Elec4621 Lab4 - T1 2020
Filter Design and Implementation

March 10, 2020

This lab is essentially a practice Matlab exercise to illustrate the concepts of filter design and implementation. In the first exercise, you will experiment with the windowing method for filter design. Importantly, you will observe the trade-off between the group delay and the quality of the approximation of the ideal impulse response. This is a practical trade-off that is often encountered in practical scenarios and can be important in real time applications, such as audio.

In the second exercise, on the other hand, you will experience the effect of quantising the filter coefficients on the performance. This also can be a critical design choice in practical applications that are either real time or where the hardware cost is critical (e.g. in environmental sensing where perhaps thousands of sensors need to be deployed and where hardware cost can be a major feasibility driver).

1. A filter has the impulse response

$$h[n] = \mathrm{sinc}\left(\frac{n}{6}\right),$$

for $-\infty < n < \infty$. Here $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

   (a) Is the filter linear? time invariant? causal?

   (b) What is the transfer function of the filter?

   (c) Plot the frequency response (both magnitude and phase) of the filter. What do you observe (**Hint**: here use a large enough number of samples of the impulse response so that it is small enough to appropriately approximate the true impulse response)?

(d) The filter is to be approximated by an implementable filter. One method to achieve this is to truncate the filter to $2L$ samples, such that the filter impulse response is defined for $-L \leq n \leq L$. investigate the frequency response for various values of L (you may, say, try $L = 8,\ 32,\ 128$ etc...).

i. Comment on the magnitude response of the filter. (what do you observe with respect to the response in 1c?)

ii. Comment on the phase response of the filter.

iii. What should the group delay be for the filter to be practically realisable?

(e) One way to improve the frequency response is to multiply the truncated impulse response by a suitable window. That is we put $h'[n] = w[n]h[n]$.

Plot the frequency response for the same values of L as above but with a Hanning window applied. Compare your results with those obtained without the application of the window. What do you observe?

2. An fourth order all-pole filter has poles at

$$p_1 = -0.51432,\ p_2 = -0.4417,\ p_3 = 0.9562e^{j0.45\pi},\ p_4 = 0.9581e^{-j0.45\pi}.$$

(a) What is the transfer function of the filter?

(b) Plot the magnitude and phase responses of the filter.

(c) Suppose that only 4 bit registers are available to store the values of the poles and zeroes.

i. Now suppose you decide to quantise the magnitude and phase of the poles and zeroes, such that 4 bits are used for the value of the magnitude and another 4 bits for the phase. Determine the quantized poles and zeroes. Calculate and plot the magnitude and phase responses. Compare to your answer in part 2b.

ii. Suppose, on the other hand, you decide to write the poles and zeroes in the form $z_R + jz_I$ and to quantise the real and imaginary parts instead. What is the resulting transfer function? Plot the corresponding magnitude and phase responses and compare to your answers above.

(d) The filter is to be implemented in cascade form, as shown in Figure 1. Determine the coefficients used by this implementation.

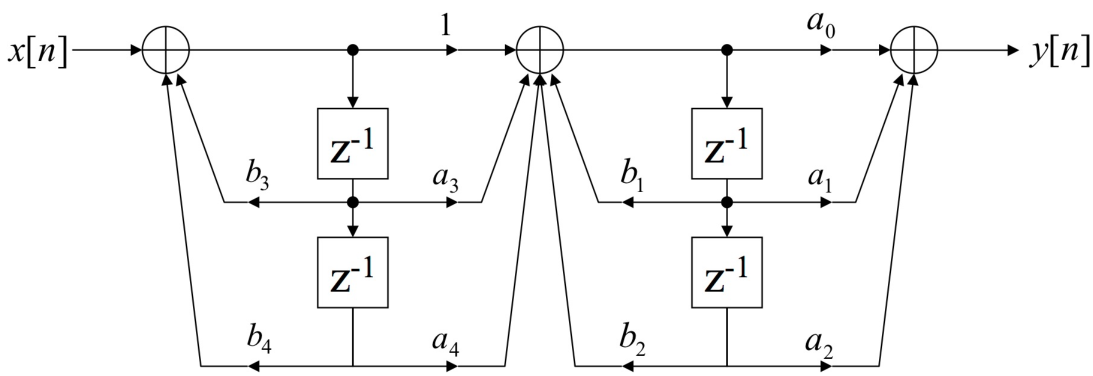

Figure 1: Cascade filter implmentation.

(e) Suppose that the coefficients a and b are now to be quantised using the 4 bits as before.

i. What is the resulting transfer function?

ii. Determine the poles and zeroes and show them on a pole-zero plot along with the other values from the previous parts above.

iii. plot the magnitude and phase responses and compare to your results above.