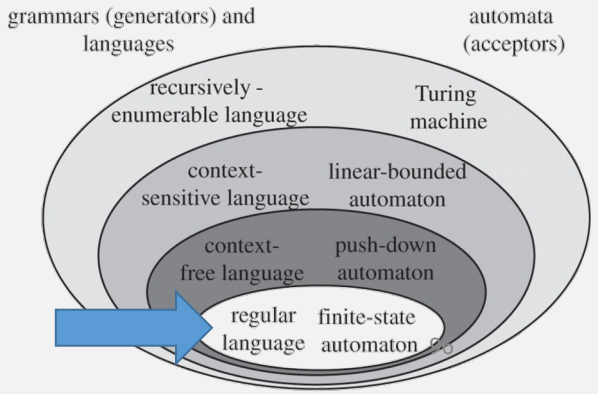
Regular Languages



Logistic Stuff

- Exam 1: 10AM – 8PM Saturday Oct 16
- Exam 2: 10AM – 8PM Saturday Nov 27 (???)
- See Schedule
- Thoughts?

- Homework Due Tonight.
- General questions I can answer?

The Computation Model *Informally*

- Computer = some finite automata
- Program = input string of chars
- Start in “start state”
- 1 char at a time, follow transition table to change states
- Result =
  - “Accept” if stops at an “Accept” state
  - “Reject” otherwise

Last Time: Finite Automata, Formally

5 components

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta: Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

Last Time: Computation, Formally

- A finite automata $M = (Q, \Sigma, \delta, q_0, F)$ is a computer
- We "run" on M an input string $w = w_1 w_2 \cdots w_n$, e.g. "1101"
- M **accepts** w if there is sequence of states r_0,...,r_n in Q where:
  - $r_0 = q_0$ **(start in "start" state)**
  - $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$ **("next" states follow transition table)**
  - $r_n \in F$ **(last state is an "accept" state)**

Last Time:

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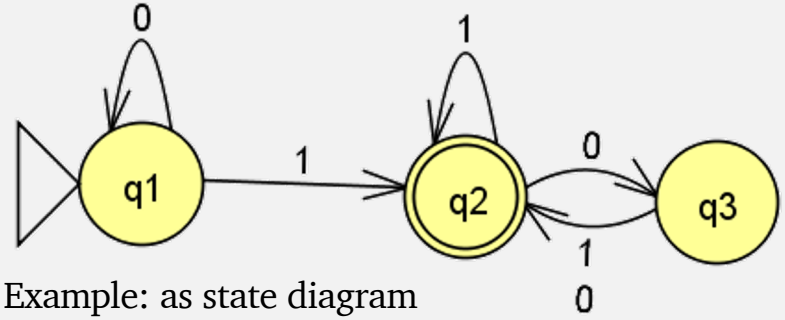


Example: as state diagram

Example: as formal description

$M_1 = (Q, \Sigma, \delta, q_1, F)$, where

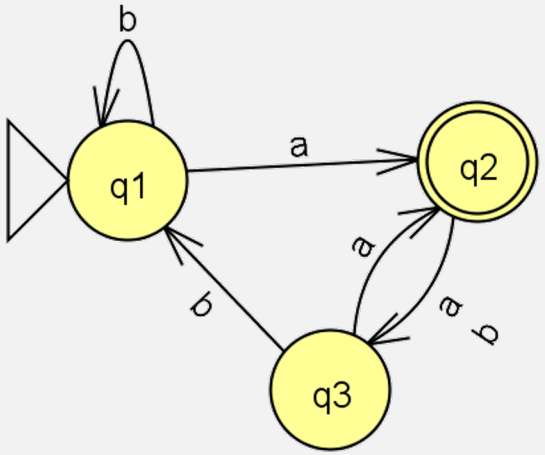
1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

In-class exercise 1

- Come up with a formal description of the following machine:



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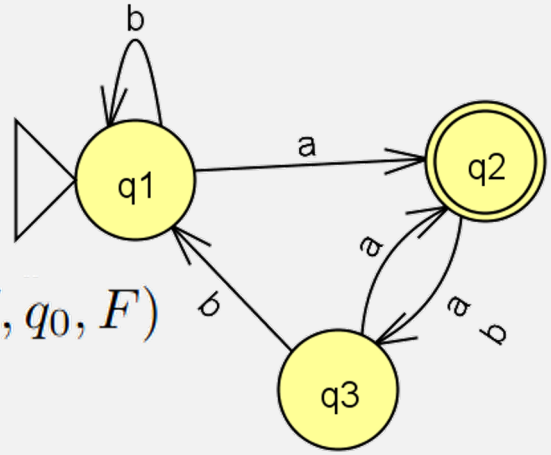
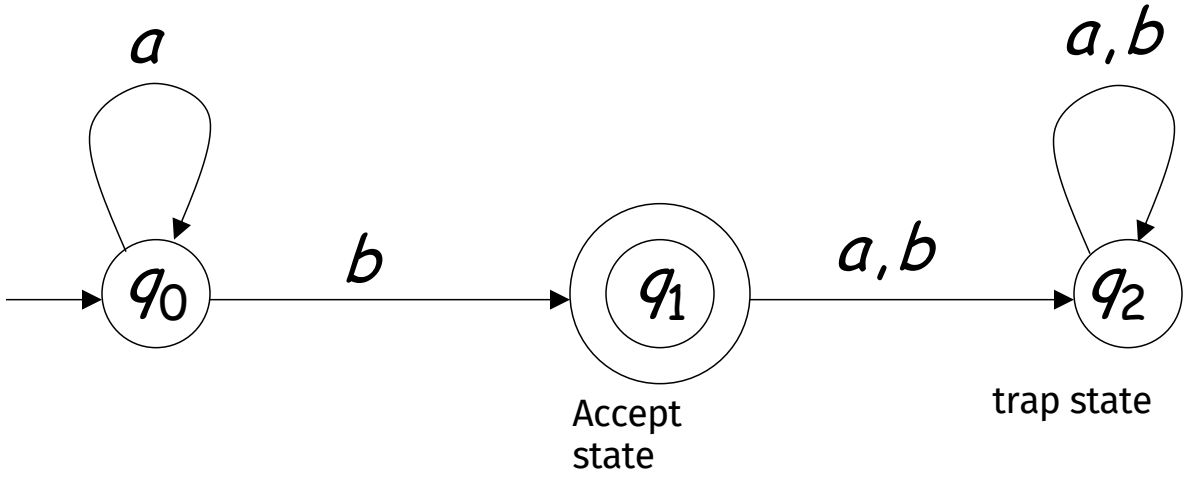
In-class exercise 1: solution



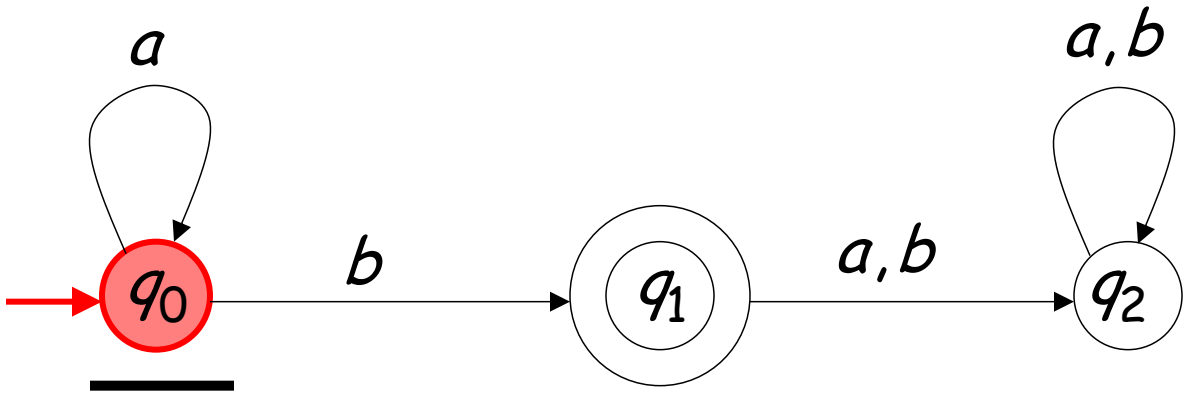
$$M = (Q, \Sigma, \delta, q_0, F)$$

- Q = {q1, q2, q3}
- Σ= {a, b}
- Delta
  - δ(q1,a) = q2
  - δ(q1,b) = q1
  - δ(q2,a) = q3
  - δ(q2,b) = q3
  - δ(q3,a) = q2
  - δ(q3,b) = q1
- q_0 = q1
- F = {q2}

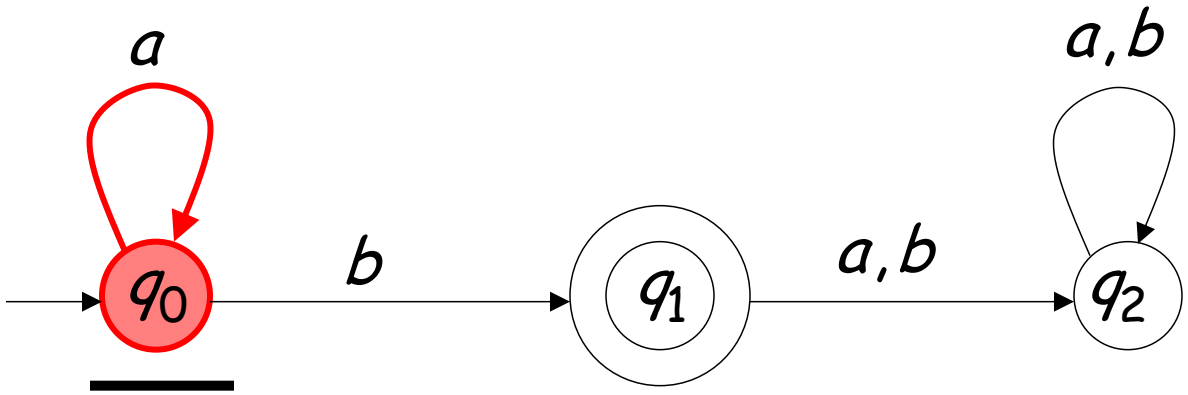
Another Example

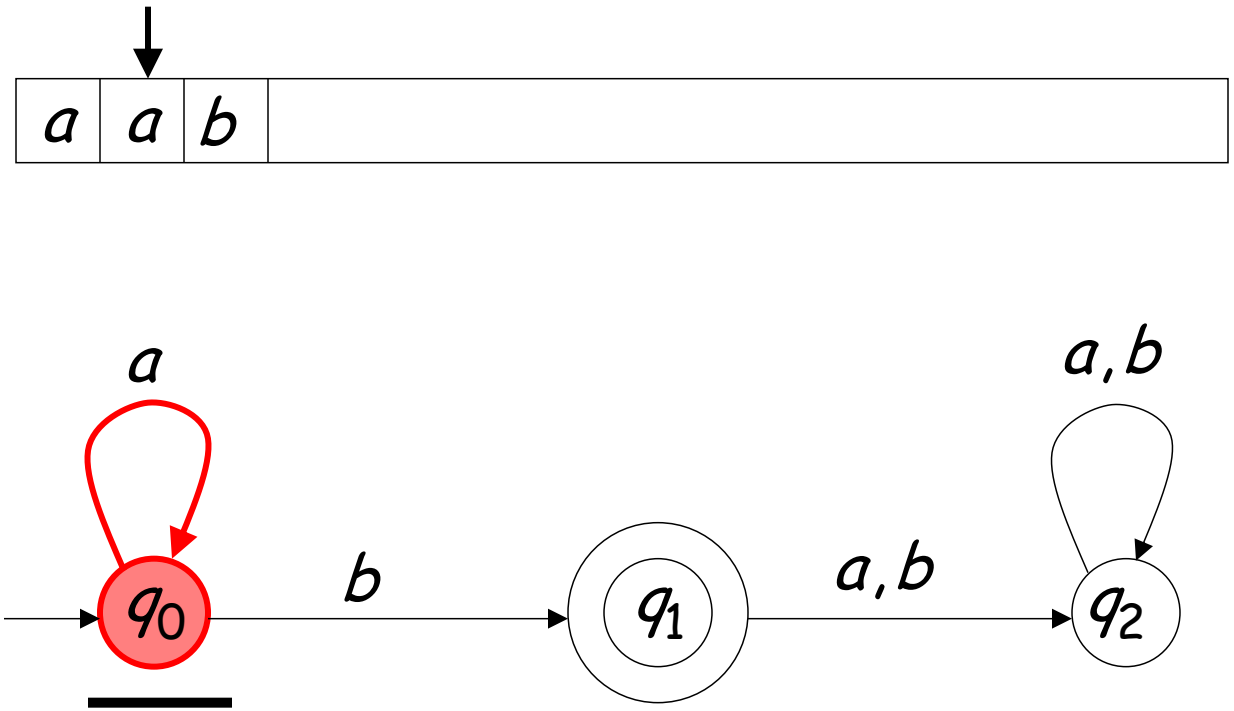


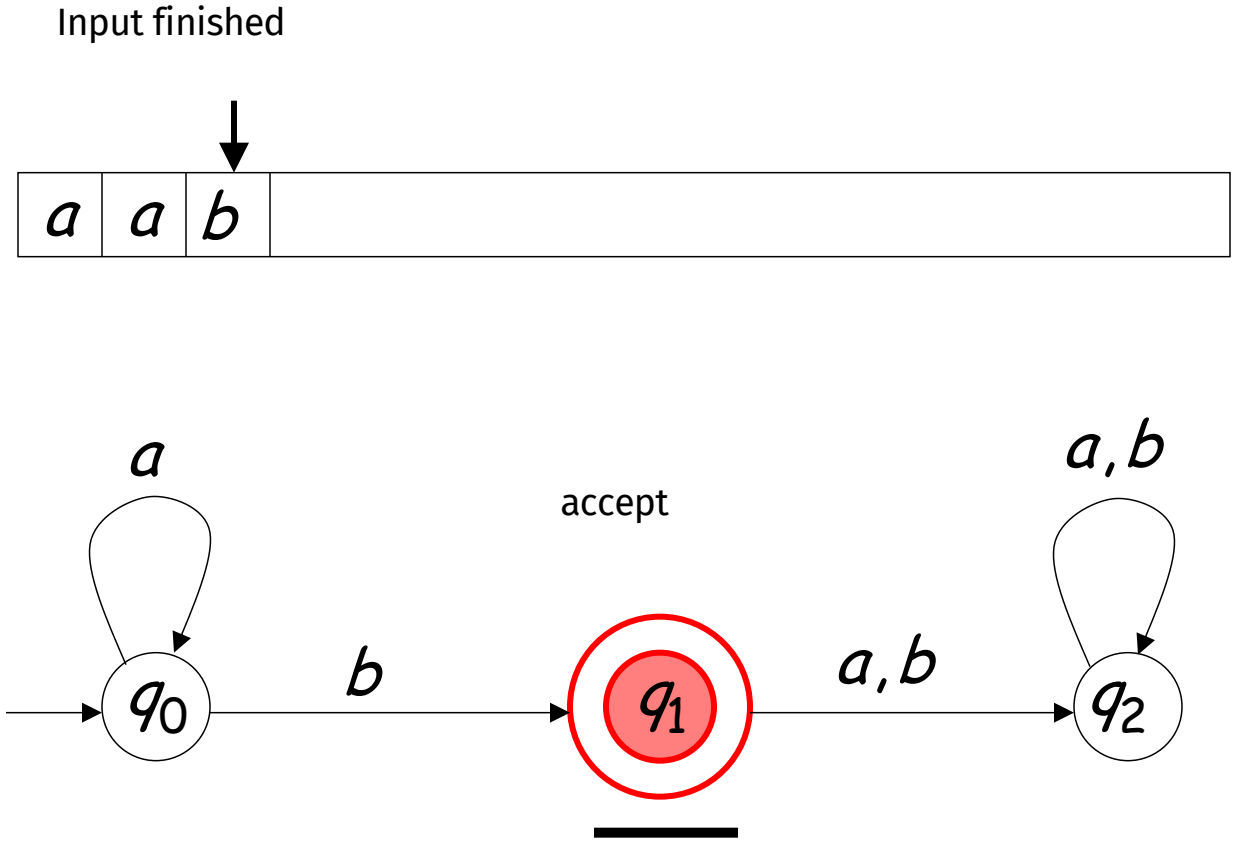
Input String




a
a
b

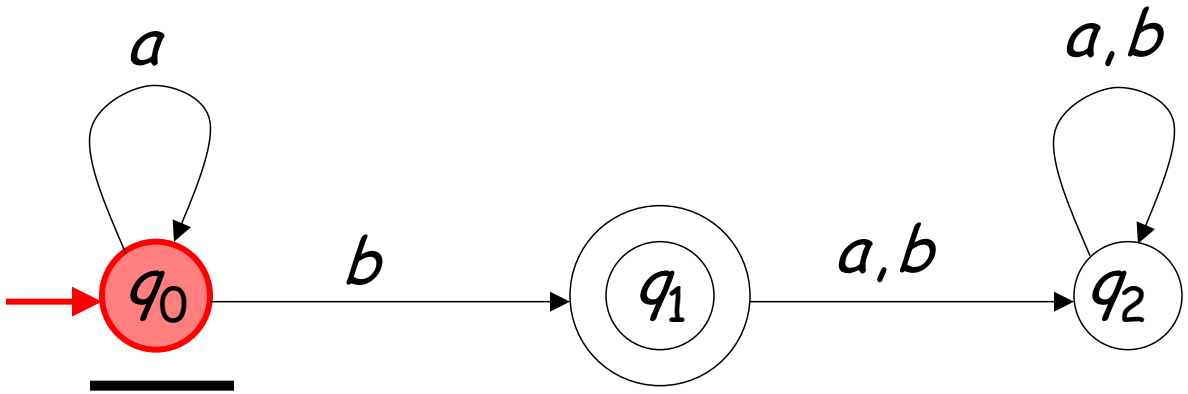

a
b
a, b
a, b
q_0
q_1
q_2

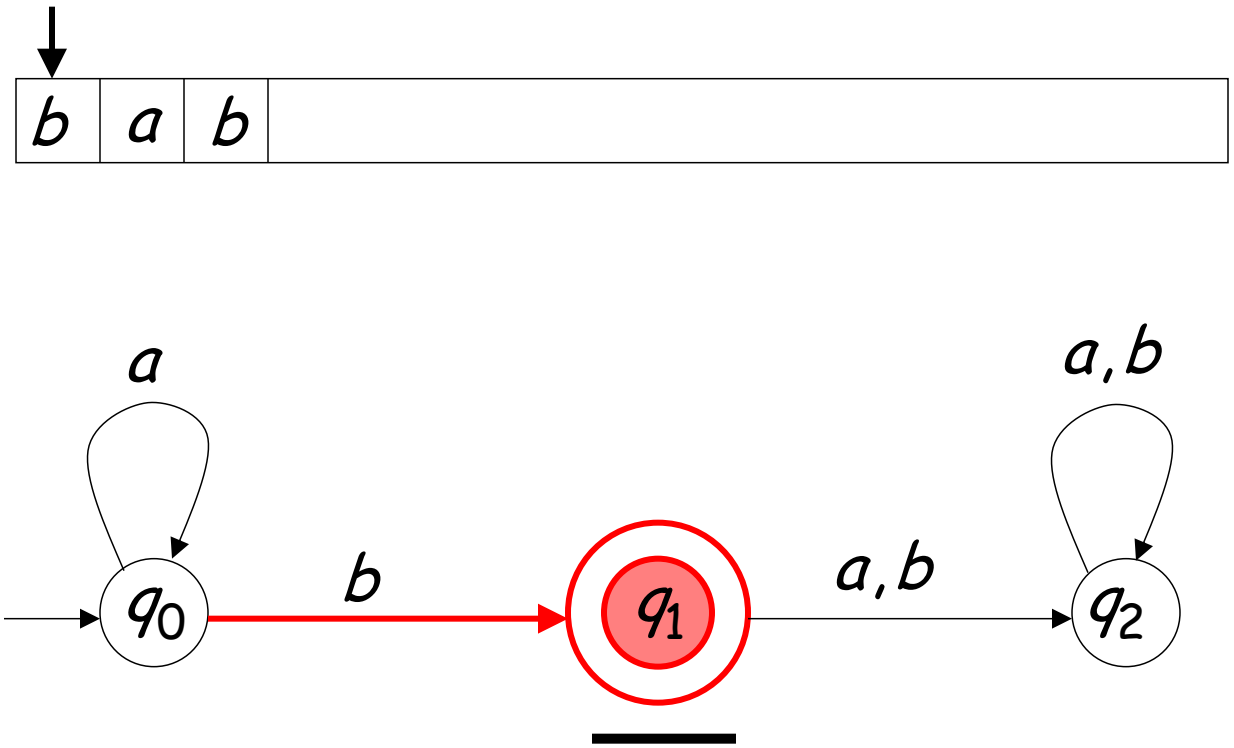

a
a
b
a
b
a, b
a, b
q0
q1
q2

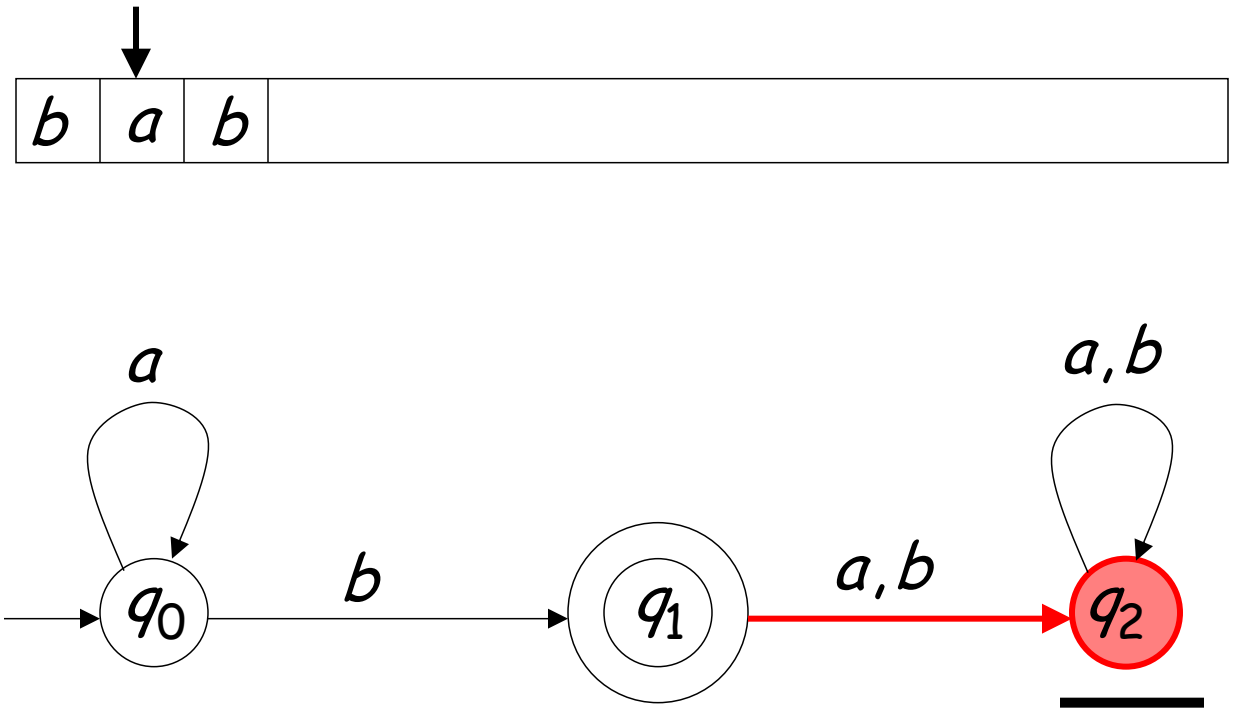

Input finished
a
a
b
a
accept
a, b
b
a, b
q_0
q_1
q_2

A rejection case

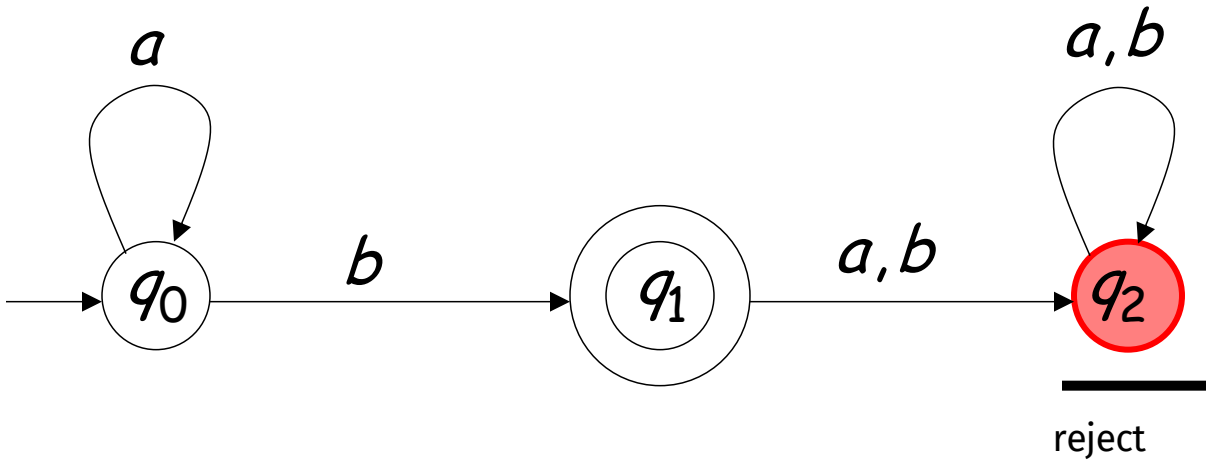
Input String




b
a
b
a
a, b
b
a, b
q_0
q_1
q_2

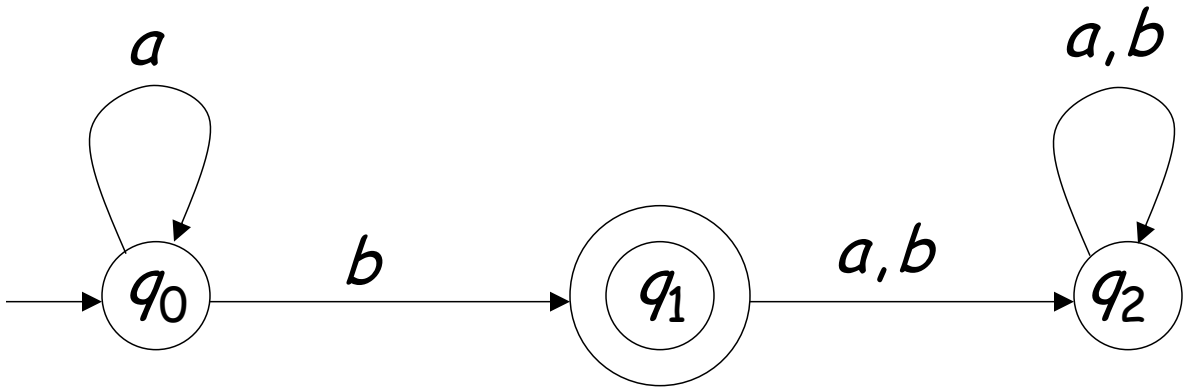

b
a
b
a
a, b
a, b
b
q_0
q_1
q_2

Input finished



Language Accepted:

$$L = \{a^n b : n \geq 0\}$$



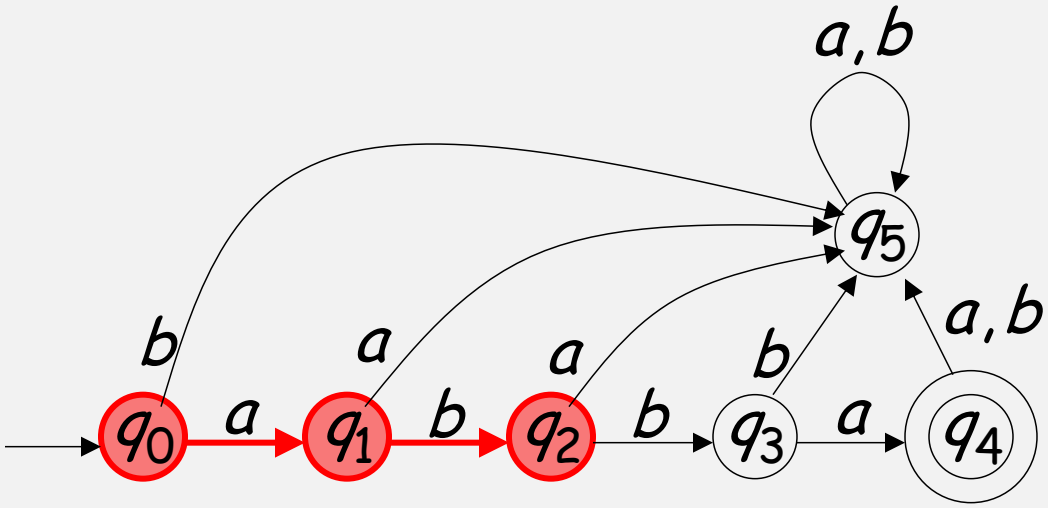
Extended Transition Function

- $\delta^* : Q \times \Sigma^* \to Q$

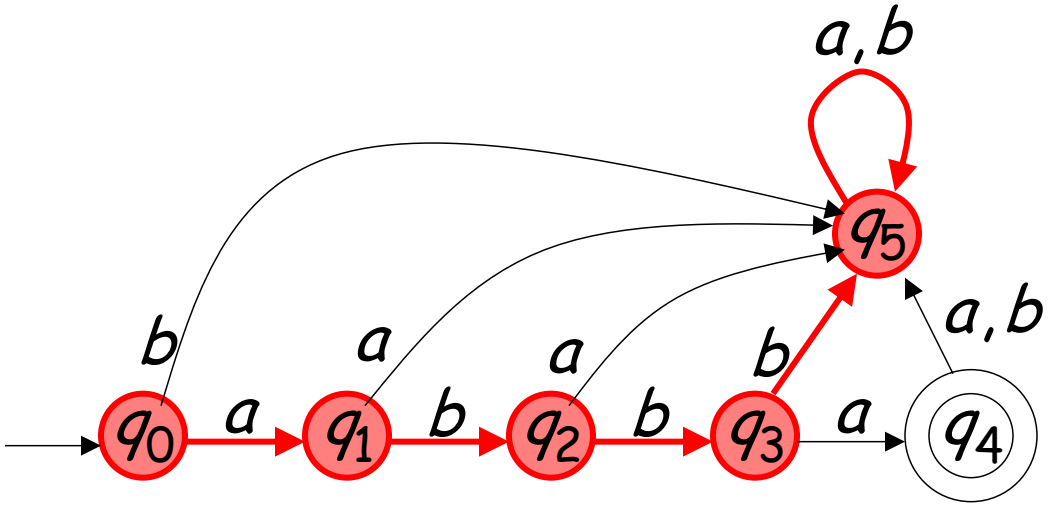
$$\delta^*(q, w) = q'$$

Describes the resulting state
after scanning string w from state q

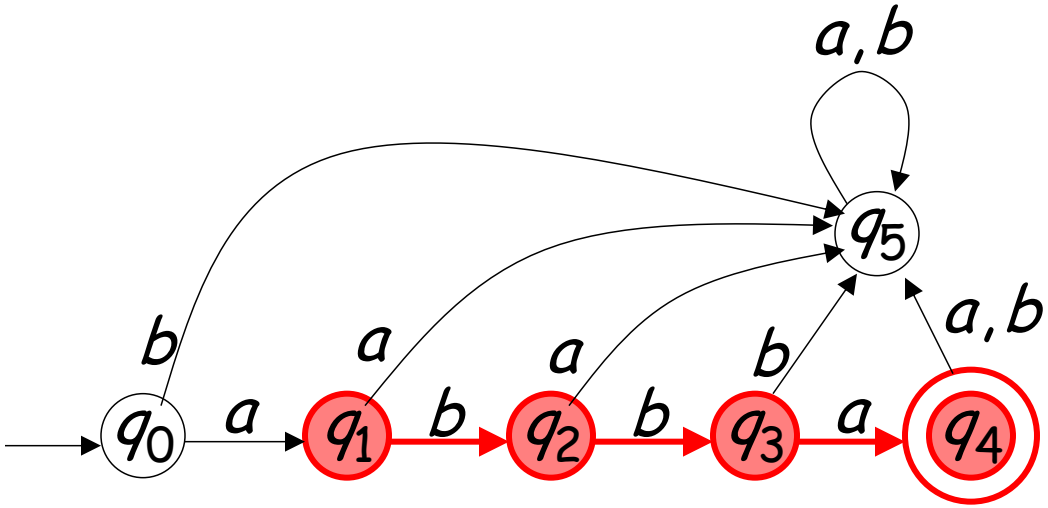
- Example: $\delta^*(q_0, ab) = q_2$



$$\delta^*(q_0, abbbaa) = q_5$$



$$\delta^*(q_1, bba) = q_4$$



Special case:

for any state: $\delta^*(q,\varepsilon) = q$

A ***language*** is a set of strings.

Terminology

- M ***accepts*** w
- M ***recognizes language*** A

$$\text{if } A = \{w | \, M \text{ accepts } w\}$$

“the set of all …” **“such that …”**

A ***language*** is a set of strings.

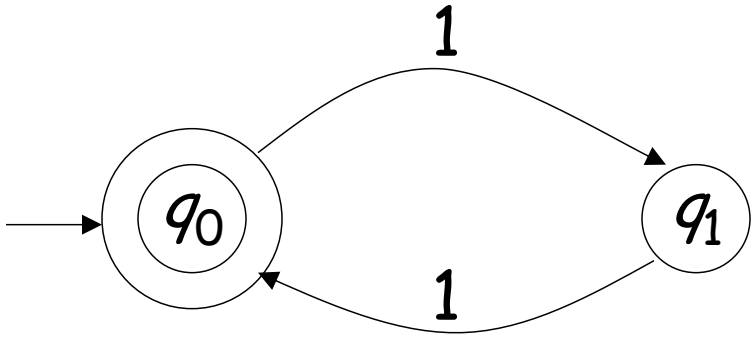
Terminology

- M ***accepts*** w
- M ***recognizes language*** A

$$\text{if } A = \{w | \ M \text{ accepts } w\}$$

Another Example

Alphabet: $\Sigma = \{1\}$



Language Accepted:

$$EVEN = \{x : x \in \Sigma^* \text{ and } x \text{ is even}\}$$
$$= \{\varepsilon, 11, 1111, 111111, \ldots\}$$

Regular Languages

A language is called a ***regular language*** if some finite automaton recognizes it.

- Definition:
- A language *S* is regular if there is
- a DFA M that accepts it (i.e. L(M) = S)

- The languages accepted by all DFAs
- form the family of regular languages

A ***language*** is a set of strings.

M ***recognizes language*** A
if $A = \{w | \ M \text{ accepts } w\}$

A language, regular or not?

- If given: Finite Automata M
  - We know: the language recognized by M is a regular language

- If given: some Language A
  - Is A is a regular language?
    - Not necessarily

There exist languages which are not Regular:

$$L = \{a^n b^n : n \geq 0\}$$

$$ADDITION = \{x + y = z \ : x = 1^n, y = 1^m, z = 1^k, \\ n + m = k\}$$

There is no DFA that accepts these languages

(we will prove this in a later class)

If given: some Language A

How do we determine, i.e., *prove*, that A is a regular language?

Kinds of Mathematical Proof

- Proof by construction
  - Construct the mathematical object in question
- Proof by contradiction

- Proof by induction

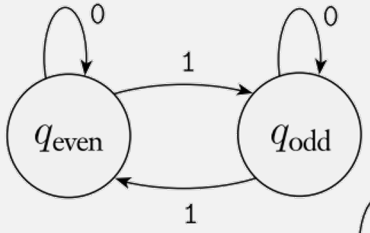
Designing Finite Automata: Tips

- Input may only be read once
- Must decide accept/reject after that
- States = the machine's **memory**!
  - Finite amount of memory: must be allocated in advance
  - Think about what information must be remembered.
- (For DFAs) Every state/symbol pair must have a transition
- Example: machine accepts strings with odd number of 1s

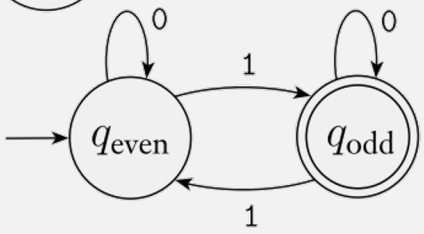
Design a DFA: accepts strs with odd # 1s

- States:
  - 2 states:
    - seen even 1s so far
    - seen odds 1s so far

- Alphabet: 0 and 1
- Transitions:



- Start / Accept states?



In-class exercise 2

- Prove that this language is a regular language:
  - {w | w has exactly three 1's}
  - i.e., design a finite automata that recognizes it!

- Where Σ= {0, 1},

- Remember:

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HW1 Pre-game

- In ToC we primarily learn about abstract mathematical objects
- But we may use code as a way to explore these math objects
- So it's important to understand the distinction: math vs code
- E.g., a set is an abstract mathematical object
  - contains other math objects like: strings, nums, characters, and other sets!
- A set's (data) representation in code can take many forms:
  - e.g., a list, an array, a space-separated string (hw0)

Math vs Representation, Examples

Abstract Math Concept	Possible Data Representation
Numbers	
Set	
Tuple (i.e., a small finite set)	
Function, i.e., a set of pairs	
Finite automata	

Math vs Representation, Examples

Abstract Math Concept	Possible Data Representation
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Function, i.e., a set of pairs	Function, dict, map, hash, tree
Finite automata	XML str, <your choice here>

Sketch out “run” fn pseudocode:

Check-in Quiz

See Gradescope