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MATH3423

QUIZ 2

1. Let $\{X_i : i = 1, \dots, n\}$ be a rs from a distribution with
$$P(X = 0) = 1 - P(X = 1) = \frac{\theta}{2} + \frac{1}{4},$$
where $\theta \in [0, 1]$ is unknown.
   a. [8 marks] Find the MLE $\hat{\theta}$ of θ.
   b. [2 marks] Find the MLE of $P(X = 0)$.

2. Consider a rs $\{X_1, X_2\}$ from a distribution with a pdf given by
$$f_X(x|\theta) = \frac{3x^2}{\theta^3} I_{\{0<x<\theta\}},$$
where the unknown parameter θ is finite and positive.
   a. [6 marks] Are $\hat{\theta}_1 = \frac{2}{3}(X_1 + X_2)$ and $\hat{\theta}_2 = \frac{7}{6}\max(X_1, X_2)$ unbiased for θ?
   b. [4 marks] Which one of $\hat{\theta}_1$ and $\hat{\theta}_2$ is better? Please justify your answer.