



INTERNATIONAL
BUSINESS SCHOOL

Annuities, perpetuities and Loan amortisation

Principles and Practices of
Business Finance

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Agenda

1. Present value of multiple cash flows
2. Perpetuity
3. Annuity
 - a) Present value of an annuity
 - b) Future value of an annuity
4. Ordinary Annuity and Annuity due
5. Amortizing Loans

Present Value of Multiple Cash Flows

- Present Values can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \dots$$

$$\sum \frac{C_t}{(1+r)^t}$$

Present Value of Multiple Cash Flows

Example

Your auto dealer gives you the choice to pay \$15,500 cash now, or make three payments: \$8,000 now and \$4,000 at the end of the following two years. If your cost of money is 8%, which do you prefer?



Perpetuity & Annuity ¹

Annuity

A **series of equal payments** made at **regular intervals** over a **fixed period**.

***Example:** A 20-year mortgage with fixed monthly payments.*

Perpetuity

A **series of equal payments** made at **regular intervals** that continue **indefinitely**.

***Example:** A scholarship fund that provides a fixed amount of annual financial aid to students indefinitely.*

Perpetuities

Present Value of Perpetuity Formula

$$PV = \frac{C}{r}$$

C = cash payment/receipt

r = interest rate/discount rate

Perpetuities

Example - Perpetuity

In order to create an endowment, which pays \$100,000 per year forever, how much money must be set aside today if the rate of interest is 10%?

$$PV = \frac{100,000}{0.10} = \$1,000,000$$

Problems:

- ❖ How does PV of perpetuity change if the interest rate changes?

Present Value of Annuity

The Present Value of an Annuity represents the current worth of a series of future periodic payments, discounted at a specific interest rate.

Formula:

$$PV = C \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Where:

PV = Present Value of the annuity

C = Cash flow per period

r = Discount rate per period

t = Number of periods

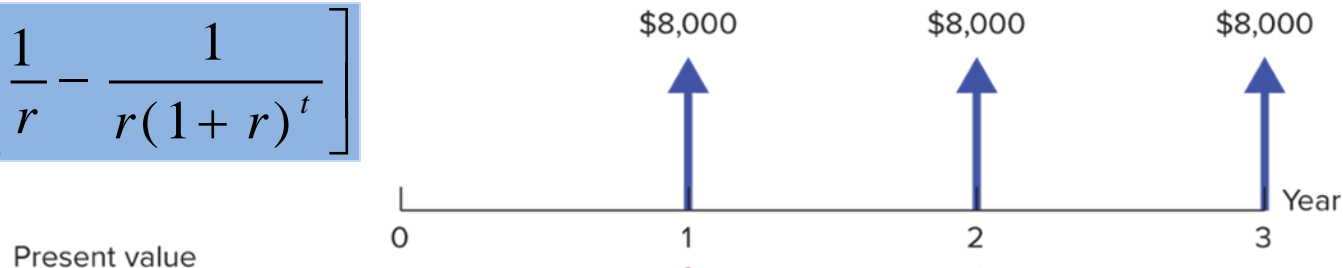
Present Value of Annuity

Example

You are purchasing a car. You are scheduled to make 3 annual installments of \$8,000 per year. Given a rate of interest of 10%, what is the price you are paying for the car (that is what is the PV)?

This is an annuity—use the Present Value of Annuity formula to calculate its present value.

$$PV = C \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$



$$PV = \$8,000 \times \left[\frac{1}{.10} - \frac{1}{.10(1+.10)^3} \right]$$

$$PV = \$19,894.82$$

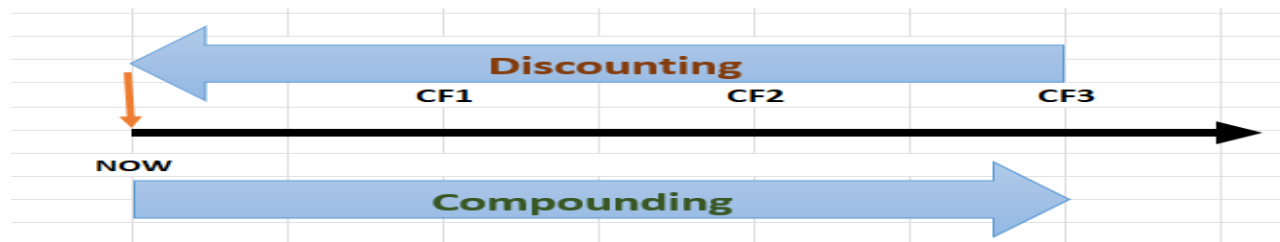
Present Value of an Annuity

Problems regarding the previous example:

- ❖ What is the theoretical difference between \$ 24,000 (3X \$ 8,000) and \$ 19,894.82?
- ❖ Why are the two amounts incomparable?
- ❖ Can we apply annuity if the installments are not equal?

Important rules related to Time Value of Money

- Rule1: Only values at the same point in time can be compared or combined.
- Rule2: To calculate a cashflow's future value, we must compound it.
- Rule3: To calculate the present value of a future cashflow, we must discount it.



Time Periods and Discounting

Assume that the investor received €500 **semiannually** for three years when the discount rate (APR) was 10%. Calculate the present value of this annuity.

First, draw a timeline for this annuity. Then, identify the values of C (cash flow), r (discount rate per period), and t (number of periods).

Time Periods and Discounting

Illustration: Assume that the investor received €500 **semiannually** for three years when the discount rate (APR) was 10%. Calculate the present value of this annuity.

r and *t* should be aligned in terms of time framework: annual, semiannual, quarterly, weekly, daily

$$PV = 500 \times \left[\frac{1}{0.05} - \frac{1}{0.05(1+0.05)^6} \right]$$

PV of annuity= €2,537.85

Future Value of an Annuity

FV of annuity based on the main formula of FV that we learnt last week:

$$FV = p \times (1 + r)^t$$

The FV annuity is simply the present value of the annuity multiplied by $(1 + r)^t$

$$FV_{\text{annuity}} = PV_{\text{annuity}} \times (1+r)^t = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right] \times (1 + r)^t$$

Future Value of an Annuity

Example - Future Value of annual payments

*You plan to save \$4,000 every year for 20 years and then retire.
Given a 10% rate of interest, what will be the FV of your retirement account?*

$$FV = 4,000 \times \left[\frac{1}{0.1} - \frac{1}{0.1(1+0.1)^{20}} \right] \times (1+0.1)^{20}$$

$$FV = \$229,100$$

Annuity Types

- ① **Ordinary Annuity** – Payments occur at the **end** of each period.

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

- ② **Annuity Due** – Payments occur at the **beginning** of each period.

In practice, if the problem does not specify whether payments occur at the beginning or end of the period, we assume it is an **Ordinary Annuity.*

$$PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1 + r)$$

$$FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1 + r)$$

Amortizing Loans

- In amortizing loans, regular constant payments made by the borrower (typically monthly), to fully payback the loan by the end of the loan term.
- The regular payments consist of two parts:
 1. Interest
 2. Principal repayment

Home Mortgages Example

- Suppose that a house costs \$125,000 and that the buyer puts down 20% of the purchase price in cash, borrowing the remaining \$100,000 from a mortgage lender. What is the monthly payment to amortise the loan if the loan is to be paid back in 30 years? APR on loans is 12%.

Solution

- Altogether 360 monthly payments
($30 \times 12 = 360$)
- The monthly payments need to be set in a way so that they have a PV of \$100,000.
→ PV of annuity formula to be used

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

- $100,000 = C \times \left[\frac{1}{0.01} - \frac{1}{0.01(1+0.01)^{360}} \right]$

- **$C = \$1,028.61$ (this is the same every month)**

Amortizing Loans

- Monthly (or other periodic) payment consists of a **part used to pay interest** on the loan, and the **other part is used to reduce the amount of the loan (principal repayment)**.
- Interest part calculation = **monthly interest × outstanding balance on the loan**
- Part used to reduce the amount of the loan: **monthly payment – interest part (the remaining part of the monthly payment after interest is deducted)**

Home Mortgages Example Cont.

- Before paying the first monthly payment, how much is the outstanding amount of the loan? It is \$100,000
- *Monthly payment* = \$1,028.61

- What part of the **first** monthly payment is interest?

$$\text{Interest} = 100,000 \times 0.01 = \$1,000$$

- How much is used to reduce the mortgage?

$$\text{Reducing the debt} = 1,028.61 - 1,000 = \$28.61$$

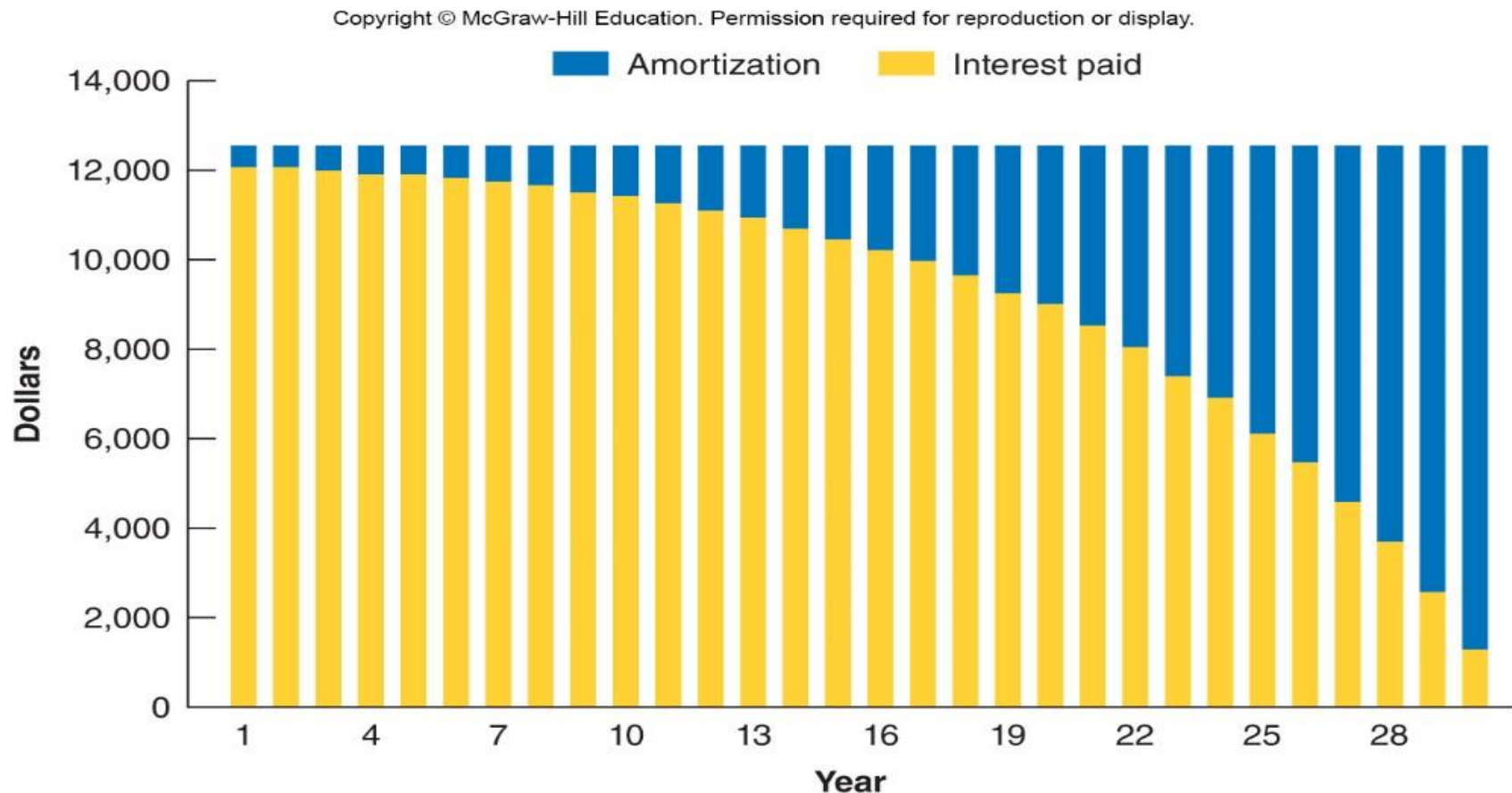
- How much is the outstanding debt/loan after the first payment?

$$\text{The outstanding loan} = \$100,000 - \$28.61 = \$99,971.39$$

Home Mortgages Example Cont.

- Before paying the second monthly payment, the outstanding amount of the loan is \$99,971.39
- *Monthly payment* = \$1,028.61
- What part of the **first** monthly payment is interest?
Interest = $\$99,971.39 \times 0.01 = \999.71
- How much is used to reduce the mortgage?
Reducing the debt = $1,028.61 - \$999.71 = \28.9
- How much is the outstanding debt/loan after the second payment?
The outstanding loan = $\$99,971.39 - \$28.9 = \$99,888.49$

- The loan is progressively paid off →
The interest fraction of each payment decreases steadily.
The fraction used to repay the loan increases steadily.



Mortgage amortization. Mortgage payments are split into interest payment and debt amortization.