Quantum Mechanics as the Least Expected Utility strategies of Differential Game Theory

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Here, I will derive Quantum Mechanics as being the probabilistic strategies of Differential Game Theory that minimize the expected change in utility function between any two times. I.e. Quantum Mechanics follows a "Principle of Least Expected Utility".

Note: We can derive Classical Physics as being the deterministic strategies of Differential Game Theory that minimizes the (actual) change in utility function between any two times. I.e. Classical Mechanics follow a "Principle of Least Utility".

In Differential Game Theory, the current Utility function is an integral over the strategy used by an agent between the start moment and the current moment. We will use the symbol S to denote the Utility function.

$$Utility(t) = S(t) = \int^{x(t),t} Ldt$$

Though in Differential Game Theory, each agent attempts to achieve the greatest possible Expected Utility Function, for physics, we consider the strategies that completely fail at this and instead achieve the worst possible change in utility function between any 2 points in time.

For Classical Physics, we consider worst-case deterministic strategies, which minimize the change in Utility Function between any two points in time.

Such a worst-case strategy will stabilize the change in utility function.

$$\forall t_2, t_1, \delta[S(t_2) - S(t_1)] = 0$$

Given our definition of

$$S(t_2) - S(t_1) = \int_{x(t_1),t_1}^{x(t_2),t_2} Ldt$$

the Euler-Lagrange equations of Classical Mechanics follows.

However, in Differential Game Theory, strategies can be probabilistic, not only deterministic. Our worst case strategies, therefore, minimize the change in expected utility function, which stabilitizes the change in expected utility function.

$$\forall t_2, t_1, \delta E[S(t_2) - S(t_1)] = 0$$
$$\forall t_2, t_1, \delta E[\int_{x(t_1),t_1}^{x(t_2),t_2} Ldt] = 0$$

This part needs work. Postulated form of Expectation Value without Explanation:

$$
\begin{aligned}
E[\int_{x(t_1),t_1}^{x(t_2),t_2} Ldt] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)\int_{x_1 t_1}^{x_2,t_2}(Ldt)\psi(x_1;t_1)dx_1dx_2 \\
&= \langle\phi(t_2)|\int_{x_1 t_1}^{x_2,t_2}(Ldt)|\phi(t_1)\rangle
\end{aligned}
$$

Now going through the algebra

$$
\begin{aligned}
0 &= \delta E[S(t_2)-S(t_1)] \\
&= \delta\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)\int_{x_1 t_1}^{x_2,t_2}(Ldt)\psi(x_1;t_1)dx_1dx_2 \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta[\phi^*(x_2;t_2)\int_{x_1 t_1}^{x_2,t_2}(Ldt)\psi(x_1;t_1)]dx_1dx_2
\end{aligned}
$$

$$
\begin{aligned}
0 = &\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)[(\frac{\partial}{\partial x}^{\dagger}\delta x(t_2)+\frac{\partial}{\partial t}^{\dagger}\delta t_2)S(t_2) \\
&+\int_{x_1 t_1}^{x_2,t_2}\delta(Ldt) \\
&+S(t_1)(\delta x(t_1)\frac{\partial}{\partial x}+\delta t_1\frac{\partial}{\partial t})]\psi(x_1;t_1)dx_1dx_2
\end{aligned}
$$

Above line needs some explanation. Why does S(t_2) and S(t_1) occur? Reason has to do with x and t being limits of an integral.

$$
\begin{aligned}
0 = &\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)[(\frac{\partial}{\partial x}^{\dagger}\delta x(t_2)+\frac{\partial}{\partial t}^{\dagger}\delta t_2)S(t_2) \\
&+\int_{x_1 t_1}^{x_2,t_2}[\frac{\partial Ldt}{\partial x}\delta x+\frac{\partial Ldt}{\partial dx}\delta dx+\frac{\partial Ldt}{\partial t}\delta t+\frac{\partial Ldt}{\partial dt}\delta dt] \\
&+S(t_1)(\delta x(t_1)\frac{\partial}{\partial x}+\delta t_1\frac{\partial}{\partial t})]\psi(x_1;t_1)dx_1dx_2
\end{aligned}
$$

$$
\begin{aligned}
0 = &\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)[(\frac{\partial}{\partial x}^{\dagger}\delta x(t_2)+\frac{\partial}{\partial t}^{\dagger}\delta t_2)S(t_2) \\
&+\int_{x_1 t_1}^{x_2,t_2}[(\frac{\partial Ldt}{\partial x}-\frac{d\partial Ldt}{\partial dx})\delta x+(\frac{\partial Ldt}{\partial t}-\frac{d\partial Ldt}{\partial dt})\delta t]+[\frac{\partial Ldt}{\partial dx}\delta x+\frac{\partial Ldt}{\partial dt}\delta t]_{t_1}^{t_2} \\
&+S(t_1)(\delta x(t_1)\frac{\partial}{\partial x}+\delta t_1\frac{\partial}{\partial t})]\psi(x_1;t_1)dx_1dx_2
\end{aligned}
$$

Defining

$$
\begin{aligned}
Fdt &= \frac{\partial L dt}{\partial x} \\
Qdt &= -\frac{\partial L dt}{\partial t} \\
p &= \frac{\partial L dt}{\partial dx} \\
E &= -\frac{\partial L dt}{\partial dt}
\end{aligned}
$$

And substituting

$$
\begin{aligned}
0 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} & \phi^*(x_2; t_2)[(\frac{\partial}{\partial x}^\dagger \delta x(t_2) + \frac{\partial}{\partial t}^\dagger \delta t_2) S(t_2) \\
&+ \int_{x_1 t_1}^{x_2, t_2} [(Fdt - dp)\delta x - (Qdt - dE)\delta t] + [p\delta x - Et]_{t_1}^{t_2} \\
&+ S(t_1)(\delta x(t_1) \frac{\partial}{\partial x} + \delta t_1 \frac{\partial}{\partial t})] \psi(x_1; t_1) dx_1 dx_2
\end{aligned}
$$

Gives us

$$
\begin{aligned}
F &= \frac{dp}{dt} \\
Q &= -\frac{dE}{dt}
\end{aligned}
$$

$$
\begin{aligned}
p &= \frac{\partial}{\partial x}^\dagger S = -S \frac{\partial}{\partial x} \\
E &= -\frac{\partial}{\partial t}^\dagger S = S \frac{\partial}{\partial t}
\end{aligned}
$$

In the case where S is approximately a constant throughout the time period considered (in particular, where $S = i\hbar$), we get Quantum Mechanics. Note, however, that we have not specified between the Heisenberg or Schrodinger Pictures yet.

Extensions to Consider:

1. In differential game theory, each agent has its own utility function, so we should consider the case where every particle in quantum mechanics has its own action and its own wavefunctions.

2. Nothing in here says that the variables have to be real valued. By allowing complex valued variables, we can have non-unitary evolutions in quantum mechanics, which allows changes in the entropy.

3. A while back, I found something I call "jagged calculus", which separates out left and right derivatives. By going through the above derivation in terms of jagged calculus, we separate force into push and pull.