NE770 HW 1

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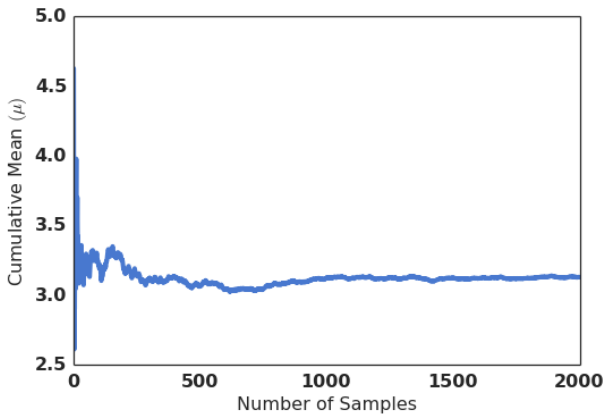
The code accompanying this report can be found at . The source for this report and for generation of the plots can be viewed here.

Problem 1

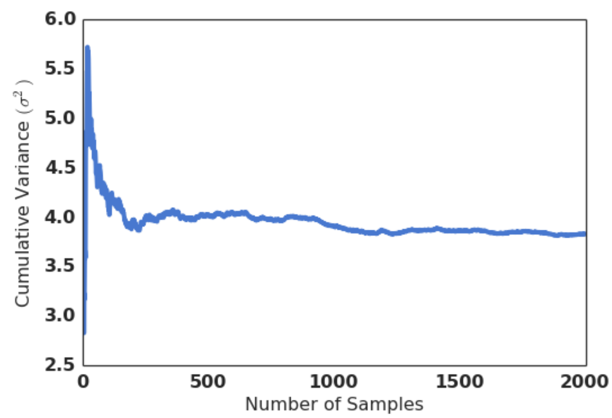
This problem is to evaluate the following integral via simple Monte Carlo.

$$\int_0^\pi \theta \sin(\theta) d\theta \tag{1}$$

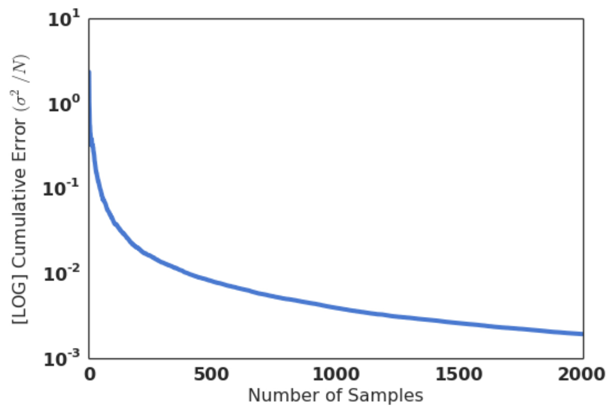
Using 2000 samples, the Monte Carlo estimate for the integral in eq. (1) is 3.1261762, with an error variance of $\hat{\sigma}^2 = \sigma^2/N = .0019117841$. The actual value for eq. (1) evaluates to $\pi \approx 3.1415927$, which is an error of $-.01541650$ and is less than $\hat{\sigma} = .0437239534$. Figure 1 shows the mean, variance and error variance for the Monte Carlo estimate of eq. (1) as a function of the number of samples N. It can be seen in figs. 1a and 1b that the estimates for the integral and the variances stabilize around $N > 500$, while fig. 1c indicates that the solution is accurate to two significant figures starting around $N > 200$.



(a) Cumulative mean



(b) Cumulative variance



(c) Cumulative error (log scale)

Figure 1: Plots of μ, σ^2 and σ^2/N with increasing number of samples

Problem 2

Here, we attempt to sample the following functions:

$$\chi(E) = .453\exp(-E/.965)\sinh(2.29)^{\frac{1}{2}} \tag{2}$$

$$\nu(E) = \begin{cases} 2.42 + .066E, & E \leq 1 \\ 2.349 + .15E, & E > 1 \end{cases} \tag{3}$$

Equation (2) describes the energy spectrum of fission neutrons and is a probability density, while eq. (3) gives the average number of neutrons emitted per fission initiated at energy E and is proportional to a probability density. Both are defined for the half-open interval $E \in [0, \infty)$; we seek to determine the mean fission neutron energy and the mean number of neutrons released per fission. By definition, the mean μ for a given (potentially unnormalized) probability distribution $p(x)$ with support $\mathcal{D}$ can be computed via the relation:

$$\mu_x = \frac{\int_{\mathcal{D}} x \cdot p(x) dx}{\int_{\mathcal{D}} p(x) dx} \tag{4}$$

We first apply eq. (4) to eqs. (2) and (3) and evaluate numerically using an adaptive Gauss-Kronrod quadrature implemented in QUADPACK. This produces the values in table 1, which serve as reference values to compare the sampling methods against.

	μ	Quadrature error
$\chi(E)$	1.9819186	2.2991644E − 8
$\nu(E)$	5.3985461	2.4323015E − 9

Table 1: Means for χ and ν evaluated by quadrature