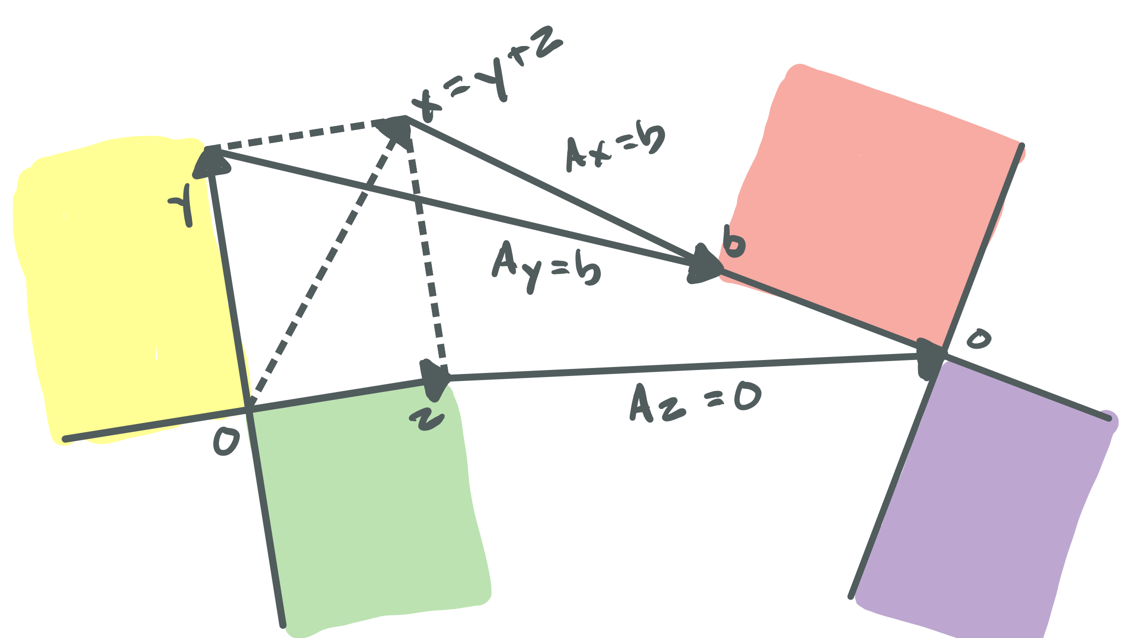
Motivations:

1. Pre-req for AI/ML
2. Fundamental math for CS
3. Regular math exercises

Four Central Problems of Linear Algebra

$Ax = b$	n by n	Linear Systems
$Ax = b$	m by n	Least Squares
$Ax = \lambda x$	n by n	Eigenvalues
$Av = \sigma u$	m by n	Singular values

Four Fundamental Subspaces for Matrix A

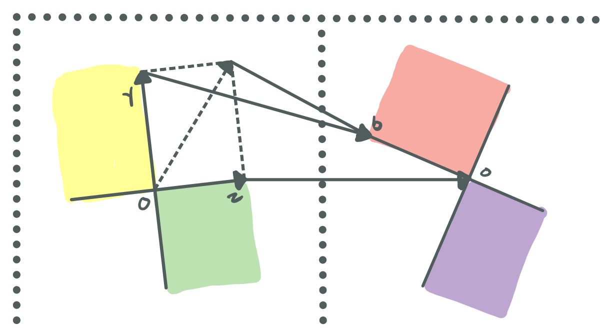


1. The dimensions of the four subspaces.
2. The orthogonality of the two pairs.
3. The best bases for all four subspaces.

Preface

- Example programs are in:
  - Python
  - Julia
  - MatLab
  - Java
  - Maple
  - Mathematica

Front Cover



Row Space + nullspace

Column Space + nullspace of A^T

Vector A example:

$$\begin{array}{cc} & \text{n-dimension} \\ & n=3 \\ \begin{array}{c}\text{m-dimension} \\ m=4\end{array} & \begin{bmatrix} 0 & 1 & 2 \\ 5 & 2 & 1 \\ 7 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$

The crucial operation in linear algebra is to take linear combinations of column vectors. This is the result of matrix-vector multiplication.

The inverse matrix has a connection to calculus.

Ax is a combination of the columns of A.

$Ax = b$ is asking for a combination (vector x) that produces b.

Topics/Structure:

- Vectors + Dot Products
- Row + Column picture of $Ax = b$
  - Algebra of Matrices
  - Elimination
- Subspaces
  - The Fundamental Theorem of Linear Algebra
- Least Squares
- Determinants
  - Cramer's Rule
  - Inverse Matrices
- Eigenvalues

- diagonalizing a symmetric matrix
- Singular values and singular vectors
- Linear transformations
- Complex vectors and matrices
  - The Fourier Matrix
- Applications
- Computing
- Probability and statistics

"Too Much Calculus" essay.

Vectors and matrices have become the language to know. 🔥

Symmetric Matrix

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Orthogonal Matrix

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Triangular Matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Matrix Alphabet:

A - Any
B - Basis
C - Cofactor
D - Diagonal
E - Elimination
F - Fourier
H - Hadamard
I - Identity
J - Jordan
K - Stiffness
L - Lower Triangular
M - Markov
N - Nullspace
P - Permutation
P - Projection
Q - Orthogonal
R - Upper Triangular
R - Reduced Echelon
S - Symmetric
T - Linear Transformation
U - Upper Triangular
U - Left Singular Vectors
V - Right Singular Vectors
X - Eigenvector
Λ - Eigenvalue
Σ - Singular Value