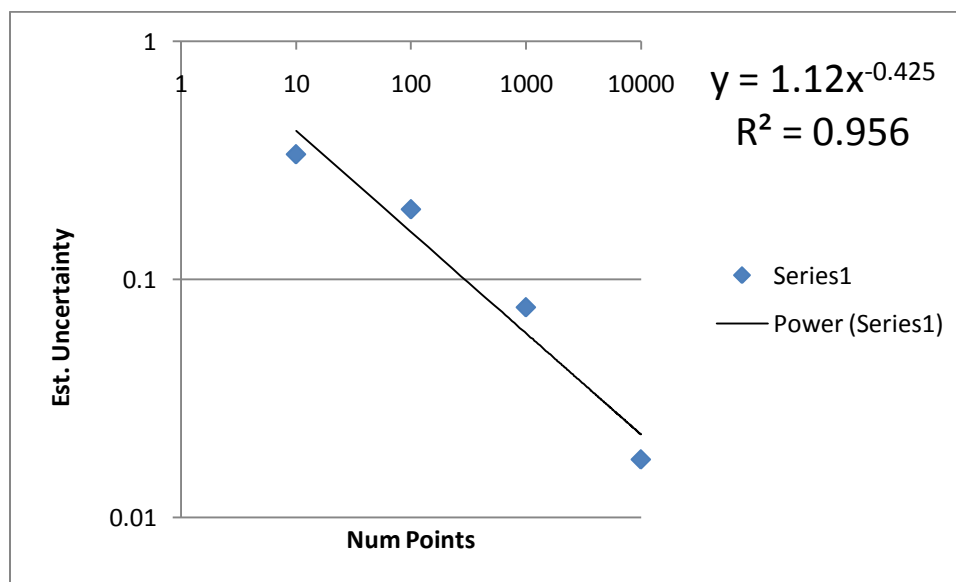


### Q3

Below is a table listing the average computed value of  $\pi$ , as well as the standard deviation in the calculation:

Num Points	avg	std dev
10	3.12	0.334664
100	3.168	0.196774
1000	3.144	0.076368
10000	3.13448	0.017569

One sees a trend moving (in general) closer to the exact value of  $\pi=3.1416\dots$  with increasing number of points  $N$ . One also sees that the estimated uncertainty (standard deviation in this case) in the calculation is decreasing with  $N$ , which is to be expected. Below is a plot of that uncertainty as a function of the number of points:



A best fit line to this data suggests that the uncertainty (standard deviation in this case) is decreasing (approximately) as  $1/\sqrt{N}$ , where  $N$  is the number of points used. This is to be expected from a random sampling. Using this information, we can estimate how many points would be required for an uncertainty of  $10E-20$ :  $\text{Error} = 1/\sqrt{N}$ , or for  $\text{Error} \approx 10E-20$ ,  $N \approx 10E38$ ! This is an enormous number of points, and demonstrates how inefficient a random sampling is in integrating simple functions, such as the area of a circle.