

MODULE *Assumes*  
EXTENDS *Integers, Sequences*

You can run this as a model using “No behavior spec” mode  
Single line comment

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ASSUME
   $\wedge \text{TRUE} = \text{TRUE}$ 
   $\wedge \neg \text{FALSE} = \text{TRUE}$ 

Jason  $\triangleq$  “jason”
ASSUME
  Jason = “jason”

record  $\triangleq$  [name  $\mapsto$  “jason”, age  $\mapsto$  2]
ASSUME
   $\wedge \text{record.name} = \text{“jason”}$ 
   $\wedge \text{record.name} \neq \text{“foo”}$ 

ASSUME
   $\forall F \in \{\text{TRUE}\} : F = F$ 

ASSUME
   $\forall F \in \{\text{FALSE}\} : F = F$ 

ASSUME  $\Rightarrow$  means “implies”, as in  $A \Rightarrow B$  is “(not  $A$ ) OR  $B$ ”
   $\text{FALSE} \Rightarrow \text{TRUE} = \text{TRUE}$ 

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   $\text{TRUE} \Rightarrow \text{FALSE} = \text{FALSE}$ 

ASSUME
   $\text{TRUE} \equiv \text{TRUE}$ 

ASSUME
   $\text{FALSE} \equiv \text{FALSE}$ 

ASSUME
   $\forall F, G \in \{\text{TRUE}, \text{FALSE}\} : (F \Rightarrow G) \equiv \neg F \vee G$ 

ASSUME
   $\{1, 2, 2, 2, 3\} = \{1, 2, 3\}$ 
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ASSUME

$$\{1, 2, 3, 3, 4, 4\} \setminus \{4\} = \{1, 2, 3\}$$

ASSUME

$$\exists x \in \{3, 4, 5\} : x = 5$$

ASSUME

$$\{1, 3\} \subseteq \{3, 2, 1\}$$

$$IsPrime(x) \triangleq x > 1 \wedge \neg \exists d \in 2 \dots (x-1) : x \% d = 0$$

For all  $y$  in  $S$  such that  $y$  is not prime or  $y$  is less than or equal to  $x$

$$\begin{aligned} LargestPrime(S) &\triangleq \text{CHOOSE } x \in S : \\ &\quad \wedge IsPrime(x) \\ &\quad \wedge \forall y \in S : \\ &\quad \quad IsPrime(y) \Rightarrow y \leq x \\ &\quad \text{or } y > x \Rightarrow \neg IsPrime(y) \end{aligned}$$

ASSUME

$$LargestPrime(\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}) = 7$$

$$IsEven(x) \triangleq x \% 2 = 0$$

$$\begin{aligned} LargetEven(S) &\triangleq \text{CHOOSE } x \in S : \\ &\quad \wedge IsEven(x) \\ &\quad \wedge \forall y \in S : \\ &\quad \quad IsEven(y) \Rightarrow y \leq x \end{aligned}$$

ASSUME

$$LargetEven(\{1, 2, 3, 4, 5, 5, 5\}) = 4$$

ASSUME

$$\forall x \in \{\} : \text{FALSE}$$

ASSUME

$$\forall x \in \{\} : \text{TRUE}$$

ASSUME

$$\forall x \in \{\} : 7$$

ASSUME

$$\forall x \in \{\text{FALSE}\} : \text{TRUE}$$

ASSUME

$$\forall x \in \{\text{TRUE}\} : \text{TRUE}$$

ASSUME

$$(\forall x \in \{\text{FALSE}\} : \text{FALSE}) = \text{FALSE}$$

$$\begin{aligned} \text{IsCommutative}(\text{Op}(-, -), S) &\triangleq \forall x \in S : \\ &\quad \forall y \in S : \text{Op}(x, y) = \text{Op}(y, x) \end{aligned}$$

$$\begin{aligned} \text{Add}(x, y) &\triangleq x + y \\ \text{Divide}(x, y) &\triangleq x \div y \end{aligned}$$

ASSUME

$$\text{IsCommutative}(\text{Add}, \{1, 2, 3\})$$

ASSUME

$$\text{IsCommutative}(\text{Divide}, \{1, 2, 3\}) = \text{FALSE}$$

ASSUME

$$\text{IsCommutative}(\text{Divide}, \{1, 2, 3\}) \Rightarrow \text{FALSE}$$

ASSUME

$$\text{IsCommutative}(\text{Divide}, \{1, 2, 3\}) \Rightarrow \text{TRUE}$$

ASSUME

$$\neg \text{IsCommutative}(\text{Divide}, \{1, 2, 3\})$$

ASSUME

$$\neg \exists x \in \{1, 3, 5\} : \text{IsEven}(x)$$

$$\begin{aligned} \text{Pick}(S) &\triangleq \text{CHOOSE } s \in S : \text{TRUE} \\ \text{RECURSIVE } \text{SetReduce}(-, -, -) \\ \text{SetReduce}(\text{Op}(-, -), S, \text{value}) &\triangleq \text{IF } S = \{\} \text{ THEN } \text{value} \\ &\quad \text{ELSE LET } s \triangleq \text{Pick}(S) \\ &\quad \text{IN } \text{SetReduce}(\text{Op}, S \setminus \{s\}, \text{Op}(s, \text{value})) \end{aligned}$$

$$\begin{aligned} \text{Sum}(S) &\triangleq \text{LET } \_op(a, b) \triangleq a + b \\ &\quad \text{IN } \text{SetReduce}(\_op, S, 0) \end{aligned}$$

ASSUME

$$\text{Sum}(\{1, 2, 3\}) = 6$$

$$\text{Min}(S) \triangleq \text{CHOOSE } x \in S : \forall y \in S : x \leq y$$

ASSUME

$$\text{Min}(\{5, 3, 7, 10, 2, 9\}) = 2$$

$$\text{Max}(S) \triangleq \text{CHOOSE } x \in S : \forall y \in S : x \geq y$$

ASSUME

$$\text{Max}(\{4, 6, 1, 2, 9, 3, 5\}) = 9$$

ASSUME  
 $\langle 1, 2, 3 \rangle \in Seq(\{1, 2, 3\})$

ASSUME  
 $\langle 4 \rangle \notin Seq(\{1, 2, 3\})$

ASSUME  
 $\langle 1, 2, 3, 4 \rangle \notin Seq(\{1, 2, 3\})$

Sets of tuples.

$chessboard\_squares \triangleq \{“a”, “b”, “c”, “d”, “e”, “f”, “g”, “h”\} \times (1 .. 8)$

ASSUME  
 $\wedge \langle “a”, 1 \rangle \in chessboard\_squares$   
 $\wedge \langle “a”, 2 \rangle \in chessboard\_squares$   
 $\wedge \langle “a”, 3 \rangle \in chessboard\_squares$   
 $\wedge \langle “a”, 4 \rangle \in chessboard\_squares$

$jason \triangleq (1 .. 2) \times \{“Jason”, “DeBolt”\}$

ASSUME  
 $\wedge \langle 1, “Jason” \rangle \in jason$   
 $\wedge \langle 2, “Jason” \rangle \in jason$   
 $\wedge \langle 1, “DeBolt” \rangle \in jason$   
 $\wedge \langle 2, “DeBolt” \rangle \in jason$

$digits \triangleq \{“one”, “three”\} \times \{“two”, “four”\}$

ASSUME  
 $\wedge \langle “one”, “two” \rangle \in digits$   
 $\wedge \langle “three”, “four” \rangle \in digits$

$A \triangleq \{1\}$   
 $B \triangleq \{2\}$   
 $C \triangleq \{3\}$

ASSUME  
 $\wedge \langle 1, 2, 3 \rangle \in A \times B \times C$   
 $\wedge \langle 1, \langle 2, 3 \rangle \rangle \in A \times (B \times C)$   
 $\wedge \langle \langle 1, 2 \rangle, 3 \rangle \in (A \times B) \times C$

Structures.

Structures are hashes. They have keys and values. You specify them as  $[key \mapsto value]$  and query them with either  $[“key”]$  or  $.key$ . Both are legal and valid.

$SomeHash \triangleq [x \mapsto 1, y \mapsto \{2, 3\}]$

ASSUME

$\wedge SomeHash.x = 1$   
 $\wedge SomeHash["x"] = 1$   
 $\wedge SomeHash.y = \{2, 3\}$   
 $\wedge SomeHash["y"] = \{2, 3\}$   
 $\wedge DOMAIN\ SomeHash = \{ "x", "y" \}$

Aside from that, there's one extra trick structures have. Instead of  $key \mapsto value$ , you can do  $key : set$ . In that case, instead of a structure you get the set of all structures which have, for each given key, a value in the set.

$SetOfStructures \triangleq [x : \{1\}, y : \{2, 3, 4\}]$

If you use  $:$  syntax and any of the values are not sets, then the entire construct is invalid. In other words, while  $[a: \{1\}, b: \{2, 3\}]$  is the above set,  $[a: 1, b: \{2, 3\}]$  will throw an error if you try to use it.

ASSUME

$\wedge [x \mapsto 1, y \mapsto 2] \in SetOfStructures$   
 $\wedge [x \mapsto 1, y \mapsto 3] \in SetOfStructures$   
 $\wedge [x \mapsto 1, y \mapsto 4] \in SetOfStructures$

Type Composition

Any type can be squeezed inside any other type.

$crazy \triangleq [a \mapsto \{\langle \rangle, \langle 1, 2, 3 \rangle, \langle 3, 2, 1 \rangle\}, b \mapsto \langle [a \mapsto 0] \rangle]$

A function of keys mapping to sets of tuples or of keys mapping to tuples of functions.

ASSUME

$crazy.b[1].a = 0$  Remember that tuples are 1 indexed.

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\ \* Modification History  
 \ \* Last modified Sun Apr 21 16:44:59 PDT 2019 by jasondebolt  
 \ \* Created Sat Apr 20 20:01:34 PDT 2019 by jasondebolt