A/B Testing of Auctions

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(with Shuchi Chawla and Denis Nekipelov)

Yandex Events — August 7, 2018

A Grand Challenge for CS

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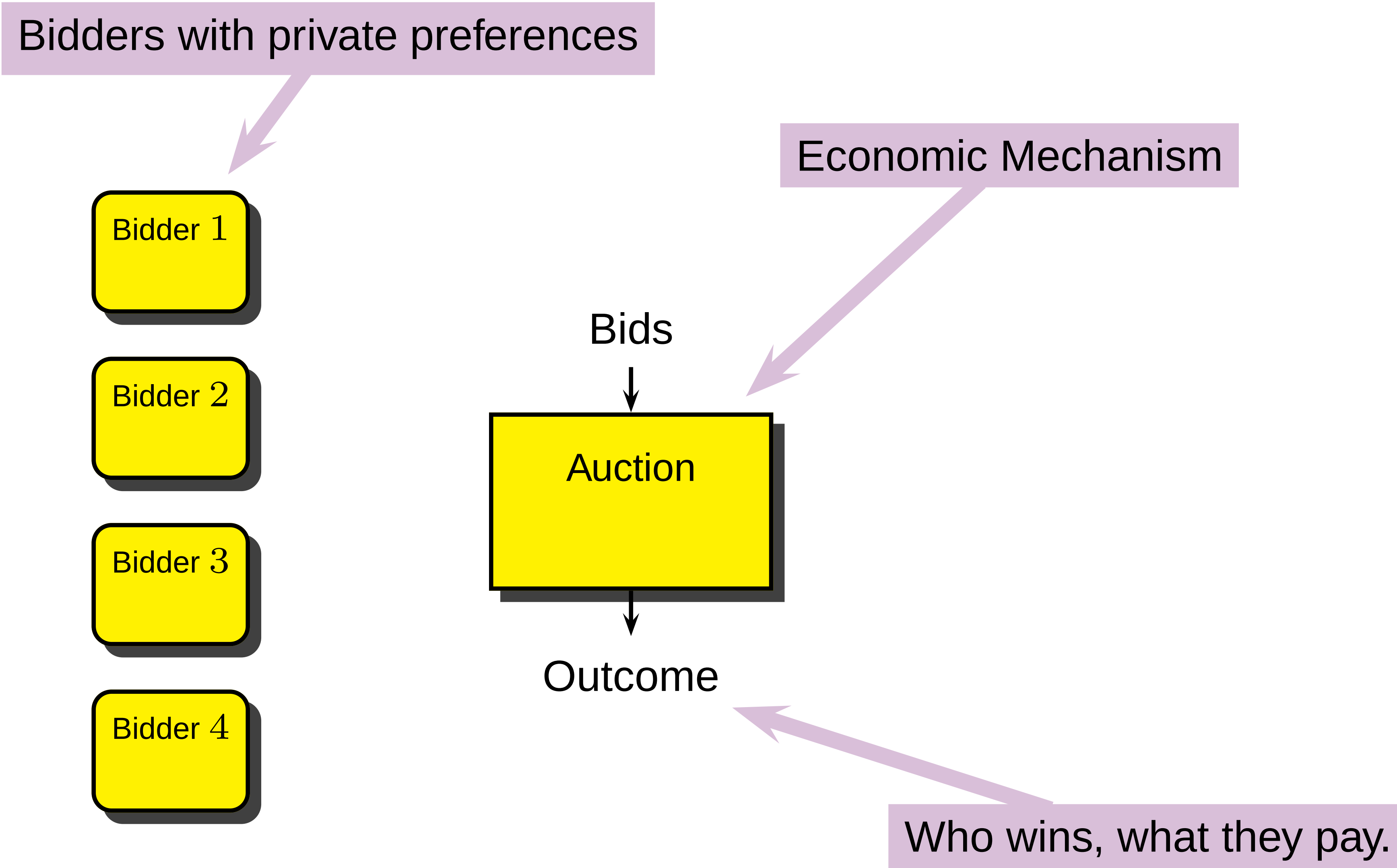
- computational primitive: local/individual/strategic optimization.
- objective: good global outcomes
- a key application area: “online markets”

  uber, airbnb, twitter, stackexchange, tinder, . . .

Example: First-price Auction



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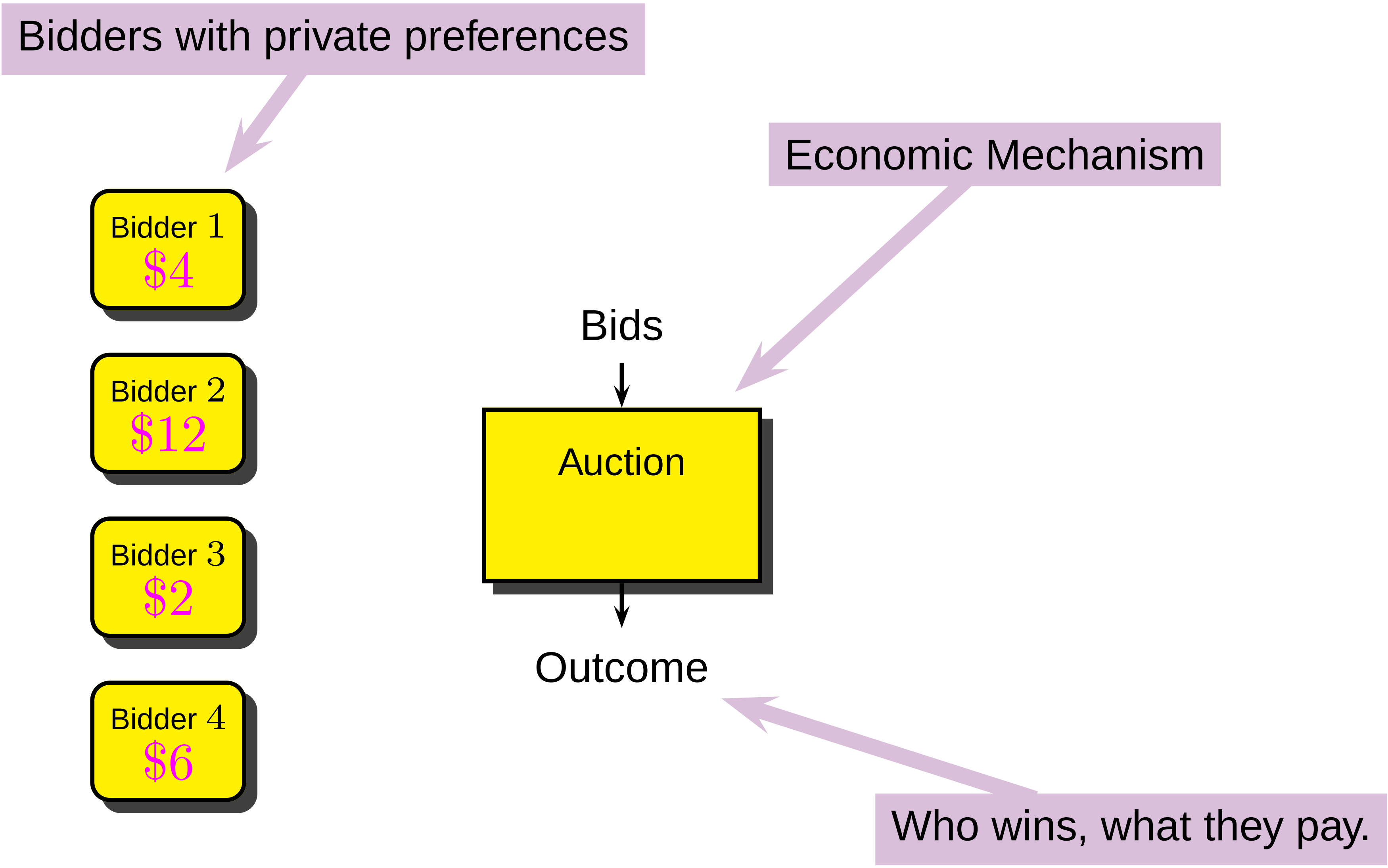

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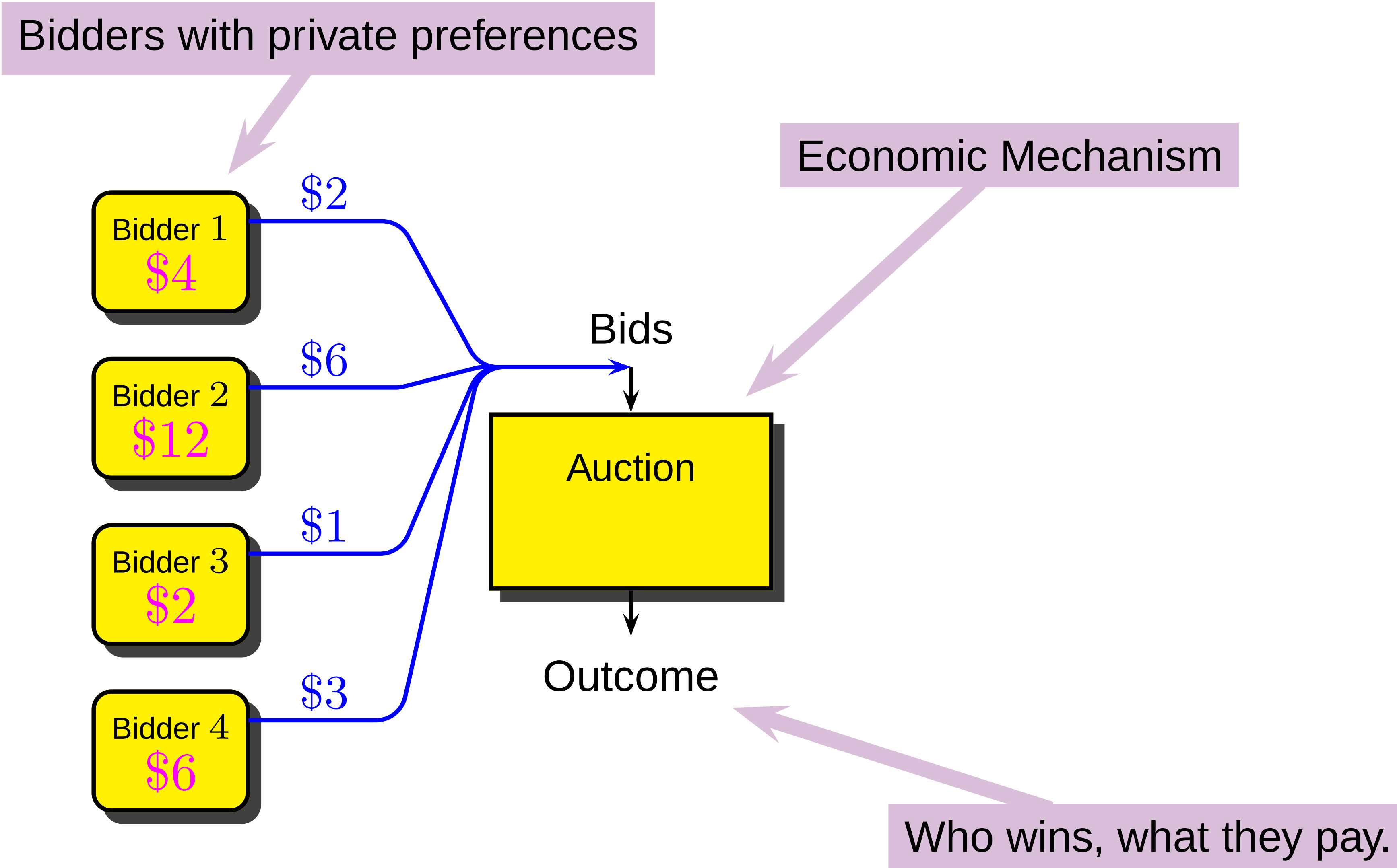

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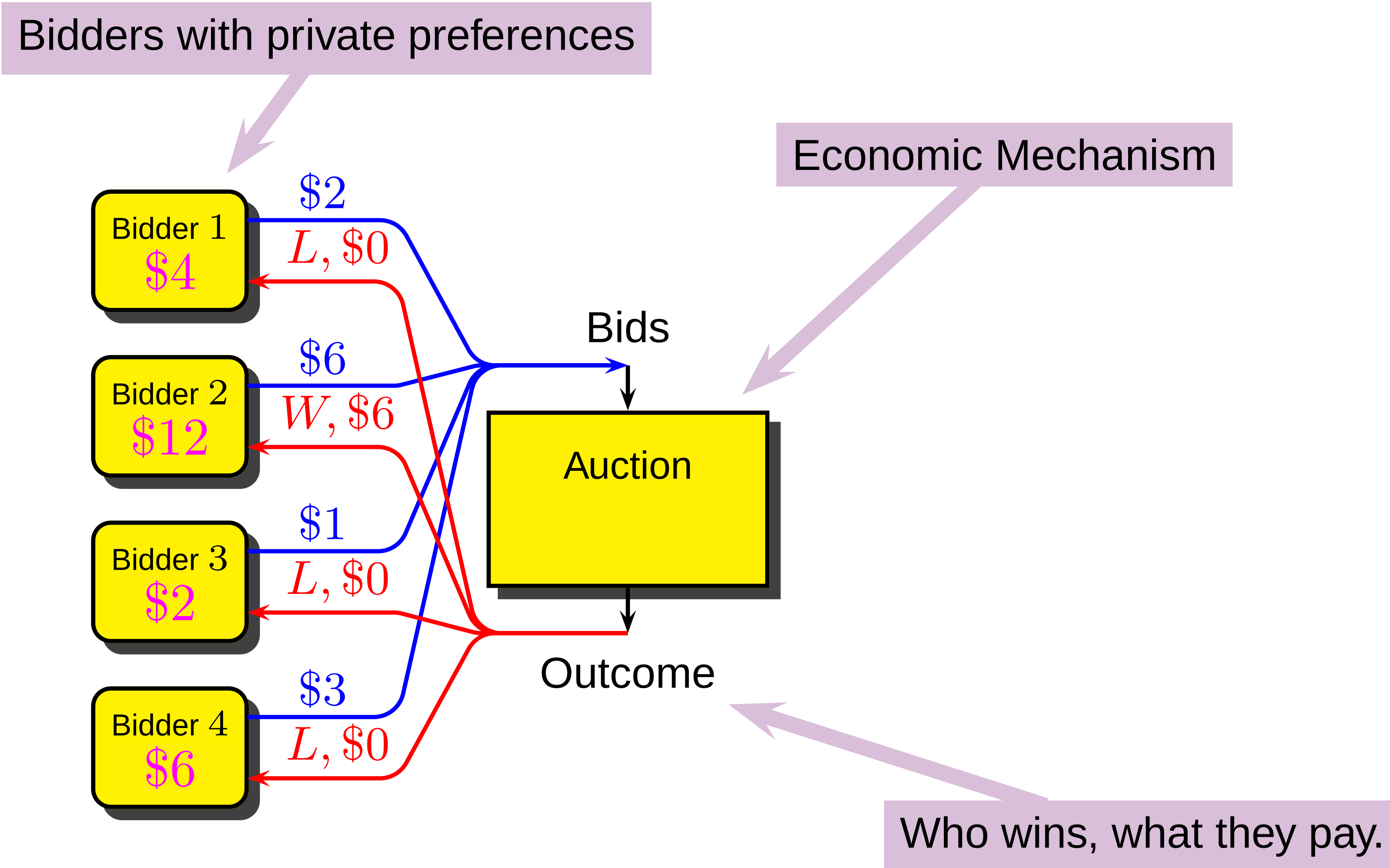

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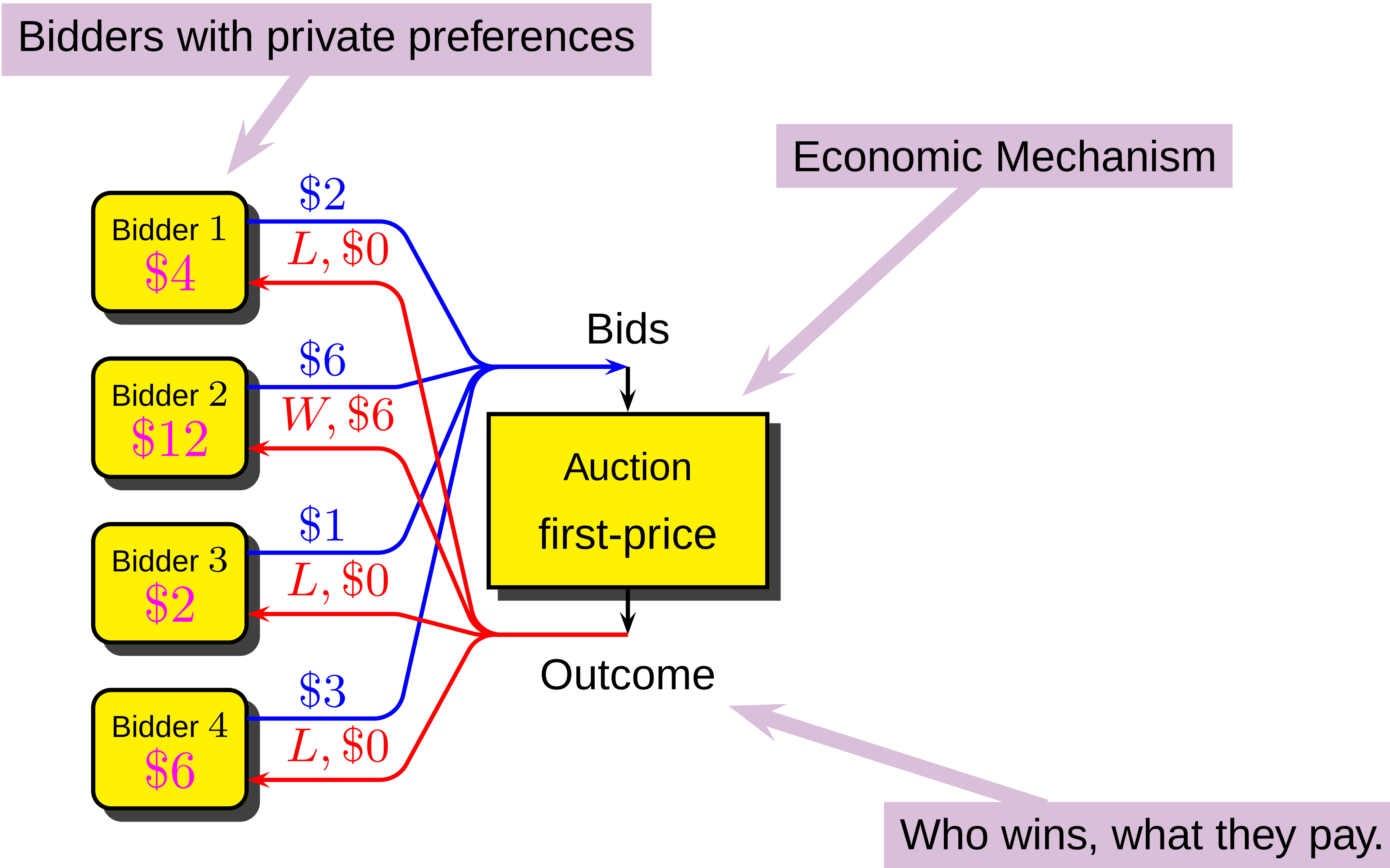

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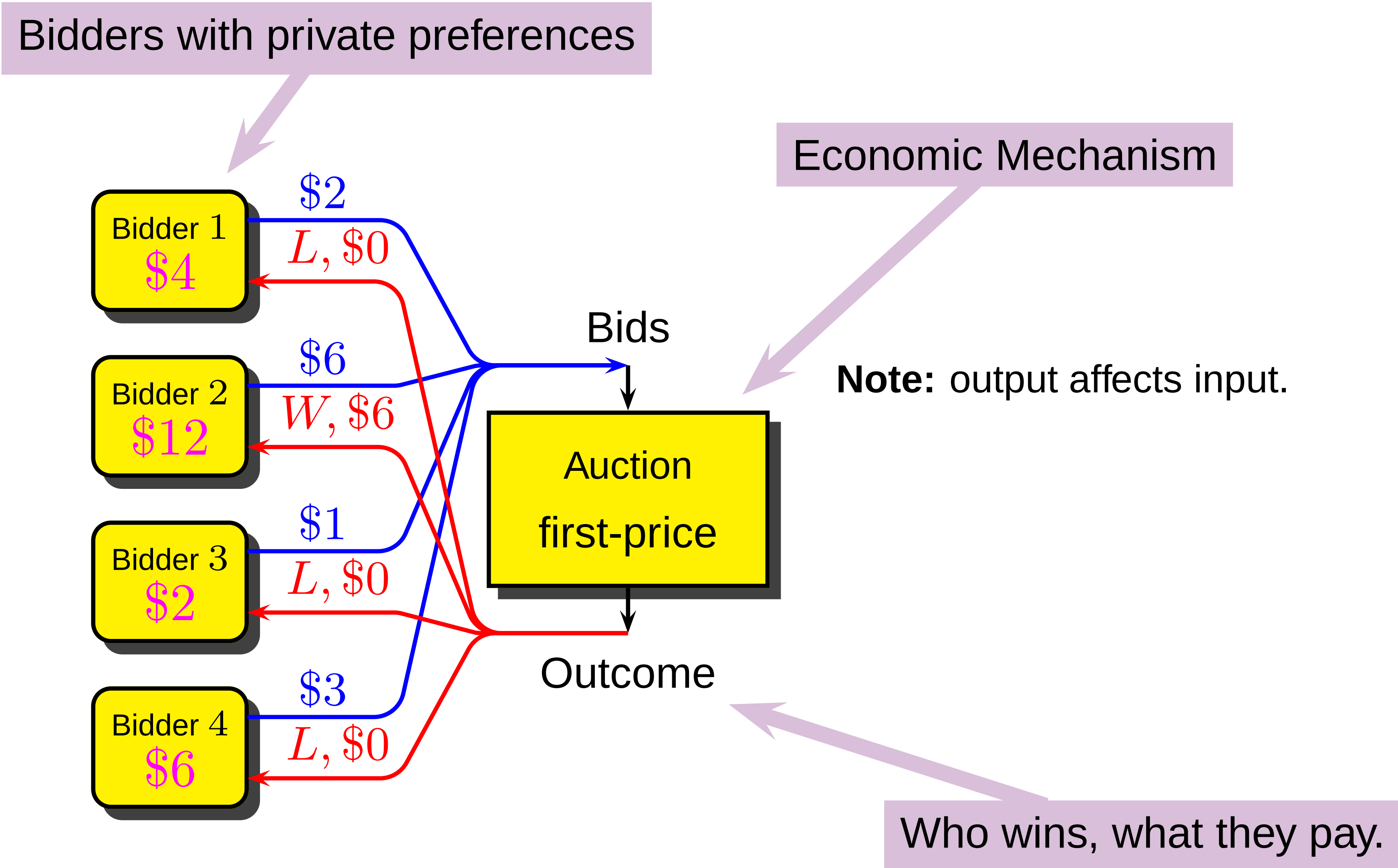

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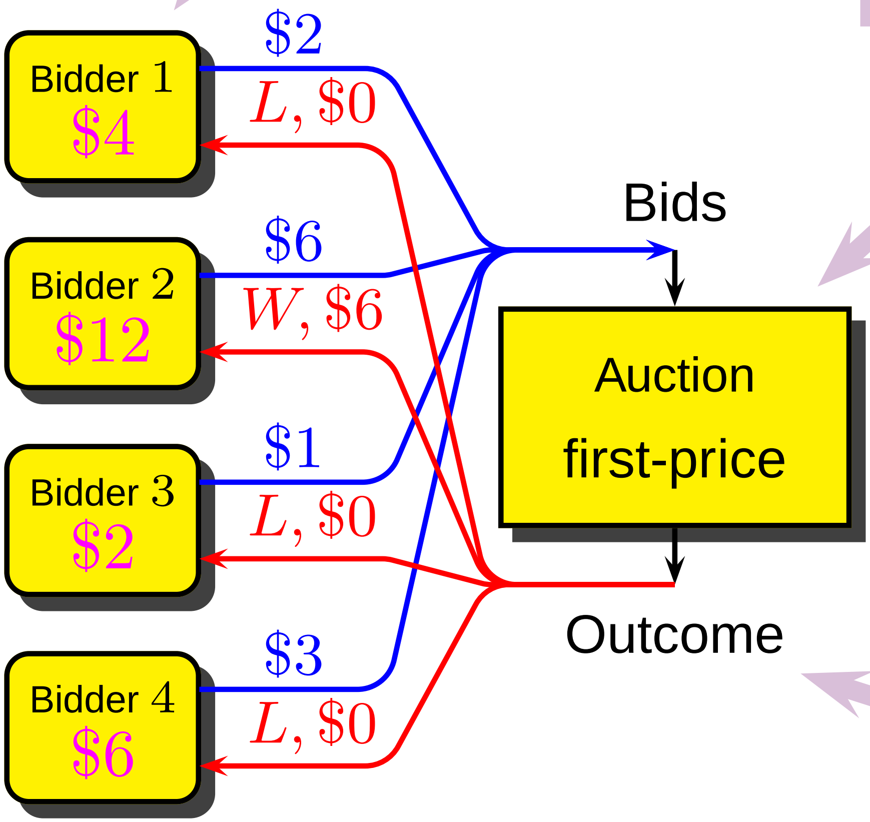
Bidders with private preferences

Economic Mechanism



Note: output affects input.

Challenges: need to rethink classical

- algorithms
- data science

Who wins, what they pay.

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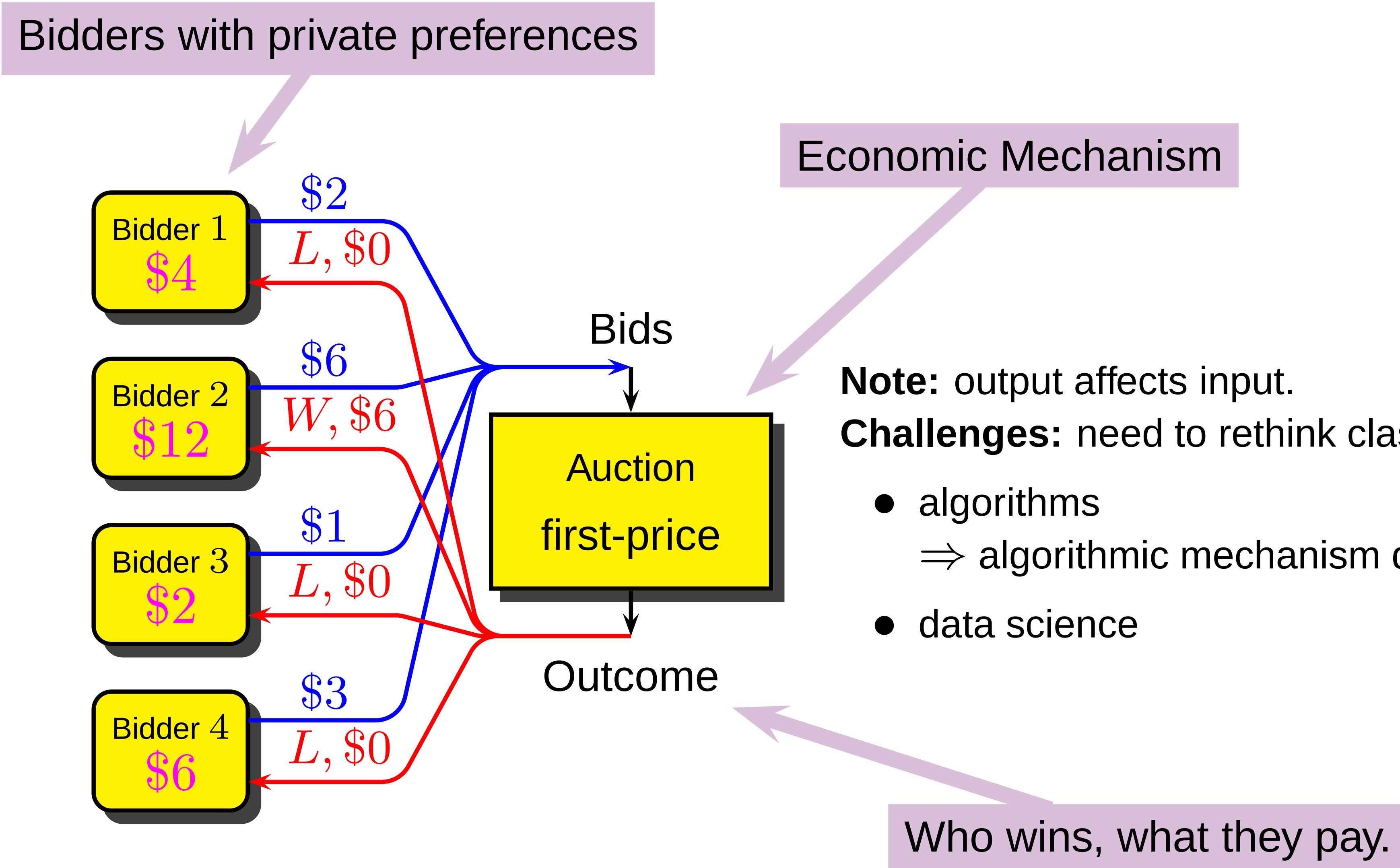

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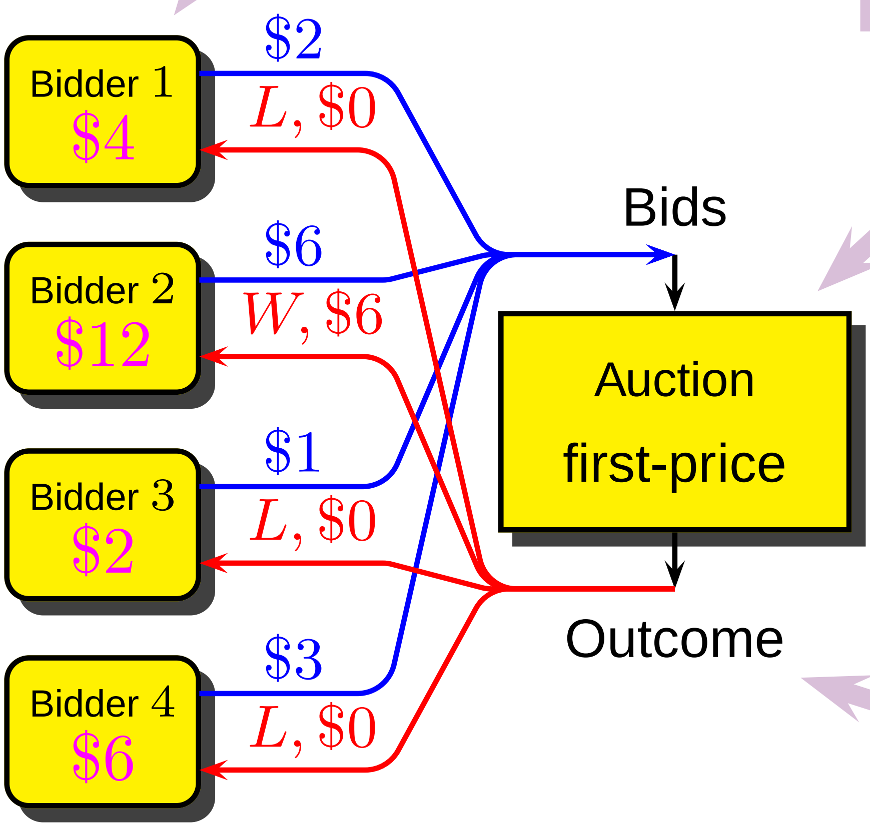

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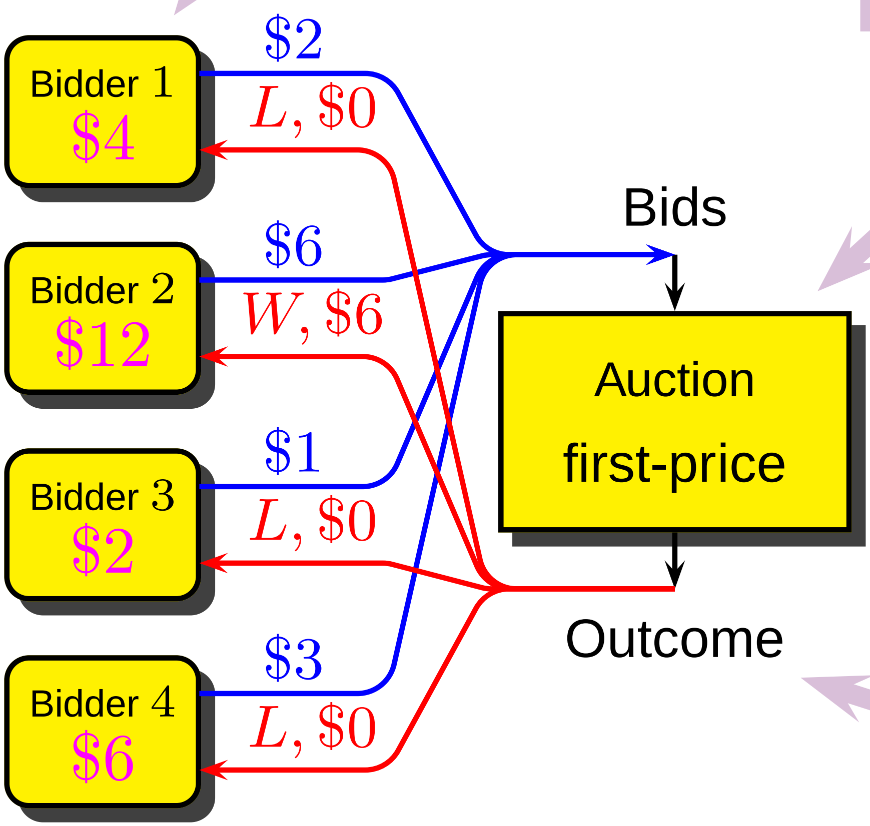

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  $\Rightarrow$ algorithmic mechanism design
- data science [since 2014; this talk]

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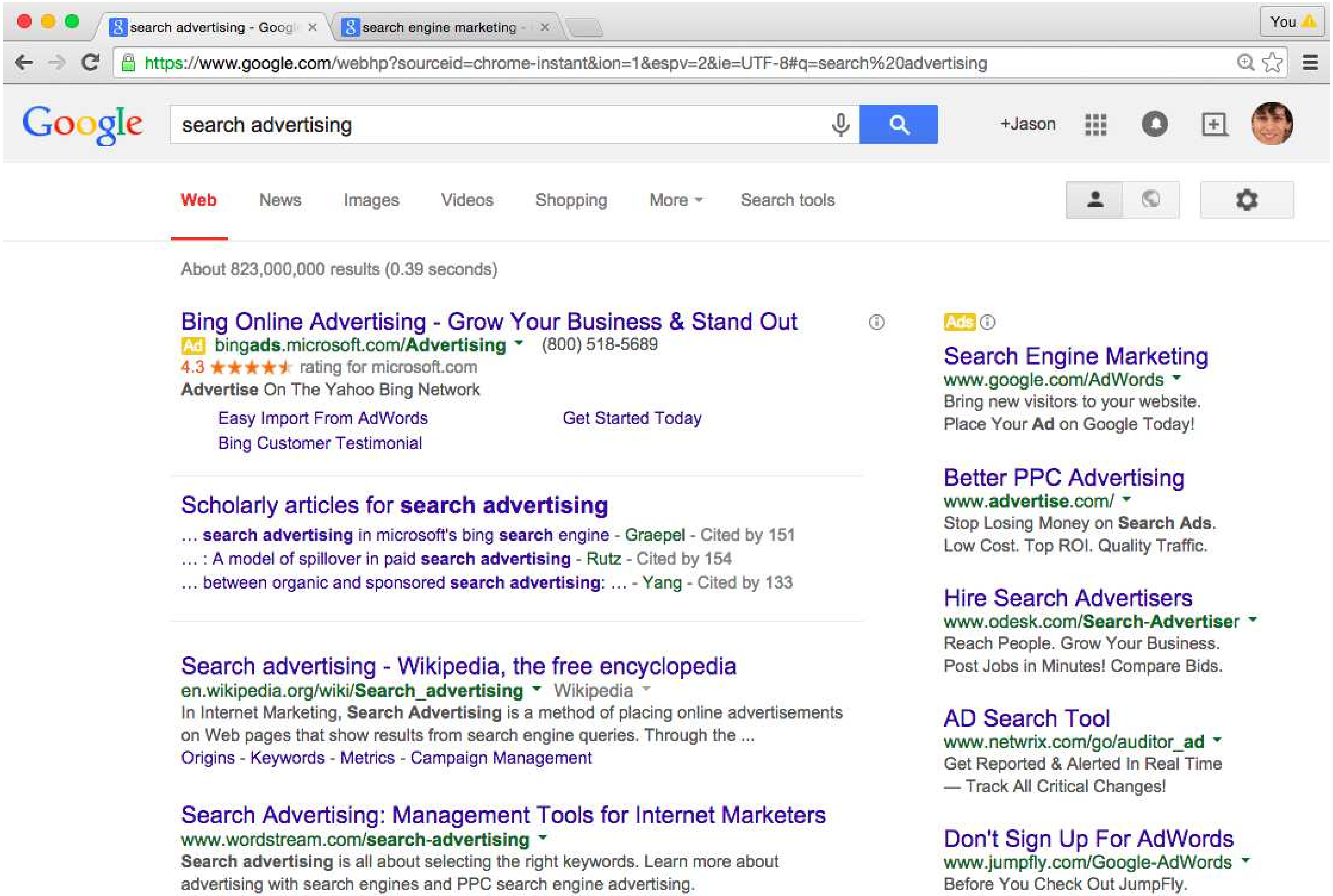
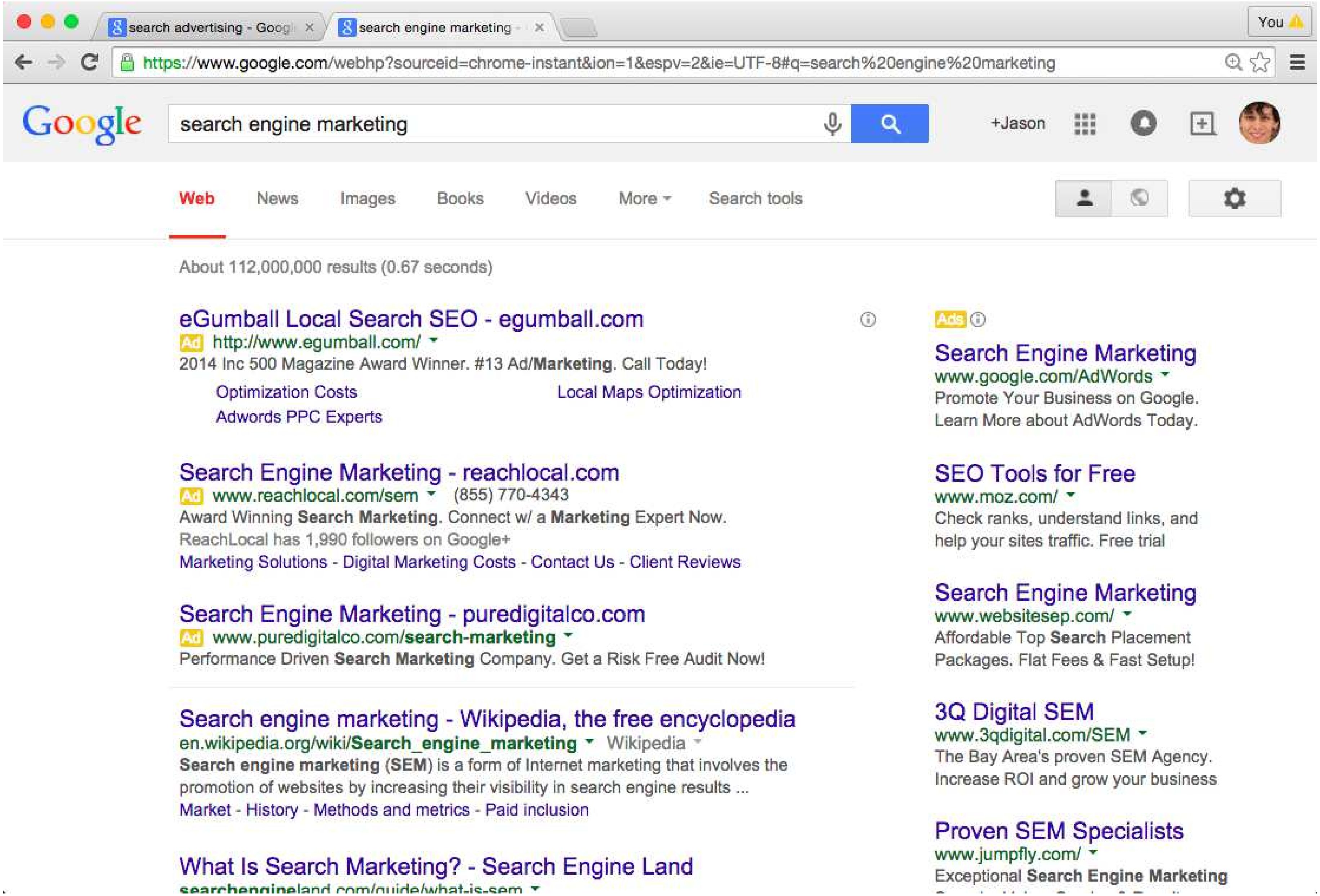
Motivating Example
Search Engine Advertising

one mainline ad



three mainline ads



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Example: estimating ads *click-through rates*.

1. user searches.
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Example: estimating *auction revenue*.

1. advertisers bid.
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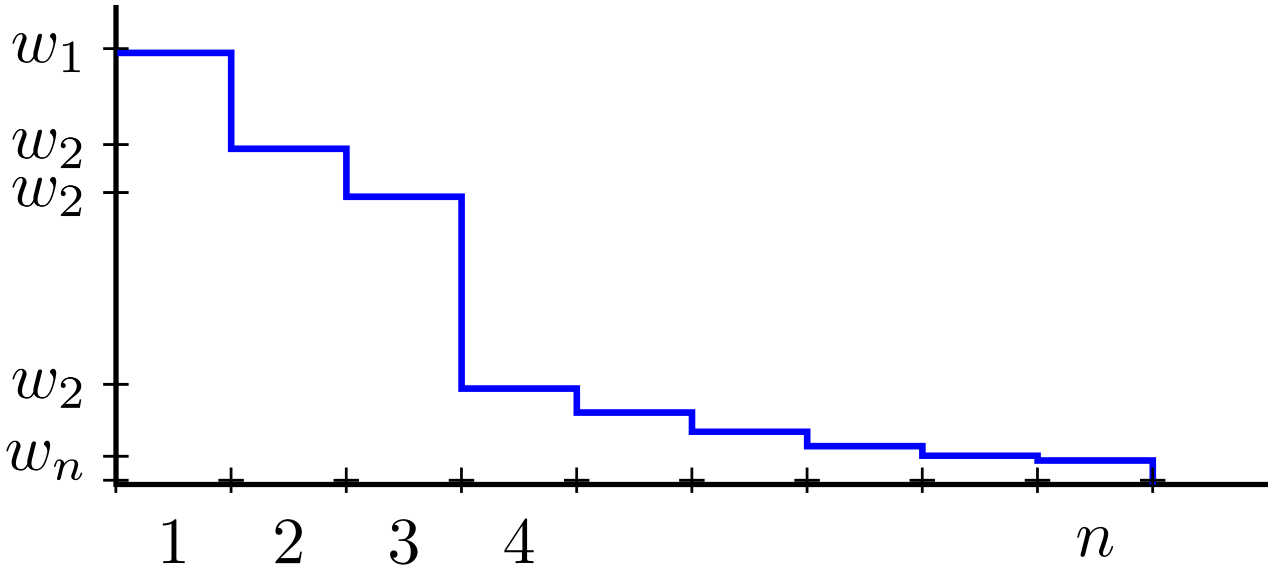
Note: bids in A/B test are neither for A nor B, but C $= 0.5$A $+ 0.5$B.

First-price Position Auctions

"First-price" Position Auction: [Varian '06; Edelman et al. '07]

- n bidders, n positions, click probabilities $\boldsymbol{w}$ with $w_1 \geq \ldots \geq w_n$.
- bidders assigned to positions in order of bid.
- bidders pay bid if clicked.



Example: A $=$ three mainline ads;

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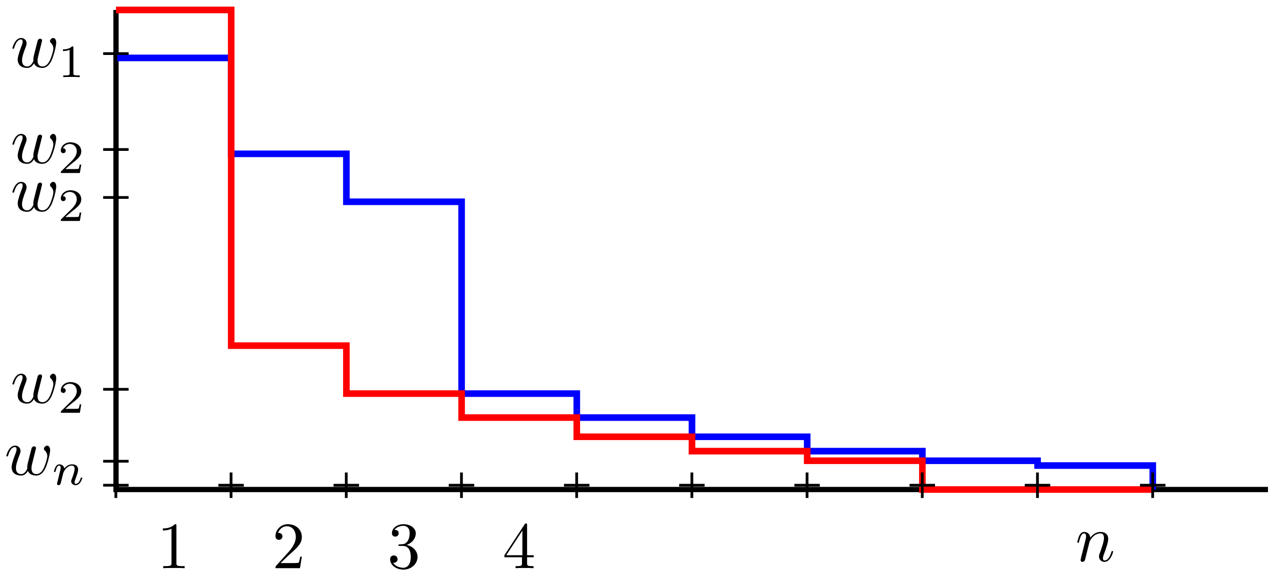

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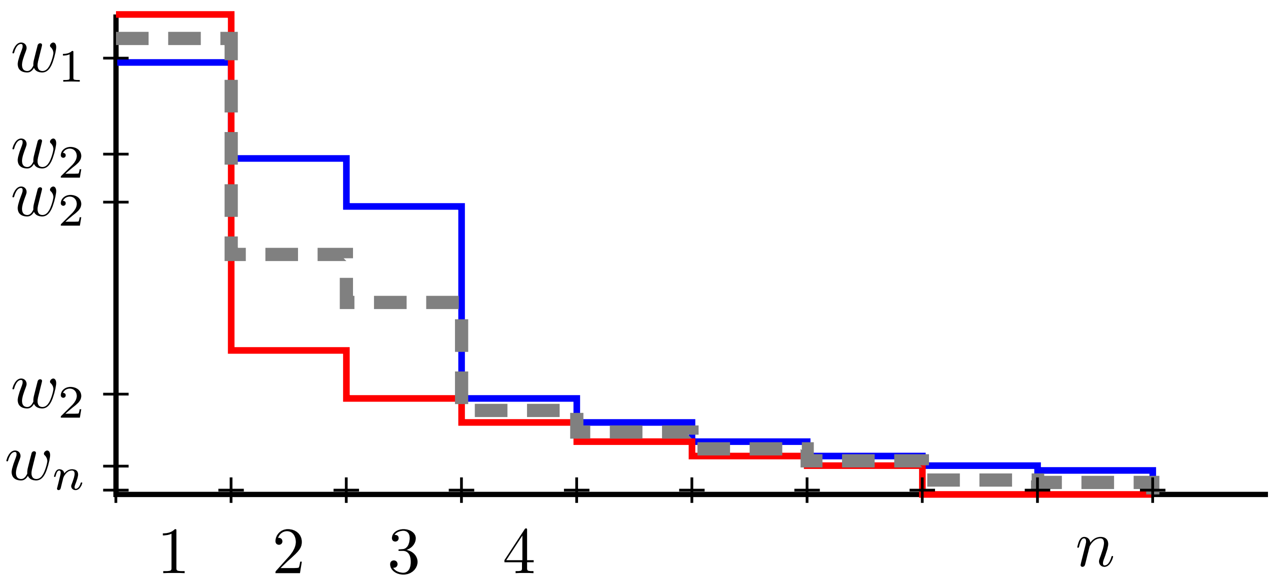

Example: A $=$ three mainline ads; B $=$ one mainline ad; **C** $=$ mix.

Toy Example

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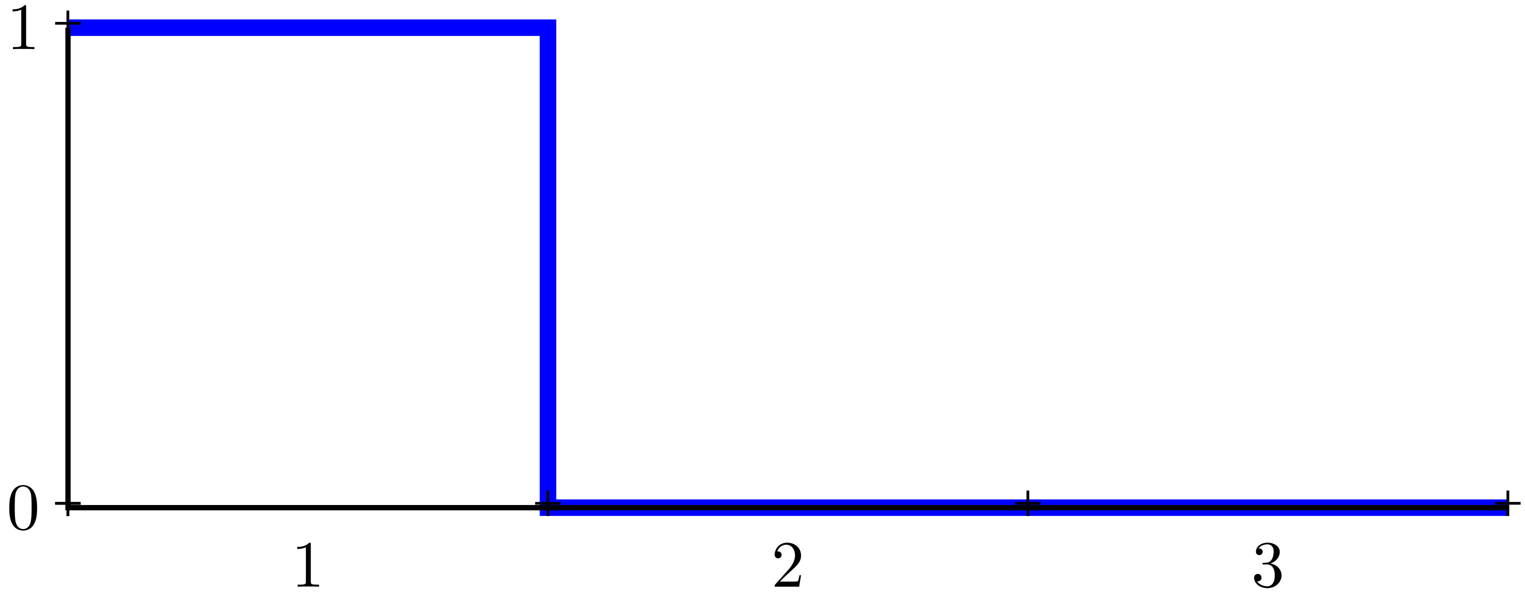
- Auction A: one unit.



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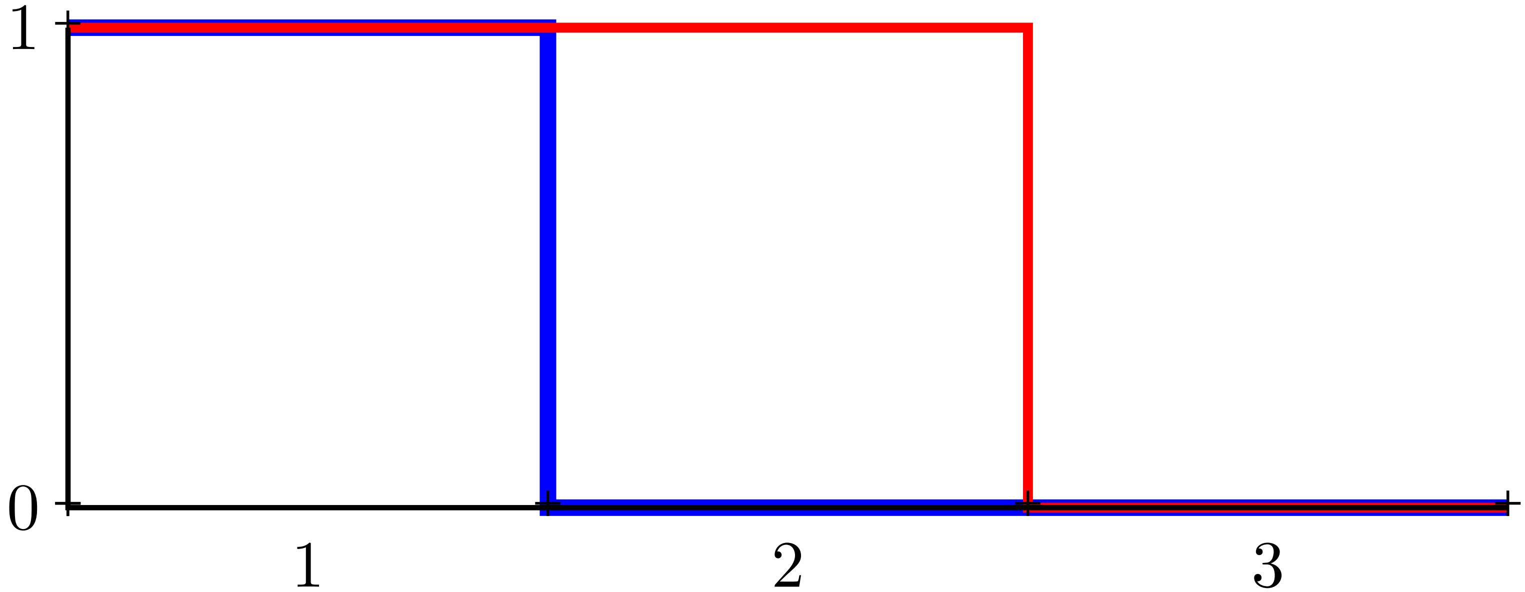
- Auction A: one unit.
- Auction B: two units.



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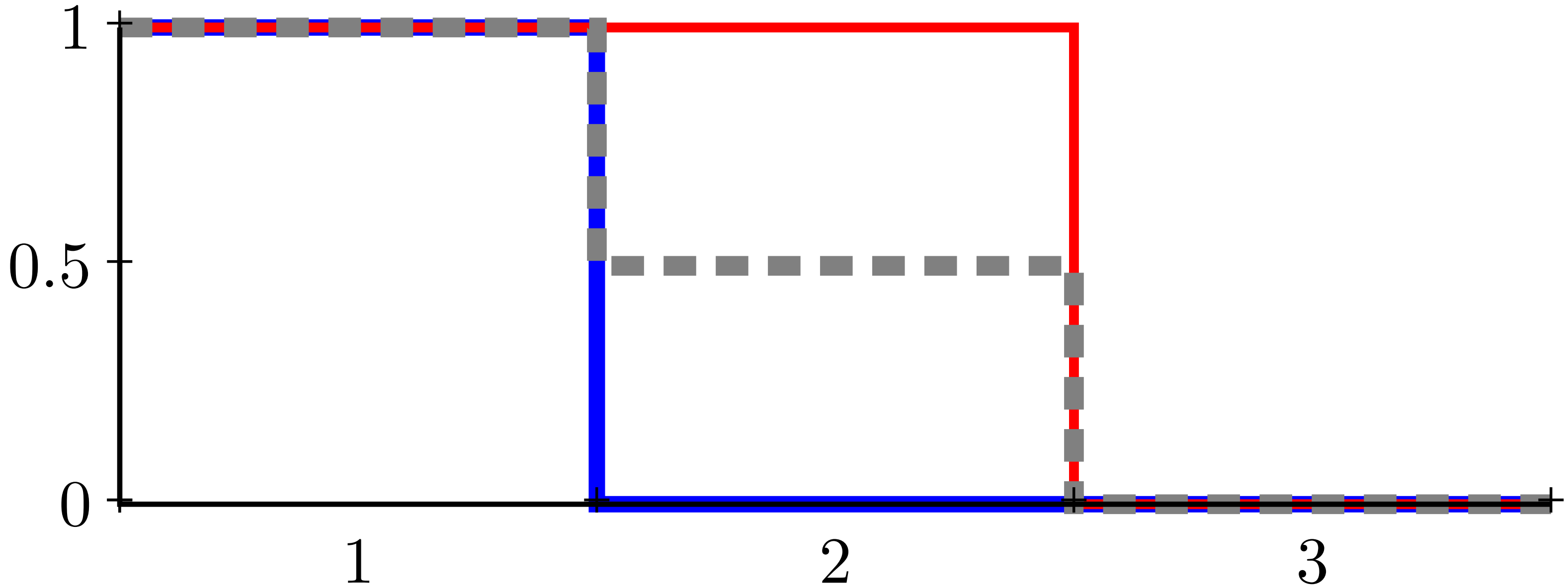
- Auction A: one unit.
- Auction B: two units.
- Auction C: mix 0.5A $+0.5$B.



Improper A/B Test: C $= 0.5$A $+ 0.5$B

Auction	Bid 1	Bid 2	Bid 3	Rev C
1A	0.74	0.34	0.11	0.74
2A	0.08	0.86	0.50	0.86
3B	0.69	0.83	0.46	1.53
4B	0.53	0.03	0.77	1.30
5A	0.91	0.49	0.54	0.91
6A	0.44	0.35	0.92	0.92
7A	0.86	0.97	0.85	0.97
8B	0.21	0.10	0.30	0.51
⋮	⋮	⋮	⋮	⋮
200B	0.13	0.30	0.98	1.28
Average				0.98

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Auction	Bid 1	Bid 2	Bid 3	Rev C	Est A	Est B
1A	0.74	0.34	0.11	0.74	0.74	0.00
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3B	0.69	0.83	0.46	1.53	0.00	1.53
4B	0.53	0.03	0.77	1.30	0.00	1.30
5A	0.91	0.49	0.54	0.91	0.91	0.00
6A	0.44	0.35	0.92	0.92	0.92	0.00
7A	0.86	0.97	0.85	0.97	0.97	0.00
8B	0.21	0.10	0.30	0.51	0.00	0.51
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
200B	0.13	0.30	0.98	1.28	0.00	1.28
Average				0.98	0.75	1.19

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1A	0.74	0.34	0.11	0.74	0.74	0.00
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Note: Improper A/B test always shows A $<$ B.

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1A	0.74	0.34	0.11	0.74	0.74	0.00	0.99	0.00
2A	0.08	0.86	0.50	0.86	0.86	0.00	1.14	0.00
3B	0.69	0.83	0.46	1.53	0.00	1.53	0.00	1.20
4B	0.53	0.03	0.77	1.30	0.00	1.30	0.00	1.08
5A	0.91	0.49	0.54	0.91	0.91	0.00	1.21	0.00
6A	0.44	0.35	0.92	0.92	0.92	0.00	1.23	0.00
7A	0.86	0.97	0.85	0.97	0.97	0.00	1.30	0.00
8B	0.21	0.10	0.30	0.51	0.00	0.51	0.00	0.48
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
200B	0.13	0.30	0.98	1.28	0.00	1.28	0.00	0.95
Average				0.98	0.75	1.19	1.01	0.96

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8B	0.21	0.10	0.30	0.51	0.00	0.51	0.00	0.48
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200B	0.13	0.30	0.98	1.28	0.00	1.28	0.00	0.95
Average				0.98	0.75	1.19	1.01	0.96

Note: Improper A/B test always shows A $<$ B.

Missing effect: more units $\Rightarrow$ lower bids.

Outline

0. Improper A/B testing.
1. Overview of Results.
2. Economic inference (get values from bids).
3. Auction revenue analysis (revenue from values).
4. Direct estimation of revenue from bids.

Results Overview

Results: N bid samples, n positions, auction B with probability ϵ.

1. Can estimate revenue of A and B directly from bids in C.

2. Revenue estimator is a *weighted order statistic.*

3. "Revenue B" estimator has error: $O\left(\frac{1}{\sqrt{N}} n \log \frac{n}{\epsilon}\right)$.

   Note: "Ideal A/B test" error: $O\left(\frac{1}{\sqrt{N}} n \frac{1}{\sqrt{\epsilon}}\right)$.

4. A universal B test.

5. Can optimize revenue over all feasible position auctions.

Simulation Results (Normalized)

Theoretical Bound: error is $O\left(\frac{1}{\sqrt{N}}\, n \log \frac{n}{\epsilon}\right)$. ($\epsilon$ prob. on B)

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$n =$	$N =$				
	10^1	10^2	10^3	10^4	10^5
2^1	0.1215	0.1150	0.1169	0.1177	0.1196
2^2	0.0814	0.0605	0.0582	0.0596	0.0642
2^3	0.0779	0.0653	0.0652	0.0672	0.0661
2^4	0.0690	0.0621	0.0612	0.0646	0.0623
2^5	0.0566	0.0522	0.0494	0.0508	0.0487
2^6	0.0425	0.0358	0.0355	0.0356	0.0349
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Note: constant with N as expected; dependence on n is not tight.

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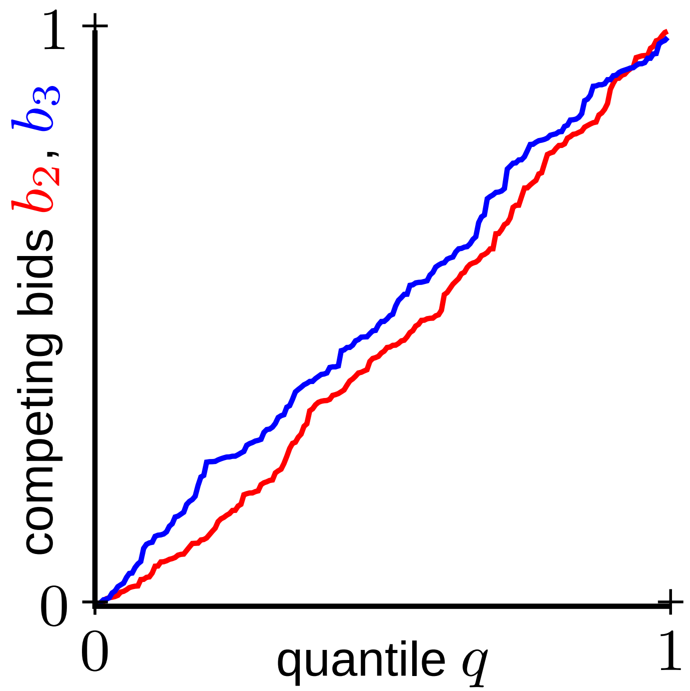

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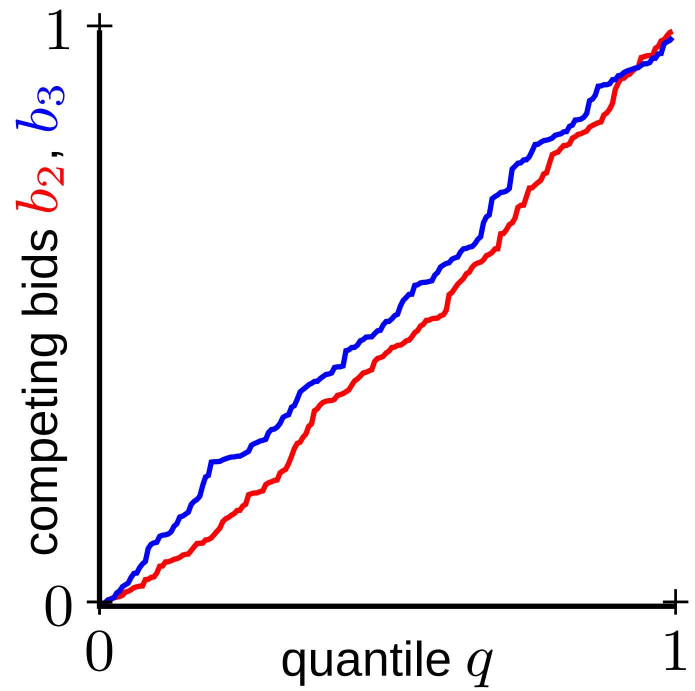

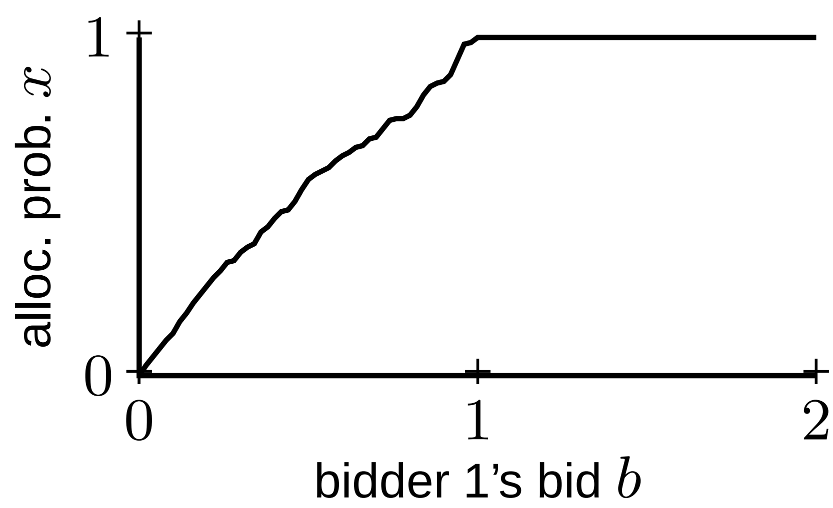

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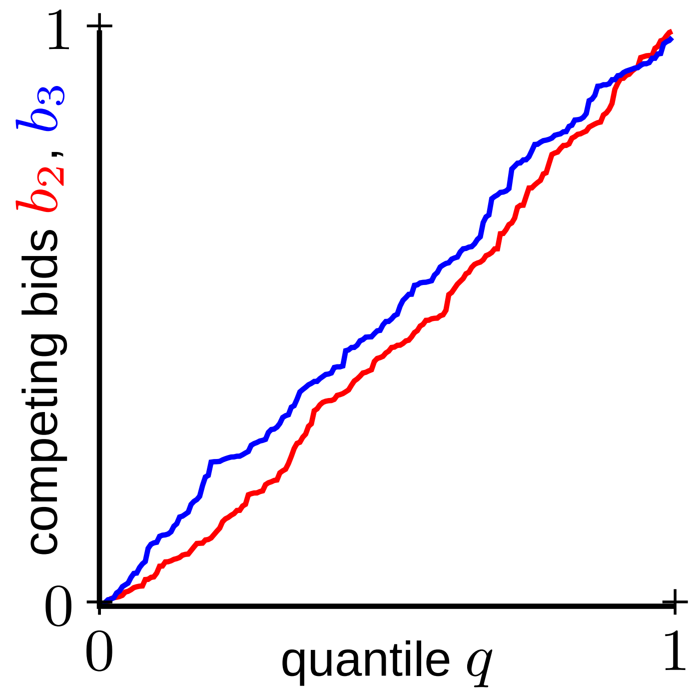
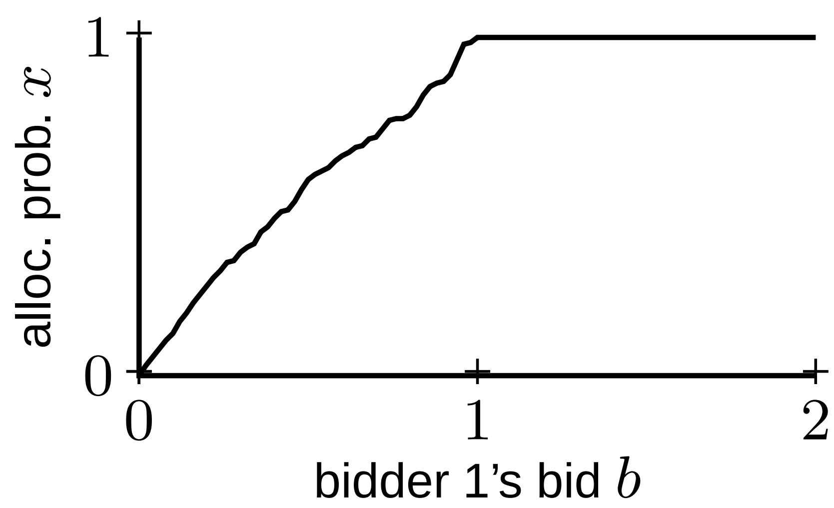


Approach:

1. given bid distribution, solve for bid strategy,
2. invert bid strategy to get bidder's value for item from bid.



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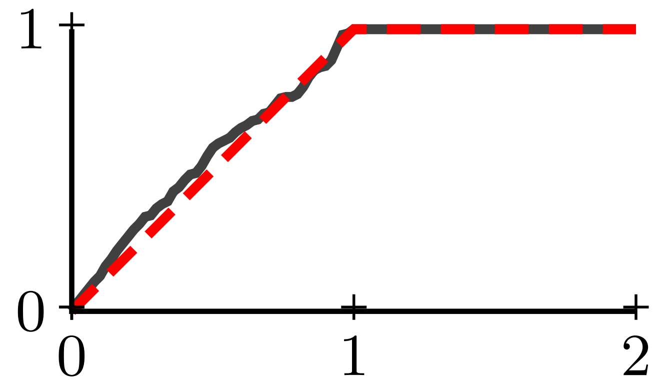

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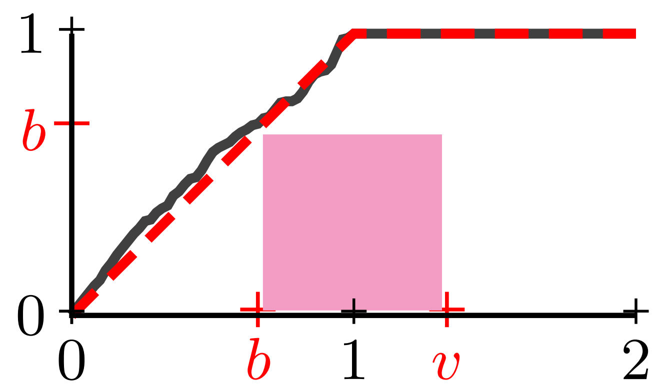

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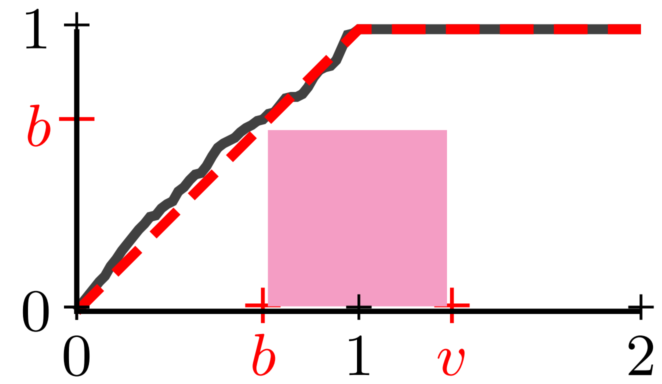

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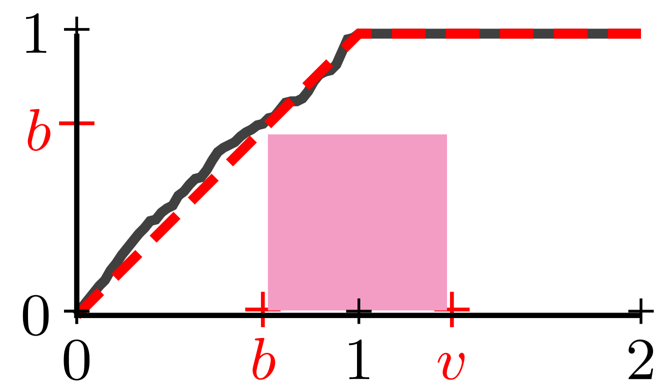

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- to maximize, take derivative $\frac{d}{db}$ and set to zero, solve

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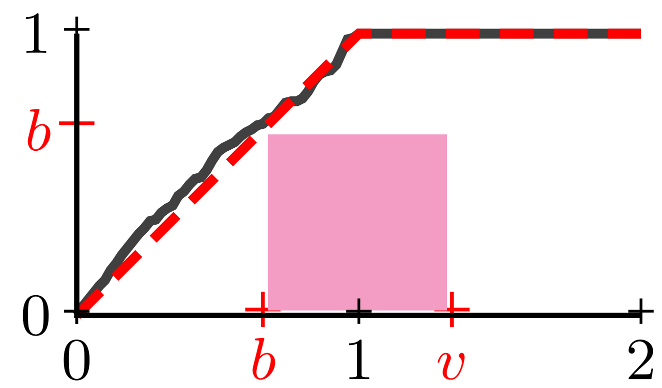

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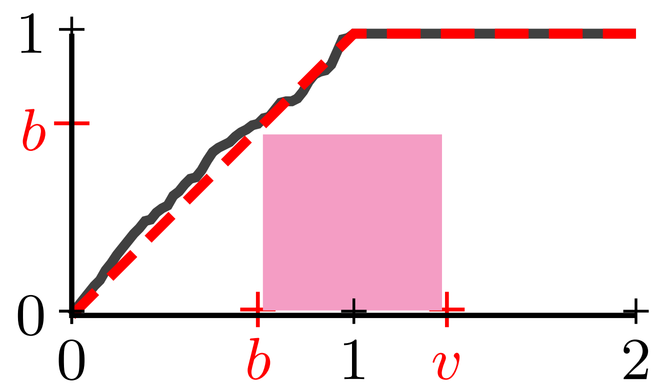
Bidder's Bid Optimization



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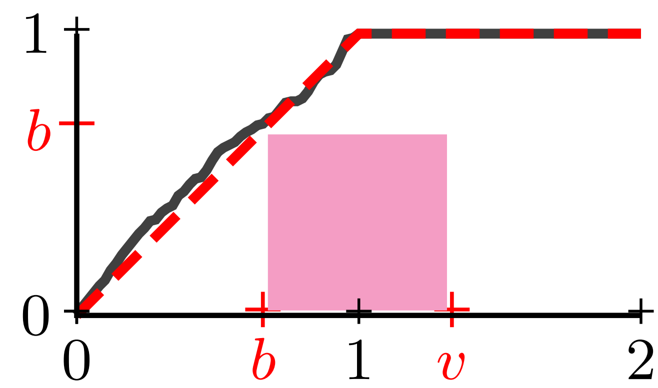

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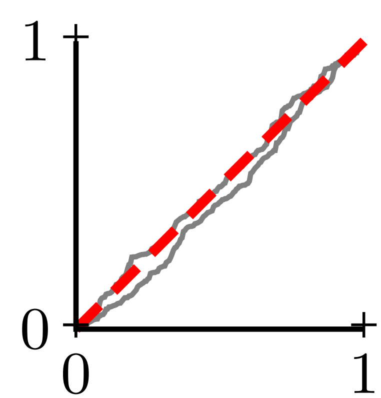

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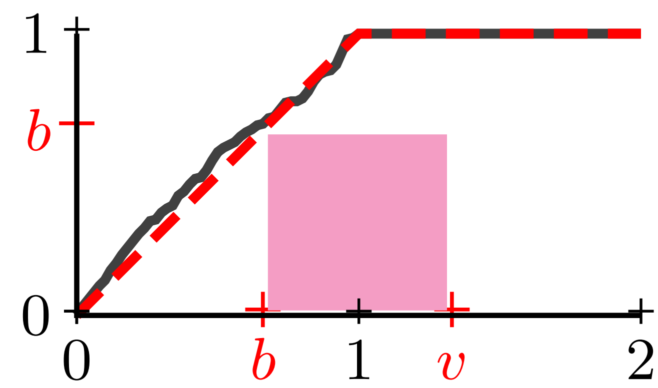

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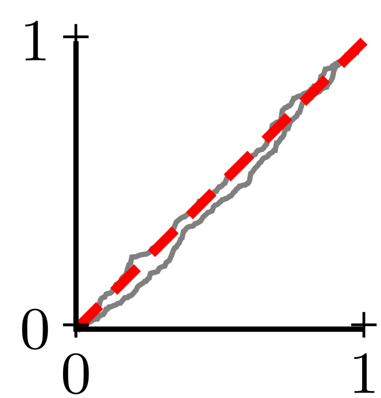

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6.	Auction revenue is $n \times$ per-agent revenue.	C's Revenue $= 1$

Revenue of A vs B

Question: Values are $U[0, 2]$, compare A and B's revenues.

Recall: C $= 0.5$A $+ 0.5$B; and $\mathbf{Rev}[x] = \int_0^1 R(q)\, x'(q)\, dq$.

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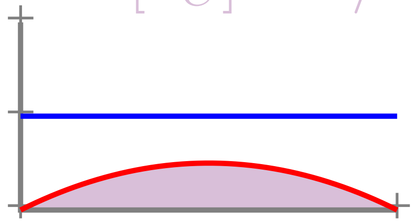
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 
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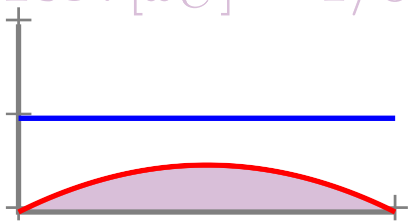
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 		
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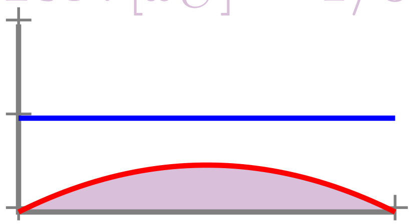
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 	$x_A(q) = q^2$	
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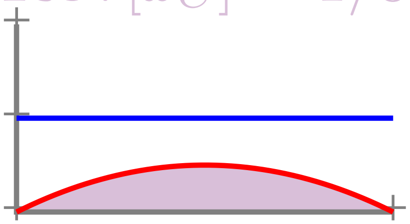
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 	$x_A(q) = q^2$ $x'_A(q) = 2q$	
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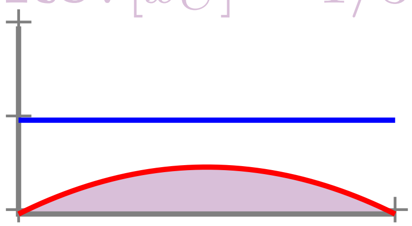
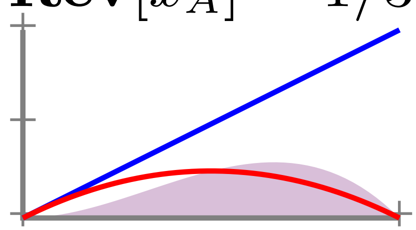
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 	$x_A(q) = q^2$ $x'_A(q) = 2q$ $\mathbf{Rev}[x_A] = 1/3$ 	
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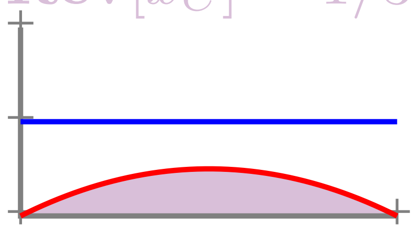
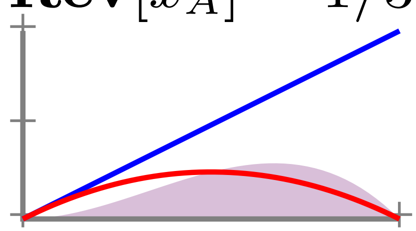
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 	$x_A(q) = q^2$ $x'_A(q) = 2q$ $\mathbf{Rev}[x_A] = 1/3$ 	
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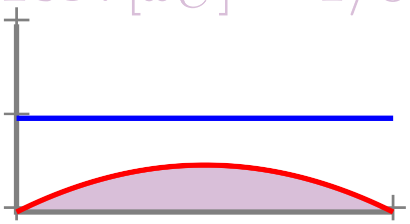	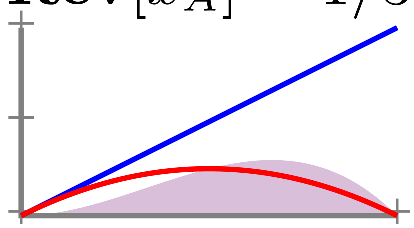	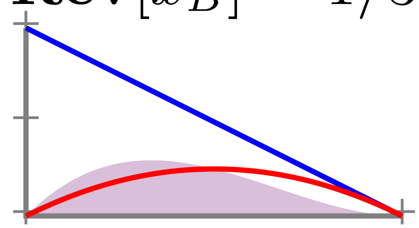
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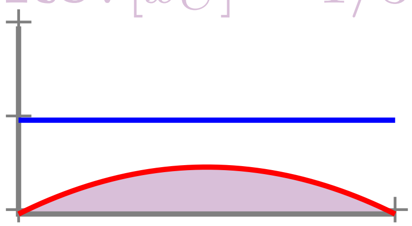
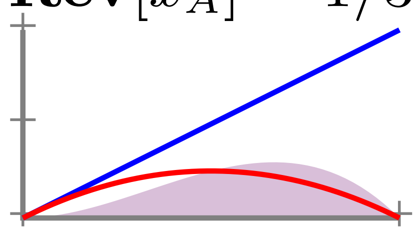
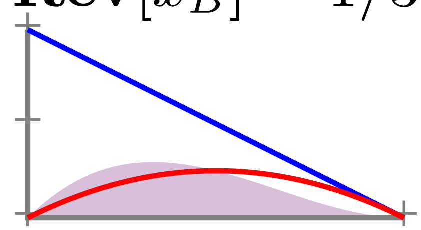
$x_C(q) = q$ $x'_C(q) = 1$ $\mathbf{Rev}[x_C] = 1/3$ 	$x_A(q) = q^2$ $x'_A(q) = 2q$ $\mathbf{Rev}[x_A] = 1/3$ 	$x_B(q) = 2q - q^2$ $x'_B(q) = 2 - 2q$ $\mathbf{Rev}[x_B] = 1/3$ 
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Generally: Revenue is A $>$ B or A $<$ B.

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$\Rightarrow$ revenue $\hat{R}_B$ estimator has rate worse than $\sqrt{N}$.

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Step 3: Bound $\int_0^1 |W_{A,B}(q)|\,dq$.

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$$\hat{R}_B = \int_0^1 \hat{v}(q)\,(1-q)\,x'_B(q)\,dq.$$

Step 1: Combine:

$$\hat{R}_B = \int_0^1 \left(\hat{b}_C(q) + \frac{x_C(q)\,\hat{b}'_C(q)}{x'_C(q)}\right)(1-q)\,x'_B(q)\,dq$$

Step 2: Simplify with integration by parts (Define $W_{A,B}$):

$$\hat{R}_B = \int_0^1 W_{A,B}(q)\,\hat{b}_C(q)\,dq$$

Step 3: Bound $\int_0^1 |W_{A,B}(q)|\,dq$.

Step 4: Estimator for N sorted bids is $\hat{R}_B \approx \sum_i W_{A,B}(\frac{i}{N-1})\,\hat{b}_{i,C}$.

Results Overview

Results: N bid samples, n positions, auction B with probability ϵ.

1. Can estimate revenue of A and B directly from bids in C.
2. Revenue estimator is a *weighted order statistic.*
3. "Revenue B" estimator has error: $O\left(\frac{1}{\sqrt{N}}\, n \log \frac{n}{\epsilon}\right)$.
4. A universal B test.
5. Can optimize revenue over all feasible position auctions.

A Grand Challenge for CS

A Grand Challenge: understand and guide computation in the wild

- computational primitive: local/individual/strategic optimization.
- objective: good global outcomes
- a key application area: “online markets”

  uber, airbnb, twitter, stackexchange, tinder, . . .