assignment06

Jason Grahn

2/12/2019

Chapter 11 Page 402:

# 11.9 A

Use the sons data of Table 3.8…

sons <- read.table(here::here("assignment06/T3\_8\_SONS.DAT")) %>%   
 rename(y1 = V1, # head length  
 y2 = V2, # head breadth  
 x1 = V3, # head length  
 x2 = V4) # head breadth  
  
head(sons, 5)

## y1 y2 x1 x2  
## 1 191 155 179 145  
## 2 195 149 201 152  
## 3 181 148 185 149  
## 4 183 153 188 149  
## 5 176 144 171 142

#normalize the data  
sons.std <-sweep(sons, 2, sqrt(apply(sons,2,var)), FUN="/")  
head(sons.std, 5)

## y1 y2 x1 x2  
## 1 19.56600 21.02287 17.82824 21.60972  
## 2 19.97576 20.20908 20.01942 22.65295  
## 3 18.54160 20.07345 18.42583 22.20585  
## 4 18.74648 20.75161 18.72463 22.20585  
## 5 18.02941 19.53092 17.03144 21.16262

#subgroup the data into first and second sons  
first.son <- sons.std %>% select(y1, y2)  
second.son <- sons.std %>% select(x1, x2)

## (a) Find the canonical correlations between (,) and (, )·

### the sample correlation matrix

S <- cov(sons.std)  
  
#find the sample correlation matrix for either son  
R11 <-cor(first.son)  
R11

## y1 y2  
## y1 1.0000000 0.7345555  
## y2 0.7345555 1.0000000

R22 <-cor(second.son)  
R22

## x1 x2  
## x1 1.0000000 0.8392519  
## x2 0.8392519 1.0000000

### b) the characteristic equation

# correlate column by column for the 4 columns   
 ## first from each  
R12 <-c(cor(first.son[,1],   
 second.son[,1]),   
 ## first and second  
 cor(first.son[,1],   
 second.son[,2]),  
 ## second and first  
 cor(first.son[,2],   
 second.son[,1]),  
 ## second and second  
 cor(first.son[,2],   
 second.son[,2]))  
  
# coerce r12 into a 2x2 matrix  
R12 <-matrix(R12, ncol=ncol(R22), byrow=T)   
  
# R21 is the transpose of R12  
R21 <-t(R12)

# we have to find the eigenvectors and values in order to write the formulas, so this   
# WILL INCLUDE part C  
  
# Finding the E1 and E2 matrices:  
E1 <-solve(R11) %\*% R12 %\*% solve(R22) %\*% R21  
# and their eigenvector  
e1vectors <- round(eigen(E1)$vectors,2)  
e1vectors

## [,1] [,2]  
## [1,] 0.73 -0.70  
## [2,] 0.69 0.71

E2 <-solve(R22) %\*% R21 %\*% solve(R11) %\*% R12  
e2vectors <- round(eigen(E2)$vectors,2)  
e2vectors

## [,1] [,2]  
## [1,] -0.68 -0.71  
## [2,] -0.73 0.71

So the final formulas are:

u1 = 0.73[y1] + 0.69[y2]  
v1 = -0.68[x1] - 0.71[y2]  
u2 = -0.70[y1] + 0.71[y2]  
v2 = -0.73[x1] + 0.71[y2]

### c) eigenvalues

eigen(E1)$values

## [1] 0.621744734 0.002887956

eigen(E2)$values

## [1] 0.621744734 0.002887956

#they are the same!

### Finally, the cannonical correlation

canon.corr <-sqrt(eigen(E1)$values)  
canon.corr

## [1] 0.7885079 0.0537397

# 2. Chapter 11 Page 402: #11.9 part (c)

## Test the significance of each canonical correlation.

# known facts  
n <- nrow(sons)  
p <- 2  
q <- 2  
k <- 2  
  
#make $V\_H$  
vh <- q  
vh

## [1] 2

#make $V\_E$  
ve <- n - 1 - q  
ve

## [1] 22

We reject **IF** <= .

# lambda is 1 - the canonical correlation eigenvalue  
test1 <- 1 - canon.corr[1]  
test2 <- 1 - canon.corr[2]  
  
# look at Table A.9 for the critical value  
# table A.9 shows the critical value for vh = 2 and ve = 22 is .643  
CriticalValue <- .643  
result1 <- test1 <= CriticalValue  
result2 <- test2 <= CriticalValue

It is TRUE that the lambda value (0.2114921) of canonical correlation **1** (0.7885079) is less than or equal to the critical value 0.643; so we reject and conclude that the first canonical correlation is significant and could use this relationship for building out multivatiate model.

But it is FALSE that the lambda value (0.9462603) of canonical correlation **2** (0.0537397) is less than or equal to the critical value 0.643; so we fail to reject and conclude that the second canonical correlation is NOT significant.