

A/B Testing of Auctions

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— A Grand Challenge for CS —

A Grand Challenge: understand and guide computation in the wild

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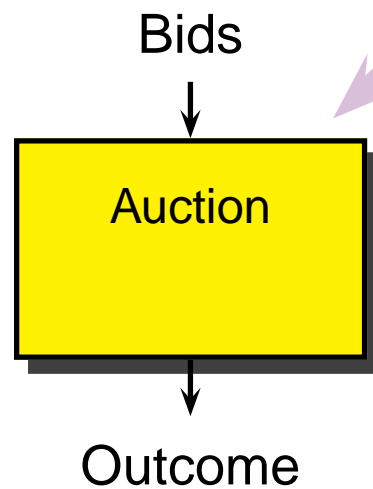
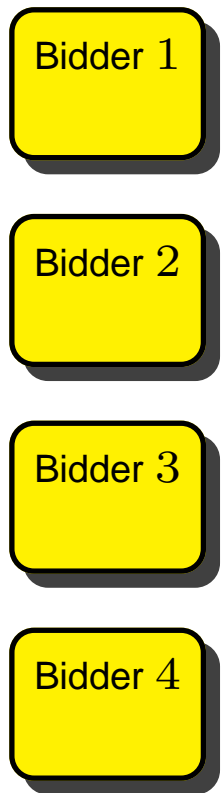
A Grand Challenge for CS

A Grand Challenge: understand and guide computation in the wild

- **computational primitive:** local/individual/strategic optimization.
- **objective:** good global outcomes
- **a key application area:** “online markets”
uber, airbnb, twitter, stackexchange, tinder, ...

Example: First-price Auction

Bidders with private preferences



Economic Mechanism

Who wins, what they pay.

Example: First-price Auction

Bidders with private preferences

Bidder 1
\$4

Bidder 2
\$12

Bidder 3
\$2

Bidder 4
\$6

Bids

Auction

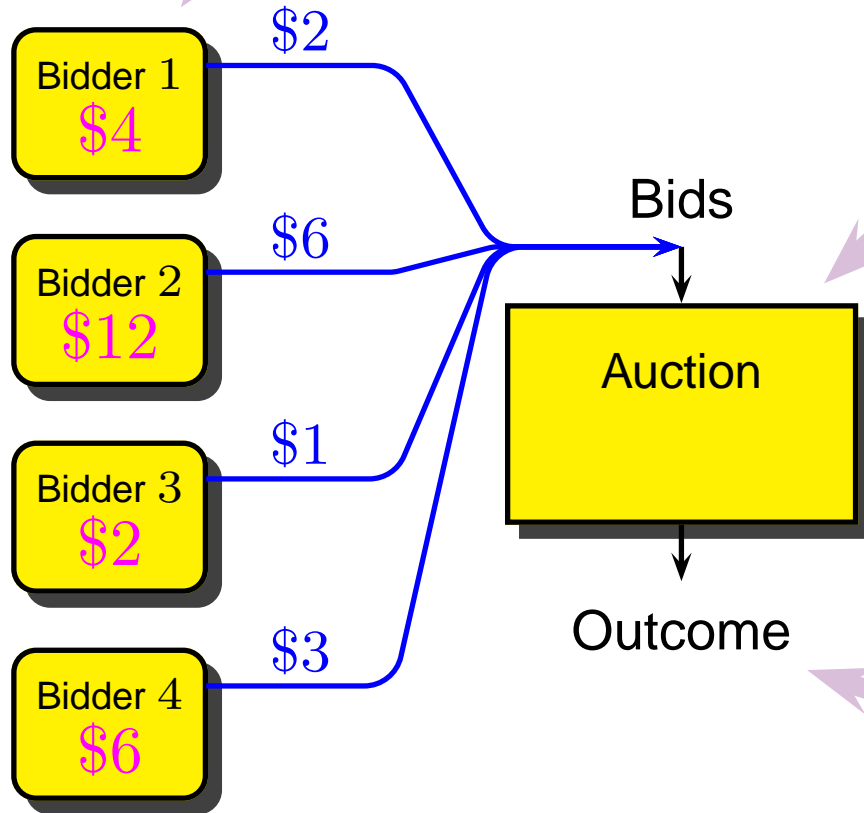
Outcome

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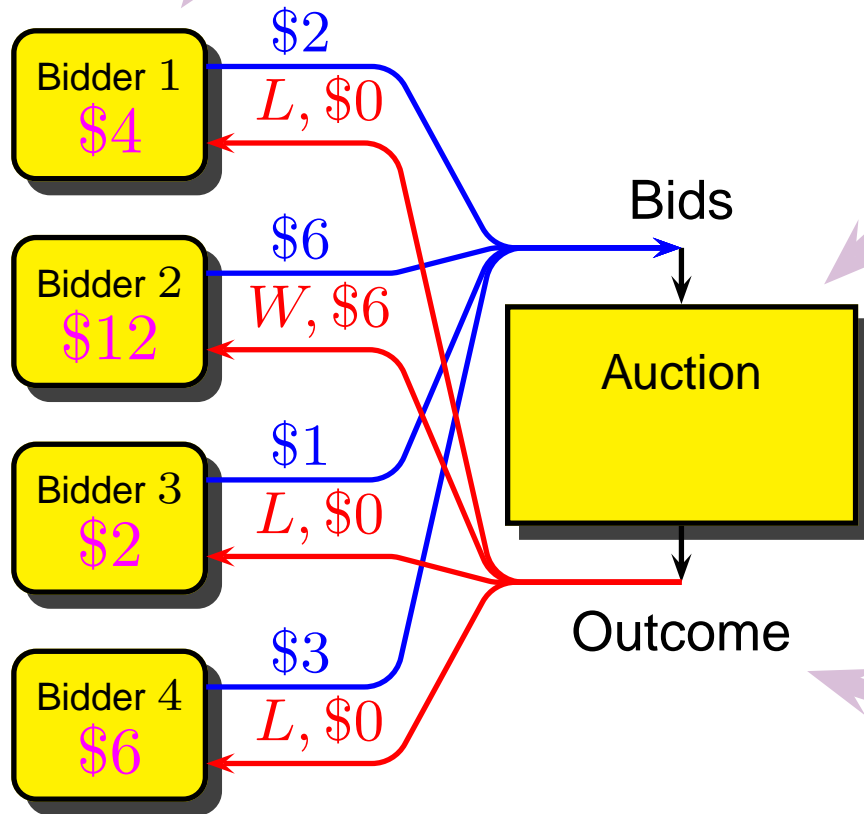


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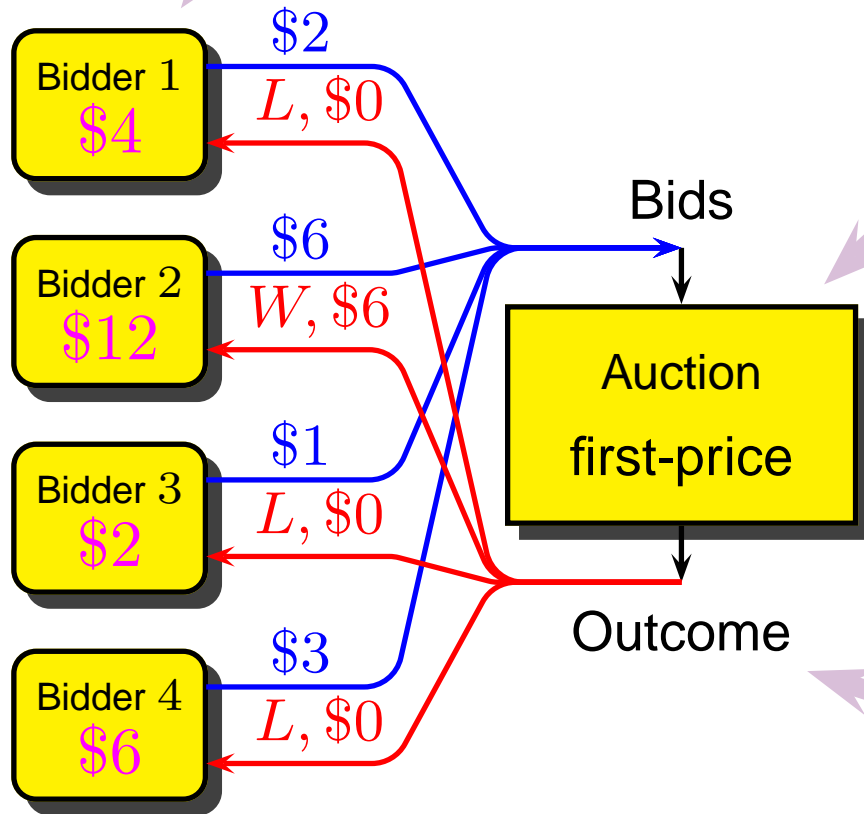


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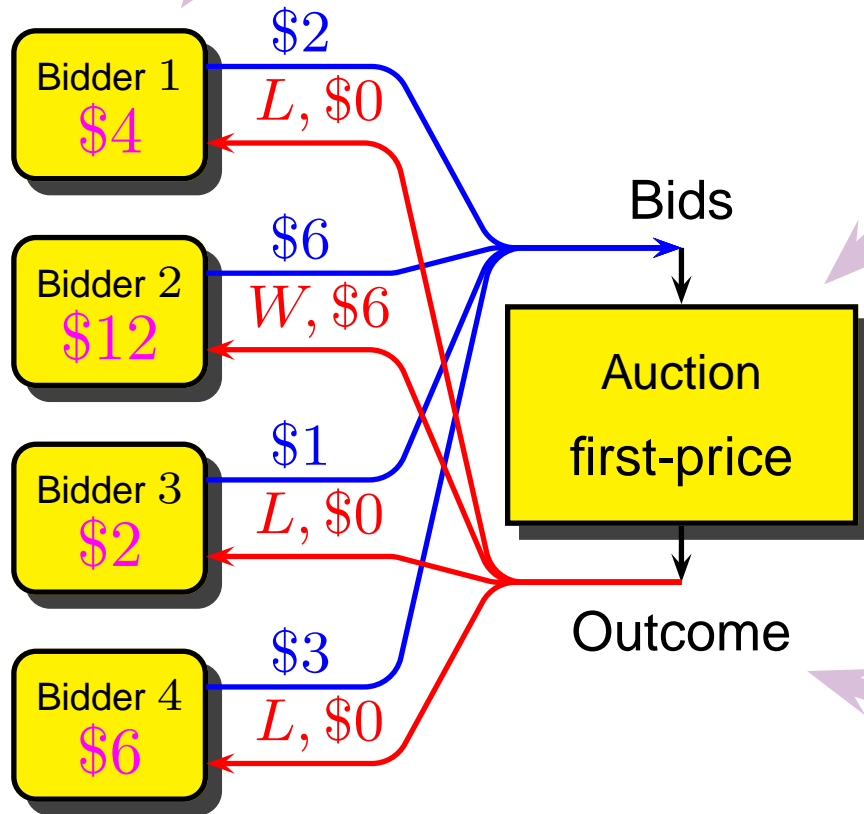


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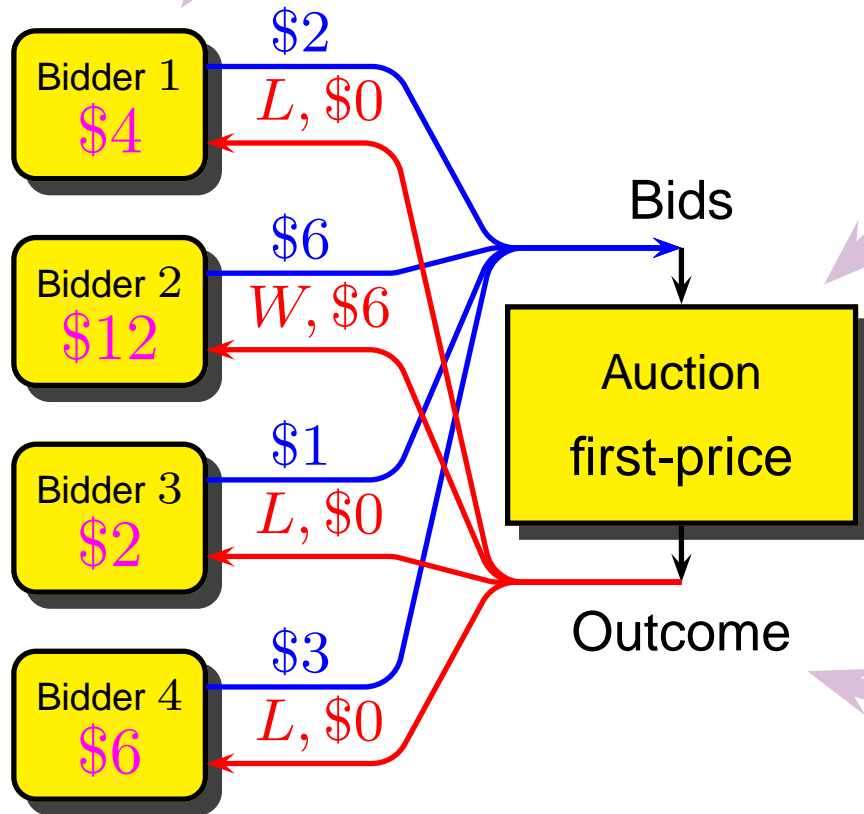
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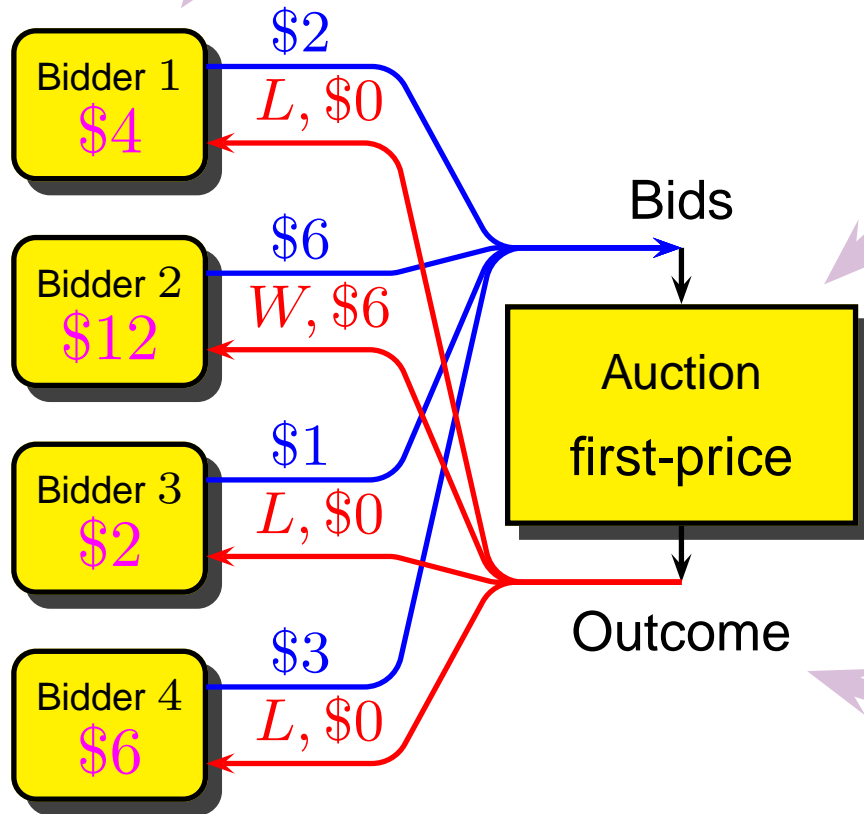
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Challenges: need to rethink classical

- algorithms
- data science

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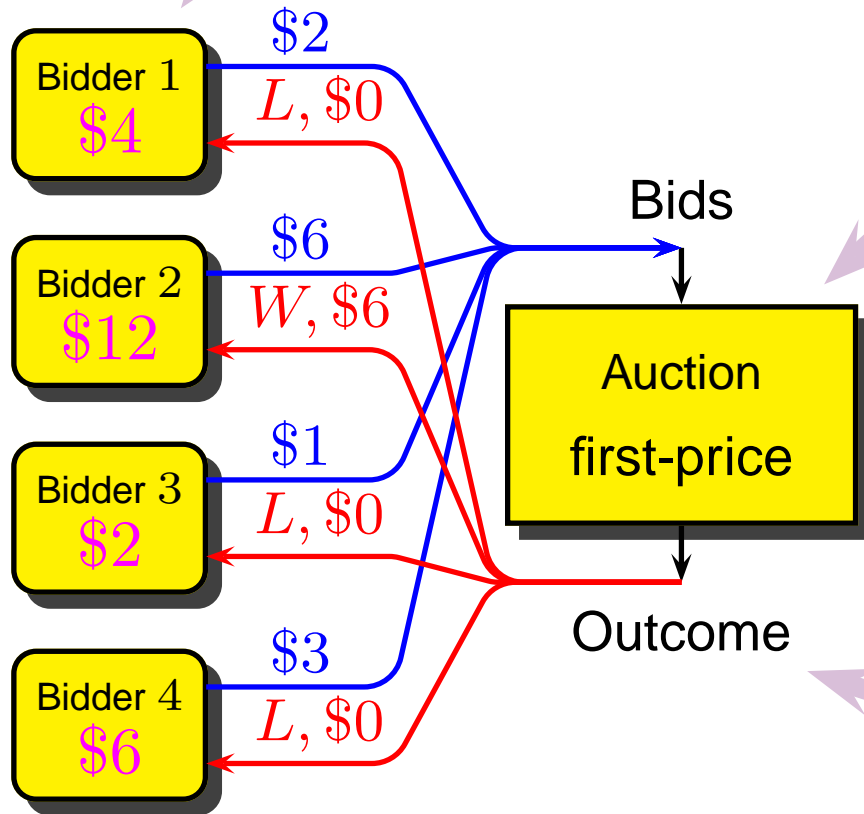
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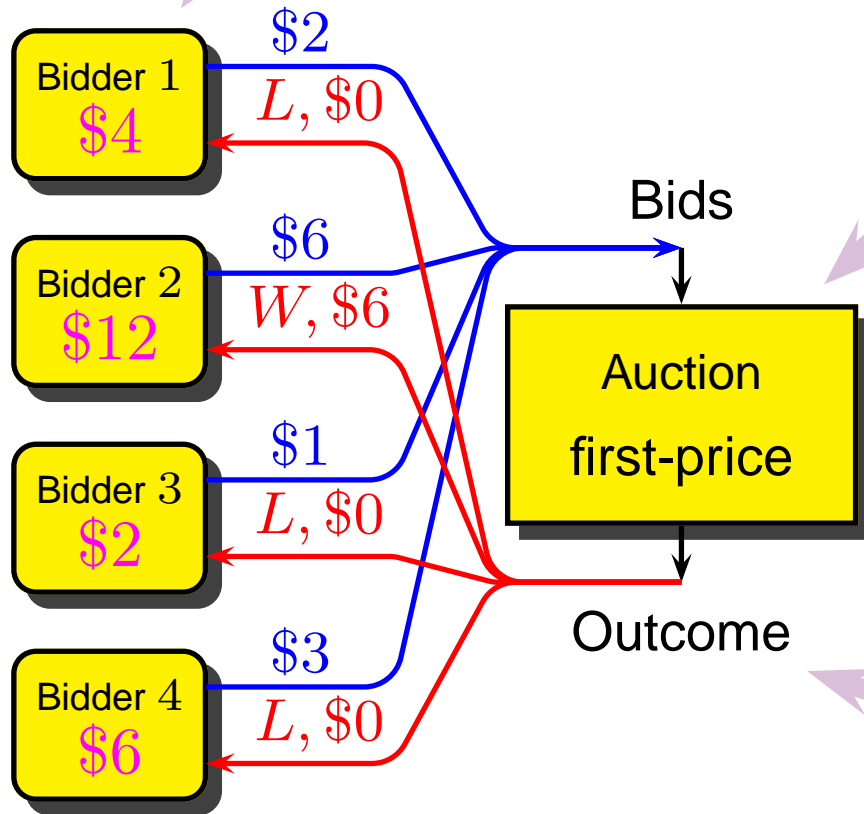
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Economic Mechanism

Note: output affects input.

Challenges: need to rethink classical

- algorithms [since 1999]
⇒ algorithmic mechanism design
- data science [since 2014; this talk]

Who wins, what they pay.

Motivating Example

Search Engine Advertising

one mainline ad

The screenshot shows a Google search results page for the query "search advertising". The browser's address bar shows the URL: <https://www.google.com/webhp?sourceid=chrome-instant&ion=1&espy=2&ie=UTF-8#q=search%20advertising>. The search bar contains the text "search advertising". Below the search bar, the results are categorized into "Web", "News", "Images", "Videos", "Shopping", and "More". The "Web" category is selected, showing approximately 823,000,000 results in 0.39 seconds.

The results are divided into two columns. The left column contains organic search results, and the right column contains paid advertisements.

Organic Search Results (Left Column):

- Bing Online Advertising - Grow Your Business & Stand Out**
bingads.microsoft.com/Advertising (800) 518-5689
4.3 ★★★★★ rating for microsoft.com
Advertise On The Yahoo Bing Network
Easy Import From AdWords Get Started Today
Bing Customer Testimonial
- Scholarly articles for search advertising**
... search advertising in microsoft's bing search engine - Graepel - Cited by 151
... : A model of spillover in paid search advertising - Rutz - Cited by 154
... between organic and sponsored search advertising: ... - Yang - Cited by 133
- Search advertising - Wikipedia, the free encyclopedia**
en.wikipedia.org/wiki/Search_advertising Wikipedia
In Internet Marketing, Search Advertising is a method of placing online advertisements on Web pages that show results from search engine queries. Through the ...
Origins - Keywords - Metrics - Campaign Management
- Search Advertising: Management Tools for Internet Marketers**
www.wordstream.com/search-advertising
Search advertising is all about selecting the right keywords. Learn more about advertising with search engines and PPC search engine advertising.

Advertisements (Right Column):

- Search Engine Marketing**
www.google.com/AdWords
Bring new visitors to your website.
Place Your Ad on Google Today!
- Better PPC Advertising**
www.advertise.com/
Stop Losing Money on Search Ads.
Low Cost. Top ROI. Quality Traffic.
- Hire Search Advertisers**
www.odesk.com/Search-Advertiser
Reach People. Grow Your Business.
Post Jobs in Minutes! Compare Bids.
- AD Search Tool**
www.netwrix.com/go/auditor_ad
Get Reported & Alerted In Real Time
— Track All Critical Changes!
- Don't Sign Up For AdWords**
www.jumpfly.com/Google-AdWords
Before You Check Out JumpFly.

three mainline ads

The screenshot shows a Google search results page for the query "search engine marketing". The browser's address bar shows the URL: <https://www.google.com/webhp?sourceid=chrome-instant&ion=1&espv=2&ie=UTF-8#q=search%20engine%20marketing>. The search bar contains the text "search engine marketing". Below the search bar, the results are categorized by "Web", "News", "Images", "Books", "Videos", "More", and "Search tools". The search results show "About 112,000,000 results (0.67 seconds)".

The results are divided into two columns. The left column contains organic search results, and the right column contains paid advertisements.

Organic Search Results (Left Column):

- eGumball Local Search SEO - egumball.com**
Ad <http://www.egumball.com/> 2014 Inc 500 Magazine Award Winner. #13 Ad/Marketing. Call Today!
Optimization Costs Local Maps Optimization
Adwords PPC Experts
- Search Engine Marketing - reachlocal.com**
Ad www.reachlocal.com/sem (855) 770-4343
Award Winning Search Marketing. Connect w/ a Marketing Expert Now.
ReachLocal has 1,990 followers on Google+
Marketing Solutions - Digital Marketing Costs - Contact Us - Client Reviews
- Search Engine Marketing - puredigitalco.com**
Ad www.puredigitalco.com/search-marketing
Performance Driven Search Marketing Company. Get a Risk Free Audit Now!
- Search engine marketing - Wikipedia, the free encyclopedia**
en.wikipedia.org/wiki/Search_engine_marketing Wikipedia
Search engine marketing (SEM) is a form of Internet marketing that involves the promotion of websites by increasing their visibility in search engine results ...
Market - History - Methods and metrics - Paid inclusion
- What Is Search Marketing? - Search Engine Land**
searchengineland.com/guide/what-is-sem

Advertisements (Right Column):

- Search Engine Marketing**
www.google.com/AdWords
Promote Your Business on Google.
Learn More about AdWords Today.
- SEO Tools for Free**
www.moz.com/
Check ranks, understand links, and help your sites traffic. Free trial
- Search Engine Marketing**
www.websitesep.com/
Affordable Top Search Placement Packages. Flat Fees & Fast Setup!
- 3Q Digital SEM**
www.3qdigital.com/SEM
The Bay Area's proven SEM Agency.
Increase ROI and grow your business
- Proven SEM Specialists**
www.jumpfly.com/
Exceptional Search Engine Marketing

— A/B testing —

Question: how many mainline ads to show?

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Example: estimating ads *click-through rates*.

1. user searches.
2. page layout A or B is shown.
3. user reacts.

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Example: estimating *auction revenue*.

1. advertisers bid.
2. searches randomized to page layout A or B.
3. outcome reported.

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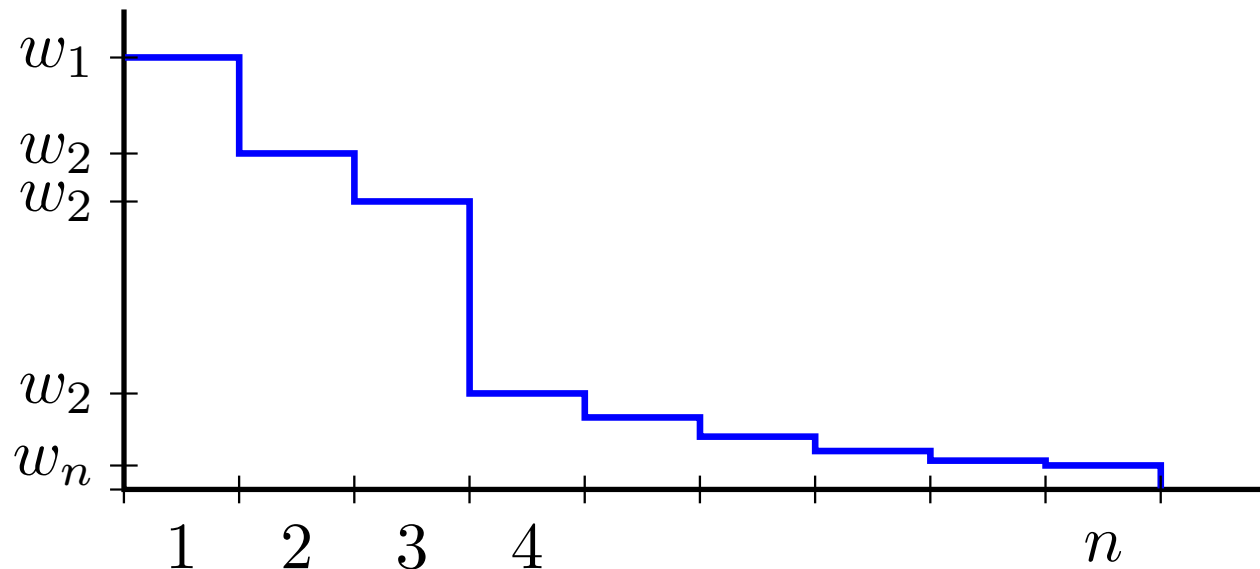
1. advertisers bid.
2. searches randomized to page layout A or B.
3. outcome reported.

Note: bids in A/B test are neither for A nor B, but $C = 0.5A + 0.5B$.

First-price Position Auctions

“First-price” Position Auction: [Varian '06; Edelman et al. '07]

- n bidders, n positions, click probabilities w with $w_1 \geq \dots \geq w_n$.
- bidders assigned to positions in order of bid.
- bidders pay bid if clicked.

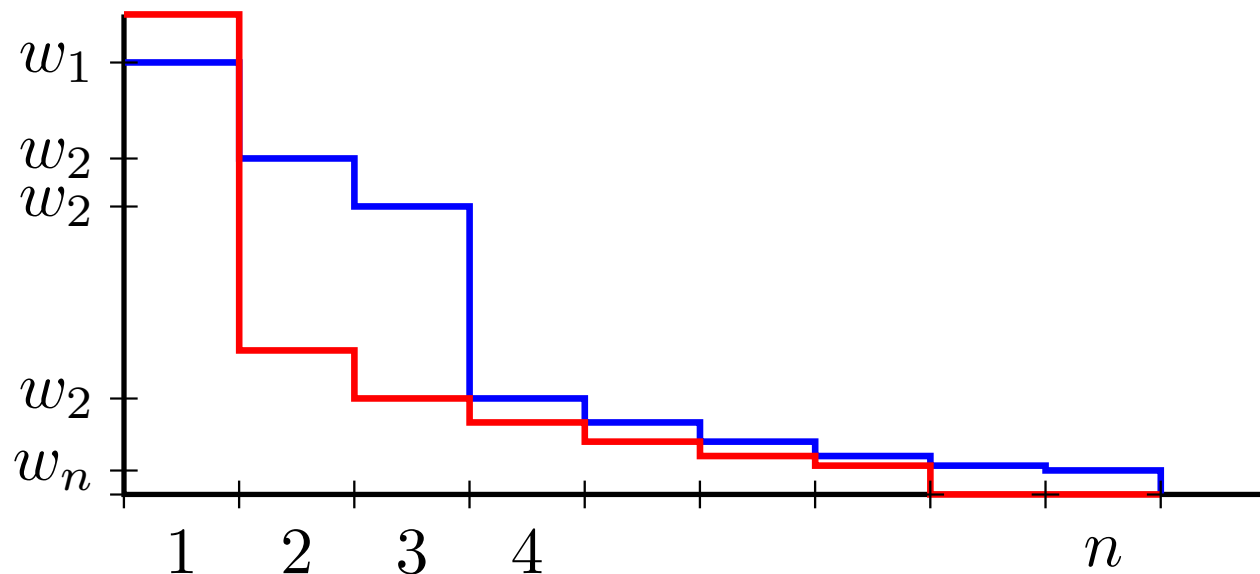


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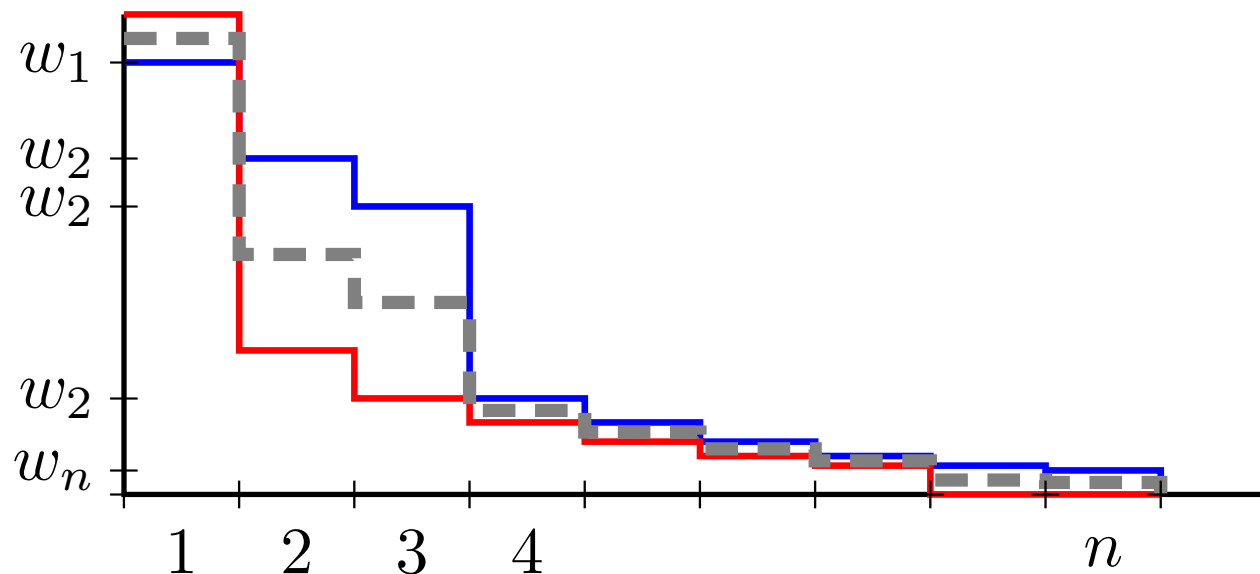


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Example: **A** = three mainline ads; **B** = one mainline ad; **C** = mix.

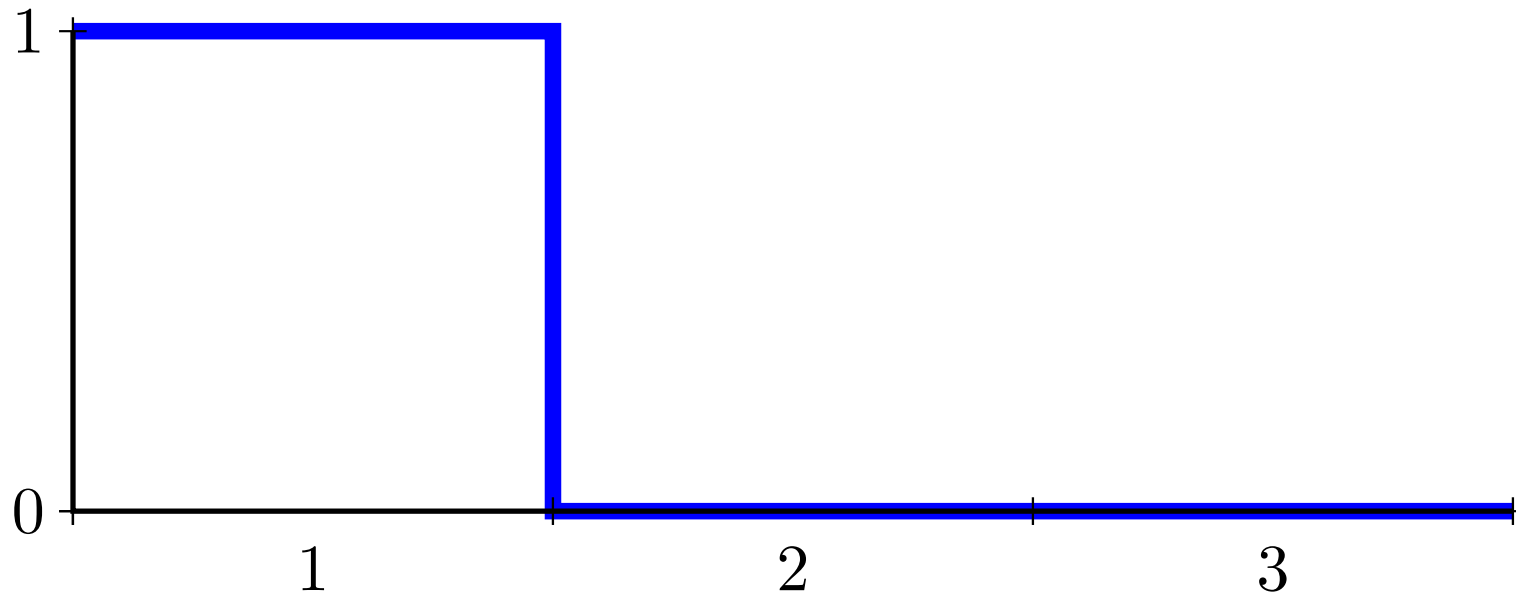
Toy Example

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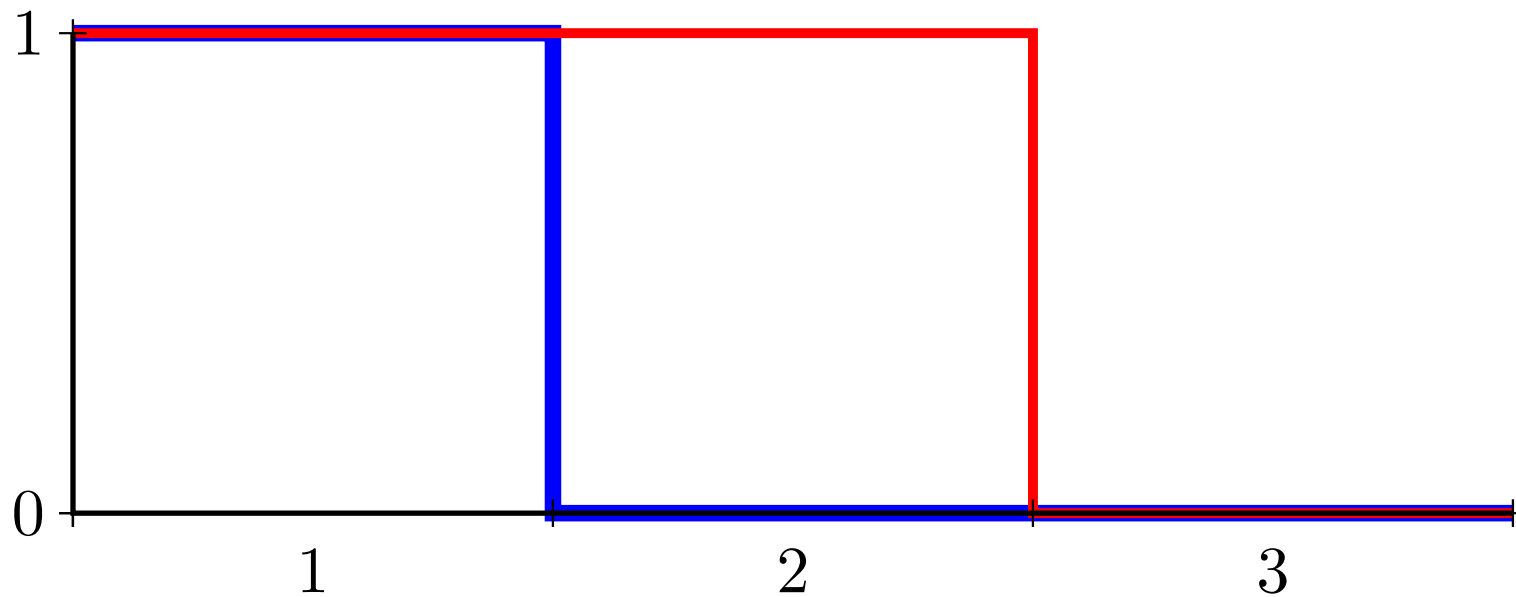
- Auction A: one unit.



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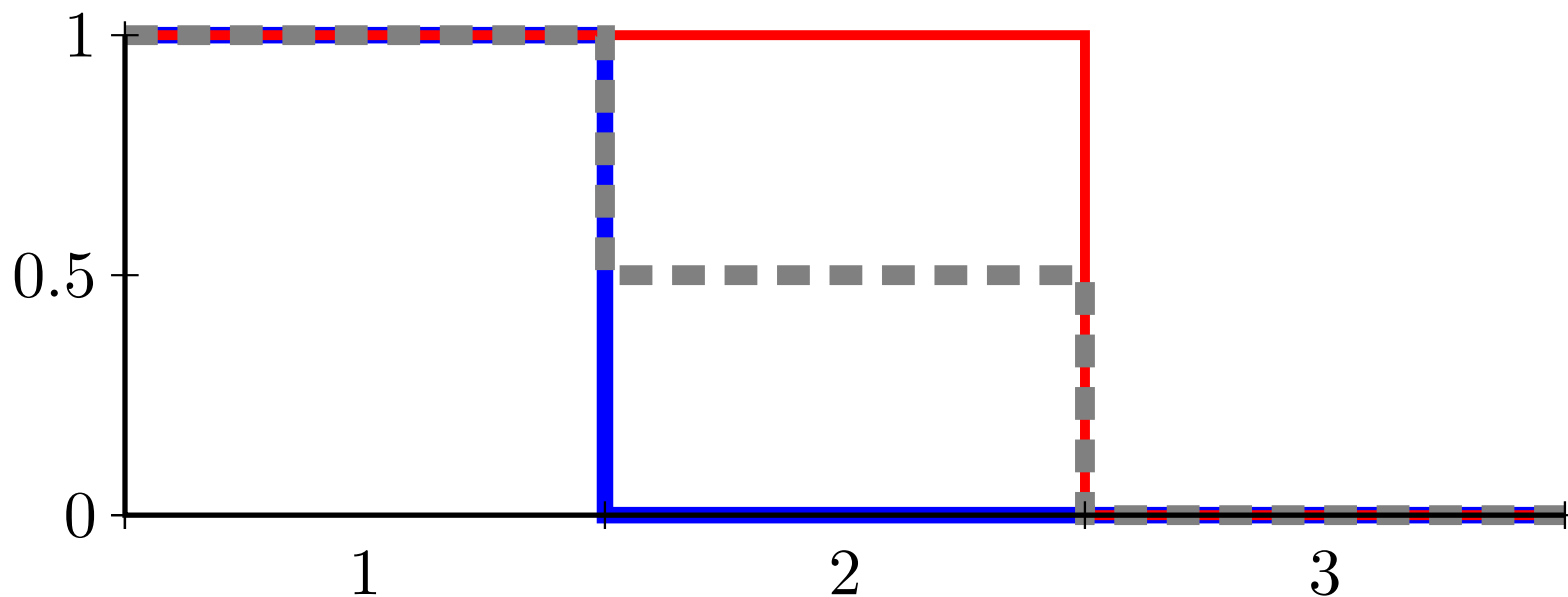
- Auction A: one unit.
- Auction B: two units.



Toy Example

Toy Example: three bidders, highest-bidders win, first-price.

- Auction A: one unit.
- Auction B: two units.
- Auction C: mix $0.5A + 0.5B$.



Improper A/B Test: $C = 0.5A + 0.5B$

Auction	Bid 1	Bid 2	Bid 3	Rev C
1A	0.74	0.34	0.11	0.74
2A	0.08	0.86	0.50	0.86
3B	0.69	0.83	0.46	1.53
4B	0.53	0.03	0.77	1.30
5A	0.91	0.49	0.54	0.91
6A	0.44	0.35	0.92	0.92
7A	0.86	0.97	0.85	0.97
8B	0.21	0.10	0.30	0.51
⋮	⋮	⋮	⋮	⋮
200B	0.13	0.30	0.98	1.28
Average				0.98

Improper A/B Test: $C = 0.5A + 0.5B$

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1A	0.74	0.34	0.11	0.74	0.74	0.00
2A	0.08	0.86	0.50	0.86	0.86	0.00
3B	0.69	0.83	0.46	1.53	0.00	1.53
4B	0.53	0.03	0.77	1.30	0.00	1.30
5A	0.91	0.49	0.54	0.91	0.91	0.00
6A	0.44	0.35	0.92	0.92	0.92	0.00
7A	0.86	0.97	0.85	0.97	0.97	0.00
8B	0.21	0.10	0.30	0.51	0.00	0.51
⋮	⋮	⋮	⋮	⋮	⋮	⋮
200B	0.13	0.30	0.98	1.28	0.00	1.28
Average				0.98	0.75	1.19

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1A	0.74	0.34	0.11	0.74	0.74	0.00
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Note: Improper A/B test always shows $A < B$.

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2A	0.08	0.86	0.50	0.86	0.86	0.00	1.14	0.00
3B	0.69	0.83	0.46	1.53	0.00	1.53	0.00	1.20
4B	0.53	0.03	0.77	1.30	0.00	1.30	0.00	1.08
5A	0.91	0.49	0.54	0.91	0.91	0.00	1.21	0.00
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8B	0.21	0.10	0.30	0.51	0.00	0.51	0.00	0.48
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
200B	0.13	0.30	0.98	1.28	0.00	1.28	0.00	0.95
Average				0.98	0.75	1.19	1.01	0.96

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200B	0.13	0.30	0.98	1.28	0.00	1.28	0.00	0.95
Average				0.98	0.75	1.19	1.01	0.96

Note: Improper A/B test always shows $A < B$.

Missing effect: more units \Rightarrow lower bids.

Outline

0. Improper A/B testing.
1. Overview of Results.
2. Economic inference (get values from bids).
3. Auction revenue analysis (revenue from values).
4. Direct estimation of revenue from bids.

Results Overview

Results: N bid samples, n positions, auction B with probability ϵ .

1. Can estimate revenue of A and B directly from bids in C.
2. Revenue estimator is a *weighted order statistic*.
3. “Revenue B” estimator has error: $O\left(\frac{1}{\sqrt{N}} n \log \frac{n}{\epsilon}\right)$.

Note: “Ideal A/B test” error: $O\left(\frac{1}{\sqrt{N}} n \frac{1}{\sqrt{\epsilon}}\right)$.

4. A universal B test.
5. Can optimize revenue over all feasible position auctions.

Simulation Results (Normalized)

Theoretical Bound: error is $O\left(\frac{1}{\sqrt{N}} n \log \frac{n}{\epsilon}\right)$. (ϵ prob. on B)

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$n =$	$N =$				
	10^1	10^2	10^3	10^4	10^5
2^1	0.1215	0.1150	0.1169	0.1177	0.1196
2^2	0.0814	0.0605	0.0582	0.0596	0.0642
2^3	0.0779	0.0653	0.0652	0.0672	0.0661
2^4	0.0690	0.0621	0.0612	0.0646	0.0623
2^5	0.0566	0.0522	0.0494	0.0508	0.0487
2^6	0.0425	0.0358	0.0355	0.0356	0.0349
2^7	0.0230	0.0281	0.0241	0.0248	0.0253

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2^5	0.0566	0.0522	0.0494	0.0508	0.0487
2^6	0.0425	0.0358	0.0355	0.0356	0.0349
2^7	0.0230	0.0281	0.0241	0.0248	0.0253

Note: constant with N as expected; dependence on n is not tight.

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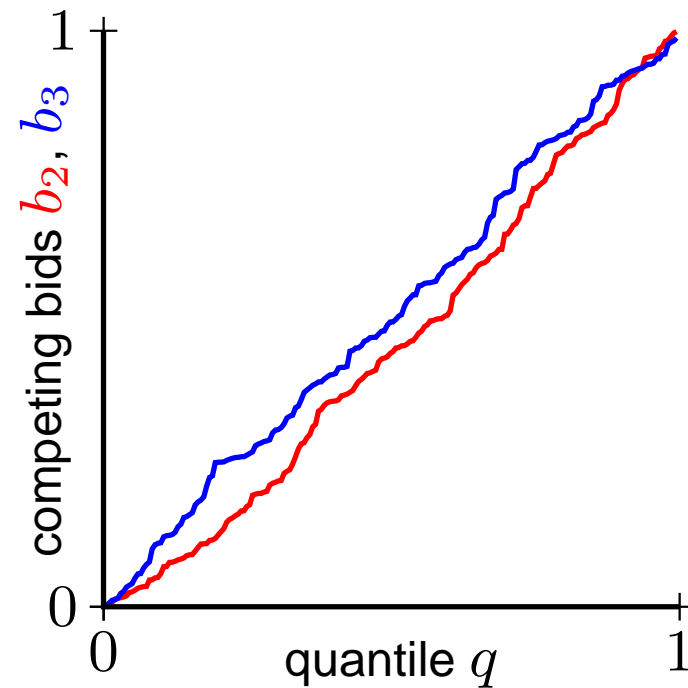
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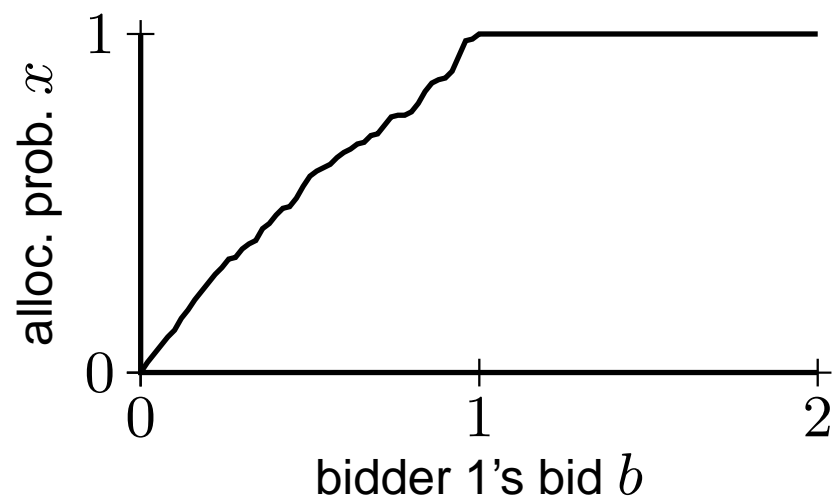
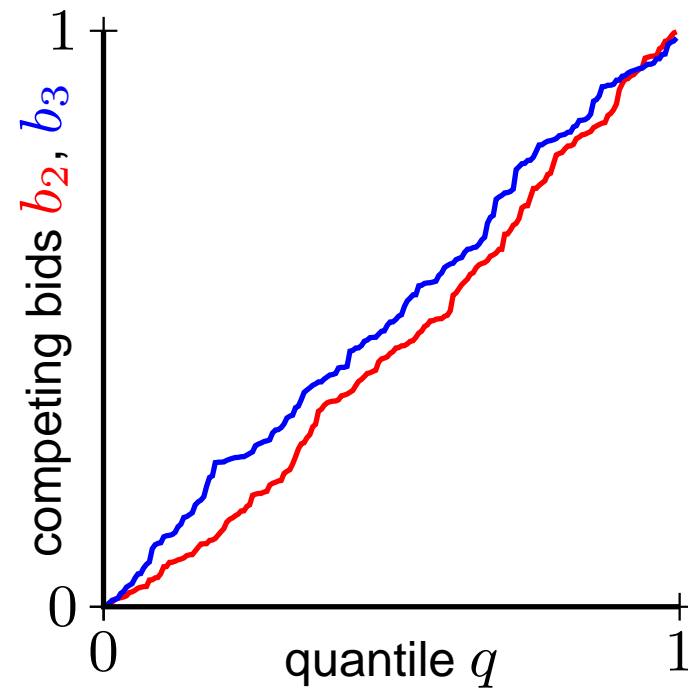


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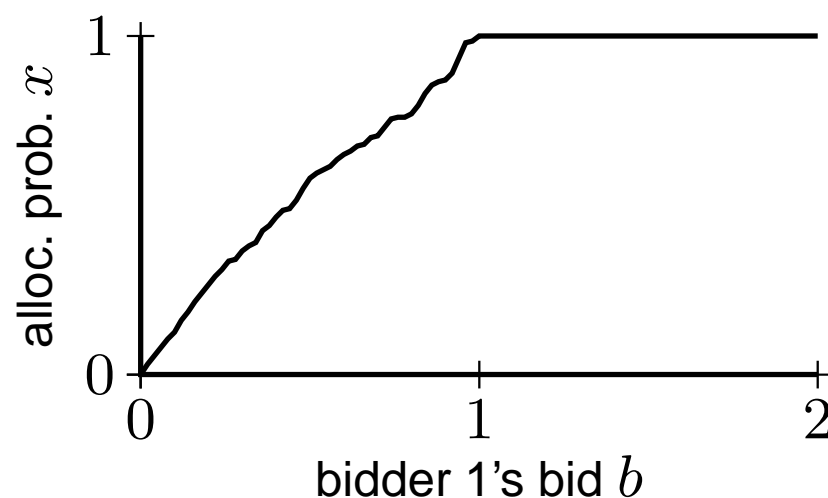
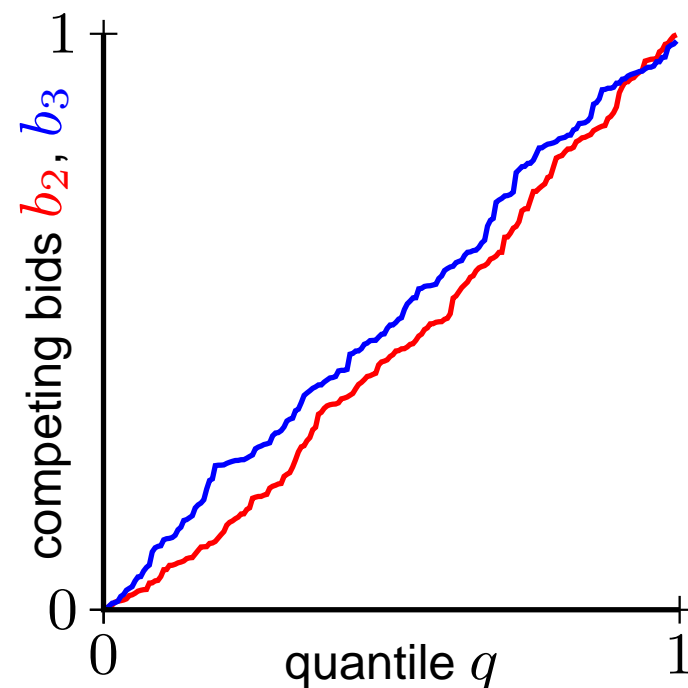
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Approach:

1. given bid distribution, solve for bid strategy,
2. invert bid strategy to get bidder's value for item from bid.



Bidder's Bid Optimization

Example: How should bidder 1 bid in Auction C?

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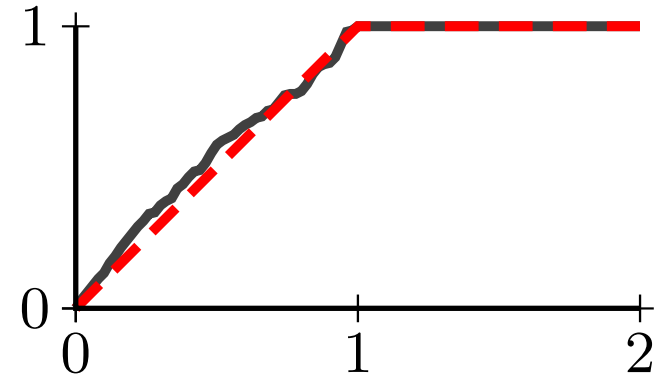
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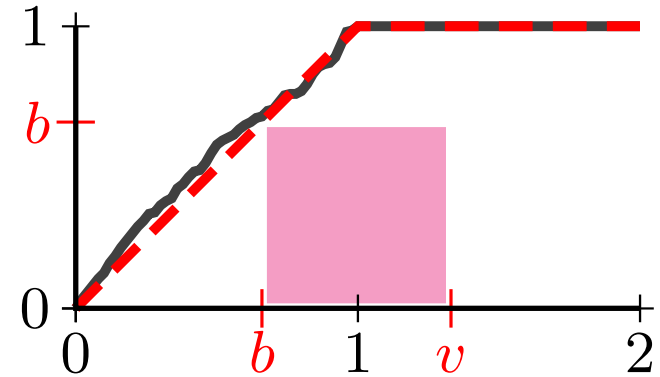


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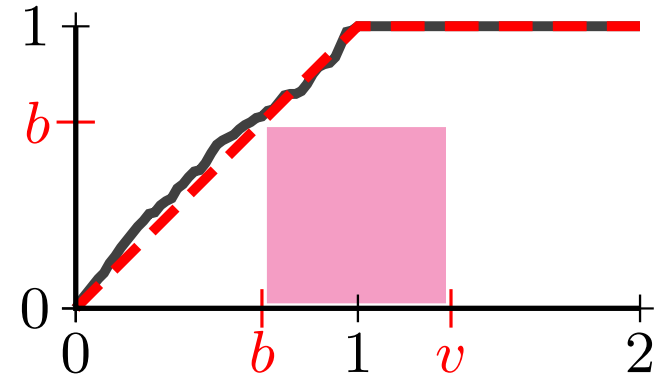


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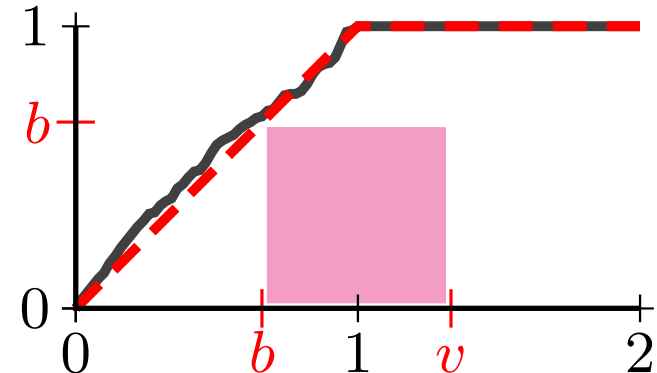


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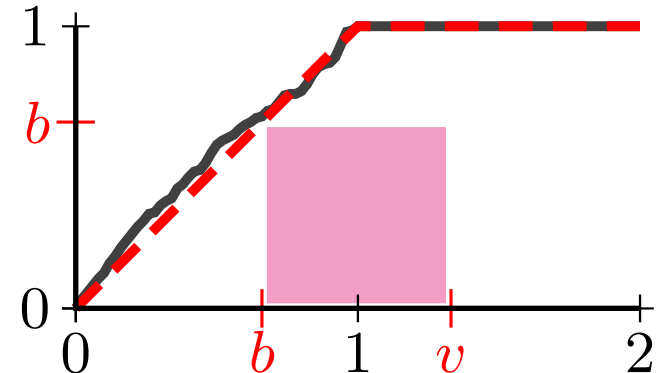
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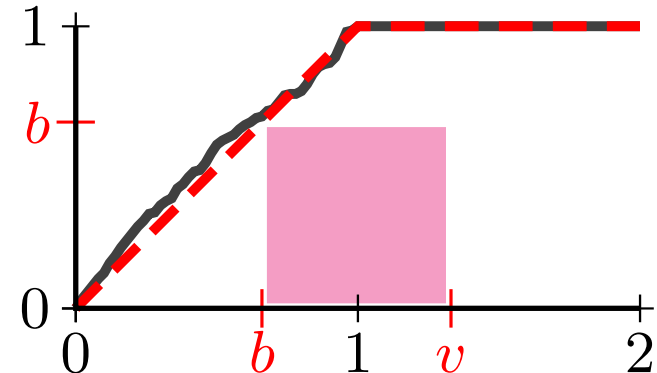
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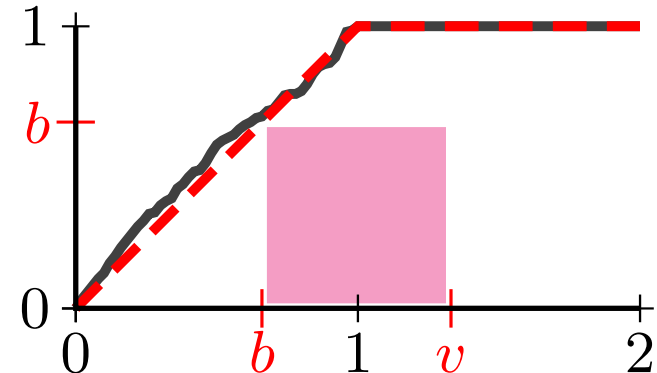
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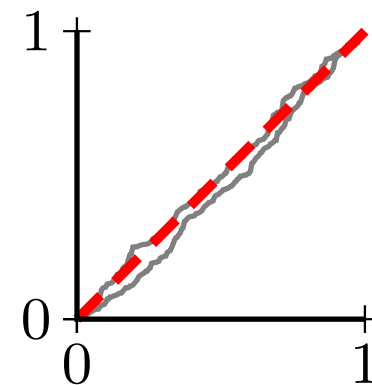


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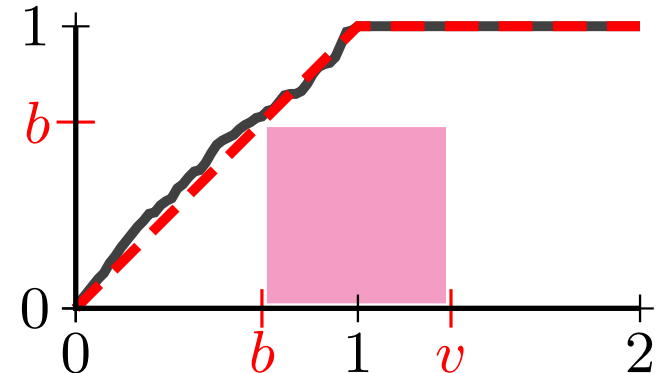
Recall: Bids uniform on $[0, 1]$.



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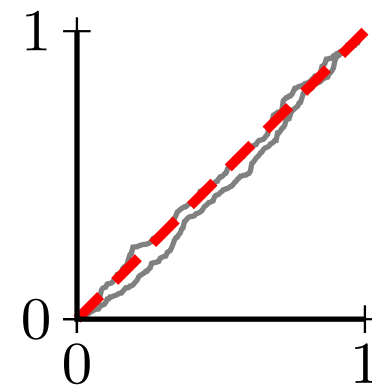
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Conclusion 2: Values are uniform on $[0, 2]$.



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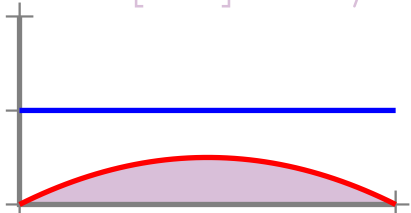
Revenue of A vs B

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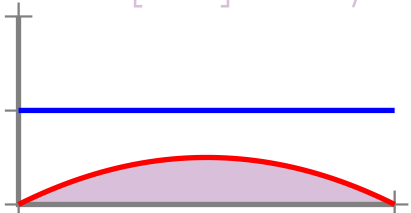
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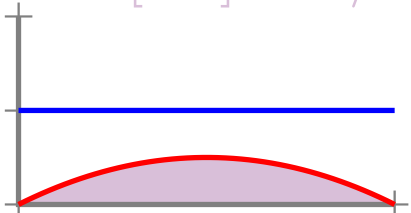
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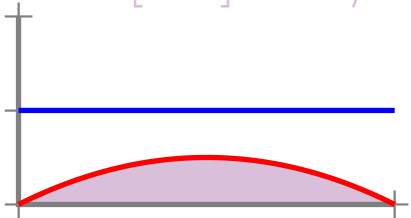
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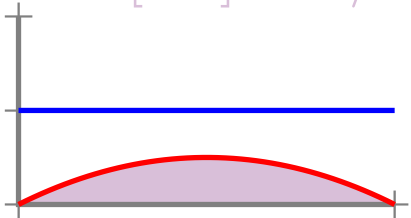
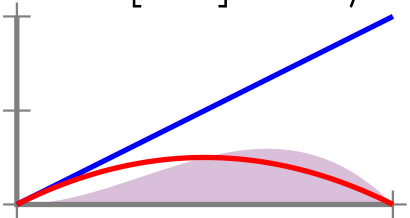
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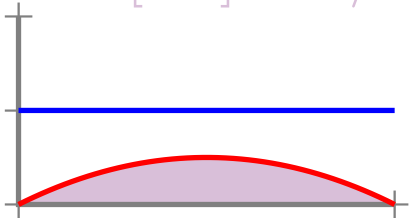
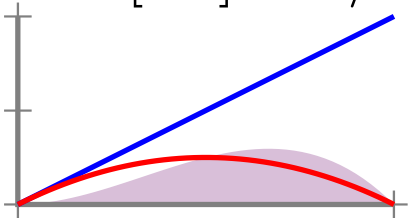
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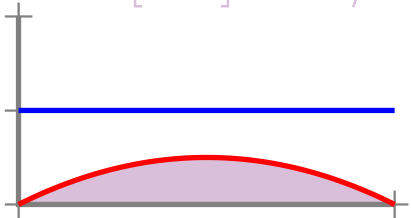
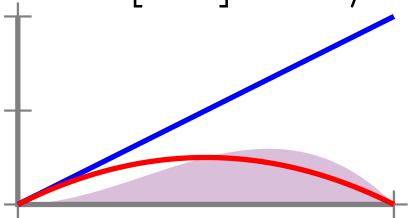
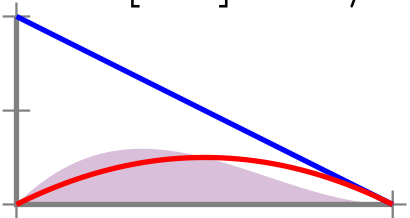
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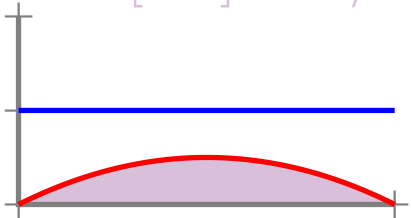
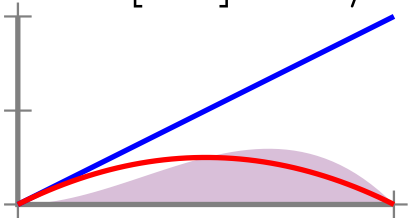
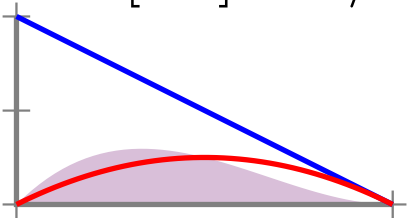
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Generally: Revenue is $A > B$ or $A < B$.

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Inference Equation: for first price auction C:

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- \Rightarrow revenue \hat{R}_B estimator has rate worse than \sqrt{N} .

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$$\hat{R}_B = \int_0^1 \left(\hat{b}_C(q) + \frac{x_C(q) \hat{b}'_C(q)}{x'_C(q)} \right) (1 - q) x'_B(q) dq$$

Step 2: Simplify with integration by parts (Define $W_{A,B}$):

$$\hat{R}_B = \int_0^1 W_{A,B}(q) \hat{b}_C(q) dq$$

Direct Approach

Inference Equation: for first price auction C:

$$\hat{v}(q) = \hat{b}_C(q) + \frac{x_C(q) \hat{b}'_C(q)}{x'_C(q)}$$

Auction Theory: Expected revenue of auction B is:

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Step 4: Estimator for N sorted bids is $\hat{R}_B \approx \sum_i W_{A,B}\left(\frac{i}{N-1}\right) \hat{b}_{i,C}$.

Results Overview

Results: N bid samples, n positions, auction B with probability ϵ .

1. Can estimate revenue of A and B directly from bids in C.
2. Revenue estimator is a *weighted order statistic*.
3. “Revenue B” estimator has error: $O\left(\frac{1}{\sqrt{N}} n \log \frac{n}{\epsilon}\right)$.
4. A universal B test.
5. Can optimize revenue over all feasible position auctions.

A Grand Challenge for CS

A Grand Challenge: understand and guide computation in the wild

- **computational primitive:** local/individual/strategic optimization.
- **objective:** good global outcomes
- **a key application area:** “online markets”
uber, airbnb, twitter, stackexchange, tinder, ...