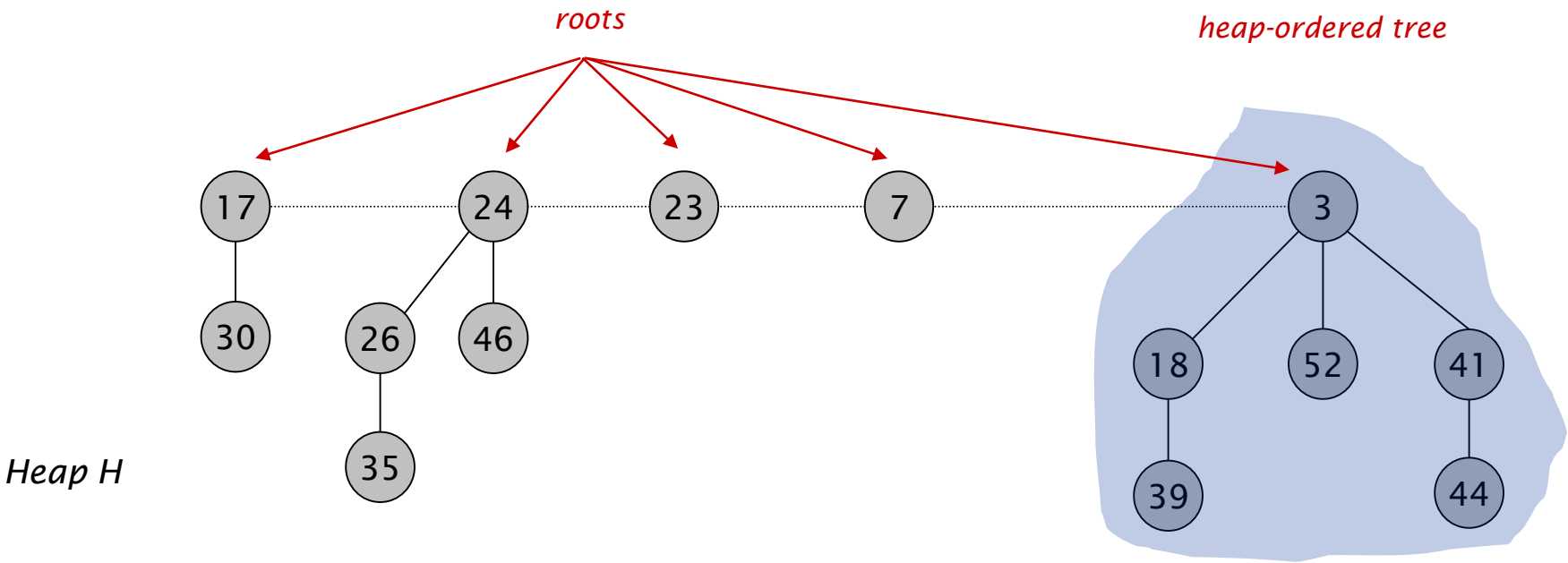
Fibonacci Heaps
from Princeton

Fibonacci Heaps: Structure

each parent smaller than its children

Fibonacci heap.

- Set of heap-ordered trees.
- Maintain pointer to minimum element.
- Set of marked nodes.



Fibonacci Heaps: Structure

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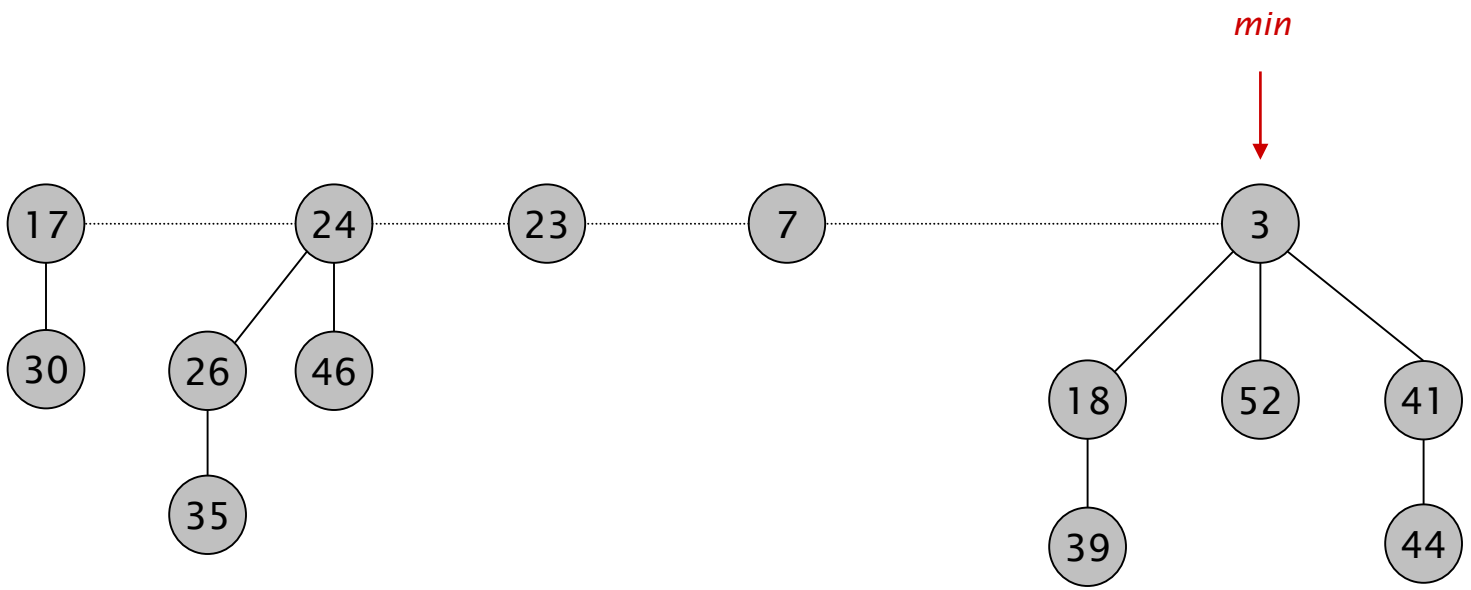
find-min takes O(1) time



Heap H

Fibonacci Heaps: Structure

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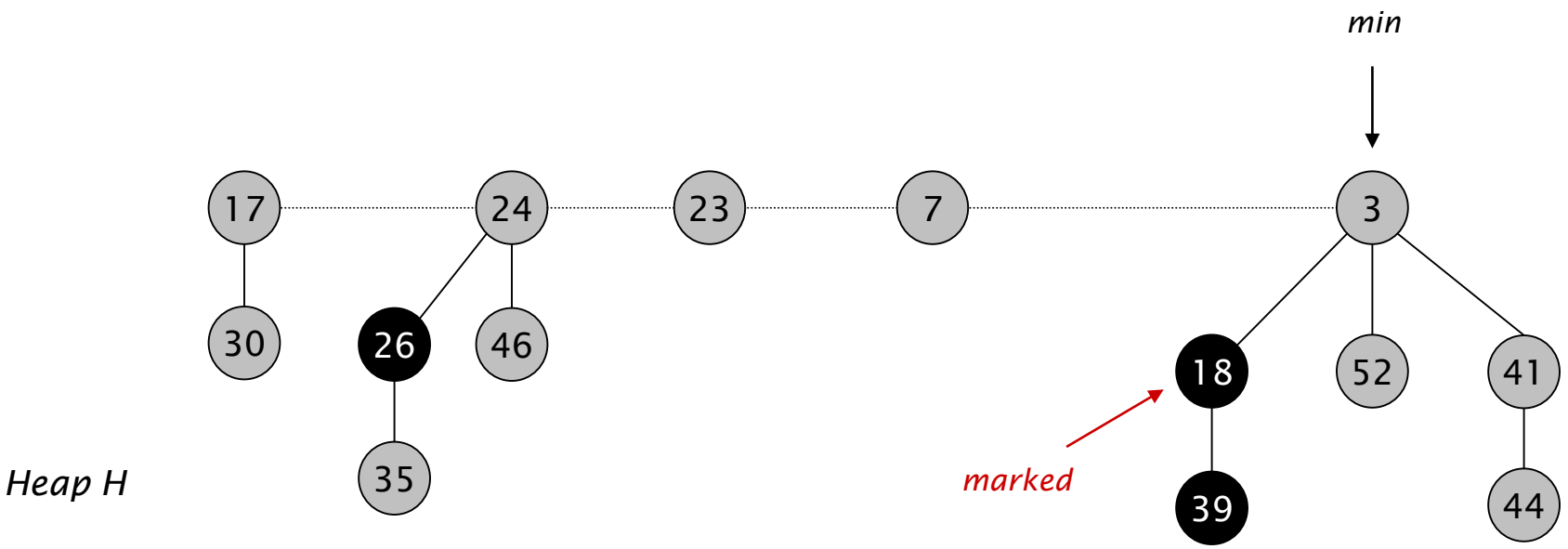
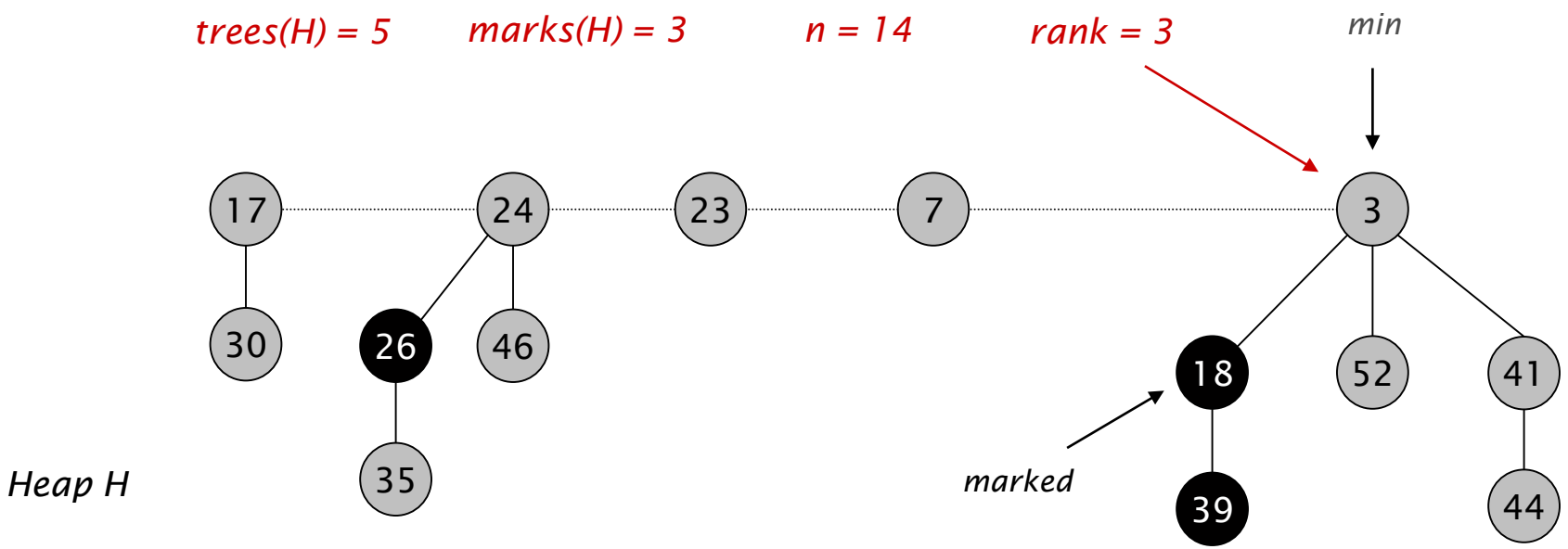
use to keep heaps flat (stay tuned)



Fibonacci Heaps: Notation

Notation.

- *n* = number of nodes in heap.
- *rank(x)* = number of children of node *x*.
- *rank(H)* = max rank of any node in heap *H*.
- *trees(H)* = number of trees in heap *H*.
- *marks(H)* = number of marked nodes in heap *H*.

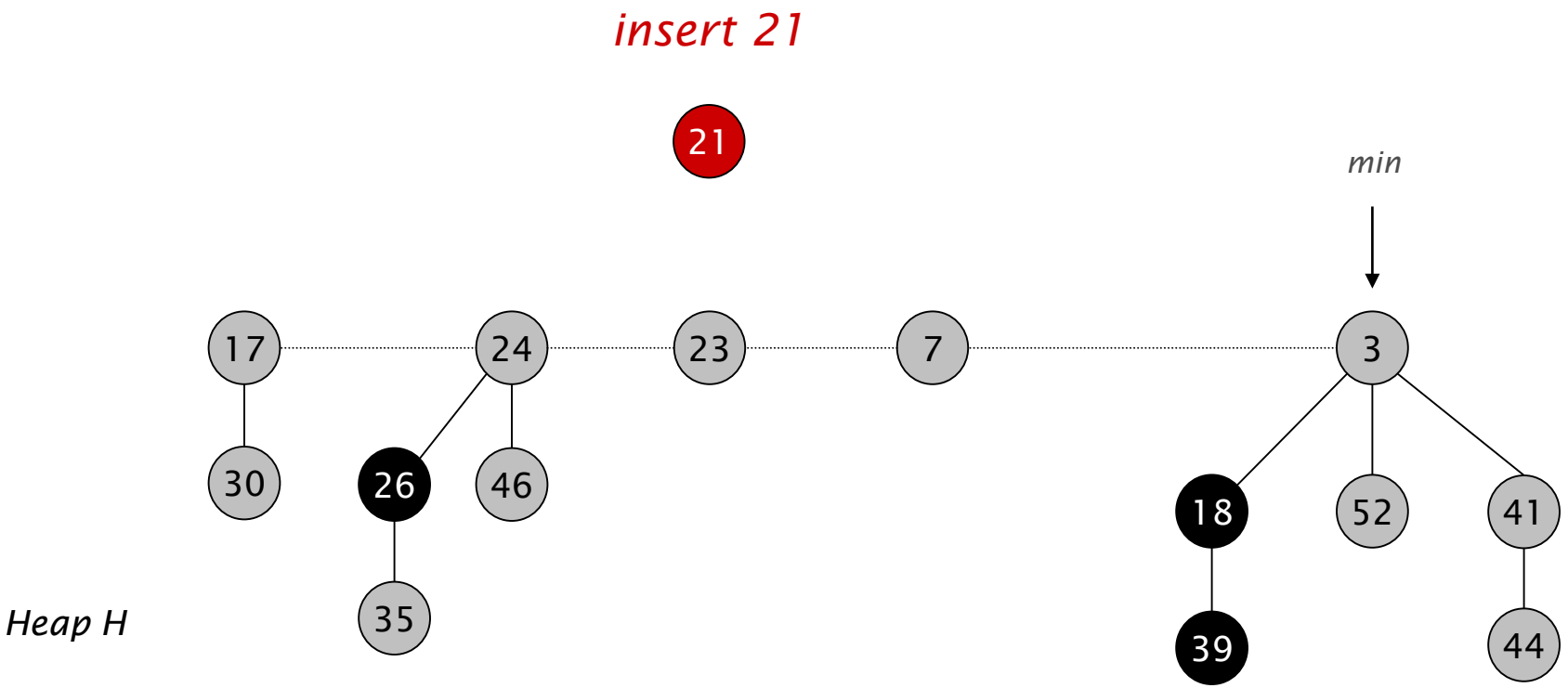


Insert

Fibonacci Heaps: Insert

Insert.

- Create a new singleton tree.
- Add to root list; update min pointer (if necessary).



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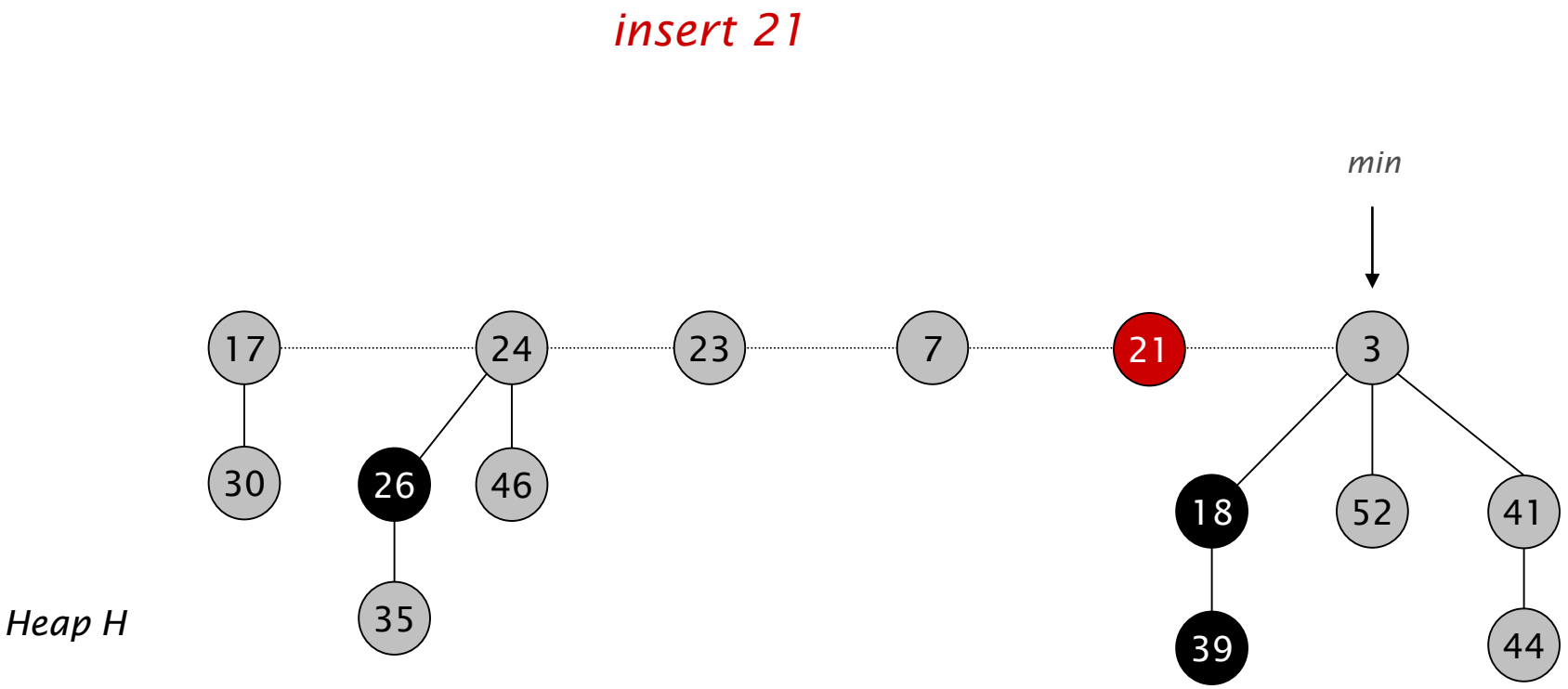
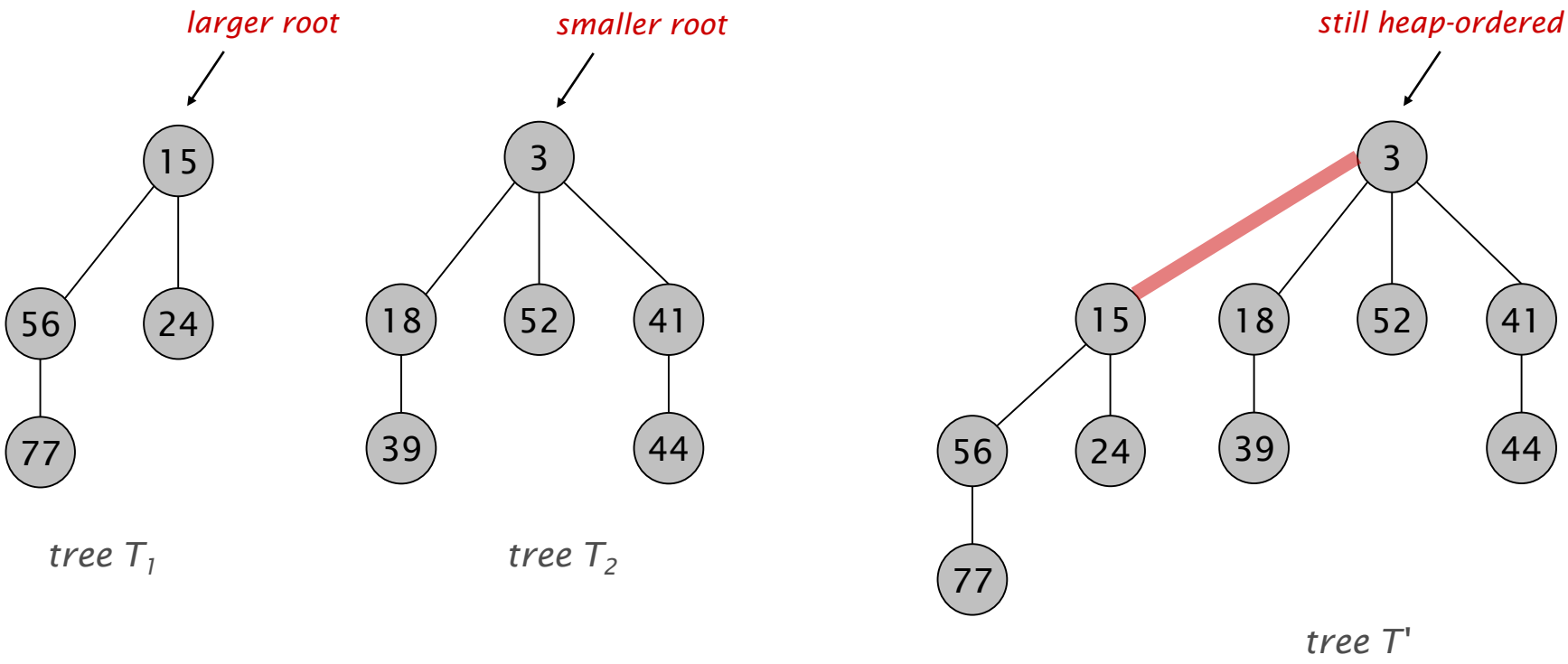


Delete Min

Linking Operation

Linking operation. Make larger root be a child of smaller root.



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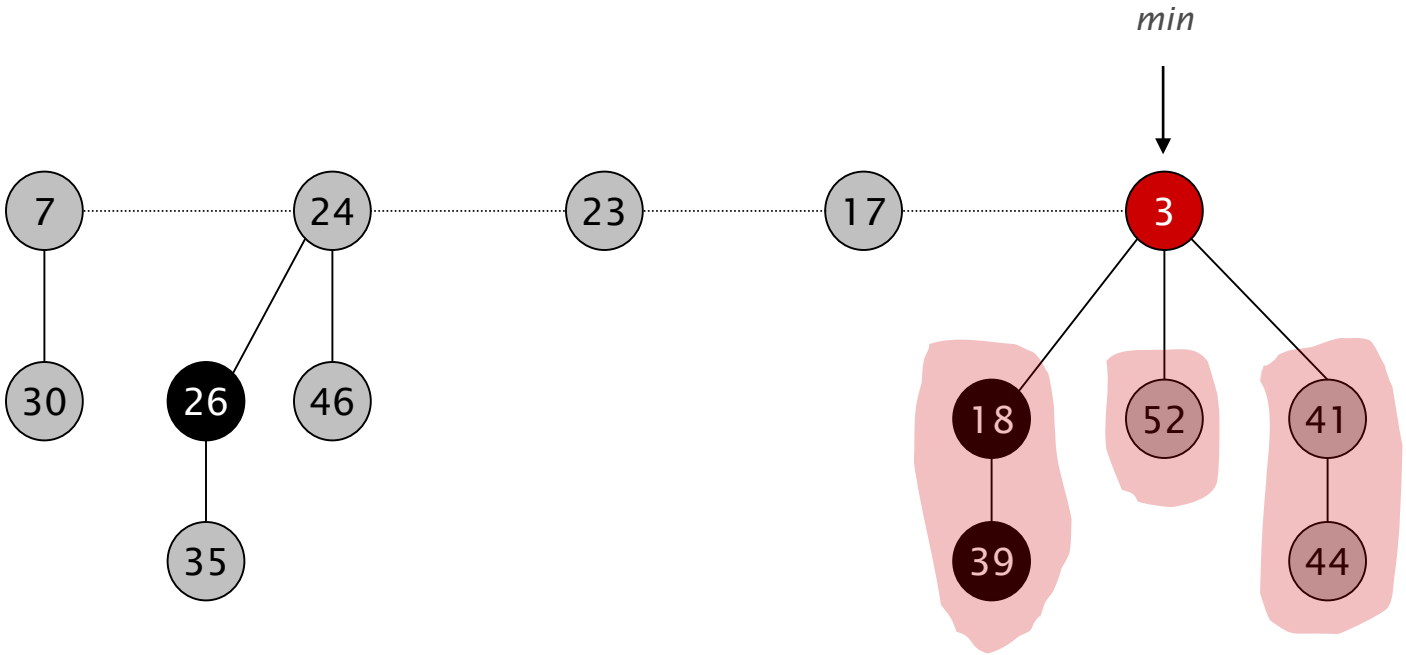
- Delete min; meld its children into root list; update min.
- Consolidate trees so that no two roots have same rank.



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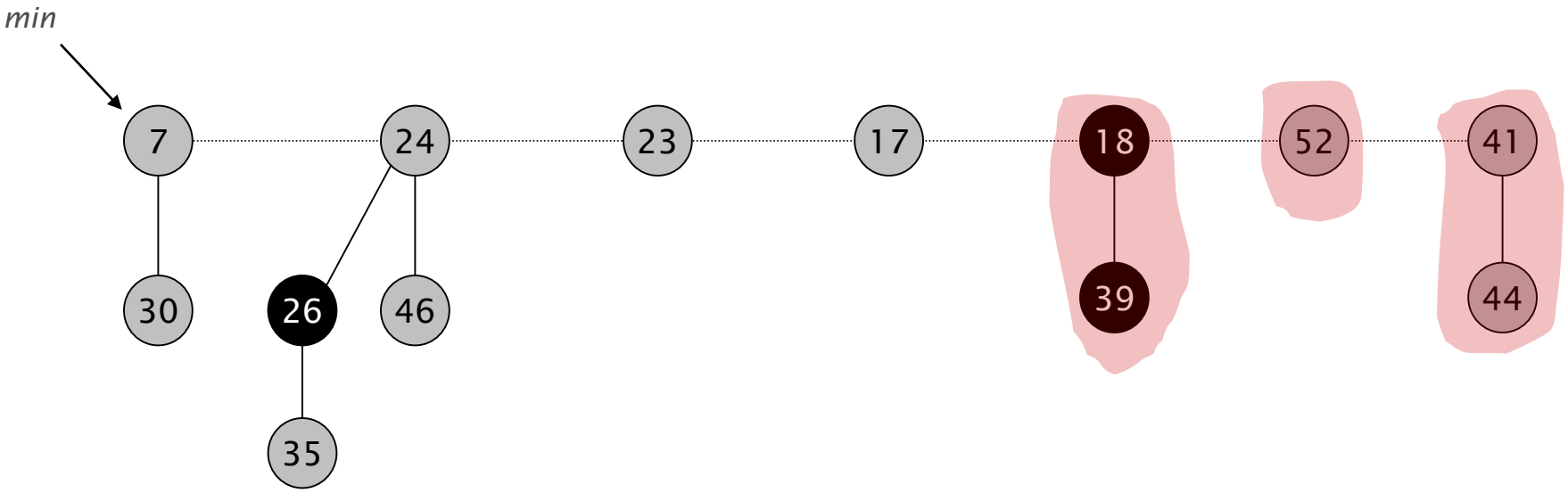

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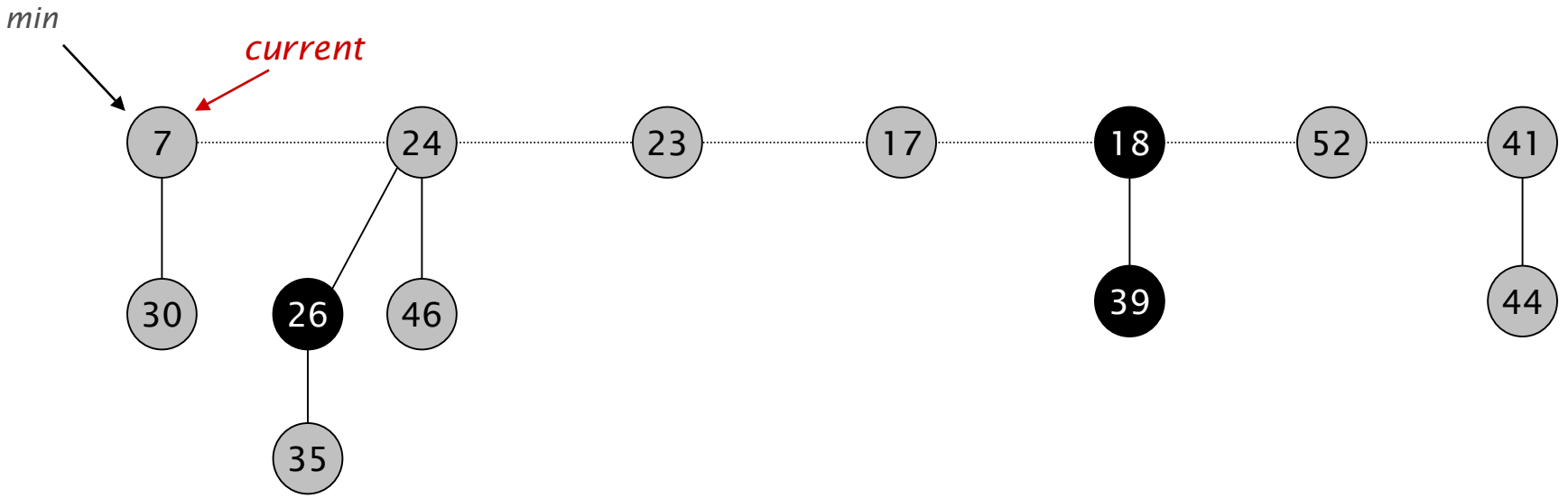

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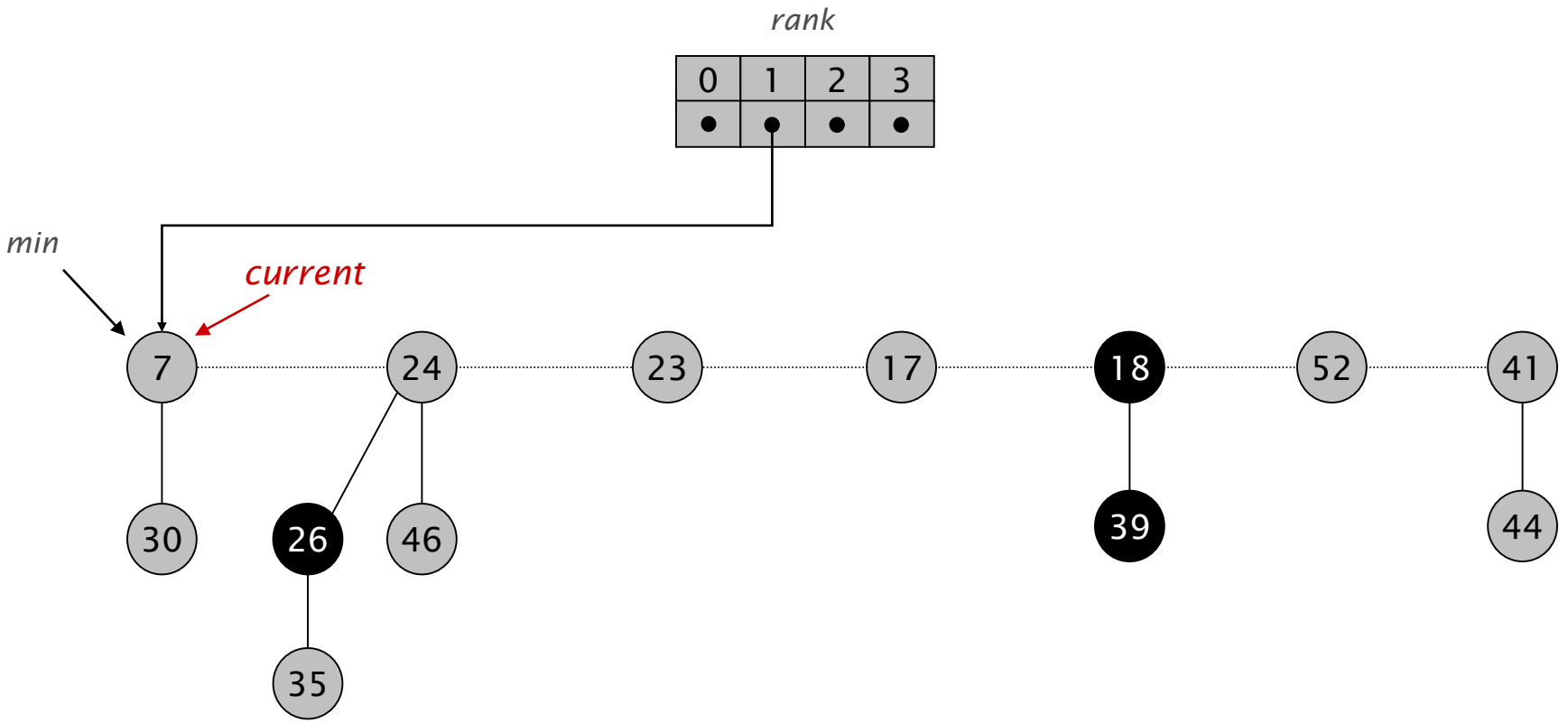

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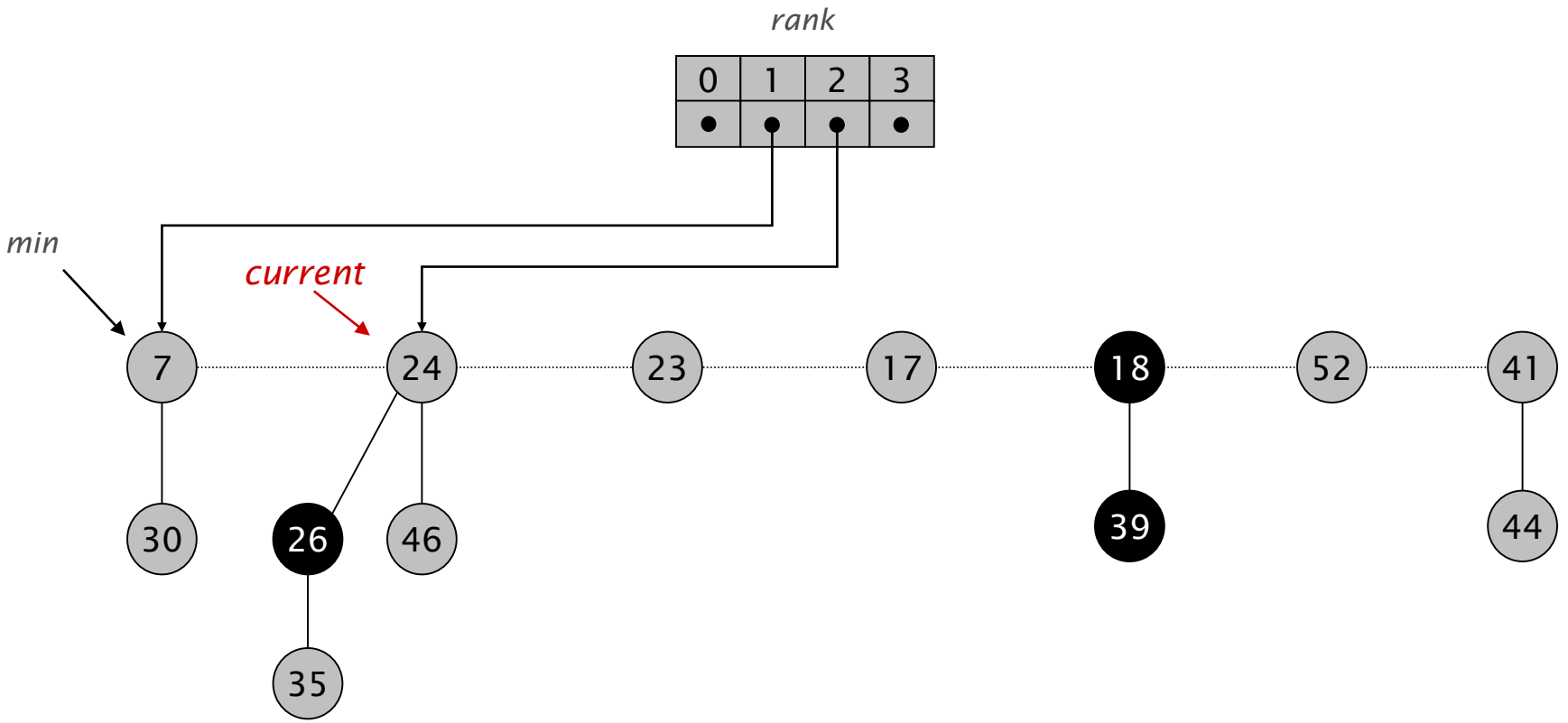

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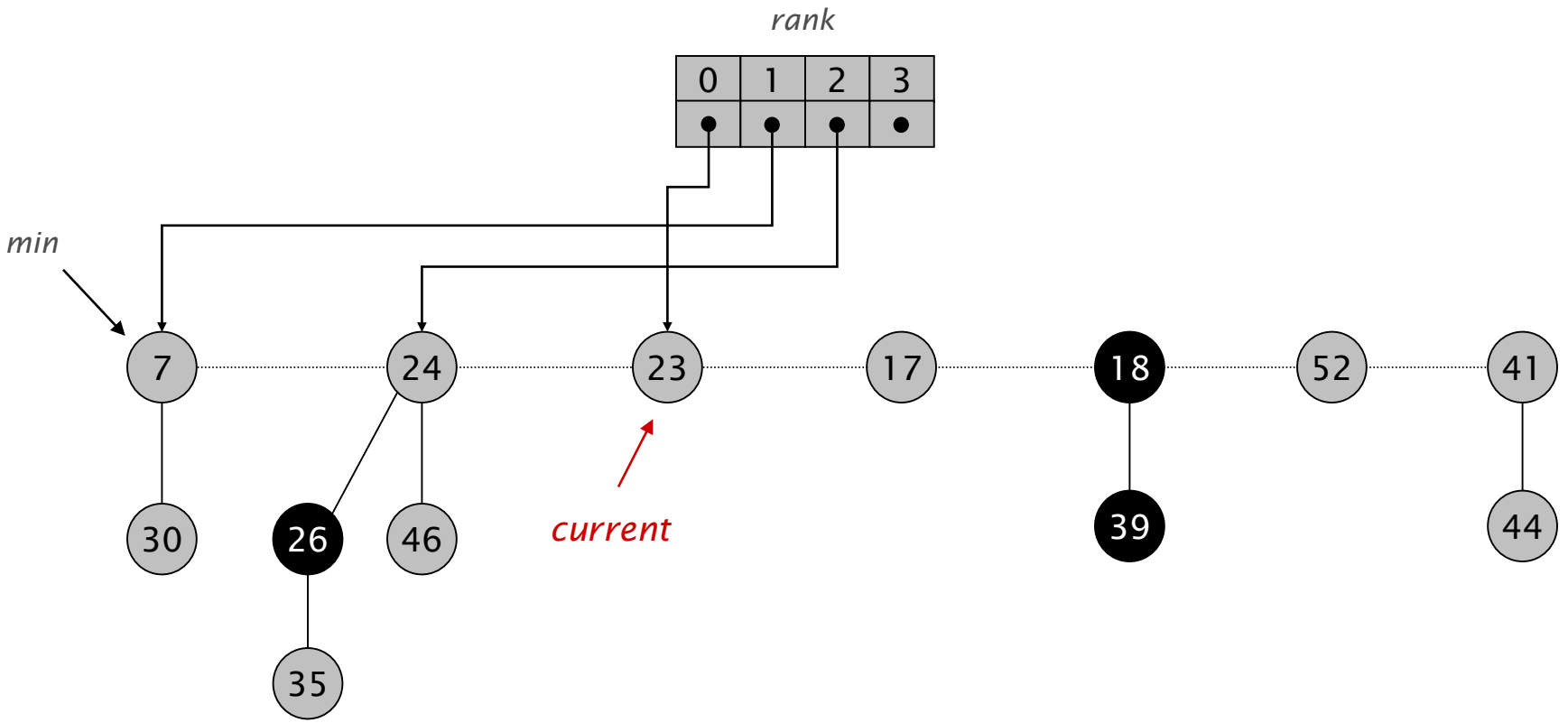

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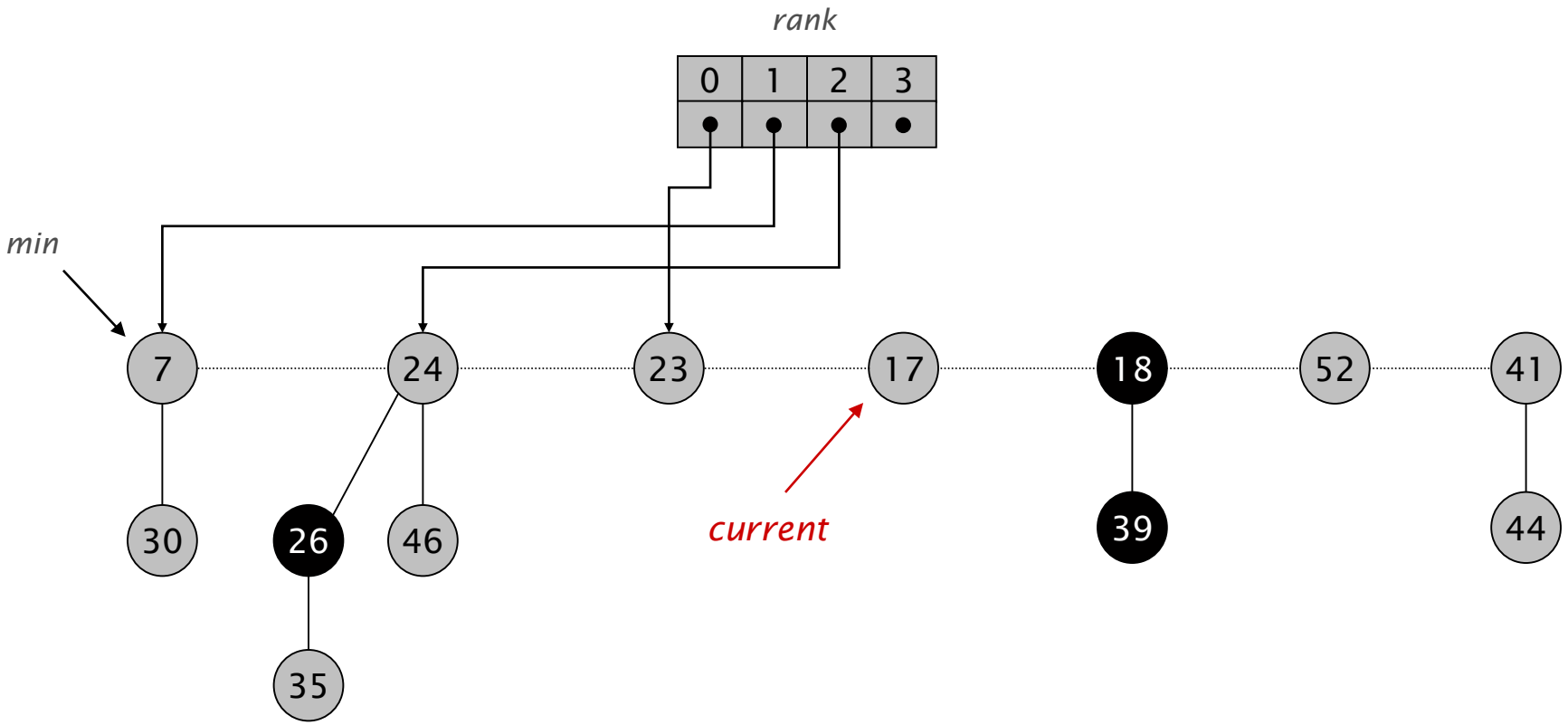

link 23 into 17

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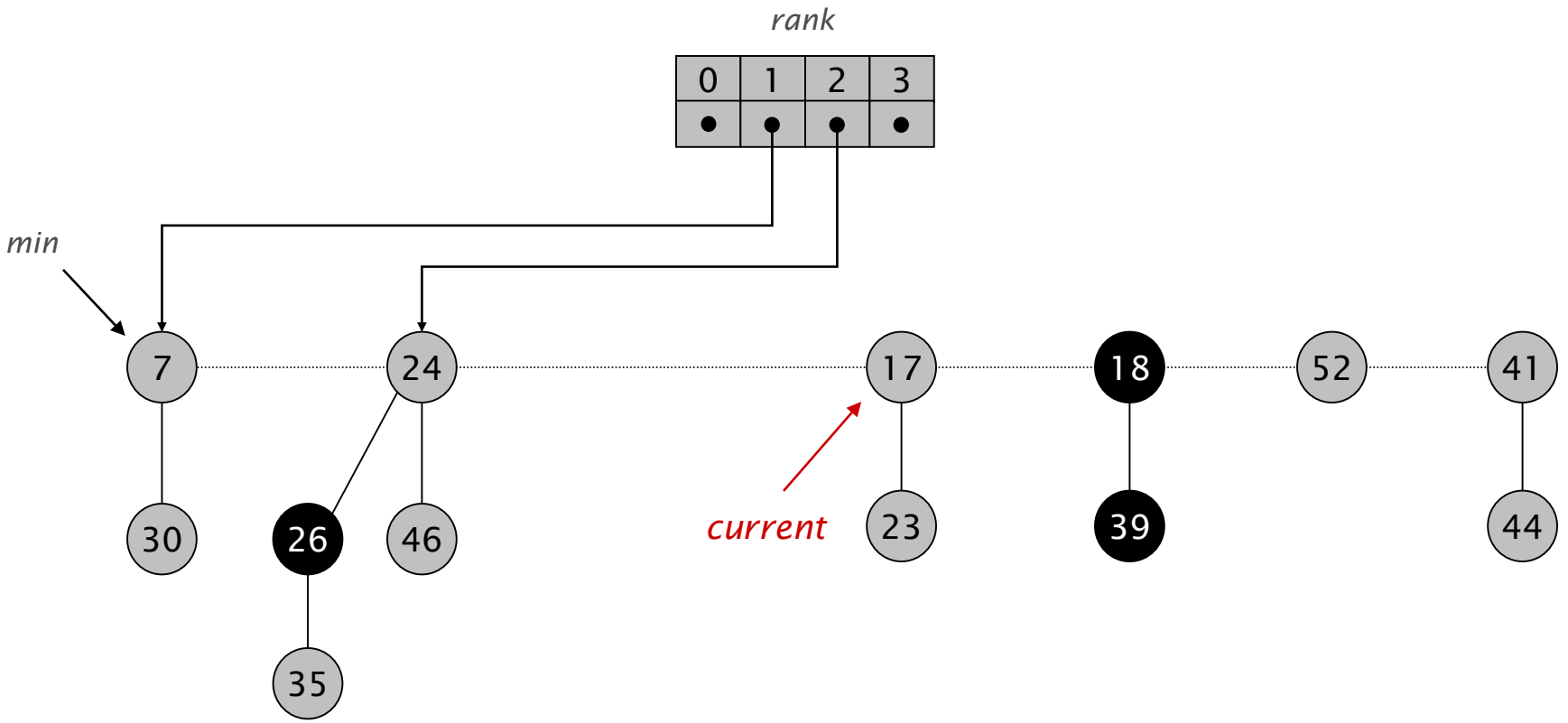

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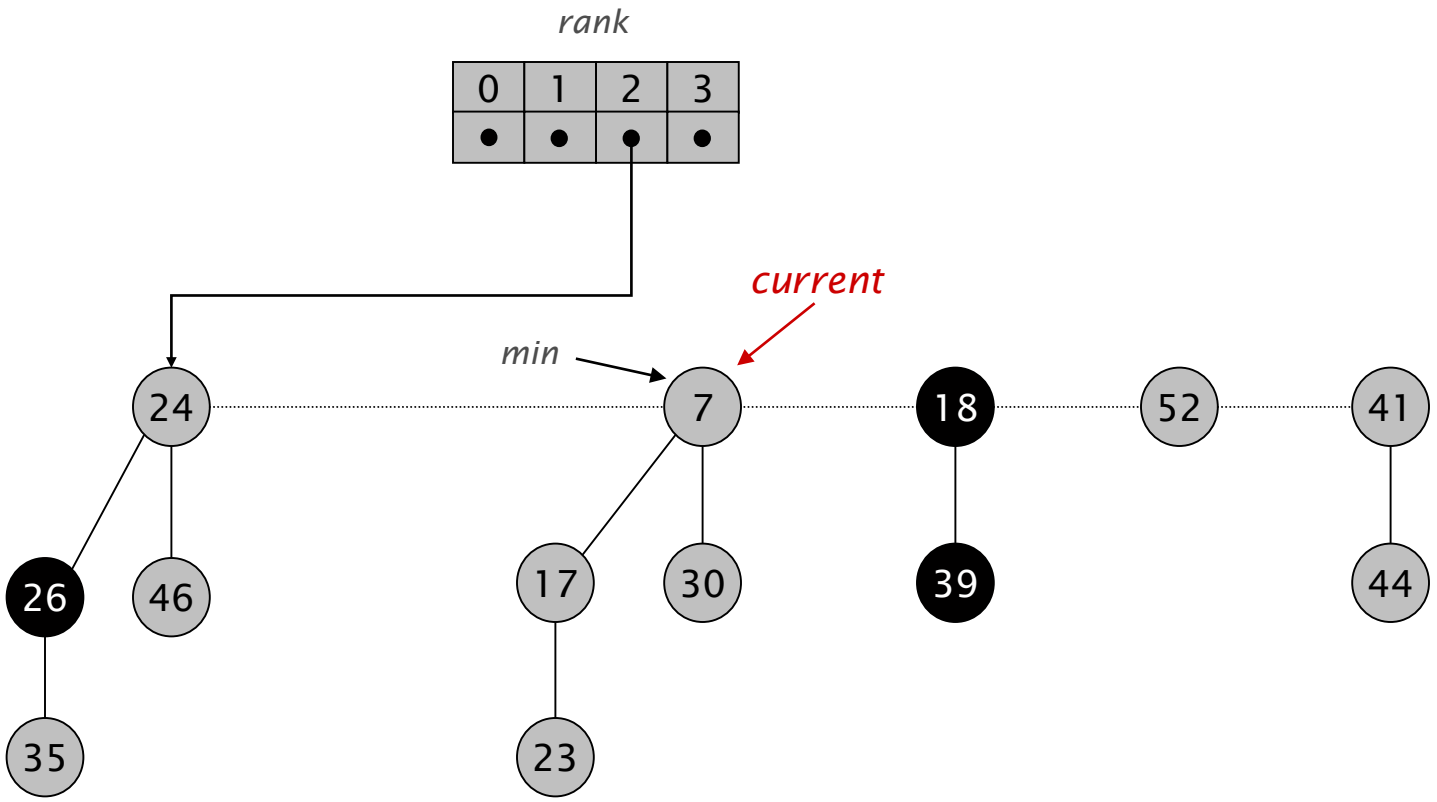

link 24 into 7

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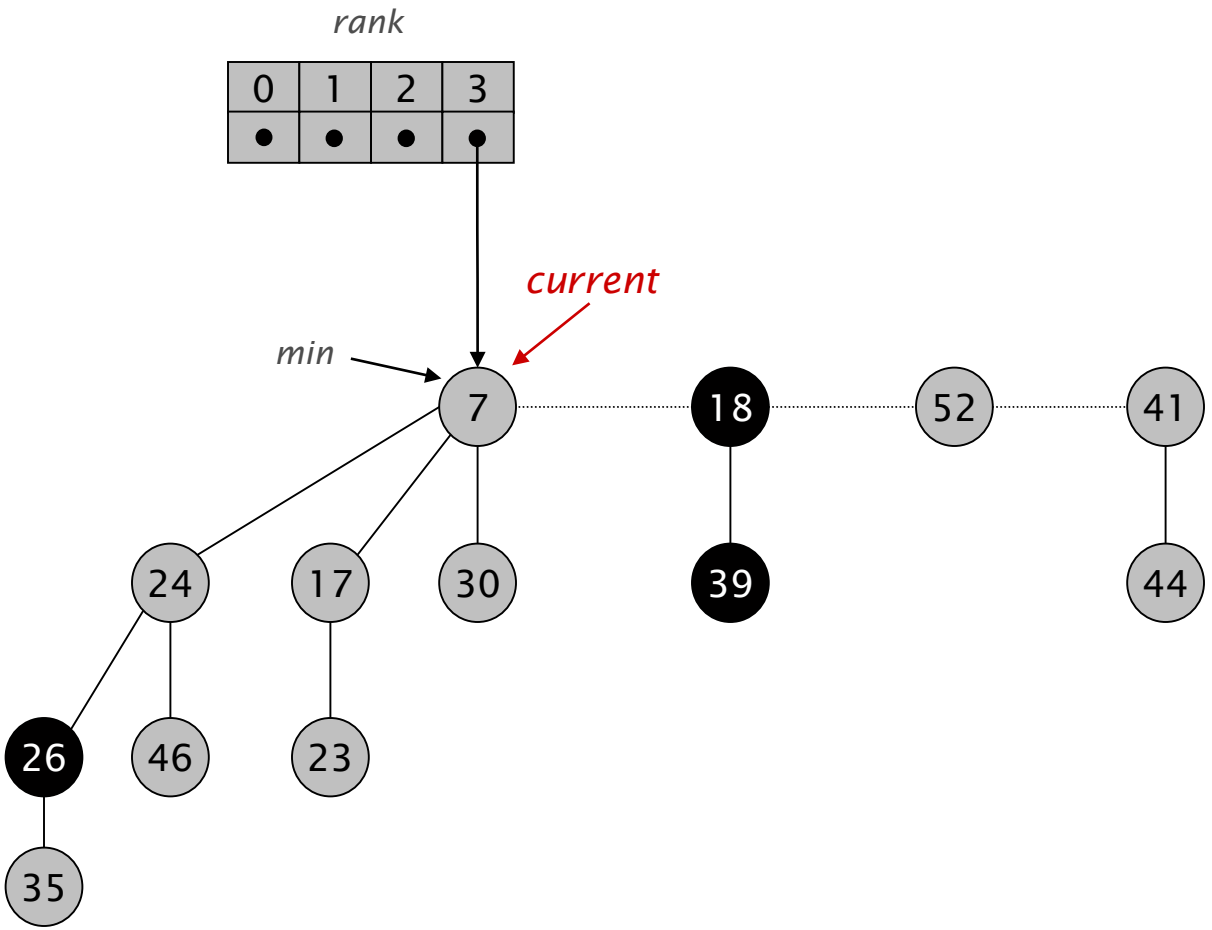

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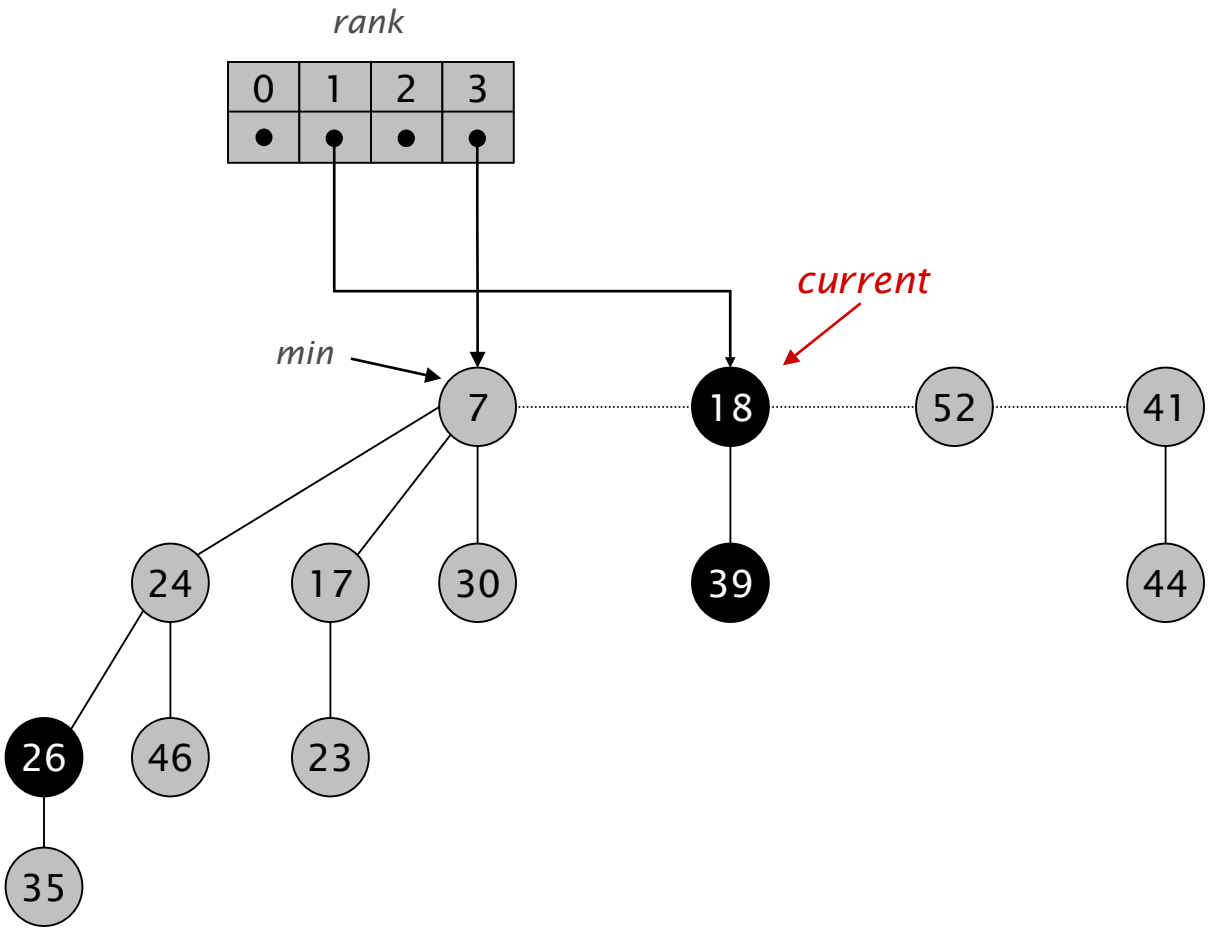

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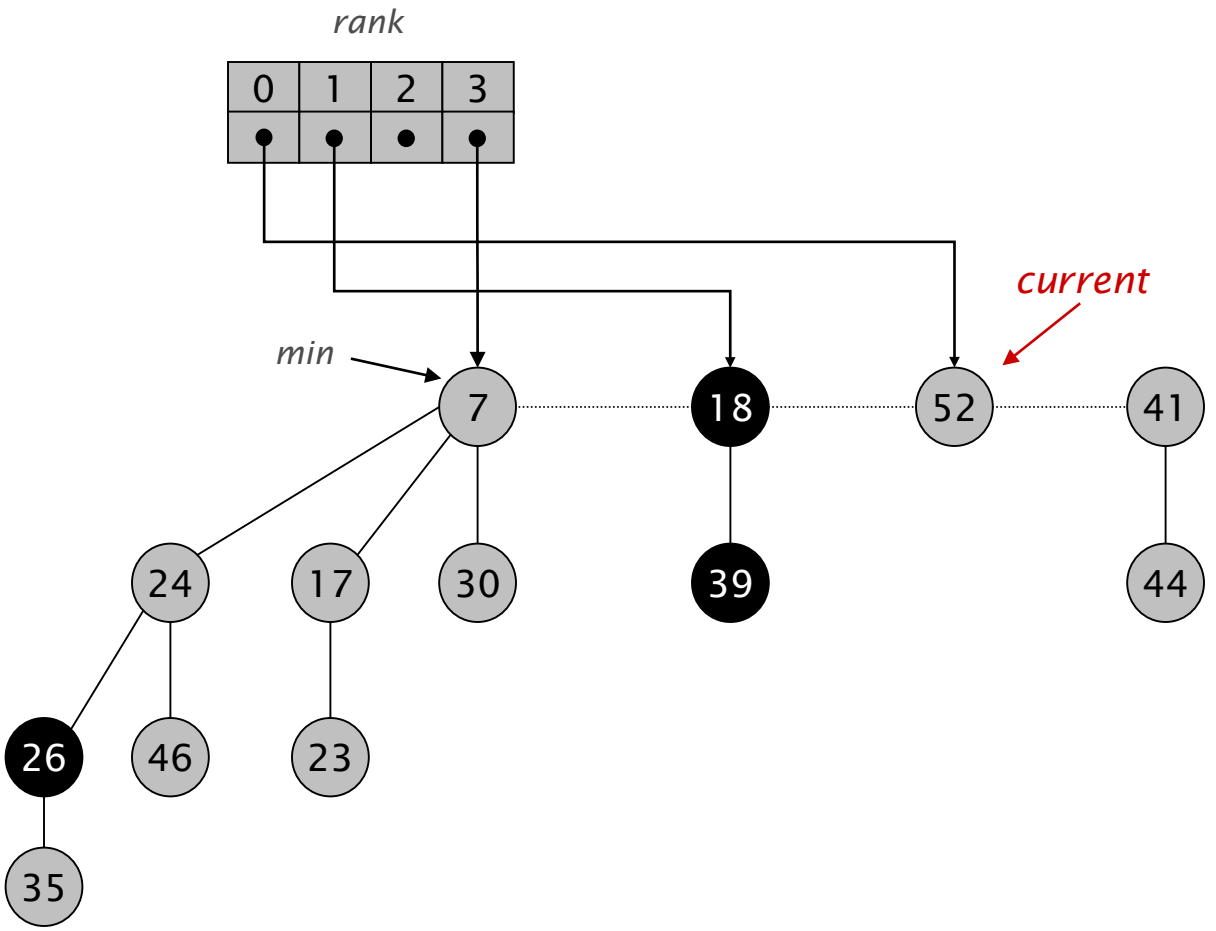

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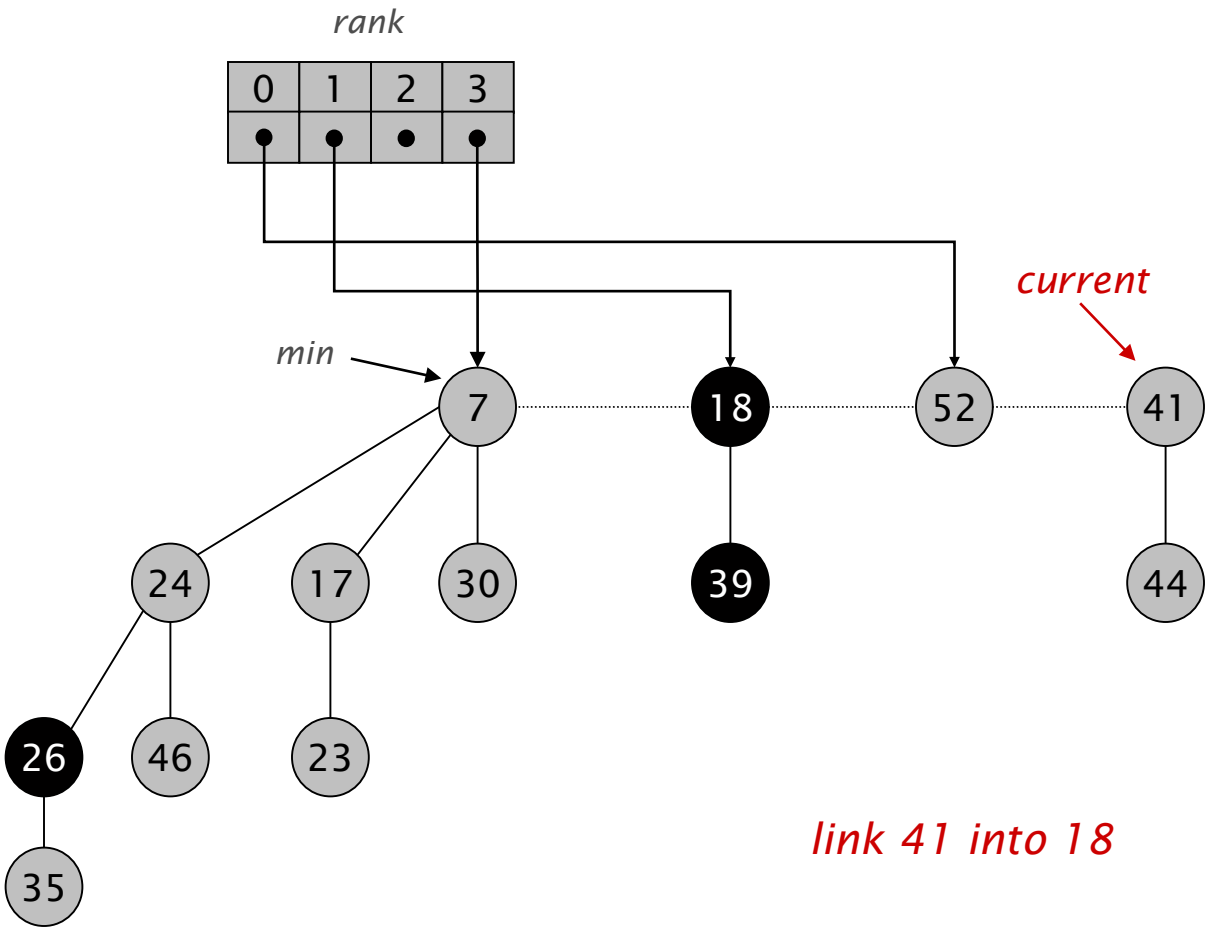

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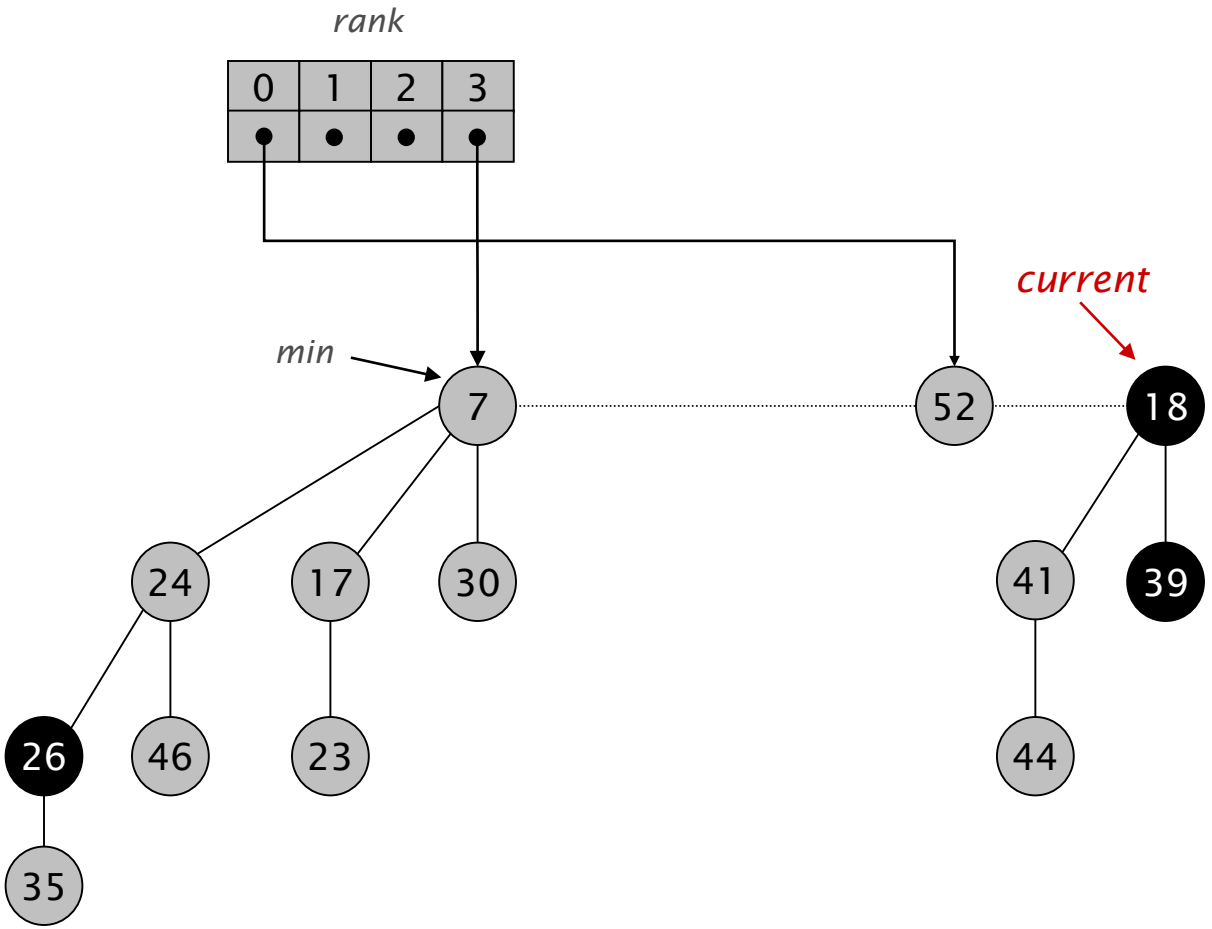

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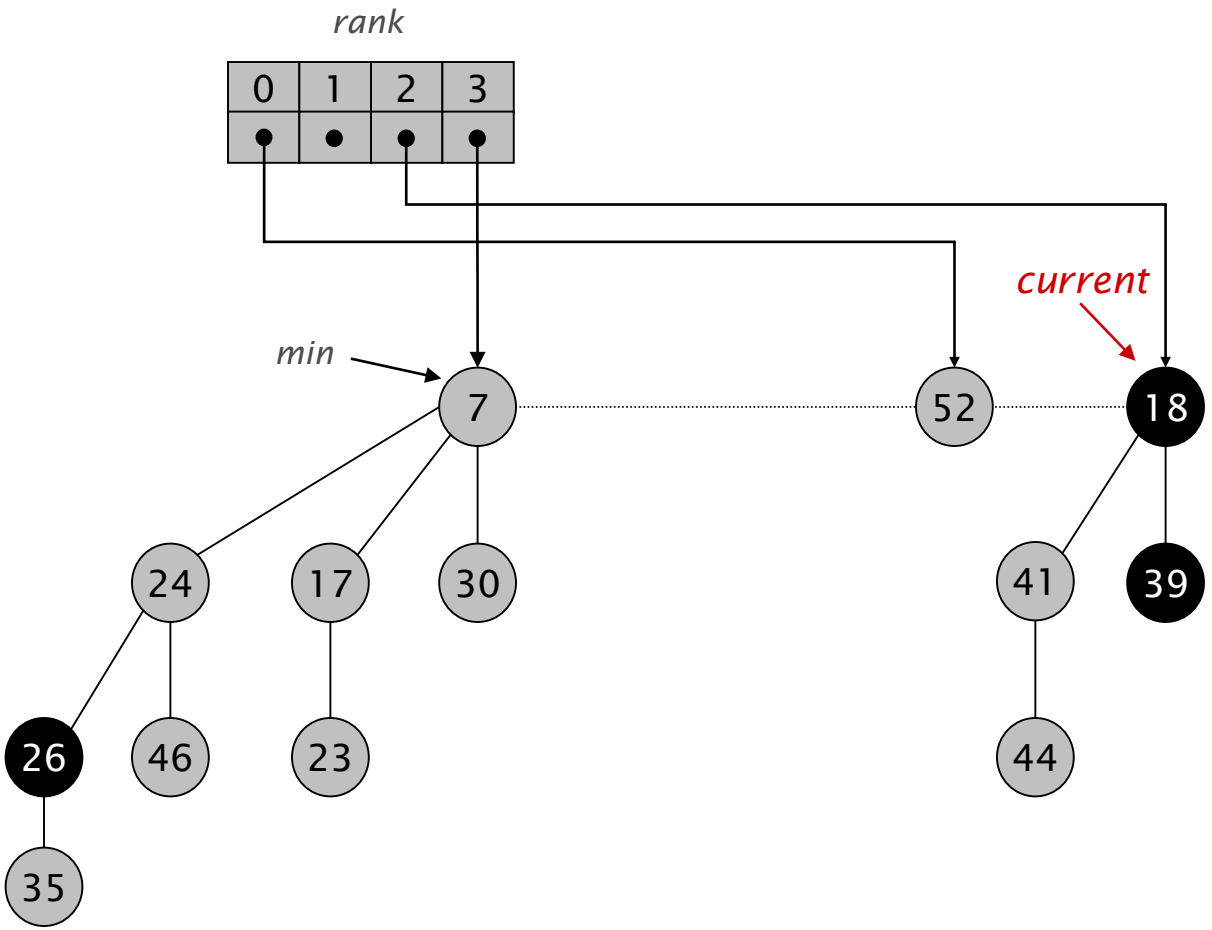

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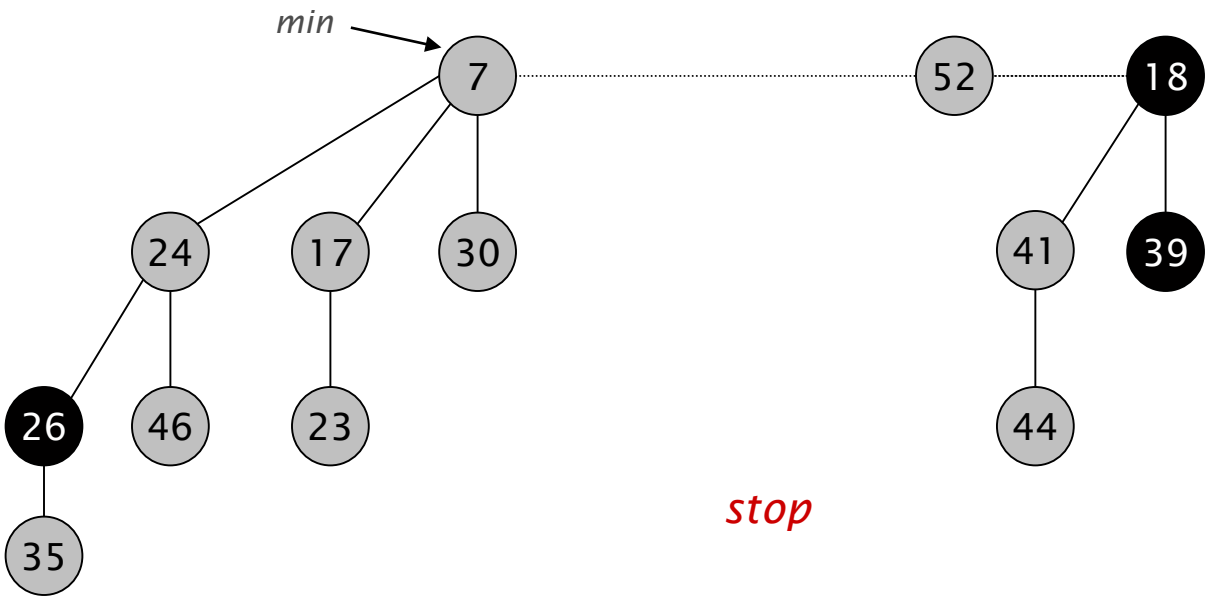
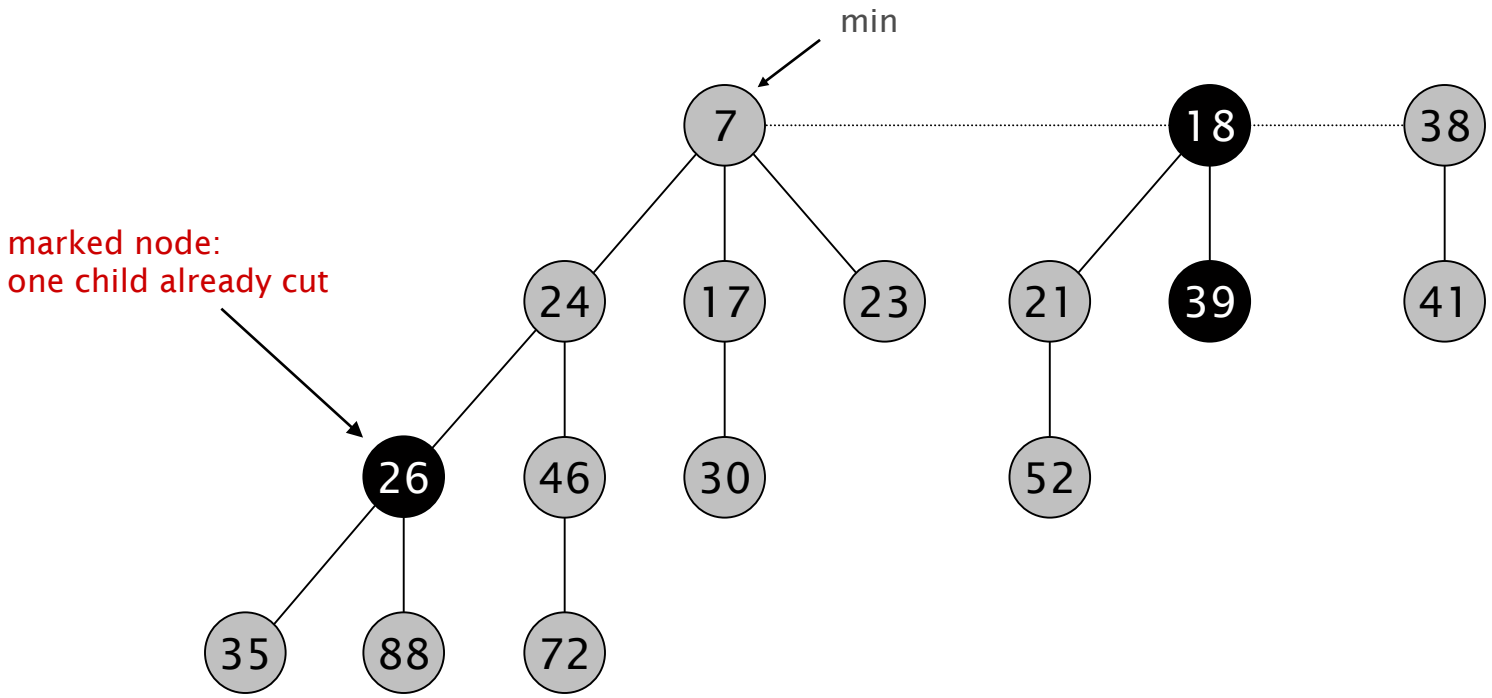


Decrease Key

Fibonacci Heaps: Decrease Key

Intuition for decreasing the key of node *x*.

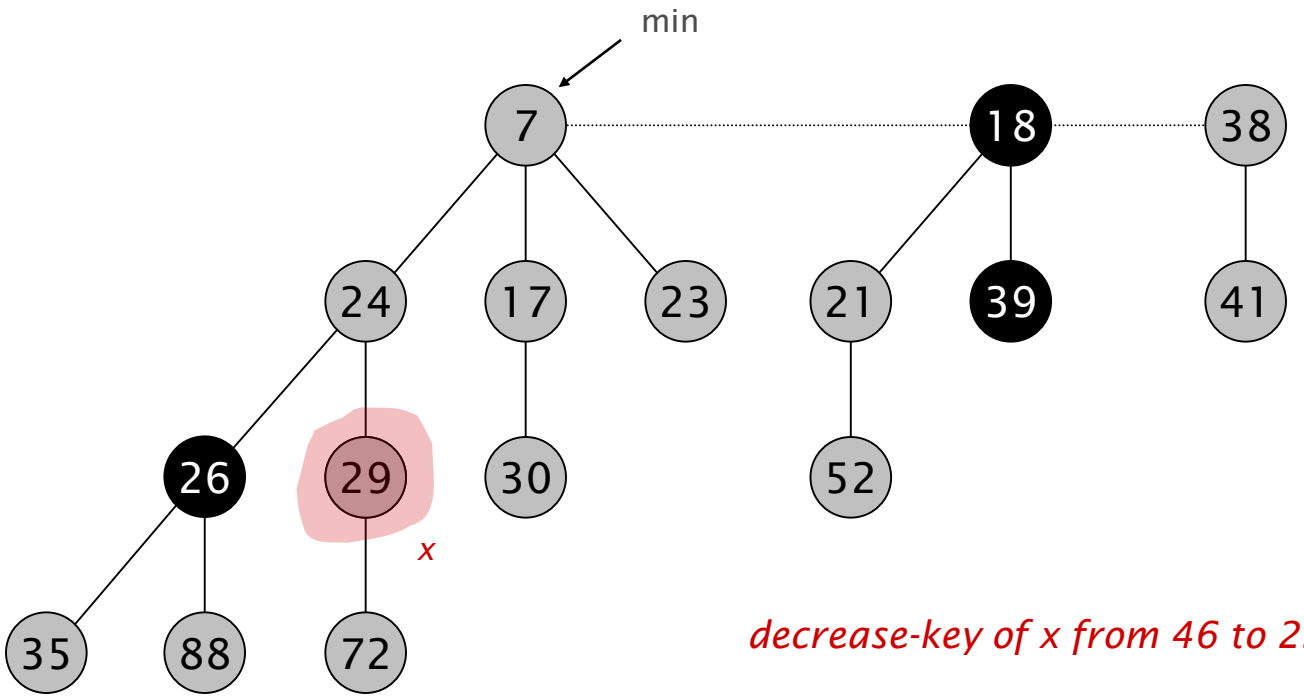
- If heap-order is not violated, just decrease the key of *x*.
- Otherwise, cut tree rooted at *x* and meld into root list.
- To keep trees flat: as soon as a node has its second child cut, cut it off and meld into root list (and unmark it).



Fibonacci Heaps: Decrease Key

Case 1. [heap order not violated]

- Decrease key of *x*.
- Change heap min pointer (if necessary).



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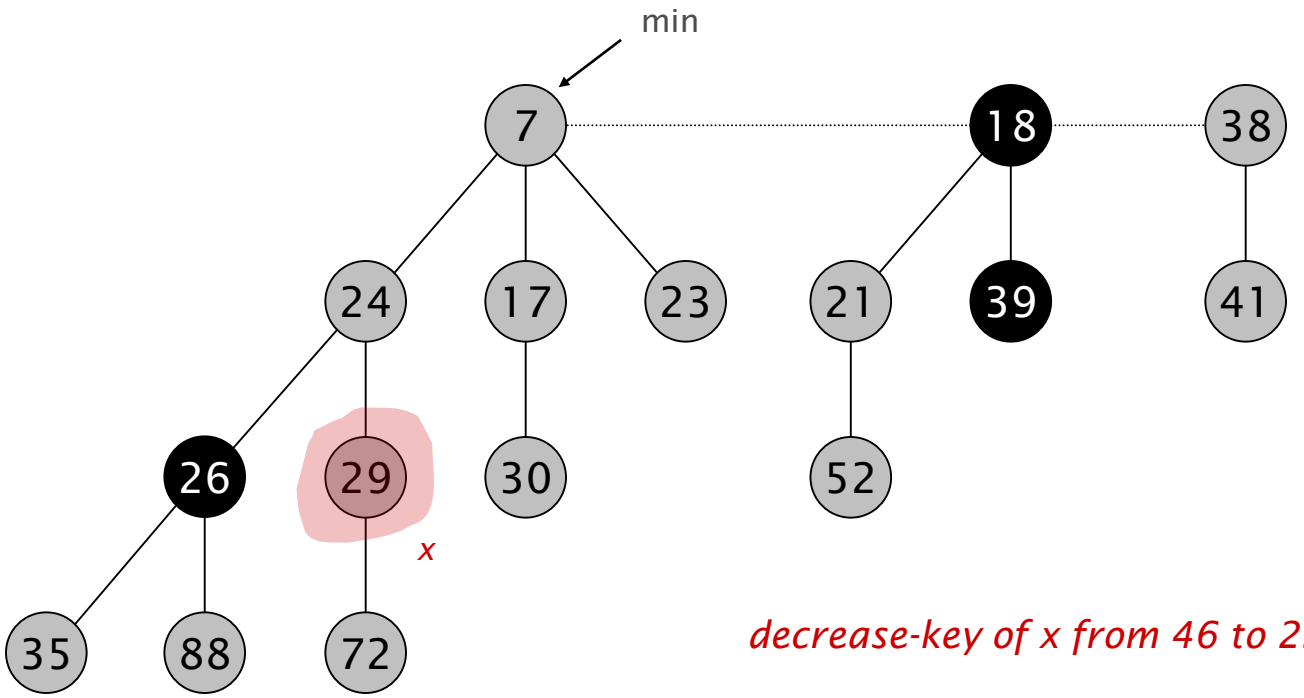

Fibonacci Heaps: Decrease Key

Case 2a. [heap order violated]

- Decrease key of *x*.
- Cut tree rooted at *x*, meld into root list, and unmark.
- If parent *p* of *x* is unmarked (hasn't yet lost a child), mark it;
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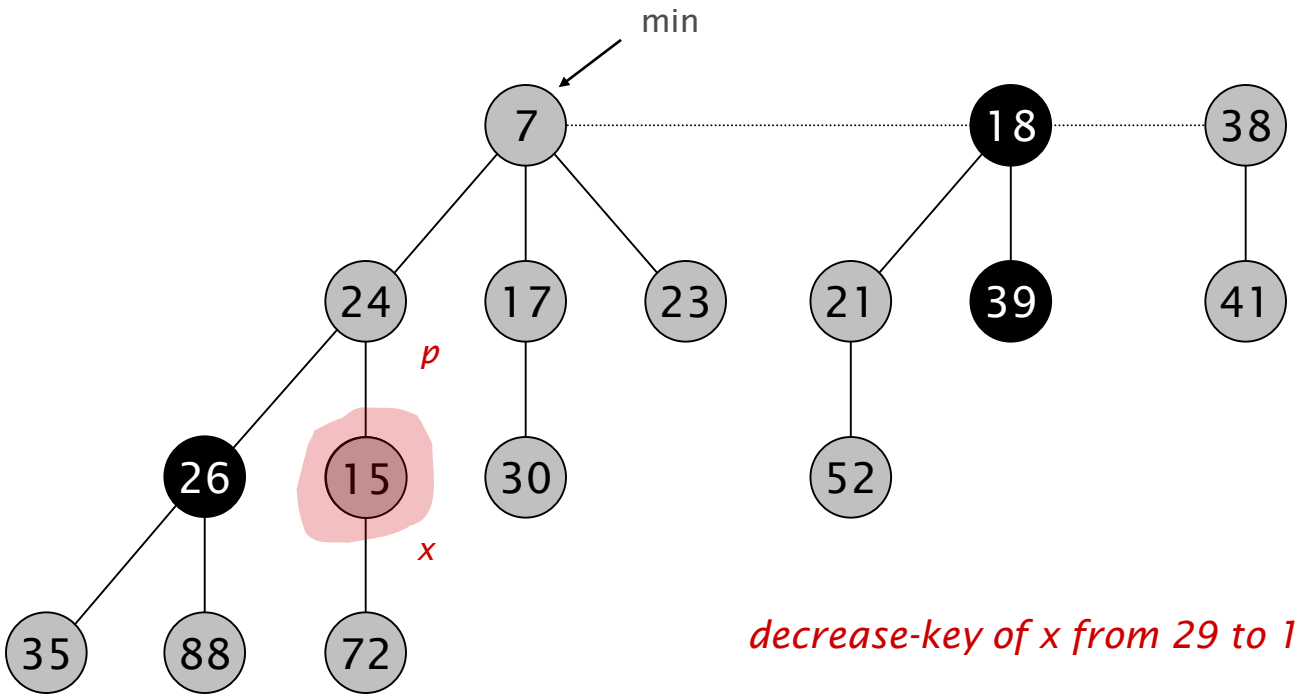

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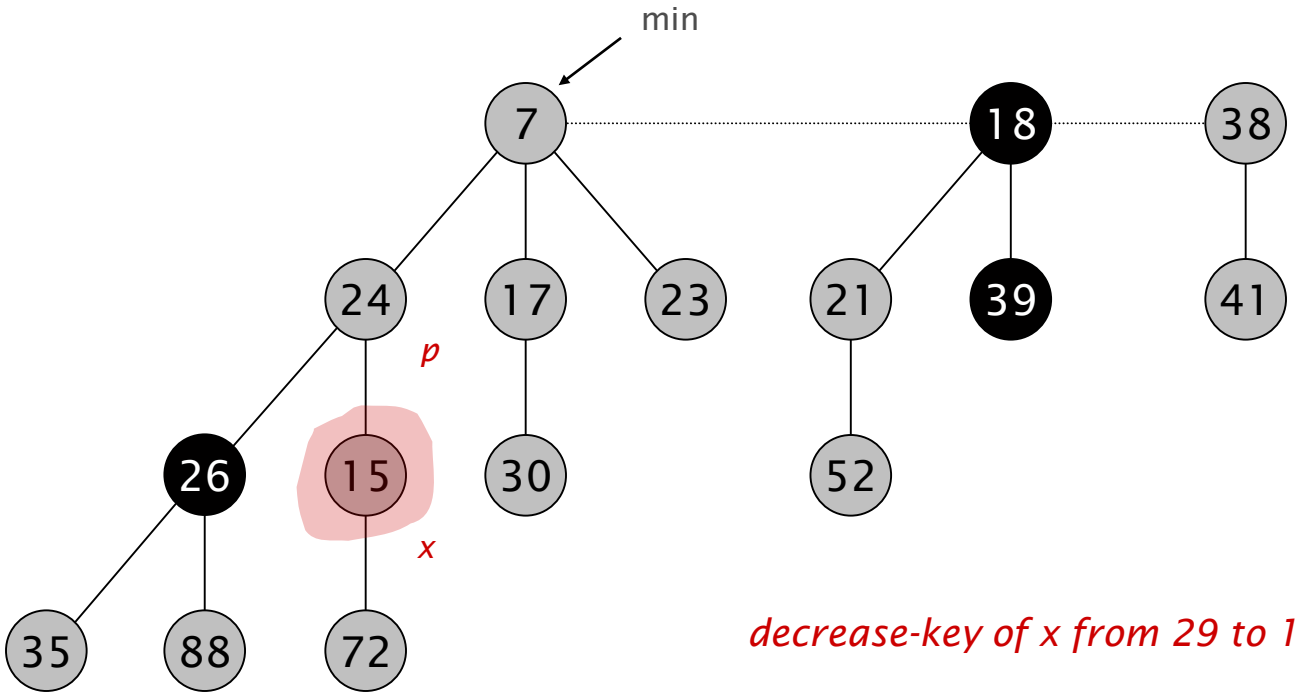

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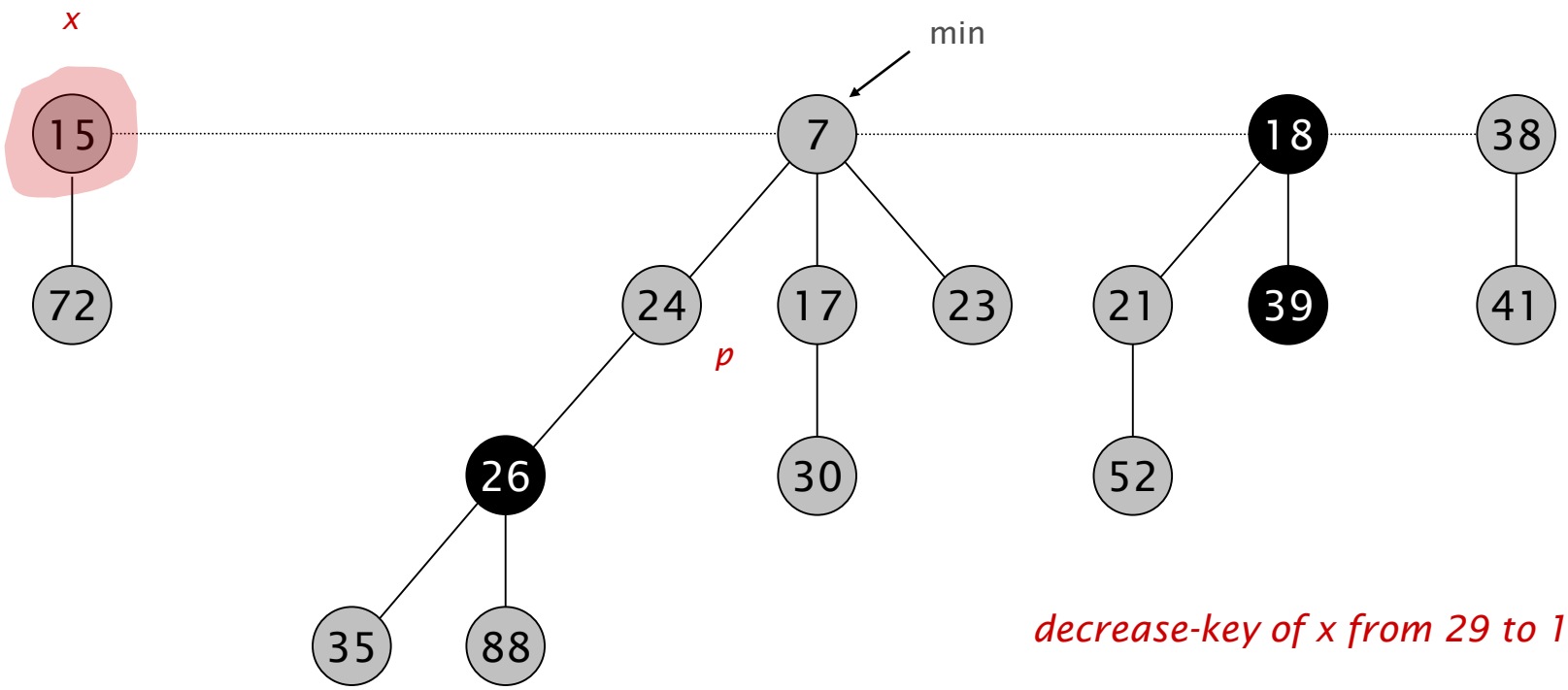

decrease-key of x from 29 to 15

Fibonacci Heaps: Decrease Key

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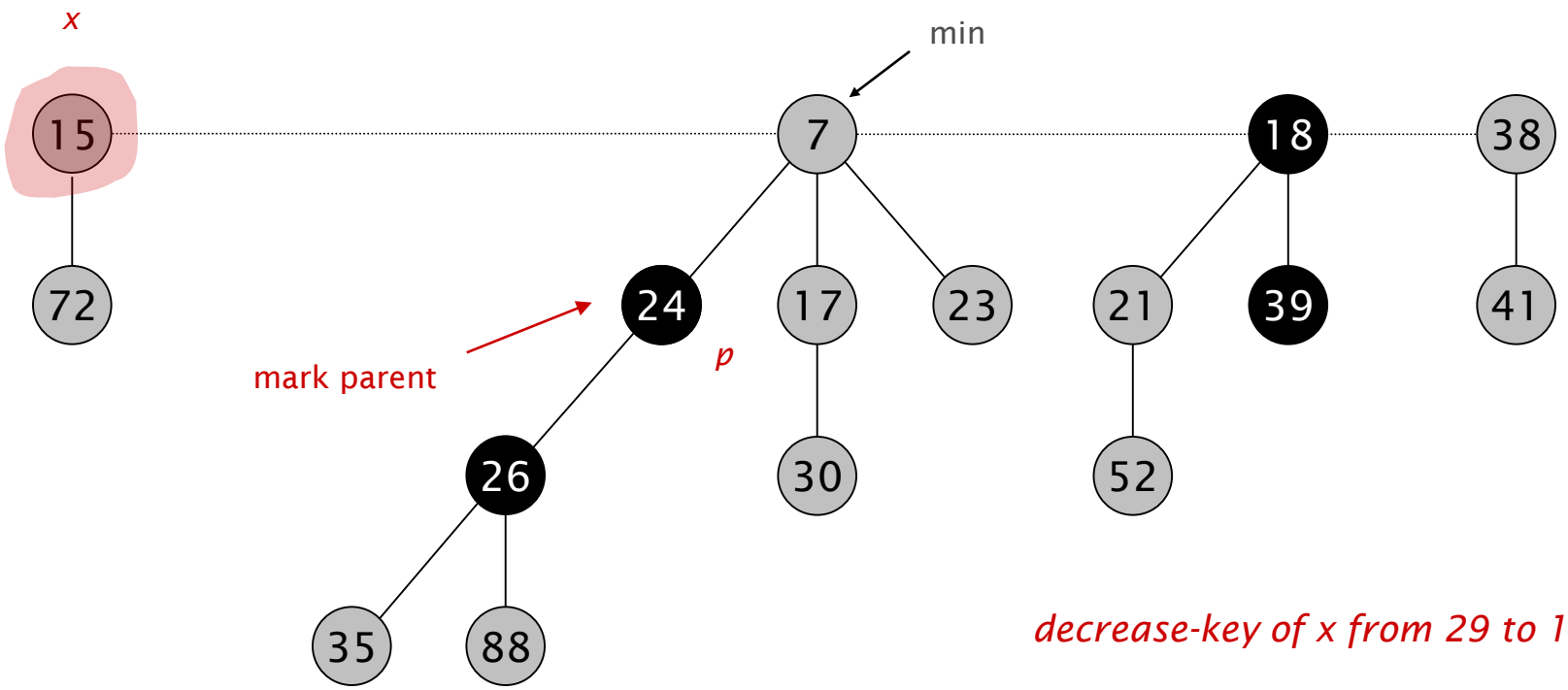

Fibonacci Heaps: Decrease Key

Case 2b. [heap order violated]

- Decrease key of x.
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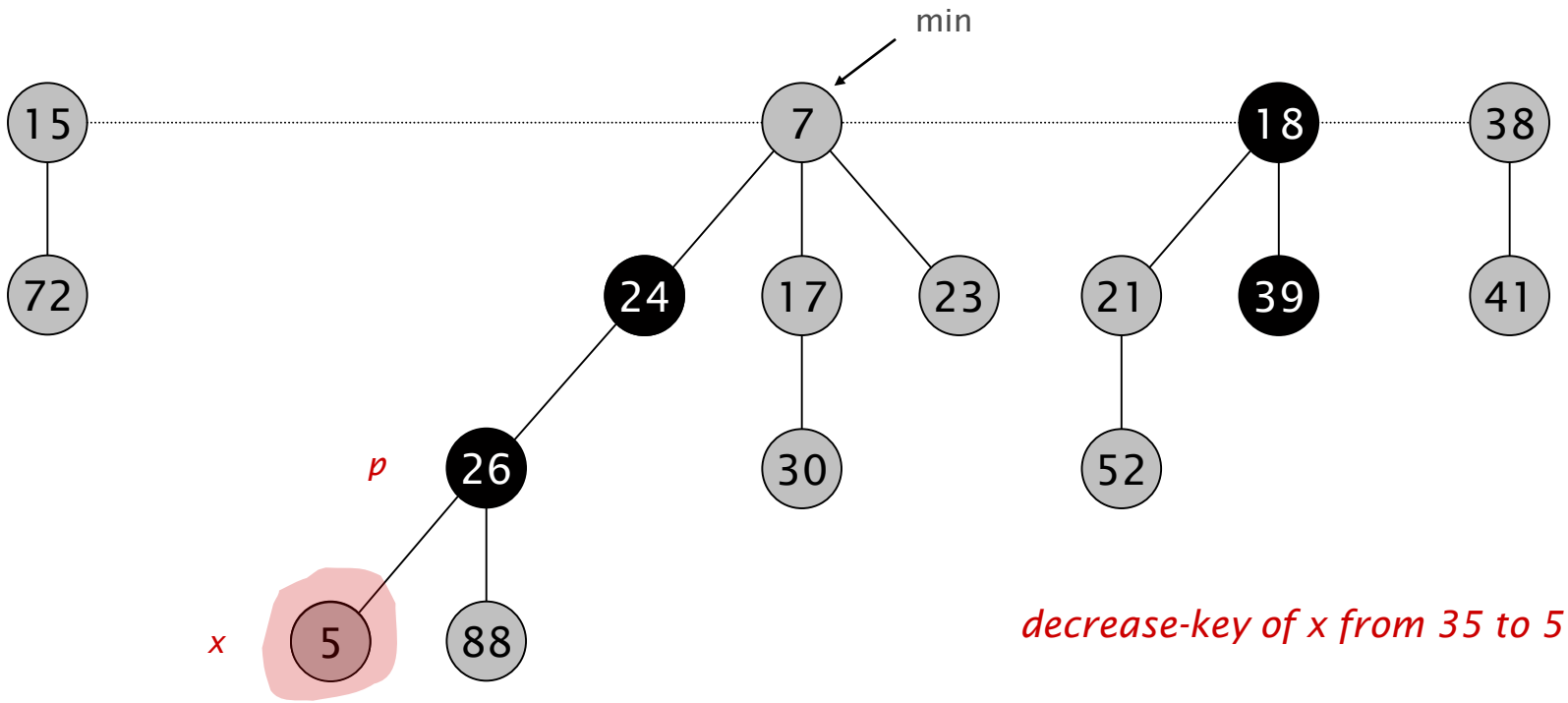

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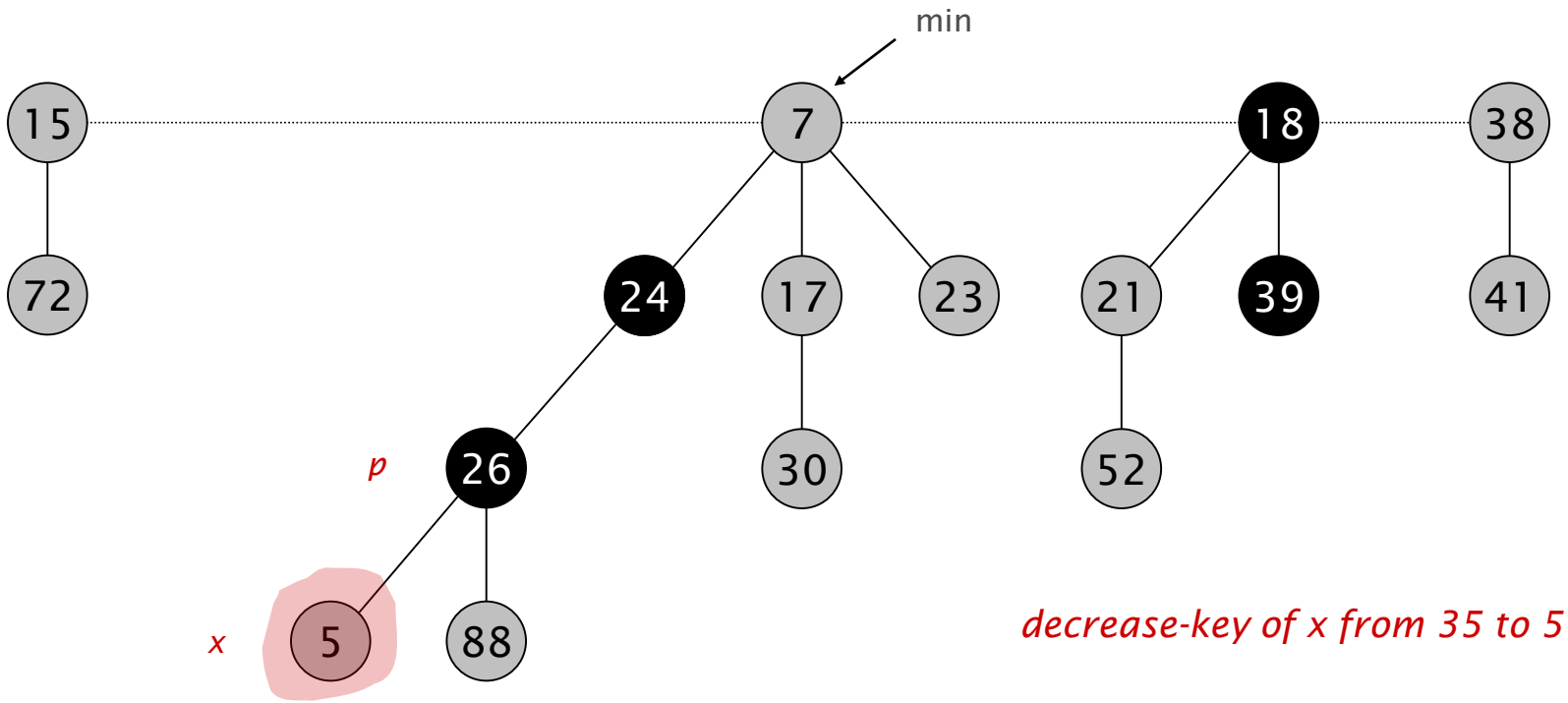

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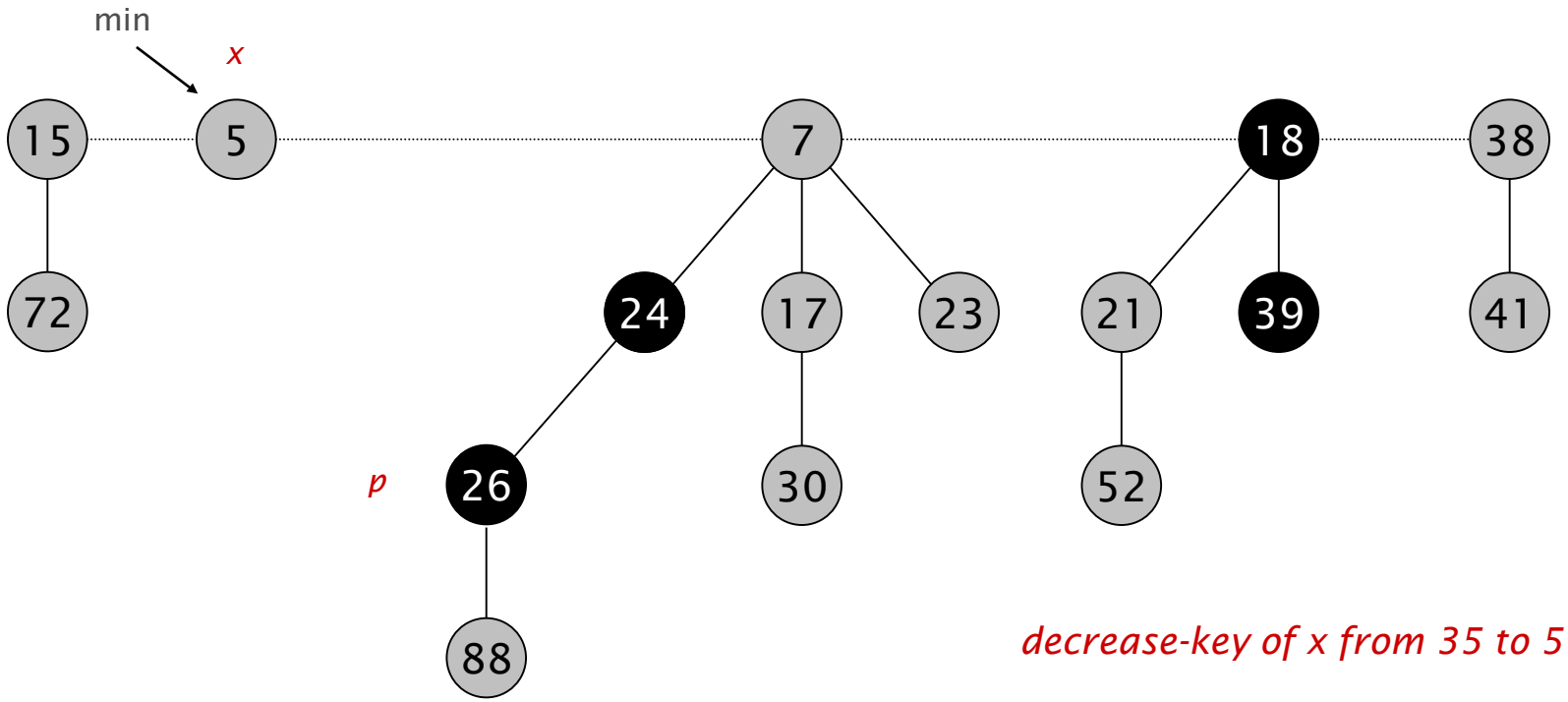

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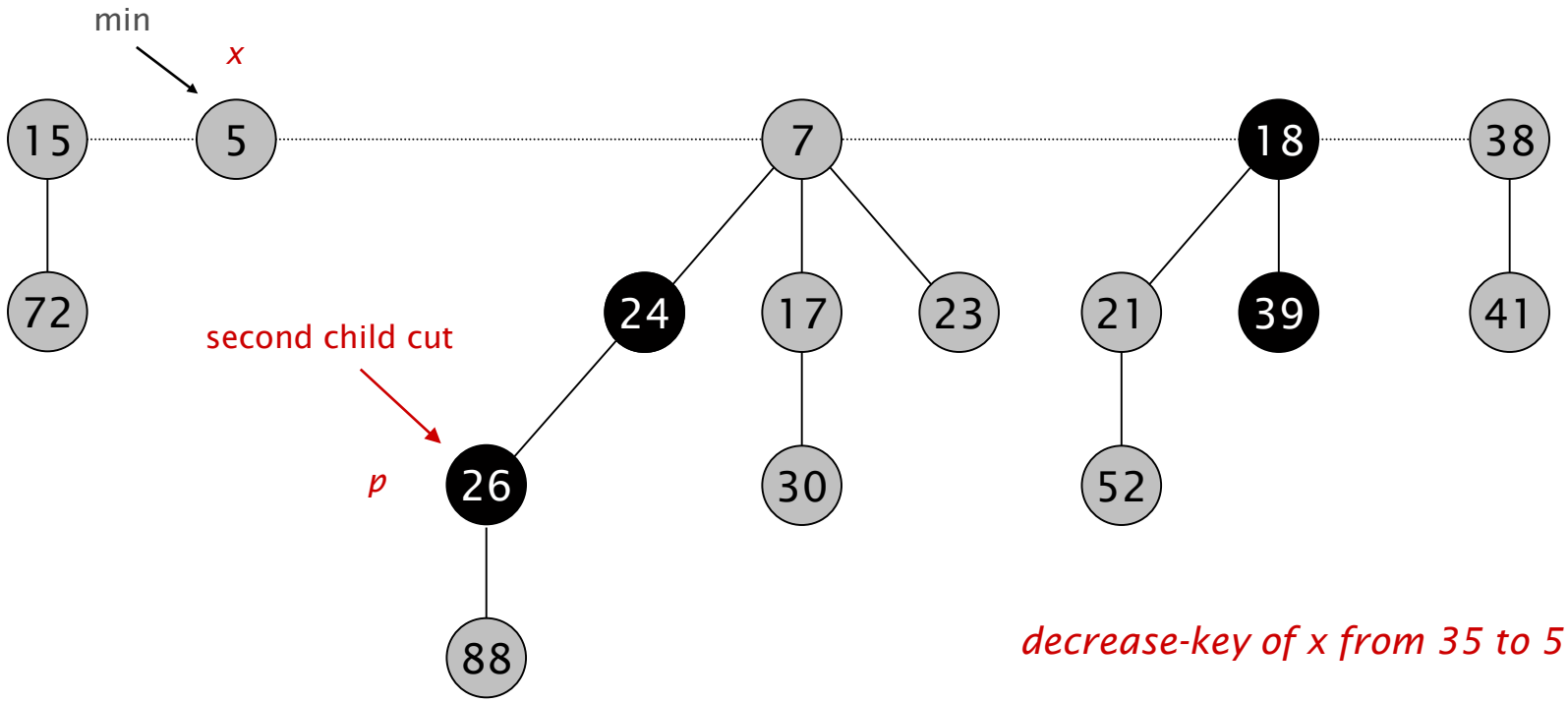

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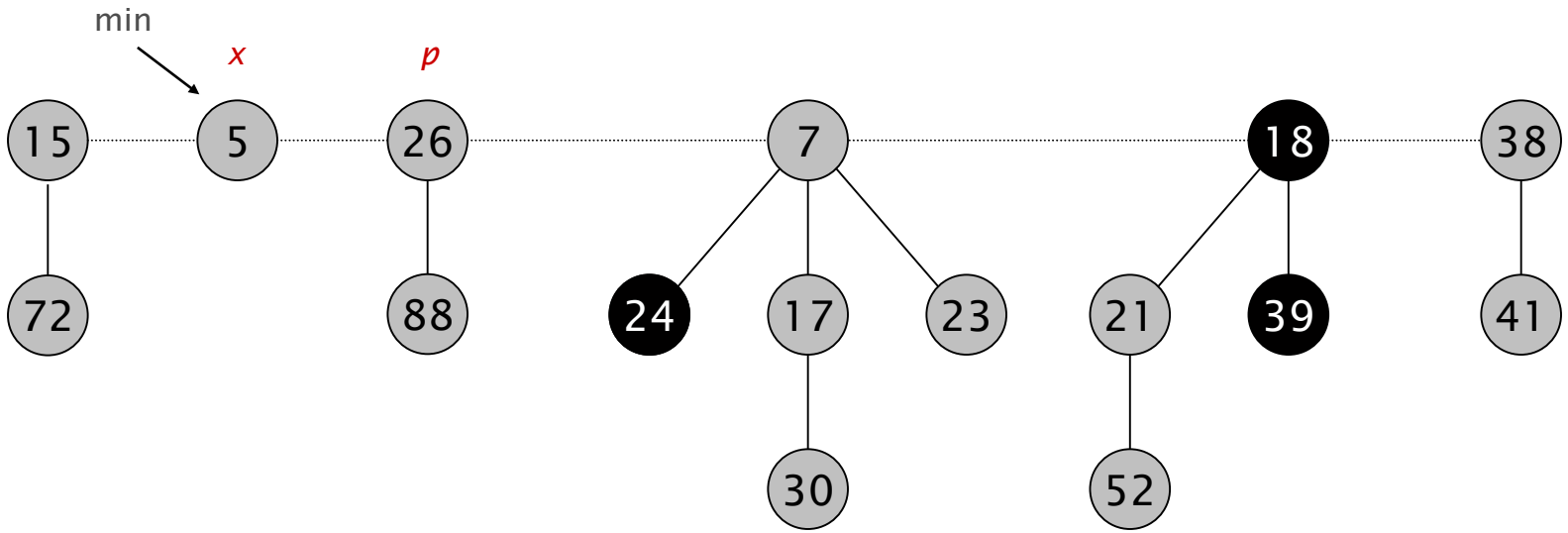

decrease-key of x from 35 to 5

Fibonacci Heaps: Decrease Key

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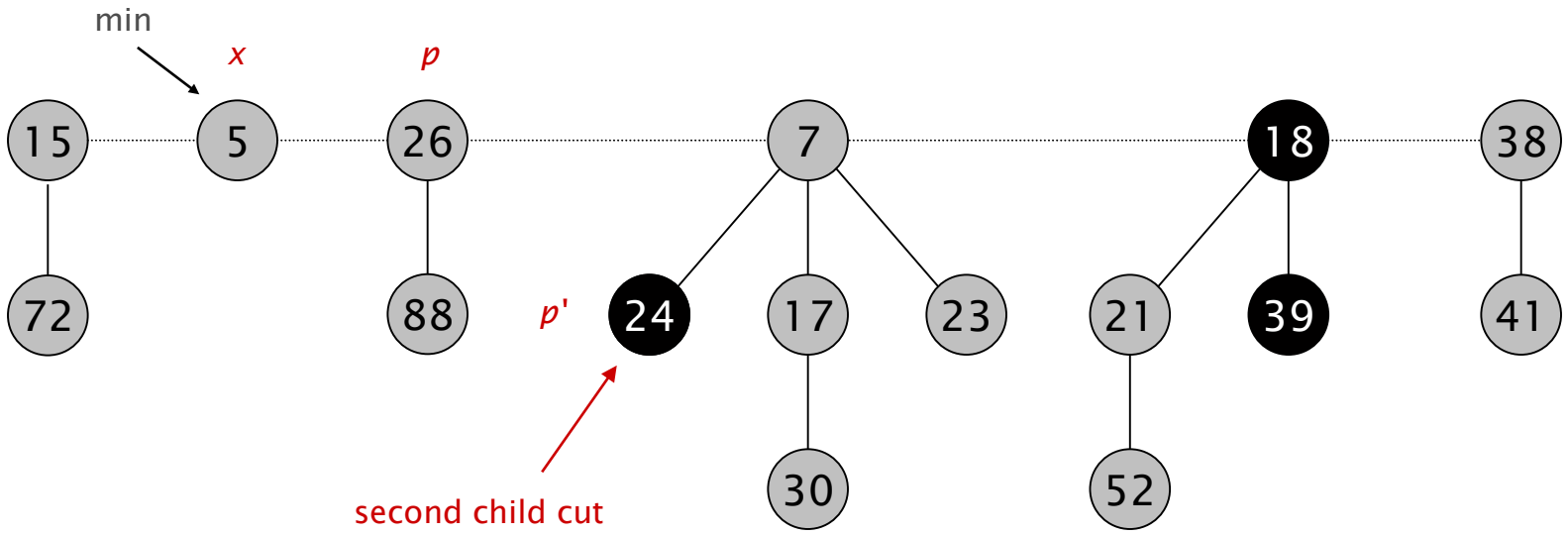

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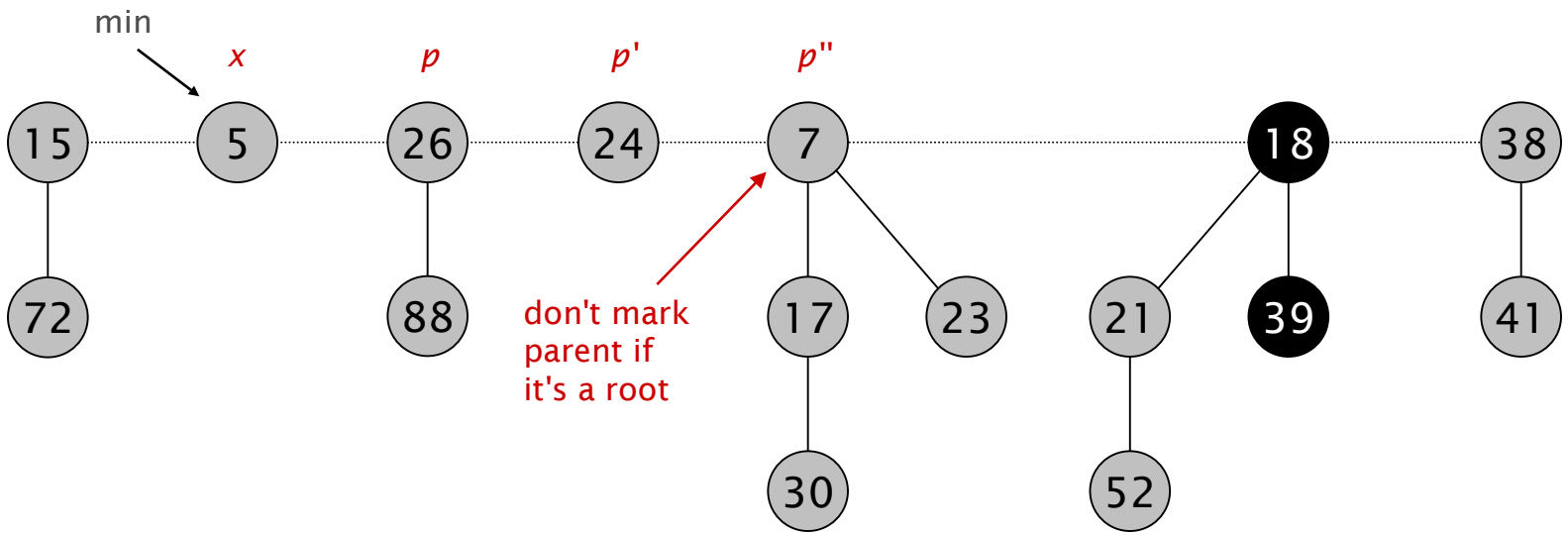
Case 2b. [heap order violated]

- Decrease key of *x*.
- Cut tree rooted at *x*, meld into root list, and unmark.
- If parent *p* of *x* is unmarked (hasn't yet lost a child), mark it;
  Otherwise, cut *p*, meld into root list, and unmark

  (and do so recursively for all ancestors that lose a second child).



decrease-key of x from 35 to 5