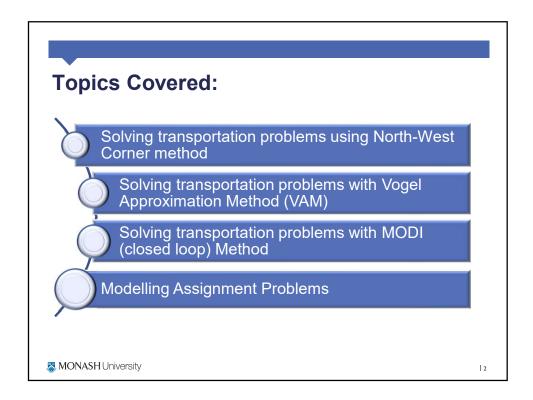
## FIT3158 Business Decision Modelling

## Lecture 6

Network Modelling (Part 2)



### Introduction

#### Please recall ...

- A <u>network model</u> is one which can be represented by a set of nodes, a set of arcs, and functions (e.g., costs, supplies, demands, etc.) associated with the arcs (also called edges) and/or nodes (also called vertices).
- Each of these models can be formulated as a linear programming problem and solved by general purpose linear programming (LP) codes.



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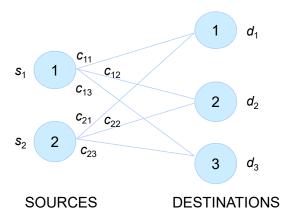
### Introduction

- One of the most important applications of quantitative analysis in solving business problems is the physical distribution of products.
- Great cost savings can be achieved by more efficient routing, distribution and scheduling of goods and services from one node (<u>source</u>, where the supply is) to the required destination (<u>sink</u>, where the demand is).
- The <u>transportation problem</u> seeks to minimize the total shipping costs of transporting goods from *m* origins (each with a supply s<sub>i</sub>) to *n* destinations (each with a demand d<sub>j</sub>), when the unit shipping cost from an origin, i, to a destination, j, is c<sub>ii</sub>.

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## **Transportation Problem**

 The following is a <u>network representation</u> of a transportation problem with two sources and three destinations



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Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning

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## **Transportation Problem - LP Formulation**

The linear programming formulation in terms of the amounts shipped from the origins to the destinations,  $x_{ij}$ , can be written as:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \le s_{i} \quad for \ i = 1, 2, ..., m \quad \text{Supply}$$

$$\sum_{i=1}^{m} x_{ij} = d_{j} \quad for \ i = 1, 2, ..., n \quad \text{Demand}$$

$$x_{ij} \ge 0 \quad \text{for all } i \text{ and } j$$

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## **Transportation Problem**

LP Formulation Special Cases

- Total supply exceeds total demand:
  - No modification of LP formulation is necessary.
- Total demand exceeds total supply:

Add a dummy origin with supply equal to the shortage amount. Assign a zero shipping cost per unit. The amount "shipped" from the dummy origin (in the solution) will not actually be shipped.



Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning

# **LP Formulation Special Cases**

The following special-case modifications to the linear programming formulation can be made:

❖ Minimum shipping guarantee from *i* to *j*:

$$x_{ij} \geq L_{ij}$$

❖ Maximum route capacity from *i* to *j*:

$$x_{ij} \leq L_{ij}$$

Unacceptable route:

Remove the corresponding decision variable.

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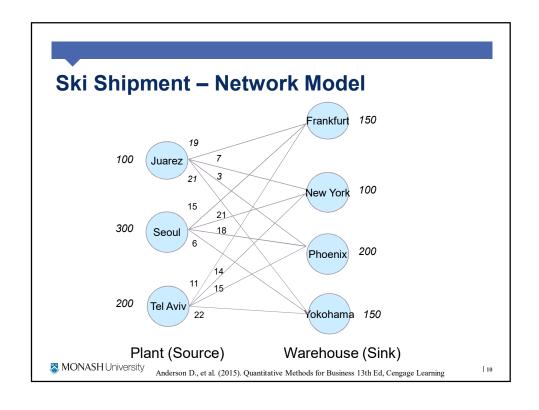
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# **Example: Ski Shipment Scheduling**

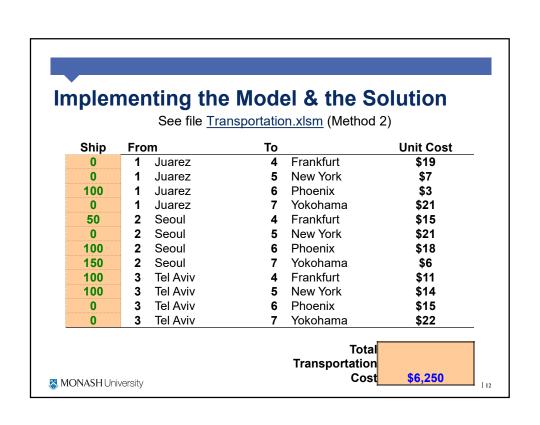
From		То	Warehou	se	
Plant	Frankfurt	New York	Phoenix	Yokohama	Capacity
Juarez	19	7	3	21	100
Seoul	15	21	18	6	300
Tel Aviv	11	14	15	22	200
Demand	150	100	200	150	

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	4	Plant	Frankfurt	New York	Phoenix	Yokohama	Capacity	/
	5	Juarez	19	7	3	21	100	
	6	Seoul	15	21	18	6	300	
	7	Tel Aviv	11	14	15	22	200	
	8	Demand	150	100	200	150	C(min)	/
SUM(B	12:E	314)		Solution			\$6,250	)
	10	From	*	To Wa	rehouse			LIM/D40.F
	M	Plant	Frankfurt	New York	Phoenix	Yokohama	Tota	UM(B12:E
	12	Juarez	0	0	100	0	100	
	13	Seoul	50	0	100	150	300	
	14	Tel Aviv	100	100	0	0	200	A
	15	Total	150	100	200	150	* S	olution



# **Transportation Models**

We will now look at some of the techniques used to solve transportation problems.

- **❖ Northwest Corner Method**
- ❖ Vogel's Approximation Method (VAM)
- ❖ MODI (The Closed-Loop Method)



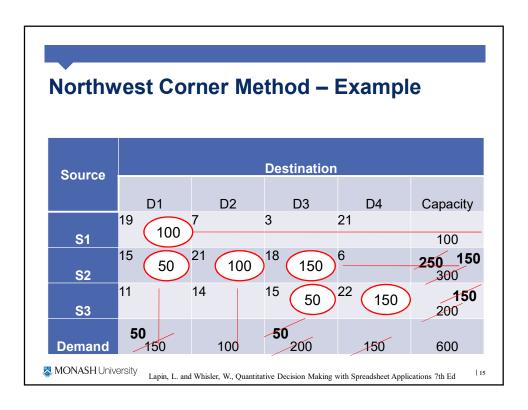
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### **Northwest Corner Method**

### Algorithm:

- Start in the top left hand (or Northwest) corner.
- Allocate the maximum supply possible to demand.
- Adjust the row and column entries.
- If demand is met, move to next column.
- If supply is exhausted, move to next row.
- Move from top left → bottom right

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### **Northwest Corner Method – Solution**

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D1	100	19	1900
S2 → D1	50	15	750
S2 → D2	100	21	2100
S2 → D3	150	18	2700
S3 → D3	50	15	750
S3 → D4	150	22	3300
			\$11.500

This is quite a 'simplistic' technique. It does not take the costs into consideration.

The solution generated is far from optimal – compare this with Slide 11 or 12 (where solution is far cheaper).

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Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed

# **Vogel's Approximation Method (VAM)**

This method was originally used for ammunition distribution.

### The Basic Principle:

In choosing a route,

- Try to avoid high cost routes.
- Will be implicitly making decisions about alternative routes.
- Does not only consider direct costs but also the next best alternative.



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# **Vogel's Approximation Method (VAM)**

### Algorithm:

- 1. Calculate the potential opportunity loss for rows. The opportunity loss is conservatively estimated as the difference between the lowest cost cell and the next lowest cost cell.
- 2. Do the same thing for columns.
- 3. Locate the highest potential opportunity loss. Break ties arbitrarily.
- 4. Allocate the maximum supply possible to the minimum cost cell in the row or column located in (3).
- 5. Adjust rows and columns.
- 6. Iterate

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		•				
0			Destinatio	n		
Source	D1	D2	D3	D4	Capacity	
<b>S</b> 1	1 <u>9</u>	7	3 100	21	100	7-3=4 0
S2	15 50	21	18 100	6 150	150 <b>3</b> 00	15-6=9 18-15=3
S3	11_100	14 100	15	22	_100 _ <b>20</b> 0	14-11=3 15-11 <del>=</del> 4
Demand	50 -150	100 14-7=7	100 <b>200</b>	150	600	

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D3	100	3	300
S2 → D1	50	15	750
S2 → D3	100	18	1800
S2 → D4	150	6	900
S3 → D1	100	11	1100
S3 → D2	100	14	1400
			\$6,250

## **MODI (Closed-Loop Method)**

### Also known as:

- Modified Distribution Method; or
- Modified Dantzig Iteration Algorithm

### What we have noticed so far:

If there are 3 sources (N) and 4 destinations (M), the total number of allocations is 3 + 4 - 1 = 6

So, for non-degenerate solutions, there will always be: N + M - 1 allocations.



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## **MODI (Closed-Loop Method)**

### <u>Algorithm:</u>

(Recall that the  $\boldsymbol{C}_{ij}$  are the edge costs.)

- 1. Generate a basic feasible solution (e.g., using Northwest Corner or Vogel's Approximation Method [VAM]).
- 2. Derive  $R_i + K_j = C_{ij}$  for any cell with a shipment

(where  $R_i$  = Row Indicators,  $K_j$  = Column Indicators). By convention, we always set  $R_1$  = 0

3. Calculate the  $C_{ij}$  –  $(R_i + K_j)$  values for cells with no shipment.

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## **MODI (Closed-Loop Method)**

- 4. Put a "+" sign in the most negative cell.
  - a) If there is more than 1 negative, choose the biggest reduction. Break ties arbitrarily.
  - b) If there are no cells with a negative  $C_{ij} (R_i + K_j)$  values  $\Rightarrow \underline{STOP}$  the solution is optimal.
- 5. Form a closed loop.
- 6. Determine maximum adjustment and modify solution.
- 7. Iterate

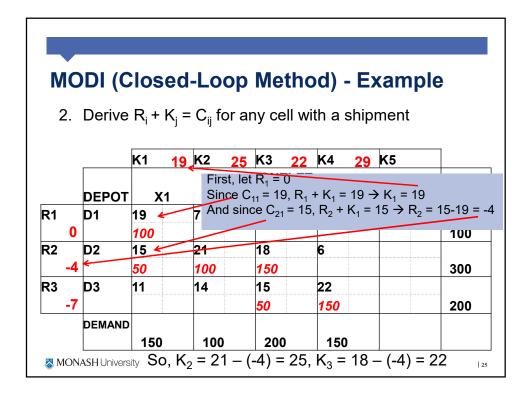
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12:

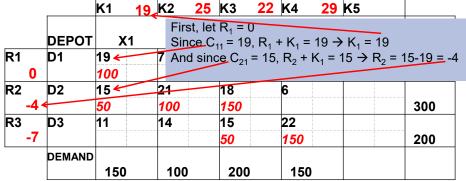
# **MODI (Closed-Loop Method) - Example**

1.Start with a basic feasible solution – let's use the one from Northwest Corner Method (see above, approx. slide 15).

Source		[	) Destination	n	
Jource	D1	D2	D3	D4	Capacity
S1	19 100	7	3	21	100
S2	15 50	21 100	18 150	6	300
<b>S</b> 3	11	14	15 50	22 150	200
Demand MONASHUM	150	100	200	150	600



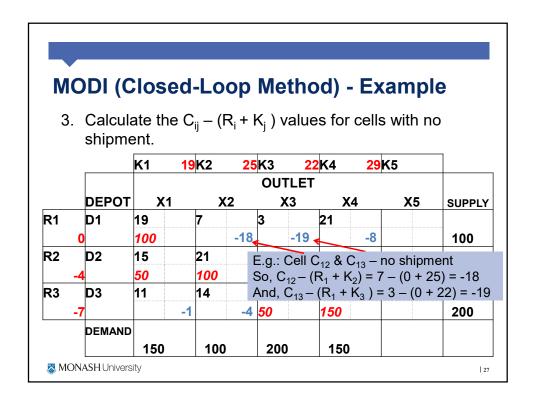
# MODI (Closed-Loop Method) - Example 2. Derive $R_i + K_j = C_{ij}$ for any cell with a shipment

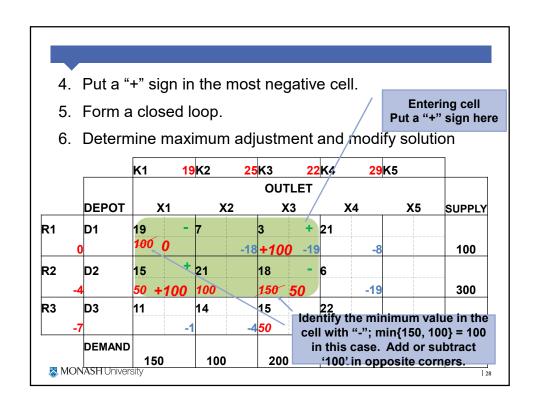


There are (M+N-1) allocations. And, setting  $R_1 = 0$ , there are (M+N)-1 values of  $R_i$  and  $K_j$  to find.

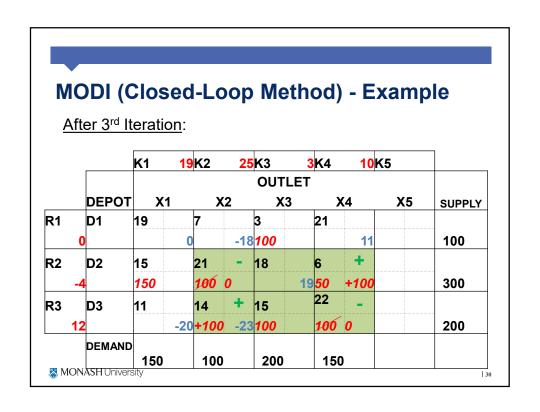
So,  $K_1 = 19 - 0 = 19$ ,  $R_2 = 15 - 19 = -4$ ,  $K_2 = 21 - (-4) = 25$ ,  $K_3 = 18 - (-4) = 22$ ,  $R_3 = 15 - 22 = -7$ ,  $K_4 = 22 - (-7) = 29$  (and this completes step 2). We now go to step 3 and calculate  $C_{ij} - (R_i + K_j)$  values for cells with no shipment.

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Δft								/			
	er 2 <sup>nd</sup> I	terati	on:								
			<u> </u>								
		<b>K</b> 1	0	K2	6	<b>K</b> 3	3	K4	10K	5	
						OU.	ΓLET				
	DEPOT	X	1	X	2	)	<b>(</b> 3	Х	<b>4</b>	X5	SUPPLY
₹1	D1	19		7		3		21			
C			19		1	100			11		100
₹2	D2	15		21		18	-	6	+		
15	5	150		100		<b>50</b>	0	+50	-19		300
₹3	D3	11		14		15	+	22	_		
••			-1		-4	<b>50</b>	+50	150	100		200
12		1									



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		K1	-4	K2	2	K3	3	K4	-13K	5	
						OUT	LET				
	DEPOT	Х	1	Х	2	Х	(3	Х	4	X5	SUPPLY
R1	D1	19		7		3		21			
0			23		5	100			34		100
R2	D2	15	-	21		18	+	6			
19		150	<del>50</del>	0		+100	-4	150			300
R3	D3	11	+	14		15	-	22			
12		+100	3	100		100	0		23		200

		<u>ration</u> :					
						+ K <sub>j</sub> ) valu	ıes <del>→</del>
<u>:</u>	<u> </u>	- the so	olution is	optimai			
		K1	<mark>0</mark> K2	<mark>3</mark> K3	<mark>3</mark> K4	- <mark>9</mark> K5	
				OUT	LET		
	DEPOT	X1	X2	X	3 X	4 X	SUPPLY
R1	D1	19	7	3	21		
0			19	4100		30	100
R2	D2	15	21	18	6		
15		<b>50</b>		3100	150		300
R3	D3	11	14	15	22		
				1	1	1	

# **MODI (Closed-Loop Method) – Solution**

To supply:	Quantity:	Unit Cost:	Total Cost:
S1 → D3	100	3	300
S2 → D1	50	15	750
S2 → D3	100	18	1800
S2 → D4	150	6	900
S3 → D1	100	11	1100
S3 → D2	100	14	1400
			\$6,250

MODI always gives the optimal solution.

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## Some issues to take note of:

- 1. When supply does not equal demand
  - When demand < supply, add a dummy column for demand to make up for the difference.

	X1	X2	Dummy	Supply
D1	8	9	0	200
D2	12	7	0	200
Demand	200	100	100	

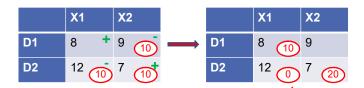
 Similarly, when demand > supply, add a dummy row for supply

In fact, the first step in doing any allocations is to check whether the demand and supply are equal.

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### Some issues to take note:

When there's a case of degeneracy
 E.g.:



We now ended up having 2 allocations, thus breaking the (M + N - 1) rule.

To handle such a situation, we put a zero (0) in one of the cells.

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# **Assignment Problem**

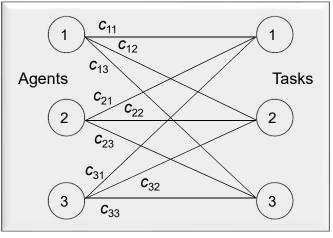
- An <u>assignment problem</u> seeks to minimize the total cost assignment of *m* workers to *m* jobs, given that the cost of worker *i* performing job *j* is c<sub>ij</sub>.
- It assumes all workers are assigned and each job is performed.
- An assignment problem is a special case of a <u>transportation</u> <u>problem</u> in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.
- The <u>network representation</u> of an assignment problem with three workers and three jobs is shown on the next slide.

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Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning

# **Assignment Problem**

Network Representation



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# **Assignment Problem**

Linear Programming Formulation

Using the notation:

$$x_{ij} = \begin{cases} 1 & \text{if agent } i \text{ is assigned to task } j \\ 0 & \text{otherwise} \end{cases}$$

 $c_{ij}$  = cost of assigning agent i to task j

continued  $\longrightarrow$ 

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## **Assignment Problem**

Linear Programming Formulation (continued)

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad for j = 1, 2, ..., n \text{ Tasks}$$

$$\sum_{j=1}^{n} x_{ij} \le 1 \quad for i = 1, 2, ..., m \text{ Agents}$$

$$x_{ij} \ge 0 \text{ for all } i \text{ and } j$$

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## **Assignment Problem**

- LP Formulation Special Cases
  - Number of agents exceeds the number of tasks:

Extra agents simply remain unassigned.

Number of tasks exceeds the number of agents:

Add enough dummy agents to equalize the number of agents and the number of tasks. The objective function coefficients for these new variables would be zero.

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Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning

## **Assignment Problem**

- LP Formulation Special Cases (continued)
  - The assignment alternatives are evaluated in terms of revenue or profit:

Solve as a maximization problem.

• An assignment is unacceptable:

Remove the corresponding decision variable.

• An agent is permitted to work *t* tasks:

$$\sum_{j=1}^{n} x_{ij} \le t \quad \text{for } i = 1, 2, ..., m \text{ Agents}$$

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## **Assignment Problem: Example**

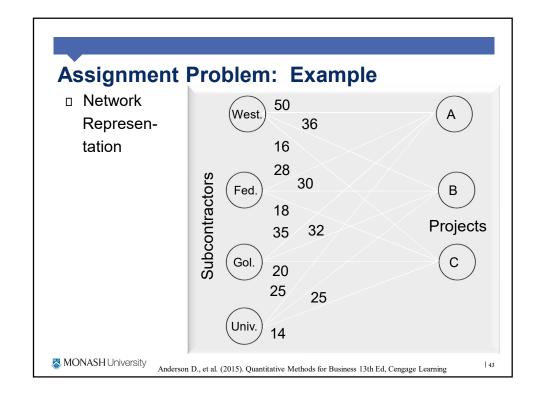
An electrical contractor pays her subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

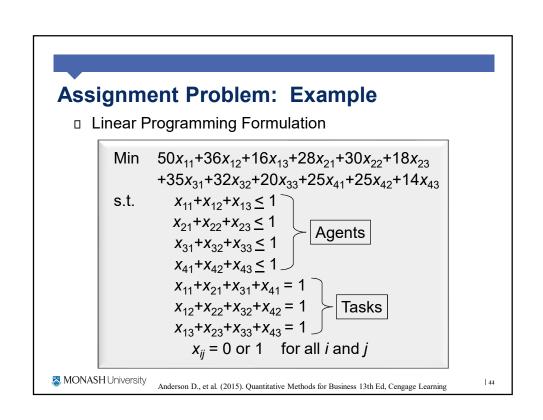
	<u>Projects</u>		
Subcontractor	<u>A</u>	<u>B</u>	<u>C</u>
Westside Federated	50 28	36 30	16 18
Goliath	35	32	20
Universal	25	25	14

How should the contractors be assigned so that total mileage is minimized?

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# Implementation of Model & Solution:

See file <u>Assignment Problems.xlsx</u> (Contractor Assignment )

	-					•	,
				Ma	trix of Indic	ators	
		Projects		A	В	C	(Sum <= 1)
	Matrix of	Subcontractors	Westside	0	0	1	1
			Fe de rate d	1	0	0	1
	Indicators		Goliath	0	0	0	0
			Universal	0	1	0	1
			(Sum = 1)	1	1	1	
	35	Projects		A	В	C	
		Subcontractors	Westside	50	36	16	
			Fe de rate d	28	30	18	
	weights		Goliath	35	32	20	
			Universal	25	25	14	
	Matrix of Products	Projects		A	В	С	
		Subcontractors	Westside	0	0	16	
			Fe de rate d	28	0	0	
			Goliath	0	0	0	
			Universal	0	25	0	
			Obje				
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<b>™</b> MONAS	Products	Subcontractors  Projects	Federated Goliath Universal Westside Federated Goliath Universal	50 28 35 25 A 0 28 0 0 ective Func	36 30 32 25 B 0	16 18 20 14 C 16 0 0	

# **Assignment Problem: Example**

■ The optimal assignment is:

SubcontractorProjectDistanceWestsideC16FederatedA28Goliath(unassigned)UniversalB25Total Distance= 69 miles

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# Job Assignment based on maximising preferences

- We have 10 people and we wish to assign each person one job to do. Each person makes a list of 3 preferences and we assign the jobs accordingly.
- This problem is probably getting quite close to the size that you could do with the solver in practice and it provides a good example of how the solver works.
- By observing partial solutions, we can see that the solver initially relaxes the constraint that indicators be integers and gradually enforces this condition as a solution is approached.



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## **Setup: Input Data**

• Input data, shown coded as a table of weights.

Preferences				Preferences coded 10 = first, 5 = second, 1 = thir								
Person	<b>P1</b>	P2	P3	1	2	3	4	5	6	7	8	9
a	2	7	8	0	10	0	0	0	0	5	1	0
b	2	4	5	0	10	0	5	1	0	0	0	0
c	9	8	6	0	0	0	0	0	1	0	5	10
d	6	9	1	1	0	0	0	0	10	0	0	5
e	6	4	7	0	0	0	5	0	10	1	0	0
f	6	7	9	0	0	0	0	0	10	5	0	1
g	6	8	7	0	0	0	0	0	10	1	5	0
h	1	7	5	10	0	0	0	1	0	5	0	0
I	3	7	4	0	0	10	1	0	0	5	0	0
i	1	9	8	10	0	0	0	0	0	0	1	9

See file <u>Assignment Problems.xlsx</u> (Job Choices)

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### **Indicator Matrix**

• Final settings showing indicator values and constraint values.

	1	2	3	4	5	6	7	8	9	10	Check
	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
Check	1	1	1	1	1	1	1	1	1	1	

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The Travelling Salesperson Problem
A salesperson wants to find the least costly route for visiting clients in n different cities, visiting each city exactly once before returning home.

n	(n-1)!
3	2
5	24
9	40,320
13	479,001,600
17	20,922,789,888,000
20	121,645,100,408,832,000

Ragsdale, C.. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e), Cengage

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Ragsdale (2021, 9e), chap. 8, sec. 8.14

### **Example: The Traveling Salesperson Problem**

 Wolverine Manufacturing needs to determine the shortest distance for a drill bit to drill 9 holes in a fiberglass panel.

See file TSP.xlsm

<u>Note</u>: This is a Non-linear Programming (NLP) problem.

Ragsdale, C.. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) , Cengage Learning. Ragsdale (9e, 2021), chap. 8

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### **End of Lecture 6**

### References:

Ragsdale, C. (2021). 9th edition, chapter 5,

Ragsdale, C. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) Cengage Learning: Chapter 8

Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 12

Anderson D., et al. (2015). Quantitative Methods for Business 13th Ed, Cengage Learning: Chapter 10

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### **Homework**

- > Go through today's lecture examples :
  - ✓ Familiarise yourself with the different algorithms used:
    - Northwest Corner Method
    - ❖ Vogel's Approximation Method
    - MODI (Closed-loop) Method
  - ✓ Understand how the spreadsheets are being modeled for Assignment problems and Transportation problems
- Readings for next Lecture:
- C. T. Ragsdale (9<sup>th</sup> edn), chapter 8, secs. 8.4 8.5, Economic Order Quantity (EOQ)
- Lapin, L. and Whisler, W., Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 15 - Inventory Decisions under Certainty



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### Tutorial 5 this week:

### **Network Modelling:**

- The Shortest Route Problem
- Maximal Flow Problem
- Minimal Spanning Tree Problem

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