The Fundamental Problem of Linear Algebra

Solve a system of linear equations

n equations, n unknowns

- Row Picture
- ★ Column Picture
- Matrix Form

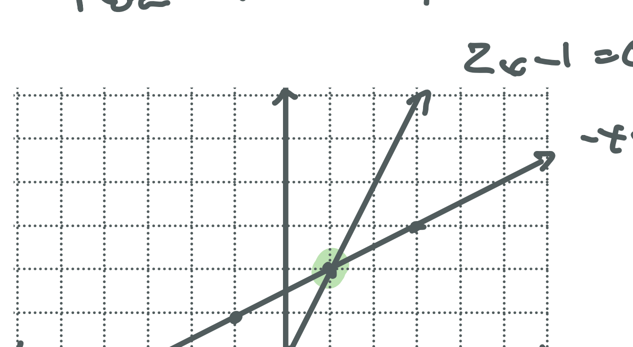
Examples:

2 equations 2 unknowns

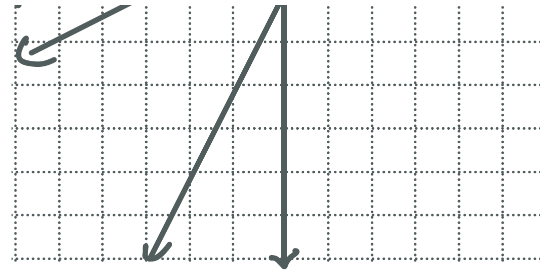
Row Form:
$$\begin{aligned} 2x - y &= 0 \\ -x + 2y &= 3 \end{aligned}$$

Matrix Form:
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$
$$A \mathbf{x} = b$$

Row Picture:



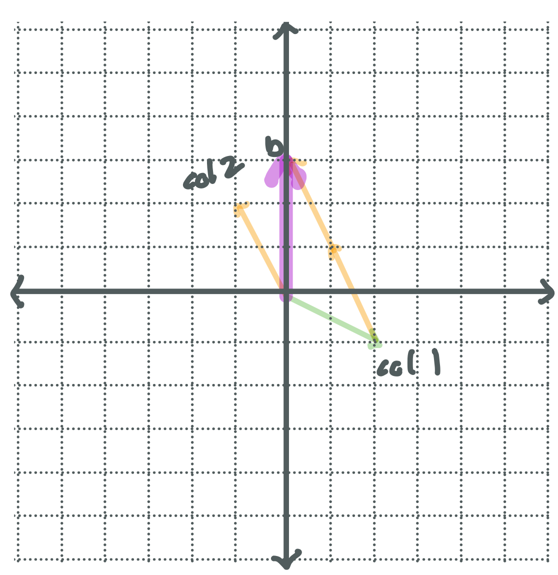
$x = 1$
$y = 2$



Column Picture

$$x\begin{bmatrix}2\\-1\end{bmatrix} + y\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

linear combination



All combinations give you the xy plane.

3 equations, 3 unknowns

$$\begin{aligned} 2x - y \quad\quad &= 0 \\ -x + 2y - z &= -1 \\ -3y + 4z &= 4 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Three Planes intersect at one point.

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix} + y\begin{bmatrix}-1\\2\\-3\end{bmatrix} + z\begin{bmatrix}0\\-1\\4\end{bmatrix} = \begin{bmatrix}0\\-1\\4\end{bmatrix}$$

$x=0,\ y=0,\ z=1$

This matrix A can solve for any b.

Matrix

$$Ax = b$$

Vector　　　vector

$$\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}10\\6\end{bmatrix} = \begin{bmatrix}12\\7\end{bmatrix}$$

A singular matrix can not solve for every b so it doesn't fill the entire space.

In this case columns of the matrix are linearly dependent.