1

$\frac{1}{a}$

* 0.0006374845

chopping $= 0.6374845 \times 10^{-3}$

$= 0.6374 \times 10^{-3}$

Normalised form

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| + | 6 | 3 | 7 | 4 | - | 0 | 3 |

* 5749855743

Rounding $= 0.5749855743 \times 10^{10}$

$= 0.5750 \times 10^{10}$

Normalised form

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| + | 5 | 7 | 5 | 0 | + | 1 | 0 |

(ii) * $0.255640 \times 10^{1}$

we have to use chopping/rounding

$= 0.2556 \times 10^{1}$

Absolute Error $= 0.255640 \times 10^{1} - 0.2556 \times 10^{1}$

$= 0.00004 \times 10^{1} = 4 \times 10^{1-5} = 4 \times 10^{-4}$

* 2.55640

we have to use chopping/rounding

$= 2.55600$

Relative error $= (2.55640 - 2.55600)/2.55640$

$\simeq 1.56 \times 10^{-4}$

(iii) ~~$x_1 = 0.6705 \times 10^{-99}$ and $x_2 = 0.6685$~~

$x_1 = 0.6705 \times 10^{-99}$ and $x_2 = 0.6685 \times 10^{-99}$

To compute the value of $x_1 - x_2$

result $x = x_1 - x_2 = 0.0020 \times 10^{-99} = 0.2000 \times 10^{-101}$

But the exponent $-101$ of the result is less than the smallest exponent $-99$ that can be stored in our standard format.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| +/- | $D_1$ | $d_2$ | $d_3$ | $D_4$ | +/- | $e_1$ | $e_0$ |

The error, due to the fact the the result cannot be stored.

(iv.) $a = 0.1101 \times 10^{3}$ and $b = -0.3326 \times 10^{-4}$

$c = a \div b$

Solⁿ As $a \neq 0 \neq b$, therefore

$m \leftarrow (0.1101) \div (-0.3326) = -0.36106\ldots$

(un-normalized 8 digit register) and

$e \leftarrow 3 - (-4) = +7$

as $m < 1$, no adjustments of m and e are required

On normalization $m \leftarrow -0.3611$ (using rounding)

and $e = 7$,

we get $m = 0.3021$. Therefore,

$c = -0.3021 \times 10^{7}$.

(v) $a = 586309 = 0.586309 \times 10^6$
$b = 0.20000 \times 10^{99}$

$c = a \times b = m \times 10^e$, where $m$ is in normalized floating form, then

if $0.1 \le |m|$ then $m = m_1 \times m_2$,
$e \leftarrow e_1 + e_2 = 6 + 99 = 105$

else $m = m_1 \times m_2 \times 10$ and
$e \leftarrow e_1 + e_2 - 1 = 6 + 99 - 1 = 104$

In both cases $e$ cannot be stored in the 2-decimal digit space allotted to $e$
or, alternatively, we can show the overflow through the following argument

$e = 6 + 99 = 105 \ge 99$ cannot be stored in the 2 decimal

Underflow: we know that in our 4-digit mantissa and 2-digit exponent, underflow occurs when the exponent, which is an integer for floating number, is (strictly) less than 99.

(vi) if the sequence of 8 bits in the exponent, is 1010 1010 with binary value $128 + 32 + 8 + 2 = 170$, but contributes $170 - 127 = 43$, i.e., the value contributed by the mantissa is multiplied $2^{43}$. The number 127 is called the Bias.



(b) In this case, $f(x) = \sqrt{(x^2+1)} - 1$, can be re-written as
$f(x) = x^2 / \sqrt{(x^2+1)} + 1$, (using
$a - b = \frac{a^2 - b^2}{a+b}$, if $a \ne -b$), which

does not involve subtraction of nearly two equal numbers.
$f(0.25) = (0.25)^2 / \sqrt{(0.25)^2 + 1)} + 1$

$= 0.0625 / (1.03 + 1) \approx 0.0308.$
Thus, just by re-formulation of the function, hence, of the algorithm, we get much better approximation of the correct result. The source of error was the form / algorithm for the function. In such cases, the earlier algorithm is called unstable algorithm.

(c) As the $n^{th}$ derivative of $(1-2x)^{-1}$, i.e.,
$f(x) = (1-2x)^{-1}$
$f(0) = (1 - 2 \times 0)^{-1} = 1$
$f'(x) = \frac{d(1-2x)^{-1}}{dx} = -1\{(1-2x)^{-1-1} \times -2\}$
$= \{+2(1-2x)^{-2}\}$
$f'(0) = \{+2(1-0)^{-2}\} = 1 \times 2 = 2$
$f''(x) = \frac{d\, 2(1-2x)^{-2}}{dx} = 2\frac{d(1-2x)^{-2}}{dx}$
$= 2 \times \{-2(1-2x)^{-3} \times -2\}$

$= 2x + (1-2x)^{-3} = 8(1-2x)^{-3}$

$f''(0) = 8(1-2\times 0)^{-3} = 8$

$f'''(x) = 8\{-3(1-2x)^{-4} \times -2\}$

$= 8 \times 6(1-2x)^{-4}$

$= 48(1-2x)^{-4}$

$f'''(0) = 48(1-2\times 0)^{-4}$

$= 48 \times 1 = 48$

Now Maclaurin series is

$f(0) + \left(\frac{x}{1!}\right)f'(0) + \left(\frac{x^2}{2!}\right)f''(0) + \left(\frac{x^3}{3!}\right)f'''(0) + \dots$

$= 1 + \frac{x}{1!} \times 2 + \frac{x^2}{2!} \times 8 + \frac{x^3}{3!} \times 48 + \dots$

$= 1 + \frac{2x}{1!} + \frac{8x^2}{2!} + \frac{48x^3}{3!} + \dots$

put $x = 0.1$

$= 1 + \frac{2 \times 0.1}{1!} + \frac{8(0.1)^2}{2!} + \frac{48(0.1)^3}{3!} + \dots$

$= 1 + 0.2 + 0.04 + 0.008 = 1.248$

(4 significant digits).

The truncation error is

$\left(\frac{x^4}{4!}\right)f''''(0) + \left(\frac{x^5}{5!}\right)f'''''(0) \dots$



(d) Taylor series :- is a representation of a function as an infinite sum of terms that are calculated from the value of the function's derivatives at a single point.

Ex:- Find the Taylor series for $(1-x)^{-1}$ at $x = 0$

Sol$^n$ As the $n^{th}$ derivative of $(1-x)^{-1}$, i.e.,

$d^n((1-x)^{-1})/dx^n = n!(1-x)^{-(n+1)}$

Therefore, the value of $d^n((1-x)^{-1})/dx^n$ at $x = 0$ is $n!$

Using Taylor series, we get

$1 + \left(\frac{x}{1!}\right)1! + \left(\frac{x^2}{2!}\right)2! + \left(\frac{x^3}{3!}\right)3! + \dots$

giving the geometric series. Thus, the value of $(1-x)^{-1}$ around $x = 0$ is given by the infinite series

$1 + x + x^2 + x^3 + \dots$

put $x = 0.1$

$1 + (0.1) + (0.1)^2 + (0.1)^3 + \dots$

Truncation error

approximate the value of $(1-x)^{-1}$ at $x = 0.1$

$1 + (0.1) + (0.1)^2 + (0.1)^3 = 1.111.$

The truncation error in the value of $(1-x)^{-1}$, say at $x = 0.1$ is $0.000111\dots$

Q-2
(a)
$$x+y+6z=6$$
$$7x+3y-4z=4$$
$$2x-7y+3z=21$$

Matrix form

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 6 \\ 7 & 3 & -4 & 4 \\ 2 & -7 & 3 & 21 \end{array}\right]$$

Using partial pivoting
$R_1 \leftrightarrow R_2$ and $R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 7 & 3 & -4 & 4 \\ 2 & -7 & 3 & 21 \\ 1 & 1 & 6 & 6 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_3$
$R_3 \rightarrow R_1 - 7R_3$

$$\left[\begin{array}{ccc|c} 7 & 3 & -4 & 4 \\ 0 & -9 & -9 & 9 \\ 0 & -4 & -46 & -38 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 7 & 3 & -4 & 4 \\ 0 & -4 & -46 & -38 \\ 0 & -9 & -9 & 9 \end{array}\right]$$

$R_3 \rightarrow R_2 - \frac{4}{9} \times R_3$

$$\left[\begin{array}{ccc|c} 7 & 3 & -4 & 4 \\ 0 & -4 & -46 & -38 \\ 0 & 0 & -42 & -42 \end{array}\right]$$



Now the equation

$$7x+3y-4z=4 \quad \text{—— (I)}$$
$$-4y-46z=-38 \quad \text{—— (II)}$$
$$-42z=-42 \quad \text{—— (III)}$$

from eq$^n$ - (III)

$$+42z=+42$$
$$z=1$$

put $z=1$ in eq$^n$ - (II)

$$-4y-46z=-38$$
$$-4y-46\times 1=-38$$
$$-4y=-38+46$$
$$-4y=8$$
$$y=-2$$

put $z=1$ and $y=-2$ in eq$^n$ - (I)

$$7x+3y-4z=4$$
$$7x+3\times(-2)-4\times 1=4$$
$$7x-6-4=4$$
$$7x-10=4$$
$$7x=4+10$$
$$7x=14$$
$$x=2$$

Here
$$x=2$$
$$y=-2$$
$$z=1$$

(b).
$$4x+y-2z=15$$
$$x-6y+2z=-10$$
$$-2x+4y+8z=-24$$

(i) Jacobi Method

$$4x+y-2z=15$$
$$x=\frac{15-y+2z}{4}$$
$$x-6y+2z=-10$$
$$\cancel{-10+6y}$$
$$y=\frac{-10-x-2z}{-6}$$
$$=\frac{10+x+2z}{6}$$
$$-2x+4y+8z=-24$$
$$z=\frac{-24+2x-4y}{8}$$

Iteration-I Taking $x=y=z=0$

$$x=\frac{15-0+0}{4}=\frac{15}{4}=3.75$$
$$y=\frac{+10+0+0}{6}=\frac{10}{6}=1.666$$
$$z=\frac{-24+0-0}{8}=\frac{-24}{8}=-3$$
$$=(3.75,1.66,-3)^T$$



Iteration-(II)

$x=3.75,\ y=1.66,\ z=-3$

$$x=\frac{15-1.66+2\times(-3)}{4}=\frac{7.34}{4}=1.835$$
$$y=\frac{+10+3.75+2\times(-3)}{6}=\frac{7.75}{6}=1.29$$
$$z=\frac{-24+2\times3.75-4\times1.66}{8}=\frac{-23.14}{8}=-2.89$$
$$=(1.835,1.29,-2.89)^T$$

The exact solution is $(2,1,-3)^T$.
you may note that Iteration-(II) is a good approximation to the exact solution compared to $(2,1,-3)^T$.

(ii) Gauss-Seidel method

$$x=\frac{15-y+2z}{4}$$
$$y=\frac{+10+x+2z}{6}$$
$$z=\frac{-24+2x-4y}{8}$$

Iteration-1 Taking $x=y=z=0$

$$x=3.75$$
$$y=\frac{10+3.75+2\times0}{6}=\frac{13.75}{6}=2.29$$
$$z=\frac{-24+2\times3.75-4\times2.29}{8}=\frac{-25.66}{8}=-3.20$$
$$(3.75,2.29,-3.20)^T$$

Iteration - III

$x = 3.75,\ y = 2.29,\ z = -3.20$

$$x = \frac{15 - 2.29 + 2\times(-3.20)}{4} = \frac{6.31}{4} = 1.57$$

$$y = \frac{10 + 1.57 + 2\times(-3.20)}{6} = \frac{5.17}{6} = 0.86$$

$$z = \frac{-24 + 2\times 1.57 - 4\times(0.86)}{8} = \frac{-24.3}{8} = -3.03$$

$$(1.57,\ 0.86,\ -3.03)^T$$

Iteration - III is a good approximation to the exact solution compared to $(2, 1, -3)^T$

Q-3

$f(x) = 4x^3 - 6x^2 - 8x + 11 = 0$

(a) Regula-falsi method

$f(0) = 11 > 0$

$f(1) = 4\times1 - 6\times1 - 8\times1 + 11$
$= 4 - 6 - 8 + 11 = 1 > 0$

$f(2) = 4\times(2)^3 - 6\times(2)^2 - 8\times2 + 11$
$= 32 - 24 - 16 + 11 = 3 > 0$

✓ $f(-1) = 4(-1)^3 - 6\times(-1)^2 - 8(-1) + 11$
$= -4 - 6 + 8 + 11 = 9 > 0$

✓ $f(-2) = 4(-2)^3 - 6(-2)^2 - 8(-2) + 11$
$= -32 - 24 + 16 + 11$

$= -29 < 0$



The negative root lies between $-1$ and $-2$

$x_1 = -1$
$y_1 = f(x_1) = f(-1) = 9$

$x_2 = -2$
$y_2 = f(x_2) = f(-2) = -29$

Iteration - I

$$x = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{-1(-29) - (-2)\times 9}{-29 - 9}$$

$$= \frac{47}{-38} = -1.236 = -1.24$$

$y = f(x)$

$\Rightarrow f(-1.24) = 4(-1.24)^3 - 6(-1.24)^2 - 8(-1.24) + 11$
$= -7.63 - 9.23 + 9.92 + 11 = 4.06$

Iteration - II

$x_1 = -1.24,\ y_1 = 4.06$
$x_2 = -2,\ y_2 = -29$

$$x = \frac{-1.24\times(-29) - (-2)\times 4.06}{-29 - 4.06} = \frac{44.08}{-33.06}$$

$= -1.33$

$y = f(-1.33) = 4(-1.33)^3 - 6\times(-1.33)^2 - 8(-1.33) + 11$
$= -9.41 - 10.61 + 10.64 + 11 = 1.62$

Iteration-III

$x_1 = -1.33, \quad y_1 = 1.62$

$x_2 = -2, \quad y_2 = -29$

$$x = \frac{-1.33(-29) - (-2)\times 1.62}{-29 - 1.62} = \frac{41.81}{-30.62}$$

$$= -1.36$$

$$y = f(-1.36) = 4(-1.36)^3 - 6(-1.36)^2 - 8(-1.36) + 11$$

$$= -10.06 - 11.10 + 10.88 + 11 = 0.72$$

Iteration (IV)

$x_1 = -1.36, \quad y_1 = 0.72$

$x_2 = -2, \quad y_2 = -29$

$$x = \frac{-1.36\times(-29) + (2)\times 0.72}{-29 - 0.72} = \frac{40.88}{29.72}$$

$$= -1.36$$

Since value of $x$ repeats we take the root as

$$x = -1.36$$

(b) Newton-Raphson method

$$f(x) = 4x^3 - 6x^2 - 8x + 11 = 0$$

$$f'(x) = 12x^2 - 12x - 8$$



N-R scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{4x_n^3 - 6x_n^2 - 8x_n + 11}{12x_n^2 - 12x_n - 8}$$

$$= \frac{12x_n^3 - 12x_n^2 - 8x_n - 4x_n^3 + 6x_n^2 + 8x_n - 11}{12x_n^2 - 12x_n - 8}$$

$$= \frac{8x_n^3 - 6x_n^2 - 11}{12x_n^2 - 12x_n - 8}$$

Let, $x_0 = 0$

Iteration-I

$$x_{0+1} = \frac{8x_0^3 - 6x_0^2 - 11}{12x_0^2 - 12x_0 - 8} = \frac{8\times 0 - 6\times 0 - 11}{12\times 0 - 12\times 0 - 8}$$

$$x_1 = \frac{-11}{-8} = 1.37$$

Iteration-II

$$x_{1+1} = \frac{8x_1^3 - 6x_1^2 - 11}{12x_1^2 - 12x_1 - 8} = \frac{8(1.37)^3 - 6(1.37)^2 - 11}{12(1.37)^2 - 12(1.37) - 8}$$

$$x_2 = \frac{20.57 - 11.26 - 11}{22.52 - 16.44 - 8} = \frac{-1.69}{-1.92} = 0.88$$

Iteration-III

$$x_{2+1} = \frac{8(0.88)^3 - 6(0.88)^2 - 11}{12(0.88)^2 - 12(0.88) - 8} = \frac{-10.2}{-9.27}$$

$$x_3 = 1.10$$

**Iteration - IV**

$$x_{3+1} = \frac{8(1.10)^3 - 6(1.10)^2 - 11}{12(1.10)^2 - 12(1.10) - 8} = \frac{-7.61}{-6.68}$$

$$x_4 = 1.14$$

**Iteration V**

$$x_{4+1} = \frac{8(1.14)^3 - 6(1.14)^2 - 11}{12(1.14)^2 - 12(1.14) - 8} = \frac{-6.95}{-6.09}$$

$$x_5 = 1.14$$

Since value of $x_5$ repeats we take the root as

$$x = 1.14$$

## (c) Bisection method

$f(x) = 4x^3 - 6x^2 - 8x + 11$

$f(-1) = 9$

$f(-2) = -29$

the negative root lies between $-1$ and $-2$

let $x_1 = -1$ and $x_2 = -2$

**Iteration - I**

$$x = \frac{x_1 + x_2}{2} = \frac{-1-2}{2} = \frac{-3}{2} = -1.5$$

$$f(-1.5) = 4(-1.5)^3 - 6(-1.5)^2 - 8(-1.5) + 11$$
$$= -13.5 - 13.5 + 12 + 11 = -4$$



**Iteration - II**

$x_1 = -1$ and $x_2 = -1.5$

$$x = \frac{-1-1.5}{2} = \frac{-2.5}{2} = -1.25$$

$$f(-1.25) = 4(-1.25)^3 - 6(-1.25)^2 - 8(-1.25) + 11$$
$$= -7.81 - 9.37 + 10 + 11 = 3.82$$

| Iteration No. | $x = \frac{x_1+x_2}{2}$ | $f(x)$ | $x_1$ | $x_2$ |
|---|---|---|---|---|
| 1 | $-1.5$ | $-4$ | $-1$ | $-1.5$ |
| 2 | $-1.25$ | $3.82$ | $-1.5$ | $-1.25$ |

we can choose the root as

$$x = \frac{-1-1.25}{2} = \frac{-2.75}{2} = -1.37$$

## (d) Secant method

$f(x) = 4x^3 - 6x^2 - 8x + 11 = 0$

Let, $x_0 = 0$ and $x_1 = 1$

$y_0 = f(x_0) = f(0) = 11$

$y_1 = f(x_1) = f(1) = 4 - 6 - 8 + 11 = 1$

**Iteration - I**

$$x_{n+1} = \frac{x_{n-1} y_n - y_{n-1} x_n}{y_n - y_{n-1}} \quad \text{where } n = 1, 2, 3 \ldots$$

$$x_{1+1} = \frac{x_0 y_1 - y_0 x_1}{y_1 - y_0}$$

$$x_2 = \frac{0\times1 - 11\times1}{1-11} = \frac{-11}{-10} = 1.1$$

$y_2 = f(x_2)$

$= f(1.1) = 4(1.1)^3 - 6(1.1)^2 - 8(1.1) + 11$
$= 5.32 - 7.26 - 8.8 + 11 = 0.26$

Iteration-II

$x_1 = 1,\ x_2 = 1.1$
$y_1 = 1,\ y_2 = 0.26$

$$x_{2+1} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$$

$$x_3 = \frac{1\times0.26 - 1\times1.1}{0.26-1} = \frac{0.26-1.1}{0.26-1}$$
$$= \frac{-0.84}{-0.74} = 1.14$$

~~Iteration-(III)~~ $y_3 = f(x_3)$

$= f(1.14) = 4(1.14)^3 - 6(1.14)^2 - 8(1.14) + 11$
$= 5.92 - 7.8 - 9.12 + 11 = 0$

root $\underline{x = 1.14}$



Q4. (a)

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x$ | 1 | 3 | 6 | 10 |
| $f(x)$ | 1 | 7 | 31 | 91 |

Lagrange interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0)$$
$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2)$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$= \frac{(x-3)(x-6)(x-10)}{(1-3)(1-6)(1-10)}\times1 + \frac{(x-1)(x-6)(x-10)}{(3-1)(3-6)(3-10)}\times7$$
$$+ \frac{(x-1)(x-3)(x-10)}{(6-1)(6-3)(6-10)}\times31 + \frac{(x-1)(x-3)(x-6)}{(10-1)(10-3)(10-6)}\times91$$

$$= (x-3)(x-6)\left\{\frac{(x-10)}{-90} + \frac{(x-1)}{252}\times91\right\}$$
$$+ (x-1)(x-10)\left\{\frac{(x-6)}{42}\times7 + \frac{(x-3)}{-60}\times31\right\}$$

$$= x^2-6x-3x+18\left\{\frac{(10-x)}{90} + \frac{(91x-91)}{252}\right\}$$
$$+ x^2-10x-x+10\left\{\frac{(x-6)}{6} - \frac{(31x-93)}{60}\right\}$$

Now

$$f(3.5) = (3.5)^2 - 9(3.5) + 18 \left\{ \frac{10-3.5}{90} + \frac{91(3.5)-91}{252} \right\}$$

$$+ (3.5)^2 - 11(3.5) + 10 \left\{ \frac{(3.5-6)}{6} - \frac{31(3.5)-93}{60} \right\}$$

$$= 12.25 - 31.5 + 18 \left\{ \frac{6.5}{90} + \frac{227.5}{252} \right\} +$$

$$12.25 - 38.5 + 10 \left\{ \frac{-2.5}{6} - \frac{15.5}{60} \right\}$$

$$= -1.25 \{ 0.072 + 0.923 \} +$$

$$(-16.25) \{ -0.417 - 0.258 \}$$

$$= -1.25 \times 0.995 + 16.25 \times 0.675$$

$$= -1.244 + 10.969 = 9.725$$

---

(b)

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x$ | 4 | 16 | 36 | 81 |
| $y = f(x)$ | 1 | 3 | 5 | 8 |

$$x = f^{-1}(y) = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} \times x_1 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} x_2 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} x_3$$

put $y = 7$

$$= \frac{(7-3)(7-5)(7-8)}{(1-3)(1-5)(1-8)} \times 4 + \frac{(7-1)(7-5)(7-8)}{(3-1)(3-5)(1-8)} \times 16$$

$$+ \frac{(7-1)(7-3)(7-8)}{(5-1)(5-3)(5-8)} \times 36 + \frac{(7-1)(7-3)(7-5)}{(8-1)(8-3)(8-5)} \times 81$$

$$= \frac{4 \times 2 \times (-1) \times 4}{(-2)(-4)(-7)} + \frac{6 \times 2 \times (-1) \times 16}{2 \times (-2)(-7)}$$

$$+ \frac{6 \times 4 \times (-1) \times 36}{4 \times 2 \times (-3)} + \frac{6 \times 4 \times 2 \times 81}{7 \times 5 \times 3}$$

$$= \frac{-32}{-56} + \frac{-192}{28} + \frac{-864}{-24} + \frac{3888}{105}$$

$$= 0.571 - 6.857 + 36 + 37.028$$

$$= 66.742$$

## Q-5

(a) year (x) : 1998 2003 2008 2013 2018
population (y) : 19 40 79 142 235

| i. | $x_i$ | $y_i$ | 1st diff | 2nd diff. | 3rd diff | 4th diff |
|---|---|---|---|---|---|---|
| 0 | 1998 | 19 | | | | |
| | | | 21 | | | |
| 1 | 2003 | 40 | | 18 | | |
| | | | 39 | | 6 | |
| 2 | 2008 | 79 | | 24 | | |
| | | | 63 | | 6 | |
| 3 | 2013 | 142 | | 30 | | |
| | | | 93 | | | |
| 4 | 2018 | 235 | | | | |



Using Stirling's formula, taking

$x_0 = 2008$ nearest of 2007

$x_p = x = 2007$ , $h = 2003 - 1998 = 5$

so, that $p = \frac{x_p - x_0}{h} = \frac{-1}{5} = -0.2$

$$y_p = y_0 + p \cdot \frac{1}{2}\left(\delta y_{\frac{1}{2}} + \delta y_{-\frac{1}{2}}\right) + \frac{p^2}{2!}\delta^2 y_0 + \frac{(p+1)p(p-1)}{3!} \times \frac{1}{2}\left(\delta^3 y_{\frac{1}{2}} + \delta^3 y_{-\frac{1}{2}}\right) + \frac{p^2(p^2-1^2)}{4!}\delta^4 y_0 + \cdots$$

$$\boxed{\cancel{= 79 + (-0.2) \times \frac{1}{2}(30 + 24) + \frac{(-0.2)^2}{2} \times 24 + \frac{(-0.2+1)(-0.2)(-0.2-1)}{6} \times \frac{1}{2}(}}$$

$$= 79 + (-0.2) \times \frac{1}{2}(63 + 39) + \frac{(-0.2)^2}{2} \times 24 + \frac{(-0.2+1)(-0.2)(-0.2-1)}{6} \times \frac{1}{2}(6+6) + 0$$

$$= 79 - 10.2 + 0.48 + 0.192 = 69.472$$

(ii) ~~y = f(x)~~

Newton's FD (forward formula) :-

~~f(x) =~~

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

~~$x_0 = 1998$~~

(ii) $x_0 = 1998$

Since $x_p = 2000$ is near upper end of the table, we use FD formula. choose $x_0 = 1998$ so, that

$$p = \frac{x_p - x_0}{h} = \frac{2000-1998}{5} = \frac{2}{5} = 0.4$$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 19 + (0.4)\times 21 + \frac{0.4(0.4-1)}{2}\times 18 + \frac{0.4(0.4-1)(0.4-2)}{6} \times 6$$

$$= 19 + 8.4 + (-2.16) + 0.384$$

$$= 25.624$$

(iii) Since $x_p = 2015$ is near the lower end of the table we use BD formula choosing

$x_0 = 2018$

i.e. $$p = \frac{x_p - x_0}{h} = \frac{2015-2018}{5} = -\frac{3}{5} = -0.6$$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p+1)}{2!}\Delta^2 y_0 + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_0$$

$$= 235 + (-0.6)\times 93 + \frac{(-0.6)(-0.6+1)}{2}\times 30 + (-0.6)(-0.6+1)(-0.6+2) \times 6$$



$$= 235 - 55.8 - 0.12\times 30 + (-0.336)$$

$$= 175.264$$

(b) Newton's FD Formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \times \Delta^3 y_0 + \cdots$$

Now

$$y_1 - y_0 = \Delta y_0 = \nabla y_1 = \delta y_{\frac{1}{2}}$$

$$y_2 - y_1 = \Delta y_1 = \nabla y_2 = \delta y_{\frac{3}{2}}$$

$$\Delta y_1 - \Delta y_0 = \Delta(\Delta y_0) = \Delta^2 y_0$$

$$= \nabla y_2 - \nabla y_1 = \nabla(\nabla y_2) = \nabla^2 y_2$$

$$= \delta y_{\frac{3}{2}} - \delta y_{\frac{1}{2}} = \delta(\delta y_1) = \delta^2 y_1$$

$$y_p = y_0 + p\,\delta y_{\frac{1}{2}} + \frac{p(p-1)}{2!}\times \delta^2 y_1 + \frac{p(p-1)(p-2)}{3!} \times \delta^3 y_{\frac{3}{2}} + \cdots$$

(24)

Q-6

(a)

| i | $x_i$ | $y_i$ | 1st diff | 2nd diff | 3rd diff |
|---|---|---|---|---|---|
| 0 | 2 | 5.00 | | | |
| | | | 2.75 | | |
| 1 | 2.5 | 7.75 | | 0.5 | |
| | | | 3.25 | | 0 |
| 2 | 3 | 11.00 | | 0.5 | |
| | | | 3.75 | | |
| 3 | 3.5 | 14.75 | | | |

choosing $x_0 = 2$, $x = x_p = 2.5$

$$p = \frac{2.25 - 2.0}{0.5} = 0.5$$

First Derivatives

$$y'(x) = \frac{1}{h}\left\{\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2-6p+2}{6}\Delta^3 y_0\right\}$$

$$= \frac{1}{0.5}\left\{2.75 + \frac{2(0.5)-1}{2} \times 0.5 + 0\right\}$$

$$= \frac{1}{0.5}\left\{2.75 + 0\right\}$$

$$= 5.5$$

Second Derivatives

$$y''(x) = \frac{1}{h^2}\left\{\Delta^2 y_0 + (p-1)\Delta^3 y_0\right\}$$

$$= \frac{1}{(0.5)^2}\left\{0.5 + (0.5-1) \times 0\right\}$$

$$= \frac{1}{0.25} \times 0.5 = 2$$

$$y = x^2 + x - 1$$

$$y' = 2x + 1$$

put $x = 2.25$

$$= 2 \times 2.25 + 1 = 4.50 + 1$$

$$y'(x) = 5.50$$

$$y'' = 2$$

$$y''(x) = 2.$$

Truncation Error (TE) $= -\frac{h}{2} y''(x)$

$$= -\frac{0.5}{2} \times 2 = -0.5$$

Actual error $= 5.50 - 5.50 = 0$

(b) ~~$y'(x) =$~~ Using ~~the Lagrange's~~ interp formula

~~$y'(x) = 2x_0 - x_1 - x_2$~~

2.6

$$P(x) = \frac{(x-2.5)(x-3)(x-3.5)}{(2-2.5)(2-3)(2-3.5)} \times 5 + $$

$$\frac{(x-2)(x-3)(x-3.5)}{(2.5-2)(2.5-3)(2.5-3.5)} \times 7.75 + \frac{(x-2)(x-2.5)(x-3.5)}{(2.5-2)(3-2.5)(3-3.5)}$$

$$\times 11 + \frac{(x-2)(x-2.5)(x-3)}{(3.5-2)(3.5-2.5)(3.5-3)} \times 14.75$$

$$= (x-2.5)(x-3)\left\{\frac{5(x-3.5)}{-0.75} + \frac{14.75(x-2)}{0.75}\right\}$$

$$+ (x-2)(x-3.5)\left\{\frac{7.75(x-3)}{0.25} + \frac{11(x-2.5)}{-0.125}\right\}$$

$$= (x^2 - 3x - 2.5x + 7.5)\left\{\frac{-5(x-3.5) + 14.75(x-2)}{0.75}\right\}$$

$$+ (x^2 - 3.5x - 2x + 7)\left\{\frac{7.75(x-3) - 2\times 11(x-2.5)}{0.25}\right\}$$

$$= (x^2 - 5.5x + 7.5)\left\{\frac{-5x + 17.5 + 14.75x - 29.5}{0.75}\right\}$$

$$+ (x^2 - 5.5x + 7)\left\{\frac{7.75x - 23.25 - 22x + 5}{0.25}\right\}$$

$$= (x^2 - 5.5x + 7.5)\left\{\frac{9.75x - 12}{0.75}\right\} + (x^2 - 5.5x + 7)$$

$$\left\{\frac{-14.25x - 18.25}{0.25}\right\}$$

(27)

$= (x^2 - 5.5x + 7.5)\{13x - 16\} + (x^2 - 5.5x + 7)\{-57x - 7\}$

$= 13x^3 - 71.5x^2 - 97.5x - 16x^2 + 88x - 120$

$+ (-57x^3) + 313.5x^2 - 399x - 73x^2 + 401.5x$

$- 511$

$= -44x^3 + 153x^2 - 7x - 631$

$\underline{p'(x)}$

$= -3 \times 44x^2 + 2 \times 153x - 7$

$= -132x^2 + 306x - 7$

$p'(2.25) = -132(2.25)^2 + 306(2.25) - 7$

$= -668.25 + 688.5 - 7$

$= 13.25$

$p''(x) = -6 \times 44x + 306$

$p''(2.5) = -264x + 306$

$= -264 \times 2.5 + 306 = -354$

Q.7

(a) $\int_0^6 (2x^3+5x^2-11)\,dx$

Sol^n

$y(x) = f(x) = 2x^3+5x^2-11$

taking 12 equal subintervals using

$n = 12, \quad b = 6, \quad a = 0$

$nh = b - a$

$h = \frac{b-a}{n} = \frac{6-0}{12} = \frac{6}{12} = 0.5$

Now

$f(0) = -11$

$f(0.5) = 2\times(0.5)^3 + 5(0.5)^2 - 11 = -9.5$

$f(1) = 2+5-11 = -4$

$f(1.5) = 2(1.5)^3 + 5(1.5)^2 - 11 = 7$

$f(2) = 2(2)^3 + 5(2)^2 - 11 = 25$

$f(2.5) = 2(2.5)^3 + 5(2.5)^2 - 11 = 51.5$

$f(3) = 2(3)^3 + 5(3)^2 - 11 = 88$

$f(3.5) = 2(3.5)^3 + 5(3.5)^2 - 11 = 136$

$f(4) = 2(4)^3 + 5(4)^2 - 11 = 197$

$f(4.5) = 2(4.5)^3 + 5(4.5)^2 - 11 = 272.5$

$f(5) = 2(5)^3 + 5(5)^2 - 11 = 364$

$f(5.5) = 2(5.5)^3 + 5(5.5)^2 - 11 = 473$

$f(6) = 2(6)^3 + 5(6)^2 - 11 = 601$

Trapezoidal Rule

$\int_a^b f(x)\,dx = \frac{h}{2}\{y_0 + y_n + 2(y_1 + y_2 + \cdots y_n)\}$



$\cancel{= \frac{0.5}{2}\{-11+601+2(-9.5)\}}$

$= \frac{0.5}{2}\left[-11+601+2\{-9.5-4+7+25+51.5+88+136+197+272.5+364+473\}\right]$

$= \frac{0.5}{2}\left[590 + 2\{1600.5\}\right]$

$= \frac{0.5}{2}\left[590 + 3201.0\right]$

$= \frac{0.5}{2} \times 3791 = 947.75$

(b.) Simpson's 1/3 Rule

$\int_a^b f(x) = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})\right]$

$= \frac{0.5}{3}\left[-11+601+4(-9.5+7+51.5+136+272.5+473) + 2(-4+25+88+197+364+[illegible])\right]$

$= \frac{0.5}{3}\left[590 + 4(930.5) + 2(670)\right]$

$= \frac{0.5}{3}\left[590 + 3722 + 1340\right]$

$= \frac{0.5}{3} \times 5652 = 942$

30

Q.8

(a) $y' = 1 + x^2 y$

Euler's Method

$y_{n+1} = y_n + h f_n$

$= y_n + h(1 + x^2 y)_n$

$= y_n + h + h x^2 y_n$

$= y_n(1 + h x_n^2) + h$

$y(0) = 1, \ h = 0.25$

$y_{0+1} = y_0(1 + h x_0^2) + h$

$\Rightarrow y_1 = 1(1 + h \times 0) + 0.25 = 1.25$

$y_{1+1} = y_1(1 + h x_1^2) + h$

$\Rightarrow y_2 = 1.25(1 + 0.25(0.25)^2) + 0.25$

$= 1.25(1 + 0.015) + 0.25$

$= 1.51$

$y_{2+1} = y_2(1 + h x_2^2) + h$

$= 1.51(1 + 0.25(0.5)^2) + 0.25$

$= 1.51(1 + 0.063) + 0.25$

$\Rightarrow y_3 = 1.85$

$y_{3+1} = y_3(1 + h x_3^2) + h$

$= 1.85(1 + 0.25(.75)^2) + 0.25$

$= 2.36$

$y(0) = 1, \ h = 0.1$

(ii) $y_{n+1} = y_n(1 + h x_n^2) + h$

$y_{0+1} = y_0(1 + h x_0^2) + h$

$\Rightarrow y_1 = 1(1 + 0.1 \times 0) + 0.1 = 1.1$

$y_{1+1} = y_1(1 + h x_1^2) + h$

$= 1.1(1 + 0.1(0.1)^2) + 0.1$

$\Rightarrow y_2 = 1.20$

$y_{2+1} = y_2(1 + h x_2^2) + h$

$= 1.20(1 + h(0.2)^2) + 0.1$

$\Rightarrow y_3 = 1.30$

$y_{3+1} = y_3(1 + h x_3^2) + h$

$= 1.30(1 + 0.1(0.3)^2) + 0.1$

$\Rightarrow y_4 = 1.41$

$y_{4+1} = y_4(1 + h x_4^2) + h$

$= 1.41(1 + 0.1(0.4)^2) + 0.1$

$\Rightarrow y_5 = 1.53$

$y_{5+1} = 1.53(1 + 0.1(0.5)^2) + 0.1 = 1.67$

$= 1.67(1 + 0.1(0.6)^2) + 0.1 = 1.830$

$y_{6+1} = 1.67(1 + 0.1(0.6)^2) + 0.1 = 2.01$

$y_{7+1} = 1.83(1 + 0.1(0.7)^2) + 0.1 = 2.24$

$y_{8+1} = 2.01(1 + 0.1(0.8)^2) + 0.1 = 2.52$

$y_{9+1} = 2.24(1 + 0.1(0.9)^2)$

Q-8
(b) $\cancel{y' = 1 + x^2 y}$
$y' = xy + x^2$

i) Runge-kutta (R-K) Method of $O(h^2)$

$y(0) = 1$, $h = 0.2$

$$\boxed{\begin{aligned} y^*_{n+1} &= y_0 + h f(x_n, y_n) \\ y_{n+1} &= y_n + \frac{h}{2}\{f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\} \end{aligned}}$$

Sol^n

$h = 0.2$

$f(x, y) = xy + x^2$, $x_0 = 0$, $y_0 = 1$

$x_1 = x_0 + h = 0.2$

$k_1 = h f(x_n, y_n) = 0.2 \times \{0 \times 1 + 0\} = 0$

$k_2 = 0.2\{0.2(1+0) + (0.2)^2\}$
$= 0.2\{0.2 + 0.04\} = 0.2 \times 0.24$
$= 0.048$

$y_{0+1} = y_0 + \frac{1}{2}(k_1 + k_2)$

$= 1 + \frac{1}{2}(0 + 0.048) = 1 + 0.024$

$y_1 = 1.024$

$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$, $x_3 = 0.6$

$k_1 = h f(x_2, y_1)$



$= 0.2\{0.4 \times 1.024 + (0.4)^2\}$

$= 0.2\{0.569\} = 0.114$

$\cancel{k_2 = 0.2\{1.024(0.4+0.2) +}$

$k_2 = 0.2\{(1.024 + 0.114)(0.4+0.2)^2 + (0.4+0.2)^2\}$

$= 0.2\{1.138 \times 0.36 + 0.36\}$

$= 0.2 \times 0.769 = 0.154$

$y_{1+1} = y_1 + \frac{1}{2}\{k_1 + k_2\}$

$= 1.024 + \frac{1}{2}\{0.114 + 0.154\}$

$= 1.024 + \frac{1}{2} \times 0.268$

$\Rightarrow y_2 = 1.024 + 0.134 = 1.158$

$x_4 = 0.8$, $y_2 = 1.158$

$k_1 = 0.2\{0.8 \times 1.158 + (0.8)^2\}$
$= 0.2\{0.926 + 0.64\} = 0.313$

$y_{2+1} = y_2 + k_1 = 1.158 + 0.313$

$= 1.471$

ii) R-K method of $O(h^4)$

$h = 0.2$, $f(x, y) = xy + x^2$, $x_0 = 0$, $y_0 = 1$

$x_1 = 0.2$

$k_1 = 0$

$\cancel{k_2 = 0.048}$

$$k_2 = hf\left(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1\right)$$

$$= 0.2\left\{\left(0.2 + \frac{0.2}{2}\right)(1 + 0) + \left(0.2 + \frac{0.2}{2}\right)^2\right\}$$

$$= 0.2\{0.3 + 0.09\}$$

$$= 0.078$$

$$k_3 = hf\left(x_2 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_2\right)$$

$$= 0.2\left\{(0.4 + 0.1)\left(1 + \frac{0.078}{2}\right) + (0.4 + 0.1)^2\right\}$$

$$= 0.2\{0.5 \times 1.039 + 0.25\}$$

$$= 0.154$$

~~$k_4 = hf\left(x_3 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_3\right)$~~

$$k_4 = hf(x_3 + h, y_0 + k_3)$$

$$= 0.2\{(0.6 + 0.2)(1 + 0.154) + (0.8)^2\}$$

$$= 0.2\{0.923 + 0.64\} = 0.313$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= y_0 + \frac{1}{2}\left(0 + 2(0.078) + 2(0.154) + 0.313\right)$$

$$= 1 + \frac{1}{2}(0.156 + 0.308 + 0.313)$$

$$= 1 + 0.388 = 1.388$$