Analytical expressions for view factors

The view factor from a surface A_1 to another surface A_2 is written F_{12} and is defined as the fraction of the total radiant flux leaving A_1 that is incident upon A_2. The view factor from a plane point source (point view factor) is obtained by integration over A_2 while the mean configuration factor from a line source or a finite source is an average of the point view factor over the line source or finite source, respectively.

As a further test for the View3D code, an expression can be derived for the obstructed view factor of surfaces 1 and 6 for the L-shaped room ("Test 3" from Pinney and Beans (1988)).

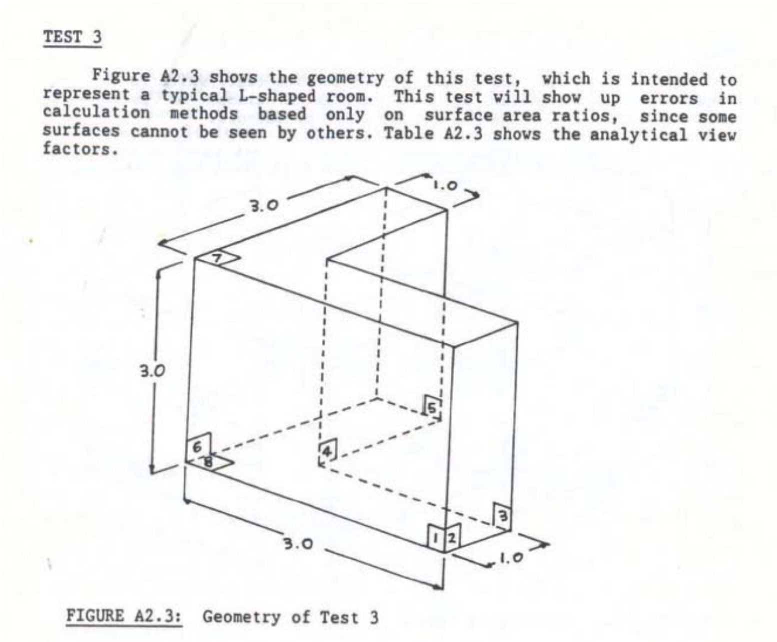
TEST 3

Figure A2.3 shows the geometry of this test, which is intended to represent a typical L-shaped room. This test will show up errors in calculation methods based only on surface area ratios, since some surfaces cannot be seen by others. Table A2.3 shows the analytical view factors.

FIGURE A2.3: Geometry of Test 3

A top view (Figure 1) shows that Surface 6 is unobscured from points on Surface 1 that are within 1.5 length units from the corner. Beyond 1.5 units, Surface 3 obscures a portion of Surface 6 from the view of Surface 1.

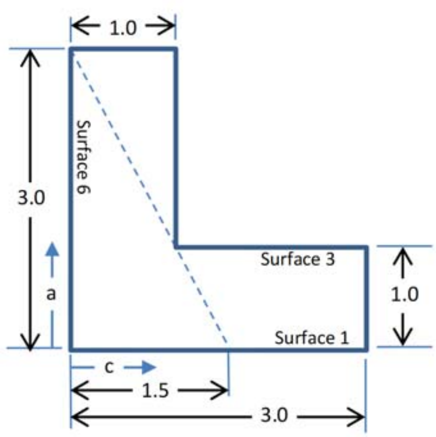

Figure 1: Top View of L-Shaped Room
Reference: Pinney and Beans (1988)

From inspection of Figure 1, the length a of the unobscured portion of Surface 6 as seen by a point source on Surface 1 at a distance c from the corner may be written as:

$$a(c)=\begin{cases} 3 & , \quad c\le 1.5 \\ \dfrac{c}{c-1} & , \quad c>1.5 \end{cases} \tag{1}$$

Hamilton and Morgan (1952) report an expression for the view factor $F_{dA_1-A_2}$ of a line source dA_1 and a plane rectangle A_2 which intersects the plane of dA_1 at an angle Φ.

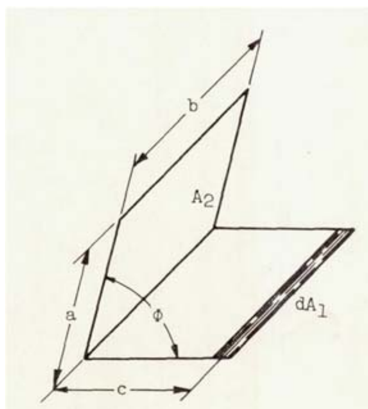

Figure 2: Line source and plane
Reference: Hamilton and Morgan (1952)

In terms of variables $N=a/b$ and $L=c/b$, the view factor $F_{dA_1-A_2}$ may be written as:

$$\begin{aligned} F_{dA_1-A_2} = \frac{1}{\pi}\Bigg\{ & \tan^{-1}\left(\frac{1}{L}\right)+\frac{L}{2}\sin^2\Phi\,\ln\left[\frac{L^2\left(L^2-2NL\cos\Phi+1+N^2\right)}{\left(1+L^2\right)\left(L^2-2NL\cos\Phi+N^2\right)}\right]- \\ & L\sin\Phi\cos\Phi\left[\frac{\pi}{2}-\Phi+\tan^{-1}\left(\frac{N-L\cos\Phi}{L\sin\Phi}\right)\right]+ \\ & \cos\Phi\sqrt{1+L^2\sin^2\Phi}\left[\tan^{-1}\left(\frac{N-L\cos\Phi}{\sqrt{1+L^2\sin^2\Phi}}\right)+\tan^{-1}\left(\frac{L\cos\Phi}{\sqrt{1+L^2\sin^2\Phi}}\right)\right]+ \\ & \frac{N\cos\Phi-L}{\sqrt{L^2-2NL\cos\Phi+N^2}}\tan^{-1}\left(\frac{1}{\sqrt{L^2-2NL\cos\Phi+N^2}}\right)\Bigg\} \end{aligned} \tag{2}$$

In terms of N and L, the expression in Equation (1) takes the form:

$$N(L) = \begin{cases} 1 & , \quad L \le 1/2 \\ \dfrac{L}{3L-1} & , \quad L > 1/2 \end{cases} \tag{3}$$

The view factor F_{16} for Surface 1 and all of Surface 6 (both the obscured and unobscured portions) may be found from Equation (2) by setting $\Phi = \pi/2$, substituting the expression for $N(L)$ from Equation (3), and integrating from L = 0 to L = 1:

$$\begin{aligned} F_{16} &= \int_{L=0}^{1} F_{dA_1-A_6}\, dL \\ &= \int_{L=0}^{1} dL\, \frac{1}{\pi}\left\{ \tan^{-1}\left(\frac{1}{L}\right) + \frac{L}{2}\ln\left[\frac{L^2\left(1+L^2+N^2\right)}{\left(1+L^2\right)\left(L^2+N^2\right)}\right] - \frac{L}{\sqrt{L^2+N^2}}\tan^{-1}\left(\frac{1}{\sqrt{L^2+N^2}}\right)\right\} \end{aligned} \tag{4}$$

where $dA_1 = (1)\, dL$ and, for clarity, we omit the explicit dependence of N on L. Equation (4) may be solved symbolically or numerically to yield the result

$$F_{16} = 0.182356\,. \tag{5}$$