Elec4621 Lab6 - T1 2020

This lab is essentially a practice Matlab exercise in some of the basics of statistical signal processing.

In Matlab one may use the functions `rand` to generate a number from a uniform distribution and and `randn` to obtain a number from a Gaussian distribution.

1. Suppose that $N = 32$ samples are obtained from a white Gaussian noise process with mean zero and variance 0.25. Use the `randn` function to generate $K = 256$ different realisations of the random process **x** (each of length $N = 32$).

   Now you will obtain the ensemble statistics of x. Calculate the following:

   (a) mean.

   (b) variance.

   (c) autocorrelation.

   (d) covariance.

   Furthermore, use the command `surf` (you may also use `imagesc`) to plot the autocorrelation and covariance.

2. In this part, the statistics are obtained using time averages. Equivalently, this can be understood as taking the time sequence and breaking it into blocks of suitable length. These blocks can then be considered the realisations (in a similar way to part 1).

   Now suppose that we only have a single realisation of x, say $N = 256$ samples. Generate such a realisation in Matlab. Calculate the statistics in part 1. For the autocorrelation, calculate the lags $-32 \leq m \leq 32$.

3. The power spectral Density is the Fourier Transform of the autocorrelation.

   Looking first at part 1:

   (a) What do you expect the PSD of the signal to look like?

   (b) Plot the magnitude squared of the spectrum of each realisation. What do you observe?

   (c) Plot the FT of the autocorrelation on the same axes.

       Record your observations.

4. Repeat the process for the signal in part 2.

5. Consider the filter

$$H(z) = \frac{z^2 - 1.5z + 1}{z^2 - 0.4z + 0.4}. \tag{1}$$

   (a) Obtain frequency response of the filter. Plot the magnitude and phase responses.

   (b) Apply the filter to each of the realisations in part 1.

      i. Calculate the statistics and plot them as you have done in part 1.a. Comment on your results.

      ii. Plot the magnitude spectra of the outputs (each of the realisations), and their averages (*note that you cannot average the magnitude but the square of it - why?*). What do you see? Plot the averaged Plot the PSD (FT of the autocorrelation). What do you see now?

   (c) Now apply the filter to the sequence in part 2. Repeat the process of calculating the statistics of the sequence and its PSD. Comment on your results.

Now suppose that we have a system with an unknown transfer function. Can you suggest, based on your observations in this lab, a way to determine the unknown transfer function? Provide your reasoning.