Glossary of Linear Algebra Terms

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basis for a subspace:
A basis for a subspace W is a set of vectors $\mathbf{v}1, \ldots, \mathbf{v}k$ in W such that:

1. $\mathbf{v}1, \ldots, \mathbf{v}k$ are linearly independent; and
2. $\mathbf{v}1, \ldots, \mathbf{v}k$ span W.

characteristic polynomial of a matrix:
The characteristic polynomial of a n by n matrix A is the polynomial in t given by the formula $\det(A - t*I)$.

column space of a matrix:
The column space of a matrix is the subspace spanned by the columns of the matrix considered as vectors. See also: row space.

consistent linear system:
A system of linear equations is consistent if it has at least one solution. See also: inconsistent.

defective matrix:
A matrix A is defective if A has an eigenvalue whose geometric multiplicity is less than its algebraic multiplicity.

diagonalizable matrix:
A matrix is diagonalizable if it is **dimension** of a subspace:
The dimension of a subspace W is the number of vectors in any basis of W. (If W is the subspace $\{\mathbf{0}\}$, we say that its dimension is 0.)

row **echelon form** of a matrix:
A matrix is in row echelon form if:

1. all rows that consist entirely of zeros are grouped together at the bottom of the matrix; and
2. the first (counting left to right) nonzero entry in each nonzero row appears in a column to the right of the first nonzero entry in the preceding row (if there is a preceding row).

reduced row **echelon form** of a matrix:
A matrix is in reduced row echelon form if:

1. the matrix is in row echelon form;
2. the first nonzero entry in each nonzero row is the number 1; and
3. the first nonzero entry in each nonzero row is the only nonzero entry in its column.

eigenspace of a matrix:
The eigenspace associated with the eigenvalue c of a matrix A is the null space of $A - c*I$.

eigenvalue of a matrix:
An eigenvalue of a n by n matrix A is a scalar c such that $A*\mathbf{x} = c*\mathbf{x}$ holds for some nonzero vector **x** (where **x** is an n-tuple). See also: eigenvector.

eigenvector of a matrix:
An eigenvector of a n by n matrix A is a nonzero vector **x** such that $A*\mathbf{x} = c*\mathbf{x}$ holds for some scalar c. See also: eigenvalue.

equivalent linear systems:
Two systems of linear equations in n unknowns are equivalent if they have the same set of solutions.

homogeneous linear system:
A system of linear equations $A*\mathbf{x} = \mathbf{b}$ is homogeneous if $\mathbf{b} = \mathbf{0}$.

inconsistent linear system:
A system of linear equations is inconsistent if it has no solutions. See also: consistent.

inverse of a matrix:
The matrix B is an inverse for the matrix A if $A*B = B*A = I$.

invertible matrix:
A matrix is invertible if it has an inverse.

least-squares solution of a linear system:
A least-squares solution to a system of linear equations $A*\mathbf{x} = \mathbf{b}$ is a vector **x** that minimizes the length of the vector $A*\mathbf{x} - \mathbf{b}$.

linear combination of vectors:
A vector **v** is a linear combination of the vectors $\mathbf{v}1, \ldots, \mathbf{v}k$ if there exist scalars $a1, \ldots, ak$ such that $\mathbf{v} = a1*\mathbf{v}1 + \ldots + ak*\mathbf{v}k$.

linearly dependent vectors:
The vectors $\mathbf{v}1, \ldots, \mathbf{v}k$ are linearly dependent if the equation $a1*\mathbf{v}1 + \ldots + ak*\mathbf{v}k = \mathbf{0}$ has a solution where not all the scalars $a1, \ldots, ak$ are zero.

linearly independent vectors:
The vectors $\mathbf{v}1, \ldots, \mathbf{v}k$ are linearly independent if the only solution to the equation $a1*\mathbf{v}1 + \ldots + ak*\mathbf{v}k = \mathbf{0}$ is the solution where all the scalars $a1, \ldots, ak$ are zero.

linear transformation :
A linear transformation from V to W is a function T from V to W such that:

1. $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all vectors **u** and **v** in V; and
2. $T(a*\mathbf{v}) = a*T(\mathbf{v})$ for all vectors **v** in V and all scalars a.

algebraic **multiplicity** of an eigenvalue:
The algebraic multiplicity of an eigenvalue c of a matrix A is the number of times the factor $(t\text{-}c)$ occurs in the characteristic polynomial of A.

geometric **multiplicity** of an eigenvalue:
The geometric multiplicity of an eigenvalue c of a matrix A is the dimension of the eigenspace of c.

nonsingular matrix:

An n by n matrix A is nonsingular if the only solution to the equation $A*\mathbf{x} = \mathbf{0}$ (where $\mathbf{x}$ is an n-tuple) is $\mathbf{x} = \mathbf{0}$. See also: singular.

null space of a matrix:
The null space of a m by n matrix A is the set of all n-tuples $\mathbf{x}$ such that $A*\mathbf{x} = \mathbf{0}$.

null space of a linear transformation:
The null space of a linear transformation T is the set of vectors $\mathbf{v}$ in its domain such that $T(\mathbf{v}) = \mathbf{0}$.

nullity of a matrix:
The nullity of a matrix is the dimension of its null space.

nullity of a linear transformation:
The nullity of a dimension of its null space.

orthogonal set of vectors:
A set of n-tuples is orthogonal if the dot product of any two of them is 0.

orthogonal matrix:
A matrix A is orthogonal if A is invertible and its **orthogonal** linear transformation:
A linear transformation T from V to W is orthogonal if $T(\mathbf{v})$ has the same length as $\mathbf{v}$ for all vectors $\mathbf{v}$ in V.

orthonormal set of vectors:
A set of n-tuples is orthonormal if it is orthogonal and each vector has length 1.

range of a matrix:
The range of a m by n matrix A is the set of all m-tuples $A*\mathbf{x}$, where $\mathbf{x}$ is any n-tuple.

range of a linear transformation:
The range of a linear transformation T is the set of all vectors $T(\mathbf{v})$, where $\mathbf{v}$ is any vector in its domain.

rank of a matrix:
The rank of a matrix is the number of nonzero rows in any row equivalent matrix that is in row echelon form.

rank of a linear transformation:
The rank of a linear transformation (and hence of any matrix regarded as a linear transformation) is the dimension of its range. Note: A theorem tells us that the two definitions of rank of a matrix are equivalent.

row equivalent matrices:
Two matrices are row equivalent if one can be obtained from the other by a sequence of elementary **row operations**:
The elementary row operations performed on a matrix are:

- interchange two rows;
- multiply a row by a nonzero scalar;
- add a constant multiple of one row to another.

row space of a matrix:
The row space of a matrix is the subspace spanned by the rows of the matrix considered as vectors. See also: **similar** matrices:

Matrices A and B are similar if there is a square nonsingular matrix S such that $S^(-1)*A*S = B$.

singular matrix:
An n by n matrix A is singular if the equation $A*\mathbf{x} = \mathbf{0}$ (where $\mathbf{x}$ is an n-tuple) has a nonzero solution for $\mathbf{x}$. See also: nonsingular.

span of a set of vectors:
The span of the vectors $\mathbf{v}1, \ldots, \mathbf{v}k$ is the subspace V consisting of all linear combinations of $\mathbf{v}1, \ldots, \mathbf{v}k$. One also says that the subspace V is spanned by the vectors $\mathbf{v}1, \ldots, \mathbf{v}k$ and that these vectors span V.

subspace:
A subset W of n-space is a subspace if:

1. the zero vector is in W;
2. $\mathbf{x}+\mathbf{y}$ is in W whenever $\mathbf{x}$ and $\mathbf{y}$ are in W; and
3. $a*\mathbf{x}$ is in W whenever $\mathbf{x}$ is in W and a is any scalar.

symmetric matrix:
A matrix is symmetric if it equals its transpose.