COMM611_assignment_1_probability_theory

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Assignment 1

Q1

Toss a fair coin 4 times, what is the probability of getting at least one head?

Q2

Toss a fair coin 10 times, what is the probability of 3 heads?

Q3

A box contains 2 red balls, 5 blue balls, and 10 yellow balls.

1. Draw two balls simultaneously, what is the probability of getting 2 yellow balls?
2. Each ball in this box has a unique number label. Drawing 3 balls out of this box, how many possible combinations we can get for the 3 drawn balls (ignoring the order of the balls drawn)?

Q4

In a class of 30 students, 18 like math, 12 like sciences, and 8 like both. Given a student likes science, what is the probability he/she also likes math?

Q5

A test for HIV has 98% accuracy rate (98% of the sick people will correctly tested positive and 98% of the healthy people will correctly tested negative). The disease will affect 1 person every 10,000 people. If a person tested positive, what is the probability that this person is truly affected?

Q6

Expected value A small business sells two products, Product A and Product B. Each day, the number of sales for each product is random and independent. The details are as follows:

- The number of daily sales of **Product A** (denoted X) follows this distribution:

  $$P(X = 0) = 0.2,\ P(X = 1) = 0.5,\ P(X = 2) = 0.3$$

  Each sale generates a profit of $20.

- The number of daily sales of **Product B** (denoted Y) follows this distribution:

$$P(Y = 0) = 0.3,\ P(Y = 1) = 0.4,\ P(Y = 2) = 0.3$$

  Each sale generates a profit of \$15.

Questions

1. Compute the expected number of sales for Product A ($E(X)$) and Product B ($E(Y)$).
2. Find the expected daily profit for each product ($E(Profit_A)$ and $E(Profit_B)$), considering the profit per sale.
3. Calculate the total expected daily profit for both products combined.
4. If the business introduces a flat daily operational cost of \$50, write the expression for the **net profit** and compute its expected value.

Q7

1. Calculate the variance for the sales for Product A and Product B in Q6.1.
2. Calculate the variance for the expected daily profit for each product in Q6.2.
3. Calculate the variance for the expected daily profit for both products combined in Q6.3.

Q9

A survey records the number of books read by 50 students over the summer. The data is summarized in the following frequency table:

Number of Books (x)	Frequency (f)
0	8
1	12
2	15
3	10
4	5

Questions

1. Draw a bar plot for the frequency table.
2. What is the expected value for the number of books that were read by a student?
3. Assume this empirical observed probability distribution is truly the theoretical probability distribution for the number of books, what is the variance of the number of books read by a student?

Q10

You are playing Aeroplane chess with your friends with a fair 6 sided dice. In this game, if a player roll a number 6 in this dice, it is considered a success and you can start moving your airplane out of the base.

Now consider you and 6 other players are rolling the dice in a row one by one for one round.

a. What is the probability of 4 players rolled a six in the first round?
b. What is the probability of at least 4 players rolled a six in the first round?
c. What is the probability of the number of players rolled a six is less than 6 and more than 2?
d. What is the expected value of number of players that moved their airplane in the first round?
e. What is the variance of number of players that moved their airplane in the first round?

Q11

Draw the probability mass function of the variable (number of players sucecssfully moved their airplane in the first round) in Q10.

Q12

For a random variable X, where $X \sim N(2, 100)$.

1. What is the probability $P(X <= 2)$?
2. What is the probability $P(X >= 2)$?
3. Write down the R code to calculate the probability $P(-10 \leq X \leq 50)$ *Recall the cumulative probability function $P(X \leq x)$ for normal variable, R code is pnorm(x, mean=μ, sd=σ)*

Q13

For a random variable X, where $X \sim F_{1,20}$.

1. Write down the R code to calculate the probability $P(X <= 2)$?
2. Write down the R code to calculate the probability $P(X >= 2)$?
3. Write down the R code to calculate the probability $P(1 \leq X \leq 3)$

Recall the cumulative probability function $P(X \leq x)$ for F distributed, R code is pnorm(x, df1=d_1, df2=d_2)

Q13

For a Z distributed random variable $X \sim N(0, 1)$. Given that $P(X \leq -1) = 0.158$

1. What is the probability $P(X \geq 1)$? Do it without consulting softwares or tables.
2. What is the probability $P(0 \leq X \leq 1)$? Do it without consulting softwares or tables.

Q14

For a normal distributed random variable $X \sim N(2, 10)$. Given that $P(X \leq -17.6) = 0.025$. Find the value k where $P(-17.6 \leq X \leq k) = 0.95$.