Power-Law Distributions of Particle Concentration in Free-Surface Turbulence

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Outline

I. Experiment Description/Overview

II. Particle Evolution

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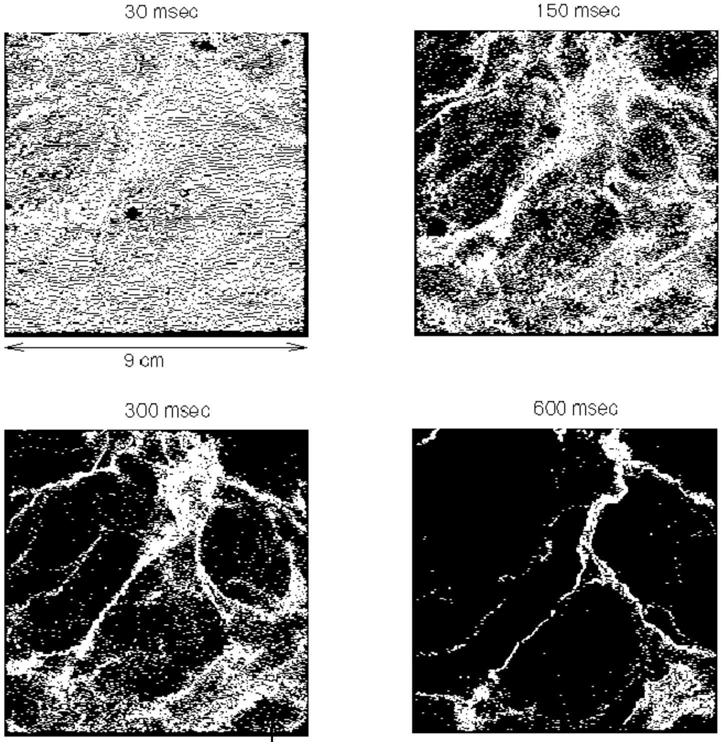
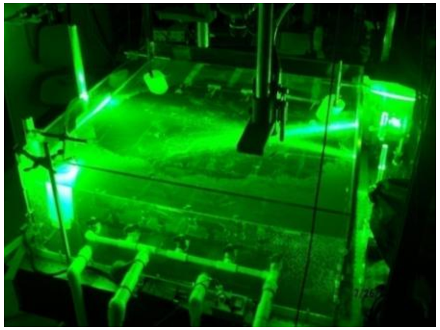
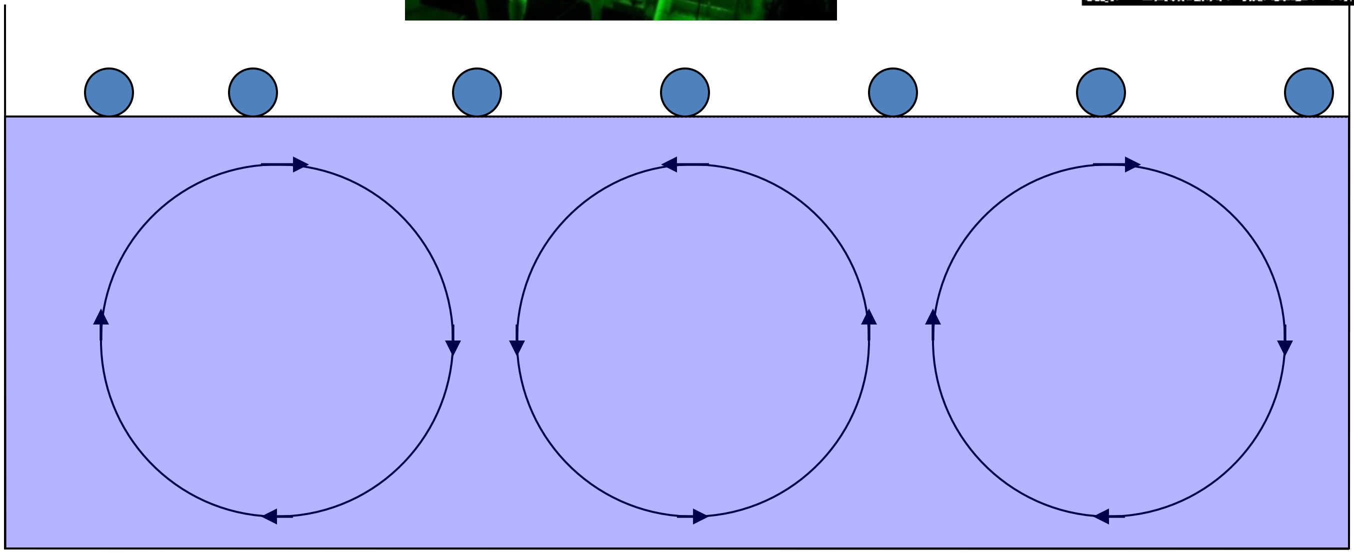
VI. Conclusion

I. Experiment Overview

- Experiment: track floating particles on the surface of a turbulent sea.
- Models transport of pollutants, chemical species, etc. which are PASSIVE and BOUYANT.
- Floaters flee fluid upwellings (sources) and cluster around fluid downwellings (sinks).







- Inertial particles cluster in 3D incompressible turbulence:

$$\overrightarrow{v_p}(\vec{x},t) \neq \overrightarrow{v_f}(\vec{x},t)$$

G. Falkovich, Nature 419, 151 (2002).

- In same spirit, one may then write the velocity of the floaters:

$$\overrightarrow{v_p}(x,y,t) = \overrightarrow{v_f}(x,y,z=0,t)$$

- Floaters cluster due to effective compressibility:

$$C = \left\langle \left(\vec{\nabla}_2 \cdot \vec{v} \right)^2 \right\rangle / \left\langle \left(\vec{\nabla}_2 \vec{v} \right)^2 \right\rangle$$

C=0 (incompressible)
C=1 (irroational)
C≈0.5 (experiment)

Parameter	
Integral Scale:	$\ell_0 = \int dr \frac{\langle v_{\|\|}(x+r)v_{\|\|}(x)\rangle}{\langle (v_{\|\|}(x))^2\rangle}$
Dissipative (Kolmogorov) length scale:	$\eta = \left(\frac{\nu^3}{\varepsilon_{diss}}\right)^{1/4}$
Energy Dissipation Rate:	$\varepsilon_{diss} = 10\nu\left\langle\left(\frac{\partial v_x}{\partial x}\right)^2\right\rangle$
Taylor microscale:	$\lambda = \sqrt{\frac{v^2{}_{rms}}{\langle (\partial v_x / \partial x)^2\rangle}}$
Taylor microscale Reynolds:	$\mathrm{Re}_\lambda = \frac{v_{rms}\lambda}{\nu}$
RMS Velocity:	$v_{rms} = \sqrt{\langle v^2\rangle - \langle v\rangle^2}$
Large Eddy Turnover time:	$\tau_0 = \frac{\ell_0}{v_{rms}}$
Compressibility:	$C = \frac{\langle (\nabla_2 \cdot \vec{v})^2\rangle}{\langle (\nabla_2 \vec{v})^2\rangle}$

- Results from K41:

For: $\eta < r < \ell_0$ "Inertial range"

$$\langle \delta v(r)^2\rangle = \langle \left(v_{||}(x+r) + v_{||}(x)\right)^2\rangle \sim \varepsilon_{diss}^{2/3} r^{2/3}$$

This is approximately what is seen at the surface:
J.R. Cressman et al., New J. Phys. ***6****, 53 (2004).*

For: $r < \eta$ "Dissipative range"

$$\langle \delta v(r)^2\rangle \sim r^2$$

Flow is viscous (laminar).

- Free-surface clustering and interital clustering thus share many similarities. In dissipative range:

$$\left\langle \delta v(r)^2 \right\rangle \propto r^2$$

E. Balkovsky, arXiv.chaodynp. 9912027 (1999).

-Predicts a power-law behavior of the PDF of concentration in dissipative range:

$$\Pi(n_r) \sim n_r^{-\beta} \qquad \beta < 1$$

J. Bec, Phys. Rev. Lett. 92, 224501 (2004).

-Predicts a multi-fractal distribution of particles in dissipative range:

$$\langle n_r^p \rangle \sim r^{\alpha_p} \qquad \alpha_p \neq p\alpha_1$$

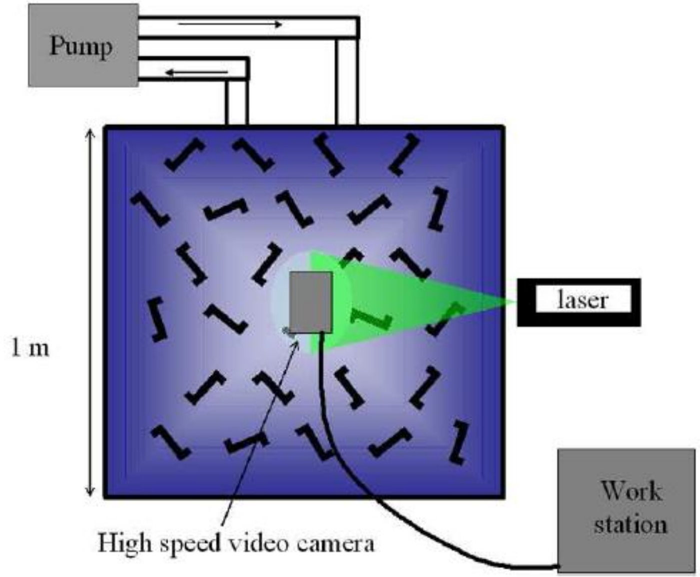
- In inertial range:

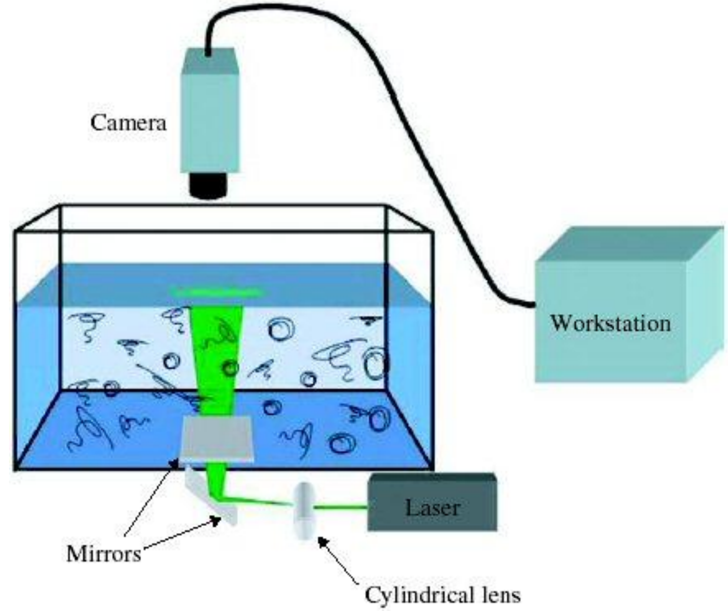
$$\left\langle \delta v(r)^2 \right\rangle \propto r^{2/3}$$

- Field is rough, and not amicable to analytical techniques.

A. Pumir, Phys. Rev. E. 77, 066304 (2008).

-DNS of 2D compressible turbulence which shows particles concentrate on multifractal in BOTH dissipative and inertial range.





- Turbulence generated in tank of water (*ν=0.01 cm^2/s*) 1mx1mx30cm

-Injection source far from surface, waves do not exceed 0.5mm.

J.R. Cressman et al., New J. Phys. ***6****, 53 (2004).*

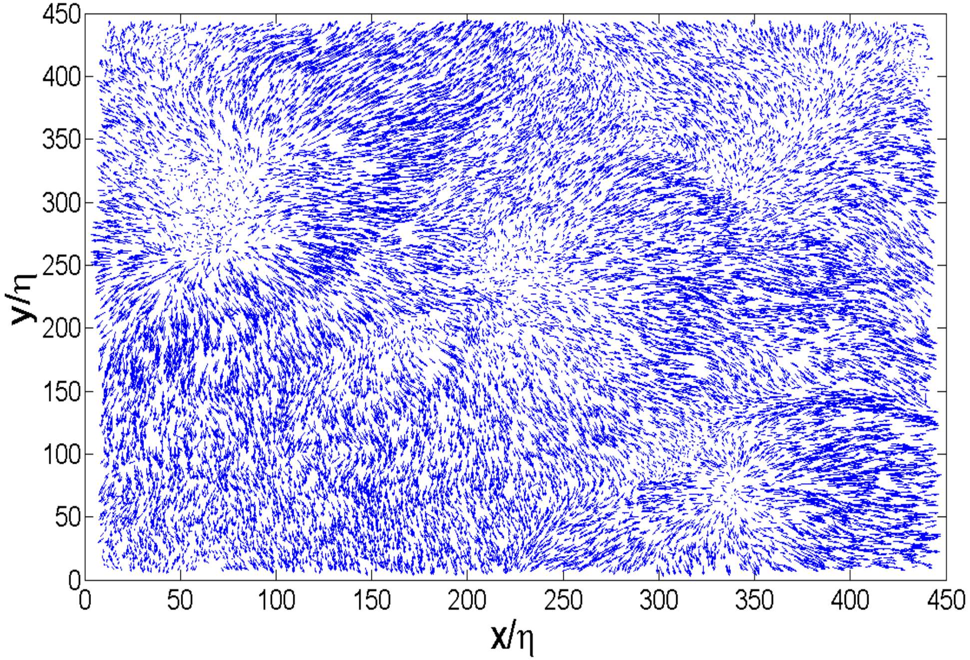
- Use bouyant (s.g. 0.25) “small” (0.05mm) particles: $St = \tau_s v_{rms}/a \cong 0.1$

- Camera speed set at 100Hz, Images processed using special PIV program.

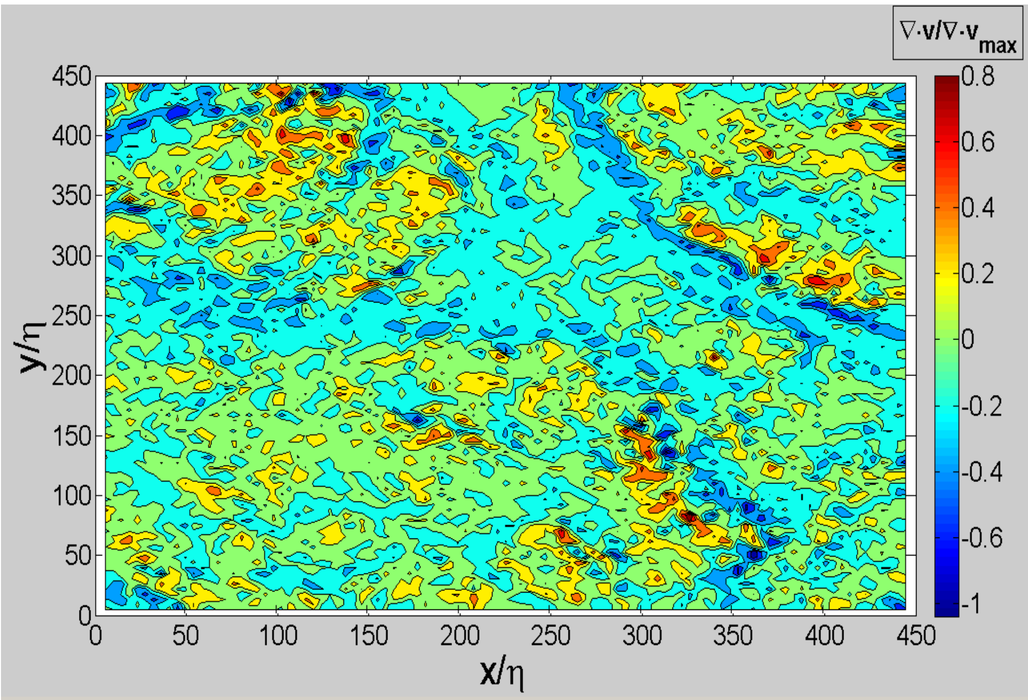
-Typically data taken for 5-10s, producing 500-1000 unique velocity vector fields.

- Camera field of view (FOV) spans L=9cm, each pixel is then 0.1mm (approx. 2η).

-Can thus resolve the entire inertial range of flow and measure dissipative scale effects.



- On average 23k vectors/field, yields average vector spacing δx=2.5 η.



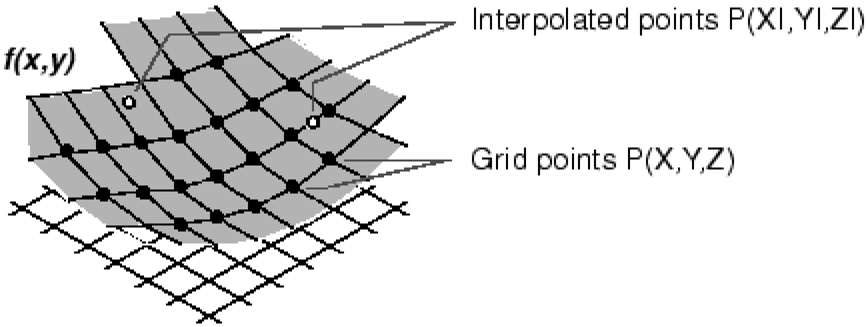
- Divergence field normalized by absolute value of maximum divergence.

Measured Turbulent Parameters

Parameter	Equation	Measured Value
Taylor microscale:	$\lambda = \sqrt{\dfrac{v^2_{rms}}{\langle (\partial v_x / \partial x)^2 \rangle}}$	$0.46 - 0.47 cm$
Taylor microscale Reynolds:	$\mathrm{Re}_\lambda = \dfrac{v_{rms}\lambda}{\nu}$	$150 - 170$
Integral Scale:	$\ell_0 = \int dr \dfrac{\langle v_{\|\|}(x+r) v_{\|\|}(x) \rangle}{\langle (v_{\|\|}(x))^2 \rangle}$	$1.45 - 1.5 cm$
Large Eddy Turnover time:	$\tau_0 = \dfrac{\ell_0}{v_{rms}}$	$0.43 - 0.5 s$
Energy Dissipation Rate:	$\varepsilon_{diss} = 10\nu \left\langle \left(\dfrac{\partial v_x}{\partial x} \right)^2 \right\rangle$	$5.9 - 6.1 cm^2 / s^3$
Dissipative (Kolmogorov) length scale:	$\eta = \left(\dfrac{\nu^3}{\varepsilon_{diss}} \right)^{1/4}$	$0.02 cm$
RMS Velocity:	$v_{rms} = \sqrt{\langle v^2 \rangle - \langle v \rangle^2}$	$3.3 cm / s$
Compressibility:	$C = \langle (\vec{\nabla}_2 \cdot \vec{v})^2 \rangle / \langle (\vec{\nabla}_2 \vec{v})^2 \rangle$	0.49 ± 0.03

II. Particle Evolution

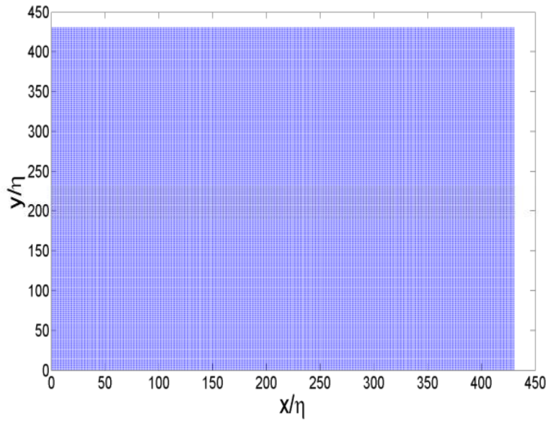
$$\frac{d\vec{x}}{dt} = \vec{v}(\vec{x}_i(t), t)$$



- Particles are evolved using a cubic-spline interpolation scheme to achieve dissipative length resolution . Criteria: *$\delta x < \pi\eta$*

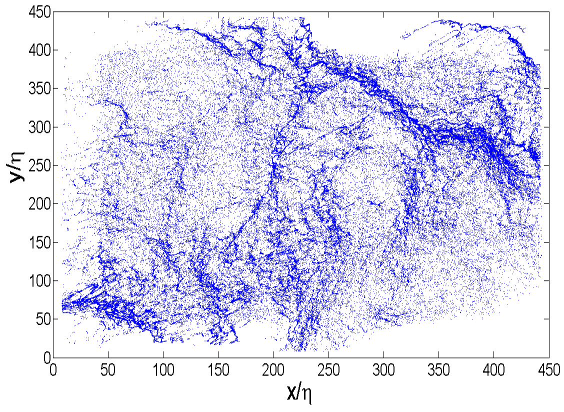
*- S. Pope, J. Comp. Phys. **79**, 373 (1988).*

- Our data: $\delta x = 2.5\ \eta$, results insensitive using $\delta x = 4\eta$.



$$\vec{x}_i(t) = (x_i(t), y_i(t))$$

- Evolve 4×10^5 homogeneous distribution at t=0s. Results insensitive using 10^5 to 4×10^5.
- Steady-state reached in ~1s (2 LETTs).



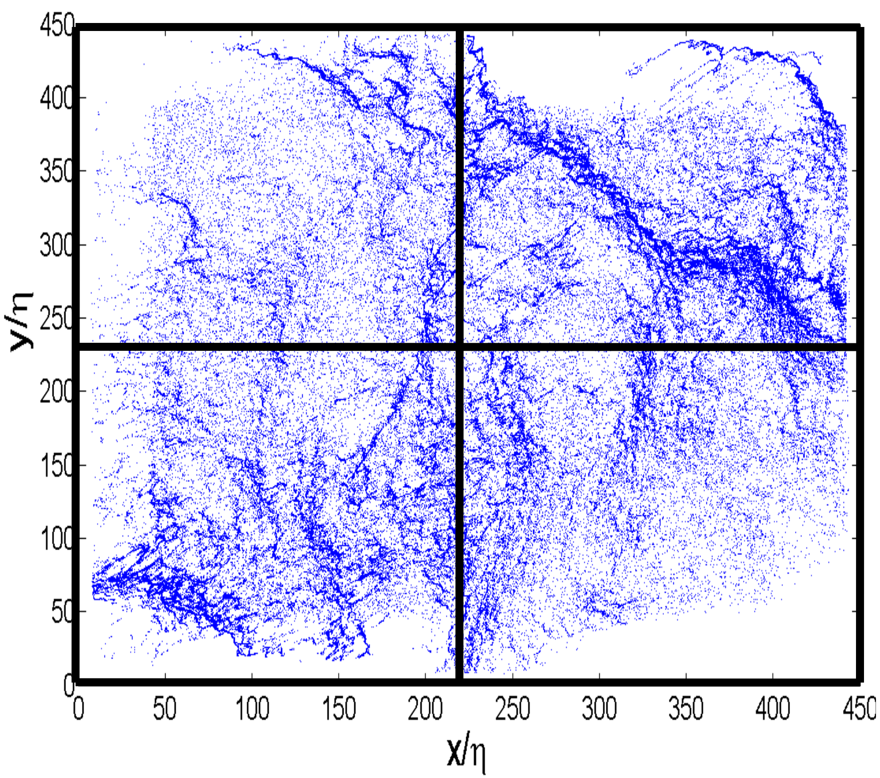
III. Coarse-Graining Procedure

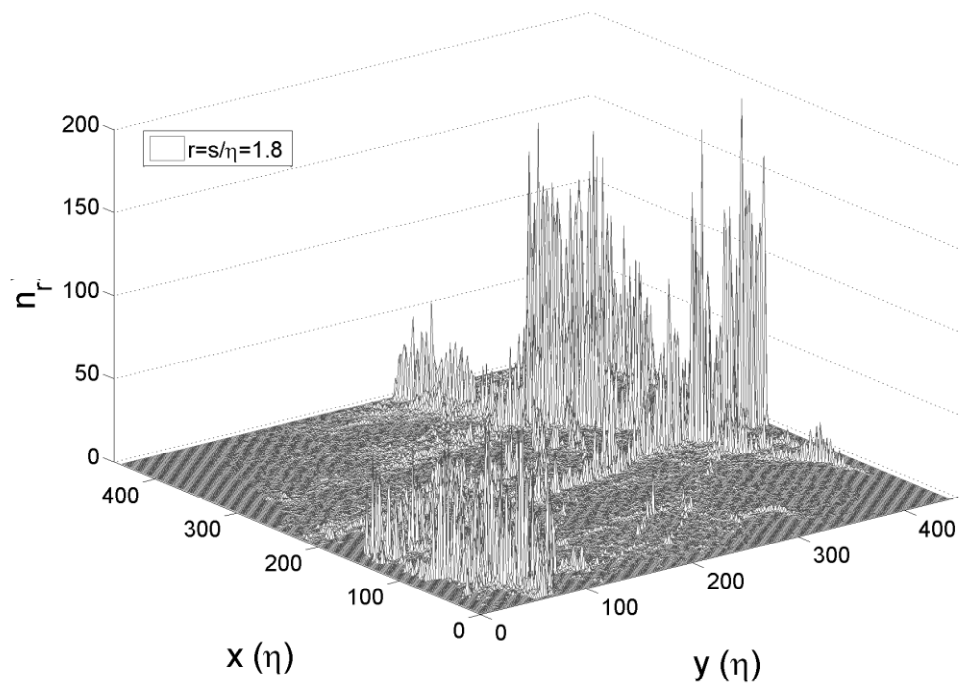
Coarse-Grained (Eulerian) Concentration: $n_r = \dfrac{N_r}{\langle N_r \rangle}$ $\quad \langle n_r \rangle = \left\langle \dfrac{N_r}{\langle N_r \rangle} \right\rangle = 1$

$$\langle N(r,t) \rangle = N_t (s/L)^2$$

s $\qquad r = s/\eta$

x

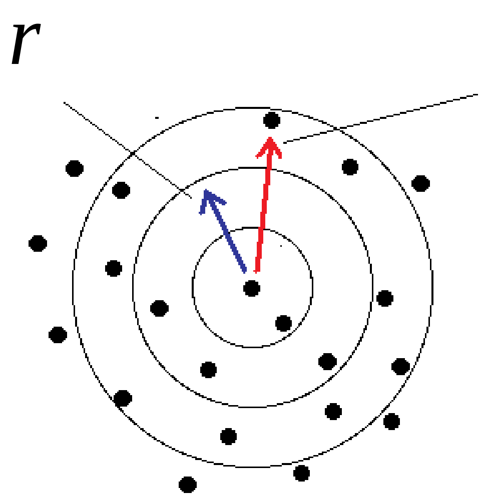




Lagrangian Coarse-Grained Concentration

- Coarse-Grained (Lagrangian) Concentration:

$$c_r(t) = \frac{N_L(r,t)}{\langle N(r,t)\rangle} \qquad c_r(t) = \frac{1}{\langle N(r,t)\rangle}$$



•The Eulerian and Lagrangian frames are equivalent with a weighting:

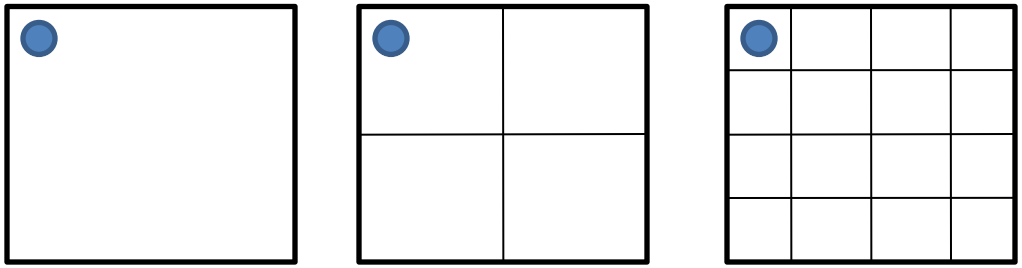
-P. Grassberger, Phys. Rev. Lett. ***50****, 346 (1983).*

$$\left\langle c_r^{p} \right\rangle = \left\langle n_r^{p+1} \right\rangle \qquad \Pi(c_r) = \Pi(n_r)\,n_r$$

•Easier to obtain accurate results in Lagrangian frame, and the two frames have been checked to be equivalent.

•The moments evolve to a statistically steady state in approx. 1 s (2 LETTs). All analysis done on steady-state distributions!

IV. Concentration Statistics



$\langle n_r{}^2 \rangle = 1$ $\quad \langle n_r{}^2 \rangle = 4$ $\quad \langle n_r{}^2 \rangle = 16$

$$\langle n_r{}^2 \rangle \sim r^{-\alpha_2}$$

$$\langle c_r \rangle \sim r^{-\alpha_2}$$

Point: $\alpha_2 = 2$

Line: $\alpha_2 = 1$

Surface: $\alpha_2 = 0$

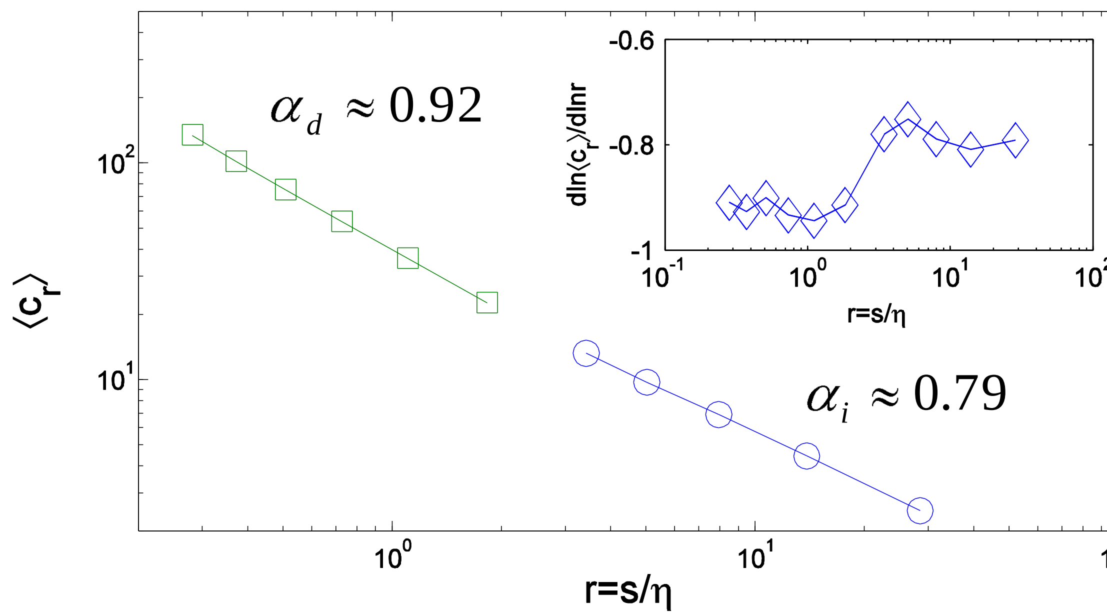
-For fractal distributions:

$$\langle n_r{}^m \rangle \propto r^{-\alpha_m}$$

α_m is a non-interger, “Strange Attractor”

-Result of 2 scaling regimes in inertial/dissipative ranges consistent with:

A. Pumir, Phys. Rev. E. 77, 066304 (2008).



Dissipative Range Concentration PDF

-For power-law distributions: $P(x) \propto x^{-\beta}$

$\Pi(c_r) \propto c_r$ $\quad \Pi(n_r) \propto const.$

$$c_r(t) = \frac{1}{\langle N(r,t) \rangle}$$

$$\langle N(r,t) \rangle = N_t (s/L)^2$$

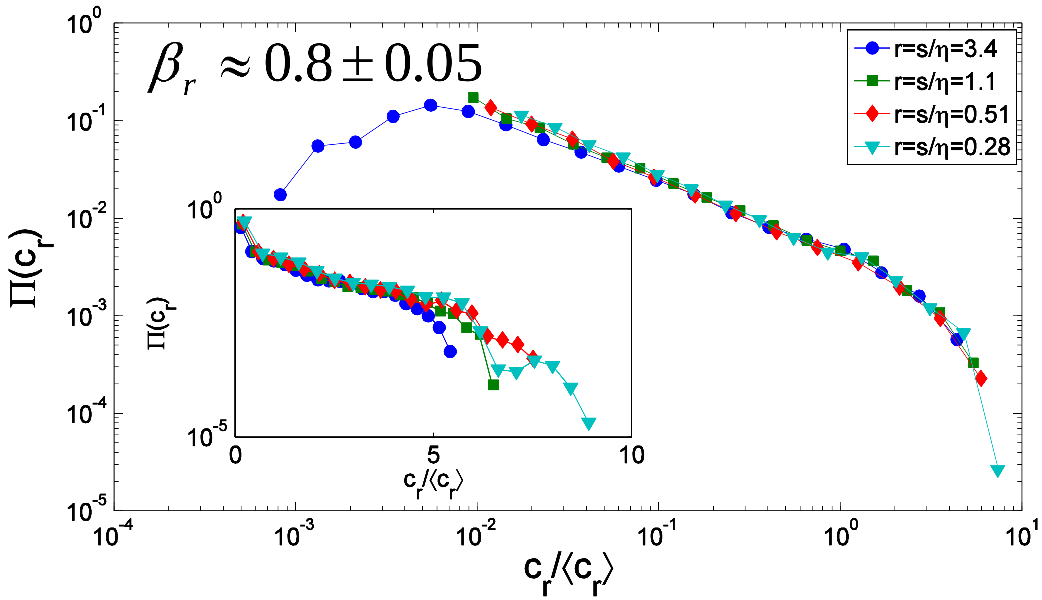
-At large c_r, observed fall-off is faster than algebraic.

E. Balkovsky, arXiv.chaodynp. 9912027 (1999).

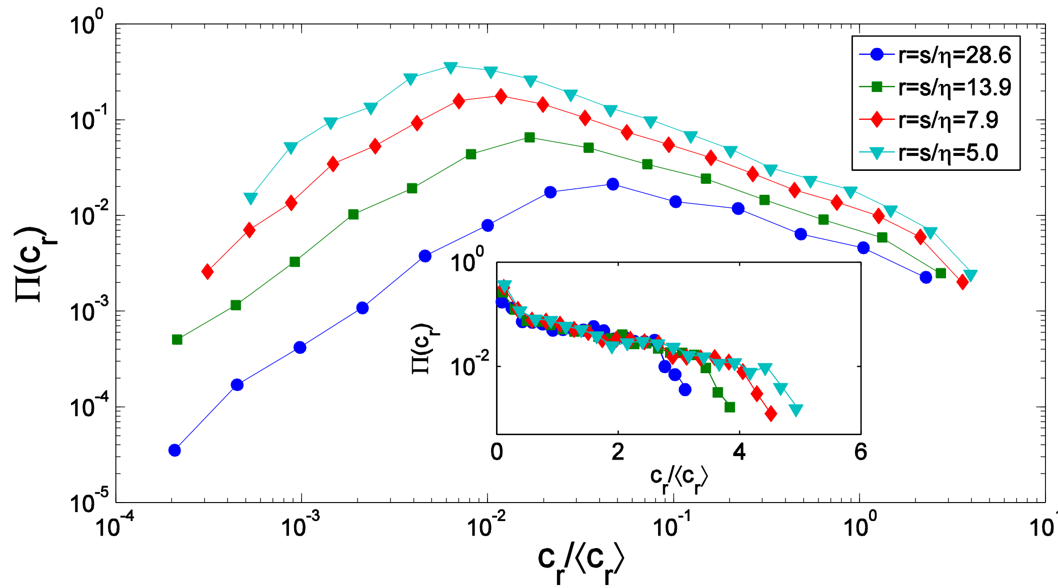
$\Pi(n_r) \sim n_r^{-\beta} \qquad \beta < 1$

Encountered in many different fields:

M.E.J. Newman, Contemporary Phys. ***46****, 323 (2005).*



Inertial Range Concentration PDF



-Inertial range results qualitatively consistent with:

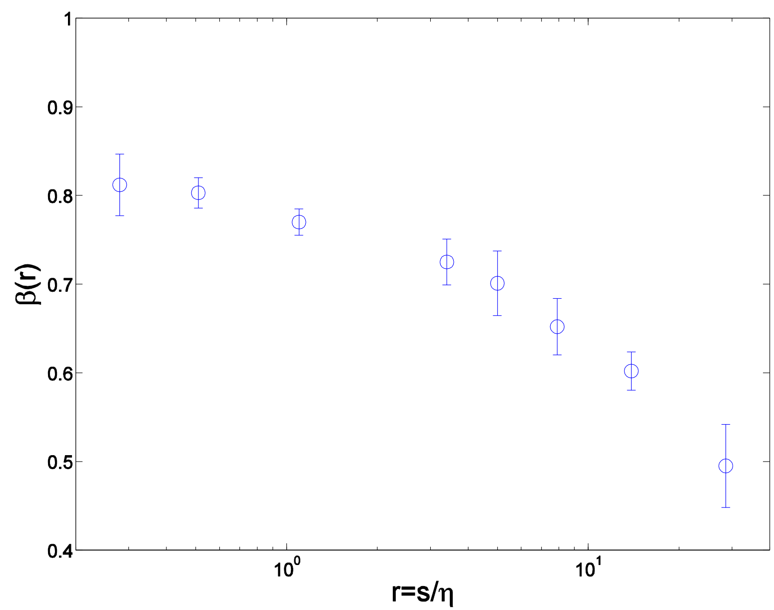
A. Pumir, Phys. Rev. E. 77, 066304 (2008).

-Exponent of power-law behavior varies with scale *r*

-For large cr, observed fall-off is again faster than algebraic.

-Very small cr wing is observed with: $\Pi(c_r) \propto c_r$

$$\Pi(n_r) \propto const.$$



VI. Discussion: Multi-Fractal Distributions

Imagine we have scaling of the concentration moment(s):

$$\left\langle c_r^p \right\rangle \propto r^{-\alpha_p}$$

A multifractal is defined as:

$$\alpha_{p+1} \neq p\alpha_p$$

Consider an Eulerian concentration PDF:

$$\Pi(n_r) \neq f(r)$$

One can then show, if one moment has scaling:

$$\left\langle n_r^p \right\rangle \propto r^{-\alpha_p}$$

The other moments are related:

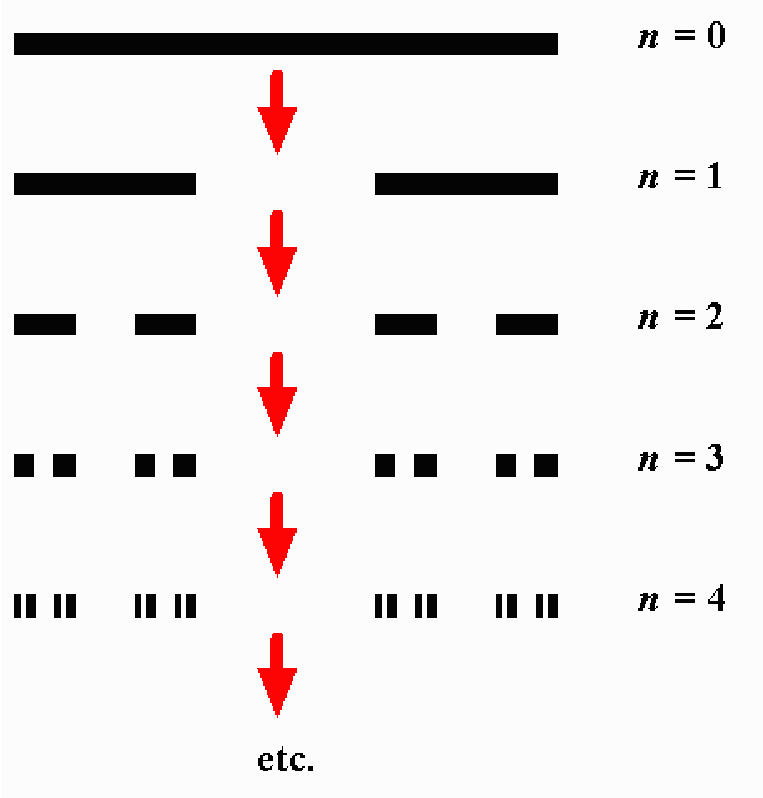
$$\left\langle n_r^{2p} \right\rangle \propto r^{-p\alpha_p}$$

Alternatively, if the PDF *does* depend on *r*:

$$\Pi(n_r) = f(r)$$

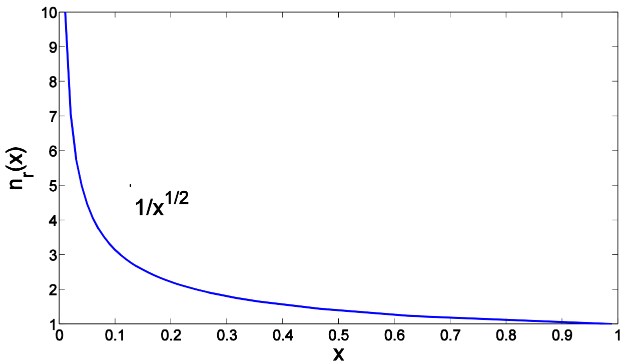
And one moment has scaling, then:

$$\alpha_{p+1} \neq p\alpha_1$$



$$\left\langle c_r^p \right\rangle = \left\langle n_r^{p+1} \right\rangle$$

Caustics



$$x = x_0^2 \qquad n_r(x) = \frac{1}{x^{1/2}}$$

$$\Pi(n_r) \sim {n_r}^{-3}$$

-Interesting case is Burger's map (shocks): $x = x_0 + t(-x_0 + x_0^3)$

-Which produces two singularities: $\Pi(n_r) \sim {n_r}^{-3} \quad \Pi(n_r) \sim {n_r}^{-2}$

Observed in: *M. Wilkinson, Europhys. Lett.* ***71****, 186 (2005) and P. Denissenko, Phys. Rev. Lett.* ***97****, 244501 (2006).*

-This is likely NOT what is seen in this experiment:

$$\Pi(n_r) \sim {n_r}^{-\beta} \quad \beta = f(r) \quad \beta < 2$$

Particles follow fluid flow "exactly", surface flow governed by 3D Navier-Stokes.

Major results of this work:

- The first moment of the (Lagrangian) coarse-grained concentration exhibits unique scale-free behavior in the inertial and dissipative ranges.
- Particles concentrate on multifractal in *both* inertial and dissipative ranges.
- The PDFs of the coarse-grained concentration evolve quantitatively differently in the inertial and dissipative ranges.

-Results in dissipative range in (qualitative) agreement with:

J. Bec, Phys. Rev. Lett. 92, 224501 (2004).

E. Balkovsky, arXiv.chaodynp. 9912027 (1999).

A. Pumir, Phys. Rev. E. 77, 066304 (2008).

-Results in inertial range in (qualitative) agreement with:

A. Pumir, Phys. Rev. E. 77, 066304 (2008).

-Results most likely NOT due to caustics, as in:

M. Wilkinson, Europhys. Lett. ***71****, 186 (2005)*

P. Denissenko, Phys. Rev. Lett. ***97****, 244501 (2006).*

VI. Conclusion

- Floating particles on turbulent surface cluster into multi-fractal distributions in both inertial and dissipative ranges.
- Effect <u>not</u> due to surface waves or inertia.
- Likely cause is <u>not</u> caustics.
- Power-law distribution describes clustering AND expulsion, which requires better understanding of surface flow.

Questions?

Collaborator’s (Dr.’s W.I. Goldburg, M.M. Bandi, and A. Pumir) would like to thank G. Falkovich and K. Gawedzki for very helpful discussions. Funding was provided by the US National Science Foundation grant # DMR-0604477 and by the French ANR (contract DSPET) and by IDRIS. This work was partially carried out under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396. Dr. A. Pumir thanks the French ANR (contract DSPET), and IDRIS for support.