- **Angular Speed and Related Concepts**: The instructor discusses angular speed, emphasizing its importance in physics and its relationship to rotation. Angular speed is defined as the amount of rotation (in radians) per unit time, similar to linear speed but applied to rotational motion . For example, spinning 4π radians in one minute corresponds to an angular speed of 4π radians per minute . The relationship between linear speed and angular speed involves the radius: linear speed = distance / time, and angular speed = rotation / time . The instructor highlights the importance of unit conversions, such as radians to degrees and miles to feet, to accurately compute these speeds .

- **Standard Position and Terminal Side**: An angle in standard position has its vertex at the origin, initial side along the positive x-axis, and the terminal side passing through a point on the coordinate plane . Any point on the terminal side (except the origin) can be chosen to determine the angle's trig functions. The instructor emphasizes selecting the simplest point, often given, such as (4,5), to facilitate calculations . The distance from the origin to this point, called R, is calculated using the Pythagorean theorem: $R = \sqrt{x^2 + y^2}$.

- **Trig Functions and Their Definitions**: The six trig functions are defined based on the point (x, y) on the terminal side and the radius R:
    - $\sin\theta = \frac{y}{R}$
    - $\cos\theta = \frac{x}{R}$
    - $\tan\theta = \frac{y}{x}$
    - The other three are cosecant, secant, and cotangent, reciprocals of sine, cosine, and tangent respectively .

- **Finding Coordinates and Rationalizing**: To find the trig functions, first determine the point's coordinates and R. For example, with point (4,5), $R = \sqrt{4^2 + 5^2} = \sqrt{41}$. The ratios are then computed, and irrational denominators are rationalized as needed .

- **Multiple Points on the Terminal Side**: Any point along the same terminal side (excluding the origin) yields the same trig ratios because they share the same terminal side. For instance, scaling (4,5) to (8,10) doubles the distance but maintains the same ratios for sine, cosine, and tangent . The instructor advises choosing points that simplify calculations, like (4,5), and notes that the ratios remain consistent regardless of the point chosen on the terminal side .

- **Important Definitions and Memorization**: Students are expected to memorize six key definitions related to angles in standard position and their trig functions, including the relationships involving the point coordinates and R . Memorizing these is crucial for success, and practice rewriting these formulas helps retention.

- **Avoiding Common Mistakes**: The instructor strongly discourages using Sokotoa in this context, emphasizing that it only applies to right triangles and is unreliable for general angles. Instead, the formal definitions based on the point and radius are preferred for accuracy and consistency .

- **Practice and Application**: Students should practice drawing angles in standard position, labeling the angle and terminal side, and calculating the six trig functions for given points. They should also be able to find other points on the same terminal side by scaling, ensuring ratios stay the same . The instructor emphasizes rewriting formulas repeatedly to memorize them and understanding when to use the Pythagorean theorem versus other methods for calculating R .

- **Key Concepts Recap**:

  - Angles in standard position have their vertex at the origin with the initial side along the positive x-axis.
  - The terminal side passes through a point (x, y), and R is the distance from the origin to that point.
  - Trig functions are ratios involving x, y, and R.
  - Any point on the terminal side (except the origin) can be used to compute these ratios.
  - Rationalize denominators when necessary, and practice finding multiple points on the same terminal side.

- **Additional Notes**: The instructor stresses the importance of understanding the definitions over memorization alone, and warns against relying on shortcuts like Sokotoa outside their limited applicable cases. They also highlight the importance of labeling angles correctly and including all necessary components when drawing in standard position .

Course Material Summary

Angles and Circles

- When dealing with angles in standard position, spinning the angle creates a circle with radius R. The point on the terminal side of the angle lies on this circle (e.g.,).
- A *unit circle* has radius 1, making it easier to find coordinates of points corresponding to specific angles, such as 45°, 30°, 60°, etc. For example, at 45°, the coordinates are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ ().
- Memorizing key points on the circle for special angles (30°, 45°, 60°) simplifies locating points on the terminal side without complex calculations ().

Choosing Circle Radius

- While the unit circle (radius = 1) is common, using a circle with radius 2 can make calculations more straightforward, especially when dealing with points like

$(\pm\sqrt{3}, \pm 1)$ scaled appropriately ().
- The key is understanding that points on any circle can be scaled to match the desired radius, and the coordinates are proportional to the radius ().

Finding Coordinates on the Terminal Side

- To find a point on the terminal side of an angle, you can:
  - Use the *unit circle* and memorize key points.
  - Use the *Pythagorean theorem* with a right triangle to determine coordinates.
  - Draw the angle and identify the quadrant to determine the signs of x and y ().
- For angles not on the axes, calculate the reference angle and then find the corresponding point, adjusting for the quadrant ().

Trigonometric Ratios and Their Calculations

- The six basic trig functions are:
  - $\sin\theta = \frac{y}{r}$
  - $\cos\theta = \frac{x}{r}$
  - $\tan\theta = \frac{y}{x}$
  - $\csc\theta = \frac{r}{y}$
  - $\sec\theta = \frac{r}{x}$
  - $\cot\theta = \frac{x}{y}$
- To evaluate these, find the point (x, y) on the terminal side and the radius r. Then substitute into the formulas and simplify ().

Handling Large or Negative Angles

- For angles greater than 2π or negative angles, determine how many rotations or fractions of rotations are involved. For example, 12.5 rotations correspond to $12.5 \times 2\pi$ radians, ending on the negative x-axis ().
- Use reference angles and quadrant analysis to find the point and trig ratios for these angles ().

Special Triangles and Memorization

- Memorize the key ratios for 30°, 45°, and 60° angles, such as:
  - 30°: $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
  - 45°: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
  - 60°: $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- These points help quickly determine coordinates and trig ratios without extensive calculations ().

Practical Tips

- Always draw the angle in standard position to visualize the quadrant and the point.
- Use the Pythagorean theorem to find missing coordinates when necessary.
- Remember that the radius can be scaled; the key is proportionality.
- Practice converting between degrees and radians, and understand how multiple rotations affect the terminal side position.
- Recognize that some angles produce the same trig ratios even if their measures differ, due to symmetry ().

Summary of Key Concepts

- Circles with different radii can be used to find points on the terminal side.
- Coordinates are derived from the radius and reference angles.
- Trig functions are ratios of coordinates and radius.
- Large or negative angles require quadrant analysis and reference angles.
- Memorization of key points and ratios accelerates problem-solving.

References