Assignment 2

Please submit your solution (in pdf) on Canvas before 4pm on April 15, 2020.

Question 1: Consider a random sample $\{X_1, \dots, X_n\}$ of size $n > 1$ with an unknown mean $\mu \in (-\infty, \infty)$ and unknown variance $\sigma^2 \in (0, \infty)$. Show that S_n^2 has a smaller MSE than S_{n-1}^2.

Question 2: Consider a random sample $\{X_1, \dots, X_n\}$ of size $n > 2$ from $U[0, \theta]$, where θ is positive and finite. We found that $X_{(n)}$ is the MLE of θ, and it is easy to see that $2\bar{X}$ is the MME of θ. In the sense of MSE, which one is better? Please justify your answer.

Question 3: Consider a random sample $\{X_1, \dots, X_n\}$ of size $n > 1$ from a uniform distribution on an interval $\left[\mu - \sqrt{3}\sigma,\ \mu + \sqrt{3}\sigma\right]$, where $\mu \in (-\infty, \infty)$ and $\sigma \in (0, \infty)$ are unknown. Find

a) the MMEs of μ and σ, and
b) the MLEs of μ and σ.

Question 4: Consider a random sample $\{X_1, \dots, X_n\}$ from a distribution with a pdf defined by

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1}, & for\ x \in (0,1) \\ 0, & otherwise \end{cases},$$

where $0 < \theta < \infty$. Let $g(\theta) = 1/\theta$.

a) Find the MLE of $g(\theta)$.
b) Is the MLE an unbiased estimator of $g(\theta)$?

Given that the regularity conditions hold, then

c) Find the C-R inequality for $g(\theta)$.
d) Show that the MLE found in (a) is the UMVUE of $g(\theta)$.