A Multi-Phase Direct Collocation Program for Descent Trajectory Optimization

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1 Overview

1.1 Trajectory Leg, Collocation Node

One phase represents on trajectory leg. Each trajectory leg is comprised of a set of collocation points (or collocation nodes), and a total length of time for a defined dynamical system to progress through all of the collocation nodes along the leg.

Each collocation point consists of a state, $\vec{X}$, and control, $\vec{U}$. At each node k with state $\vec{X}_k$ and control $\vec{U}_k$ along a trajectory leg, the system experiences a dynamical response, $\dot{\vec{X}}_k$:

$$\dot{\vec{X}}_k = \vec{f}_k(\vec{X}_k, \vec{U}_k) = \vec{f}_k(\vec{C}_k) \tag{1}$$

If a collocation point at node k (of N total nodes) is represented by $\vec{C}_k = [\vec{X}_k, \vec{U}_k]$, a trajectory leg $\vec{L}_i$, as a component of an entire trajectory from leg $i = 0 \dots P$, can be represented programmatically as:

$$\vec{L}_i = \left\{T_i, \left[\vec{C}_1, \vec{C}_2, \dots, \vec{C}_N\right]\right\} \tag{2}$$

1.2 Trajectory Leg Constraints

1.2.1 Boundary Constraints

Boundary constraints are represented at the start of each trajectory leg and at the end of each trajectory leg:

$$\vec{h}_{0_i} = \vec{h}_{0_i}(\vec{L}_1, \vec{L}_2, \dots, \vec{L}_P) \qquad i \in \{1, ..., P\} \tag{3}$$

$$\vec{h}_{f_i} = \vec{h}_{f_i}(\vec{L}_1, \vec{L}_2, \dots, \vec{L}_P) \qquad i \in \{1, ..., P\} \tag{4}$$

1.2.2 Defect Constraints

Using a Hermite-Simpson transcription, for a specific trajectory leg i, a defect constraint $\vec{\Delta}_{k\to k+1}$, from collocation node $\vec{C}_k = [\vec{X}_k, \vec{U}_k]$ to $\vec{C}_{k+1} = [\vec{X}_{k+1}, \vec{U}_{k+1}]$, with a fixed time delta h_i in between each node, is represented as:

$$\vec{\Delta}_{k\to k+1} = \vec{X}_k - \vec{X}_{k+1} + \frac{h_i}{6}\left[f(\vec{C}_k) + 4f(\vec{C}_c) + f(\vec{C}_{k+1})\right] \tag{5}$$

$$\vec{C}_c = [\vec{X}_c, \vec{U}_c] \tag{6}$$

$$\vec{U}_c = \frac{1}{2}\left(\vec{U}_k + \vec{U}_{k+1}\right) \tag{7}$$

$$\vec{X}_c = \frac{1}{2}\left(\vec{X}_k + \vec{X}_{k+1}\right) + \frac{h_i}{8}\left[f(\vec{C}_k) - f(\vec{C}_{k+1})\right] \tag{8}$$

1.2.3 Path Constraints

Path constraints are typically difficult to implement for a collocation-type transcription because of the requirement to interpolate dynamics and check constraints through non-node continuous locations. For that reason, the ability to implement path constraints is as of now available to the user for each collocation node, but the user should be aware that these path constraints are only evaluated at collocation nodes, rather than at each continuous state along a trajectory.

Path constraints are represented at each collocation node k in a specific trajectory leg as:

$$\vec{g}_k = \vec{g}_k(\vec{X}_k, \vec{U}_k) = \vec{g}_k(\vec{C}_k) \qquad k \in \{1, ..., N\} \tag{9}$$

Because the Hermite-Simpson transcription still represents a linear control interpolation between control $\vec{U}_k$ and $\vec{U}_{k+1}$, it is safe to bound control path, such as limiting a magnitude of a thrust vector direction to unity. For path constraints of state, an example of a well-behaved path constraint would be an altitude constraint on a lunar descent trajectory, where it is known that the most optimal trajectory typically does not involve close dips to low altitudes followed by a return to a higher altitude. The author plans to eventually study state path constraints in more detail.

2 User Required Inputs

2.1 Required User Function Definitions

A user must specify the following, for each trajectory leg:

- $\vec{X}$, the structure of the state vector,
- $\vec{U}$, the structure of the control vector,
- $\vec{f}(\vec{X}, \vec{U})$, the system dynamics of a collocation node,
- $J(\vec{L}_1, \ldots, \vec{L}_P)$, objective as a function of all trajectory legs,
- $\vec{g}_0(\vec{L}_1, \ldots, \vec{L}_P)$, initial boundary constraint of this specific leg as a function of all trajectory legs,
- $\vec{g}_f(\vec{L}_1, \ldots, \vec{L}_P)$, final boundary constraint of this specific leg as a function of all trajectory legs,
- $\vec{h}_e(\vec{L}_1, \ldots, \vec{L}_P)$, the path equality constraints at each node in the trajectory leg, including:
  - Thrust direction vector unity magnitude constraints,
- $\vec{h}_i(\vec{L}_1, \ldots, \vec{L}_P)$, the path inequality constraints at each node in the trajectory leg, including:
  - Throttle magnitude constraints,
  - Altitude restrictions,
- $t_{lb} \le T \le t_{ub}$, time bound for specific trajectory leg,
- $\vec{X}_{lb} \le \vec{X} \le \vec{X}_{ub}$, state bound for each collocation node,
- $\vec{U}_{lb} \le \vec{U} \le \vec{U}_{ub}$, control bound for each collocation node,
- Partials:
  - $X = \frac{\partial J}{\partial(\vec{L}_1, \ldots, \vec{L}_P)}$, objective wrt. trajectory,
  - $A = \frac{\partial \vec{f}}{\partial \vec{X}}$, dynamics wrt. state,
  - $B = \frac{\partial \vec{f}}{\partial \vec{U}}$, dynamics wrt. control,
  - $C_0 = \frac{\partial \vec{g}_0}{\partial(\vec{L}_1, \ldots, \vec{L}_P)}$, initial boundary constraint wrt. trajectory,
  - $C_f = \frac{\partial \vec{g}_f}{\partial(\vec{L}_1, \ldots, \vec{L}_P)}$, final boundary constraint wrt. trajectory,
  - $D_e = \frac{\partial \vec{h}}{\partial(\vec{L}_1, \ldots, \vec{L}_P)}$, path equality constraint wrt. trajectory,
  - $D_i = \frac{\partial \vec{h}}{\partial(\vec{L}_1, \ldots, \vec{L}_P)}$, path inequality constraint wrt. trajectory,

2.2 Decision Vector Structure

Let's configure an example of a burn-coast-burn ($P = 3$) lunar descent trajectory. Each leg will have $N = 15$ collocation nodes. Each collocation node will consist of a state vector of 7 elements (position, velocity, mass), and a control vector of 4 elements (thrust unit direction, throttle). Each collocation node will be combined from the state and control vector in the following fashion:

$$\vec{C}_k = \underset{(7)\times(1)}{\begin{bmatrix} \vec{R} \\ \vec{V} \\ m \\ \vec{u} \\ \eta \end{bmatrix}} \tag{10}$$

Note that the constraint to normalize the control vector direction $\vec{u} = \hat{u}$ will be set in the path constraints. An entire trajectory leg i of P with N collocation points is constructed as:

$$\vec{L}_i = \underset{(1+7N_i)\times(1)}{\begin{bmatrix} T \\ \vec{C}_1 \\ \vec{C}_2 \\ \vdots \\ \vec{C}_N \end{bmatrix}} \tag{11}$$

The entire decision vector, i.e. the entire trajectory, is then configured as:

$$\vec{O} = \underset{(P+7P(N_1+...+N_P))\times(1)}{\begin{bmatrix} \vec{L}_1 \\ \vec{L}_2 \\ \vdots \\ \vec{L}_P \end{bmatrix}} \tag{12}$$

The system dynamics of a collocation node:

$$\vec{f}(\vec{C}) = \vec{f}(\vec{X}, \vec{U}) = \underset{(7)\times(1)}{\begin{bmatrix} \vec{V} \\ \frac{T_m \eta}{m} \hat{u} + \frac{\mu}{R^3} \vec{R} \\ \frac{T_m \eta}{g_0 I_{sp}} \end{bmatrix}} \tag{13}$$

The entire fitness and constraint vector for one specific trajectory leg:

$$\vec{F} = \underset{()\times(7)}{\begin{bmatrix} J(\vec{L}_1, \ldots, \vec{L}_P) \\ \vec{g}_0(\vec{L}_1, \ldots, \vec{L}_P) \\ \vec{g}_f(\vec{L}_1, \ldots, \vec{L}_P) \\ \vec{\Delta}(\vec{L}_1, \ldots, \vec{L}_P) \\ \vec{h}_e(\vec{L}_1, \ldots, \vec{L}_P) \\ \vec{h}_i(\vec{L}_1, \ldots, \vec{L}_P) \end{bmatrix}} \tag{14}$$