

# Proofs of Induction

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**CS 131 induction giving you headaches? Interview coming up?  
Need some brush-up on proofs?**

**Here are some classic problems from my CS 131 course experience  
that can help you out.**

## Problems

### Problem 1: The Wandering Robot

Let's say I have an iRobot that needs to clean up my dorm room before my roommate comes back from class.

The robot starts at position  $(0,0)$  and can move at each step up or down 1 unit, left or right 1 unit, or diagonally 1 unit.

The possible states that the robot can move are at  $(1,1)$ ,  $(1,-1)$ ,  $(-1,1)$ , or  $(-1,-1)$  [the four unit quadrants].

For scaling purposes, assume each unit is a 10 ft, which is the length of a wall in my (nearly) square dorm room.

Use strong induction on the number of steps to prove that the robot can or cannot ever reach  $(1,0)$ , or reach the end of my room.

### Problem 2: Game Night at GSU

It's game night at GSU for BU orientation. For the incoming freshmen, they are required to show financial maturity (to avoid using up those precious Convenience Points) that requires the sharing of BU swag. So, here are the pricings for the BU swag:

Item	Offer
FitRec Shirt	4 dollar medium and 2 large
Terrier Shirts	5 dollar medium and 3 large

Assume as a visiting BU student, you only have a budget for 40 dollars. Use strong induction to show for the following:

**You can buy any  $n \geq 40$  using only 3 dollar medium shirts and 6 dollar large shirts.**

### **Problem 3: Fireworks Display**

To remember July 4th fondly, let's assume that the pyrotechnics are firing the fireworks around an L-shaped dock, which the boats are tied to the dock to avoid being swept away in the Charles River during high winds in the Boston Summer.

Assume we have the fireworks display set in a  $2^n$  by  $2^n$  grid, where  $n$  is the number of feet the fireworks can reach in radius. To avoid hitting the dock, and face financial and legal ramifications, we need to be careful not to hit the L-shaped structure.

**Show by strong induction, that the pyrotechnics can safely display the fireworks for all  $n \geq 0$ , if there exists a placement in the tiled breakdown of the grid with the dock located at the center.**

#### **Problem 4 (Challenge): Geneaology Project**

Read the following Wikipedia page to get a sense of structural induction:

[https://en.wikipedia.org/wiki/Structural\\_induction](https://en.wikipedia.org/wiki/Structural_induction)

We always wanted to know how far back our generations spanned throughout history. So, let's say you wanted to find the geneaology of your family / ancestors.

Following the rules shown in the Wikipedia link, show by induction (weak/strong/structural), that:

**An geneaology tree spanning  $k$  generations can shows at least  $2^k$  persons AND at most  $2^k - 1$  persons are shown in the tree.**

Hint: Use two induction proofs, and combine them in a biconditional proof (if  $p \leftarrow q$ , then  $q \rightarrow p$ ).

**Solutions coming soon!**