

3 - (c) - 행렬의 곱셈

행렬 곱셈  $a_1 = (-1, 2)$  ,  $b_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$   
 $(1 \times 2 \text{ 행렬})$   $(2 \times 1 \text{ 행렬})$

$$a_1 \times b_1 = -1 \times 3 + 2 \times 2 = 1$$

정의

$$a = (a_1, a_2, \dots, a_n) \text{ 이고 } b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \text{ 이고}$$

$$\vec{a} \times \vec{b} = \langle a, b \rangle = a_1 b_1 + a_2 b_2 + \dots + a_n b_n = \sum_{i=1}^n a_i b_i$$

예시

$$a_1 = (-1, 2) \quad a_2 = (1, 1)$$

$$A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$A \times b_1 = \begin{pmatrix} \langle a_1, b_1 \rangle \\ \langle a_2, b_1 \rangle \end{pmatrix} = \begin{pmatrix} (-3+4) \\ (3+2) \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$a_3 = (3, 0)$$

$$A' = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad A' \times b_1 = \begin{pmatrix} \langle a_1, b \rangle \\ \langle a_2, b \rangle \\ \langle a_3, b \rangle \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

공식

$$A = \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \text{ 이고 } b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \text{ 이고}$$

$$Ab = \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{pmatrix} b = \begin{pmatrix} \langle \vec{a}_1, b \rangle \\ \langle \vec{a}_2, b \rangle \\ \vdots \\ \langle \vec{a}_m, b \rangle \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n a_{1i} b_i \\ \sum_{i=1}^n a_{2i} b_i \\ \vdots \\ \sum_{i=1}^n a_{mi} b_i \end{pmatrix} = \begin{pmatrix} a_{11}b_1 + a_{12}b_2 + \dots + a_{1n}b_n \\ a_{21}b_1 + a_{22}b_2 + \dots + a_{2n}b_n \\ \vdots \\ a_{m1}b_1 + a_{m2}b_2 + \dots + a_{mn}b_n \end{pmatrix}$$

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$$A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix} \text{ over } \mathbb{Q}$$

$$B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix} \text{ over } \mathbb{Z}_4$$

$$A \times B = \begin{pmatrix} \langle a_1, b_1 \rangle & \langle a_1, b_2 \rangle \\ \langle a_2, b_1 \rangle & \langle a_2, b_2 \rangle \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 \\ 5 & 4 \end{pmatrix}$$

01234

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix} = \begin{pmatrix} a_{11}, a_{12}, \dots, a_{1n} \\ a_{21}, a_{22}, \dots, a_{2n} \\ \vdots \\ a_{m1}, a_{m2}, \dots, a_{mn} \end{pmatrix} \text{ over } \mathbb{Q}$$

$$B = \begin{pmatrix} b_{11}, b_{12}, \dots, b_{1k} \\ b_{21}, b_{22}, \dots, b_{2k} \\ \vdots \\ b_{m1}, b_{m2}, \dots, b_{mk} \end{pmatrix} \text{ over } \mathbb{Z}_n$$

$$AB = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix} \begin{pmatrix} b_1, b_2, \dots, b_k \end{pmatrix}$$

$$= \begin{pmatrix} \langle a_1, b_1 \rangle & \langle a_1, b_2 \rangle & \dots & \langle a_1, b_k \rangle \\ \langle a_2, b_1 \rangle & \langle a_2, b_2 \rangle & \dots & \langle a_2, b_k \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle a_m, b_1 \rangle & \langle a_m, b_2 \rangle & \dots & \langle a_m, b_k \rangle \end{pmatrix}$$

$$= \langle a_p, a_q \rangle = \sum_{i=1}^n a_{pi} b_{qi}$$

$$= a_{p1} b_{1q} + a_{p2} b_{2q} + \dots + a_{pn} b_{nq}$$

$\frac{1}{2} \frac{1}{2} \frac{1}{2}$

$$AB = \begin{pmatrix} 1 & 4 \\ 0 & 0 \\ 8 & 0 \end{pmatrix} \begin{pmatrix} 0 & 10 \\ 0 & -3 & 11 \end{pmatrix}$$

$\frac{1}{2} \frac{1}{2} \frac{1}{2} B$

$\frac{1}{2} \frac{1}{2} \frac{1}{2}$

A

		$\rightarrow 0$	1	2
		$\rightarrow 0$	-3	11
$\downarrow$	$\downarrow$			
1	4	$1 \times 0$ + $4 \times 0$	$1 \times 1$ + $4 \times (-3)$	$1 \times 0$ + $4 \times 11$
0	0	$0 \times 0$ + $0 \times 0$	$0 \times 1$ + $0 \times (-3)$	$0 \times 0$ + $0 \times 11$
8	0	$8 \times 0$ + $0 \times 0$	$8 \times 1$ + $0 \times (-3)$	$8 \times 0$ + $0 \times 11$

=

$\frac{1}{2} \frac{1}{2} \frac{1}{2} B$

$\frac{1}{2} \frac{1}{2} \frac{1}{2}$

A

		$\rightarrow 0$	1	2
		$\rightarrow 0$	-3	11
$\downarrow$	$\downarrow$			
1	4	0	-3	44
0	0	0	0	0
8	0	0	8	0

$$= \begin{pmatrix} 0 & -3 & 44 \\ 0 & 0 & 0 \\ 0 & 8 & 0 \end{pmatrix}$$

스칼라  $k$  가 행렬  $A$  의 각 원소를  $k$  배 곱하여 곱셈 가능

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$$A = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

이때 이 행렬을  $k$  배 곱하면

$kA$  는 다음과 같다.

$$kA = k \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} = \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$

예제

$$A = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix} \quad \frac{1}{2} \text{ 스칼라 곱해 주면}$$

$$= \begin{pmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

행렬 곱셈의 규칙

① 행렬  $A$  와 행렬  $B$  를 곱할 때 행렬  $A$  의 열의 개수와 행렬  $B$  의 행의 개수가 같아야 함

$$\hookrightarrow A = (m \times n) \text{ 행렬}, B = (n \times l) \text{ 행렬}$$

$$\text{결과} \Rightarrow AB = (m \times l) \text{ 행렬}$$

② 곱셈의 순서에 따라 결과가 달라짐. 행렬의 크기가 같아도

$$A' = \begin{pmatrix} -1 & 2 \\ 1 & 1 \\ 3 & 0 \end{pmatrix} \quad B' = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 3 & 0 \end{pmatrix} \text{ 일때}$$

$$A'B' = \begin{pmatrix} 1 & 5 & -2 \\ 5 & 4 & 2 \\ 9 & 3 & 6 \end{pmatrix} \quad B'A' = \begin{pmatrix} 4 & 7 \\ 1 & 7 \end{pmatrix}$$

## 영행렬 Zero-matrix

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{모든 채워진 행렬}$$

0 행렬을 곱하지 않아도 영행렬이 되는 경우가 있음

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

## 단위행렬 Identity matrix

→ 항등 사상 Identity map

$A \times I = A$  와 같이 곱해도 값이 바뀌지 않는 행렬

왼쪽위, 오른쪽 아래의 대각선상의 성분이 모두 1이고, 나머지는 0인 행렬

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dots \quad \text{등}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 1 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 1 & 7 \end{pmatrix}$$