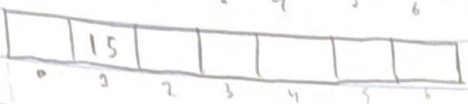
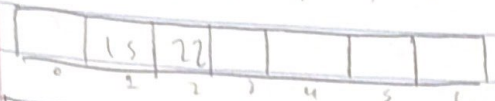
Part A

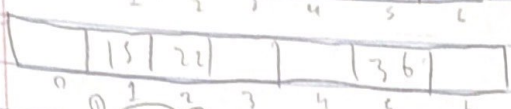
- Initialize hash table



- Insert 15 at 1, as 15%7 = 1



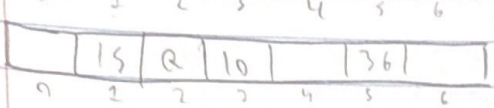
- Insert 22 at 2, as 22%7 = 1 is taken, so we do $22\%7 + 1^2 = 2$



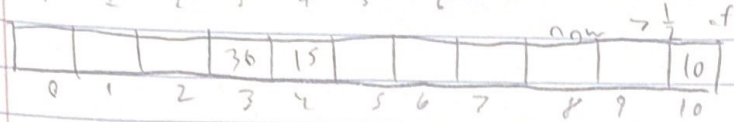
- Insert 36 at 5, as 36%7 = 1 is taken and so is $36\%7 + 1^2 = 2$, so we do $36\%7 + 2^2 = 5$

- Remove 22 by first looking at 22%7 = 1, then keep looking at $22\%7 + 1^2 = 2$. Put R in place of 22.

- Start looking for 36 at 36%7 = 1, then check at $36\%2 + 1^2 = 2$, then finally check at $36\%7 + 2^2$. Sequence: 1, 2, 5. Result: 36

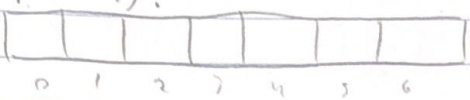


- Insert 10 at 10%7 = 3, but load factor is now $\frac{1}{2}$ of 7, so we resize & rehash

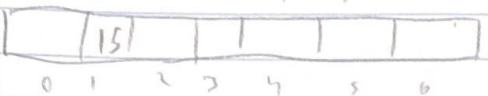


- 15%11 = 4, 10%11 = 10, 36%11 = 3, the R gets removed when we resize

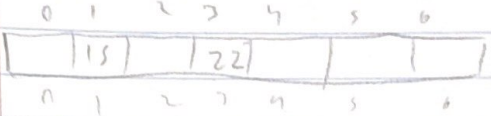
Part B:



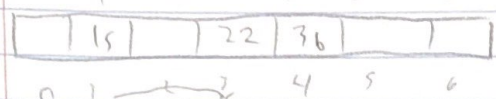
- Initialize hash table



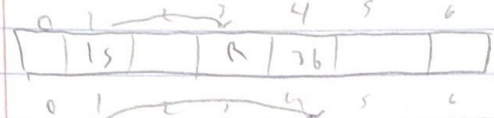
- Insert 15 at 1, as 15%7 = 1



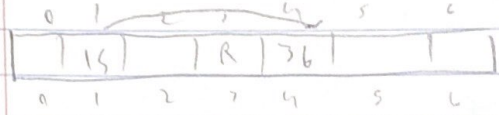
- Insert 22 at 3 as 22%7 = 1 is taken, so we insert at $22\%7 + 3 - (22\%3) = 3$



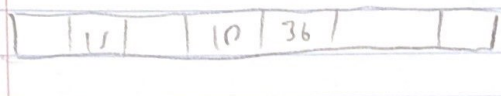
- Insert 36 at 4 as 36%7 = 1 is taken, so we insert at $36\%7 + 3 - (36\%3) = 4$



- Remove 22 by first looking at 22%7 = 1, then look at $22\%7 + 3 - (22\%3) = 3$ and find 22.



- Find 36 by first looking at 36%7 = 1, then look at $36\%7 + 3 - (36\%3) = 4$. Sequence: 1, 4 Result: 36



- Insert 10 by inserting at 10%7 = 3 and overwrite R.

Part C

For a single hash function, the probability that a bit is not set to one is $1 - 1/m$. Since we have three hash functions, this probability is $(1 - 1/m)^3$. However, since each spot has three hash functions and since we've already visited $2m/3$, the remaining $m/3$ spots must be accounted for: $(1 - 1/m)^{3m/3}$, which simplifies again to $(1 - 1/m)^m$. Since we search for 27 items, our final probability for false positive is:

$$\left((1 - \frac{1}{m})^m\right)^{27}$$

Problem 3:

1. For my large test I used Richard the third by Shakespeare, for my moderate tests one is a short story called How to Tell a True War Story by Tim O'Brien, and the other is a uniformly random text file.

2. For the large test my cache had size of 250, for both my moderate tests my cache had size of 25.

3. The large test had 173197 total rotations, the obrien test had 25059 total rotations, and the random test had 25411 total rotations.

4. The large test had 29,278 words, the obrien test had 5070 words and the random test had 5000 words.

5. The average for the large test was 5.92, for obrien it was 4.94, and for the random test the average was 5.08.

6. For the large test I removed 26776 items, for obrien I removed 4646 items, and for the random test I removed 4962 items.

7. The difference between the two moderate tests had no major differences.

8. It was interesting that the moderate cases weren't that different considering one is a coherent story and one is complete gibberish. This helps demonstrate the nature of how caches work.