

# NTPL: Nanoscale Transport Phenomena Laboratory

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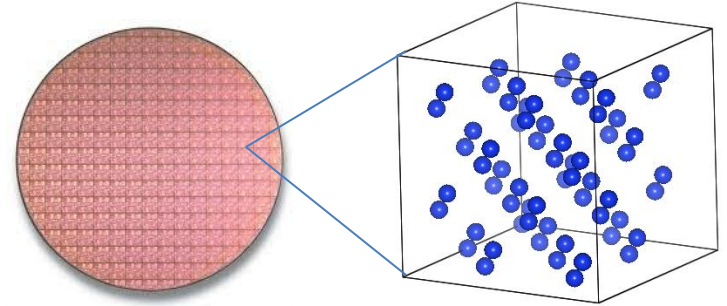
<http://ntpl.me.cmu.edu/>

4/13/2011



# Nanoscale Transport Phenomena

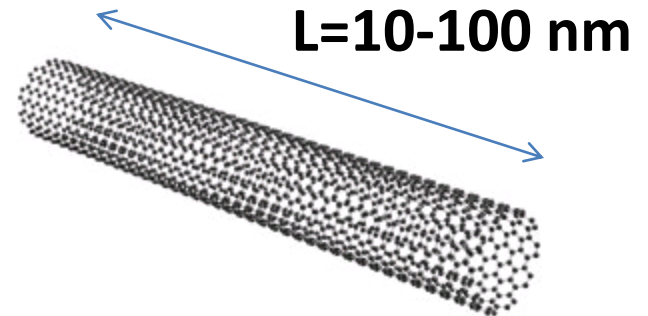
- **Bulk Properties**



- Transport Carriers: **phonons**, electrons, photons, etc.
- Statistical Properties: classical vs. quantum

- **Nanoscale Behavior**

- Transport: continuum?
- Carrier Properties: Bulk?



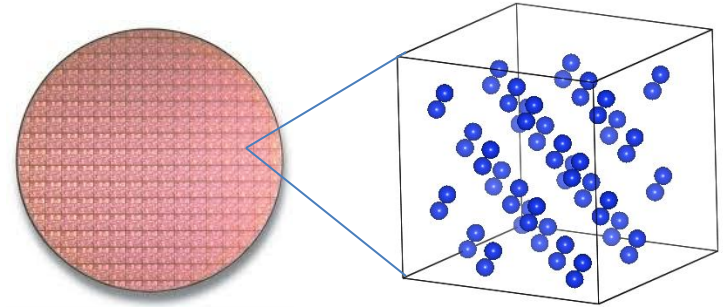
# Predicting Thermal Conductivity of Defected Systems using Spectral Energy Density

4/13/2011

# Dielectric Thermal Conductivity

- Dielectric crystal = Electrical Insulator

- Ex: Si, Ge



- **Bulk** dielectric:  $\vec{q} = -\kappa \nabla T$

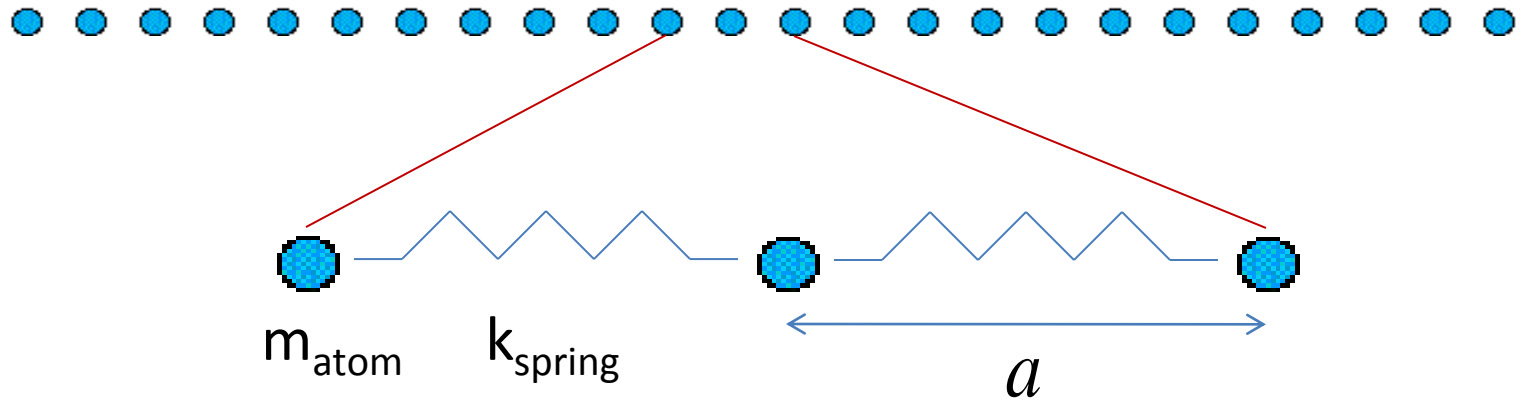
- Dielectric Thermal Conductivity:

$$K_{total} = K_{phonon} + \cancel{K_{elec}}$$

- **Phonons** are lattice vibrations...

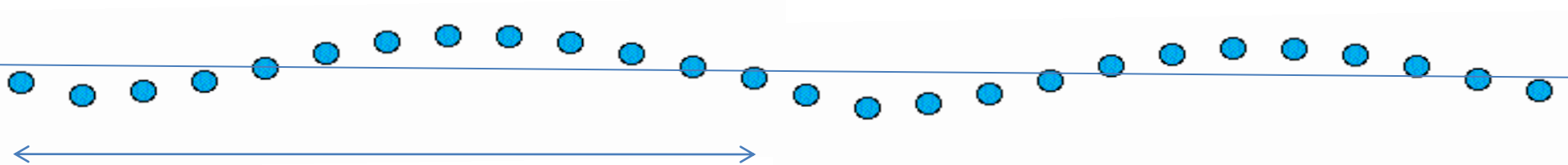
# Phonons

## 1-D model of crystal:



Lattice vibrations (**Phonons**) are travelling waves:

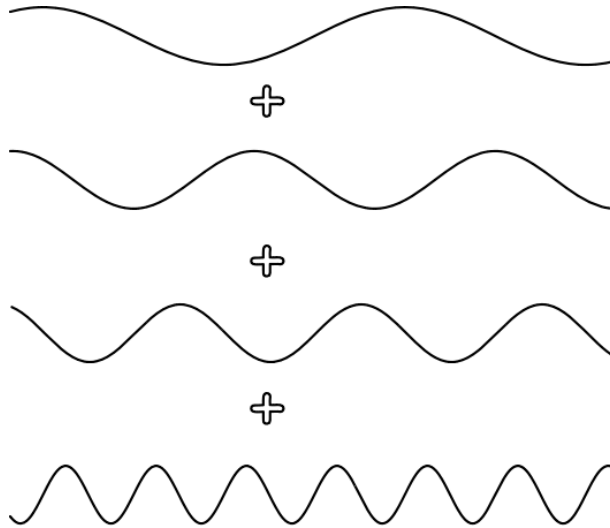
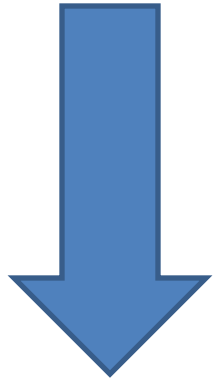
$$u(x) \propto e^{i(kx - \omega t)} \longrightarrow v_g$$



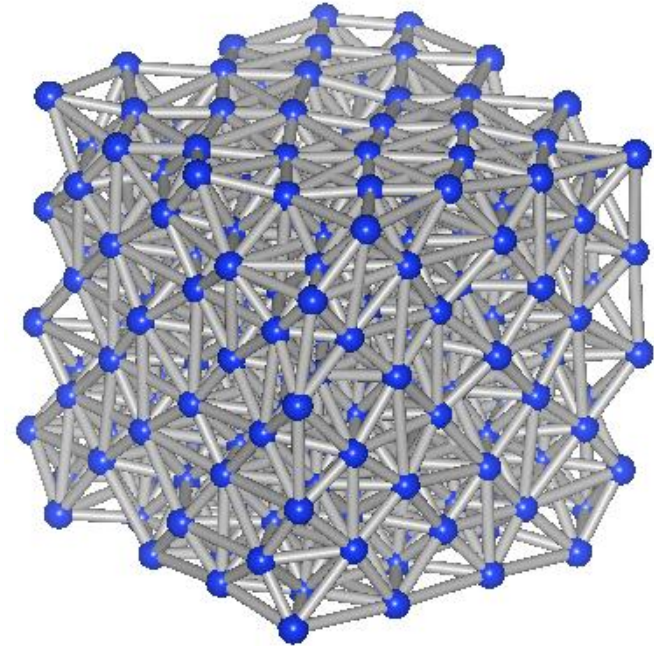
$$k = \frac{2\pi}{\lambda}, \omega$$

# Phonon Gas

$$E_{\text{phonon}} = \hbar\omega$$



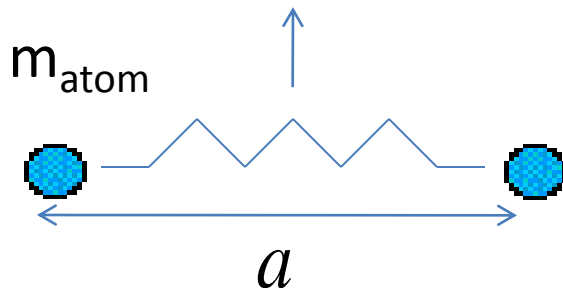
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## Phonons Interact:

$$F \propto kx + \epsilon x^3 + \dots$$

- Non-linear springs make an  
**Interacting Gas...**



# Thermal Conductivity Phonon Gas

## Kinetic Theory:

$$\vec{q} = -\kappa \nabla T$$

$T_H$   $T_C$

### Ideal Gas:

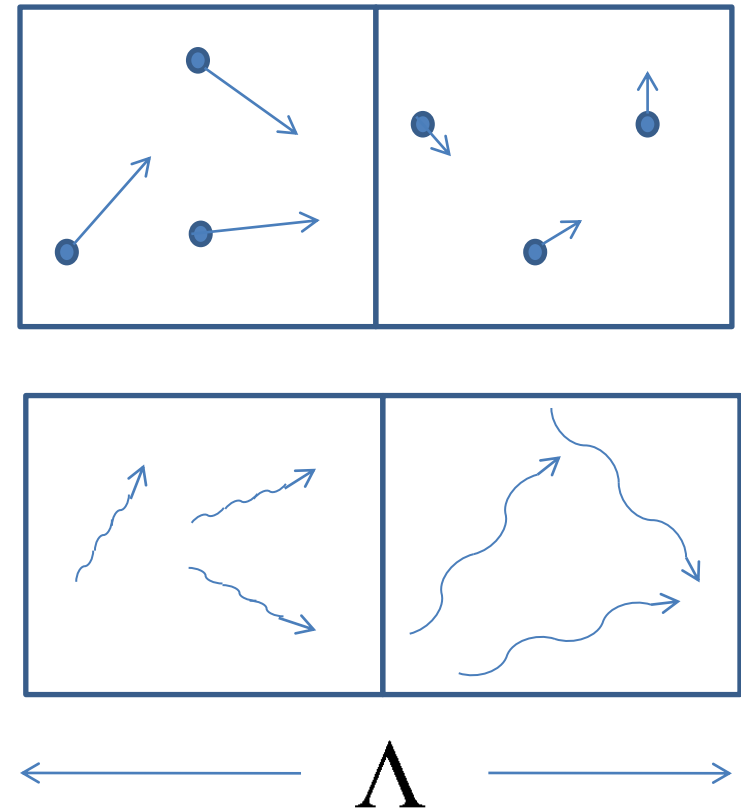
$$\kappa = \frac{1}{3} \rho c_v \langle v \rangle \Lambda$$

### Phonon Gas:

- If system  $L \gg \Lambda$ :

$$\kappa = \frac{1}{3} \rho c_v v_g \Lambda$$

**-Phonons interact:** with each other, impurities, defects, etc.



# Thermal Conductivity Phonon Gas

**Phonons interact**

$$\kappa = \frac{1}{3} \rho c_v v_g \Lambda \quad \longrightarrow \quad \kappa = \frac{1}{V} \sum_i c_v(\omega_i) v_g^2(\omega_i) \tau_i(\omega_i)$$

**Phonon Lifetime:**

$$\Lambda_i(\omega_i) = \tau_i(\omega_i) v_g(\omega_i)$$

**Properties needed for  $\kappa$  prediction:**

$$\omega_i, \tau_i(\omega_i), v_g(\omega_i)$$





# Molecular Dynamics and Spectral Energy Density

- Molecular Dynamics

$$\vec{F} = m\vec{a}$$

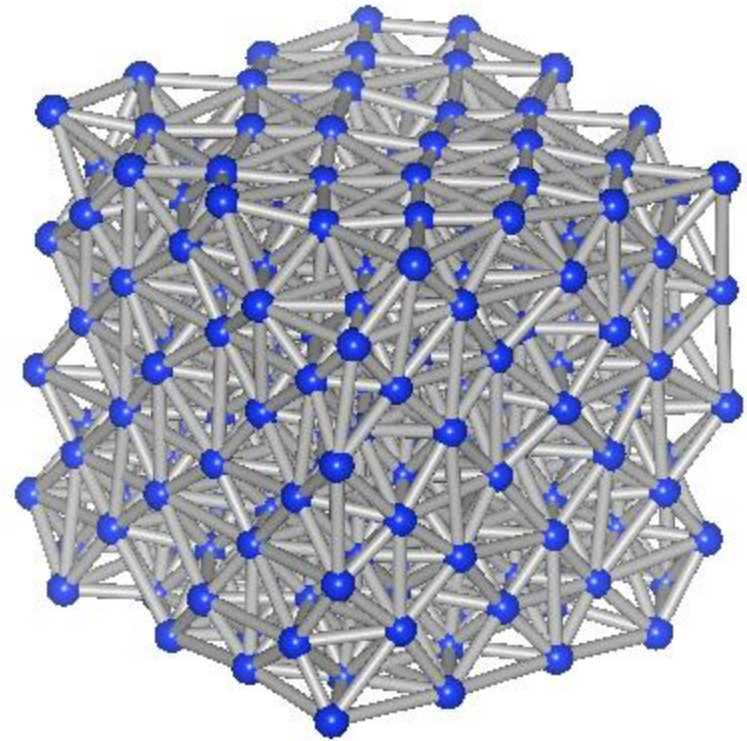
- Equilibrium/non-equilibrium properties:

$$\vec{r}(t), \vec{p}(t)$$

- Spectral Energy Density

- **Frequency, group velocity** and **lifetimes** of phonons from Molecular Dynamics.

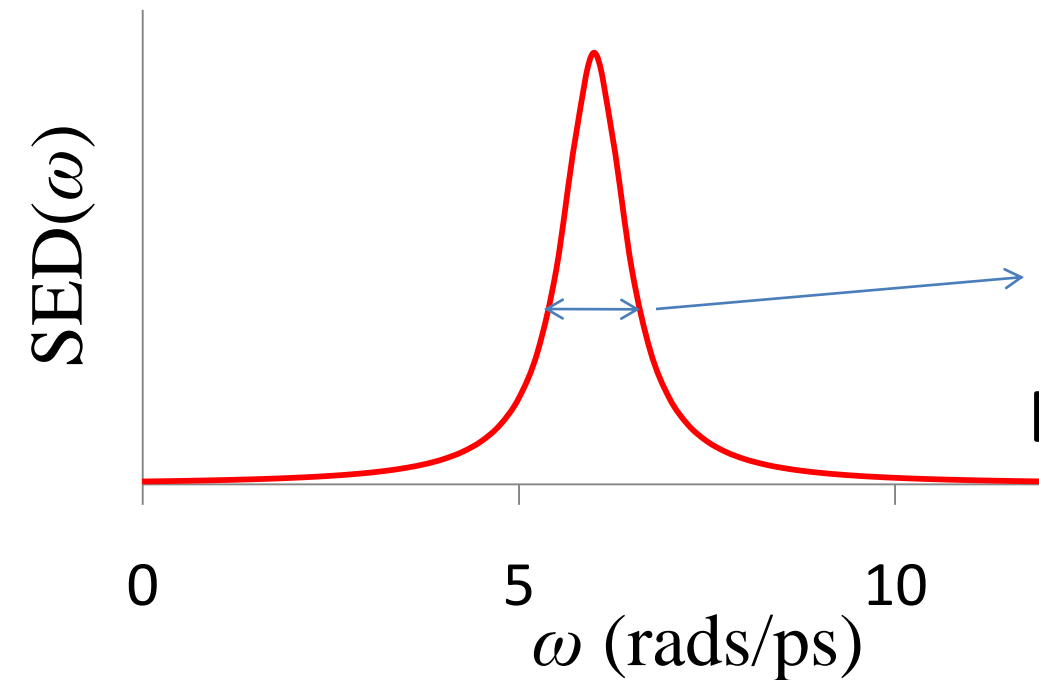
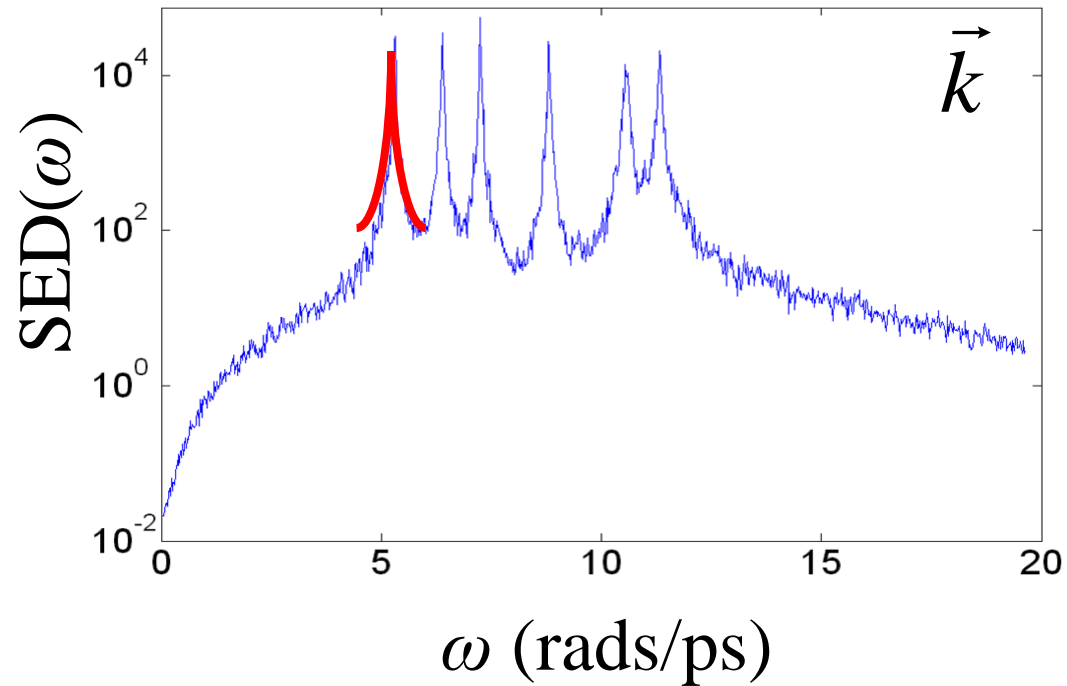
$$\omega_i, \tau_i(\omega_i), v_g(\omega_i)$$



# Spectral Energy Density

- Spectral Energy Density:  
(SED) system energy in  
frequency space.

$$\omega_i, \tau_i(\omega_i), v_g(\omega_i)$$



$$\Gamma \rightarrow \tau_i(\omega_i) = 1/\Gamma$$

Broad peak = short **lifetime**

# Thermal Conductivity Disordered Materials

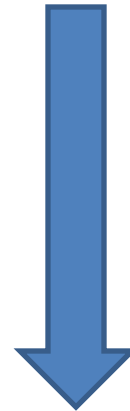
Disorder = non-periodic

Blue  $\Rightarrow$  mass=1.0

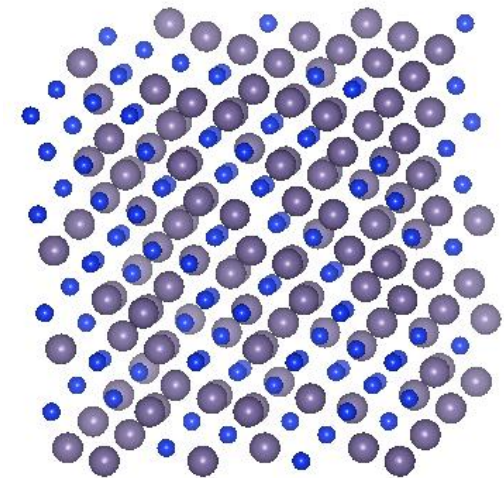
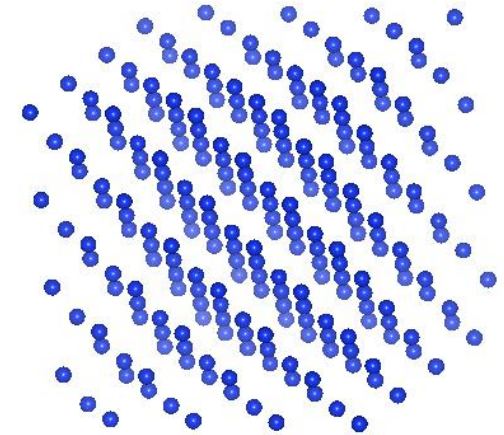
Purple  $\Rightarrow$  mass=3.0

Phonon picture still valid?

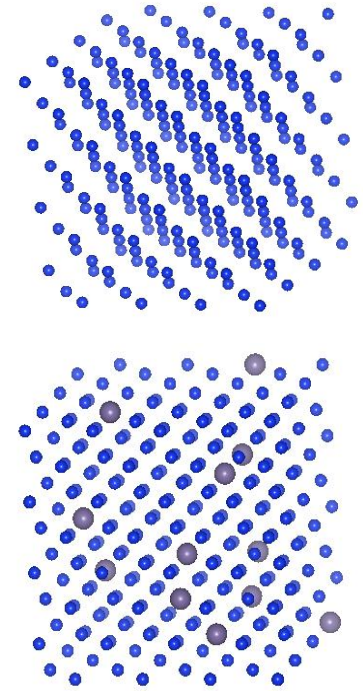
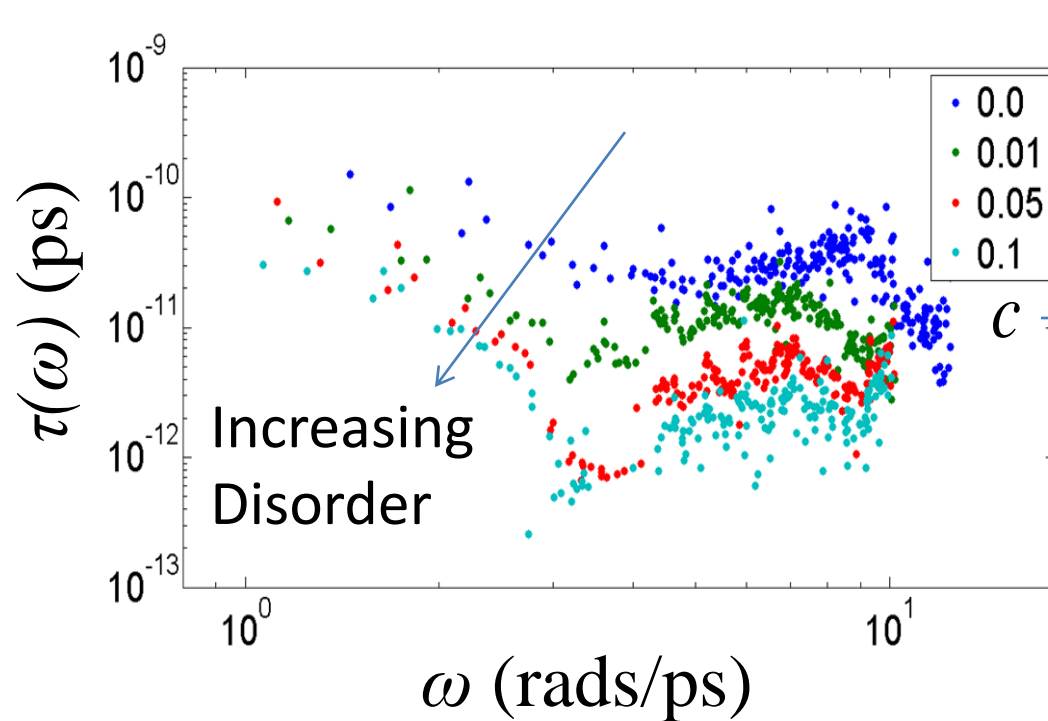
$c = 0.0$



$c = 0.5$



# Phonon Lifetimes (Mean Free Path)



**Pure**

$$\Lambda^{0.0} = 74 - 0.5 \text{ nm}$$

**Defected**

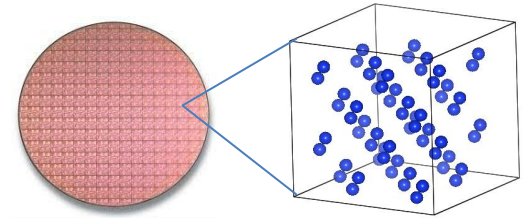
$$\Lambda^{0.1} = 11 - 0.1 \text{ nm}$$



$$L \approx \Lambda$$

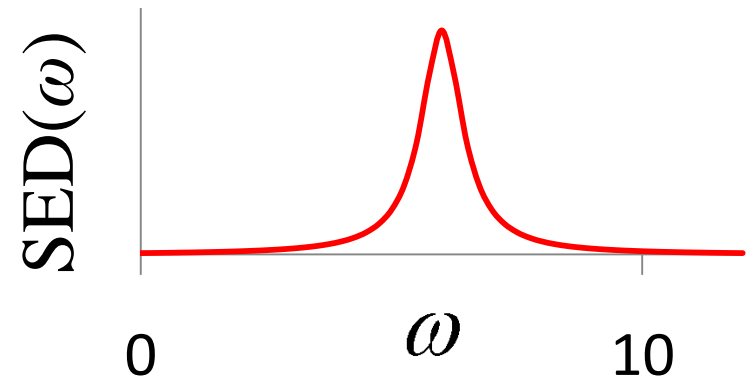


# Discussion



$$\vec{q} = -\kappa \nabla T$$

- Dielectric thermal conductivity can be described by **Kinetic Theory** (bulk system).
- **Molecular Dynamics** and **Spectral Energy Density** can measure phonon properties.
- Phonon properties can be predicted for “weakly” perturbed systems, analyzed on mode by mode basis.



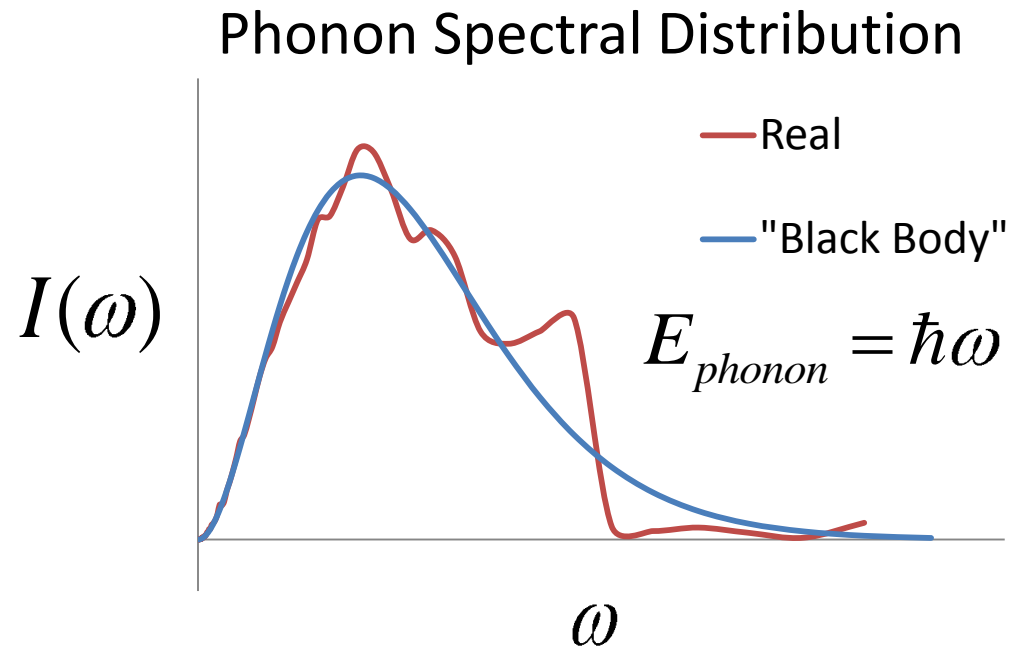
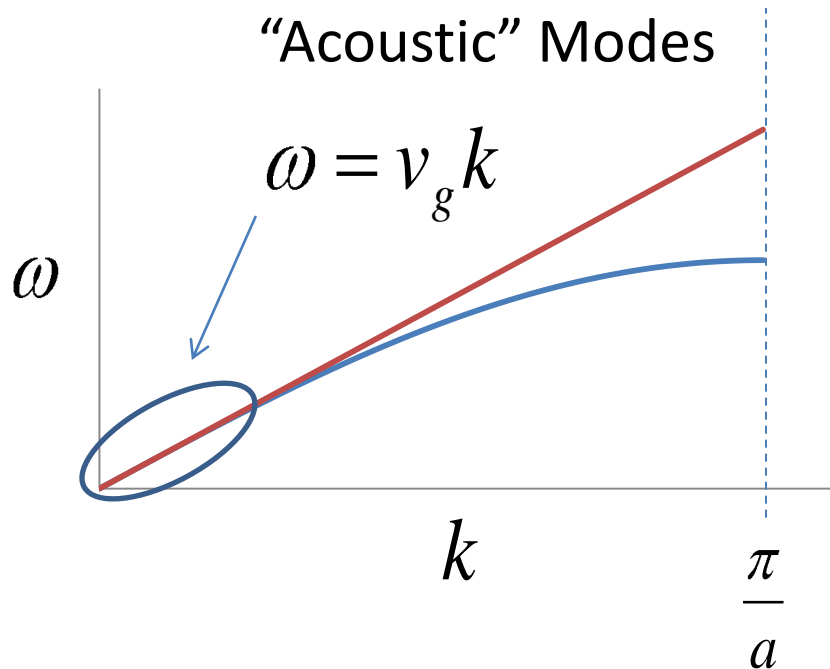
$$\omega_i, \tau_i(\omega_i), v_g(\omega_i)$$



# Questions

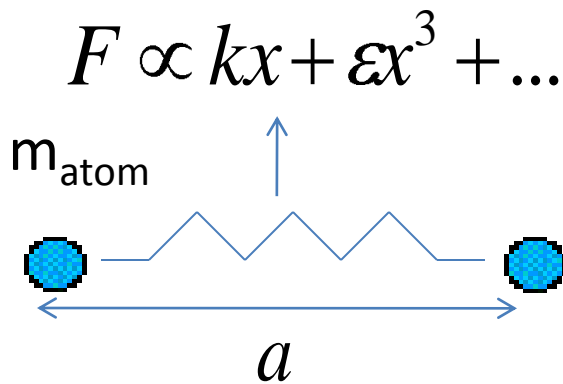


# Phonon Gas



## Phonons vs. Photons:

- Non-linear springs
- Phonons interact with each other
- **Interacting Gas...**



$$\Phi(\omega, \mathbf{\kappa}) = 2 \sum_v^{3n} \sum_{\alpha, b}^{3, n} \langle T(\mathbf{\kappa}_v^b; \omega) \rangle$$

$$= \frac{1}{4\pi\tau_0 N} \sum_b^n m_b \sum_{\alpha}^3 \left| \int_{-\tau_0}^{\tau_0} \sum_l^N \dot{u}_{\alpha}(l; t) \exp[i\mathbf{\kappa} \cdot \mathbf{r}_0(l)_0 - i\omega t] dt \right|^2$$

