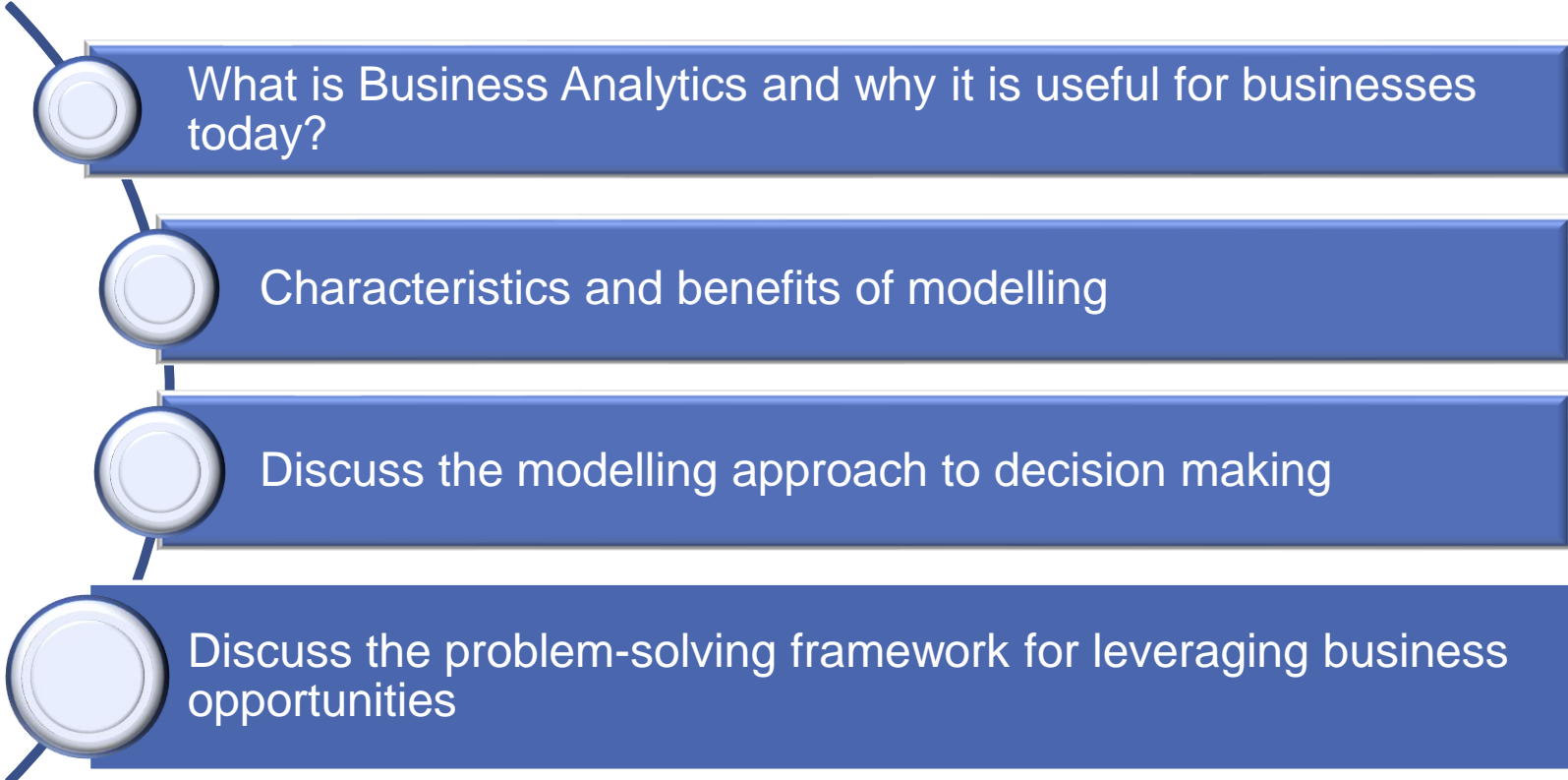# Part 1
Introduction to Modelling and Decision Analysis

What is Business Analytics and why it is useful for businesses today?

Characteristics and benefits of modelling

Discuss the modelling approach to decision making

Discuss the problem-solving framework for leveraging business opportunities

Introduction

- We face numerous decisions in life and business. Its not always easy to make good decision.

  - For any given situation there are often numerous possible courses of action.

- Decision analysis

  - A means of systematically identifying, evaluating & choosing best course of action.

- We can use computers to analyse the potential outcomes of decision alternatives.

What is Business Analytics?

- A field of study that uses computers, statistics, and mathematics to solve business problems.

- Considerable overlap with:

  - Operations Research

  - Management Science

  - Decision Science

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Home Runs in Business Analytics

- **Chevron**

    - Developed optimization tool for

        - Operational & strategic planning

        - Mixing crude oils

        - Planning capital expenditures

    - Benefits:

        - Annual savings of $1.0 billion

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Home Runs in Business Analytics

- **Dell**
    - Built analytics models to
        - Optimize hardware configurations
        - Optimize its website's design
        - Optimize promotion design and timing
    - Benefits:

        Increased profit by $142 million

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Home Runs in Business Analytics

- **Kroger**

  – Analytics team created models to determine reorder point & order up to quantities for items in 1,950 in-store pharmacies

  – Benefits:

    • Reduced prescription stock outs by 1.6 million

    • Lowered inventory by over $120 million

    • Increased revenue by $80 million

Home Runs in Business Analytics

- **National Broadcast Network Company**

  - Gov't owned broadband provider in Australia

  - Developed tool to optimize design of network servicing 8 million locations

  - Benefits:

    - Reduction in network design time

    - Savings of ~$1.7 billion

Home Runs in Business Analytics

- **Alliance for Paired Donations (APD)**

    – Transplant patients needing a kidney often have potential donors who are incompatible

    – APD finds exchanges with other patient-donor pairs to optimize paired matches

    – Benefits:

        • 220 lives saved since 2006

        • Value of savings: *priceless*

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A Modelling Approach to Decision Making

- A model is
  - A structure which has been built purposely to exhibit features and characteristics of some other objects
- Everyone uses models to make decisions
- Types of models:
  - Mental (arranging furniture)
  - Visual (blueprints, road maps)
  - Physical/Scale (aerodynamics, buildings)
  - Mathematical (what we'll be studying)

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Characteristics of Models

– Usually _simplified_ versions of things they represent

– A _valid_ model accurately represents the _relevant characteristics_ of the object or decision problem being studied
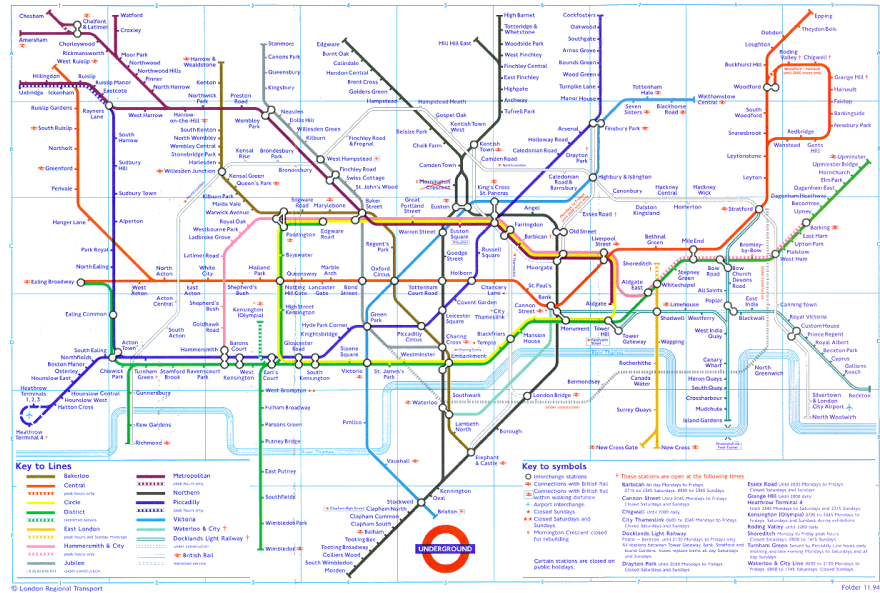
A more formal definition

- A model is a device which behaves approximately like part of the real world:

    - Train-sets, dolls, toy prams…

    - Small aircraft replicas in wind tunnels…

    - Sponge-rubber men used in car crash…

    - The solar system as a number of small balls moving in ellipses round a larger ball...

Models for Decision Making

May include:

- Mathematical equations

  - Mathematical Models

- Structured set of rules or steps to follow

  - Algorithms and Discrete Event processes

- Simulation of real events

  - Queuing models, waiting lines

- Statistical techniques

  - Data Analysis, Equation Fitting

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Modelling: Why bother?

- Real Understanding:

  – The process may reveal hidden inner relationships within the system.

  – Formal Analysis (assumptions/relationships)

  – Experimentation - can be used repeatedly

  – To large extent, independent of the data   Relisten the explaination

  – Variety of models for the same problem

Benefits of Modelling

- **Economy**
  - Less costly to analyze decision problems.

- **Timeliness**
  - Deliver needed information more quickly than their real-world counterparts.

- **Feasibility**
  - Used to do things that would be impossible in reality.

- **Insight & understanding**
  - That improves decision making.

Advantages of Using Models

- Less expensive and disruptive than experimenting with real world system

- Models enable us to ask 'what if' questions and test the sensitivity of particular values

- Models force a consistent, systematic approach to a problem

- Models require managers to be specific about constraints and goals

- Models can reduce the time taken to make decisions

Disadvantages of Using Models

- May be expensive to develop and test

- May be misused and misunderstood because of their complexity   (e.g., there may be too much variables)

- Models may down play the value of non-quantifiable information

- May oversimplify the variables of the real world

What is a "Computer Model"? = Spreadsheet model

- A set of mathematical relationships and logical assumptions implemented in a computer as an abstract representation of a real-world object or phenomenon.

- Spreadsheets provide the most convenient way for business people to build computer models

  - Spreadsheets are commonplace on most computers.

  - They are the tool of choice for today's managers.

  - They facilitate decision-making process by making it easier to play out various what-if scenarios.

Example of a Mathematical Model

Profit = Revenue - Expenses

or

Profit = f(Revenue, Expenses)

or

$$Y = f(X_1, X_2)$$

Dependent variable

Independent variables

A Generic Mathematical Model

$$Y = f(X_1, X_2, \ldots, X_n)$$

Where:

Y is DEPENDENT for the variable of X_i

Y = dependent variable

(aka bottom-line performance measure)

X_i = independent variables

(inputs having an impact on Y)

$f(\cdot)$ = function defining the relationship between Y & X_i

Mathematical Models & Spreadsheets

- Most spreadsheet models are very similar to our generic mathematical model:

$$Y = f(X_1, X_2, \ldots, X_n)$$

- Most spreadsheets have input cells (representing X_i) to which mathematical functions ($f(\cdot)$) are applied to compute a bottom-line performance measure (or Y).

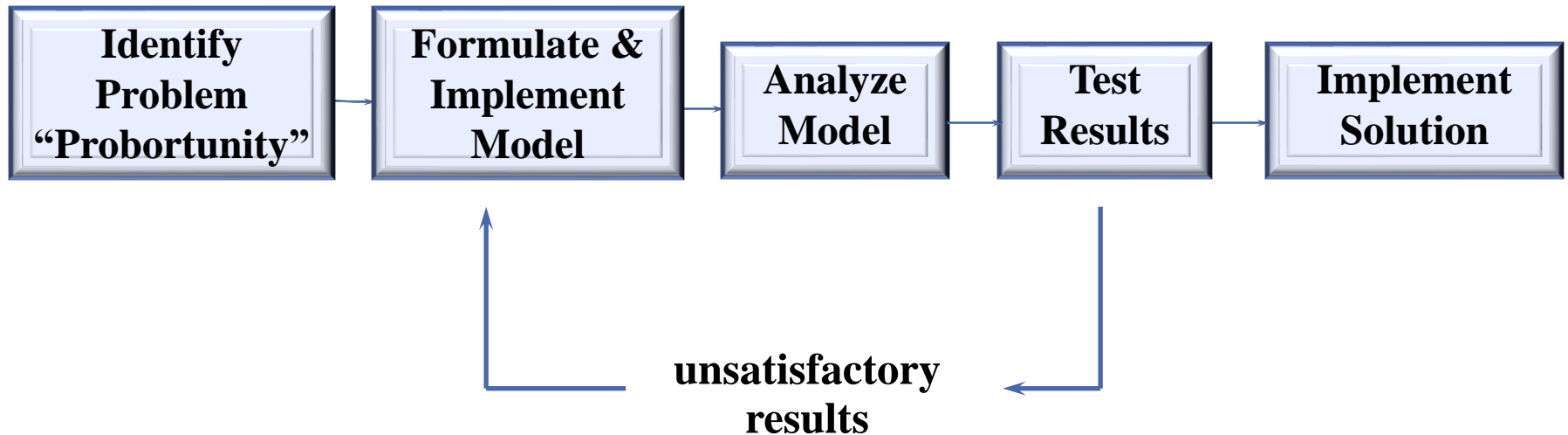# Categories of Mathematical Models

Model Category	Relationship Form of $f(\cdot)$	Independent Variables	OR/MS Techniques
Prescriptive	known, well-defined	known or under decision maker's control	LP, Networks, IP, CPM, EOQ, NLP, GP, MOLP
Predictive	unknown, ill-defined	known or under decision maker's control	Regression Analysis, Time Series Analysis, Discriminant Analysis
Descriptive	known, well-defined	unknown or uncertain	Simulation, PERT, Queueing, Inventory Models

How to Use a Model

It wont make a decision for you,
but it would help you make a decision

- Don't accept an answer without further analysis and questioning. Get better insight; question our assumption; get relationship between variables - Like BRAINSTORMING

- A mathematical model is only one of a number of tools for decision making.

- If the answer is unacceptable, the reasons for unacceptability should be spelled out and incorporated in a modified model.

- An acceptable answer is only an option.

- By successive questioning of the answers and altering the model, it is possible to clarify the options available and obtain a greater understanding of what is possible in a real situation.

The Problem-Solving Framework for Leveraging Business Opportunities

| Identify Problem "Probortunity" | → | Formulate & Implement Model | → | Analyze Model | → | Test Results | → | Implement Solution |

unsatisfactory results

Psychology of Decision Making

- Models can be used for structural aspects of decision problems.

- Other aspects cannot be structured easily, requiring intuition and judgment.

- *Caution*: Human judgment and intuition are not always rational!

- Errors in human judgment often arise because of *anchoring* and *framing* effects associated with decision problems.

Anchoring Effects

- Arise when trivial factors influence initial thinking about a problem.
- Decision-makers usually under-adjust from their initial "anchor."

anchor.

- Example:
  - What is 1x2x3x4x5x6x7x8?   Median estimate = 512

    好似好大，其實一樣

  - What is 8x7x6x5x4x3x2x1?   Median estimate = 2,250
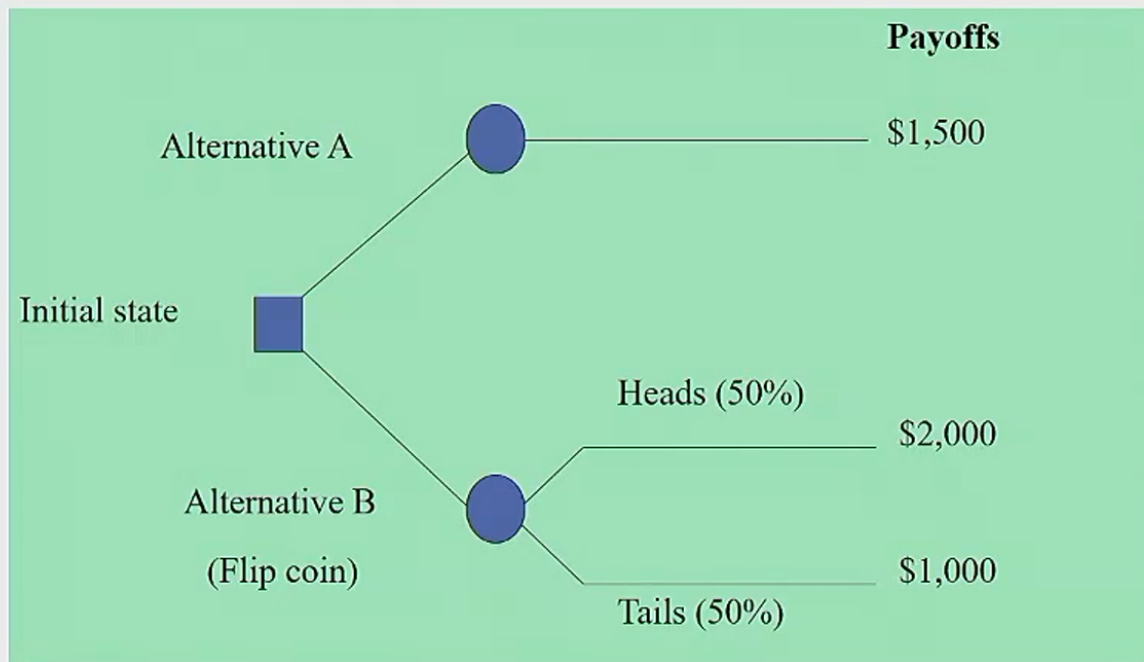
  The product is the same in both cases: 40,320

Framing Effects

- Refers to how decision-makers view a problem from a win-loss perspective.

- The way a problem is framed often influences choices in irrational ways…

Framing Effects 1

- Suppose you've been given $1000 and must choose between:

    A1. Receive $500 more immediately => Most popular

    B1. Flip a coin and receive $1000 more if heads occurs or $0 more if tails occurs.

- Now suppose you've been given $2000 and must choose between:

    A2. Give back $500 immediately

    B2. Flip a coin and give back $0 if heads occurs or give back $1000 if tails occurs. => Most popular

A Decision Tree for Both Examples

Payoffs

Alternative A — $1,500

Initial state

Alternative B (Flip coin)

Heads (50%) — $2,000

Tails (50%) — $1,000

看似不同，其實一樣

Framing Effects 2
DARK PATTERNS !

Which option would you select?

This food is 95% fat free

vs

This food consists of 5% fat

You generally obtain different answers from the same person.

Good Decisions vs. Good Outcomes

- Good decisions do not always lead to good outcomes...

- A structured, modelling approach to decision making helps us make good decisions, but can't guarantee good outcomes.

What is a good decision?

A balance among

- What we know
- What we can do
- What we want

		Outcome Quality	
		Good	Bad
Decision Quality	Good	Deserved Success	Bad Luck
	Bad	Dumb Luck	Poetic Justice

Part 2
Introduction to Optimisation and Linear Programming

Discuss the characteristics of optimisation problems

Explore the use of linear programming (LP) in solving optimisation problems

Solving LP problems using graphical approach

Introduction

- We all face decision about how to use limited resources such as:

  - Oil in the earth

  - Land for dumps

  - Time

  - Money

  - Workers

Mathematical Programming...

- A field of management science that finds the optimal, or most efficient, way of using limited resources to achieve the objectives of an individual of a business.

- *Also referred to as …* Optimisation

Applications of Optimization

- **Determining Product Mix**

  – How many of each product to produce to ==**maximise profits**== or to satisfy demand at ==**minimum cost**==?

- **Manufacturing**

  – Eg. For a circuit board, what is the drilling order that **minimises total distance** the drill bit must be moved?

- **Routing and Logistics**

  – What is the **least costly** method of transferring goods from warehouses to stores?

- **Financial Planning**

  – How much to save in superannuation to **minimise tax liability**?

Characteristics of Optimization Problems

- Decisions

  - One or more decisions that must be made

- Constraints

  - Due to limited resources

- Objectives

  - Goal that decision maker considers when making decision

  - To maximise profit or minimise cost

General Form of an Optimization Problem

MAX (or MIN): $\qquad f_0(X_1, X_2, \ldots, X_n)$

Objective Function

Subject to: $\quad f_1(X_1, X_2, \ldots, X_n) <= b_1$

$\qquad\qquad\qquad\qquad\qquad\qquad :$

$\qquad\qquad f_k(X_1, X_2, \ldots, X_n) >= b_k$

$\qquad\qquad\qquad\qquad\qquad\qquad :$

$\qquad\qquad f_m(X_1, X_2, \ldots, X_n) = b_m$

RHS

Value of constraints

LHS (Left Hand Side)

Note: If all the functions in an optimization are linear, the problem is a Linear Programming (LP) problem

Linear Programming (LP) Problems

MAX (or MIN): $c_1 X_1 + c_2 X_2 + \ldots + c_n X_n$

Subject to: $a_{11} X_1 + a_{12} X_2 + \ldots + a_{1n} X_n <= b_1$

$$:$$

$$a_{k1} X_1 + a_{k2} X_2 + \ldots + a_{kn} X_n >= b_k$$

$$:$$

$$a_{m1} X_1 + a_{m2} X_2 + \ldots + a_{mn} X_n = b_m$$

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An Example LP Problem

Blue Ridge Hot Tubs produces two types of hot tubs: Aqua-Spas & Hydro-Luxes.

	Aqua-Spa	Hydro-Lux
Pumps	1	1
Labor	9 hours	6 hours
Tubing	12 feet	16 feet
Unit Profit	$350	$300

There are 200 pumps, 1566 hours of labor, and 2880 feet of tubing available. Decision: Hw many Aqua-Spa and Hydro-Lux can be produced

5 Steps In Formulating LP Models:

1. Understand the problem.

2. Identify the decision variables.

X_1=number of Aqua-Spas to produce

X_2=number of Hydro-Luxes to produce

3. State the objective function as a linear combination of the decision variables.

MAX: $350X_1 + 300X_2$

5 Steps In Formulating LP Models (continued)

4. State the constraints as linear combinations of the decision variables.

$$1X_1 + 1X_2 <= 200 \quad \} \text{ pumps}$$

$$9X_1 + 6X_2 <= 1566 \quad \} \text{ labor}$$

$$12X_1 + 16X_2 <= 2880 \} \text{ tubing}$$

5. Identify any upper or lower bounds on the decision variables.

$$X_1 >= 0$$

$$X_2 >= 0$$

LP Model for
Blue Ridge Hot Tubs

MAX: $350X_1 + 300X_2$

S.T.: $1X_1 + 1X_2 <= 200$

$9X_1 + 6X_2 <= 1566$

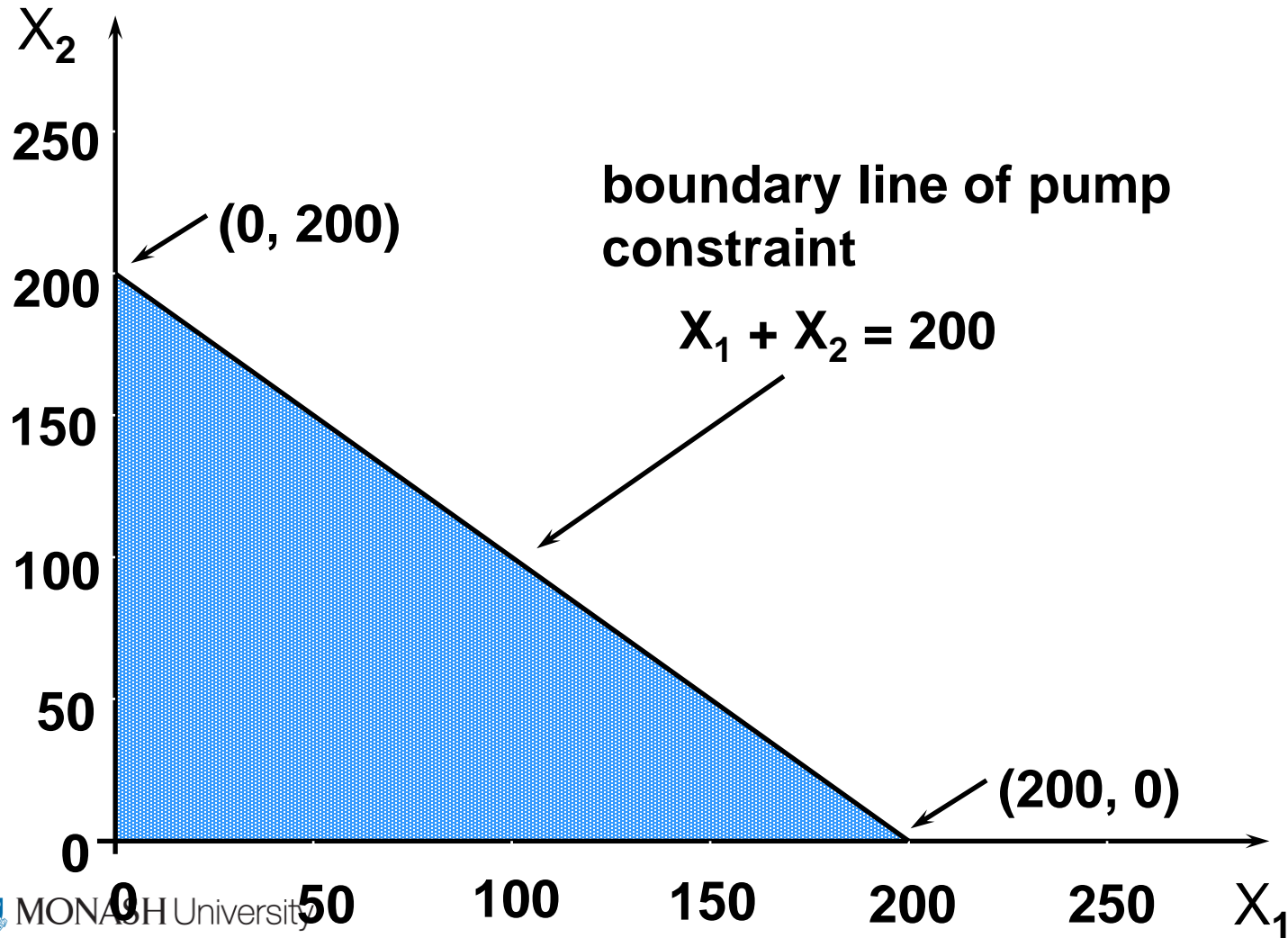$12X_1 + 16X_2 <= 2880$

$X_1 >= 0$

$X_2 >= 0$

Solving LP Problems: An Intuitive Approach

- Idea: Each Aqua-Spa (X_1) generates the highest unit profit ($350), so let's make as many of them as possible!

- How many would that be?

  - Let $X_2 = 0$

    - 1st constraint: $1X_1 <= 200$

    - 2nd constraint: $9X_1 <= 1566$ or $X_1 <= 174$

    - 3rd constraint: $12X_1 <= 2880$ or $X_1 <= 240$

- If $X_2 = 0$, the maximum amount of X_1 you could make is 174 and the total profit is $350*174 + $300*0 = $60,900

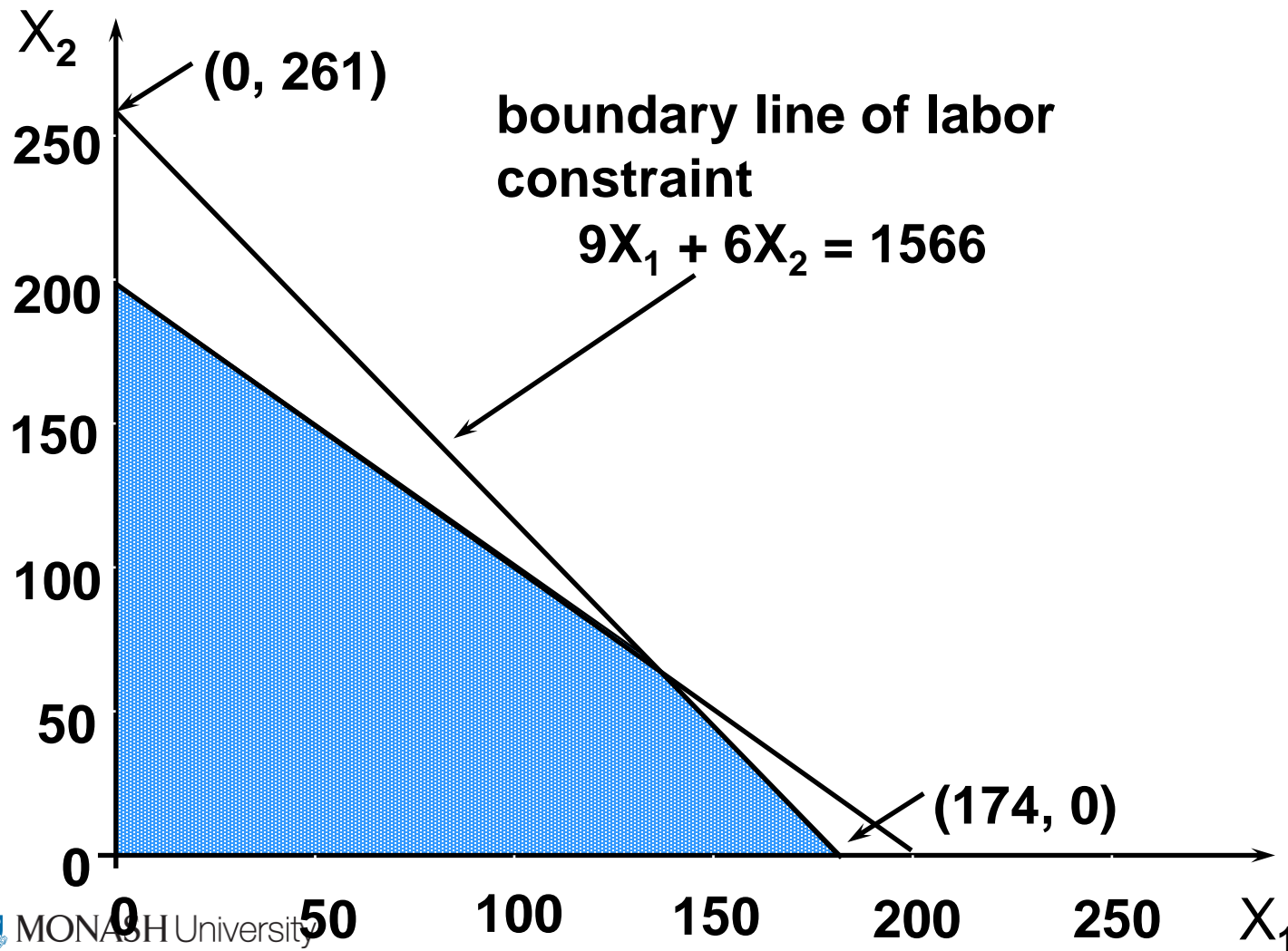- This solution is *feasible*, but is it *optimal?*

  No!

Solving LP Problems: A Graphical Approach

- The constraints of an LP problem defines its feasible region.
- The best point in the feasible region is the optimal solution to the problem.
- For LP problems with 2 variables, it is easy to plot the feasible region and find the optimal solution.
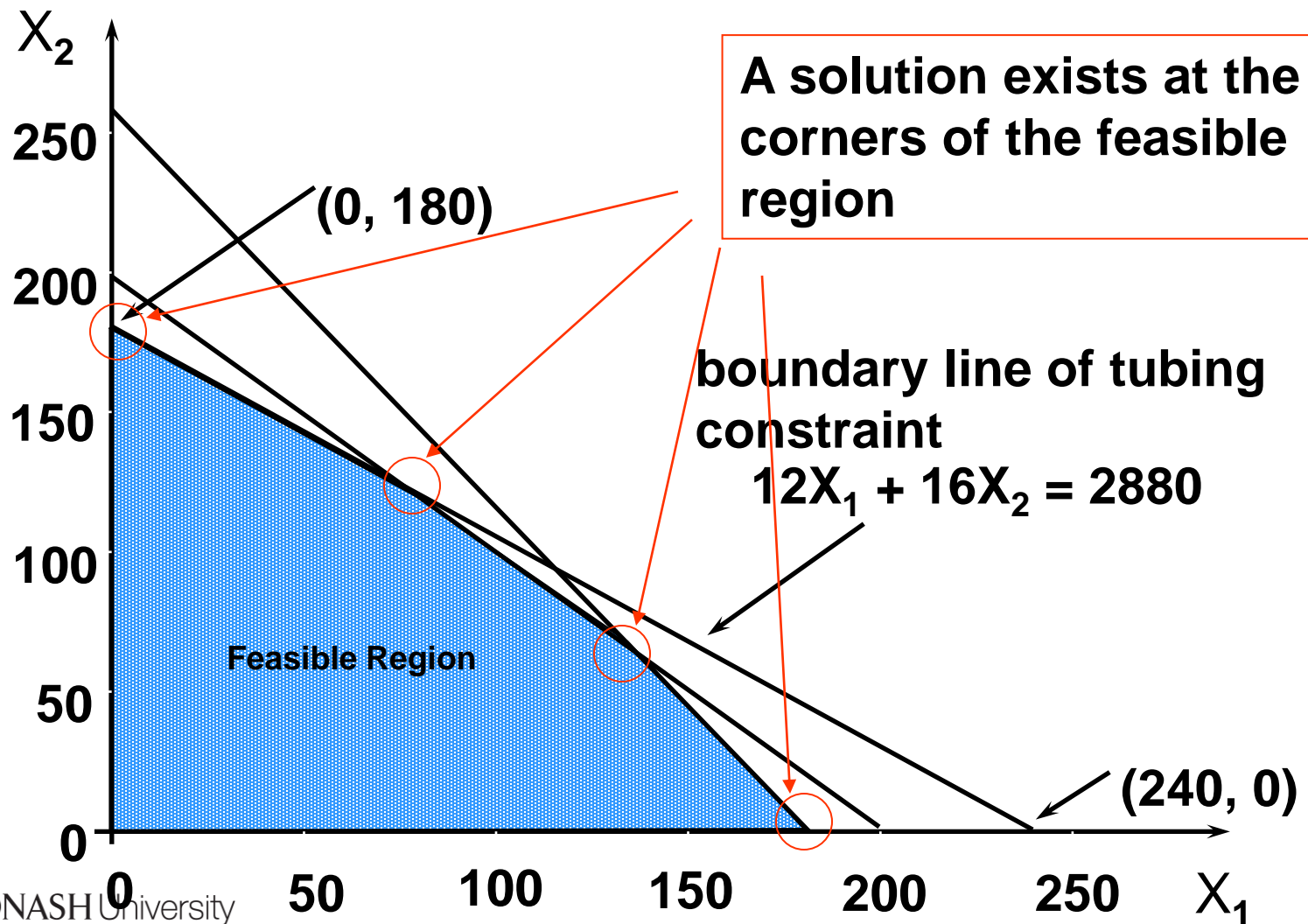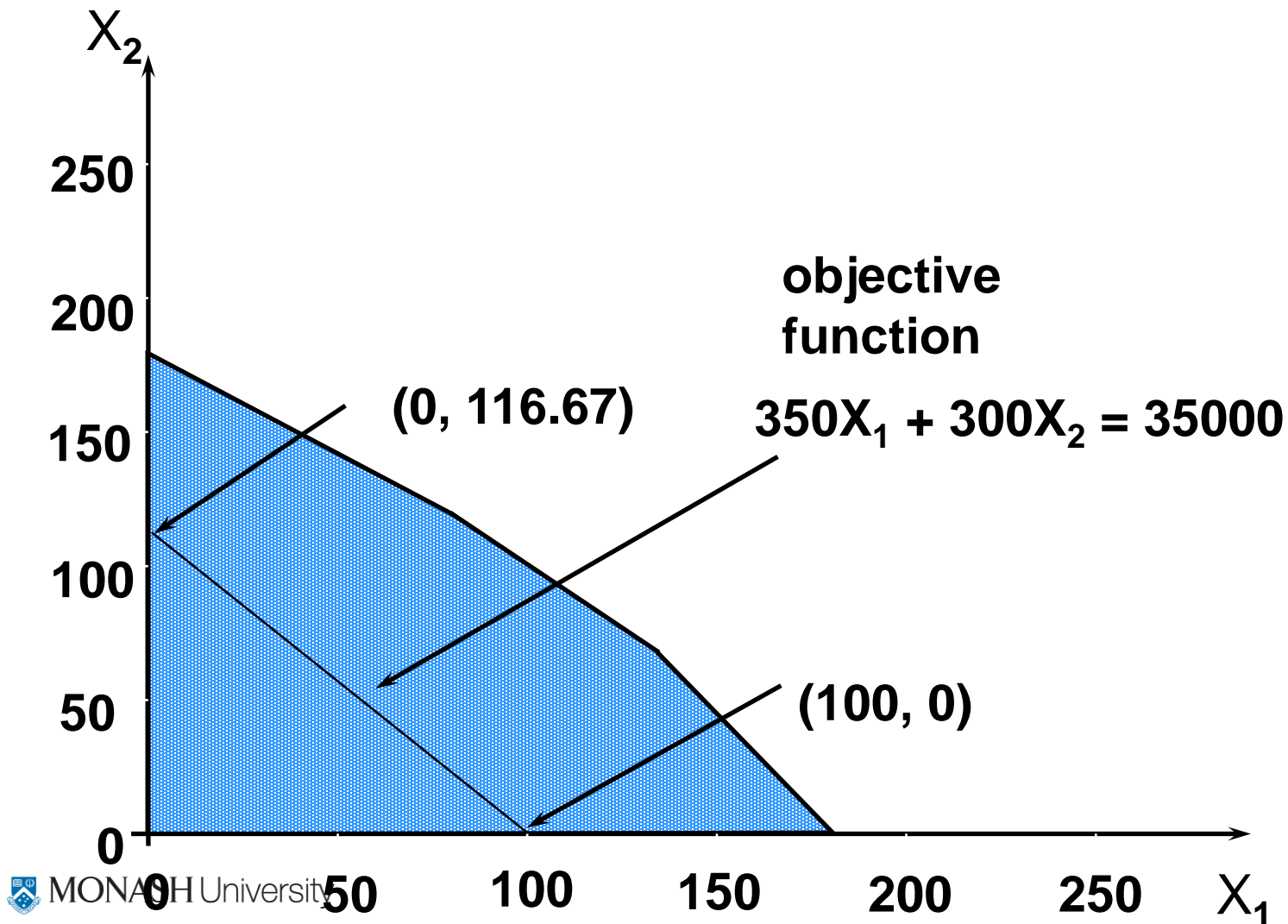
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Plotting the First Constraint

X_2

250

(0, 200)

boundary line of pump constraint

$X_1 + X_2 = 200$

200

150

100

50

(200, 0)

0

0   50   100   150   200   250 X_1

| 41

Plotting the Second Constraint

X_2

(0, 261)

boundary line of labor constraint

$$9X_1 + 6X_2 = 1566$$

250

200

150

100

50

0

(174, 0)

0   50   100   150   200   250 X_1

Plotting the Third Constraint

Feasible != Optimal

A solution exists at the corners of the feasible region

(0, 180)

boundary line of tubing constraint
$12X_1 + 16X_2 = 2880$

Feasible Region

(240, 0)

Plotting A Level Curve of the Objective Function

objective function

$350X_1 + 300X_2 = 35000$

(0, 116.67)

(100, 0)

A Second Level Curve of the Objective Function

objective function
$350X_1 + 300X_2 = 35000$

objective function
$350X_1 + 300X_2 = 52500$

(0, 175)

(150, 0)
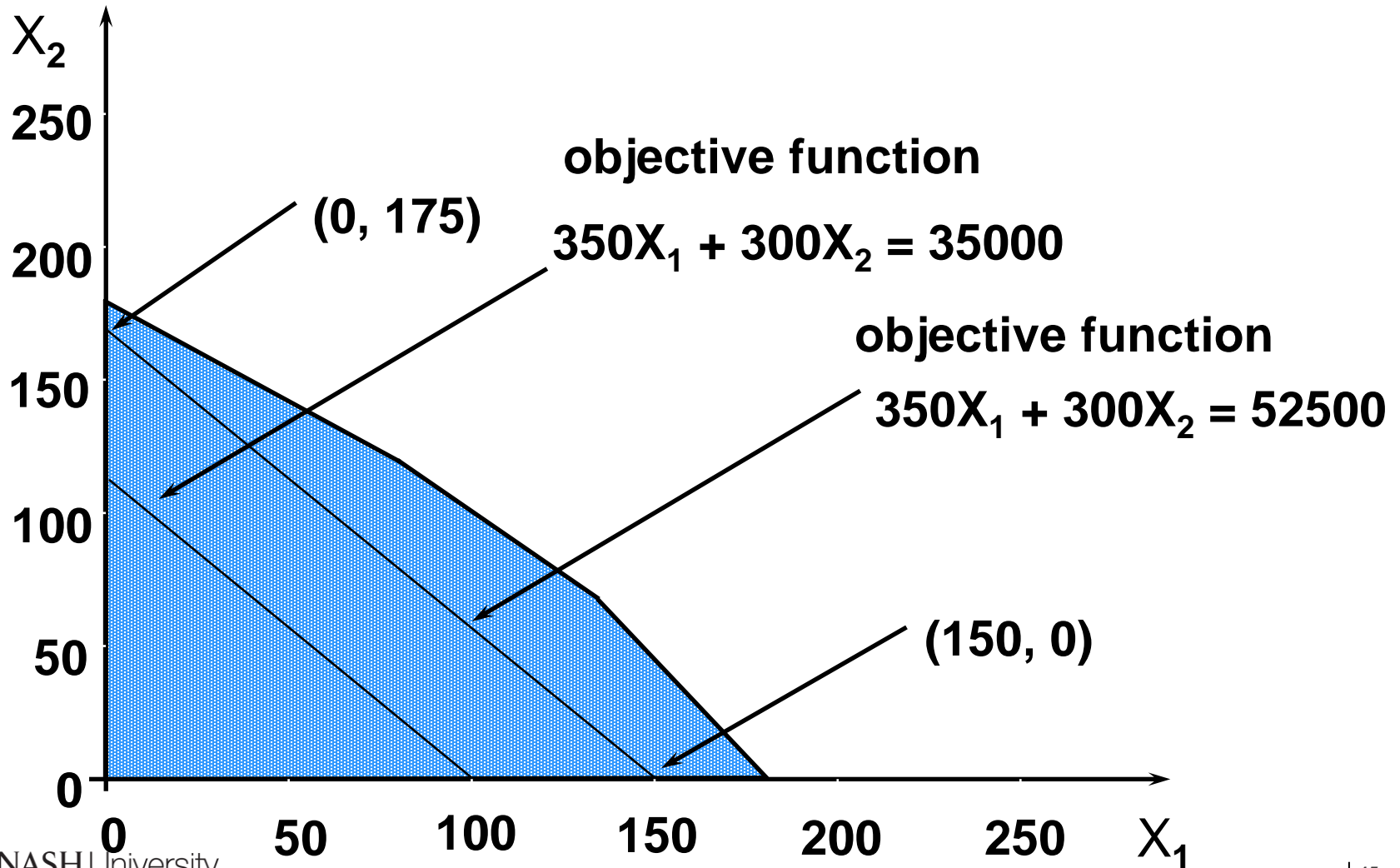
Using A Level Curve to Locate the Optimal Solution

objective function

$350X_1 + 300X_2 = 35000$

optimal solution

$350X_1 + 300X_2 = 66100$

objective function

$350X_1 + 300X_2 = 52500$
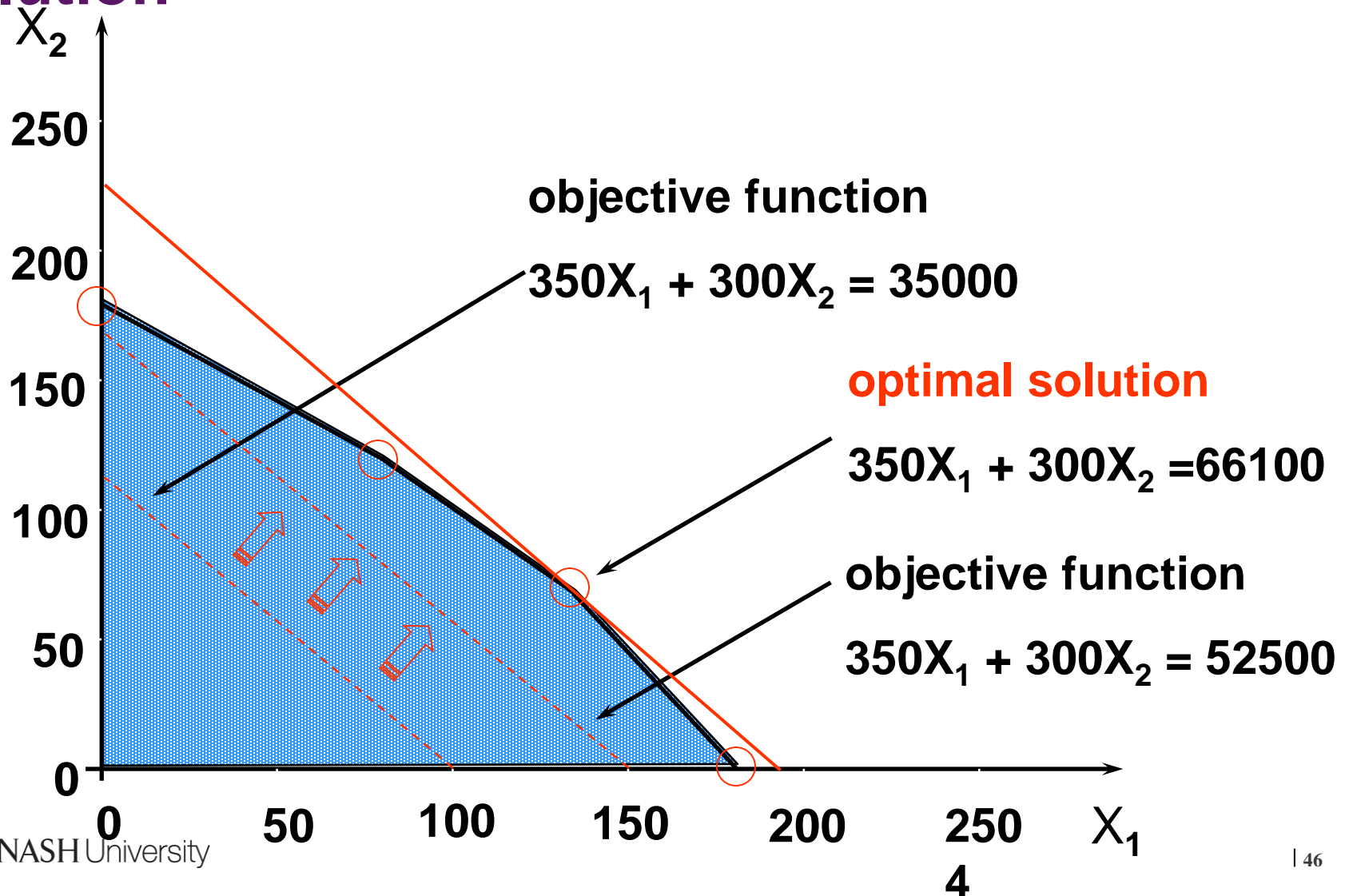
Calculating the Optimal Solution

- The optimal solution occurs where the "pumps" and "labor" constraints intersect.

- This occurs where:

$$X_1 + X_2 = 200 \qquad (1)$$

$$\text{and} \quad 9X_1 + 6X_2 = 1566 \qquad (2)$$

- From (1) we have, $X_2 = 200 - X_1$ $\qquad\qquad$ (3)

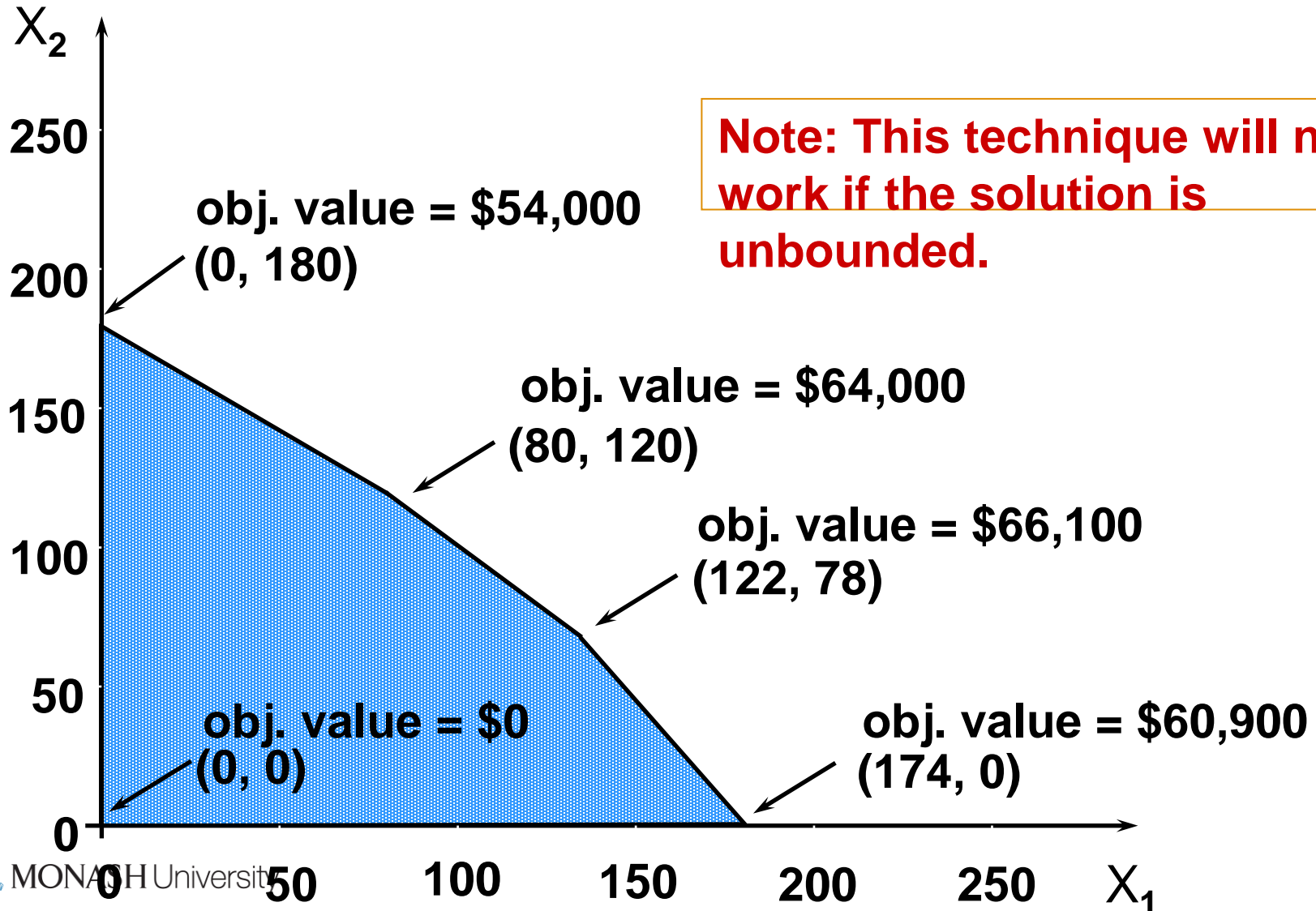- Substituting (3) for X_2 in (2) we have,

$$9X_1 + 6(200 - X_1) = 1566$$

$$\rightarrow \quad X_1 = 122$$

- So the optimal solution is:

Substituting $X_1 = 122$ in $X_2 = 200 - X_1 \rightarrow X_2 = 78$

Total Profit = \$350*122 + \$300*78 = \$66,100

Enumerating The Corner Points

Note: This technique will not work if the solution is unbounded.

obj. value = $54,000 (0, 180)

obj. value = $64,000 (80, 120)

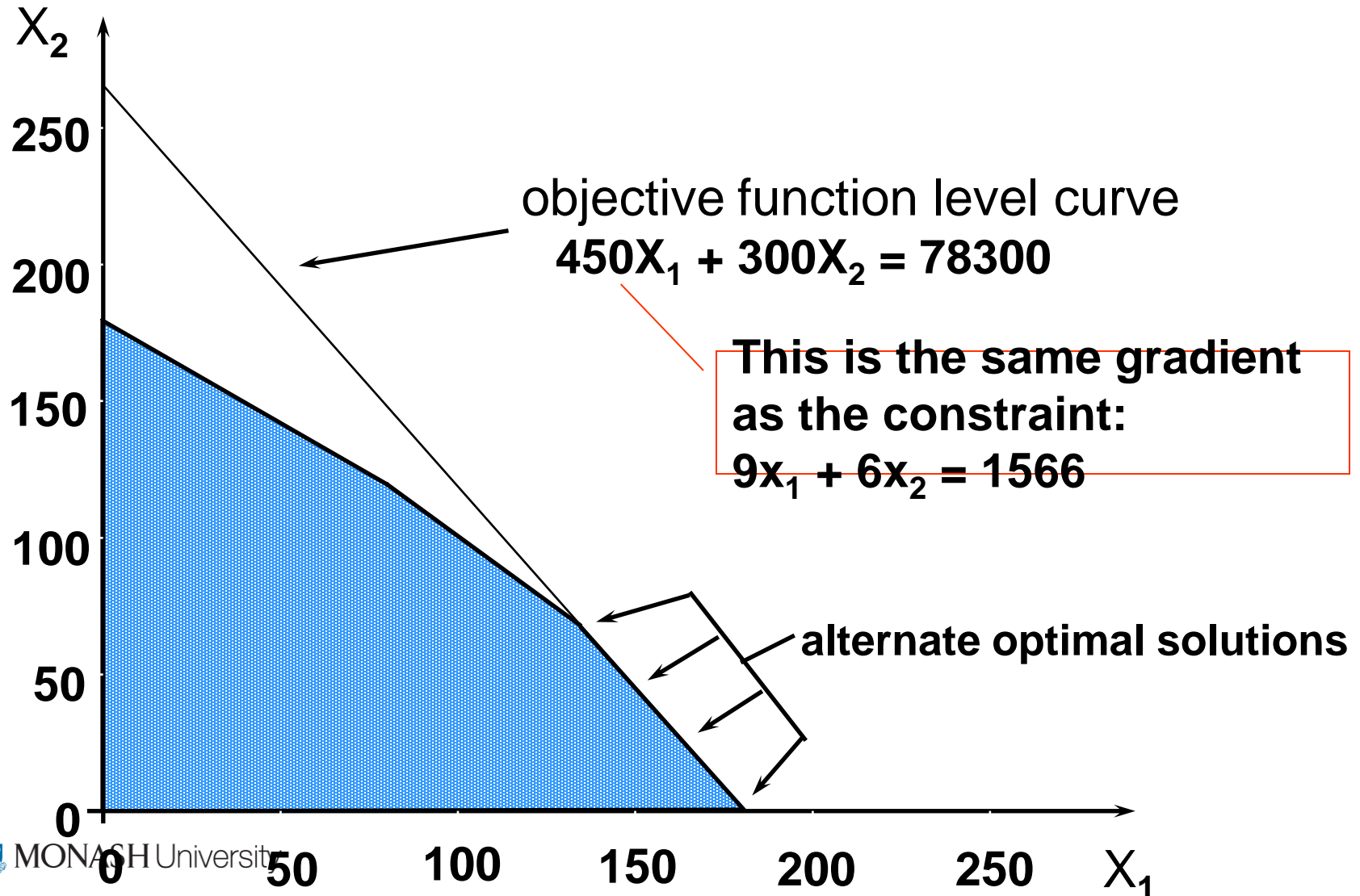obj. value = $66,100 (122, 78)

obj. value = $0 (0, 0)
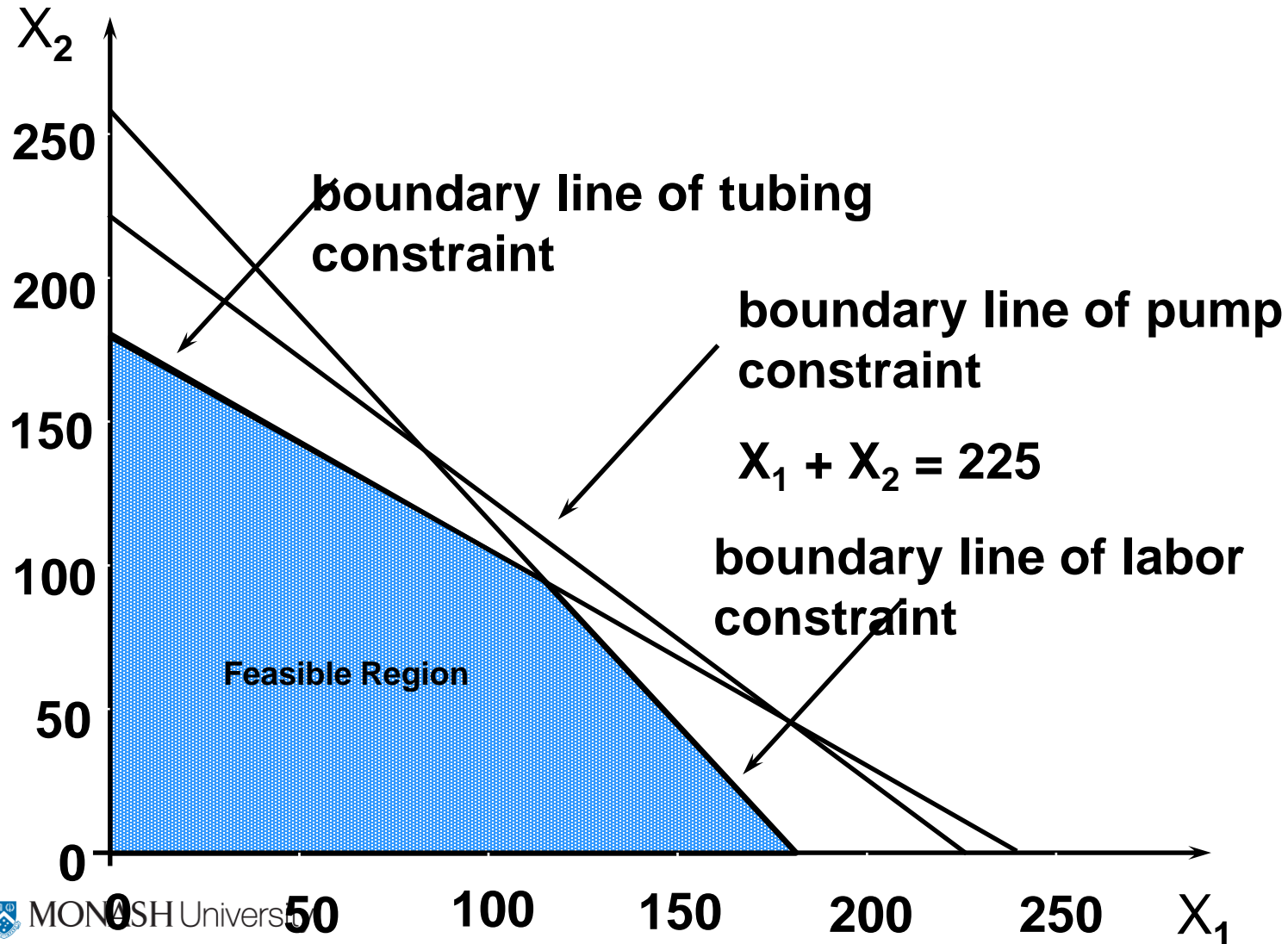
obj. value = $60,900 (174, 0)

Special Conditions in LP Models

- A number of anomalies can occur in LP problems:

  - Alternate Optimal Solutions

  - Redundant Constraints

  - Unbounded Solutions

  - Infeasibility
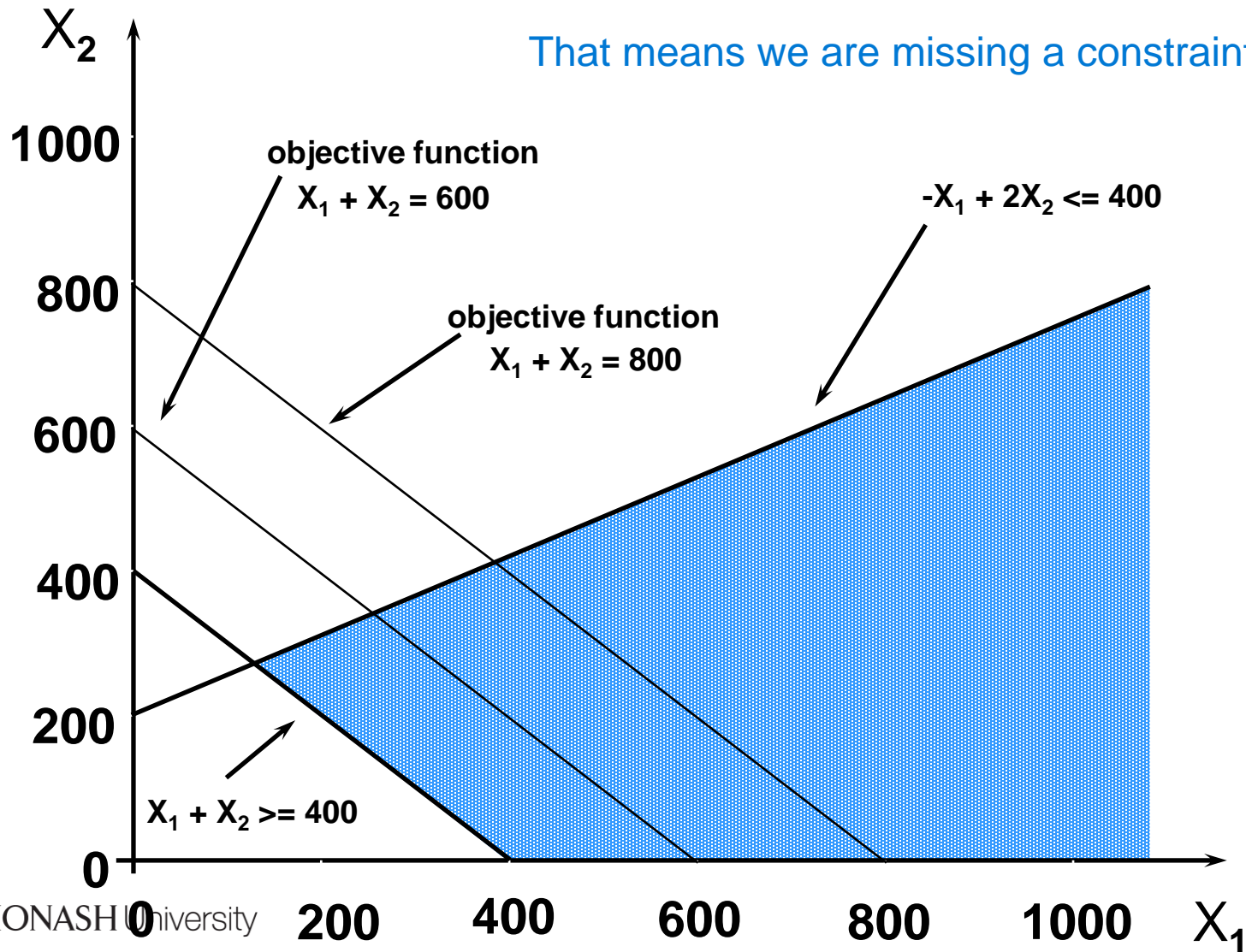
Example of Alternate Optimal Solutions

X_2

objective function level curve
$450X_1 + 300X_2 = 78300$

This is the same gradient as the constraint:
$9x_1 + 6x_2 = 1566$

alternate optimal solutions

X_1

Example of a Redundant Constraint

Example of an Unbounded Solution

That means we are missing a constraint

objective function $X_1 + X_2 = 600$

objective function $X_1 + X_2 = 800$

$-X_1 + 2X_2 <= 400$

$X_1 + X_2 >= 400$

Example of Infeasibility

Maybe you are

1) missing 1 constraint
2) Misinterpret 1 constraint

X_2

250
200 — $X_1 + X_2 >= 200$
150
100
50
0

feasible region for second constraint

feasible region for first constraint

$X_1 + X_2 <= 150$

0   50   100   150   200   250 X_1
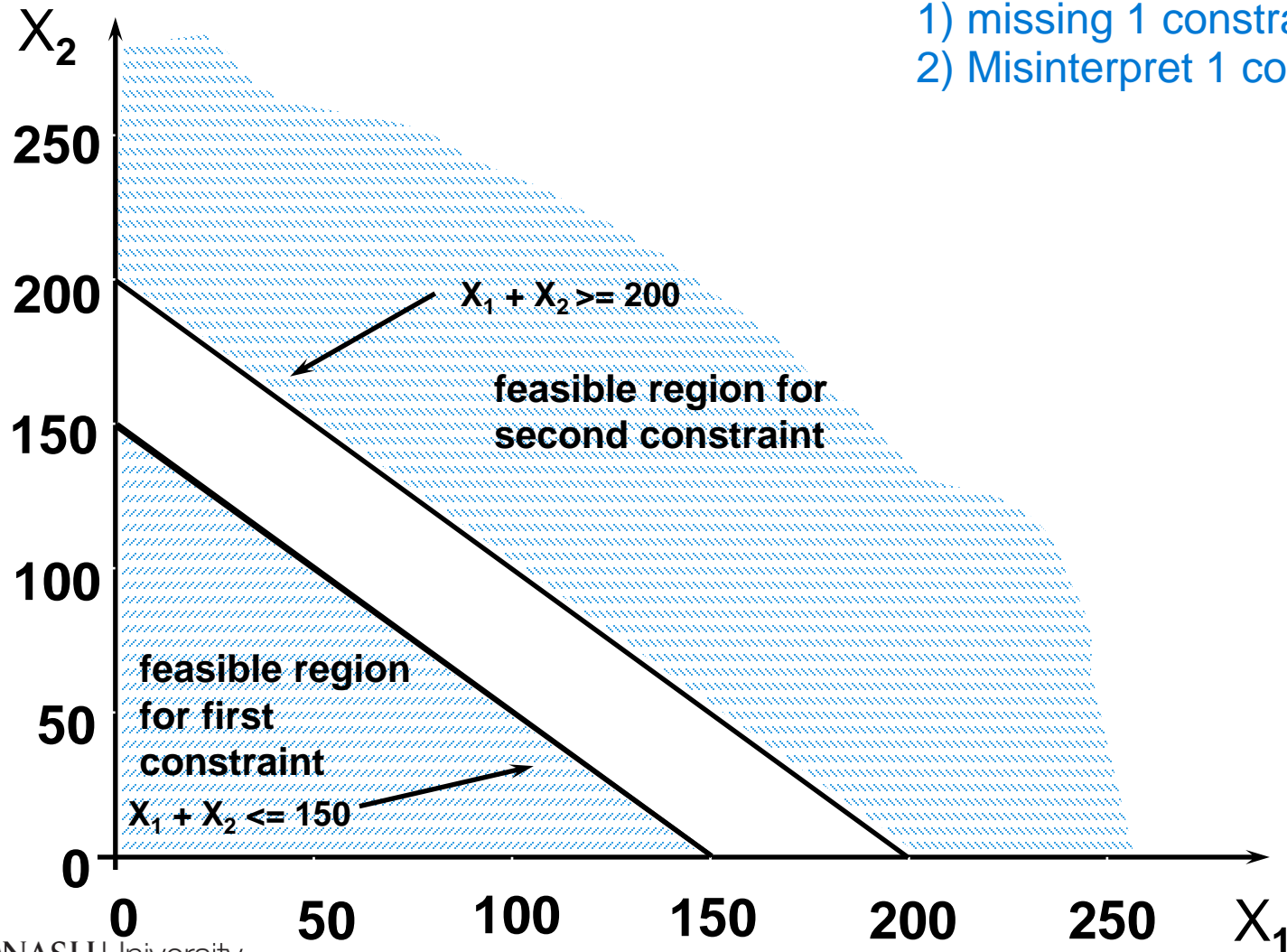
☞ *There's no tutorial this week*
Please go through the Excel exercises on Moodle on your own to brush up your Excel skills for subsequent exercises in the weeks to come.

Readings for next week's Lecture:
Ragsdale, C. T., Spreadsheet Modelling and Decision Analysis 8th Ed, Cengage Learning, 2017: Chapter 3