ELEC4621 Lab5 - T1, 2020

This lab is essentially a practice Matlab exercise to illustrate the concepts of spectral representation and give you an introduction to time-frequency analysis. Understanding the frequency content of some signals is essential to characterising how the information is contained in them. This is in turn crucial to understanding how these signals are used in practice to carry out the functions they are designed for. In this lab we will study actual bat echolocation signals.

I have already mentioned the amazing capability of echolocation in bats. Bats have, in fact, evolved their waveforms to be optimal for their tasks of navigating and hunting. Bats have shown an exceptional ability to adapt their waveforms to the task at hand. Studying bat echolocation calls can help us design better radar, sonar, and other systems.

1 Introduction

Nature has evolved very advanced solutions to many signal processing problems. Understanding the problems that nature has solved and the solutions that it has developed can greatly enhance the signal processing state-of-the-art. This field of enquiry has been referred to as “Biologically Inspired Signal Processing”.

Bats, in particular, have evolved an outstanding capability to employ sonar to navigate and to detect and hunt insects. Bats emit signals that reflect off objects in their environments and are received back for analysis. Understanding these signals is essential to give us insights into the exquisite bat sonar system.

In this activity, you are give actual bat recordings and your goal is to examine, understand, and carry out some pre-processing steps of bat echolocation calls. To this end, a number of real bat calls, that were recorded in South Africa, are provided to you.

2 Analysis of Bat Echolocation Calls

The bat echolocation calls are recorded and saved as wav files and can easily be loaded into Matlab. Each file contains the data vector and the sampling frequency. As the bat calls are all ultrasonic, they are sample at $750ksamples/s$. For each bat echolocation call, carry out the following tasks:

1. Calculate and plot the magnitude Fourier spectrum of the signal. What do you observe?

2. What is the bandwidth of the signal and what is the minimum required sampling frequency for each data record?

3. The signals can be played with a different sampling frequency simply using the command `sound(data,fs1)`, where fs1 is the new sampling frequency. Using `fs1` = 44.1 kHz, play the sound. Note what you hear. What is the mapping between the content of the original signal and the signal that is played?

4. The bat signal is time-varying. That is, much like a musical piece, different notes exist at different times. Write code to calculate the STFT (see section **??**) and plot the time varying spectrum of the signals. Use different window lengths. Comment on your results.

5. The signals are to be resampled to the minimum sampling frequency achievable. You are required to implement the system that would resample the different signals. (You are expected to do some research into how resampling is done and the issues that are involved). Report the quality of your resampling strategy using the mean squared error between the original and new signals in the frequency band of interest.

6. Plot the STFT of the resampled signal and compare it to the original signal. Comment on you results.

7. Play the resampled signal using `fs1` = 44.1 kHz. Comment on what you hear. What is the mapping in this case?

3 The Short-time Fourier Transform

Recall that the definition of the Discrete-time Fourier Transform for a signal, $x[n]$, is

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\omega}. \tag{1}$$

It is clear from the expression above that the DTFT is a global transform that employs all of the samples and any time information is lost in the transformation. To illustrate this point, consider the signal

$$x[n] = \begin{cases} A\cos(\omega n) + w[n], & \text{for } 0 \leq n \leq N-1 \\ w[n], & \text{otherwise.} \end{cases} \tag{2}$$

The DTFT of the signal would not give any information on the fact that the cosine component of the signal is localised to the interval $[0, N-1]$. In fact the spectrum would look the same if the cosine component was localised to a different interval.

In order to preserve the time information, we can consider breaking the signal into blocks and taking the DTFT. Let us see how this works for the signal above. Let

$$x_l[n] = \begin{cases} x[lN+n], & \text{for } 0 \leq n \leq N-1 \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Thus $x_l[n]$ is the $l-th$ block of $x[n]$ of size N. Now we take the DTFT of each of the blocks so that the localisation of a component to a block is preserved by our knowledge of the block index that we are processing. Looking at the cosine example above, we can see that block 0 contains the cosine and that the peak associated with it only shows up in the spectrum for that block.

The DTFT of the blocks can be expressed as

$$X_l(\omega) = \sum_{n=-\infty}^{\infty} x_l[n]e^{-jn\omega} \tag{4}$$

$$= \sum_{n=0}^{N-1} x[lN+n]e^{-jn\omega}. \tag{5}$$

Another way to obtain the localised result is to note that the signal $x_l[n]$ consists of a the block of samples between lN and $(l+1)N-1$ and shifted back to zero. The time shift is only a phase shift in the frequency domain, which disappears when we look at the magnitude spectrum. The block over the interval $[lN, (l+1)N-1]$ is also obtained by multiplying the signal by the unit rectangular window that is 1 over that interval. This concept can then be generalised by using any window rather than the rectangular (or all-or-nothing) window. Finally, we note that limiting ω to a grid of frequencies $\frac{2\pi k}{K}$ for $0 \leq k \leq K-1$ gives the DFT of each block, which we saw is a sampled version of the DTFT that can be calculated using the FFT.. This is called the short-time Fourier Transform. Note that K does not have to be equal to N. If $K = rN$, then we simply pad the blocks with zeroes to get the required length to apply the FFT.

The procedure for obtaining the STFT is to window the signal to break it into blocks and obtain the magnitude spectrum of each block. The results can then be compiled into a matrix and plotted as a three-dimensional plot against the time index l and frequency index k.