FIT3158
Business Decision Modelling

Lecture 5

- Network Modelling (Part 1)

Topics Covered:

- Formulating and solving transshipment problems
- Modelling shortest path problems
- Understanding generalised network flow problems
- Modelling maximal flow problems
- Exploring the algorithm for the minimal spanning tree problem

MONASH University

Introduction

- A number of business problems can be represented graphically as networks.
- This week we will focus on the following:
  - Transshipment Problems
  - Shortest Path Problems
  - Generalised Network Flow Problems
  - Maximal Flow Problems
  - The Minimum Spanning Tree Problem
- Next week we will look at:
  - Transportation and Assignment Problems
  - Traveling Salesman Problem (TSP)

Network Flow Problem Characteristics

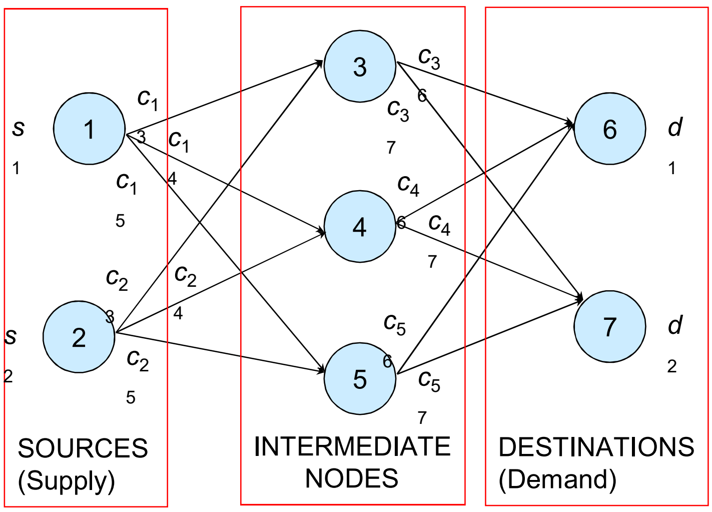
- Network flow problems can be represented as a collection of nodes connected by arcs.
- There are three types of nodes:
  - Supply
  - Demand
  - Transshipment
- We'll use negative numbers to represent supplies and positive numbers to represent demand.

➢ *The Transshipment Problem*

Transshipment Problem

- Transshipment problems are transportation problems in which a shipment may move through intermediate nodes (transshipment nodes) before reaching a particular destination node.
- Transshipment problems can be converted to larger transportation problems and solved by a special transportation program.
- Transshipment problems can also be solved by general purpose linear programming (LP) codes.
- The network representation for a transshipment problem with two sources, three intermediate nodes, and two destinations is shown on the next slide.

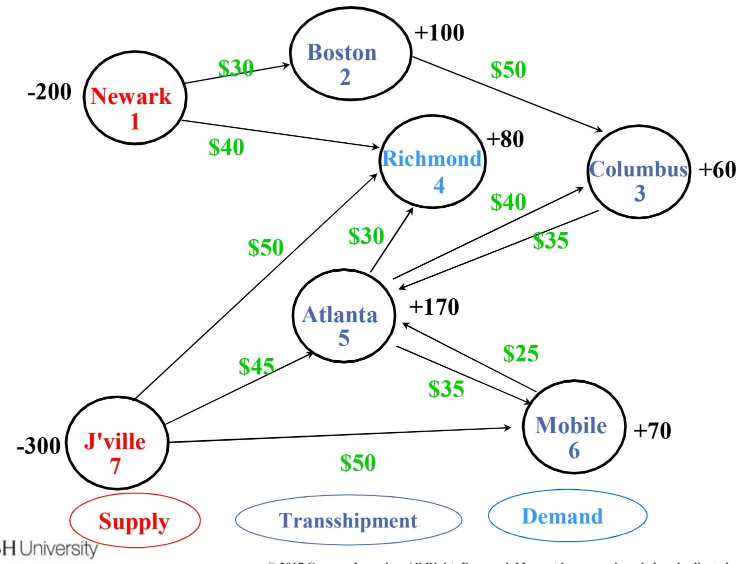
Network Representation



MONASH University

l

The Bavarian Motor Company



MONASH University

l 8

Defining the Decision Variables

For each arc in a network flow model
we define a decision variable as:

X_{ij} = the amount being shipped (or flowing) *from* node i *to* node j

For example…

X_{12} = the # of cars shipped *from* node 1 (Newark) *to* node 2 (Boston)

X_{56} = the # of cars shipped *from* node 5 (Atlanta) *to* node 6 (Mobile)

Note: The number of arcs determines
the number of variables!

Defining the Objective Function

Minimize total shipping costs.

$$
\begin{aligned}
\text{MIN:}\quad & 30X_{12} + 40X_{14} + 50X_{23} + 35X_{35} \\
& + 40X_{53} + 30X_{54} + 35X_{56} + 25X_{65} \\
& + 50X_{74} + 45X_{75} + 50X_{76}
\end{aligned}
$$

Constraints for Network Flow Problems: The Balance-of-Flow Rules

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule At Each Node:
Total Supply > Total Demand	Inflow-Outflow >= Supply or Demand
Total Supply < Total Demand	Inflow-Outflow <=Supply or Demand
Total Supply = Total Demand	Inflow-Outflow = Supply or Demand

MONASH University

Defining the Constraints

- In the Bavarian motor company (BMC) problem:
  - Total Supply = 500 cars (Supply >= Demand)
  - Total Demand = 480 cars
- For each node we need a constraint like this:
  - Inflow - Outflow >= Supply or Demand
- Constraint for node 1:
  - $-X_{12} - X_{14} >= -200$ (Note: there is no inflow for node 1!)
- This is equivalent to:
  - $+X_{12} + X_{14} <= 200$

MONASH University

Defining the Constraints

- Flow constraints

$-X_{12} - X_{14} >= -200$ } node 1

$+X_{12} - X_{23} >= +100$ } node 2

$+X_{23} + X_{53} - X_{35} >= +60$ } node 3

$+ X_{14} + X_{54} + X_{74} >= +80$ } node 4

$+ X_{35} + X_{65} + X_{75} - X_{53} - X_{54} - X_{56} >= +170$ } node 5

$+ X_{56} + X_{76} - X_{65} >= +70$ } node 6

$-X_{74} - X_{75} - X_{76} >= -300$ } node 7

- Nonnegativity conditions

$X_{ij} >= 0$ for all ij

MONASH University

Implementing the Model

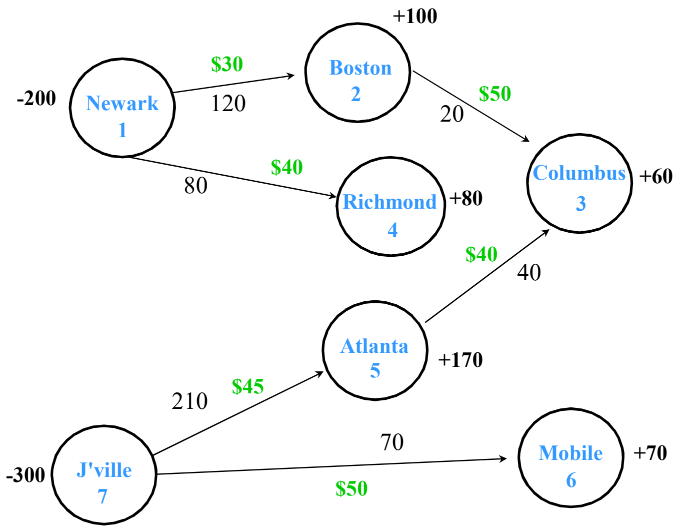
See file Network Modelling_1.xlsm (Transshipment)

Ship	From		To		Unit Cost
120	1	Newark	2	Boston	$30
80	1	Newark	4	Richmond	$40
20	2	Boston	3	Columbus	$50
0	3	Columbus	5	Atlanta	$35
40	5	Atlanta	3	Columbus	$40
0	5	Atlanta	4	Richmond	$30
0	5	Atlanta	6	Mobile	$35
0	6	Mobile	5	Atlanta	$25
0	7	Jacksonville	4	Richmond	$50
210	7	Jacksonville	5	Atlanta	$45
70	7	Jacksonville	6	Mobile	$50

Total Transportation Cost $22,350

MONASH University

Optimal Solution to the BMC Problem



MONASH University

➢ *The Shortest Path Problem*

MONASH University

The Shortest Path Problem

- Many decision problems boil down to determining the shortest (or least costly) route or path through a network.
  - Example: Emergency Vehicle Routing

- This is a special case of a transshipment problem where:
  - There is one supply node with a supply of -1
  - There is one demand node with a demand of +1
  - All other nodes have supply/demand of +0

MONASH University

| 17

Algorithm for the Shortest Route

1. Assign START a node value of 0 and shade in its node. (This is the first evaluated node)
2. Mark all arcs connecting an evaluated node to an unevaluated one.
   - Calculate for each arc the sum of its evaluated node's value and the arc length.

– Continuation…

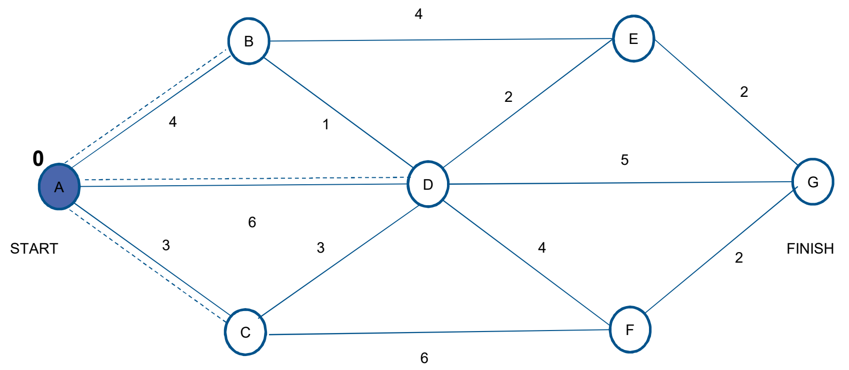
MONASH University

|

Algorithm for the Shortest Route (cont'd)

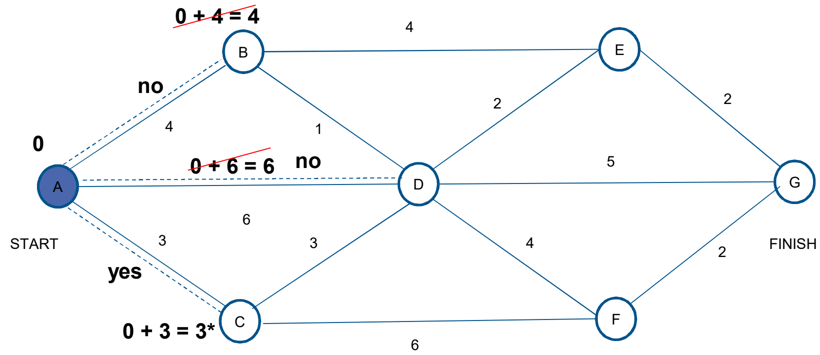
3. Select the *key* arc with the minimum sum.
   – Assign this minimum sum as the node value → this equals the minimum distance to it from the START.
   – Shade in that node which is now evaluated.
   – Place a pointer near this node, alongside the key arc, aiming at the key's arc opposite node.
   – If FINISH is not yet evaluated, return to Step 2.
4. Find the shortest route from START to FINISH.
   – The shortest route is found by tracing the pointer backward from FINISH to START.

An Example: Yellow Jacket Freightways

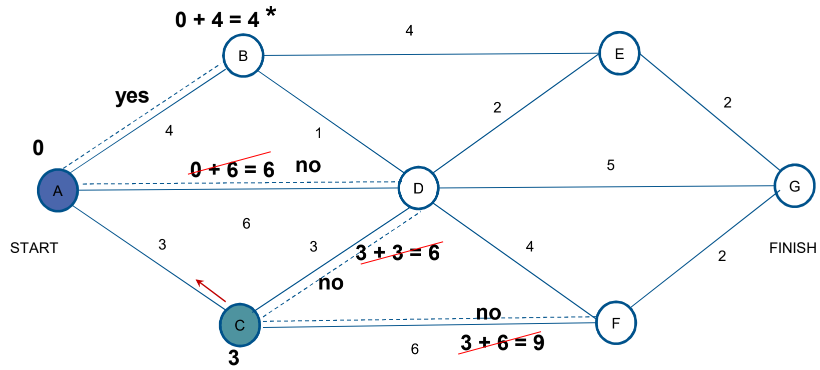


1. The START node A has been shaded in and assigned a node value of 0.
2. Mark the arcs (A,B), (A,C) & (A,D) which connect evaluated node (A) to unevaluated node (B, C & D)

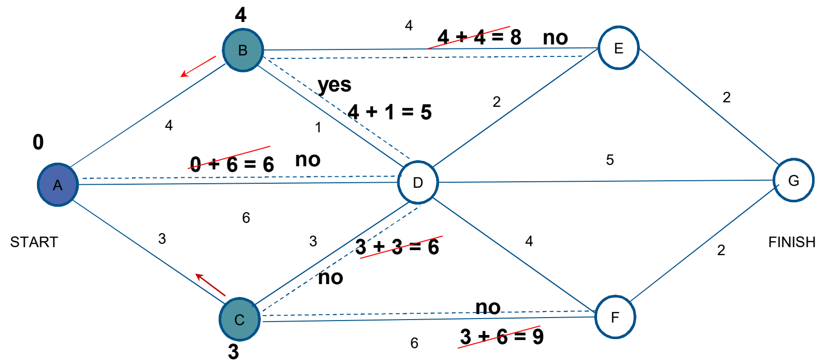
Yellow Jacket Freightways



3. Calculate for each arc the sum of its evaluated node's value and the arc length.
4. Arc (A,C) is the key arc with the minimum sum.

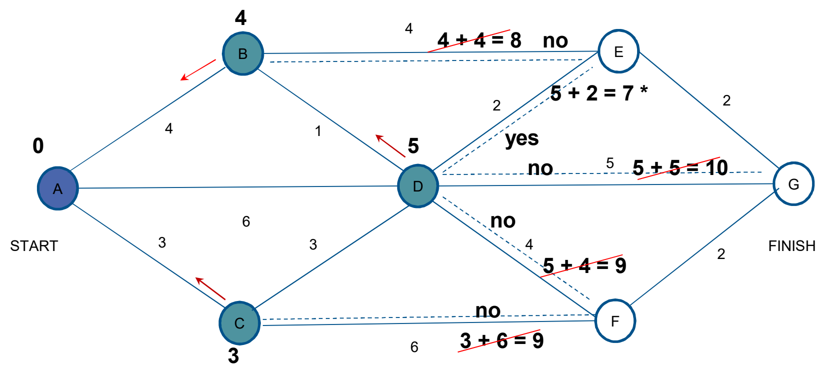


5. Node C is shaded and a permanent value 3 is entered next to node C.
6. A pointer (arrow) is placed beside the key arc (A,C) aiming back to node A, indicating direction of shortest route from START to C. → Repeat Steps 2 – 6.



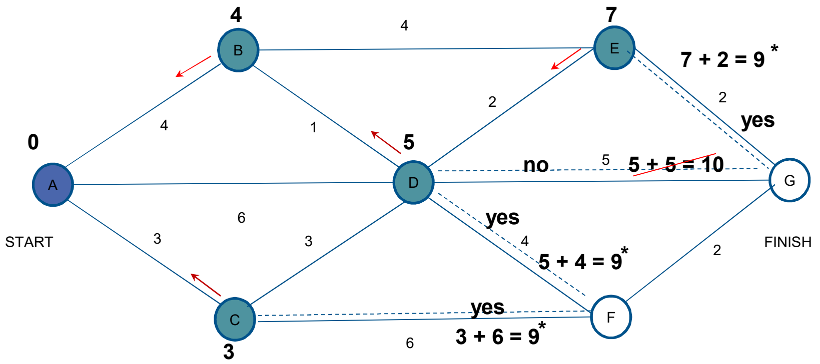
7. Node B is shaded and a permanent value 4 is entered next to node B.
8. A pointer (arrow) is placed beside the key arc (A,B) aiming back to node A.

MONASH University



9. Shade Node D and a permanent value 5 is assigned to it.
10. A pointer (arrow) is placed beside the key arc (B,D) aiming back to node B.

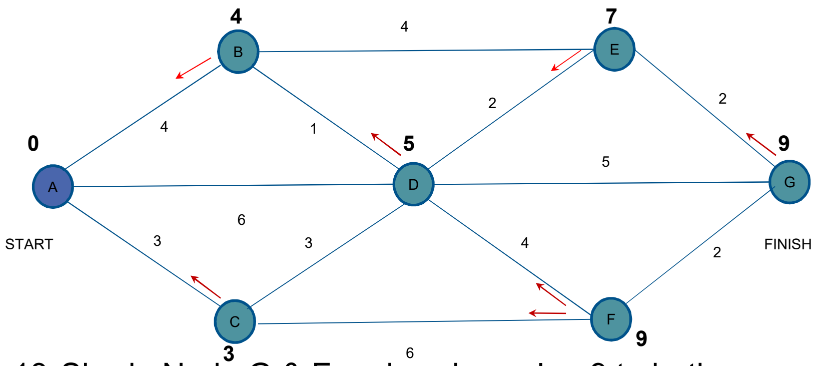
MONASH University



11. Shade Node E and assign value 7 to it.

12. A pointer (arrow) is placed beside the key arc (D,E) aiming back to node D.

Solution:



13. Shade Node G & F and assign value 9 to both.

- Shortest Route: A – B – D – E – G
- The minimum distance from START to FINISH equals the sum of the arc lengths on this path: 4+1+2+2 = 9

Solving the Problem using Spreadsheet

See file Network Modelling_1.xlsm (Shortest Route)

Yellow Jacket Freightways

Select Route?	From		To		Distance
1.0	1	A	2	B	4.0
0.0	1	A	3	C	3.0
0.0	1	A	4	D	6.0
1.0	2	B	4	D	1.0
0.0	2	B	5	E	4.0
0.0	3	C	4	D	3.0
0.0	3	C	6	F	6.0
1.0	4	D	5	E	2.0
0.0	4	D	7	G	5.0
0.0	4	D	6	F	4.0
1.0	5	E	7	G	2.0
0.0	6	F	7	G	2.0
				Total	9.0

Nodes		Net Flow	Supply/Demand
1	A	-1	-1
2	B	0	0
3	C	0	0
4	D	0	0
5	E	0	0
6	F	0	0
7	G	1	1

=SUMIF($E$7:$E$18,I8,$B$7:$B$18) -
SUMIF($C$7:$C$18,I8,$B$7:$B$18)

```
Minimize:     G20
By changing:  B7:B18
Subject to:   K7:K13=L7:L13
              B7:B18>=0
```


➢ *Generalised Network Flow Problems*

Generalised Network Flow Problems

- In some problems, a gain or loss occurs in flows over arcs.
  - Examples
    - Oil or gas shipped through a leaky pipeline
    - Imperfections in raw materials entering a production process
    - Spoilage of food items during transit
    - Theft during transit
    - Interest or dividends on investments
- These problems require some modelling changes.

MONASH University

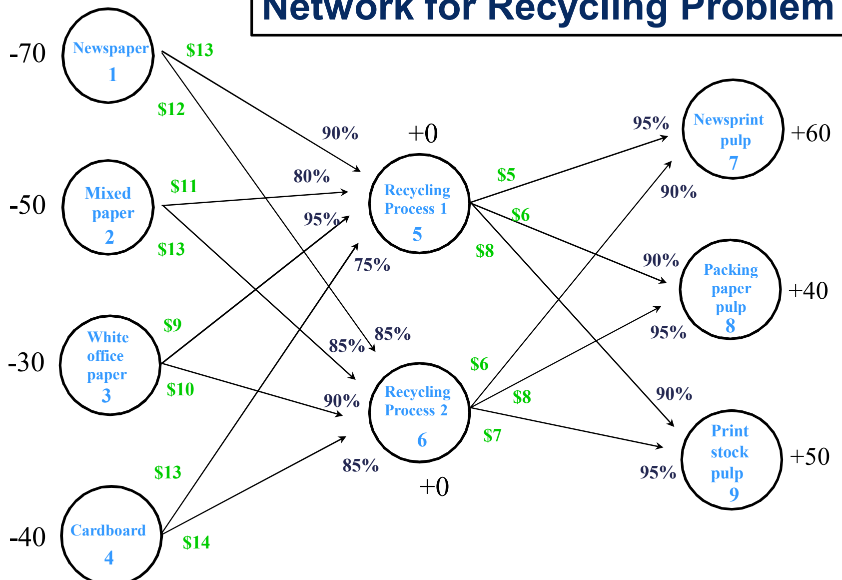
Coal Bank Hollow Recycling

	Process 1		Process 2		
Material	Cost	Yield	Cost	Yield	Supply
Newspaper	$13	90%	$12	85%	70 tons
Mixed Paper	$11	80%	$13	85%	50 tons
White Office Paper	$9	95%	$10	90%	30 tons
Cardboard	$13	75%	$14	85%	40 tons

	Newsprint		Packaging Paper		Print Stock	
Pulp Source	Cost	Yield	Cost	Yield	Cost	Yield
Recycling Process 1	$5	95%	$6	90%	$8	90%
Recycling Process 2	$6	90%	$8	95%	$7	95%
Contracted demand	60 tons		40 tons		50 tons	

MONASH University

Network for Recycling Problem



MONASH University

Defining the Objective Function

Minimize total cost.

MIN: $13X_{15} + 12X_{16} + 11X_{25} + 13X_{26}$
$+ 9X_{35} + 10X_{36} + 13X_{45} + 14X_{46}$
$+ 5X_{57} + 6X_{58} + 8X_{59} + 6X_{67} + 8X_{68} + 7X_{69}$

MONASH University

Defining the Constraints-I

- Raw Materials

$-X_{15} - X_{16} >= -70$ } node 1

$-X_{25} - X_{26} >= -50$ } node 2

$-X_{35} - X_{36} >= -30$ } node 3

$-X_{45} - X_{46} >= -40$ } node 4

Defining the Constraints-II

- Recycling Processes

$+0.9X_{15}+0.8X_{25}+0.95X_{35}+0.75X_{45}- X_{57}- X_{58}-X_{59} >= 0$ } node 5

$+0.85X_{16}+0.85X_{26}+0.9X_{36}+0.85X_{46}-X_{67}-X_{68}-X_{69} >= 0$ } node 6

MONASH University

Defining the Constraints-III

e.g., pipe leakage

- Paper Pulp

$+0.95X_{57} + 0.90X_{67} >= 60$ } node 7

$+0.90X_{58} + 0.95X_{68} >= 40$ } node 8

$+0.90X_{59} + 0.95X_{69} >= 50$ } node 9

MONASH University

Implementing the Model

See file Network Modelling_1.xlsm (Generalised Flow)

Flow From Node		Yield	Flow Into Node		Cost
43.4	1 Newspaper	0.90	39.1	5 Process 1	$13
26.6	1 Newspaper	0.85	22.6	6 Process 2	$12
50.0	2 Mixed Paper	0.80	40.0	5 Process 1	$11
0.0	2 Mixed Paper	0.85	0.0	6 Process 2	$13
30.0	3 White Office	0.95	28.5	5 Process 1	$9
0.0	3 White Office	0.90	0.0	6 Process 2	$10
0.0	4 Cardboard	0.75	0.0	5 Process 1	$13
35.4	4 Cardboard	0.85	30.1	6 Process 2	$14
63.2	5 Process 1	0.95	60.0	7 Newsprint	$5
44.4	5 Process 1	0.90	40.0	8 Packaging	$6
0.0	5 Process 1	0.90	0.0	9 Print Stock	$8
0.0	6 Process 2	0.90	0.0	7 Newsprint	$6
0.0	6 Process 2	0.95	0.0	8 Packaging	$8
52.6	6 Process 2	0.95	50.0	9 Print Stock	$7

Total Cost $3,149

MONASH University

Important Modelling Point - I

- In generalised network flow problems, gains and/or losses associated with flows across each arc *effectively* increase and/or decrease the available supply.
- This can make it difficult to tell if the total supply is adequate to meet the total demand.
- When in doubt, it is best to assume the total supply **is** capable of satisfying the total demand and use Solver to prove (or refute) this assumption.

MONASH University

Important Modeling Point - II

- If all the demand can't be met, another objective might be to meet as much of the demand as possible at minimum cost.
- To do this, modify the network as follows:
  - Add an artificial supply node with an arbitrarily large amount of supply.
  - Connect the artificial supply node to each demand node with arbitrarily large costs on each artificial arc.
  - This causes as much demand as possible to be met using real supply to minimize use of the expensive artificial supply.

➢ *Maximal Flow Problem*

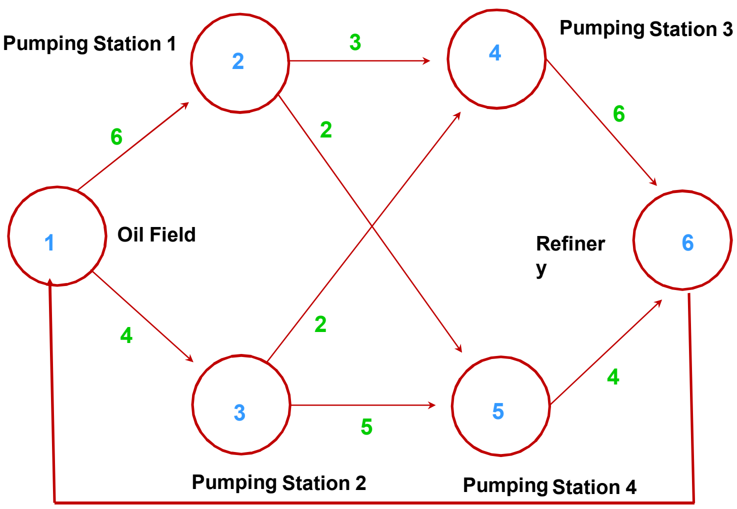
The Maximal Flow Problem

- In some network problems, the objective is to determine the maximum amount of flow that can occur through a network.
- The arcs in these problems have upper and lower flow limits.
- Examples
  - How much water can flow through a network of pipes?
  - How many cars can travel through a network of streets?

MONASH University

| 39

The Northwest Petroleum Company



MONASH University

| 40

Formulation of the Max Flow Problem

MAX: X_{61}

Subject to:
$+X_{61} - X_{12} - X_{13} = 0$
$+X_{12} - X_{24} - X_{25} = 0$
$+X_{13} - X_{34} - X_{35} = 0$
$+X_{24} + X_{34} - X_{46} = 0$
$+X_{25} + X_{35} - X_{56} = 0$
$+X_{46} + X_{56} - X_{61} = 0$

with the following bounds on the decision variables:

$0 <= X_{12} <= 6$ $0 <= X_{25} <= 2$ $0 <= X_{46} <= 6$
$0 <= X_{13} <= 4$ $0 <= X_{34} <= 2$ $0 <= X_{56} <= 4$
$0 <= X_{24} <= 3$ $0 <= X_{35} <= 5$ $0 <= X_{61} <= \text{inf}$

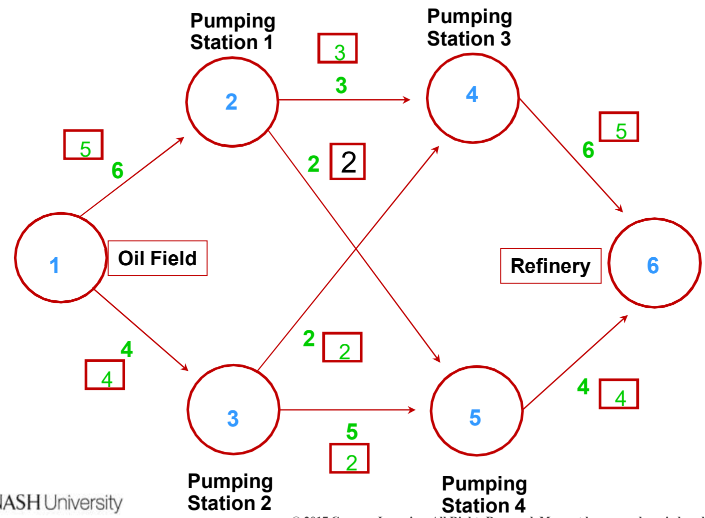
Implementing the Model

See file Network Modelling_1.xlsm (Maximum Flow)

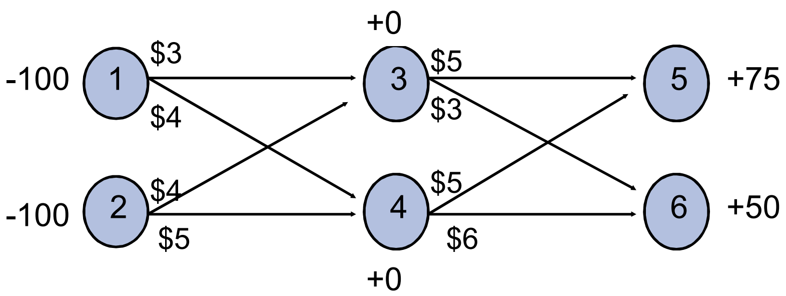
Units of Flow	-- Arcs -- From		To		Upper Bound
5.0	1	Oil Field	2	Station 1	6
4.0	1	Oil Field	3	Station 2	4
3.0	2	Station 1	4	Station 3	3
2.0	2	Station 1	5	Station 4	2
2.0	3	Station 2	4	Station 3	2
2.0	3	Station 2	5	Station 4	5
5.0	4	Station 3	6	Refinery	6
4.0	5	Station 4	6	Refinery	4
9.0	6	Refinery	1	Oil Field	9999

9.0 **Maximal Flow**

Optimal Solution

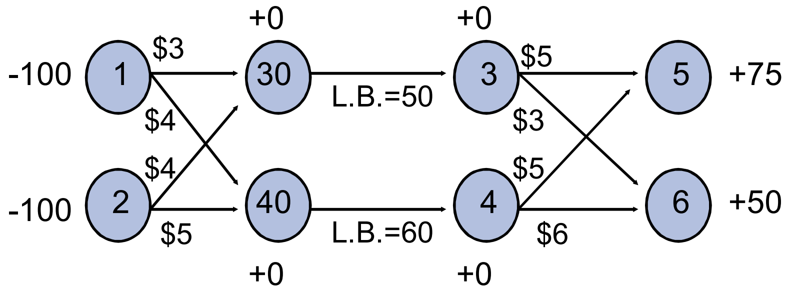


Special Modelling Considerations: Flow Aggregation



Suppose the total flow into nodes 3 & 4 must be at least 50 and 60, respectively. How would you model this?

Special Modelling Considerations: Flow Aggregation

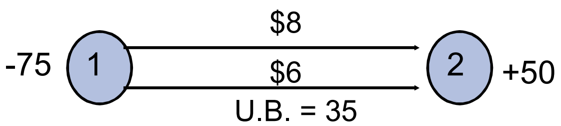


Nodes 30 & 40 aggregate the total flow into nodes 3 & 4, respectively.

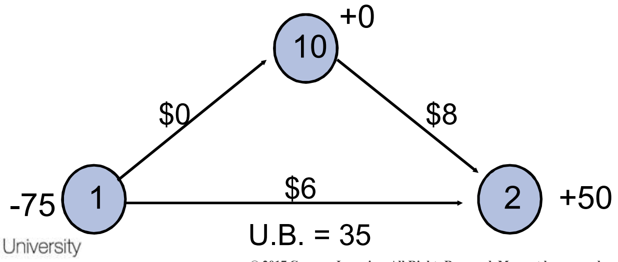
MONASH University

| 45

Special Modelling Considerations: Multiple Arcs Between Nodes



Two two (or more) arcs cannot share the same beginning and ending nodes. Instead, try...

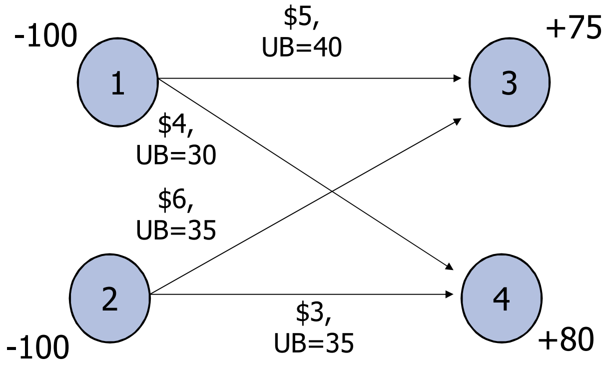


MONASH University

| 46

Special Modelling Considerations: Capacity Restrictions on Total Supply

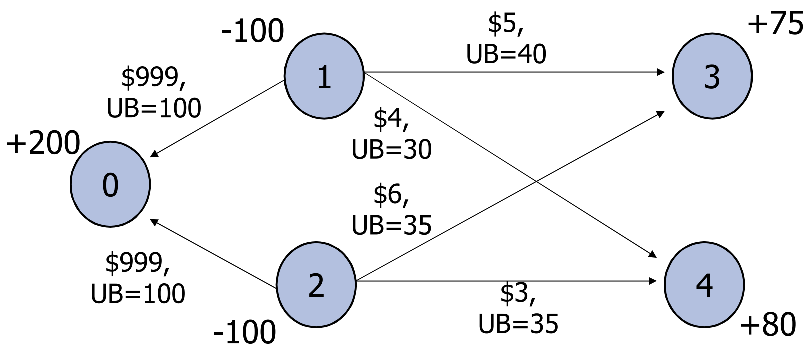
Supply exceeds demand, but the upper bounds prevent the demand from being met.



MONASH University

Special Modelling Considerations: Capacity Restrictions on Total Supply

Now demand exceeds supply. As much “real” demand as possible will be met in the least costly way.



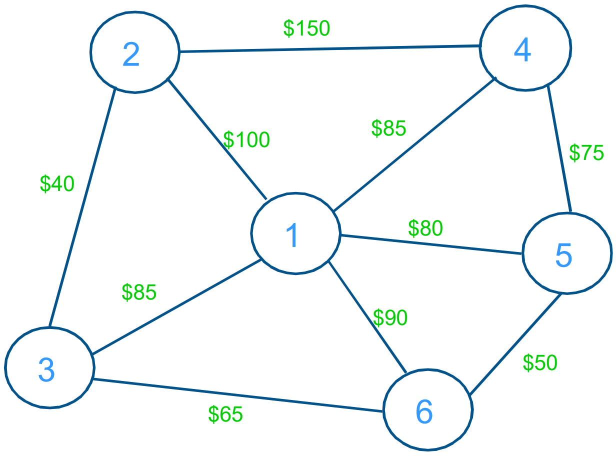
MONASH University

➢ *Minimal Spanning Tree Problem*

The Minimal Spanning Tree Problem

- For a network with n nodes, a *spanning tree* is a set of $(n\text{-}1)$ arcs that connects all the nodes and contains no loops.
- The minimal spanning tree problem involves determining the set of arcs that connects all the nodes at minimum cost.
- An efficient plan would normally not use all arcs in the original network, thereby conserving scarce resources needed in making the physical connections over the chosen linkages.
- Seemingly cannot be solved as an LP problem. However, easily solved using a manual algorithm

Minimal Spanning Tree Example: Windstar Aerospace Company



Nodes represent computers in a local area network.

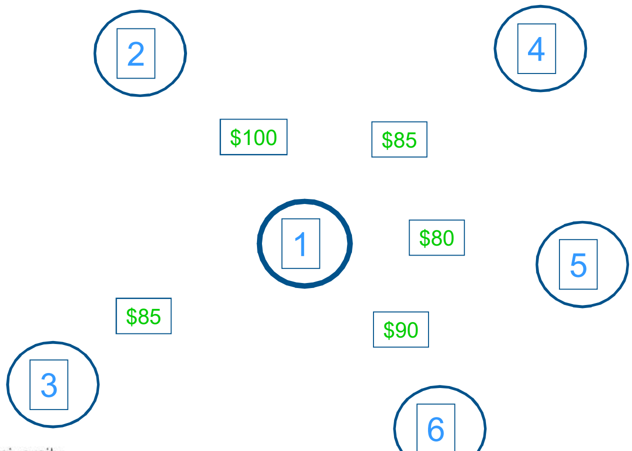
MONASH University

The Minimal Spanning Tree Algorithm

1. Select any node. Call this the current subnetwork.
2. Add to the current subnetwork the cheapest arc that connects any node within the current subnetwork to any node not in the current subnetwork. (Ties for the cheapest arc can be broken arbitrarily.) Call this the current subnetwork.
3. If all the nodes are in the subnetwork, stop; this is the optimal solution. Otherwise, return to step 2.

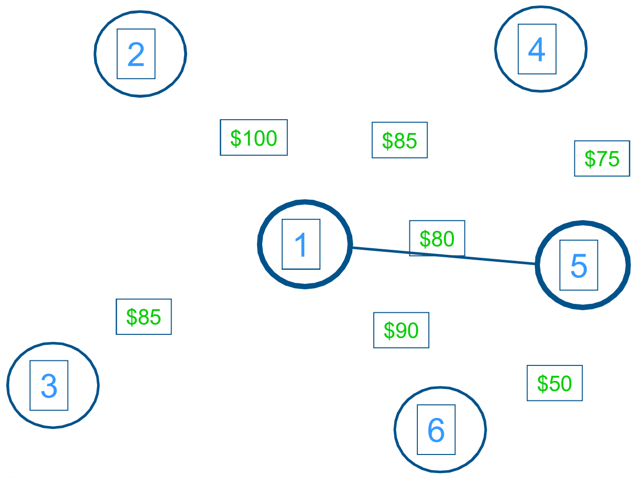
MONASH University

Solving the Problem – Step 1



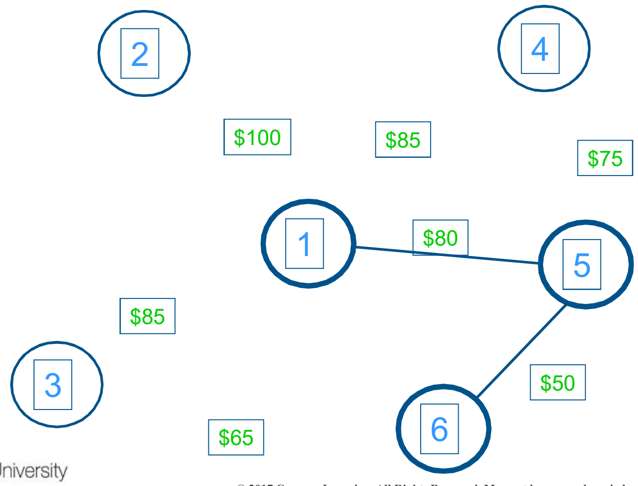
MONASH University

Solving the Problem – Step 2



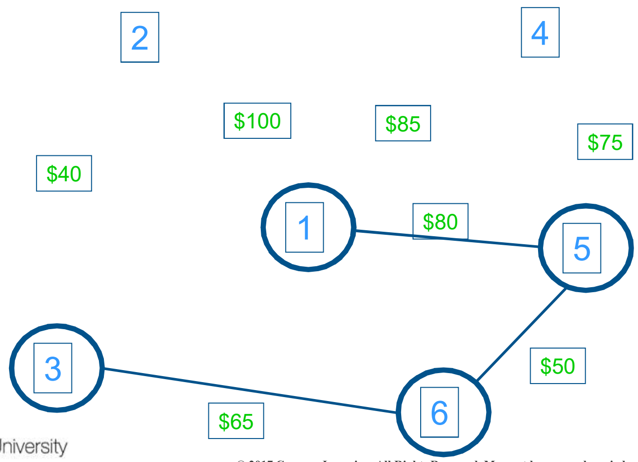
MONASH University

Solving the Problem – Step 3



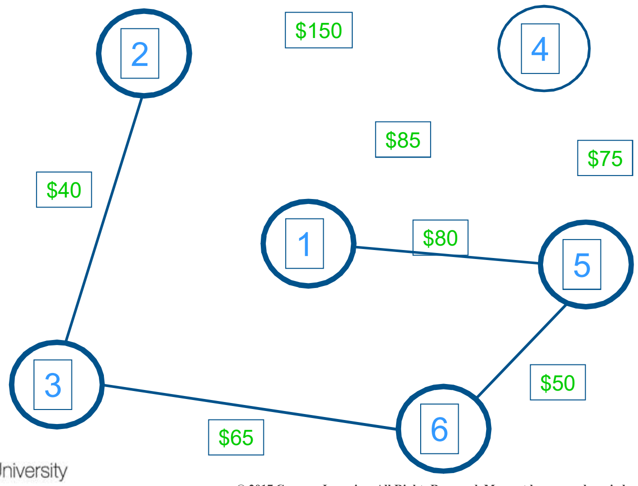
MONASH University

Solving the Problem – Step 4



MONASH University

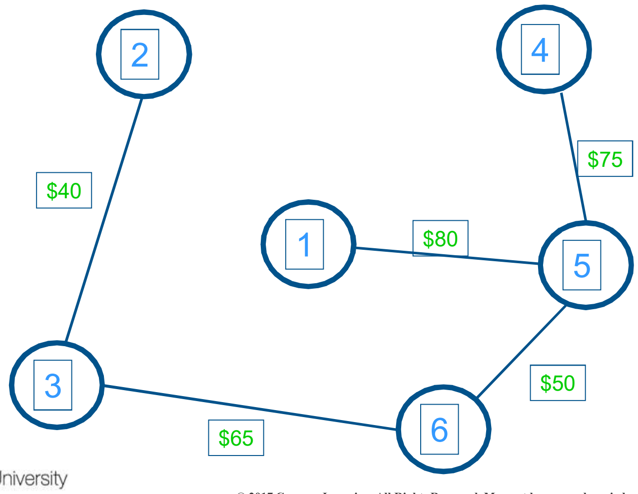
Solving the Problem – Step 5



MONASH University

| 57

Solving the Problem – Step 6



MONASH University

| 58

End of Lecture 5

References:

Ragsdale, C.. (2017, 2021). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e, 9e) Cengage Learning: Chapter 5

Homework

- Go through today's lecture examples and Ragsdale Chapter 5, to:
  - ✔ Familiarise yourself with the different algorithms used.
  - ✔ Understand how the spreadsheets are being modelled

Readings for next week's Lecture:

- Ragsdale, C. (2021). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (9e) Cengage Learning: Chapter 8 (pp 412 – 417)
- Ragsdale, C. (2017). Spreadsheet Modeling & Decision Analysis: A Practical Introduction to Business Analytics (8e) Cengage Learning: Chapter 8 (Pg 419 – 423)

Tutorial 4 this week:

- Formulating ILP Models
- Understanding the use of 'Big M' in the formulation
- Linking constraints