Solve $\begin{cases} 2x + y = 3 \\ x - 2y = -1 \end{cases}$ and find out

its "row picture" and "column picture"

$y = 3 - 2x$

$x - 2(3 - 2x) = -1$

$x - 6 + 4x = -1$

$5x = 5$

$x = 1$

$y = 3 - 2(1)$

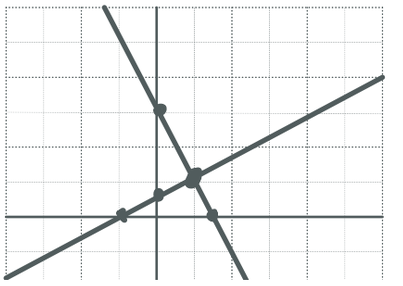
$y = 1$

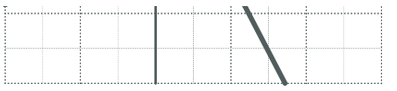
$$\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

"row picture" (dot product of rows)

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 3 \qquad \begin{bmatrix} 1 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -1$$

$2x + y = 3$ $\qquad$ $x - 2y = -1$

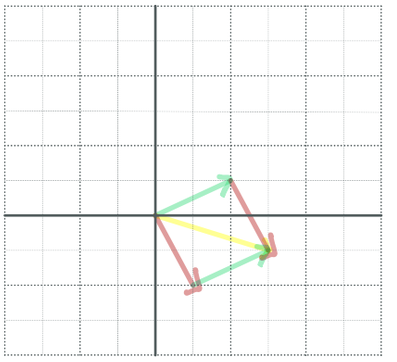




"column picture" (linear combination of columns)

$$\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$



Matrix Form

$$\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$A \quad x = b$$

$$x = A^{-1} b$$

$$A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity Matrix

I assume they call it the identity matrix because applying it to any vector yields the same vector.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$