Quantum Mechanics as the Least Expected Utility strategies of Differential Game Theory

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Here, I will derive Quantum Mechanics as being the probabilistic strategies of Differential Game Theory that minimize the expected change in utility function between any two times. I.e. Quantum Mechanics follows a “Principle of Least Expected Utility”.

Note: We can derive Classical Physics as being the deterministic strategies of Differential Game Theory that minimizes the (actual) change in utility function between any two times. I.e. Classical Mechanics follow a “Principle of Least Utility”.

In Differential Game Theory, the current Utility function is an integral over the strategy used by an agent between the start moment and the current moment. We will use the symbol S to denote the Utility function.

$$Utility(t) = S(t) = \int^{x(t),t} L dt$$

Though in Differential Game Theory, each agent attempts to achieve the greatest possible Expected Utility Function, for physics, we consider the strategies that completely fail at this and instead achieve the worst possible change in utility function between any 2 points in time.

For Classical Physics, we consider worst-case deterministic strategies, which minimize the change in Utility Function between any two points in time.

Such a worst-case strategy will stabilize the change in utility function.

$$\forall t_2, t_1, \delta[S(t_2) - S(t_1)] = 0$$

Given our definition of

$$S(t_2) - S(t_1) = \int_{x(t_1),t_1}^{x(t_2),t_2} L dt$$

the Euler-Lagrange equations of Classical Mechanics follows.

However, in Differential Game Theory, strategies can be probabilistic, not only deterministic. Our worst case strategies, therefore, minimize the change in expected utility function, which stabilitizes the change in expected utility function.

$$\forall t_2, t_1, \delta E[S(t_2) - S(t_1)] = 0$$
$$\forall t_2, t_1, \delta E[\int_{x(t_1),t_1}^{x(t_2),t_2} L dt] = 0$$

This part needs work. Postulated form of Expectation Value without Explanation:

Postulate a form of Born's Rule

$$
\begin{aligned}
E[\int_{x(t_1),t_1}^{x(t_2),t_2} Ldt] &= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)\int_{x_1t_1}^{x_2,t_2}(Ldt)\psi(x_1;t_1)dx_1dx_2}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \phi^*(x_2;t_2)\psi(x_1;t_1)dx_1dx_2} \\
&= \frac{\langle\phi(t_2)|\int_{x_1t_1}^{x_2,t_2}(Ldt)|\psi(t_1)\rangle}{\langle\phi(t_2)|\psi(t_1)\rangle} \\
&= \frac{\langle\phi(t_2)|S(t_2)-S(t_1)|\psi(t_1)\rangle}{\langle\phi(t_2)|\psi(t_1)\rangle}
\end{aligned}
$$

$\delta E[S(t_2)-S(t_1)] = 0$ is equivalent to $0 = \delta\langle\phi(t_2)|S(t_2)-S(t_1)|\psi(t_1)\rangle$
Now going through the algebra

$$
\begin{aligned}
0 &= \delta\langle\phi(t_2)|S(t_2)-S(t_1)|\psi(t_1)\rangle \\
&= \delta\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\phi^*(x_2;t_2)\int_{x_1t_1}^{x_2,t_2}(Ldt)\psi(x_1;t_1)dx_1dx_2 \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\delta[\phi^*(x_2;t_2)\int_{x_1t_1}^{x_2,t_2}(Ldt)\psi(x_1;t_1)]dx_1dx_2
\end{aligned}
$$

$$
\begin{aligned}
0 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}&\phi^*(x_2;t_2)[(\frac{\partial}{\partial x}^{\dagger}\delta x(t_2))S(t_2) \\
&+\int_{x_1t_1}^{x_2,t_2}\delta(Ldt) \\
&+S(t_1)(\delta x(t_1)\frac{\partial}{\partial x})]\psi(x_1;t_1)dx_1dx_2
\end{aligned}
$$

Above line needs some explanation. Why does S(t_2) and S(t_1) occur? Reason has to do with x and t being limits of an integral.

$$
\begin{aligned}
0 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}&\phi^*(x_2;t_2)[(\frac{\partial}{\partial x}^{\dagger}\delta x(t_2))S(t_2) \\
&+\int_{x_1t_1}^{x_2,t_2}[\frac{\partial Ldt}{\partial x}\delta x+\frac{\partial Ldt}{\partial dx}\delta dx+\frac{\partial Ldt}{\partial t}\delta t+\frac{\partial Ldt}{\partial dt}\delta dt] \\
&+S(t_1)(\delta x(t_1)\frac{\partial}{\partial x})]\psi(x_1;t_1)dx_1dx_2
\end{aligned}
$$

$$
\begin{aligned}
0 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}&\phi^*(x_2;t_2)[(\frac{\partial}{\partial x}^{\dagger}\delta x(t_2))S(t_2) \\
&+\int_{x_1t_1}^{x_2,t_2}[(\frac{\partial Ldt}{\partial x}-\frac{d\partial Ldt}{\partial dx})\delta x]+[\frac{\partial Ldt}{\partial dx}\delta x]_{t_1}^{t_2} \\
&+S(t_1)(\delta x(t_1)\frac{\partial}{\partial x})]\psi(x_1;t_1)dx_1dx_2
\end{aligned}
$$

Defining

$$Fdt = \frac{\partial L dt}{\partial x}$$
$$p = \frac{\partial L dt}{\partial dx}$$

And substituting

$$0 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \phi^*(x_2; t_2)[(\frac{\partial}{\partial x}^\dagger \delta x(t_2)) S(t_2)$$
$$+ \int_{x_1 t_1}^{x_2, t_2} [(Fdt - dp)\delta x] + [p\delta x]_{t_1}^{t_2}$$
$$+ S(t_1)(\delta x(t_1) \frac{\partial}{\partial x})] \psi(x_1; t_1) dx_1 dx_2$$

Gives us

$$F = \frac{dp}{dt}$$

$$p = \frac{\partial}{\partial x}^\dagger S = -S \frac{\partial}{\partial x}$$

In the case where S is approximately a constant throughout the time period considered (in particular, where $S = i\hbar$), we get Quantum Mechanics. Note, however, that we have not specified between the Heisenberg or Schrodinger Pictures, which is why we have equations that may evolve either the wavefunctions or the equations of motion.

Additionally, must insert Born's Rule

$$E[A] = \frac{\langle \phi | A | \psi \rangle}{\langle \phi | \psi \rangle}$$

Extensions to Consider:

1. In differential game theory, each agent has its own utility function, so we should consider the case where every particle in quantum mechanics has its own action and its own wavefunctions.

2. Nothing in here says that the variables have to be real valued. By allowing complex valued variables, we can have non-unitary evolutions in quantum mechanics, which allows changes in the entropy.

3. A while back, I found something I call "jagged calculus", which separates out left and right derivatives. By going through the above derivation in terms of jagged calculus, we separate force into push and pull.

Extension 1. Multi-Particle Case

In this extension, each particle gets its own action and its own wavefunctions. Our final equations read

$$F_i dt = \frac{\partial L_i dt}{\partial x_i}$$
$$p_i = \frac{\partial L_i dt}{\partial dx_i}$$

$$F_i = \frac{dp_i}{dt}$$

$$p_i = \frac{\partial}{\partial x_i}^{\dagger} S_i = -S_i \frac{\partial}{\partial x_i}$$

To measure any quality of a particle, you must use the particle's own wavefunction.

$\langle \phi_1(t) | x_2 | \psi_1(t) \rangle$ is particle-1's expected guess as to where particle-2 is.

$\langle \phi_1(t) | x_1 | \psi_1(t) \rangle$ is particle-1's expected position.

Has some promise to resolve the EPR paradox because each particle has its own set of wavefunctions.

Extension 2. Complex Valued Case

In this extension, any variable can be complex (and potential energy is analytic), but we only measure real values (imaginary parts of values are hidden). Imaginary masses end up moving with Brownian motion. Entropy now changes. Can describe an engine run on an ensemble of quantum particles.

Extension 3. Jagged Calculus

Force separates into push and pull. Extends the algebra of allowed potential energy functions.