

- 1) (i) The most computationally efficient way to evaluate the function  $f(x) = 2x^6 - 3x^4 + 4x^2 - 3$  is by first calculating  $x^2$ , say  $x2 = x^2$ . This number can then be multiplied by itself to yield  $x^4$  and  $x^6$ .
- (ii) The maximum range for a short signed integer in C is -2147483648 to 2147483647 (although this can depend on which machine you are using, compiler, etc.). So, for the input integers  $f(11, 28)$  the integer and double results match because the calculated value of the polynomial  $f(11, 28) < 2147483647$ . However, in the polynomial function  $f(x) = 2x^6 - 3x^4 + 4x^2 - 3$  is calculated by first calculating  $x^2$ . Even if  $x^2 > 2147483647$ , the number is capped at 2147483647, and one then has the potential to obtain negative results, as in the output file (these are marked in **bold**).
- (ii) Algorithm: To check if a number  $n$  is prime, one is only required to check if the division of  $n$  by the integers from 2 to  $\sqrt{n}$  results in another integer. 0 and 1 are not prime and 2 is prime by definition.

#### Output of Program:

$f(11) = 3499680$  (int),  $3.49968e+006$  (double). Prime: **1** (1 true, 0 false)

$f(28) = 961939773$  (int),  $9.6194e+008$  (double). Prime: **0** (1 true, 0 false)

$f(45) = \text{-584631712}$  (int),  $1.65952e+010$  (double). Prime: **0** (1 true, 0 false)

$f(397) = \text{-361058976}$  (int),  $7.83013e+015$  (double). Prime: **1** (1 true, 0 false)

$f(677) = \text{-402784224}$  (int),  $1.92558e+017$  (double). Prime: **1** (1 true, 0 false)

$f(951) = \text{-129610176}$  (int),  $1.47949e+018$  (double). Prime: **0** (1 true, 0 false)

$f(2552) = \text{-2071637763}$  (int),  $5.52477e+020$  (double). Prime: **0** (1 true, 0 false)

$f(6447) = 879593088$  (int),  $1.43607e+023$  (double). Prime: **0** (1 true, 0 false)

$f(6449) = 1282775424$  (int),  $1.43875e+023$  (double). Prime: **1** (1 true, 0 false)

$f(7411) = 1689934176$  (int),  $3.31353e+023$  (double). Prime: **1** (1 true, 0 false)

$f(7412) = 1747996477$  (int),  $3.31621e+023$  (double). Prime: **0** (1 true, 0 false)

- 2) We know that the molecular weight of water is 18.0106 g/mol. If we have only 10,000 molecules of water, then the total mass of the system is:

$$0.018 \text{ kg} \times 10^4 / 6.023 \times 10^{23} = 2.99 \times 10^{-22} \text{ kg}$$

From this, we can solve for L given the proper densities of 1000 and 100 kg/m<sup>3</sup>:

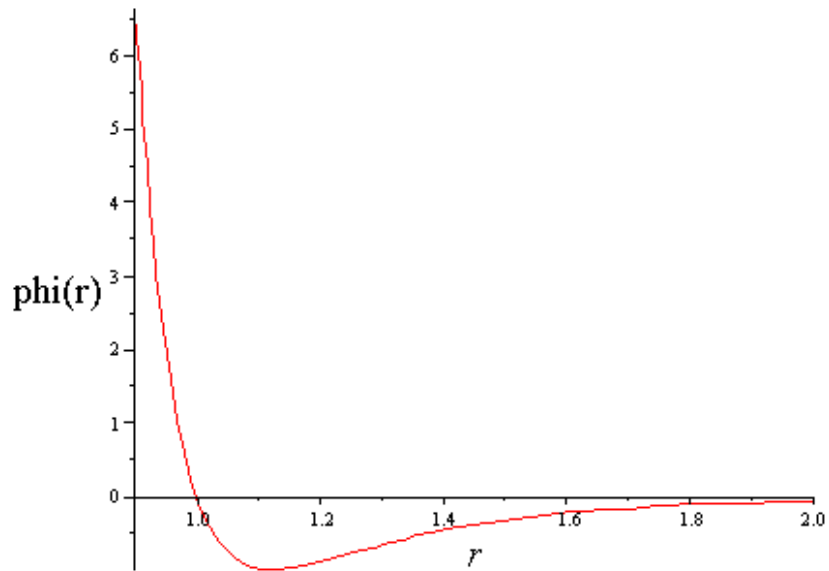
$$L = (2.99 \times 10^{-22} / 100)^{1/3} = 14.4 \text{ nm}$$

$$L = (2.99 \times 10^{-22} / 1000)^{1/3} = 6.7 \text{ nm}$$

The length L only grows as the cube root of the density, so you would increase L by a factor of  $(10)^{1/3}$ .

3)

4) a)

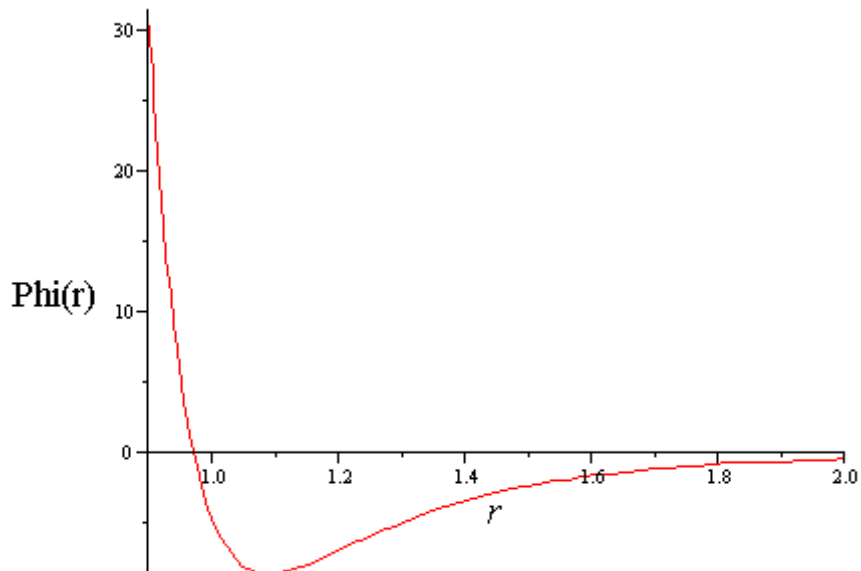


For:

$$\phi(r) = 0, r = 1$$

For:

$$\frac{\partial \phi(r)}{\partial r} = -\frac{48}{r^{13}} + \frac{24}{r^7} = 0, r = 2^{1/6} = 1.122$$



For:

$$\phi(r) = 0, r = 0.971$$

For:

$$\frac{\partial \phi}{\partial r} = -\frac{291.12}{r^{13}} + \frac{173.40}{r^7}, r = 1.09$$



5) The scale:

$$k = \frac{k_B \epsilon_{LJ}^{1/2}}{m_{LJ}^{1/2} \sigma_{LJ}^2}$$

Has the units of W/m-K.