

Introduction to Decision Analysis

- Models
 - Can help managers gain insight & understanding, but they can't make decisions.
- Decision making remains a difficult task:
 - Uncertainty regarding future events,
 - Conflicting values or objectives.

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Deciding between job offers

- Company A
 - New industry, Could boom or bust, Low starting salary -Could increase rapidly, Located near friends, family and favourite sports team
- Company B
 - Established firm, Financial strength & commitment to employees, Higher starting salary, Slower advancement opportunity, Distant location, offering few cultural or sporting activities
- Which job would you take?
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Good decisions vs good outcomes

- A structured approach to decision making:
 - Can help us make good decisions but can't guarantee good outcomes.
- It is important to appreciate that:
 - Good decisions can lead to bad outcomes and vice versa.

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Characteristics of decision problems

- Alternatives: Different courses of action
- Criteria: Factors that are important to decision maker
- <u>States of Nature</u> (State-of-the-World): Possible future events not under decision maker's control

<u>Alternatives</u>	<u>Criteria</u>	States of Nature
Work for Co. A	Salary	Co. A grows
Work for Co. B	Career potential	Co. A goes bust
Reject both offers	Location	Etc
and keep looking	Etc	

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Assumptions

- Decision maker knows what states-of-nature are possible.
- The number of possible states. n. is finite.
- States-of-nature are mutually exclusive,
- Number of actions available is finite,
- One, and only one, action must be chosen,
- For each combination of state and action, there is a unique reward.

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Classification of Problems

Due to the degree/level of uncertainty:

Decision making under certainty

- True state-of-the-world is known to decision maker (DM) before she/he makes decision

Decision making under risk

- Although true state-of-the-world is not known with certainty, <u>uncertainty can be quantified</u> by means of a probability distribution

Decision making under strict uncertainty

- Decision maker (DM) knows nothing at all about true stateof-the-world, except for which states are generally possible

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Decision Rules

Two categories:

- Non-probabilistic decision rules
 - Maximax
 - Maximin
 - Minimax regret
- Probabilistic
 - Expected Monetary Value (EMV)
 - Expected Regret or Opportunity Loss

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Problem Formulation

- A decision problem is characterized by decision alternatives, states of nature, and resulting payoffs.
- The <u>decision alternatives</u> are the different possible strategies that the decision maker can employ.
- The <u>states of nature</u> refer to future events, not under the control of the decision maker, which may occur. States of nature should be defined so that they are mutually exclusive and collectively exhaustive (i.e., every possible thing that could happen belongs to exactly one state of nature).

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Payoff Tables

- The consequence resulting from a specific combination of a decision alternative and a state of nature is a <u>payoff</u>.
- A table showing payoffs for all combinations of decision alternatives and states of nature is a <u>Payoff Table</u> or <u>Payoff</u> <u>Matrix</u>.
- Payoffs can be expressed in terms of <u>profit</u>, <u>cost</u>, <u>time</u>, <u>distance</u> or any other appropriate measure that is used as the decision criteria.

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Decision Making Without Probabilities

- Three commonly used criteria for decision making when probability information regarding the likelihood of the states of nature is unavailable are:
- \blacksquare The $\underline{Optimistic}$ approach: The $\underline{Maximax}$ payoff criterion.
- The Conservative approach: The Maximin payoff criterion.
- The <u>Opportunity Loss</u> approach: The <u>Minimax-Regret</u> criterion.

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Optimistic Approach (Maximax)

- The optimistic approach would be used by a decision maker who believed that the project had a high probability of success.
- The decision with the largest possible payoff is chosen.
- If the payoff table were in terms of costs, the decision with the lowest cost would be chosen.
- This is a strategy for a risk taking decision maker.

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Conservative Approach (Maximin)

- The conservative approach would be used by a decision maker who thought the project had a low chance of success or was a cautious decision maker.
- For each decision the minimum payoff is listed and then the decision corresponding to the maximum of these minimum payoffs is selected. (Hence, the minimum possible payoff is maximized.)
- If the payoff was in terms of costs, the maximum costs would be determined for each decision and then the decision corresponding to the minimum of these maximum costs is selected. (Hence, the maximum possible cost is minimized.)

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Minimax Regret (Opportunity Loss)

- The minimax regret approach requires the construction of a regret table or an opportunity loss table.
- This is done by calculating for each state of nature the difference between each payoff and the largest payoff for that state of nature.
- Then, using this regret table, the maximum regret for each possible decision is listed.
- The decision chosen is the one corresponding to the minimum of the maximum regrets.

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An Example: Magnolia Inns

Background info (hypothetical example):

- Hartsfield International airport in Atlanta, Georgia one of the busiest in the world. Analysts predict that traffic will continue to increase well into the future.
- Commercial development in surrounding area prevents the construction of extra runways.
- To solve the problems, plans are being developed to build another airport outside the city limits.
- Two possible locations have been identified (location A and location B). The decision as to where to build will not be made for another year.

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Magnolia Inns cont'd ...

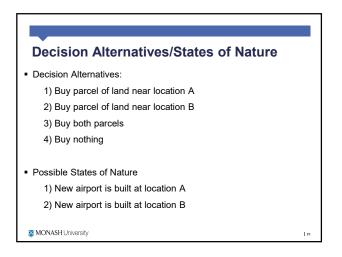
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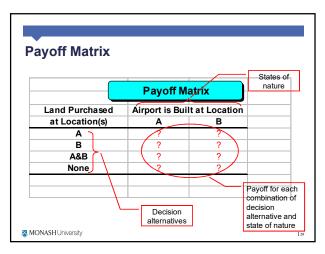
- Magnolia Inns hotel chain intends building a new hotel near the airport once the site is determined. Land values near the two possible sites for the airport are increasing due to speculation.
- Magnolia Inns have info (information) regarding:
 - The price of a suitable parcel of land for building a hotel near each possible airport site
 - The estimated present value of future cash flows that a hotel would generate at each site if the airport is ultimately located at that site.
 - The present value of the amount that Magnolia Inns believes it can sell the site for if the airport is NOT built

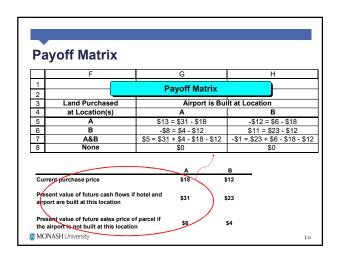
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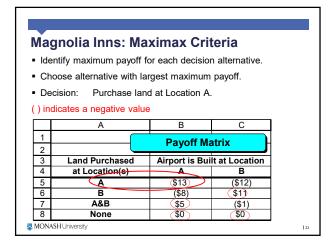
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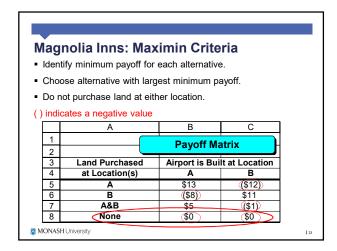
Magnolia Inns: Data See file Lecture 9.xlsm Current purchase price Present value of future cash flows if hotel and airport are built at this location (Note: All values are in millions of dollars.) MONASH University

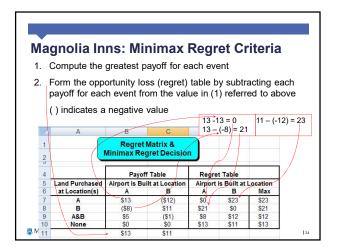












Magnolia Inns: Minimax Regret Criteria

- Identify the maximum possible regret for each alternative.
- Choose alternative with smallest maximum regret.
- Decision: Purchase both Locations A&B.

Mini	Regret Matrix & Minimax Regret Decision Rule					
	Payof	f Table	Regr	et Table		
Land Purchased	Airport is Bu	Airport is Built at Location A B		Airport is Built at Location		
at Location(s)	Α			В	Max	
Α	\$13	(\$12)	\$0	\$23	\$23	
В	(\$8)	\$11	\$21	\$0	\$21	
A&B ←	\$5	(\$1)	\$8	\$12 <	\$12	
None	\$0	\$0	\$13	\$11	\$13	

Probabilistic Methods

- Sometimes states of nature may be assigned probabilities that represent their likelihood of occurrence.
- For decision problems that occur more than once:
 - Estimate probabilities from historical data
- For decision problems that occur once off:
 - Subjective probabilities based on interviews with one or more domain experts

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Decision making using the long-run average

- Expected Value Approach
 - If probabilistic information regarding the states of nature is available, one may use the <u>expected value (EV) approach</u>.
 - Here the expected return for each decision is calculated by summing the products of the payoff under each state of nature and the probability of the respective state of nature occurring.
 - The decision yielding the <u>best expected return</u> is chosen.
 - The <u>expected value</u> refers to the long run average return, not necessarily the returns for individual cases.

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Expected Value of a Decision Alternative

- The expected value of a decision alternative is the sum of weighted payoffs for the decision alternative.
- The Expected Value (EV) or Expected Monetary Value (EMV) of decision alternative d_i is defined as:

$$EV(d_i) = \sum_{i=1}^{N} P(s_j) V_{ij}$$

Where: N = the number of states of nature

 $P(s_i)$ = the probability of state of nature s_i

 V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_i

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Magnolia Inns: Expected Monetary Value

- Let's say, the probability of the Airport being built at Location A is 0.4 and at Location B is 0.6.
- Choose the alternative with the <u>greatest EMV</u>.
 () indicates negative in table.

Payoff Matrix &
EMV Decision Rule

Land Purchased Airport is Built at Location at Location(s)
A
B
EMV
A
S13
(\$12)
(\$2.0)

A \$13 (\$12) (\$2.0)

B (\$8) \$11 \$3.4

A&B \$5 (\$1) \$1.4

None \$0 \$0 \$0 \$0.0

Expected Regret or Opportunity Loss (EOL)

• The Expected Opportunity Loss (EOL) of decision alternative d_i is defined as:

$$EOL(d_i) = \sum_{j=1}^{N} P(s_j) g_{ij}$$

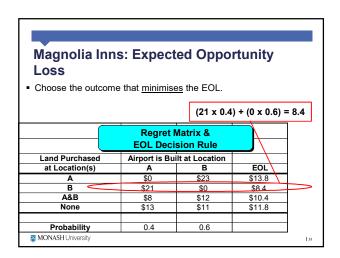
where: N = the number of states of nature

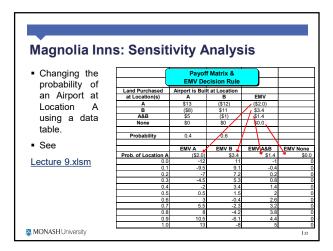
 $P(s_j)$ = the probability of state of nature s_j

 g_{ij} = the regret corresponding to decision alternative d_i and state of nature s_i

 Note: the EOL will recommend the same decision alternative as the EMV.

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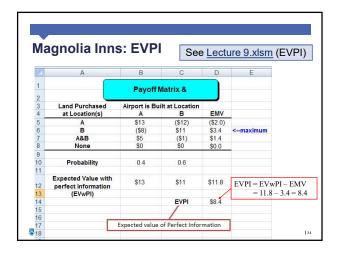




Expected Value of Perfect Information

- Frequently information is available which can improve the probability estimates for the states of nature.
- The <u>expected value of perfect information</u> (EVPI) is the increase in the expected profit that would result if one knew with certainty which state of nature would occur.
- The EVPI provides an upper bound on the expected value of any sample or survey information.

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Decision Trees

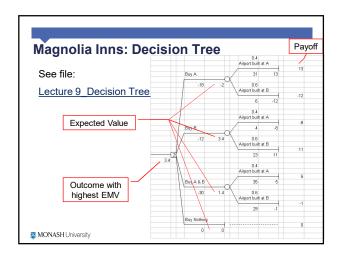
- A <u>decision tree</u> is a chronological representation of the decision problem. (These are not the same decision trees as machine learning decision trees – also sometimes called classification trees.)
- Each decision tree will use the following convention: <u>round nodes</u> will correspond to the states of nature while <u>square nodes</u> will correspond to the decision alternatives.
- The <u>branches</u> leaving each round node represent the different states of nature while the branches leaving each square node represent the different decision alternatives.
- At the end of each limb of a tree are the payoffs obtained from the series of branches making up that limb.

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Evaluating a Decision Tree

- Decision trees enable us to represent sequential decision problems in a way that allows for systematic evaluation.
- The process of evaluation (referred to as 'folding back the tree' or backward induction in the text) requires us to work backward through time, from payoff to the present.
- By evaluating the payoff of the tree at each decision point, suboptimal courses of action are identified and discarded (indicated by //).

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Sensitivity Analysis

Assume the probability of building airport in Location A varies from 0.2 to 0.8. What effect does this have on the optimal decision?

See file: Lecture 9 Decision Tree.xlsx

Probability A	EMV	Best Decision
0.2	7.2	Buy B
0.3	5.3	Buy B
0.4	3.4	Buy B
0.5	2	Buy A & B
0.6	3	Buy A
0.7	5.5	Buy A
0.8	8	Buy A

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Example Problem*

- The president of Ponderosa Records has signed a contract with a band. A recording has been made and the company must decide whether to market the group nationally.
- The CD may be marketed nationally without testing the market, in which case 50,000 units will be pressed.
- Alternatively, the company may produce a limited run of 5,000 CDs and test the market locally. If the product is a success (meaning all units are sold) the CD will then be marketed nationally.
- Probabilities and payoffs are given in the next slide.

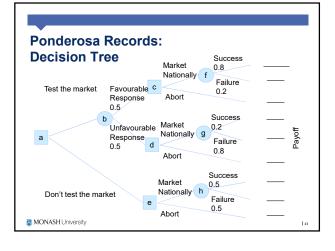
*question is from Lapin & Whisler.

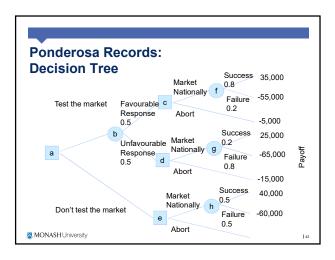
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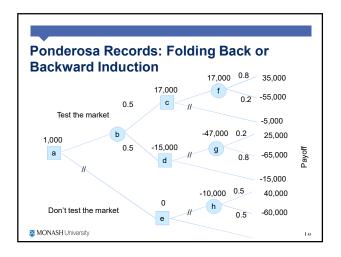
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- Without testing the market, the band's CD has a 50% chance of success. If the CD is successful locally, this is a good indicator of success nationally – being then estimated to be 80%.
- Similarly if the CD fails locally the chance of success nationally is only 20%.
- The cost of producing CDs is/are a \$5,000 fixed cost + \$1 per CD.
- The company receives \$2 for each CD sold.
- The company pays a fixed fee of \$5,000 to the band.
- Reminder: (See previous page.) The company has to decide whether to press 50,000 units without testing the market, or whether to press 5,000 first to test the market and then act on that decision. As above, the company also pays two fees (fixed cost and fixed fee).

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Bayes's Theorem and Posterior Probabilities

- Knowledge of sample or survey information can be used to revise the probability estimates for the states of nature.
- Prior to obtaining this information, the probability estimates for the states of nature are called <u>prior probabilities</u>.
- With knowledge of <u>conditional probabilities</u> for the outcomes or indicators of the <u>sample</u> or <u>survey</u> information, these prior probabilities can be revised by employing Bayes's Theorem.
- The outcomes of this analysis are called <u>posterior probabilities</u> or branch probabilities for decision trees.

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Bayes's Theorem

- Often, we begin probability analysis with initial or <u>prior probabilities</u>.
- Then, from a sample, special report, market research, or a product test we obtain some additional information.
- Given this information, we calculate revised or <u>posterior probabilities</u>.
- Bayes' theorem provides the means for revising the prior probabilities.



Computing Branch Probabilities

- Branch (Posterior) Probabilities Calculation
- Step 1: For each state of nature, multiply the prior probability by its conditional probability for the indicator – this gives the joint probabilities for the states and indicator.
- Step 2: Sum these joint probabilities over all states this gives the <u>marginal probability</u> for the indicator.
- Step 3: For each state, divide its joint probability by the marginal probability for the indicator – this gives the posterior probability distribution.

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Example: Oil Wildcatting*

Lucky Lucy is an Oil Wildcatter (a person who searches for oil). Based on 20 years of experience she estimates the probability of oil beneath Crockpot Dome. Let:

A₁ = oil below Crockpot Dome

 A_2 = no oil below Crockpot Dome

Prior Probabilities

- Using her subjective judgment: $P(A_1) = 0.2$, $P(A_2) = 0.8$
- On these data, there is a 20% probability of finding oil.
 *question is from Lapin & Whisler.

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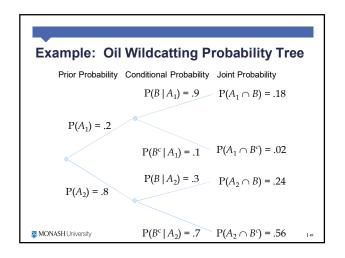
New Information

- Lucky Lucy orders a seismic survey. The petroleum engineering consultant is 90% reliable in confirming oil when there actually is oil, but only 70% reliable in predicting that there is no oil when there actually is no oil.
- Let B and B^c be the events that the tests predict / do not predict oil respectively. Then, we can represent these probabilities using set notation as:

$$P(B|A_1) = 0.9$$
 $P(B^c|A_2) = 0.7$

 We now consider how a positive/negative test result affects the probability of finding oil.

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Bayes's Theorem

 To find the posterior probability that event A_i will occur given that event B has occurred we apply <u>Bayes's theorem</u>.

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

$$= \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \dots + P(A_n)P(B|A_n)}$$

 Bayes's theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and their union is the entire sample space.

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Example: Oil Wildcatting

- Posterior Probabilities
- Given that the petroleum consultant confirms the existence of oil, we revise the prior probabilities as follows.

$$\begin{split} P(A_{1}|B) = \frac{P(A_{1})P(B|A_{1})}{P(A_{1})P(B|A_{1}) + P(A_{2})P(B|A_{2})} = \frac{(.2)(.9)}{(.2)(.9) + (.8)(.3)} \\ = 0.18/(0.18 + 0.24) = 0.18/0.42 = 3/7 \sim 0.429 \end{split}$$

- Consequently, when the test for the presence of oil is positive, the probability of oil increases from 0.2 to 0.18/0.42 = 3/7 \sim 0.429.
- Note that we expect that a negative test would decrease the probability of the presence of oil.

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Calculating Posterior Probabilities

 The simplest way of calculating posterior probabilities is in following tabular format.

	Prior	Conditional	Joint	Posterior	
States of Nature	Probabilities	Probabilities	Probabilities	Probabilities	
A1	A1 0.2		0.18	0.43	
A2	0.8	0.3	0.24	0.57	
			0.42		

States of	Prior	Conditional	Joint	Posterior		
Nature	Probabilities	Probabilities	Probabilities	Probabilities		
A1	$P(A_1)$	$P(B \mid A_1)$	$P(B \cap A_1)$	$P(A_1 B)$		
A2	$P(A_2)$	$P(B \mid A_2)$	$P(B \cap A_2)$	$P(A_2 \mid B)$		
			P(B)			

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The Expected Value of Sample Information

- What is the value of the petrol consultant's information?
- Let's assume that if Lucky Lucy is successful in finding oil then she will earn \$100,000. If she drills and does not find oil then she loses \$30,000.
- Assuming that she will not proceed if the test result is negative, we see the expected returns after a positive test result. In this case, the expected value of sample information is (3/7 x \$100,000 4/7 x \$30,000) (-\$4,000) ~ \$25,714 (-\$4,000) = \$29,714.

	Prior	Conditional	Joint	Posterior
States of Nature	Probabilities	Probabilities	Probabilities	Probabilities
A1	0.2	0.9	0.18	0.43
A2	0.8	0.3	0.24	0.57
			0.42	

I	If oil found	\$	100,000			(A)	100,000		EVSI
I	If no oil found	-\$	30,000			ှ	30,000		
ſ	Payoff	-\$	4,000			\$	25,714	\$	29,714
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End of Lecture 9

References:

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Lapin, L. and Whisler, W. (2002), Quantitative Decision Making with Spreadsheet Applications 7th Ed., Wadsworth (Thomson Learning) Belmont, 2002: Chapter 3, 4 & 5

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