one of its servers. What is the mean service time for this server.

Customers / hour	
2	3
3	4
2	3
3	3
4	2
4	3
2	4

ANSWER: 3 customers per hour gives a mean service time of 20 minutes per customer

2. Customers arrive at a store randomly, following a Poisson distribution at an average rate of 90 per hour. How many customers arrive per minute, on average?

ANSWER: 1.5 customers per minute 90/60 = 1.5

3. Customers arrive at a store randomly, following a Poisson distribution at an average rate of 90 per hour. How many customers would you expect to arrive in a 20 minute period?

ANSWER: 30 customers per 20 minute period 1.5 * 20 = 30

4. Customers arrive at a store randomly, following a Poisson distribution at an average rate of 20 per hour. What is the probability of exactly 0, 1 2, and 3 arrivals in a 15 minute period?

ANSWER: λ = 5 customers/15 minute period

Arrivals	Probability
0	0.0067
1	0.0336
2	0.0842
3	0.1404

5. A grocery clerk can serve 20 customers per hour on average and the service time follows an exponential distribution. What is the expected service time per customer?

ANSWER: 3 minutes per customer 60 / 20 =3

6. A grocery clerk can serve 20 customers per hour on average and the service time follows an exponential distribution. What is the probability that a customer's service time is less than 2 minutes?

ANSWER: T = 2/60 = 0.033 hours

$P(t<T) = 1-e^{-\mu T} = 1-e^{-20*0.033} = 1- 0.5134 = 0.4866$

7. The customer service desk at Joe's Discount Electronics store receives 5 customers per hour on average. On average, each customer requires 10 minutes for service. The customer service desk is staffed by a single person. What is the average time a customer spends in the customer service area if modeled as an M/M/1 queuing system?

why m is 6?

ANSWER: $W = 1/(\mu - \lambda) = 1 / (6 - 5) = 1$

Because every 10 mins, 1 hour = 60 mins
m = 60/10 = 6

8. A grocery store can serve an average of 360 customers per hour. The service times are exponentially distributed. The store has 4 checkout lines each of which serves 90 customers per hour. Customers arrive at the store at a Poisson rate of 240 customers per hour. The following queuing analysis spreadsheet was developed from this information.

Arrival rate	240	
Service rate	90	
Number of servers	4	(max of 40)

Utilization	66.67%
P(0), probability that the system is empty	0.0599
Lq, expected queue length	0.7568
L, expected number in system	3.4235
Wq, expected time in queue	0.0032
W, expected total time in system	0.0143
Probability that a customer waits	0.3784

1. What is the Kendall notation for this system?

   ANSWER: M/M/4

2. Based on this report what percent of the time is a grocery clerk busy serving a customer?

   ANSWER: 66.67%

3. Based on this report what is the average number of customers waiting for a checker?

   ANSWER: 0.7568

4. Based on this report how long does a customer wait before the checker begins serving them?

   ANSWER: 0.0032

5. Based on this report what is the average total time spent in line and being checked out?

   ANSWER: 0.0143

Simulation Problems

1. A news vendor has to determine how many newspapers to order to maximise profit. He pays 20c for each copy of newspaper and sells the paper for 30c each. Newspapers that are unsold by the end of the day are worthless.
   He estimates that he can sell between 6 and 10 papers per day, with varying probabilities shown in Table 1:

Demand	Probability
6	0.15
7	0.20
8	0.30
9	0.25
10	0.10

Table 1: Newspaper demand distribution

Random number
0.807752
0.915975
0.104005
0.332072
0.538625

Table 2: random numbers

i). Using the random numbers provided in Table 2, generate 5 simulations of demand.

Demand	Probability	Cumulative Probability
6		
7		
8		
9		
10		

Enter the simulated demand quantity in this table:

0.807752	
0.915975	
0.104005	
0.332072	
0.538625	

ii). Using the demand simulated in a) and assuming that the newsvendor orders 8 copies each day, what is the average profit?

		demand	
1	0.807752		
2	0.915975		
3	0.104005		
4	0.332072		
5	0.538625		

i). Using the random numbers provided in Table 2, generate 5 simulations of demand.

ANSWER:

Demand	Probability	Cumulative Probability
6	0.15	0.15
7	0.20	0.35
8	0.30	0.65
9	0.25	0.90
10	0.10	1

Enter the simulated demand quantity in this table:

0.807752	9
0.915975	10
0.104005	6
0.332072	7
0.538625	8

ii). Using the demand simulated in a) and assuming that the newsvendor orders 8 copies each day, what is the average profit?

ANSWER:

		demand	profit
1	0.807752	9	8 x 10 = 80
2	0.915975	10	8 x 10 = 80
3	0.104005	6	6 x 30 – 8 x 20 = 20
4	0.332072	7	7 x 30 – 8 x 20 = 50
5	0.538625	8	8 x 10 = 80
		Average:	**310/5 = 62**

4. Simulate the arrival of patients at Medonic Surgical using the exponential distribution using the uniform random number given in the table. Calculate the inter-arrival time with mean of two minutes. Calculate arrival time, service time and waiting time for 10 customers. Service time is a constant two minutes. What is the average waiting time?

Customer	Random #	Interarrival	Arrival Time	Service Start	Service Finish	Waiting Time
1	0.279					
2	0.975					
3	0.499					
4	0.156					
5	0.41					
6	0.627					
7	0.41					
8	0.473					
9	0.964					
10	0.963					

Average waiting time = _____

Answer:

Customer	Random #	Interarrival	Arrival Time	Service Start	Service Finish	Waiting Time
1	0.279	2.55	2.55	2.55	4.55	0.00
2	0.975	0.05	2.60	4.55	6.55	1.95
3	0.499	1.39	3.99	6.55	8.55	2.56
4	0.156	3.72	7.71	8.55	10.55	0.84
5	0.41	1.78	9.49	10.55	12.55	1.06
6	0.627	0.93	10.43	12.55	14.55	2.13
7	0.41	1.78	12.21	14.55	16.55	2.34
8	0.473	1.50	13.71	16.55	18.55	2.85
9	0.964	0.07	13.78	18.55	20.55	4.77
10	0.963	0.08	13.86	20.55	22.55	6.70

Average waiting time = **2.52**