Property	Value
Name:	IEONG, ZI LIANG JASON
Student ID:	20308345
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$P(X=0) = 1 - P(X=1) = \frac{\theta}{2} + \frac{1}{4} \Rightarrow P(X=1) = \frac{3}{4} - \frac{\theta}{2}$

$L = \prod_{i=1}^{n} \left(\left[\binom{x_i - 1}{-1} \left(\frac{\theta}{2} + \frac{1}{4}\right) \right] \times \left(x_i \right) \times \left(\frac{3}{4} - \frac{\theta}{2} \right) \right)$

$\log L = \sum_{i=1}^{n} \log \binom{x_i - 1}{-1} \left(\frac{\theta}{2} + \frac{1}{4}\right) + \log x_i \left(\frac{3}{4} - \frac{\theta}{2}\right)$

$\frac{d \log L}{d\theta} = \sum_{i=1}^{n} \frac{2}{2\theta+1} + \frac{2}{2\theta-3}$

set it to 0,

$n\left(\frac{2}{2\theta+1} + \frac{2}{2\theta-3}\right) = 0$

n_0 is # 0 in x_i

n_1 is #1 in x_i

$L = p(x=0;\theta)^{n_0} \, p(x=1;\theta)^{n_1}$ ~~where n_0, n_1 is the no. of~~

$\log L = n_0 \log\left(\frac{\theta}{2} + \frac{1}{4}\right) + n_1 \log\left(\frac{3}{4} - \frac{\theta}{2}\right)$

$\frac{d \log L}{d\theta} = n_0 \times \frac{2}{2\theta+1} + (n - n_0) \times \frac{2}{2\theta-3}$

$\theta = -\frac{1}{4}(n)(-2n_0 - 1)$ where n_0 is the no. of 0

$= -\frac{1}{4}(n)^2(-2(1-\bar{x}) - 1)$

$n_0 = n - n_1$

$\frac{n - n_1}{n} = 1 - \bar{x}$

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$f_X(x|\theta) = \frac{3x^2}{\theta^3} I\{0<x<\theta\}$

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$E(\hat{\theta}_1) = E(\frac{2}{3}(X_1+X_2)) = \frac{4}{3}\int_0^\theta x \times \frac{3x^2}{\theta^3}\, dx = \frac{4}{3} \times 3\, \frac{\theta}{4}$

$\cancel{= \frac{2}{3} \times 2\, E(X_1) = \frac{4}{3}}$ $= \theta$

$\cancel{E(\hat{\theta}_2) = \frac{7}{6} \max\{X_1, X_2\}}$

$\cancel{F_X(x|\theta) = \int_0^x \frac{3x^2}{\theta^3}\, dx}$

$F_X(x|\theta) = \frac{x^3}{\theta^3} I\{0<x<\theta)$

$f_{X_{(2)}} = 2\left(\frac{3x^2}{\theta^3}\right)\left(\frac{x^3}{\theta^3}\right)$

$E(X_{(2)}) = E(2\left(\frac{3x^2}{\theta^3}\right)\left(\frac{x^3}{\theta^3}\right))$

$= \frac{2\cdot 3}{\theta^6}\int_0^\theta x \cdot x^5\, dx = \frac{\theta^7 \times 6}{7\theta^6} = \frac{6\theta}{7}$

$\therefore E(\frac{7}{6}\max(X_1, X_2)) = \frac{7}{6} \times \frac{6}{7}\theta = \theta$ $\therefore$ unbiased.

$\therefore$ They are both unbiased.

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$$\operatorname{var}(\hat{\theta}_1) = \frac{2}{7}\int_0^\theta (x-\theta)^2 \frac{3x^2}{\theta^3}\,dx$$

$$= 2\,\frac{\theta^2}{13015} = \frac{\theta^2}{13}$$

$\cancel{\operatorname{var}(\hat{\theta}_2)}$

$$\operatorname{var}(X_{(2)}) = \frac{2\cdot 3}{\theta^6}\int_0^\theta (x-\theta)^2 x^5\,dx$$

$$= \frac{2\cdot 3}{\theta^6}\cdot\frac{\theta^8}{168}$$

$$= \frac{\theta^2}{28}$$

$$\operatorname{var}\left(\frac{6}{7}\theta_2\right) = \frac{6^2}{7^2}\cdot\frac{1}{28}\theta^2 < \operatorname{var}(\hat{\theta}_1)$$

$\therefore$ θ_2 is better