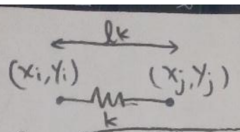
1.

1.



(i) Spring energy: $\underline{E}_s = \frac{1}{2} k\, l_k\, \varepsilon^2$, $\varepsilon = \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{l_k} - 1$

Spring force: $\underline{f}_s = (k\, l_k\, \varepsilon)\, \nabla \varepsilon$

bending energy: $\underline{E}_b = \frac{1}{2}\, \underbrace{(EI)}_{\text{Bending stiffness}}\, \frac{(K)^2}{l_k}$ curvature

bending force: $\underline{f}_b = \left(\frac{EI}{l_k} K\right) \nabla K$

(ii) inertia force: $\underline{f}_i = \frac{m}{\Delta t}\left(\frac{\underline{x}_{new} - \underline{x}_{old}}{\Delta t} - \underline{u}_{old}\right)$ $\underline{x}$: position, $\underline{u}$: velocity

$\underline{\underline{J}}_i = \frac{diag(m)}{\Delta t^2}$

$\underline{f} = \underline{f}_i + \underline{f}_s + \underline{f}_b - \underline{f}_{ext}$ external force

$\underline{\underline{J}} = \underline{\underline{J}}_i + \underline{\underline{J}}_s + \underline{\underline{J}}_b - \underline{\underline{J}}_{ext}$

For each timestep, update all free nodes $(\underline{x}_{new}, \underline{u}_{new})$ by $\underline{f}$, $\underline{\underline{J}}$:

$\underline{x}_{new}, \underline{u}_{new} = Int(\underline{x}_{old}, \underline{u}_{old}, DOF_{free})$

while err > eps:

$\underline{f}_{free} = \underline{f}\,[DOF_{free}]$ → index of free degree of freedom

$\underline{\underline{J}}_{free} = \underline{\underline{J}}\,[DOF_{free}]$

$\Delta \underline{x}_{free} = \underline{\underline{J}}_{free} \backslash \underline{f}_{free}$

$\underline{x}_{new}[DOF_{free}]$ -= $\Delta \underline{x}_{free}$

err = $\| \underline{f}_{free} \|_2$

$\underline{u}_{new} = \frac{\underline{x}_{new} - \underline{x}_{old}}{\Delta t}$

(iii) position of each nodes form the position vector

$$\underline{x} = \begin{array}{cl} & \text{index} \\ \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \\ x_N \\ y_N \end{bmatrix} & \begin{array}{l} \to 0 \\ \to 1 \\ \\ \\ \\ \\ \to 2N-1 \end{array} \end{array}$$

There are 6 DOF are fixed, so $\text{DOF}_{\text{free}} = [2, 3, \cdots, 2N-5]$

For each timestep before calculate $\underline{x}_{new}$, $\underline{u}_{new}$, we update fixed nodes

$t = 0 : \Delta t : t_f$

for $k = 1 : \text{len}(t)-1$

$\underline{x}_{old}[2N-2] = x_c[k] \rightarrow$ Control inputs

$\underline{x}_{old}[2N-1] = y_c[k] \nearrow$

$\underline{x}_{old}[2N-4] = x_c[k] - \Delta L \cos(\theta_c[k])$

$\underline{x}_{old}[2N-3] = y_c[k] - \Delta L \sin(\theta_c[k])$

$\underline{x}_{new}, \underline{u}_{new} = \text{Int}(\underline{x}_{old}, \underline{u}_{old}, \text{DOF}_{\text{free}})$

$\underline{x}_{old} = \underline{x}_{new}$, $\underline{u}_{old} = \underline{u}_{new}$.

(iv) Consider reference $\overbrace{\underline{r}_x[k], \underline{r}_y[k]}^{\text{at each time}}$, error e_x, e_y, accumulated error $e_{x,acc}$, $e_{y,acc}$.

$e_x = \underline{r}_x[0] - \underline{x}_{old}[2(\overbrace{N//2+1}^{M_i})-2]$

$e_y = \underline{r}_y[0] - \underline{x}_{old}[2(N//2+1)-1]$

$e_{x,acc} = e_x \Delta t$

$e_{y,acc} = e_y \Delta t$

for $k = 1 : \text{len}(t)-1$

$\theta_c[k] = \text{atan2}(\underline{x}_{old}[2N-1] - \underline{x}_{old}[2(M_i+1)-1], \underline{x}_{old}[2N-2] - \underline{x}_{old}[2(M_i+1)-2])$

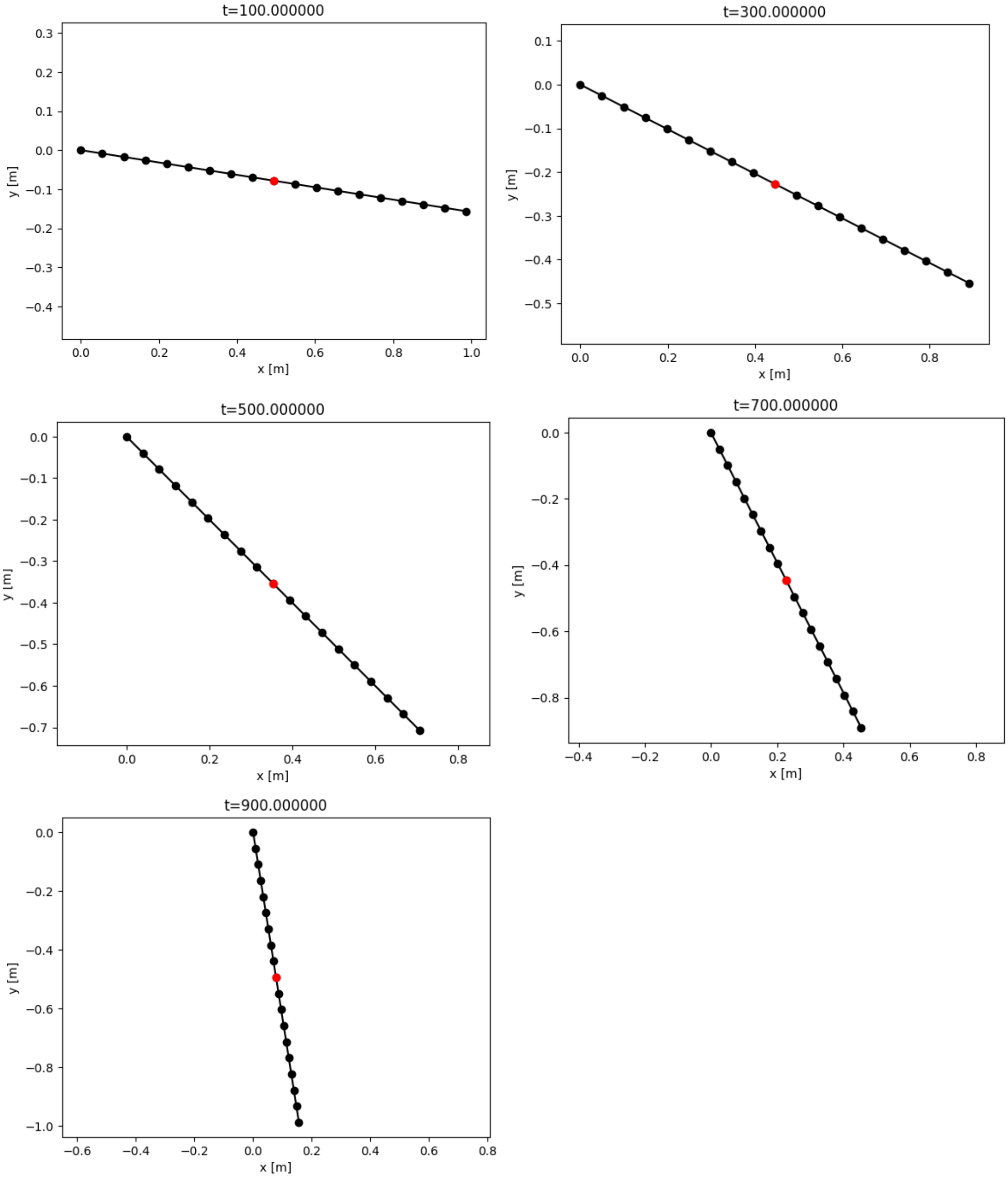
Controller

$$\boxed{\begin{aligned} x_c[k] &= x_c[k-1] + (k_{p,x} e_x + k_{i,x} e_{x,acc}) \\ y_c[k] &= y_c[k-1] + (k_{p,y} e_y + k_{i,y} e_{y,acc}) \end{aligned}}$$

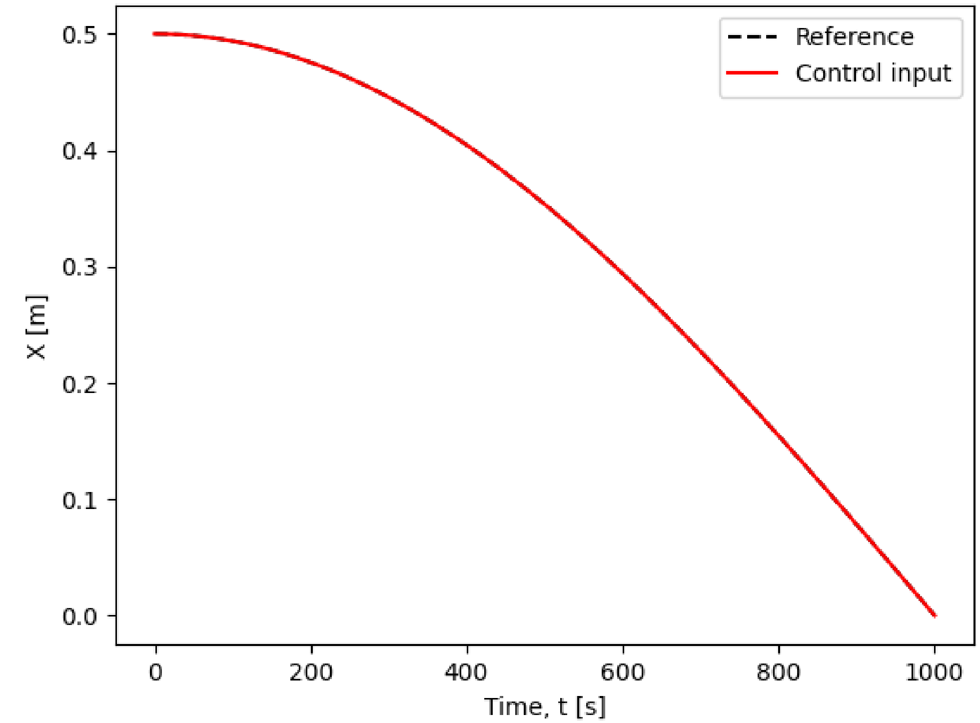
after getting $\underline{x}_{new}$: $e_x = \underline{r}_x[k] - \underline{x}_{new}[2M_i-2]$ $e_{x,acc} \mathrel{+}= e_x \cdot \Delta t$

$e_y = r_y[k] - \underline{x}_{new}[2M_i-1]$ $e_{y,acc} \mathrel{+}= e_y \Delta t$.

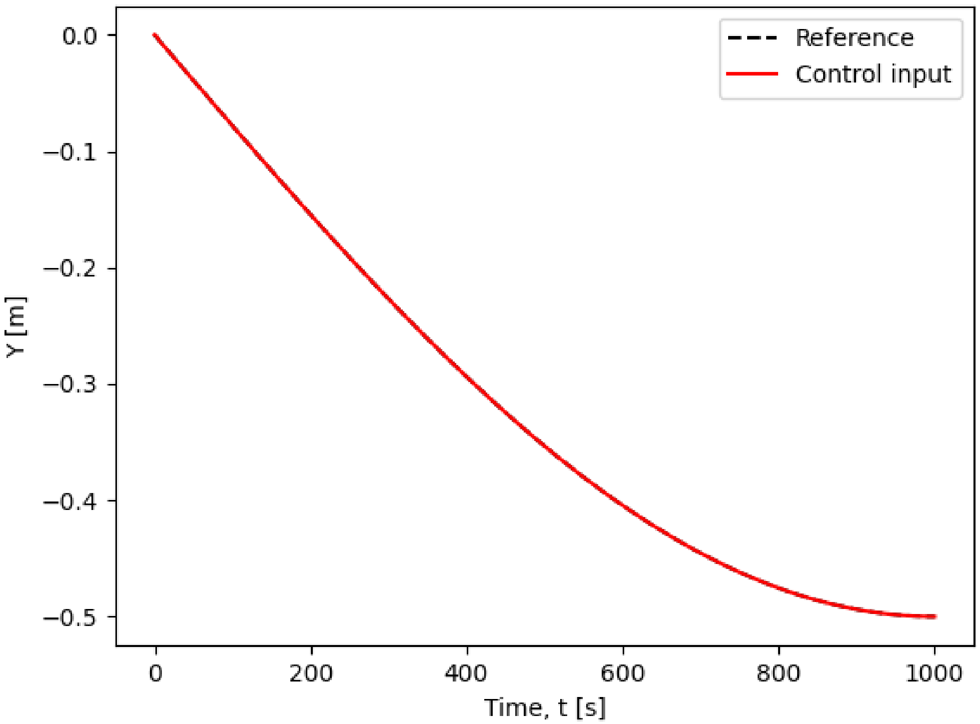
2.


t=100.000000
t=300.000000
t=500.000000
t=700.000000
t=900.000000
x [m]
y [m]

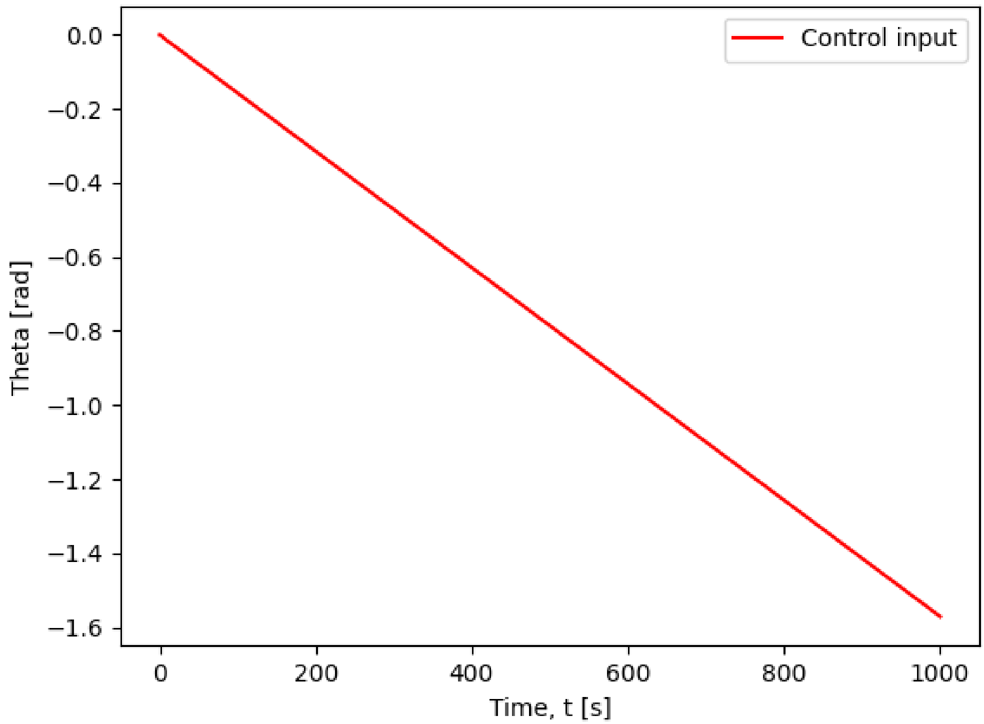
3.



Reference
Control input
X [m]
Time, t [s]



Reference
Control input
Y [m]
Time, t [s]



4. If any $x_c[k]$, $y_c[k]$, $\theta_c[k]$ is out of robot limits, set them to the robot limit (saturation).