Machine Learning: Data to Models
Assignment 2a

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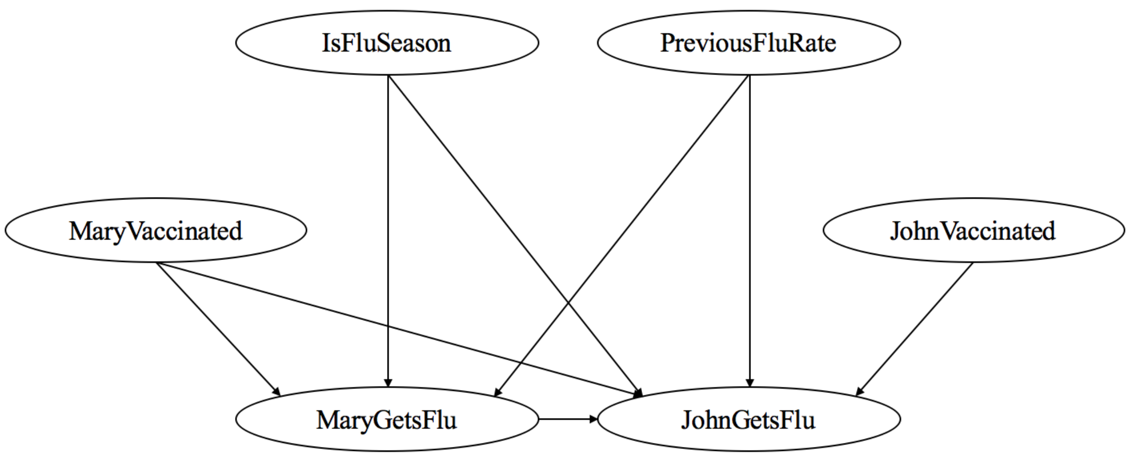
2.1 Network Manipulation [20 points]

2.1.1 Deliverables [14 points]

We applied ”Algorithm 3.2 Procedure to build a minimal I-map given an ordering” in Probabilistic Graphical Models by Daphne Koller and Nir Friedman to find the minimal I-map for this problem.



2.1.2 Analytical Question [6 points]

Ans:

In 2.1.1, we had applied the ”Algorithm 3.2 Procedure to build a minimal I-map given an ordering” to find the minimal I-map. For this problem, since a node E is eliminated from the original graph, the new independence set is a subset of the original independence set. For each independence rule that involves node E, we view the node E as an unobserved node. For the ordering of variables $\mathcal{X}$, we can still keep it the same except that we take the node E out of the order. Then we can apply the algorithm 3.2 again on the new graph using the original independence set while keeping the node E unobserved to find the minimal I-map.

2.2 Network Queries [16 points]

2.2.1. Analytical Questions

1. [8 points]

Proof: Let's prove it by contradiction. Assume altering Z's CPD cannot affect $P(X|\mathbf{Y})$, then there is no active trail from Z to X given $\mathbf{Y}$, i.e., $Z \perp X|\mathbf{Y}$. Since $\hat{Z}$ is a parent of Z, so $\hat{Z} \perp X|\mathbf{Y}$, i.e., there is no active trail from $\hat{Z}$ to X given $\mathbf{Y}$, which contradicts the condition of the statement. So the assumption doesn't hold and the statement is true. □

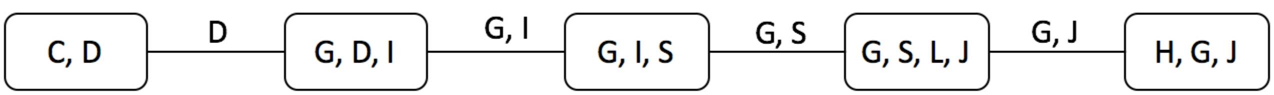
2. [8 points]

Proof: Let's prove it by contradiction. Assume altering Z's CPD can affect $P(X|\mathbf{Y})$, then there is an active trail from Z to X given $\mathbf{Y}$. Since $\hat{Z}$ is a parent of Z, we can find an active trail from $\hat{Z}$ to X given $\mathbf{Y}$ by going through $\hat{Z} \to Z$ then going through the active trail from Z to X given $\mathbf{Y}$. However, this contradicts the condition of the statement, so the assumption is false and the statement is true. □

2.3 Variable Elimination [12 points]

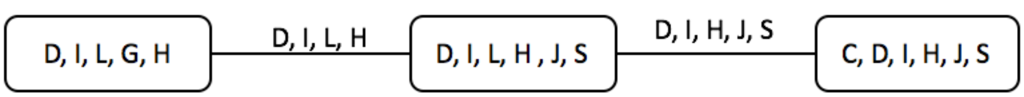
1. [8 points]

a.



A clique tree is valid if it satisfies the family preservation property and the running intersection property. For problem (a), the clique tree satisfies the preservation property because each factor ϕ is associated with a cluster C_i such that Scope[ϕ]$\subseteq C_i$. And each edge between a pair of clusters C_i and C_j is associated with a sepset (separation set) $S_{i,j} \subseteq C_i \cap C_j$. This clique also satisfied the running intersection property because for any variable X such that $X \in C_i$ and $X \in C_j$, X is also in every cluster in the path between C_i and C_j. So this clique tree is valid.

b.



It is easy to show that this clique tree in problem (b) also satisfies the family preservation property and the running intersection property so it's valid.

2. [4 points]
Solution:
$P(G,H,J) = P(H|G,J)\sum_{S,C}\sum_{L} P(J|L,S)P(L|G)\sum_{D} P(D|C)P(C)\cdot$
$\sum_{I} P(I)P(G|D,I)P(S|I)$