Valid and invalid Reasoning

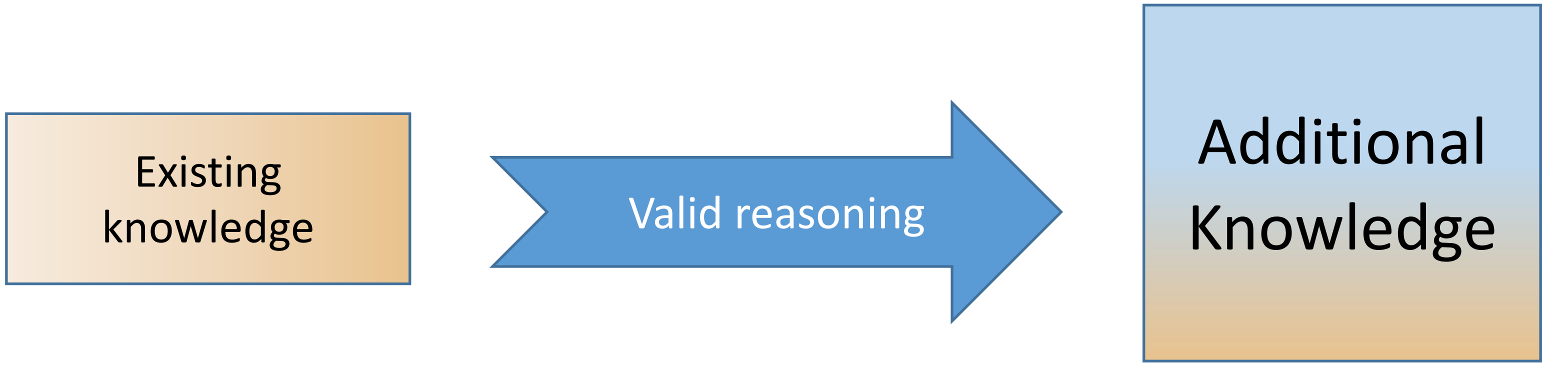
CMSC250

Reminder

- HW1 has been posted on ELMS.
- Submission on Gradescope.
  - *You must have been automatically added with your ELMS e-mail address!*
- Various submission methods.
  - *Handwritten, LaTeX, Word,…*
- Printed extra copies outside my office
  - *In a cardboard box.*
- Have had to cancel tomorrow’s office hours
- Thursday: lecture cover on circuits by Dr. Eastman

Valid reasoning

- We’ve talked about
  - **Basic Syntax:** Propositional symbols (p, q, r), operators and connectives (v, ∧, => , <=>)
  - **Handling Boolean Expressions:** Truth tables, equivalences
- Now we’re going to talk about **creating new information from existing premises** using **valid rules of reasoning!**



Template

$$\left.\begin{array}{c} \boldsymbol{premise_1} \\ \boldsymbol{premise_2} \\ \vdots \\ \boldsymbol{premise_n} \\ \hline \therefore conclusion \end{array}\right\} \text{Rule } r$$

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\therefore in $LaTeX$

$$\begin{array}{c} premise_1, \\ premise_2, \\ \vdots \\ premise_n \end{array} \xrightarrow{\text{Rule } r} \begin{array}{c} conclusion, \\ premise_1, \\ premise_2, \\ \vdots \\ premise_n \end{array}$$

Modus Ponens

- The most basic valid rule of inference is “Modus Ponens”.
- Example:
  1. All bears love honey
  2. Claws is a bear
  3. Therefore, Claws loves honey

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- Example:
  1. All bears love honey $(p \Rightarrow q)$
  2. Claws is a bear (p)
  3. Therefore, Claws loves honey (q)

Modus Ponens

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- Example:
  1. All bears love honey ($p \Rightarrow q$)
  2. Claws is a bear (p)
  3. Therefore, Claws loves honey (q)

$$
\begin{array}{l}
p \Rightarrow q \\
p \\
\hline
\therefore q
\end{array}
$$

Modus Ponens

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- Example:
  1. All bears love honey ($p \Rightarrow q$)
  2. Claws is a bear (p)
  3. Therefore, Claws loves honey (q)

So what makes modus ponens **valid?**

$$\begin{array}{l} p \Rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Determining validity

p	q	$p \Rightarrow q$
F	F	?
F	T	?
T	F	?
T	T	?

Determining validity

p	q	$p \Rightarrow q$
F	F	T
F	T	?
T	F	?
T	T	?

Determining validity

p	q	$p \Rightarrow q$
F	F	T
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T	F	?
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p	q	$p \Rightarrow q$
F	F	T
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Determining validity

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Conclusion

Premises

Determining validity

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Conclusion

Premises

- Rules where all the premises are ***True*** are called **critical rows.**
- Rules of inference where **all** critical rows have a ***True*** output are **valid** rules!

Is there “invalid” reasoning?

- Sure! Here’s one we already talked about: The **inverse error!**

$$p \Rightarrow q$$
$$\sim p$$
$$\overline{}$$
$$\therefore \sim q$$

Two critical rows…

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	F
T	T	F	F	T

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p	q	$\sim p$	$\sim q$	$p \Rightarrow q$
F	F	T	T	T
F	T	T	**F**	T
T	F	F	T	F
T	T	F	F	T

Two critical rows…

But one of them maps to *False*! ☹

So, the inverse error is **not** a valid rule of inference…

Are the following valid or invalid?

- Determine if these rules of inference are **valid** or **invalid:**

$$p \Rightarrow q$$
$$q$$
$$\therefore p$$

$$p \Rightarrow q$$
$$\sim q$$
$$\therefore \sim p$$

Are the following valid or invalid?

- Determine if these rules of inference are **valid** or **invalid:**

$$
\begin{array}{c}
p \Rightarrow q \\
q \\
\therefore p
\end{array}
\qquad\qquad
\begin{array}{c}
p \Rightarrow q \\
\sim q \\
\therefore \sim p
\end{array}
$$

Invalid: <u>Con</u>verse Error **Valid: <u>*Modus Tollens*</u>**

Quiz

- Let a rule r have premises $p_1, p_2, \dots, p_n$ and conclusion $conc$.
- Statement: r is a valid if, and only if, $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \Rightarrow conc$ is a tautology.

True

False

Quiz

- Let a rule r have premises $p_1, p_2, \ldots, p_n$ and conclusion $conc$.
- Statement: r is a valid if, and only if, $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \Rightarrow conc$ is a tautology.

True

False

- Example: Modus Ponens

p	q	$p \Rightarrow q$	$(p \wedge (p \Rightarrow q)) \Rightarrow q$
F	F	T	T
F	T	T	T
T	F	F	T
T	T	T	T

Exercise

- Let’s apply this technique to Modus Tollens and the Inverse Error!

What we know so far…

- What makes a rule valid
- What makes it invalid
- The following 4 (2 valid, 2 invalid)

Modus Ponens	Modus Tollens	Converse Error	Inverse Error
$p \Rightarrow q$ p $\therefore q$	$p \Rightarrow q$ $\sim q$ $\therefore \sim p$	$p \Rightarrow q$ q $\therefore p$	$p \Rightarrow q$ $\sim p$ $\therefore \sim q$

Here’s couple more!

- ***Disjunctive addition***

$$
\begin{array}{r}
p \\
\therefore p \vee q
\end{array}
$$

- Convince yourselves of validity!

Conjunctive addition

$$
\begin{array}{c}
p \\
q \\
\therefore p \wedge q
\end{array}
$$

Conjunctive Simplification

$$p \wedge q$$
$$\therefore p, q$$

Disjunctive Syllogism

$$p \vee q$$
$$\sim p$$
$$\therefore q$$

Hypothetical Syllogism

$$p \Rightarrow q$$
$$q \Rightarrow r$$
$$\therefore p \Rightarrow r$$

Division into cases

$$p \vee q$$
$$p \Rightarrow r$$
$$q \Rightarrow r$$
$$\therefore r$$

Proof by contradiction

$$(\sim p) \Rightarrow c$$
$$\therefore p$$

- Fundamental rule of inference, terrifically useful in formal proofs!
- Let's convince ourselves of validity!

Resolution

$$p \vee q$$
$$(\sim q) \vee z$$
$$\therefore p \vee z$$

- Resolution is terrifically important in AI!

Cheat Sheet for rules of inference

Modus Ponens	Modus Tollens	Disjunctive addition	Conjunctive addition	Conjunctive Simplification
p $p \Rightarrow q$ $\therefore q$	$\sim q$ $p \Rightarrow q$ $\therefore \sim p$	p $\therefore p \vee q$	p, q $\therefore p \wedge q$	$p \wedge q$ $\therefore p, q$
Disjunctive syllogism	**Hypothetical syllogism**	**Unit Resolution**	**Resolution**	**A bunny with a pancake on its head**
$p \vee q$ $\sim p$ $\therefore q$	$p \Rightarrow q$ $q \Rightarrow r$ $\therefore p \Rightarrow r$	$p \vee q$ $\sim q$ $\therefore p$	$p \vee q$ $\sim q \vee z$ $p \vee z$	

Complex applications of rules of deduction

- Prove that the following rule of deduction is valid **without** using truth tables:

$$
\begin{array}{l}
p \vee q \\
r \wedge \sim p \\
q \Rightarrow s \\
\hline
\therefore s
\end{array}
$$

- Alternative questioning: *Deduce* s given the premises

Complex applications of rules of deduction

1. From conjunctive simplification we have:

$$\frac{r \wedge \sim p}{\therefore r, \sim p}$$

2. From disjunctive syllogism we have:

$$\frac{\begin{array}{l} p \vee q \\ \sim p \end{array}}{\therefore q}$$

3. From modus ponens we have

$$\frac{\begin{array}{c} q \\ q \Rightarrow s \end{array}}{\therefore s}$$

Here's a tougher one!

- From the premises

1. $(\sim p) \Rightarrow (r \wedge (\sim s))$
2. $k \Rightarrow s$
3. $u \Rightarrow (\sim p)$
4. $\sim w$
5. $u \vee w$

- Deduce $\sim k$.

Solution

1. Through disjunctive syllogism we have:

$$\begin{array}{l} \sim w \\ \underline{u \lor w} \\ \therefore u \end{array}$$

2. Through modus ponens we have:

$$\begin{array}{l} u \Rightarrow (\sim p) \\ \underline{u \qquad\qquad\quad} \\ \therefore \sim p \end{array}$$

3. Through modus ponens we have:

$$\begin{array}{l} \sim p \\ \sim p \Rightarrow (r \land (\sim s)) \\ \therefore r \land (\sim s) \end{array}$$

Solution

4. Through conjunctive simplification we have:

$$\frac{r \wedge (\sim s)}{\therefore r, \sim s}$$

5. Through modus tollens we have:

$$\begin{array}{l} k \Rightarrow s \\ \underline{\sim s \quad\quad} \\ \therefore \sim k \end{array}$$

which is the desired conclusion.

Multiple proofs!

- Try this exercise on your own for a bit (not a worksheet exercise).

Given the premises

$$(\sim a) \Rightarrow (\sim b)$$
$$q \Rightarrow m$$
$$m \Rightarrow b$$
$$q$$
$$a \vee k$$
$$(\sim k) \vee (z \wedge \ell)$$

Deduce a.

Proof #1

1. Through hypothetical syllogism we have that
$$\begin{array}{l} q \Rightarrow m \\ \underline{m \Rightarrow b} \\ \therefore q \Rightarrow b \end{array}$$
2. Through modus ponens we have that
$$\begin{array}{l} q \Rightarrow b \\ \underline{q \qquad} \\ \therefore b \end{array}$$
3. From modus tollens we have that
$$\begin{array}{l} (\sim a) \Rightarrow \sim b \\ \underline{\sim(\sim b)} \\ \therefore \sim(\sim a) \end{array}$$
4. From the law of ***double negation*** (yes, of course we can still use those ☺) we have that $\sim(\sim a) \equiv a$.
5. Done.

Proof #2

1. Through resolution we have that

$$\begin{array}{l} a \vee k \\ \underline{(\sim k) \vee (z \wedge \ell)} \\ \therefore a \vee (z \wedge \ell) \end{array}$$

2. By applying De Morgan's Law we have that $(\sim z) \vee (\sim \ell) \equiv \sim(z \wedge \ell)$

3. From disjunctive syllogism (sometimes called ***unit resolution***) we have that

$$\begin{array}{l} \sim(z \wedge \ell) \\ \underline{a \vee (z \wedge \ell)} \\ \therefore a \end{array}$$

4. Done.

Worksheet time!

- Please complete exercises 1(a) and 1(b) in your worksheet!

Identifying rules of natural deduction in real-life arguments

- Socrates is a man. All men are mortal. Therefore, Socrates is mortal.

Identifying rules of natural deduction in real-life arguments

- $\underbrace{\text{Socrates is a man}}_{p}$. $\underbrace{\text{All men are mortal}}_{p \Rightarrow q}$. Therefore, $\underbrace{\text{Socrates is mortal}}_{q}$.

Identifying rules of natural deduction in real-life arguments

- $\underbrace{\text{Socrates is a man}}_{p}$. $\underbrace{\text{All men are mortal}}_{p \Rightarrow q}$. Therefore, $\underbrace{\text{Socrates is mortal}}_{q}$.

Modus ponens!

Identifying rules of natural deduction in real-life arguments

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2. **If** you are tall, **then** you can enter the team. You can also enter the team **if** you're a good shooter. **Since** I'm 7'4", I can enter the team.

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4. **If** you're a Computer Scientist, **then** you're smart. **If** you're smart, **then** you make good money. So, **if** you're a Computer Scientist, **then** you make good money.

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5. **If** you play for the Cavaliers, **then** you make good money. Chris Paul makes **good money**, **so** he plays for the Cavaliers.

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VALID

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INVALID