Exam Extra practices WITH SOLUTION

1. Eighteen years from now, 4 years of college are expected to cost $150,000.

Required:

a) How much must be deposited into an account today to fund this expense if you could earn 11% on your savings?

b) How much more must be deposited into an account today to fund this expense if you could only earn 8% rather than the 11% you had hoped to earn on your savings?

a) $150,000 / (1.11)^{18} = 22,923.33$

b) $150,000 / (1.08)^{18} = 37,537.35$

PV= 37,537.35 - 22,923.33 = 14,614.02 more must be deposited.

If only 8% interest can be earned: $37,537.35 must be deposited. So $14,614 more must be deposited.

2. Lester's just signed a contract that will provide the firm with annual cash inflows of $28,000, $35,000, and $42,000 over the next three years with the first payment of $28,000 occurring one year from today.

Required: What is this contract worth today at a discount rate of 7.25%?

$PV = (28,000 / 1.0725^1) + (35,000 / 1.0725^2) + (42,000 / 1.0725^3) = \$ 90,580.55$

This contract is worth $90,580.55 today.

3. Would you prefer a savings account that paid 7% interest compounded quarterly, 6.8% compounded monthly, 7.2% compounded weekly, or an account that paid 7.5% with annual compounding?

In order to decide this, we need to calculate the EAR for all four options:

$$EAR = [1 + (0.07 / 4)]^4 - 1 = 0.0719, \text{ or } 7.19\%$$

$$EAR = [1 + (0.068 / 12)]^{12} - 1 = 0.0702, \text{ or } 7.02\%$$

$$EAR = [1 + (0.072 / 52)]^{52} - 1 = 0.0746, \text{ or } 7.46\%$$

$$EAR = APR = 7.5\%$$

We prefer the highest one, 7.5%.

4. A couple will retire in 25 years; they plan to spend about $40,000 a year in retirement, which should last about 30 years. The APR is 6%.

a. If they make annual payments into a savings plan, how much will they need to save each

year?

First, calculate what amount they need to accumulate by the time they retire (present value of annuities for the cash flows in retirement) Then, calculate how much they should save each year to achieve this. (future value of annuities for the cash flows they save each year while they are still working) $10,035.51

$$PV = C((1 / r) - \{1 / [r(1 + r)^t]\})$$
$$C=40,000$$
$$PV = 40000\ \{(1/0.06) - (1 / (0.06(1 + 0.06)^{30}))\}$$
$$PV = 550,593.25$$

$$550,593.25 = C((1 / r) - \{1 / [r(1 + r)^t]\})\ (1 + r)^t$$
$$550,593.25 = C((1 / 0.06) - \{1 / [0.06(1 + 0.06)^{25}]\})\ (1 + 0.06)^{25}$$
$$C= 10,035.51$$

5. Today you buy a 10% coupon, 10-year maturity bond when its yield to maturity is 7%.

a. Without making any calculations, tell whether the bond is trading at a discount or ata premium today. Explain your answer.
b. Calculate the price of the bond today.
c. What is the current yield of the bond?
d. What do you expect, if everything stays the same, is the price of the bond going to increase or decrease next year? Explain your answer.

Assume that a year later the company is facing financial difficulties and the yield to maturity increases to 10.5%.

e. Without making any calculations, tell what is going to happen to the price of the bondin this case? Explain your answer.
f. What is your rate of return over the year given the increase of YTM? Explain your result.

Solution:

a. YTM is smaller than the coupon rate, so the bond is currently trading at a premium.
b. $1,210.71
c. current yield= 8.26%
d. As this is a premium bond, the bond price will decrease if everything else stays the same.
e. As the YTM increases, the bond price will decrease. (it will be trading at discount as the YTM in this case is higher than the coupon rate)
f. First calculate the P_1 ($971.77), and then the ROR. ROR= -11.48%

calculations

a) YTM is smaller than the coupon rate, so the bond is currently trading at a premium.

b) $$PV = C((1 / r) - \{1 / [r(1 + r)^t]\})$$
$$PV = 100\ ((1 / 0.07) - \{1 / [0.07(1 + 0.07)^{10}]\}) + 1,000 / (1.07)^{10}$$
$$PV = 100(14.2857 - 7.2621) + 508.35$$
$$PV = 1,210.71$$

c) Current yield = 100 / 1210.71 = 0,0826 = 8,26%
d) As the YTM increases, the bond price will decrease. (it will be trading at discount as the YTM in this case is higher than the coupon rate)
e) PV = $C((1 / r) – \{1 / [r(1 + r)^t]\})$
PV = $100\ ((1 / 0.105) – \{1 / [0.105(1 + 0.105)^9]\}) + 1{,}000 / (1.105)^9$
PV = 100(9.5238 – 3.8775) + 407.133
PV = 971.77

6. You invested $1,500 three years ago. During the three years, you earned annual rates of return of 3.8%, 8.6%, and 12.1%. What is the value of this investment today?
$1895.5

PV = 1500 * 1.038 * 1.086 * 1.121 = $1895.5

7. If Bob and Judy combine their savings of $1,260 and $975, respectively, and deposit this amount into an account that pays 2% annual interest, compounded monthly, what will the account balance be after 4 years?

$2420.98 (don't forget that you need to convert "r" and "t" to monthly!)

$R_{monthly}$ = 2% /12 = 0,1667 %
t = 4 * 12 = 48
PV = 1,260 + 975 = 2,235
FV = 2,235 $*(1 + r)^t$
FV = $2{,}235 *(1+0.001667)^{48}$ = $2420.985

8. Gabrielle just won $2.5 million in the state lottery. She is given the option of receiving a total of $1.3 million now, or she can elect to be paid $100,000 at the end of each of the next 25 years. If Gabrielle can earn 5% annually on her investments, from a strict economic point of view which option should she take?

Option 1 is worth $1.3 million.
Option 2 is worth $1.409 million.
From a strict economic point of view, it is better to choose the 100,000 per year for 25 years.

Option 1 = $ 1.3 million
Option 2 = ?
PV = $C((1 / r) – \{1 / [r(1 + r)^t]\})$
PV = $\$100{,}000* ((1 / 0.05) – \{1 / [0.05(1 + 0.05)^{25}]\})$ =
PV = 1,409,394
From a strict economic point of view, it is better to choose the 100,000 per year for 25 years.

9. Joseph is a friend of yours. He has plenty of money but little financial sense. He received a gift of $12,000 for his recent graduation and is looking for a bank in which to deposit the funds. Partners' Savings Bank offers an account with an annual interest rate of 3% compounded semiannually, whereas Selwyn's offers an account with a 2.75% annual interest rate compounded continuously. Calculate the value of the two accounts at the end of 1 year, and recommend to Joseph which account he should choose.

Compounded semiannually: $12,000\times(1+0.015)^2 = \$12362.7$
Compounded continuously (NOT needed for the midterm): $12,000\times e^{0.0275\times1} = \$12,334.58$

10. Jack and Jill have just had their first child. If college is expected to cost $150,000 per year in 18 years, how much should the couple begin depositing annually at the end of each year to accumulate enough funds to pay the first year's tuition at the beginning of the nineteenth year? Assume that they can earn a 6% annual rate of return on their investment.

$150,000 is the future value of this annuity. C=$4853.48

FV = $150,000
$FV = C((1 / r) - \{1 / [r(1 + r)^t]\}) (1 + r)^t$
$\$150,000 = C ((1/0.06) - \{1 / [0.06*(1 + 0.06)^{18}]\}) (1 + 0.06)^{18}$
$\$150,000 = C * 30,906$
$C = \$ 4853.48$

11. You can deposit $10,000 into an account paying 9% annual interest either today or exactly 10 years from today. How much better off will you be at the end of 40 years if you decide to make the initial deposit today rather than 10 years from today?

If I deposit the $10,000 today, I will have $314,094.2 at the end of 40 years.

Option 1: for 40 years
$FV = \$10,000 *(1 + r)^t$
$FV = \$10,000 *(1 + 0.09)^{40} = \$ 314,094,2$
If I deposit the $10,000 today, I will have $314,094.2 at the end of 40 years.

Option 2: I deposit 10 years later
$FV = \$10,000 *(1 + r)^t$
$FV = \$10,000 *(1 + 0.09)^{30} = \$ 132,676.78$

If I deposit 10 years later, I will have $132,676.78 at the end of 40 years.
I will be $314,094.2 - $132,676.78 = 181,417.42 dollars better off.

12.

Case	Amount of annuity	Interest rate	Deposit period (years)
A	$2,500	8%	10
B	$500	12%	6
C	$30,000	20%	5
D	$11,500	9%	8
E	$6,000	14%	30

a. Calculate the future value of the annuity, assuming that it is

(1) An ordinary annuity.

Case	**Ordinary annuity value**
A	**$36,216.4**
B	**$4,057.6**
C	**$223,248**
D	**$126,827.45**
E	**$2,140,721.08**

(2) An annuity due.

Case	**Annuity due**
A	**$39,113.72**
B	**$4,544.51**
C	**$267,897.6**
D	**$138,241.92**
E	**$2,440,422.03**

b. Compare your findings in parts a(1) and a(2). All else being identical, which type of annuity—ordinary or annuity due—is preferable? Explain why.

Clarification

Future value ordinary annuity for project A

$$FV = C((1 / r) – \{1 / [r(1 + r)t]\}) * (1 + r)^t$$

$$FV = \$2500((1 / 0.08) – \{1 / [0.08* (1 + 0.08)^{10}]\}) * (1 + 0.08)^{10}$$

FV = $ 36,216.4

Future value annuity due for project A

FV(annuity due) = FV(ordinary annuity) × (1+r)

$$= 36216.4 \times (1+0.08) = \$\ 39113.72$$

13 You plan to retire in exactly 20 years. Your goal is to create a fund that will allow you to receive $20,000 at the end of each year for the 30 years between retirement and death (a psychic told you that you would die exactly 30 years after you retire). You know that you willbe able to earn 11% per year during the 30-year retirement period.

a. How large a fund will you need when you retire in 20 years to provide the 30-year, $20,000 retirement annuity? **Present value of an annuity, where the cash flow is $20,000 annually. PV annuity= $173,875.85**

b. How much will you need today as a single amount to provide the fund calculated in part a if you earn only 9% per year during the 20 years preceding retirement? **Simple present value calculation. You will need $31,024.82.**

c. What effect would an increase in the rate you can earn both during and prior to retirement have on the values found in parts a and b? Explain.

d. Now assume that you will earn 10% from now through the end of your retirement. You want to make 20 end-of-year deposits into your retirement account that will fundthe 30-year stream of $20,000 annual annuity payments. How large do your annual deposits have to be?

First you need to calculate how large a fund will you need when you retire with the new 10% interest. You will need $188,538.29.

Then you need to calculate how much you need to deposit annually during your active year to reach this goal. You will need to deposit $3291.81 every year.

a) $PV = C((1 / r) – \{1 / [r(1 + r)^t]\})$
$PV = \$20{,}000((1 / 0.11) – \{1 / [0.11(1 + 0.11)^{30}]\})$
PV = $173,875.85
b)
$PV = \$173{,}875.85 /(1 + 0.09)^{20}) = 31{,}024.82$
c) Higher rates would result in lower PVs, lower rates in higher PVs
d1) $PV = C((1 / r) – \{1 / [r(1 + r)^t]\})$
$PV = \$20{,}000((1 / 0.10) – \{1 / [0.10(1 + 0.10)^{30}]\})$
PV = $188,538.29
d2) $FV = C((1 / r) – \{1 / [r(1 + r)^t]\}) * (1 + r)^t$
$\$188{,}538.29 = C((1 / 0.1) – \{1 / [0.1(1 + 0.1)^{20}]\}) * (1 + 0.1)^{20}$
$\$188{,}538.29 = C((10) – \{1.4864]\}) * 6{,}7275$
$\$188{,}538.29 = C (57.275)$
C = 3,291.81

14 Aunt Tillie has deposited $33,000 today in an account which will earn 10 percent annually. She plans to leave the funds in this account for seven years earning interest. If thegoal of this deposit is to cover a future obligation of $65,000, what recommendation wouldyou make to Aunt Tillie?

Aunt Tillie will not have enough in her account to cover the obligation of $65,000. Discounting the $65,000 back, we can see that she needs to deposit $33,355.28 in order to have enough money in her account when the obligation is due.

a)$FV = \$33{,}000 *(1 + r)^t$
$FV = \$33{,}000 *(1 + 0.1)^7$
FV = 64,307.66

b) $PV = \$65{,}000 / (1 + r)^t$
$PV = \$65{,}000 / (1 + 0.1)^7$
PV = $ 33,355.2 needs to be deposited

15 Simpson, Inc. is considering a five-year project that has an initial after-tax cost of $80,000. The respective future cash inflows from its project for years 1, 2, 3, 4 and 5 are: $15,000, $25,000, $35,000, $45,000 and $55,000. Simpson uses the net present value method and has a discount rate of 9%.

a. Will Simpson accept the project?

The NPV is $49,455.25, which is positive so Simpson will accept the project.

b. How would your answer change if the initial after-cost was $130,000?

In this case, the NPV is -$544.75, so Simpson should not accept the project.

a) PV = -$80,000 + $15,000 / $(1 + 0.09)^1$ + $25,000 / $(1 + 0.09)^2$ + $35,000 / $(1 + 0.09)^3$ + $45,000 / $(1 + 0.09)^4$ + $55,000 / $(1 + 0.09)^5$
PV = $49,455.25, Simpson should accept the project
b) PV = $49,455.25 – $50,000 = - $544.75, so Simpson should reject the project

16 Using a 14% cost of capital, calculate the net present value for each of the independent projects shown in the following table, and indicate whether each is acceptable.

Which project (or projects) would you accept based on the payback period rule if the cut-off period is 5 years?

years	A	B	C	D	E
0	-26000	-500000	-170000	-950000	-80000
1	4000	100000	20000	230000	0
2	4000	120000	19000	230000	0
3	4000	140000	18000	230000	0
4	4000	160000	17000	230000	20000
5	4000	180000	16000	230000	30000
6	4000	200000	15000	230000	0
7	4000		14000	230000	50000
8	4000		13000	230000	60000
9	4000		12000		70000
10	4000		11000		
NPV	**-6365.57**	**53887.93**	**-87358.34**	**116938.70**	**-11561.93**
Payback period	**6.5**	**3.875**	**Does not pay back**	**4.13**	**6.6**