Modelling and Solving LP Problems in a Spreadsheet



Goals for good spreadsheet design

Some examples in building spreadsheet models

Introduction

- Solving LP problems graphically is only possible when there are two decision variables
- Few real-world LP have only two decision variables
- Fortunately, we can now use spreadsheets to solve multi-variate LP problems
- Use spreadsheets to solve LP problems
 - The Solver in Excel.
 - The Simplex algorithm implemented in Excel's Solver searches all vertices to find the optimal solution.
- Main challenges are:
 - To formulate LP problem correctly.
 - Communicate formulation to computer accurately.
 - Use a recognised variable layout.

Software and LP Solvers

- Dozens of programs: some capable of solving problems with up to thousands variables and hundreds constraints
- Well developed interface and input-output facilities
- Special solvers callable from programming languages such as VB/Delphi etc..
- Spreadsheet solvers:
 - Analytic Solver Platform by Frontline Systems http://www.solver.com/student
 - Many Others!



The Steps in Implementing an LP Model in a Spreadsheet

- 1. Organize the data for the model on the spreadsheet.
- 2. Reserve separate cells in the spreadsheet for each decision variable in the model.
- 3. Create a formula in a cell in the spreadsheet that corresponds to the objective function.
- For each constraint, create a formula in a separate cell in the spreadsheet that corresponds to the left-hand side (LHS) of the constraint.

Let's Implement a Model for the Blue Ridge Hot Tubs Example ...

```
MAX: 350X_1 + 300X_2 } Maximise profit

S.T.: 1X_1 + 1X_2 <= 200 } pumps

9X_1 + 6X_2 <= 1566} labor

12X_1 + 16X_2 <= 2880 } tubing

X_1, X_2 >= 0 } non-negativity
```

Where:

- X₁ = # of Aqua-Spas to produce
- $X_2 = \#$ of Hydro-Luxes to produce



Implementing the Model

See file <u>Lecture2-1.xls</u>

- Note the position of:
 - Variables, Variable names,
 - Constraints (LHS), Constraint names, RHS,
 - Variable (Changing) Cells,
 - The Objective Function (Target cell)
- Formulas make extensive use of Sumproduct function.
- Set X1 and X2 to 0 or 1 as place holders while model is set up.



Implementing the model

The completed model looks like this. (Refer to <u>Lecture 2-1.xlsm</u>)

	Α	В	С	D	Е
1					
2		Blue Ridg	e Hot Tubs		
3					
4		Aqua-Spas	Hydro-Luxes		
5	Number to Make	0	0	Total Profit	
6	Unit Profits	\$350	\$300	\$0	
7				LHS	RHS
8	Constraints			Used	Available
9	Pumps Req'd	1	1	0	200
10	Labor Req'd	9	6	0	1566
11	Tubing Req'd	12	16	0	2880

How Solver Views the Model

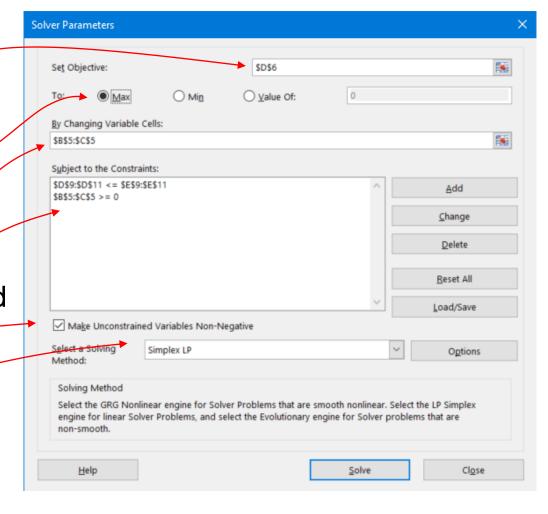
- Target cell the cell in the spreadsheet that represents the objective function
- Changing cells the cells in the spreadsheet representing the decision variables
- Constraint cells the cells in the spreadsheet representing the LHS formulas on the constraints



How the Solver views the model

Data Tab (Analyze Group)

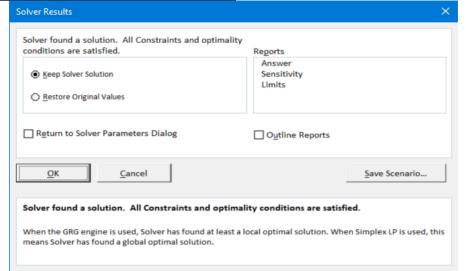
- → Solver
- Set Objective (objective function)
- Min or Max
- Changing cells (decision variables),
- Constraint cells: LHS and RHS,
- Non-negativity
- Assume linear model





The solution

	Blue Ridge	Hot Tubs		
	Aqua-Spas	Hydro-Luxes		
Number to Make	122	78	Total Profit	
Unit Profits	\$350	\$300	\$66,100	
Constraints			Used	Available
- Pumps Required	1	1	200	200
- Labor Required	9	6	1,566	1566
- Tubing Required	12	16	2,712	2880





Goals For Spreadsheet Design

- Communication A spreadsheet's primary business purpose is communicating information to managers.
- Reliability The output a spreadsheet generates should be correct and consistent.
- Auditability A manager should be able to retrace the steps followed to generate the different outputs from the model in order to understand and verify results.
- Modifiability A well-designed spreadsheet should be easy to change or enhance in order to meet dynamic user requirements.

Spreadsheet Design Guidelines - I

- Organize the data, then build the model around the data.
- Do not embed numeric constants in formulas.
- Things which are logically related should be physically related.
- Use formulae that can be copied.
- Column/rows totals should be close to the columns/rows being totaled.



Spreadsheet Design Guidelines - II

- The English-reading eye scans left to right, top to bottom.
- Use color, shading, borders and protection to distinguish changeable parameters from other model elements.
- Use text boxes and cell notes to document various elements of the model.

Make vs. Buy Decisions: The Electro-Poly Corporation

- Electro-Poly is a leading maker of slip-rings.
- A \$750,000 order has just been received.

	Model 1	Model 2	Model 3
Number ordered	3,000	2,000	900
Hours of wiring/unit	2	1.5	3
Hours of harnessing/unit	1	2	1
Cost to Make	\$50	\$83	\$130
Cost to Buy	\$61	\$97	\$145

 The company has 10,000 hours of wiring capacity and 5,000 hours of harnessing capacity.

Defining the Decision Variables

 M_1 = Number of model 1 slip rings to make in-house

 M_2 = Number of model 2 slip rings to make in-house

 M_3 = Number of model 3 slip rings to make in-house

 B_1 = Number of model 1 slip rings to buy from competitor

 B_2 = Number of model 2 slip rings to buy from competitor

 B_3 = Number of model 3 slip rings to buy from competitor



Defining the Objective Function

Minimize the total cost of filling the order.

MIN: $50M_1 + 83M_2 + 130M_3 + 61B_1 + 97B_2 + 145B_3$



Defining the Constraints

Demand Constraints

$$M_1 + B_1 = 3,000$$
 } model 1
 $M_2 + B_2 = 2,000$ } model 2
 $M_3 + B_3 = 900$ } model 3

Resource Constraints

$$2M_1 + 1.5M_2 + 3M_3 \le 10,000$$
 } wiring
 $1M_1 + 2.0M_2 + 1M_3 \le 5,000$ } harnessing

Non-negativity Conditions

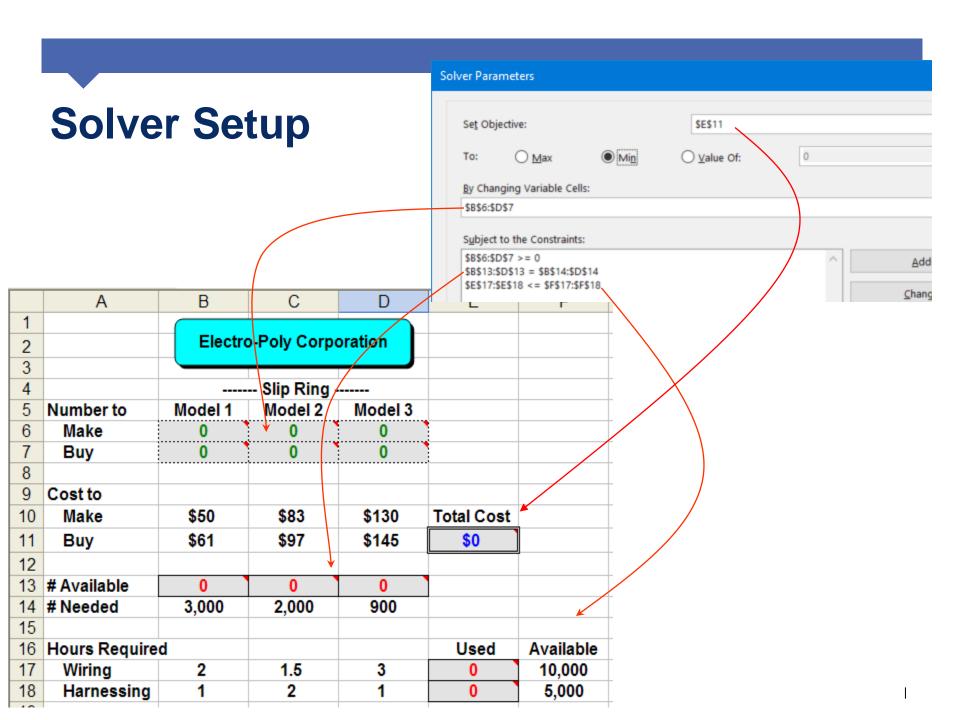
$$M_1$$
, M_2 , M_3 , B_1 , B_2 , $B_3 >= 0$



Implementing the Model

See file <u>Lecture2-2.xlsm</u>





The Solution:

	<u>Oldtioli</u>					
	Α	В	С	D	E	F
1						
2		Electro	o-Poly Corporation			
3						
4			Slip Ring -			
5	Number to	Model 1	Model 2	Model 3		
6	Make	3,000	550	900		
7	Buy	0	1,450	0		
8						
9	Cost to					
10	Make	\$50	\$83	\$130	Total Cost	
11	Buy	\$61	\$97	\$145	\$453,300	
12						
13	# Available	3,000	2,000	900		
14	# Needed	3,000	2,000	900		
15						
16	Hours Require	d			Used	Available
17	Wiring	2	1.5	3	9,525	10,000
18	Harnessing	1	2	1	5,000	5,000
10						

An Investment Problem: Retirement Planning Services, Inc.

A client wishes to invest \$750,000 in the following bonds.

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
Micro Modeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good

Investment Restrictions

- No more than 25% can be invested in any single company.
- At least 50% should be invested in long-term bonds (maturing in 10+ years).
- No more than 35% can be invested in DynaStar, Eagle Vision, and OptiPro as they are considered risky.



Defining the Decision Variables

 X_1 = amount of money to invest in Acme Chemical

 X_2 = amount of money to invest in DynaStar

 X_3 = amount of money to invest in Eagle Vision

 X_4 = amount of money to invest in MicroModeling

 X_5 = amount of money to invest in OptiPro

 X_6 = amount of money to invest in Sabre Systems



Defining the Objective Function

Maximize the total annual investment return:

MAX:
$$.0865X_1 + .095X_2 + .10X_3 + .0875X_4 + .0925X_5 + .09X_6$$

Company	† Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
Micro Modeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good



Defining the Constraints

Total amount is invested

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 750,000$$

No more than 25% in any one investment

$$X_i \le 187,500$$
, for all *i*

50% long term investment restriction.

$$X_1 + X_2 + X_4 + X_6 >= 375,000$$

35% Restriction on DynaStar, Eagle Vision, and OptiPro.

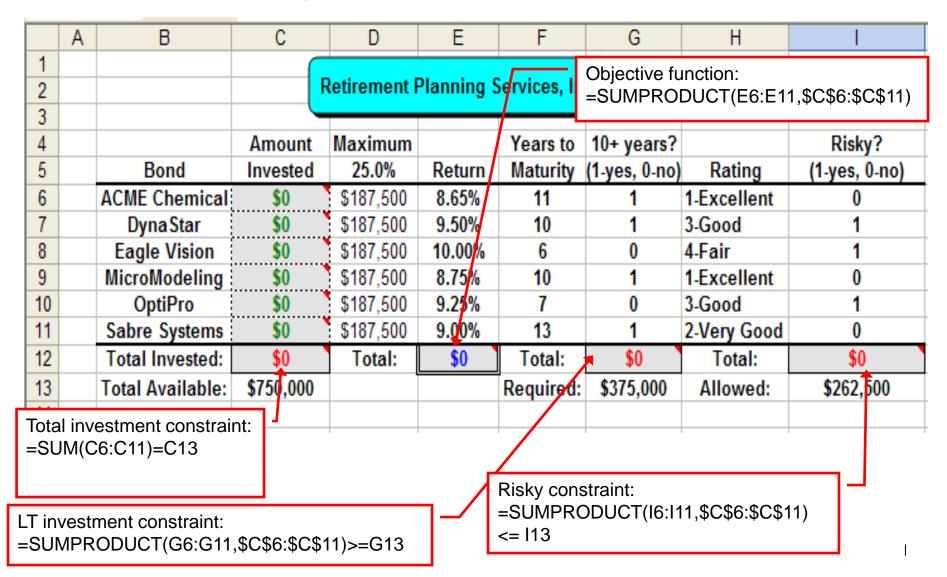
$$X_2 + X_3 + X_5 <= 262,500$$

Non-negativity conditions

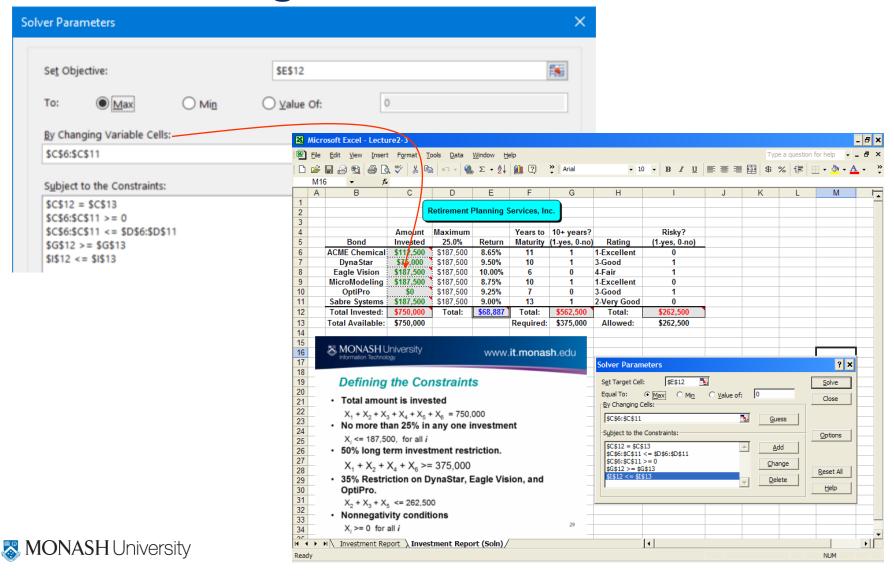
$$X_i >= 0$$
 for all i



Implementing the Model



Solver settings...and the solution:



A Blending Problem: The Agri-Pro Company

 Agri-Pro has received an order for 8,000 pounds of chicken feed to be mixed from the following feeds.

Percent of Nutrient in

Nutrient	Feed 1	Feed 2	Feed 3	Feed 4
Corn	30%	5%	20%	10%
Grain	10%	3%	15%	10%
Minerals	20%	20%	20%	30%
Cost per pound	\$0.25	\$0.30	\$0.32	\$0.15

The order must contain at least 20% corn, 15% grain, and 15% minerals.

Defining the Decision Variables

 X_1 = pounds of feed 1 to use in the mix

 X_2 = pounds of feed 2 to use in the mix

 X_3 = pounds of feed 3 to use in the mix

 X_4 = pounds of feed 4 to use in the mix



Defining the Objective Function

Minimize the total cost of filling the order.

MIN:
$$0.25X_1 + 0.30X_2 + 0.32X_3 + 0.15X_4$$



Defining the Constraints

Produce 8,000 pounds of feed

$$X_1 + X_2 + X_3 + X_4 = 8,000$$

Mix consists of at least 20% corn

$$(0.3X_1 + 0.05X_2 + 0.2X_3 + 0.1X_4)/8000 >= 0.2$$

Mix consists of at least 15% grain

$$(0.1X_1 + 0.03X_2 + 0.15X_3 + 0.1X_4)/8000 >= 0.15$$

Mix consists of at least 15% minerals

$$(0.2X_1 + 0.2X_2 + 0.2X_3 + 0.3X_4)/8000 >= 0.15$$

Non-negativity conditions

$$X_1, X_2, X_3, X_4 >= 0$$



A Comment About Scaling

- Notice the coefficient for X_2 in the 'corn' constraint is 0.05/8000 = 0.00000625
- As Solver runs, intermediate calculations are made that make coefficients larger or smaller.
- Storage problems may force the computer to use approximations of the actual numbers.
- Such 'scaling' problems sometimes prevents Solver from being able to solve the problem accurately.
- Most problems can be formulated in a way to minimize scaling errors...

Re-Defining the Decision Variables

 $X_1 =$ *thousands of pounds* of feed 1 to use in the mix

 $X_2 =$ **thousands of pounds** of feed 2 to use in the mix

 $X_3 =$ **thousands of pounds** of feed 3 to use in the mix

 $X_4 =$ **thousands of pounds** of feed 4 to use in the mix



Re-Defining the Objective Function

Minimize the total cost of filling the order.

MIN:
$$250X_1 + 300X_2 + 320X_3 + 150X_4$$



Re-Defining the Constraints

Produce 8,000 pounds of feed

$$X_1 + X_2 + X_3 + X_4 = 8$$

Mix consists of at least 20% corn

$$(0.3X_1 + 0.05X_2 + 0.2X_3 + 0.1X_4)/8 >= 0.2$$

Mix consists of at least 15% grain

$$(0.1X_1 + 0.03X_2 + 0.15X_3 + 0.1X_4)/8 >= 0.15$$

Mix consists of at least 15% minerals

$$(0.2X_1 + 0.2X_2 + 0.2X_3 + 0.3X_4)/8 >= 0.15$$

Non-negativity conditions

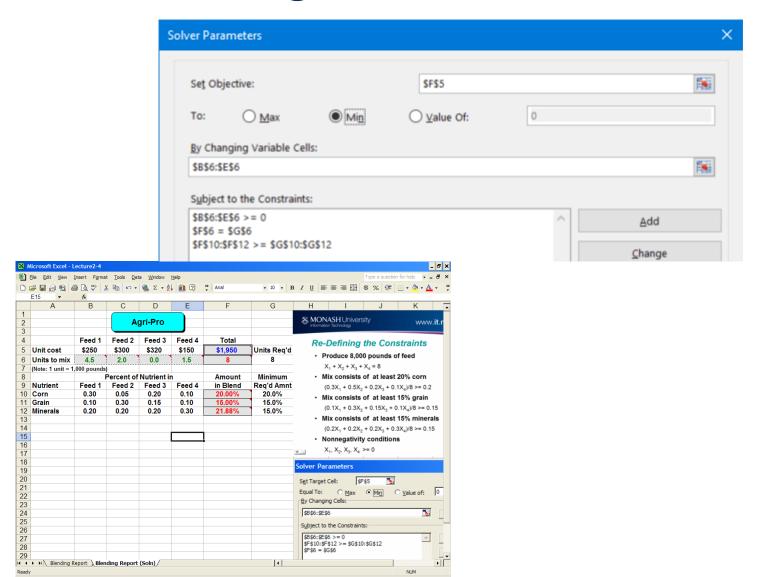
$$X_1, X_2, X_3, X_4 >= 0$$



Scaling: Before and After

- Before:
 - Largest constraint coefficient was 8,000
 - Smallest constraint coefficient was
 0.05/8 = 0.00000625.
- After:
 - Largest constraint coefficient is 8
 - Smallest constraint coefficient is
 0.05/8 = 0.00625.
- The problem is now more evenly scaled!

Solver settings...and the solution:



A Multi-Period Cash Flow Problem: The Taco-Viva Sinking Fund - I

- Taco-Viva needs a sinking fund to pay \$800,000 in building costs for a new restaurant in the next 6 months.
- Payments of \$250,000 are due at the end of months 2 and 4, and a final payment of \$300,000 is due at the end of month 6.
- The following investments may be used.

Investment		Available in Month	Months to Maturity	Yield at Maturity	
	A	1, 2, 3, 4, 5, 6	1	1.8%	
	В	1, 3, 5	2	3.5%	
	C	1, 4	3	5.8%	
	D	1	6	11.0%	

Summary of Possible Cash Flows

Cash Inflow/Outflow at the Beginning of Month

	Casi	i iiiiiow/	Outilov	w at the	Бедиш	mg or r	vionui
Investment	1	2	3	4	5	6	7
$\mathbf{A_1}$	-1	1.018					
\mathbf{B}_{1}	-1	<>	1.035				
\mathbf{C}_{1}^{T}	-1	<>	<>	1.058			
\mathbf{D}_{1}^{-}	-1	<>	<>	> <>	<>	· <>	· 1.11
${f A_2}$		-1	1.018				
$\mathbf{A_3}^-$			-1	1.018			
$\mathbf{B_3}$			-1	<>	1.035		
$\mathbf{A_4}$				-1	1.018		
$\mathbf{C_4}$				-1	<>	· <>	1.058
$\mathbf{A_5}$					-1	1.018	
\mathbf{B}_{5}					-1	<>	1.035
$\mathbf{A_6}$						-1	1.018
Req'd Payments (in \$1,000s)	\$0	\$0	\$250	\$0	\$250	\$0	\$300

Defining the Decision Variables

- A_i = amount (in \$1,000s) placed in investment A at the beginning of month i=1, 2, 3, 4, 5, 6
- B_i = amount (in \$1,000s) placed in investment B at the beginning of month i=1, 3, 5
- C_i = amount (in \$1,000s) placed in investment C at the beginning of month i=1, 4
- D_i = amount (in \$1,000s) placed in investment D at the beginning of month i=1

Defining the Objective Function

Minimize the total cash invested in month 1.

MIN:
$$A_1 + B_1 + C_1 + D_1$$



Defining the Constraints

Cash Flow Constraints

$$1.018A_{1} - 1A_{2} = 0$$
 } month 2
$$1.035B_{1} + 1.018A_{2} - 1A_{3} - 1B_{3} = 250 \text{ } month 3$$

$$1.058C_{1} + 1.018A_{3} - 1A_{4} - 1C_{4} = 0 \text{ } month 4$$

$$1.035B_{3} + 1.018A_{4} - 1A_{5} - 1B_{5} = 250 \text{ } month 5$$

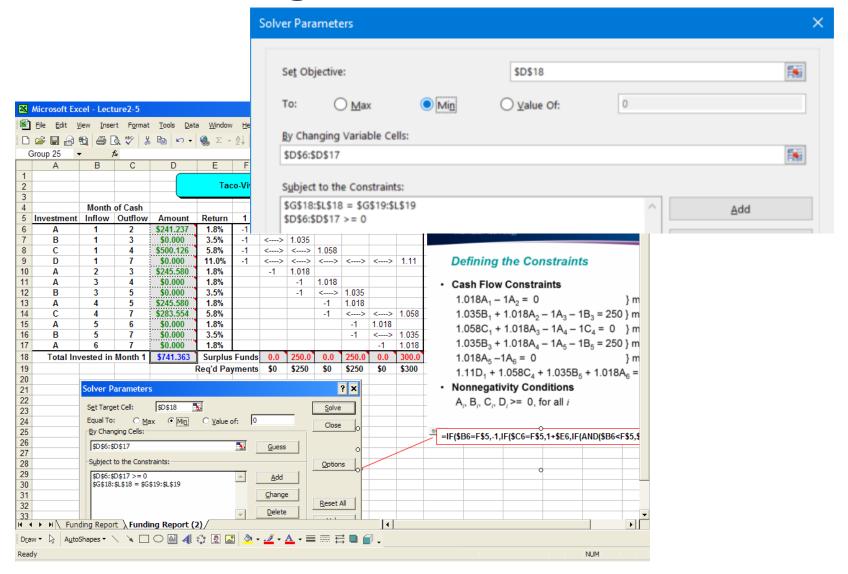
$$1.018A_{5} - 1A_{6} = 0 \text{ } month 6$$

$$1.11D_{1} + 1.058C_{4} + 1.035B_{5} + 1.018A_{6} = 300 \text{ } month 7$$

Non-negativity Conditions

$$A_i$$
, B_i , C_i , $D_i >= 0$, for all i

Solver settings...and the solution:



Tutorial 1 this week:

- LP: Graphical Method on two decision variables
 - Maximisation
 - Minimisation

Multi-variable problem formulation

