Tutorial: Foundations of Non-truthful Mechanism Design

Part I: Equilibrium Analysis Tutor: Jason Hartline

Schedule:

Part Ia: 10-10:45am (http://ec20.sigecom.org/tech/tutorial)
Part Ib: 11-11:45am (http://ec20.sigecom.org/tech/tutorial)
Exercises: 12-1pm (http://ec20.sigecom.org/tech/tutorial-exercises)
(https://tinyurl.com/non-truthful-exercises)

Protocol:

During session, panelest will answer clarifying questions in chat.

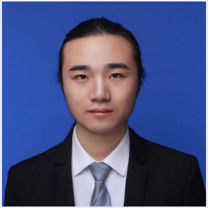
In post-session Q/A, "raise hand" to ask question.

Tutorial Cochairs

Brendan Lucier

Sigal Oren

Panelists



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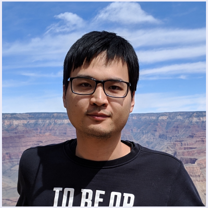

Yingkai Li

Foundations of Non-truthful Mechanism Design

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EC Tutorial 2020

Context: The Revelation Principle

Mechanism Design: identify mechanism that has good equilibrium.

Revelation principle: if exists mechanism with good equilibrium, then exists mechanism with good truthtelling equilibrium. [Myerson '81]

Proof: truthful mechanism can simulate equilibrium strategies in non-truthful mechanism.

Consequence: literature focuses on truthful mechanisms.

Issues:

- practical mechanisms are not truthful.
- not without loss for simple or prior-independent mechanisms.
- non-trivial to undo the revelation principle.

Goal: theory for non-truthful mechanism design.

Part I

Equilibrium Analysis

Warmup: Second-price Auction

Definition (Second-price Auction, SPA)

1. agents bid.
2. winner is highest bidder.
3. winner pays second-highest bid.

Thm: Truthful bidding is dominant strategy equilibrium in SPA.

Recall: Uniform Distribution $U[0,1]$

- cumulative distribution function $F(\mathrm{z}) = \mathrm{z}$
- probability density function $f(\mathrm{z}) = 1$
- **Fact:** uniform r.v.s evenly divide their interval in expectation.
  E.g., $\mathrm{v}_1, \mathrm{v}_2 \sim U[0,1] \quad \Rightarrow \quad \mathbf{E}\left[\mathrm{v}_{(1)}\right] = 2/3$, $\mathbf{E}\left[\mathrm{v}_{(2)}\right] = 1/3$

Example (Two agents, uniform values, second-price auction)

- Expected welfare in equilibrium: $\mathbf{E}\left[\mathrm{v}_{(1)}\right] = 2/3$
- Expected revenue in equilibrium: $\mathbf{E}\left[\mathrm{v}_{(2)}\right] = 1/3$

First-price Auction

Definition (First-price Auction, FPA)

1. agents bid. 2. winner is highest bidder. 3. winner pays their bid.

Qstn What are strategies? Equilibrium welfare? Equilibrium revenue?

Example (Two agents, uniform values, first-price auction)

"Guess and verify" approach:

- Guess that agent 2 bids "half of value"
- Calulate agent 1's utility with value v and bid b:

$$\mathbf{E}[\mathrm{utility}(\mathrm{v}, \mathrm{b})] = (\mathrm{v} - \mathrm{b}) \times \underbrace{\mathbf{Pr}[1 \text{ wins with bid } \mathrm{b}]}_{\mathbf{Pr}[\mathrm{b}_2 \leq \mathrm{b}] = \mathbf{Pr}[\mathrm{v}_2/2 \leq \mathrm{b}] = \mathbf{Pr}[\mathrm{v}_2 \leq 2\,\mathrm{b}] = F(2\,\mathrm{b}) = 2\,\mathrm{b}}$$

$$= (\mathrm{v} - \mathrm{b}) \times 2\,\mathrm{b} = 2\,\mathrm{v}\,\mathrm{b} - 2\,\mathrm{b}^2$$

- To maximize, take derivative $\frac{d}{d\mathrm{b}}$ and set to zero, solve.
- Optimal to bid $\mathrm{b} = \mathrm{v}/2$ $\Rightarrow$ "$b(\mathrm{v}) = \mathrm{v}/2$" is equilibrium.
- Equilibrium welfare: $\mathbf{E}\left[\mathrm{v}_{(1)}\right] = 2/3$; revenue: $\mathbf{E}[\mathrm{v}_{(1)}/2] = 2/3\,1/2 = 1/3$

Section 2

Single-dimensional Environments

References:

1. Hartline (202?) “Mechanism Design and Approximation” Chapter 2

Single-dimensional Linear Environments

Model

- agents: $\{1, \ldots, n\}$; values: $\mathbf{v} = (v_1, \ldots, v_n)$; bids: $\mathbf{b} = (b_1, \ldots, b_n)$
- linear utility: $v_i x_i - p_i$ for allocation $x_i \in [0, 1]$ and payment $p_i \in \mathbb{R}$
- feasibility constraint: $\mathcal{X} \subset [0, 1]^n$
- mechanism $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{p}})$:
  - ex post bid allocation rule: $\tilde{\boldsymbol{x}} : \mathbb{R}^n \to \mathcal{X}$
  - ex post bid payment rule: $\tilde{\boldsymbol{p}} : \mathbb{R}^n \to \mathbb{R}^n$
- welfare: $\sum_i v_i \, \tilde{x}_i(\mathbf{b})$; revenue: $\sum_i p_i(\mathbf{b})$

Example (Single-item Environments; First-price Auction)

- feasibility constraint: $\mathcal{X} = \{\mathbf{x} \subset [0, 1]^n : \sum_i x_i \leq 1\}$
- highest-bid-wins: $\tilde{\boldsymbol{x}}(\mathbf{b}) \in \operatorname{argmax}_{\mathbf{x} \in \mathcal{X}} \sum_i b_i x_i$
- winner-pays-bid: $\tilde{p}_i(\mathbf{b}) = b_i \, \tilde{x}_i(\mathbf{b})$

Encorporating Strategies and Prior Distribution

Three Stages of Mechanism Design

- ex ante: before values are drawn ($\mathbf{v}$ random)
- interim: an agent's perspective at bid time (v_i known; $\mathbf{v}_{-i}$ random)
- ex post: after bids and values are known ($\mathbf{v}$ known)

Value Allocation and Payment Rules

Compose ex post mechanism $(\tilde{x}, \tilde{p})$ and bid strategy b:

- ex post allocation rule: $x(\mathbf{v}) = \tilde{x}(b(\mathbf{v}))$
- ex post payment rule: $p(\mathbf{v}) = \tilde{p}(b(\mathbf{v}))$

Interim Allocation and Payment Rules

- $\tilde{x}_i(b_i) = \mathbf{E}_{\mathbf{v}}[\tilde{x}_i(b_i, b_{-i}(\mathbf{v}_{-i})) \mid v_i]$; $\tilde{p}_i(b_i) = \mathbf{E}_{\mathbf{v}}[\tilde{p}_i(b_i, b_{-i}(\mathbf{v}_{-i})) \mid v_i]$
- $x_i(v_i) = \mathbf{E}_{\mathbf{v}}[x_i(v_i, \mathbf{v}_{-i}) \mid v_i]$; $p_i(v_i) = \mathbf{E}_{\mathbf{v}}[p_i(v_i, \mathbf{v}_{-i}) \mid v_i]$

Note $x : \mathbb{R}^n \to \mathcal{X}$ $x : (\mathbb{R} \to [0,1])^n$ $p : \mathbb{R}^n \to \mathbb{R}^n$ $p : (\mathbb{R} \to \mathbb{R})^n$

Example: First-price Auction

Interim Allocation and Payment Rules

- $\tilde{x}_i(\mathrm{b}_i) = \mathbf{E}_{\mathbf{v}}[\tilde{\boldsymbol{x}}_i(\mathrm{b}_i, \boldsymbol{b}_{-i}(\mathbf{v}_{-i})) \mid \mathrm{v}_i]$; $\tilde{p}_i(\mathrm{b}_i) = \mathbf{E}_{\mathbf{v}}[\tilde{\boldsymbol{p}}_i(\mathrm{b}_i, \boldsymbol{b}_{-i}(\mathbf{v}_{-i})) \mid \mathrm{v}_i]$
- $x_i(\mathrm{v}_i) = \mathbf{E}_{\mathbf{v}}[\boldsymbol{x}_i(\mathrm{v}_i, \mathbf{v}_{-i}) \mid \mathrm{v}_i]$; $p_i(\mathrm{v}_i) = \mathbf{E}_{\mathbf{v}}[\boldsymbol{p}_i(\mathrm{v}_i, \mathbf{v}_{-i}) \mid \mathrm{v}_i]$

Example (two agents, uniform values, first-price auction)

equilibrium bids: $b_i(\mathrm{v}_i) = {}^{\mathrm{v}_i}/_2$; feasibility: $\mathscr{X} = \{\mathbf{x} \subset [0,1]^n : \sum_i \mathrm{x}_i \leq 1\}$

	ex post	interim
bid allocation rule	$\tilde{\boldsymbol{x}}(\mathbf{b}) \in \mathrm{argmax}_{\mathbf{x} \in \mathscr{X}} \sum_i \mathrm{b}_i\, \mathrm{x}_i$	$\tilde{x}_i(\mathrm{b}_i) = 2\,\mathrm{b}_i$
value allocation rule	$\boldsymbol{x}(\mathbf{v}) \in \mathrm{argmax}_{\mathbf{x} \in \mathscr{X}} \sum_i {}^{\mathrm{v}_i}/_2\, \mathrm{x}_i$	$x_i(\mathrm{v}_i) = \mathrm{v}_i$

Section 3

Revenue Equivalence and Applications

References:

1. Myerson (1981) "Optimal Auction Design"
2. Chawla, Hartline (2013) "Auctions with unique equilibria"
3. Hartline (202?) "Mechanism Design and Approximation" Chapter 2

Bayes-Nash Equilibrium

Definition (Bayes-Nash equilibrium, BNE)

A strategy profile $\boldsymbol{b}$ such that for all i and v_i, bidding $\mathrm{b}_i = b_i(\mathrm{v}_i)$ is a best response when other agents bid $\boldsymbol{b}_{-i}(\mathbf{v}_{-i})$ with $\mathbf{v}_{-i} \sim \boldsymbol{F}_{-i}|_{\mathrm{v}_i}$.

Example (Two agents, uniform values, first-price auction)

Strategies $\boldsymbol{b}$ as "$\forall i,\ b_i(\mathrm{v}_i) = \mathrm{v}_i/2$" is a Bayes-Nash equilibrium.

- values are $U[0, 1]$
- bids under $\boldsymbol{b}$ are $U[0, 1/2]$
- best response to bid $U[0, 1/2]$ is $b_i(\mathrm{v}_i) = \mathrm{v}_i/2$

Notation

- value profile w.o. agent i's value: $\mathbf{v}_{-i} = (\mathrm{v}_1, \ldots, \mathrm{v}_{i-1}, ?, \mathrm{v}_{i+1}, \ldots, \mathrm{v}_n)$
- conditional distribution of $\boldsymbol{F}$ given v_i: $\boldsymbol{F}_{-i}|_{\mathrm{v}_i}$ (if indep. $\boldsymbol{F}_{-i}|_{\mathrm{v}_i} = \boldsymbol{F}_{-i}$)

Characterizing Bayes-Nash Equilibrium

Proposition

$(\boldsymbol{x}, \boldsymbol{p})$ *are induced by BNE of some* $\boldsymbol{b}$, $\boldsymbol{F}$, *and* $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{p}})$ *if and only if:*

$$\forall\, i,\ \mathsf{v}_i,\ \mathsf{z}: \quad \mathsf{v}_i\, x_i(\mathsf{v}_i) - p_i(\mathsf{v}_i) \geq \mathsf{v}_i\, x_i(\mathsf{z}) - p_i(\mathsf{z})$$

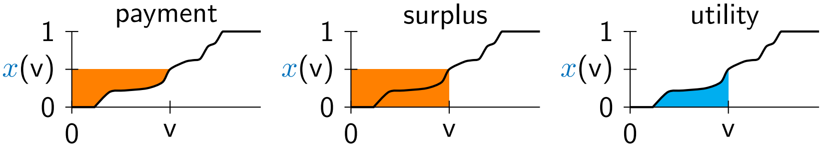
(and bids not in the range of $\boldsymbol{b}$ *are weakly dominated.)*

Theorem (Myerson '81)

$(\boldsymbol{x}, \boldsymbol{p})$ *are induced by BNE of some* $\boldsymbol{b}$, $\boldsymbol{F}$, *and* $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{p}})$ *if and only if:*

- a. *(monotonicity)* x_i *is monotonically non-decreasing*
- b. *(payment identity)* $p_i(\mathsf{v}_i) = \mathsf{v}_i\, x_i(\mathsf{v}_i) - \int_0^{\mathsf{v}_i} x_i(\mathsf{z})\, d\mathsf{z} + p_i(0)$.

(and bids not in the range of $\boldsymbol{b}$ *are weakly dominated; often* $p_i(0) = 0$*)*



Cor: revenue equivalence: same BNE allocation ⇒ same BNE revenue.

Method of Revenue Equivalence

Cor: revenue equivalence: same BNE allocation $\Rightarrow$ same BNE revenue.

Method of Revenue Equivalance

Equate two equations for payments:

a. payments from mechanism rules.
b. payments from revenue equivalance.

Two examples: for i.i.d. first-price auctions

1. solving for symmetric Bayes-Nash equilibrium
2. non-existence of symmetric Bayes-Nash equilbrium

Consequence: BNE of FPA is unique, symmetric, and welfare-maximal.

Solving for Bayes-Nash Equilibrium

Method of Revenue Equivalance

Equate two equations for payments:

a. payments from mechanism rules.
b. payments from revenue equivalance.

Thm: In i.i.d. FPA, symmetric BNE is: $b(\mathrm{v}) = \mathbf{E}\left[\mathrm{v}_{(2)} \mid \mathrm{v}_{(2)} < \mathrm{v}\right]$

Proof.

- Guess highest-valued agent wins.
- FPA is revenue equivalent to SPA.
- by mech. rules: $\mathbf{E}[\text{SPA payment for v} \mid \text{v wins}] = \mathbf{E}\left[\mathrm{v}_{(2)} \mid \mathrm{v}_{(2)} < \mathrm{v}\right]$
- by mech. rules: $\mathbf{E}[\text{FPA payment for v} \mid \text{v wins}] = b(\mathrm{v})$
- revenue equivalence: $b(\mathrm{v}) = \mathbf{E}\left[\mathrm{v}_{(2)} \mid \mathrm{v}_{(2)} < \mathrm{v}\right]$
- check guess: $\mathbf{E}\left[\mathrm{v}_{(2)} \mid \mathrm{v}_{(2)} < \mathrm{v}\right]$ is monotone in v $\square$

Non-existence of Asymmetric BNE

Restriction for Lecture: single-item auction, continuous strategies

Theorem

i.i.d. 2-agent first-price auction with (unknown) random reserve has no asymmetric equilibrium *(continuous, bounded values)*

Corollary

i.i.d. n-agent first-price auctions have no asymmetric equilibria.

Proof of Corollary.

- agent 1 and 2 face random reserve "$\max(b_3, \ldots, b_n)$"
- by theorem, their strategies are symmetric.
- same for player 1 and i.
- so all strategies are symmetric. □

Proof of Theorem

Revenue equivalence $\Rightarrow$ two formulas for agent's utility:

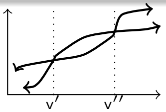
1. $u(\mathsf{v}) = (\mathsf{v} - b(\mathsf{v}))\, x(\mathsf{v})$ (first-price payment rule)
2. $u(\mathsf{v}) = \int_0^{\mathsf{v}} x(\mathsf{z})\, d\mathsf{z}$ (paymend identity / revenue equivalence)

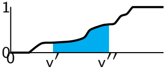
Theorem

i.i.d. 2-agent first-price auction with (unknown) random reserve has no asymmetric equilibrium *(continuous, bounded values)*

Proof (by contradiction):

- assume strategies cross twice at v' and v''
- so by Lemma, $x_1(\mathsf{v}) > x_2(\mathsf{v})$ for $\mathsf{v} \in (\mathsf{v}', \mathsf{v}'')$
- so by (2): $u_1(\mathsf{v}'') - u_1(\mathsf{v}') = \int_{\mathsf{v}'}^{\mathsf{v}''} x_1(\mathsf{z})\, d\mathsf{z}$
  $> \int_{\mathsf{v}'}^{\mathsf{v}''} x_2(\mathsf{z})\, d\mathsf{z} = u_2(\mathsf{v}'') - u_2(\mathsf{v}')$
- but by Lemma and (1): $u_1(\mathsf{v}') = u_2(\mathsf{v}')$ and $u_1(\mathsf{v}'') = u_2(\mathsf{v}'')$ $\square$



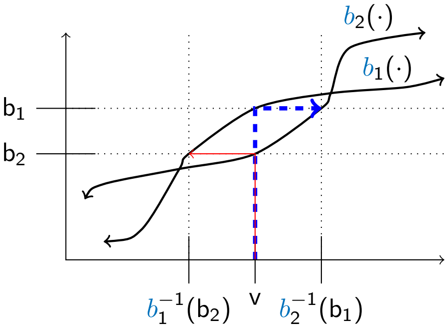


Lem: At v if $b_1(\mathsf{v}) > b_2(\mathsf{v})$ then $x_1(\mathsf{v}) > x_2(\mathsf{v})$, and equal if equal.

Proof of Lemma

Lem: At v if $b_1(\mathrm{v}) > b_2(\mathrm{v})$ then $x_1(\mathrm{v}) > x_2(\mathrm{v})$, and equal if equal.

Proof by Picture.



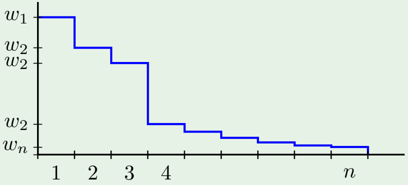
□

I.i.d. winner-pays-bid position auctions

Definition (Winner-pays-bid Position Auction [cf. Edelman, Ostrovsky, Schwarz '07])

- n positions, allocation probabilities $\mathbf{w}$ with $w_1 \geq \ldots \geq w_n$,
- agents assigned to positions in order of bid,
- agents pay bid if allocated.

Example



Theorem (Chawla, Hartline '13)

BNE of i.i.d. winner-pays-bid postion auction is unique, symmetric, and welfare-optimal.

Section 4

Robust Analysis of Equilibria

References:

1. Borodin, Lucier (2010) "Price of anarchy for greedy auctions"
2. Syrgkanis, Tardos (2013) "Composable and efficient mechanisms"
3. Hoy, Hartline, Taggart (2014) "Price of anarchy for auction revenue"
4. Dütting, Kesselheim (2015) "Algorithms against anarchy: Understanding non-truthful mechanisms"
5. Hoy, Nekipelov, Syrgkanis (2017) "Welfare guarantees from data"
6. Hartline (202?) "Mechanism Design and Approximation" Chapter 6

Winner-pays-bid Mechanisms

Definition (Winner-pays-bid Mechanism)

1. solicit bids.
2. run allocation algorithm. $\leftarrow$ which algorithms are good?
3. winners pay their bids.

Definition (Highest-bids-win)

allocate to the feasible set of agents with highest total bid.

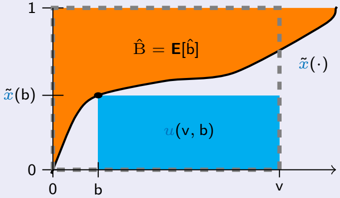
Robust Analysis of Welfare

Geometry of Best Response

"utility or competition is high"

- winner-pays-bid utility:
  $u(\mathrm{v}, \mathrm{b}) = (\mathrm{v} - \mathrm{b})\,\tilde{x}(\mathrm{b})$
- $\tilde{x}$ is cdf of rand. critical bid $\hat{\mathrm{b}}$
- expected critical bid:

$$\hat{\mathrm{B}} = \mathbf{E}_{\hat{\mathrm{b}} \sim \tilde{x}}\left[\hat{\mathrm{b}}\right] = \int_0^\infty (1 - \tilde{x}(\mathrm{z}))\, dz$$



Lem: In BNE: $u(\mathrm{v}) + \hat{\mathrm{B}} \geq {}^{e-1}/_{e}\, \mathrm{v}$

Definition (conversion ratio μ)

"high competition $\Rightarrow$ high rev"

$$\mu = \max_{\mathbf{b}} \mathrm{OPT}(\hat{\mathbf{B}}) / \mathrm{Rev}(\mathbf{b})$$

Theorem

BNE welfare is $\mu\, {}^{e}/_{e-1}$-approx.

Proof.

From lemma:

$$\mathrm{u}_i(\mathrm{v}_i) + \hat{\mathrm{B}}_i \geq {}^{e-1}/_{e}\, \mathrm{v}_i$$

For welfare-otimal $\boldsymbol{x}^\star(\mathbf{v})$:

$$\mathrm{u}_i(\mathrm{v}_i) + \hat{\mathrm{B}}_i\, x_i^\star(\mathbf{v}) \geq {}^{e-1}/_{e}\, \mathrm{v}_i\, x_i^\star(\mathbf{v})$$

Sum over agents i:

$$\mathrm{Util}(\mathbf{v}) + \mu\, \mathrm{Rev}(\mathbf{v}) \geq {}^{e-1}/_{e}\, \mathrm{OPT}(\mathbf{v})$$

Take expectations:

$$\mu\, \mathrm{Welfare} \geq {}^{e-1}/_{e}\, \mathrm{OPT} \qquad \square$$

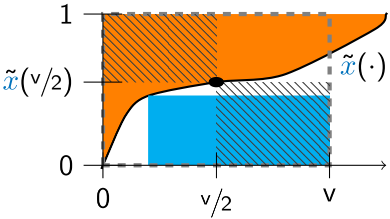
Proof of Lemma

Lemma

In BNE: $\mathrm{u}(\mathrm{v}) + \hat{\mathrm{B}} \geq {}^{e-1}\!/_{e}\,\mathrm{v}$

Proof.

- By geometry: $\mathrm{u}(\mathrm{v}) + \hat{\mathrm{B}} \geq {}^{1}\!/_{2}\,\mathrm{v}$



- More careful analysis gives ${}^{e-1}\!/_{e}$. □

Analysis of Conversion Ratio

Definition (conversion ratio μ)

"high competition $\Rightarrow$ high rev"

$\mu = \max_{\mathbf{b}} \mathsf{OPT}(\hat{\mathbf{B}}) / \mathsf{Rev}(\mathbf{b})$

Theorem

Conversion ratio of first-price auction is $\mu = 1$.

Proof.

- for any **b**
- expected critical bids are $\hat{\mathrm{B}}_i = \hat{\mathrm{b}}_i = \max_{j \neq i} \mathrm{b}_j$
- $\mathsf{Rev}(\mathbf{b}) = \max_i \mathrm{b}_i \geq \max_i \hat{\mathrm{B}}_i = \mathsf{OPT}(\hat{\mathbf{B}})$ $\square$

Properties of Conversion Ratio

- not an equilibrium property.
- tight in some environments.
- closed under simultaneous composition.
- closed under randomization.

Example: Single-minded Combinatorial Auctions

Theorem (e.g., Lucier, Borodin '10)

winner-pays-bid highest-bids-win mechanisms can have very bad equilibria.

Example (Single-minded Combinatorial Auction)

Preferences:

- m items; $m + 2$ agents.
- agent $i \in \{1, \ldots, m\}$ values bundle $\mathrm{S}_i = \{i\}$ at $\mathrm{v}_i = 1$.
- agent $h \in \{m+1, m+2\}$ values bundle $\mathrm{S}_h = \{1, \ldots, m\}$ at $\mathrm{v}_h = 1$.

A Nash equilibrium:

- agents $h \in \{m+1, m+2\}$ bid $\mathrm{b}_h = 1$ (one wins, one loses)
- agents $i \in \{1, \ldots, m\}$ bid $\mathrm{b}_i = 0$ (all lose)
- all agent utilities $= 0$ for bids ≤ 1.

Nash welfare $= 1$; optimal welfare $= m$.

Conversation ratio is $\mu = m$:

$\hat{\mathrm{B}}_i = 1$; $\hat{\mathrm{B}}_h = 1$; $\mathrm{OPT}(\hat{\mathbf{B}}) = m$; $\mathrm{Rev}(\mathbf{b}) = 1$.

Greedy Single-minded Combinatorial Auction

Definition (Greedy Winner-pays-bid Mechanism)

1. bidders bid, 2. allocate greedily by $\phi_i(\mathrm{b}_i)$, 3. winners pay their bids.

Theorem (Hartline, Hoy, Taggart '14)

Conversion ratio μ of greedy winner-pays-bid mechanism equals approximation ratio β of greedy algorithm.

Theorem (Lehmann, O'Callaghan, Shoham '02)

Greedy by $\mathrm{b}_i/\sqrt{|\mathrm{S}_i|}$ winner-pays-bid algorithm is $\beta = \sqrt{m}$ approximation.

Corollary (cf. Borodin, Lucier '10)

The BNE welfare of the greedy winner-pays-bid mechanism is $\sqrt{m}\, {}^{e}/_{e-1}$.

Theorem (Dütting, Kesselheim '15)

Conversion ratio μ for any winner-pays-bid single-minded CA is $\Omega(\sqrt{m})$.

Qstn How can near optimal non-truthful mechaisms be designed?

Parts II and III

Part II: Non-truthful Sample Complexity

1. Counterfactual Estimation
2. I.i.d. Position Auctions
3. General Reduction to I.i.d. Position Auctions

Part III: Simplicity, Robustness, & the Revelation Gap

1. Revelation Gap
2. Implementation Theory