CS 475 Machine Learning: Homework 6
Graphical Models 2

Due: Friday December 4, 2015, 11:59pm

50 Points Total Version 1.0

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1 Analytical (20 points)

1. (10 points) Causal Effect Identification Use do-calculus to give an identifying expression for $p(x_5 \mid \mathrm{do}(x_3))$ in terms of $p(x_1, x_2, x_3, x_4, x_5)$ in a hidden variable causal model in Fig. 1. Note: your expression is not allowed to refer to h_1, h_2, h_3 as those variables are not observed.

Ans:

$P(X_5|do(x_3))$
$=^p \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5, x_1, x_2, x_4|do(x_3))$
$=^c \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, x_4, do(x_3))P(x_1, x_2, x_4|do(x_3))$
$=^c \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, x_4, do(x_3))P(x_4|x_1, x_2, do(x_3))P(x_1, x_2|do(x_3))$
$=^c \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, x_4, do(x_3))P(x_4|x_1, x_2, do(x_3))P(x_2|x_1, do(x_3))P(x_1|do(x_3))$
$=^3 \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, x_4, do(x_3))P(x_4|x_1, x_2, do(x_3))P(x_2|x_1, do(x_3))P(x_1)$
$=^2 \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, x_4, do(x_3))P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^2 \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, do(x_4), do(x_3))P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^3 \sum_{x_1}\sum_{x_2}\sum_{x_4} P(X_5|x_1, x_2, do(x_4))P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^{\text{add } x_3'} \sum_{x_1}\sum_{x_2}\sum_{x_4}\sum_{x_3'} P(x_5, x_3'|x_1, x_2, do(x_4))P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^c \sum_{x_1}\sum_{x_2}\sum_{x_4}\sum_{x_3'} P(X_5|x_3', x_1, x_2, do(x_4))P(x_3'|x_1, x_2, do(x_4))P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^2 \sum_{x_1}\sum_{x_2}\sum_{x_4}\sum_{x_3'} P(X_5|x_3', x_1, x_2, x_4)P(x_3'|x_1, x_2, do(x_4))P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^3 \sum_{x_1}\sum_{x_2}\sum_{x_4}\sum_{x_3'} P(X_5|x_3', x_1, x_2, x_4)P(x_3'|x_1, x_2)P(x_4|x_1, x_2, x_3)P(x_2|x_1, do(x_3))P(x_1)$
$=^3 \sum_{x_1}\sum_{x_2}\sum_{x_4}\sum_{x_3'} P(X_5|x_3', x_1, x_2, x_4)P(x_3'|x_1, x_2)P(x_4|x_1, x_2, x_3)P(x_2|x_1)P(x_1)$

2. (10 points) Structure Learning We have data on four variables x_1, x_2, x_3, x_4, and running a set of hypothesis tests we learned that the following set of conditional independences hold in our data: $(x_2 \perp\!\!\!\perp x_3 \mid x_1)$, $(x_4 \perp\!\!\!\perp x_1 \mid x_2, x_3)$. Assuming our data was generated by a DAG model (Bayesian network), give the set of DAGs consistent with

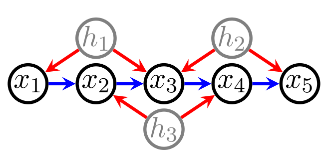
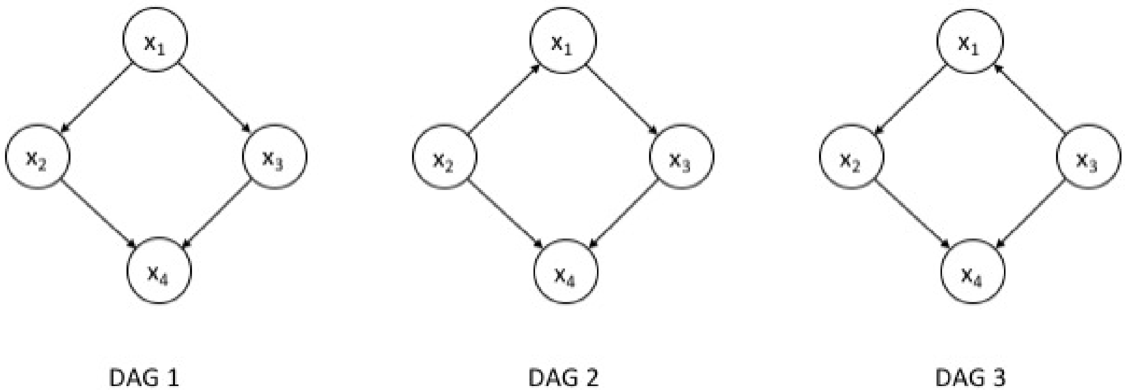


Figure 1: A hidden variable causal DAG.

the set of independences we see. Are there any causal effects with a single outcome (e.g. $p(x_i \mid \text{do}(\mathbf{x}))$, where $\mathbf{x} = \{x_j \mid i \neq j\}$) that could be identified by the same expression, *regardless* of which DAG in the set is the true DAG?

Ans:

The correponding DAGs are shown below.



The three DAG graphs above have the same skeleton and the same unshielded collider that is node X_4. Therefore, regardless which one is the real graph, they will result in a single outcome by the theorem "Verma and Pearl". They are observational equivalence