B+-trees

Perhaps the most widely used index ever!

CMSC 420

B+-trees were not covered in Spring 2019, yet it is very important that you go through these slides so that you can understand the real story behind <K,V> pairs, the nature of a range query, and why it's an excellent idea to lower the height of your tree-based index so that you can index into more disk-resident data with a smaller spatial cost in memory!

The true story: <K, V> pairs

- So far, we have only cared about storing **`Comparable`**s such that:
  - **Search** is optimized.
  - **Insert** is optimized.
  - **Delete** is optimized.
- We have required that all those elements are **`Comparable`**s because… we need to compare them.

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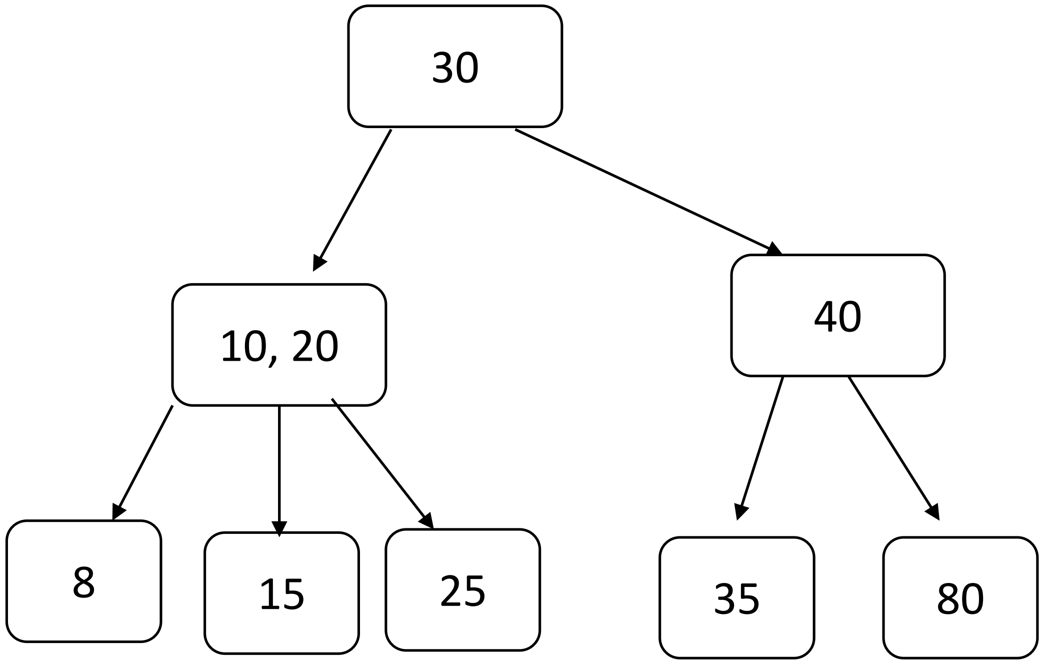
- We have required that all those elements are **`Comparable`**s because… we need to compare them.
- **But does this mean that you can only store things that have a 1-1 mapping with the naturals?**

The true story: <K, V> pairs

- **Clearly not**, we can store images, files, database table records…
- **The true story is as follows:** We have been dealing only with Comparable **keys,** where what we might want is a **complex data value.**
- By employing a sufficiently large key dictionary (e.g integers), we can associate every value V with some unique key K. **It is assumed that K points to V in $\mathcal{O}(1)$!** *(Fair assumption* ***as long as we are in memory****)*
- **All** our work on data structures so far **has concerned this organization of keys**, since once we find a key, we can access the associated value in $\mathcal{O}(1)$ ☺

The true story: <K, V> pairs



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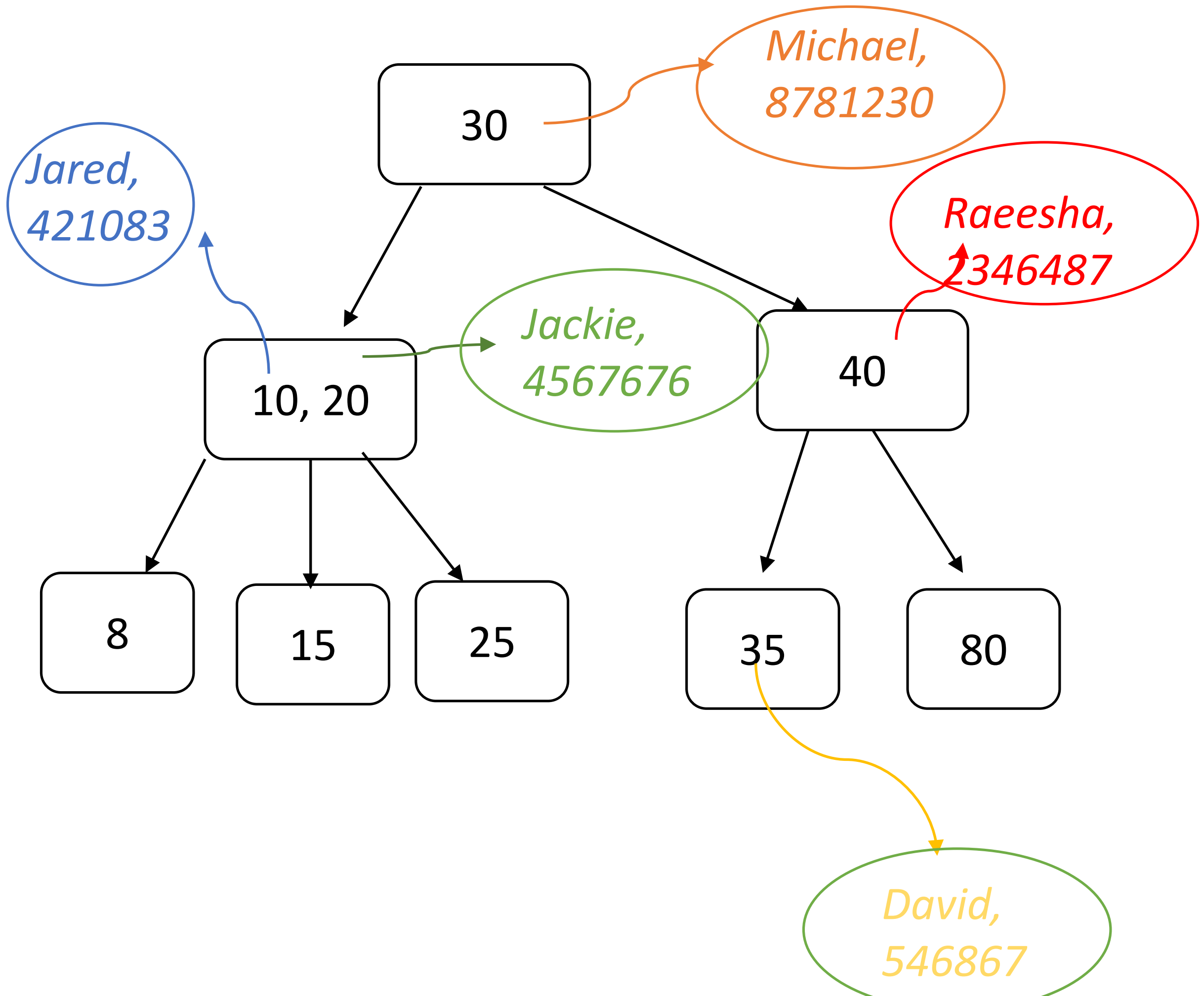

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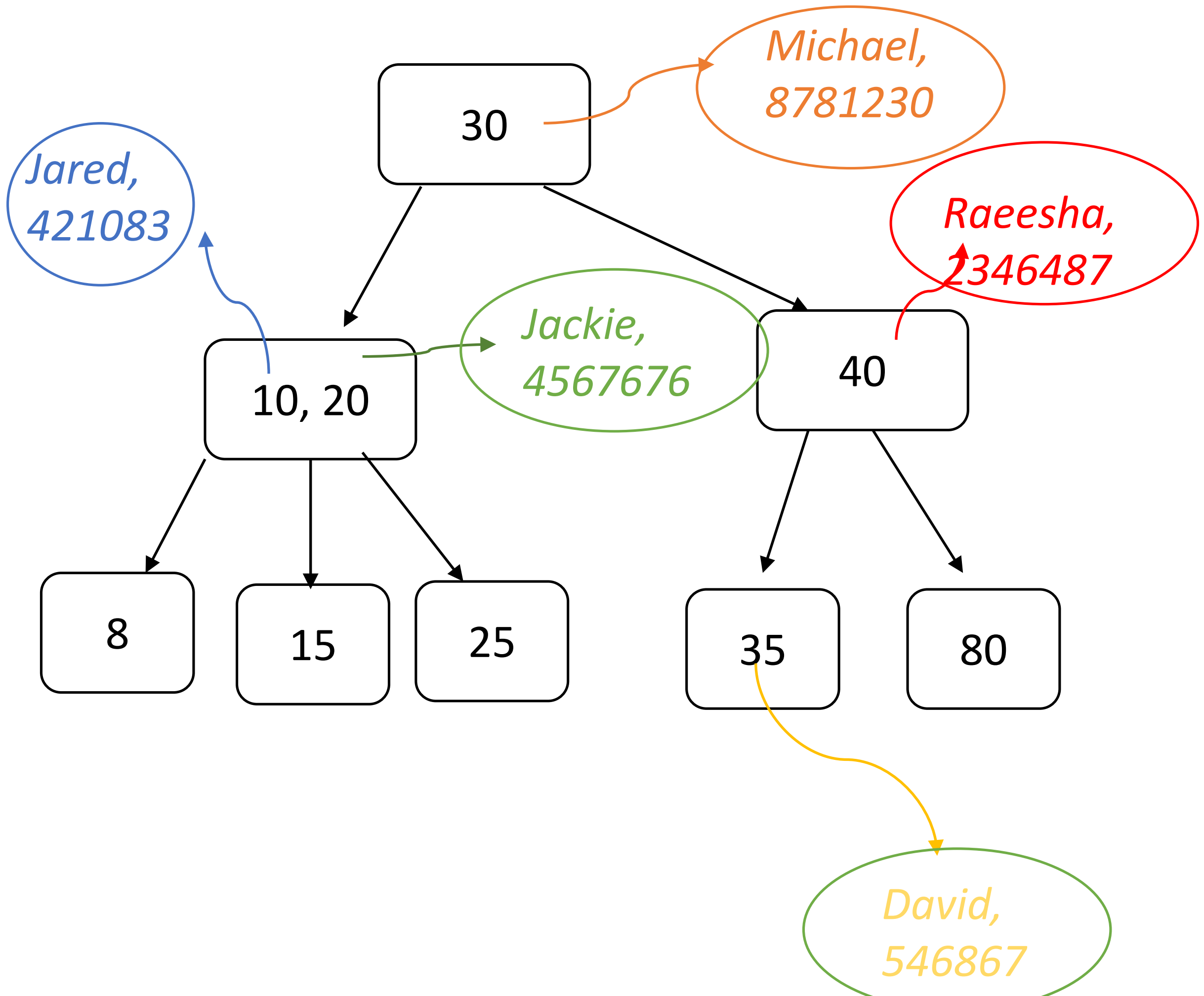
20	Jackie	4567676
…	…	…
30	Michael	8781230
…	…	…
35	David	546867
…	…	…
40	Raeesha	2346487
…	…	…

The true story: <K, V> pairs



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$\underbrace{\text{Name} \quad \text{UID}}_{\text{This is what we want (the value)}}$

Relational Databases

ID	Name	UID	Major	Campus resident	Rooms with	Parking space ID	Year
10	Jared	421083	POLI	N	NULL	89343	3
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This entire table is on disk!

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Speaking of disks….

Rotational Hard disks and pages

- Disk space is divided into so-called **pages.**
  - Typical size: **4KB**
- **PULLING PAGES FROM DISK TAKES A LOT OF TIME!**
  - Fastest seek time for enterprise HDDs: **4ms**
  - **Orders of magnitude worse** than RAM (in the μs) and registers/cache (in the ns)
- **Page Fault:** Access of data pointed to by our program, which of course runs in main memory, but the data itself **resides on disk.**
- Don't confuse with cache miss: an address sought by our program was not in cache, but might be found in main memory.

Rotational Hard disks and pages

- Operating Systems allow applications to allocate buffers for reading and writing *EXACTLY* **PAGE_SIZE** kilobytes big.
  - *Usually, PAGE_SIZE=4KB*
- The application can then do whatever it needs with the data. If a change of the data needs to persist on disk, the **entire page** will be flushed to disk.
  - Even if exactly one byte was changed.
- Beneficial for buffers to **persist in memory** if possible (leverage locality), to avoid going back to disk.

Rotational Hard disks and pages

- Disk fragmentation: Under or over-utilization of disk pages by an application.
  - Under-utilization: pages are largely empty and space is wasted.
  - Over-utilization: the application takes up a lot of new hard disk pages, and indices have to be updated (this costs time)
- We will not talk about how we can control and optimize disk space (beyond scope).

Example Queries

TABLE STUDENTS

ID	***Name***	***UID***	***Major***	***Campus resident***	***Rooms with***	***Parking space ID***	***Year***
10	*Jared*	*421083*	*POLI*	*N*	*NULL*	*89343*	*3*
…	…	…					
20	*Jackie*	*4567676*	*NULL*	*Y*	*40*	*243654*	*1*
…	…	…					
30	*Michael*	*8781230*	*ENGR*	*N*	*NULL*	*23465*	*4*
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1. Show me all fields from **all** records in the table . (`SELECT * from STUDENTS`)

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4KB

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1. Show me all fields from **all** records in the table . (`SELECT * from STUDENTS`)

- Brute-force: Pay seek cost sc for all the page faults ($\lceil {}^{N}/_{4096} \rceil$ for N bytes of disk space required for table).
- Also, pay exactly $r = {}^{N}/_{rec\_size}$ total time for printing every single record.
- Only good news: Note that you don't have to allocate N bytes of main memory for the result of this query: when you've printed the data of an entire page, use the same buffer for the next page (no need to write anything to disk either).

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CAN WE DO BETTER?

Yes

No

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CAN WE DO BETTER?

Yes No

- This query asks for **everything** and **anything**!
- Sooner or later, we **will** pay the price!
- Even if you build a Red-Black Tree over IDs… you still **have to scan all pages**!

1. Show me all fields from **all** records in the table . (`SELECT * from STUDENTS`)

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2. Show me all fields from **students ID is between 740 and 1043**. (`SELECT * from STUDENTS WHERE ID >=740 AND ID <= 1043`)

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CAN WE DO BETTER NOW?

Yes

Foreshadowing does, like, nothing for me

2. Show me all fields from **students ID is between 740 and 1043**. (`SELECT * from STUDENTS WHERE ID >=740 AND ID <= 1043`)

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We will need an index over IDs!

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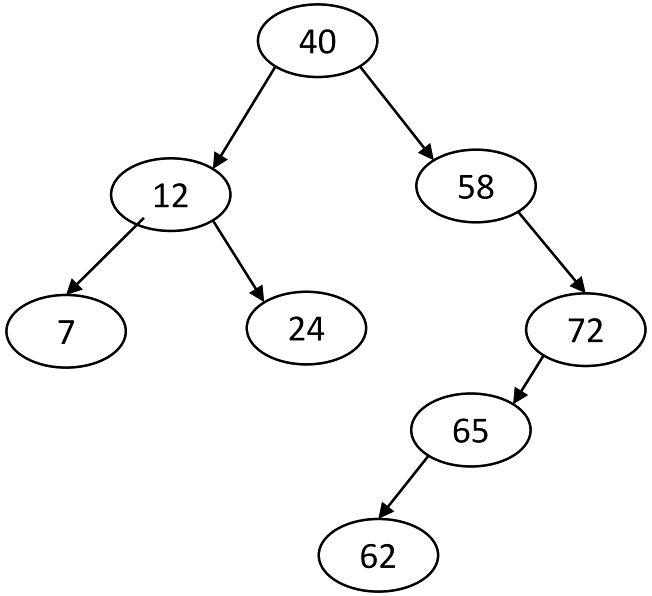
Warmup Exercise

- Take 10 to implement the **void** method

`rangeSearch(Node n, int min, int max, List<Integer> list)`,

such that we can perform range search on this BST!



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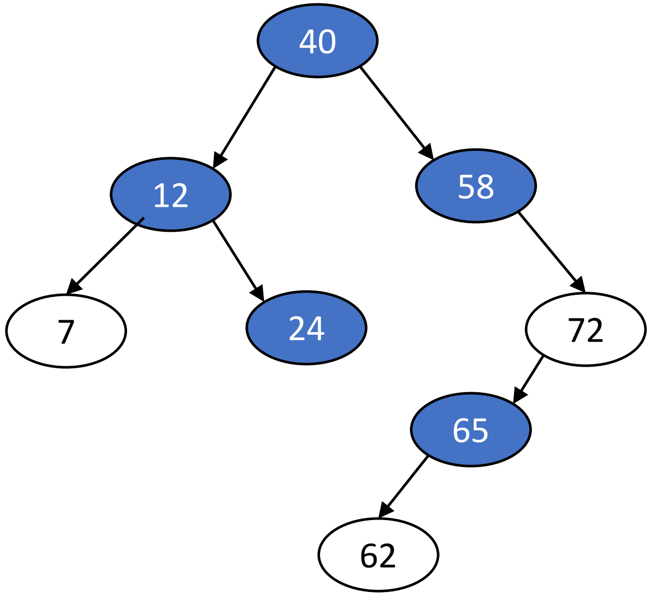
[12, 65]



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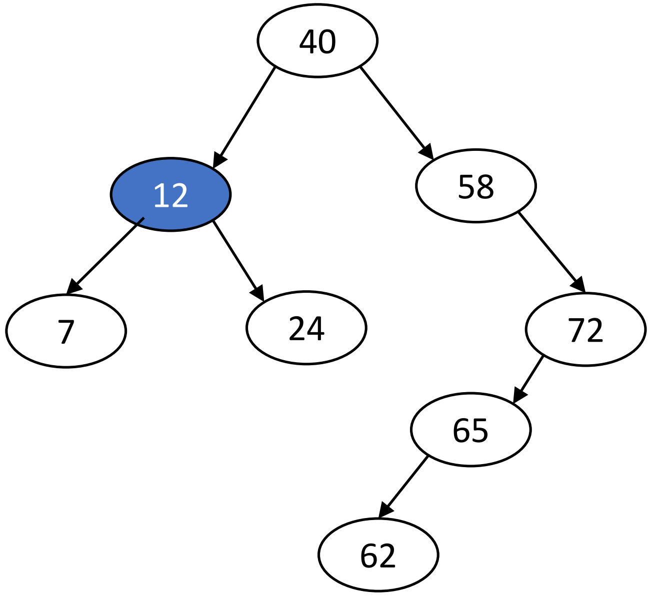

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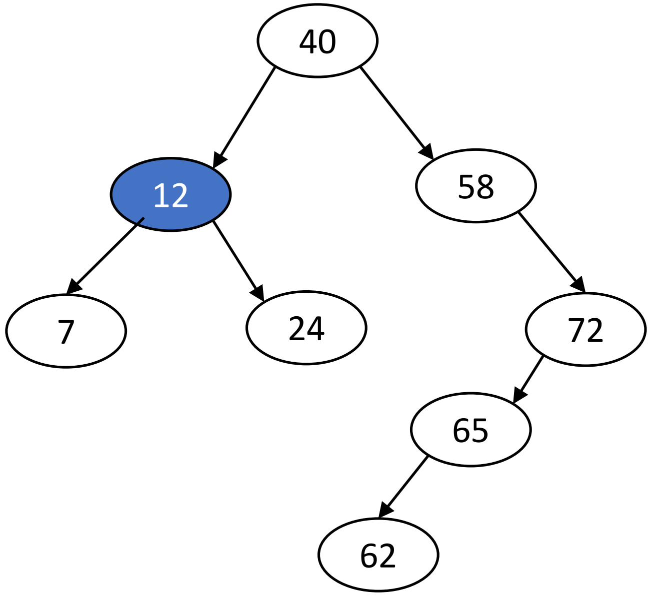
[11, 12]

(11 is not a key in this tree…)



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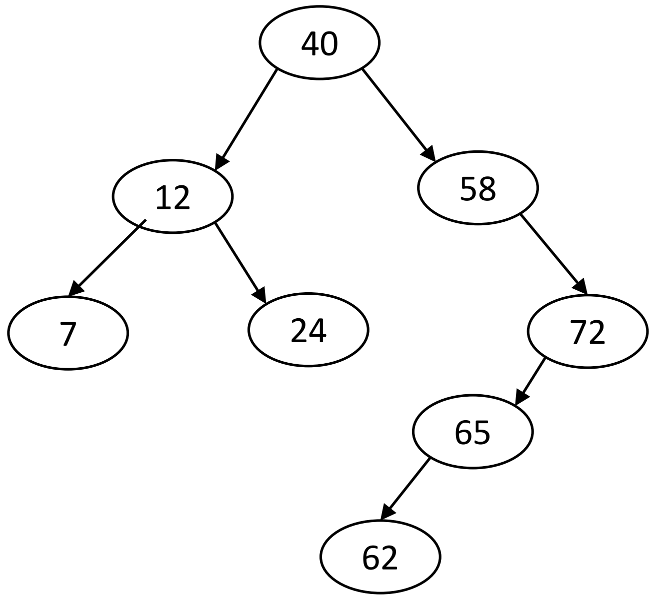
[70, 20]

Just throw your favorite exception!



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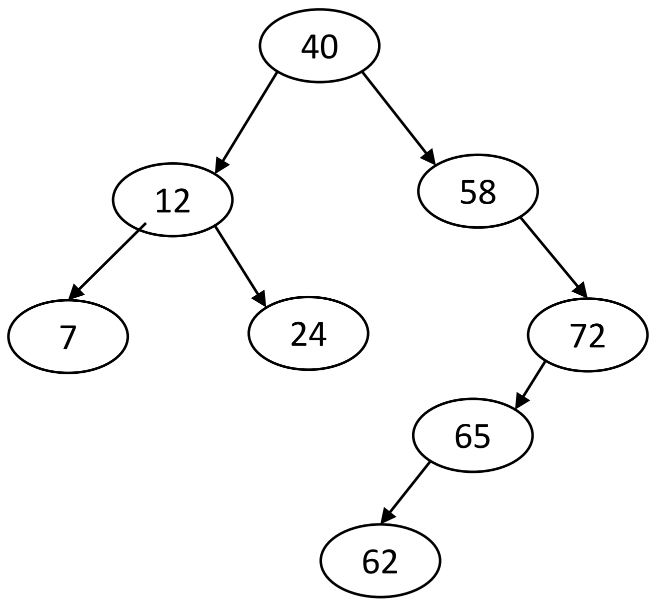
Example Call:

```
ArrayList<T> list = new ArrayList();
rangeSearch(root, min, max, list)
```



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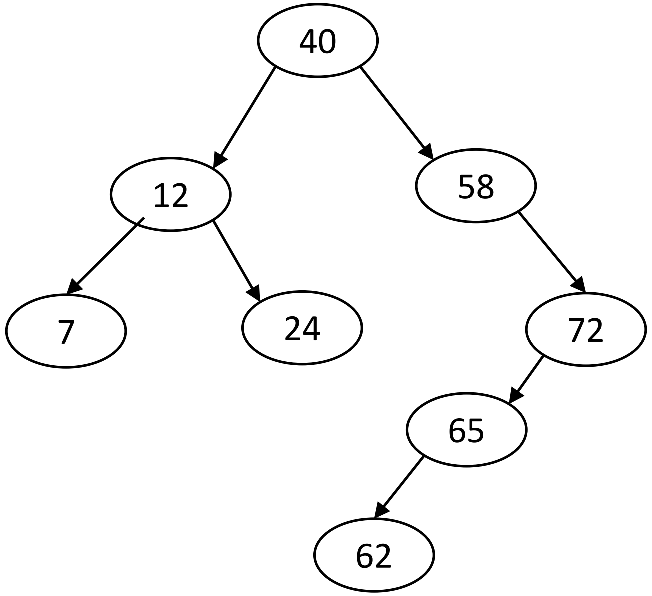
rangeSearch(Node n, int min, int max, List<Integer> list),

such that we can perform range search on this BST!

```
// This will fail if min and max are null, but whatever
private void rangeSearch(Node n, T min, T max, List<T> list){
    assert min.compareTo(max) <= 0 : "Min ought to be smaller than or equal to max.";
    if(n == null)
        return;
    if(n.key.compareTo(min) > 0 && n.key.compareTo(max) < 0){
        rangeSearch(n.left, min, n.key, list);
        list.add(n.key);
        rangeSearch(n.right, n.key, max, list);
    } else if(n.key.compareTo(min) == 0) {
        list.add(n.key);
        rangeSearch(n.right, n.key, max, list);
    } else if(n.key.compareTo(min) < 0) {
        rangeSearch(n.right, n.key, max, list);
    } else if(n.key.compareTo(max) == 0 ) {
        rangeSearch(n.left, min, n.key, list);
        list.add(n.key); // ! If you want the range sorted, the addition should be
after the recursive call.
    } else if(n.key.compareTo(max) > 0 ) {
        rangeSearch(n.left, min, n.key, list);
    }
}
```



Key-Value Dictionaries are database indices

- We can do this with a BST. We know how to do range search, so…
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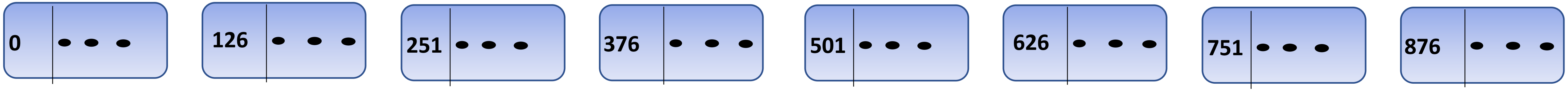
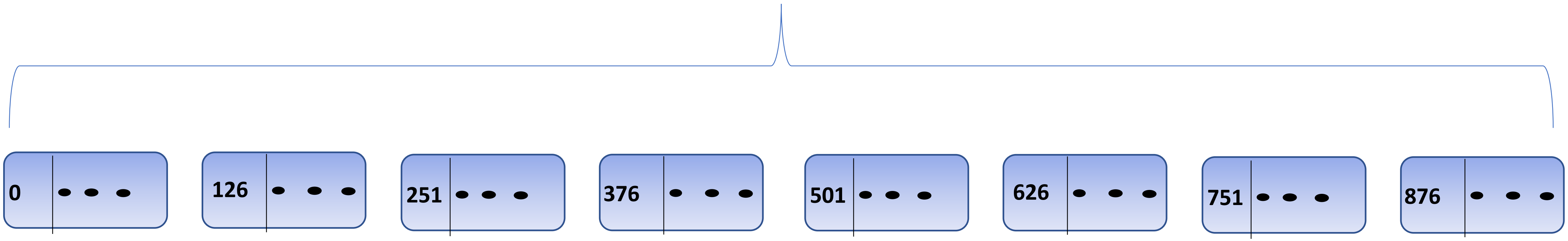
- We can do this with a BST. We know how to do range search, so…
- Let's assume that we build our index as a BST…
  - A bottom-up approach seems natural!
- Assume 1000 records of 32 bytes each.
  - $\frac{1000 \times 32\ B}{4KB} = 8$ pages
- Key: BUILDING the dictionary is expensive.
  - But once it's in place, **reading** and **writing** data, even when it lies on disk, will be much faster than having nothing in place.

Building a BST index, bottom-up



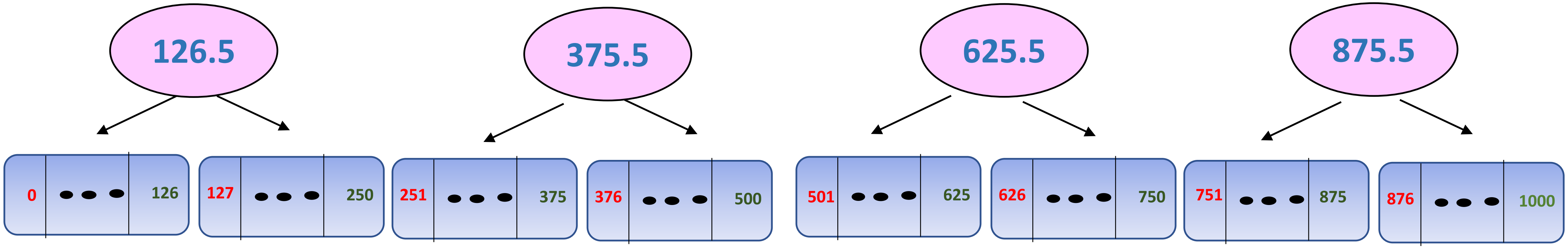
Building a BST index, bottom-up

All of those are on disk!



Building a BST index, bottom-up

- To separate the disk pages, we need an appropriate separator.
- Answer: Take the **average** of the **first** and **last** ids of every page!
  - ➢ Other values could also work, e.g median. Average is just easy.



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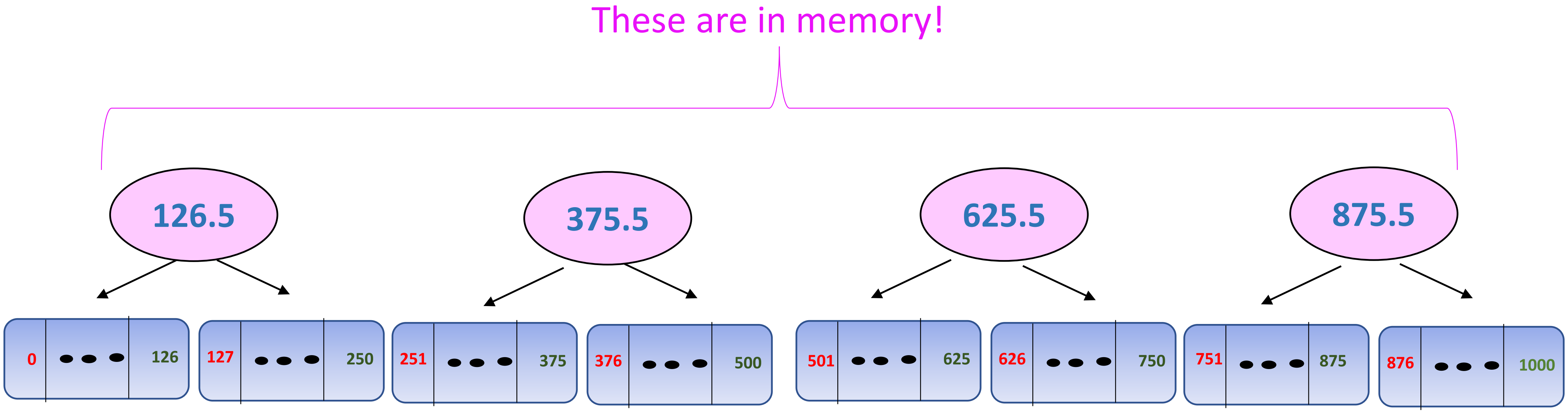
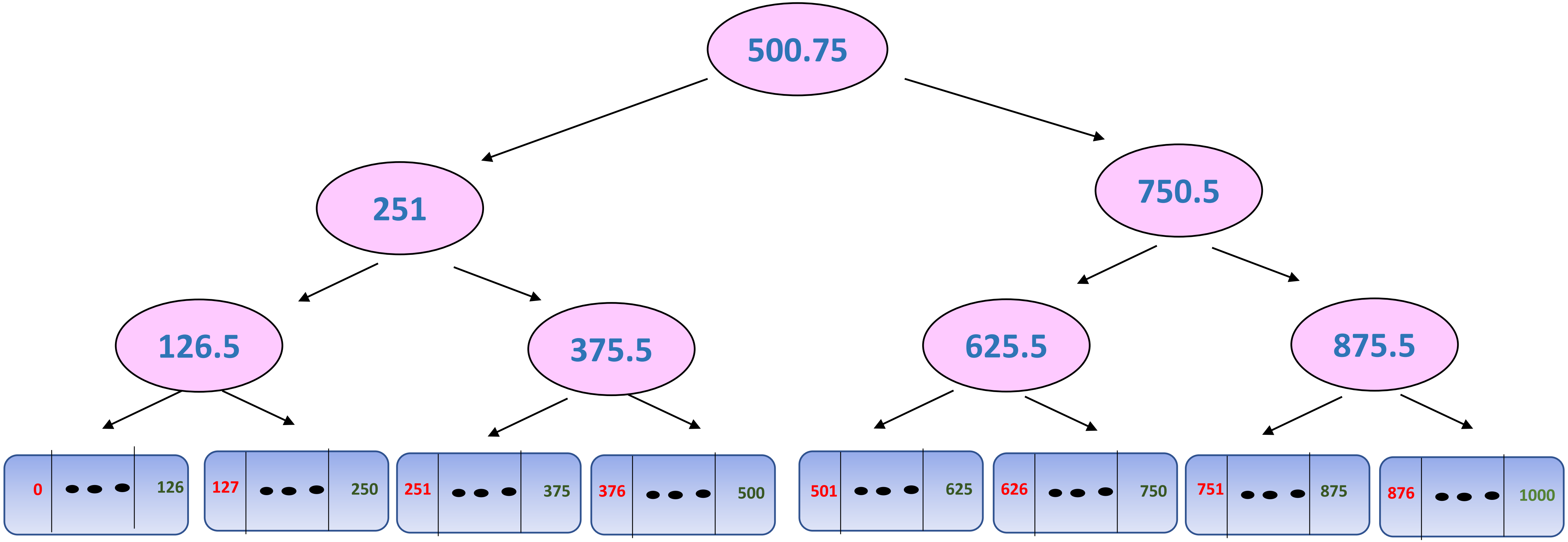


Building a BST index, bottom-up

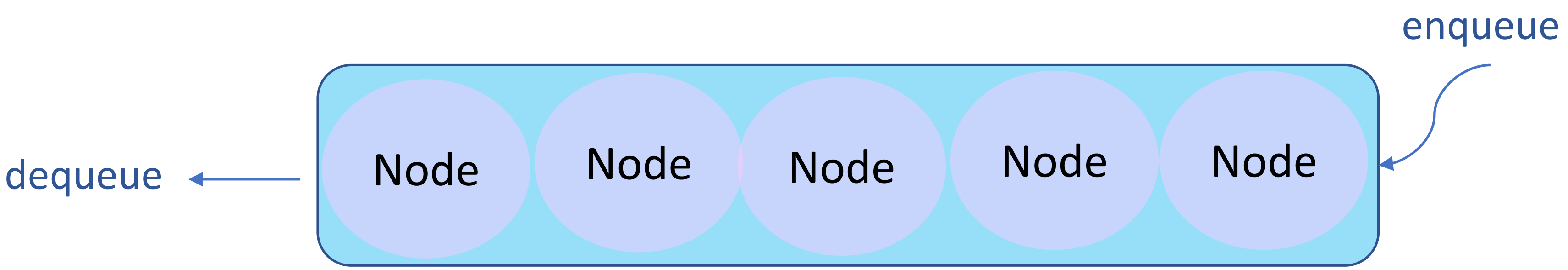
- We can repeat the exact same process for the rest of the nodes!



Some code that does this

```
class Node {
    T key;
    Node left, right;
}
```

Node

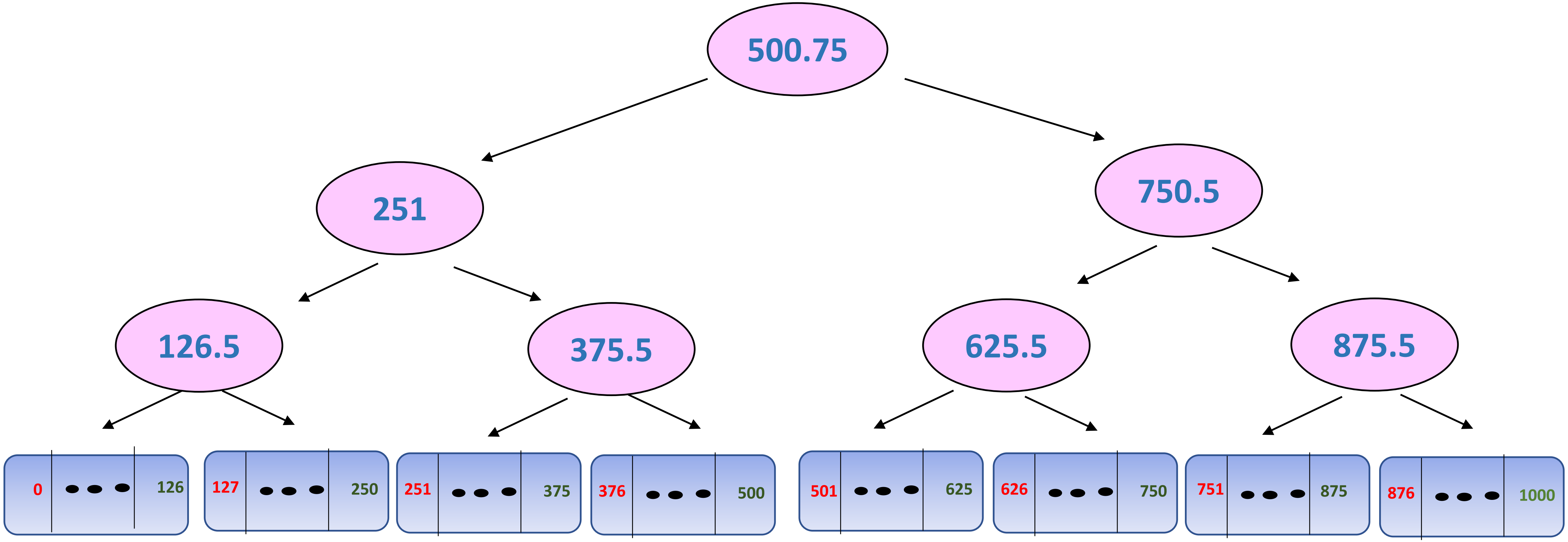



```
FIFOQueue<Node<T>>
```

```
FifoQueue<Node<T>> q = new FifoQueue<T>();
nodes.forEach(q::enqueue); // Assume that all nodes are in some Iterable structure
while(q.size() > 2){
    Node n1 = q.dequeue(), n2 = q.dequeue();
    q.enqueue(new Node((n1.key + n2.key) / 2, n1, n2));
} // At this point we only have two subtrees and have to connect them to a root node (q.size() == 2 is an invariant)
Node n1 = q.dequeue(), n2 = q.dequeue();
Node root = new Node((n1.key + n2.key) / 2, n1, n2));
```

Using our index

```
SELECT * from STUDENTS WHERE ID > 250 AND ID <= 700
```



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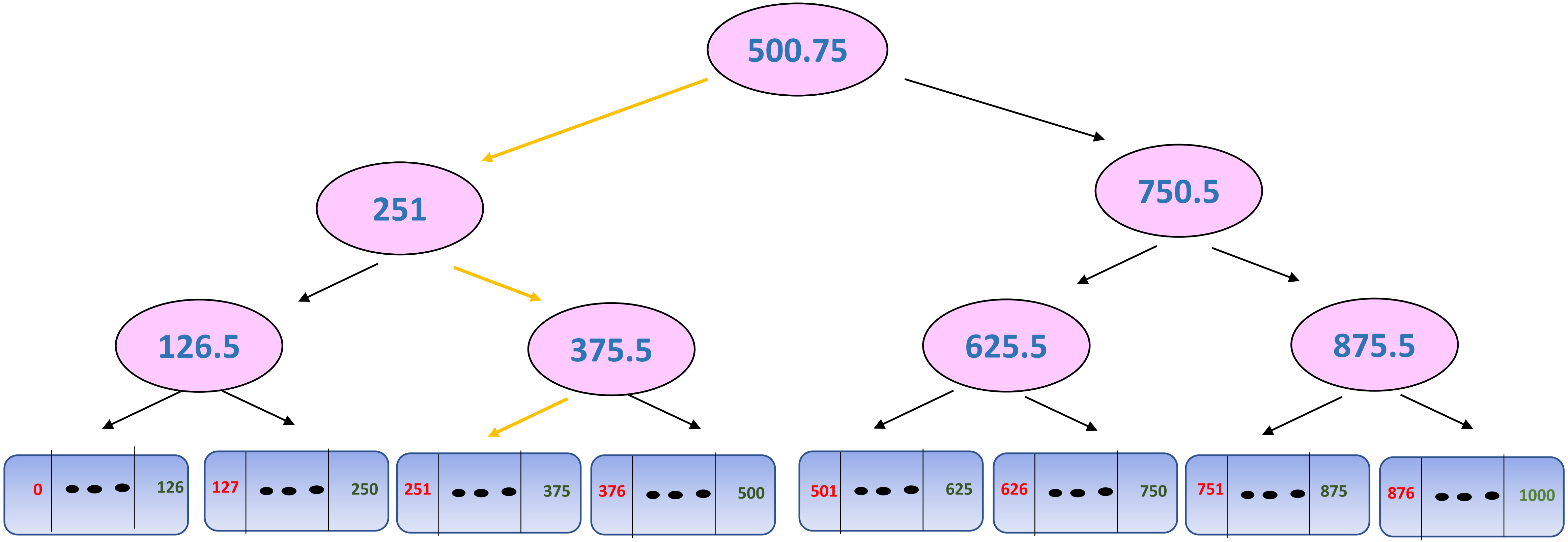

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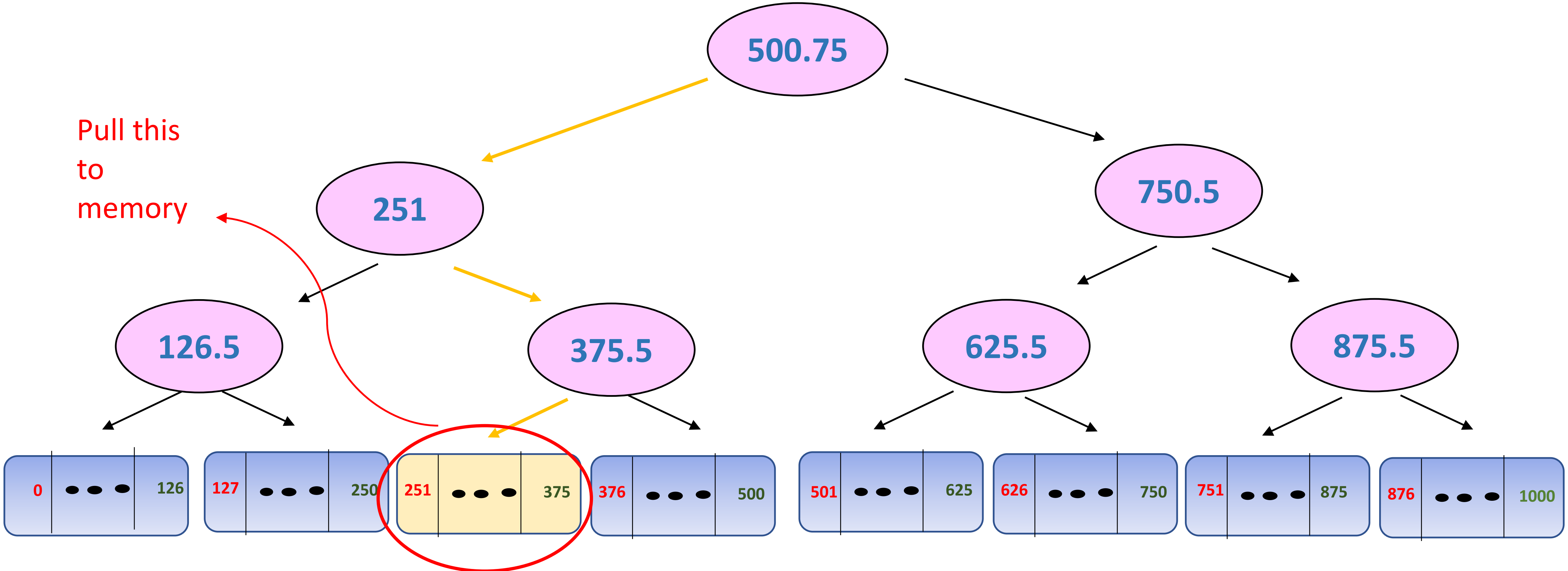

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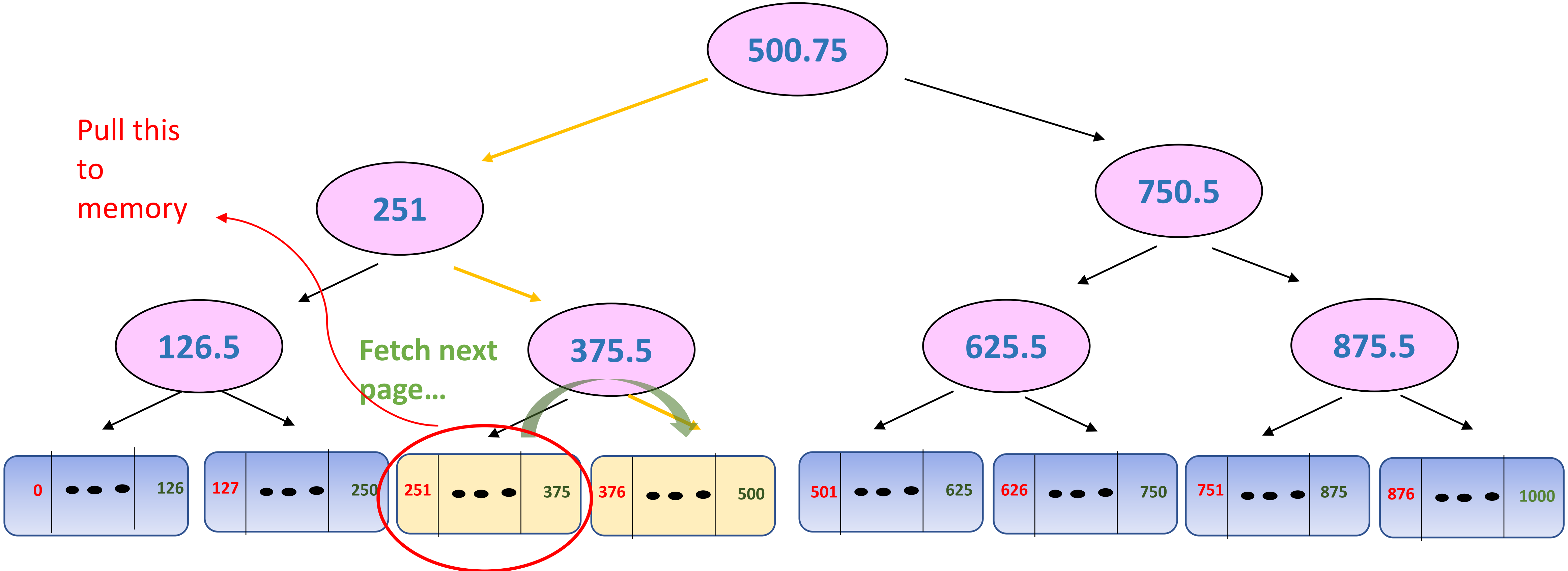

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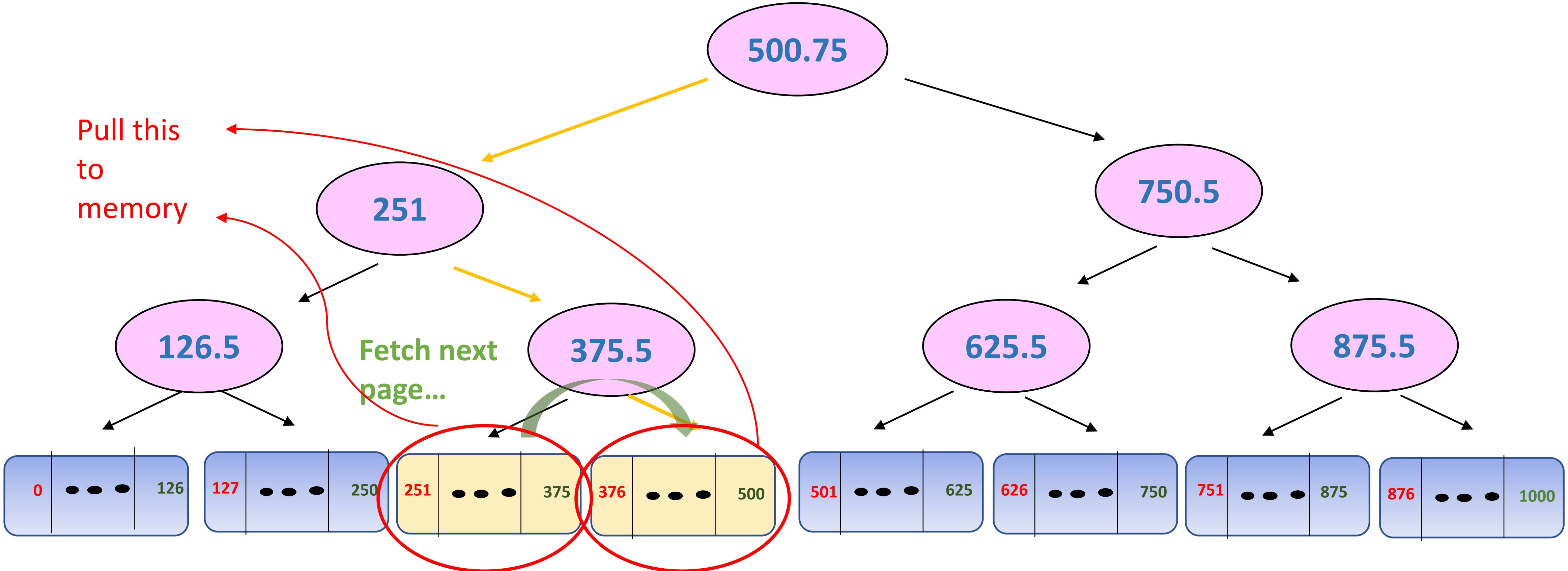

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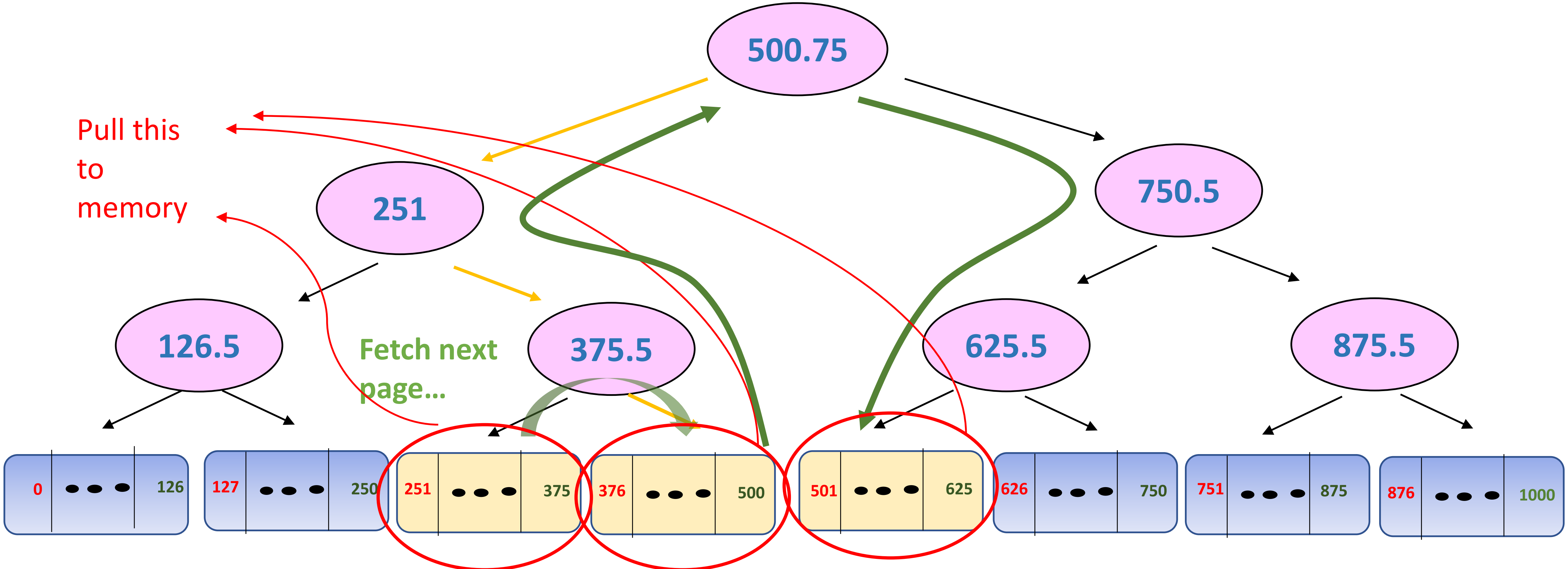

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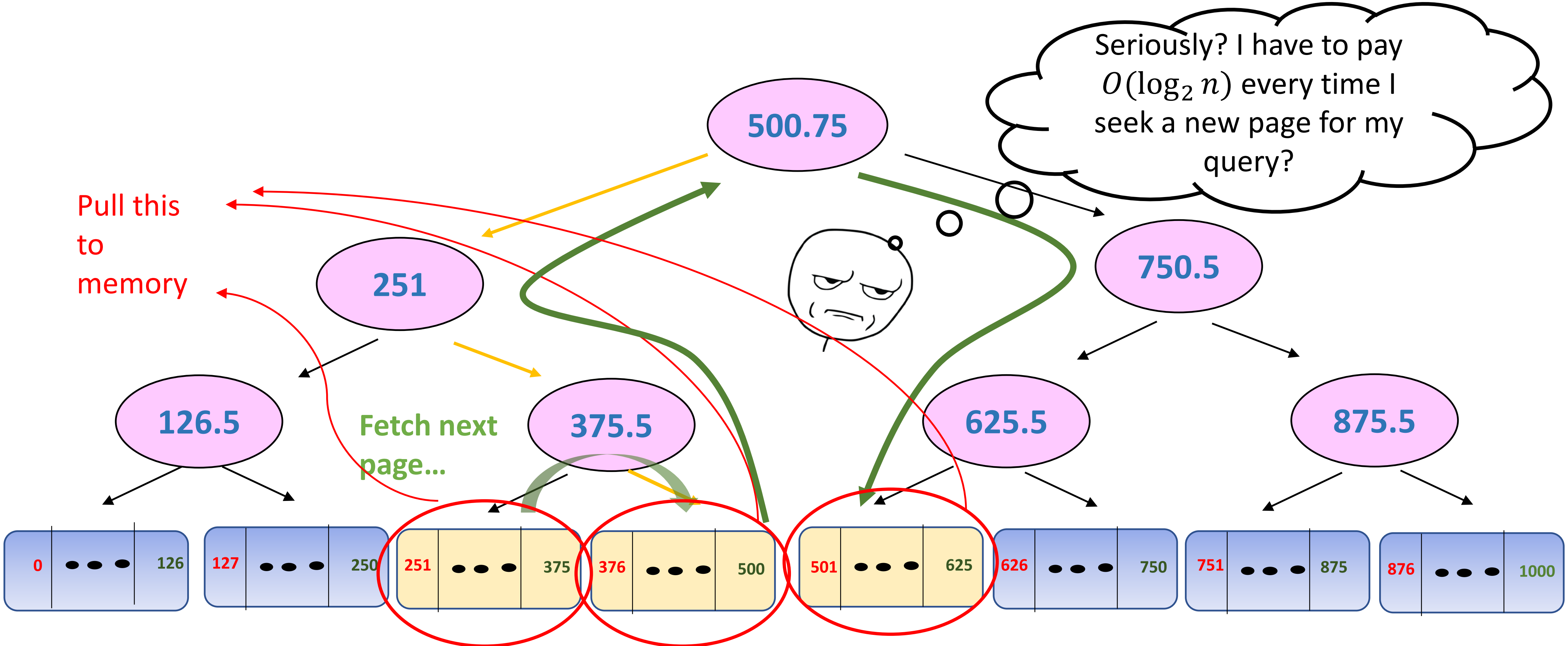

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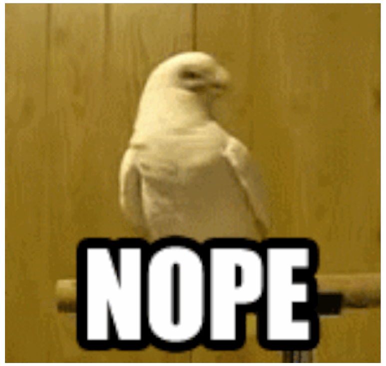
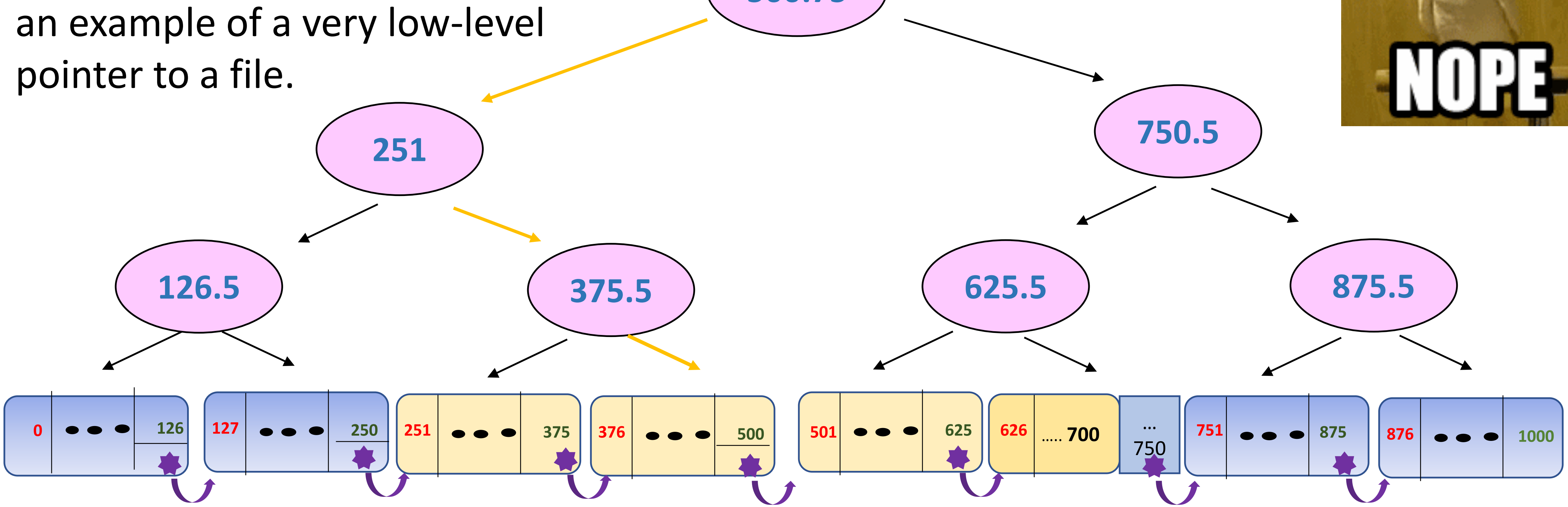
- **Store pointers on disk!**
- Look up C's `FILE*`("File-star") for an example of a very low-level pointer to a file.





Space Analysis (Main Memory)

- We have 7 nodes with 2 references and 1 float each.
  - 7 * 16 + 28 = 140 bytes total.
- Total index size: 140 bytes $= 0.4375\%$ of database size.
- 0.4375% sounds quite good, but what happens when our database is 128 GB in size? ☹
  - Then, 0.525% of 128GB is **672MB**.
  - **We can do better ☺**

Example of our new index

- Let's assume $p = 4$ (fanout of nodes = 4).
  - Non-root nodes hold between $\left\lceil \frac{p}{2} \right\rceil - 1 = 1$ and $p - 1 = 3$ keys.
  - Root can have between 1 and 3 keys.

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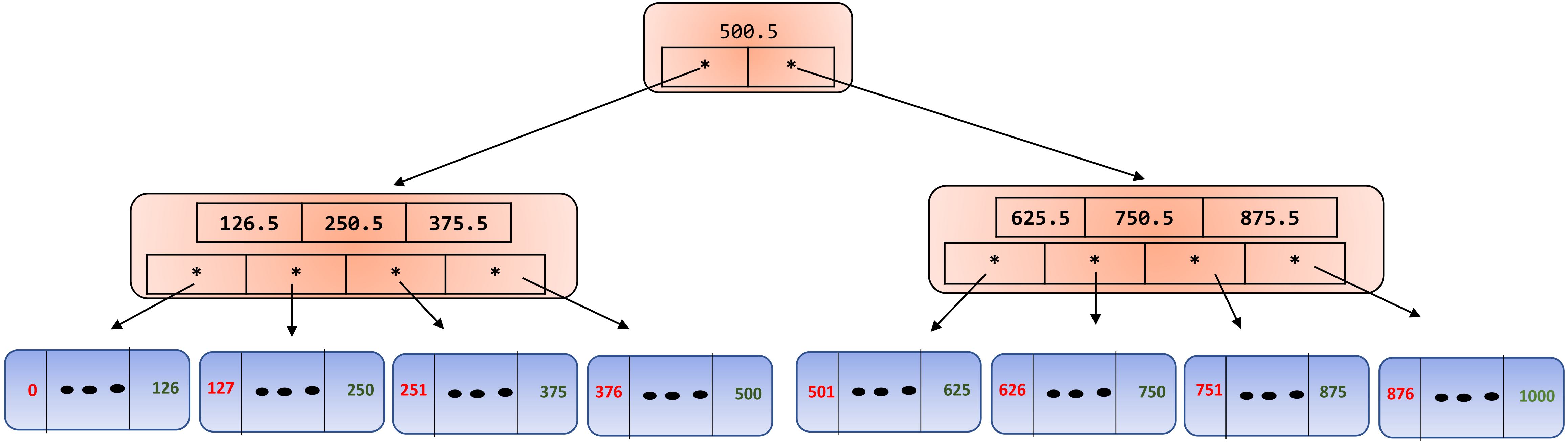

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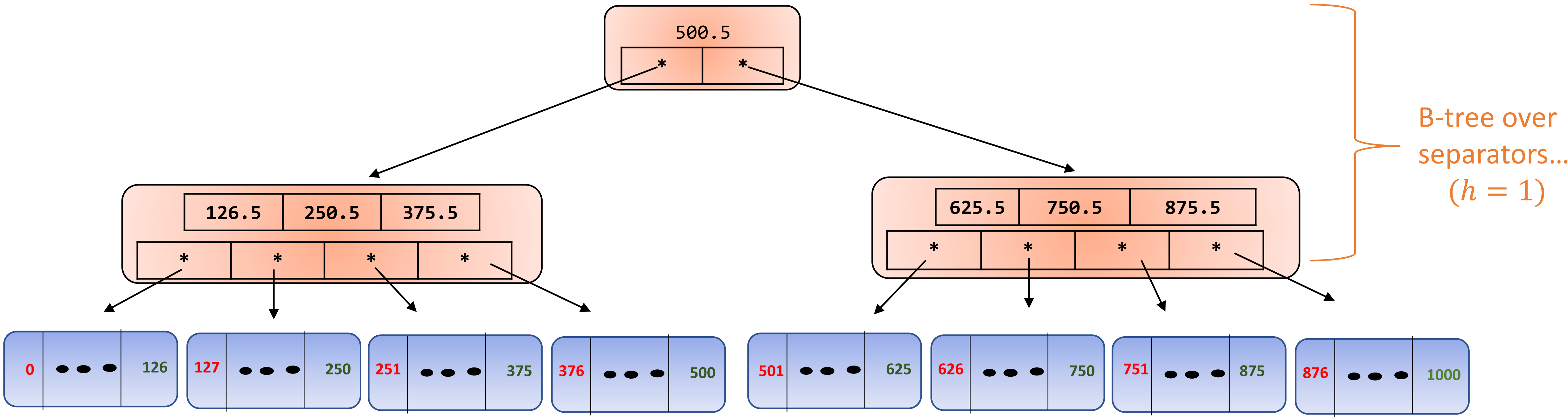

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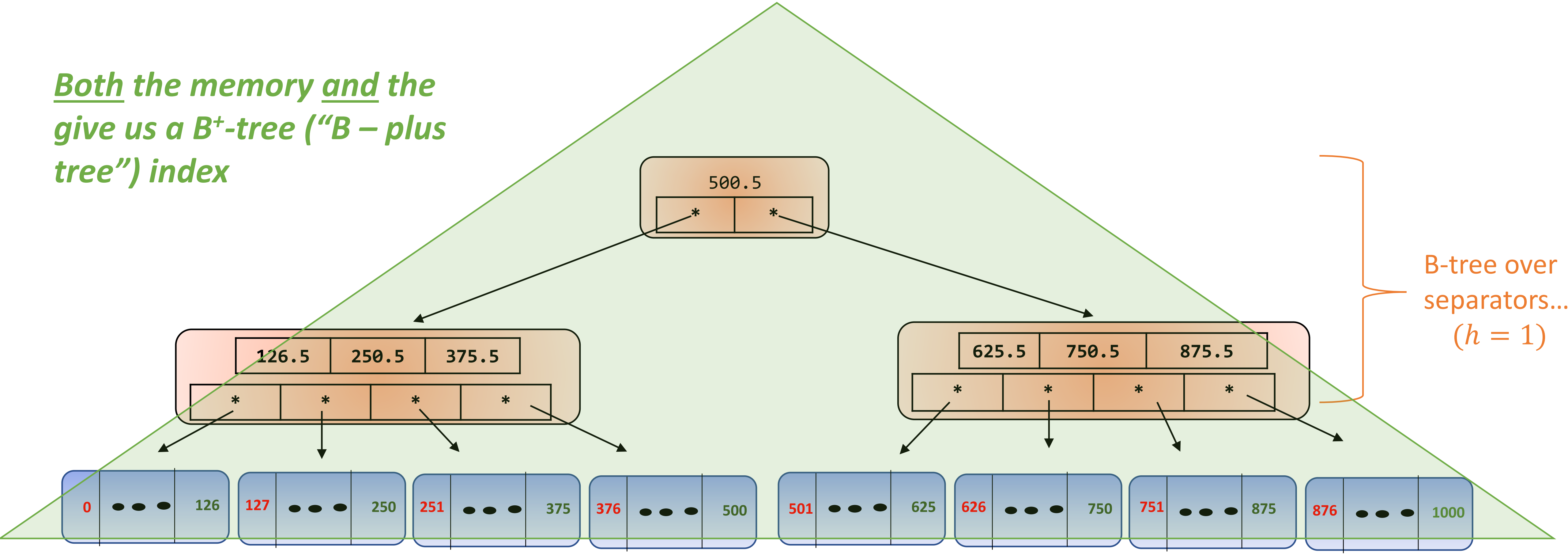
Both the memory and the give us a B+-tree ("B – plus tree") index



Comparison to BST index

- Spatial Cost of B^+- Tree index = 2 * leaf_node_cost + root_cost
- Leaf_node_cost = 3 * `sizeof(float)` + 4 *`sizeof(ref)` = 3 * 4 + 4 * 8 = 44 bytes
- root_cost = sizeof(float) + 2 * sizeof(ref) = 4 + 16 = 20 bytes
- So Spatial Cost = 2 * 44 + 20 = 104 bytes < 140 bytes which was the cost for the BST index!
  - Wait till you see the spatial benefit in larger databases…

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- Suppose my page size is 4KB (standard for most commercial PCs)

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The B-Tree component…

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$\approx 128KB$

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3. So $8 \times 64 = 8^3 = 512 = 2^9$ pages total
4. Therefore, $DB_{size} = 2^9 * 2^2\ KB = 2MB$

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While our B+-trees spatial cost is:

$(8 * 8 + 7 * 4)(1 + 8 + 8^2) =$ $6717\text{B} = 6.717KB \approx$ 0.336% of DB_{size}!

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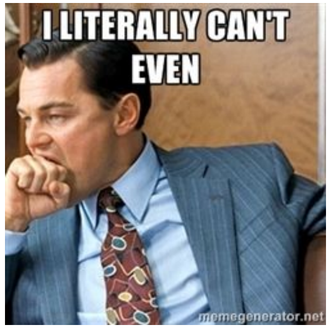
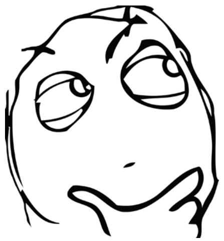


0.289% of DB SIZE!!



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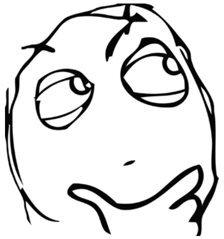
- For the same size DB, what would the size of a BST-based index be?



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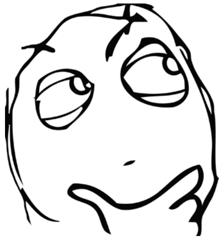

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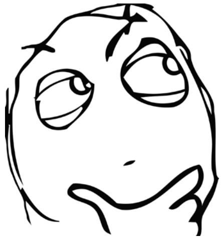

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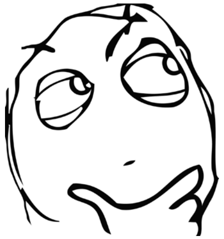

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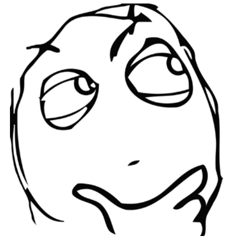

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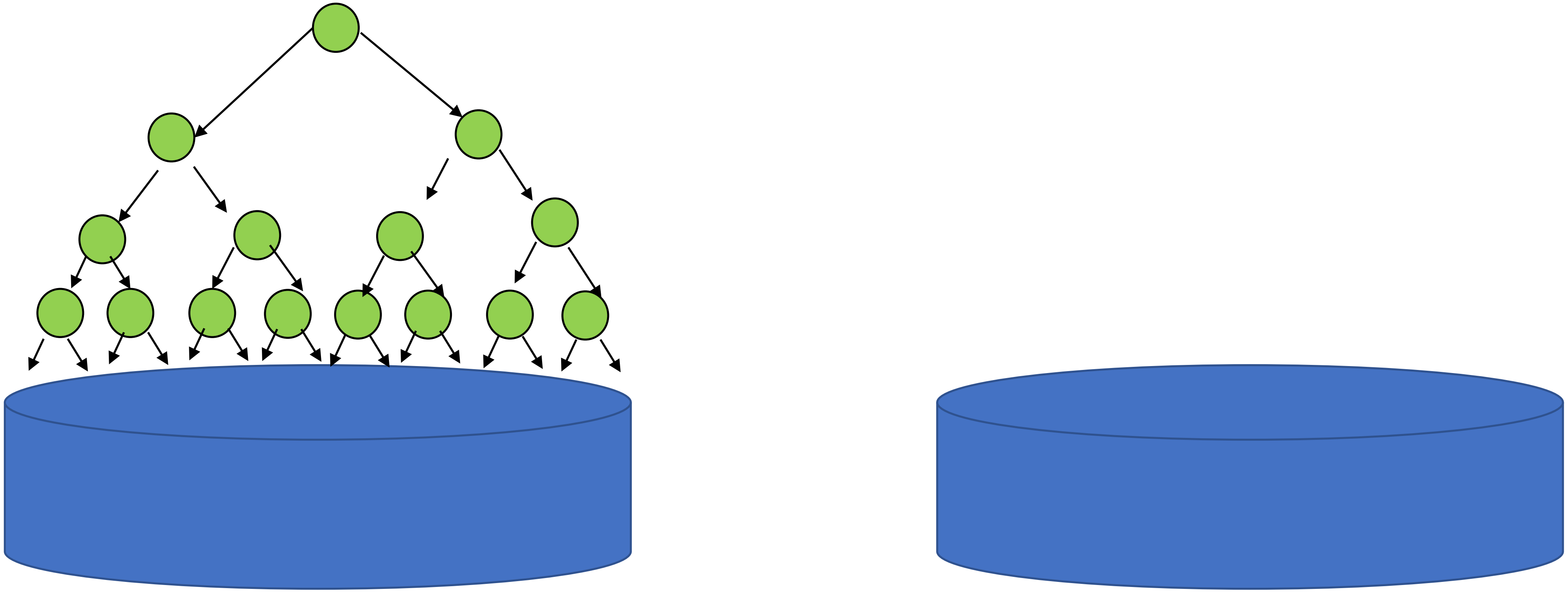
Total cost= **5,368,709,100 bytes = 5.3687091 GB** $\approx$ **0.502% of DB Size!**

So why does this happen?

- **Mathematical reason #1:** Because you have many leaves ℓ, and a small value of $p = 2$, so you need a large value for h to satisfy $\ell = p^h$!
  - This leads to a large height for the tree and an increased number of (large) summands in the sum of the geometric progression!
- **Mathematical reason #2:** Because the sum of the geometric progression when the value of h is large has many more (exponential) terms!
- **Intuitively**: Because, to cover a large DB, binary trees are **too tall for their own good**: a significant number of their subtrees can be collapsed into B-Tree nodes with appropriate separators.
  - Several subtrees are thus **redundant** and add to an **intractable storage cost for the index.**

Mnemonic rule

- When indexing into a large DB….



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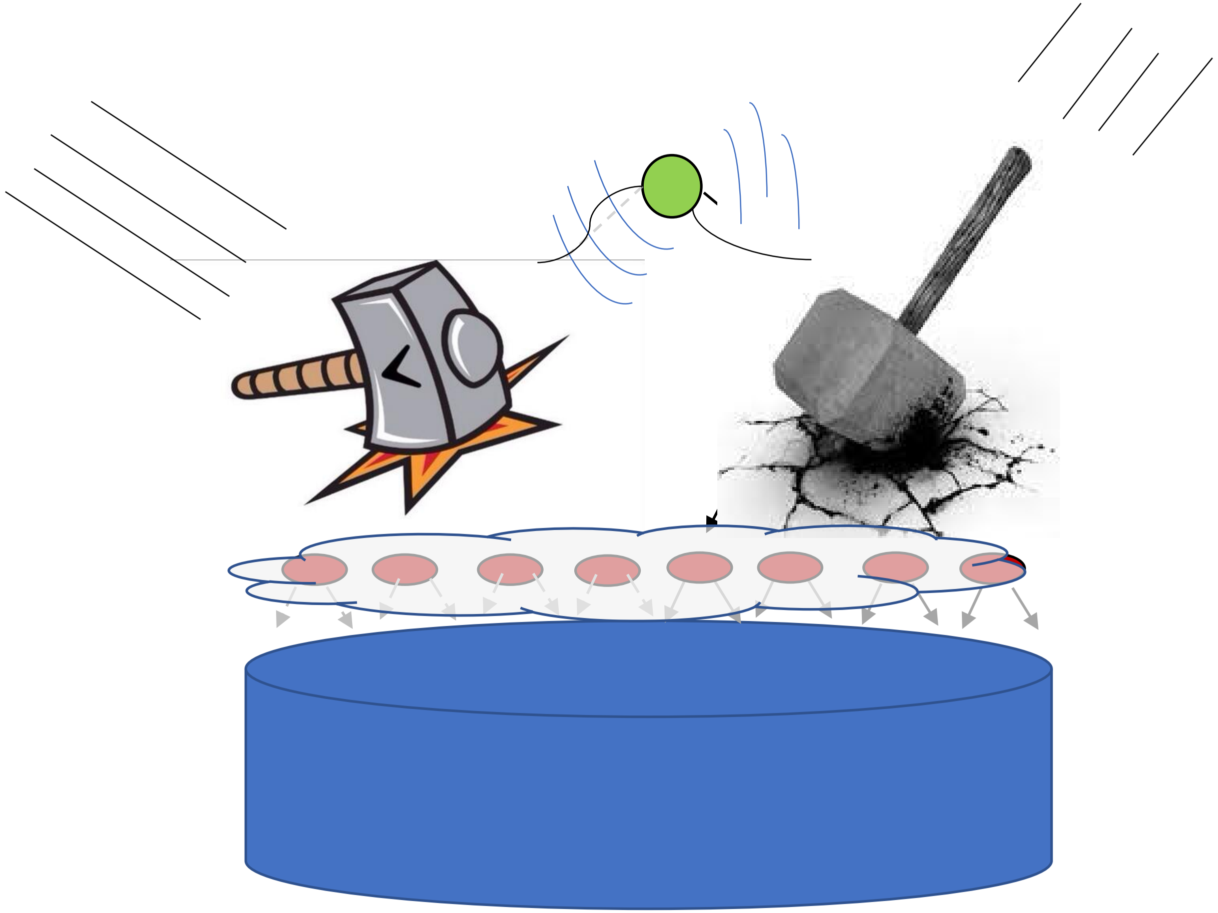
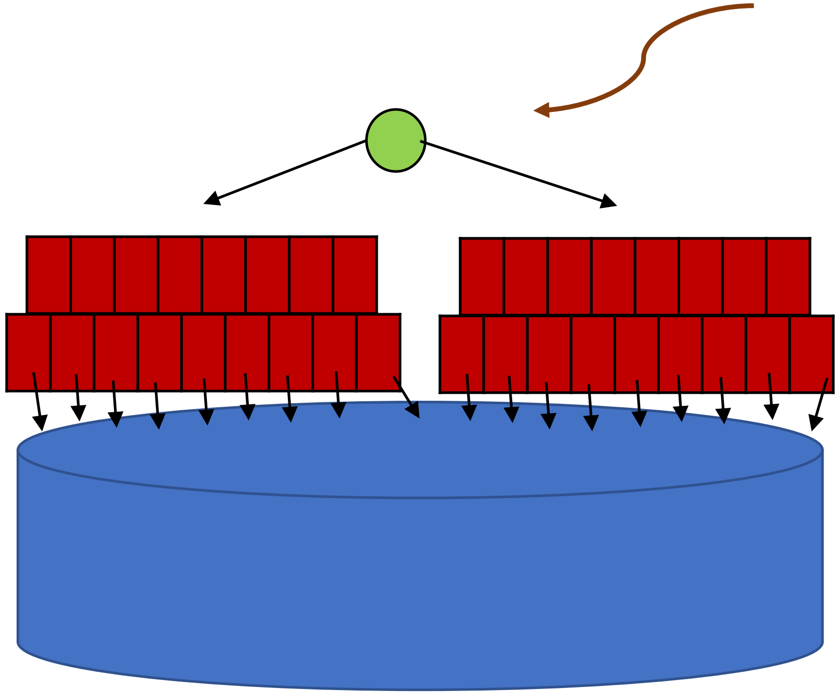


Go short and fat!



What about different page sizes?

- Oracle's UltraSPARC07 arc defines 8KBs of page size.
- *How does that affect the binary tree index's size?*
- Again, original size of data = 2^{30}KB
- Since page size is 8KB, there are $2^{30}/_8 = 2^{27}$ pages
- So we need 2^{26} leaves, for a tree of height 26 (just one less… ☹)
  - So we're not helped much ☹

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- What about the B-Tree of fanout $p = 4$?
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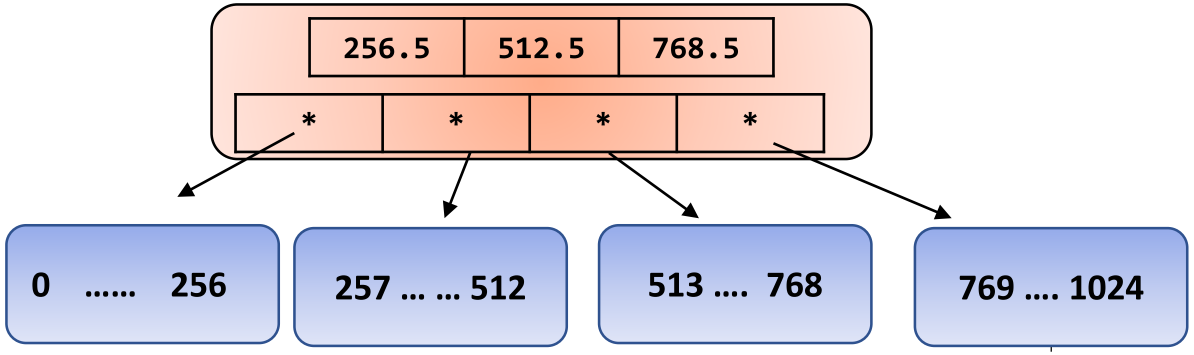

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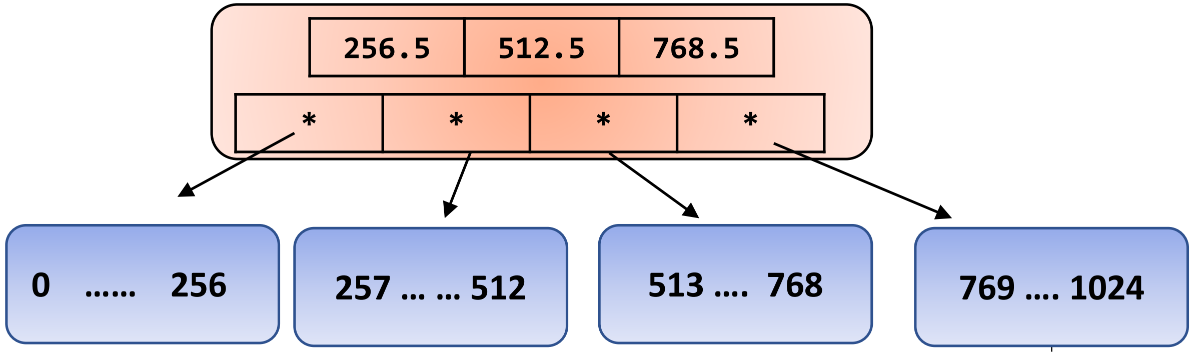

Memory cost = 3 * `sizeof(float)` + 4*`sizeof(ref)` = 44 bytes = 0.1375% of DB size

Deciding on fan-out and height

- Suppose that we have a database D of size $|D| = 4\ GB$, stored in a drive with page size $PS = 4KB$.
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- But! $\ell = p^h \Rightarrow h = \log_p \ell = \log_8 142857 = 5.7 \Rightarrow h = 6$

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0.675% of DB_Size!

Formulae

- Given the size of the database, D, **<u>in GB</u>**, and the page size PS **<u>in KB</u>**, we have #pages pg:

$$pg \leftarrow \frac{D \times 10^6}{PS} \quad (1)$$

- There are p pages pointed to by a leaf, so the #leaves ℓ:

$$\ell \leftarrow \frac{pg}{p} \overset{(1)}{=} \frac{D \times 10^6}{PS * p} \quad (2)$$

Formulae

- But since the tree is built bottom-up, it is a "perfect" p-tree, so we know that

$$\ell = p^h \qquad (3)$$

- Combining (3) with (2) allows us to connect the crucial parameters p and h in one formula:

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Given p

$$\boxed{h = \log_p \frac{D \times 10^6}{PS} - 1}$$

Given h

$$\boxed{p = \sqrt[h+1]{\frac{D \times 10^6}{PS}}}$$

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- Given both p and h and assuming 4-byte separators and 8-byte references, we have:

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Cost of node:
$12p - 4 = 4(p - 1) + 8p$

Number of nodes:
Sum of geometric progression

Applying our formulae

- $p^{h+1} = \frac{D \times 10^6}{PS}$ (identity that connects p and h)
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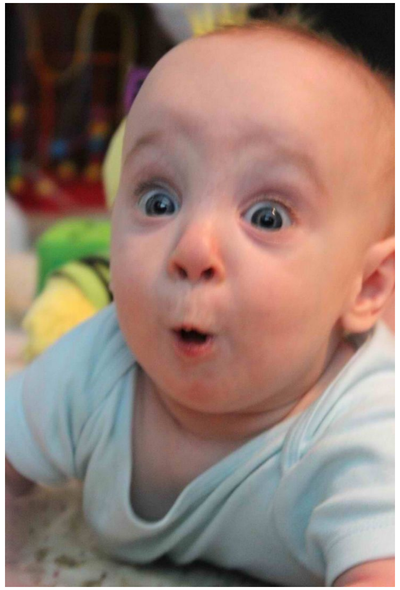
- Let $D = 1TB$ and $PS = 4KB$. Suppose we want $h = 3$. Then,

$p^4 = \frac{D \times 10^6}{PS} \Rightarrow p^4 = \frac{10^9}{4} = 250 * 10^6 \Rightarrow p \approx 126$ and our spatial cost is $(12 * 126 - 4) * \frac{126^4 - 1}{126 - 1} = 3{,}040{,}699{,}532$ bytes $\approx 3.04\text{GB}$

Applying our formulae



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(Expected, the height increased!)