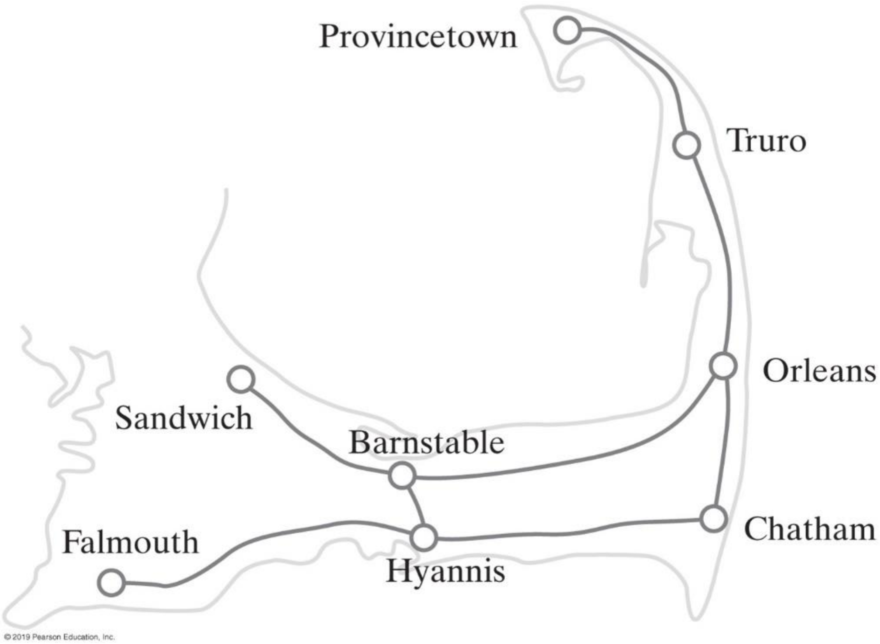
Class 12 – Graphs

CSIS 3475 Data Structures and Algorithms

Graphs

- A road map is a graph
- Definition of a graph:
    - A collection of distinct vertices and distinct edges
- In the map to the right
    - The circles are vertices or nodes
    - The lines are called edges
- A subgraph is a portion of a graph that is itself a graph

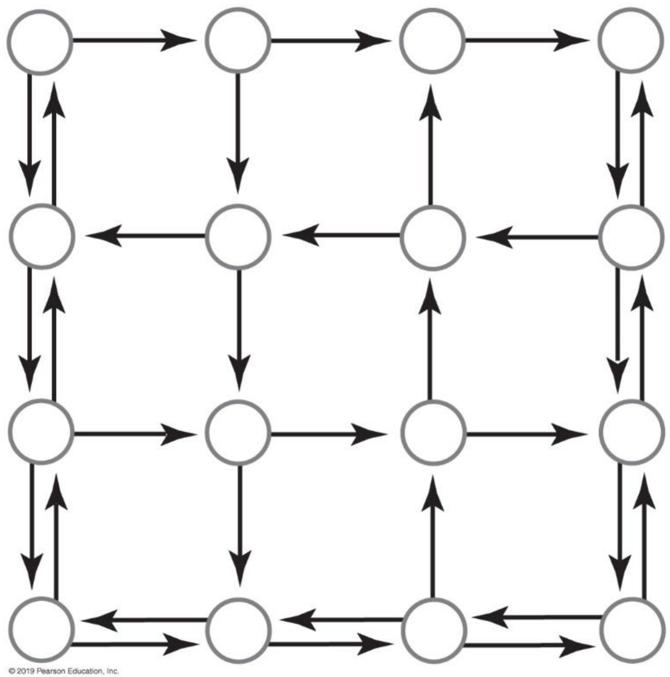


A portion of a road map

Connected Graphs

- A graph that has a path between every pair of distinct vertices is connected
- A complete graph has an edge between every pair of distinct vertices.
- Undirected graphs can be
  - Connected
  - Complete
  - Disconnected

Graphs



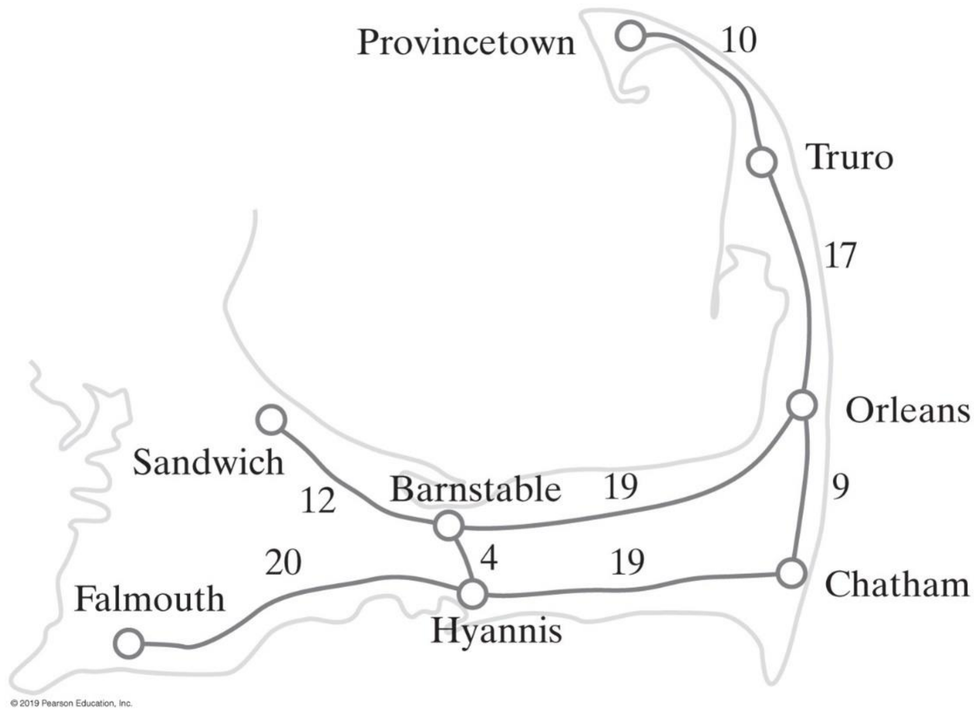


FIGURE 29-2 A directed graph representing a portion of a city's street map

- FIGURE 29-3 A weighted graph

Paths

- A path between two vertices in a graph is a sequence of edges
- A path in a directed graph must consider the direction of the edges
  - Called a directed path
- The length of a path is the number of edges that it comprises.
- A cycle is a path that begins and ends at the same vertex

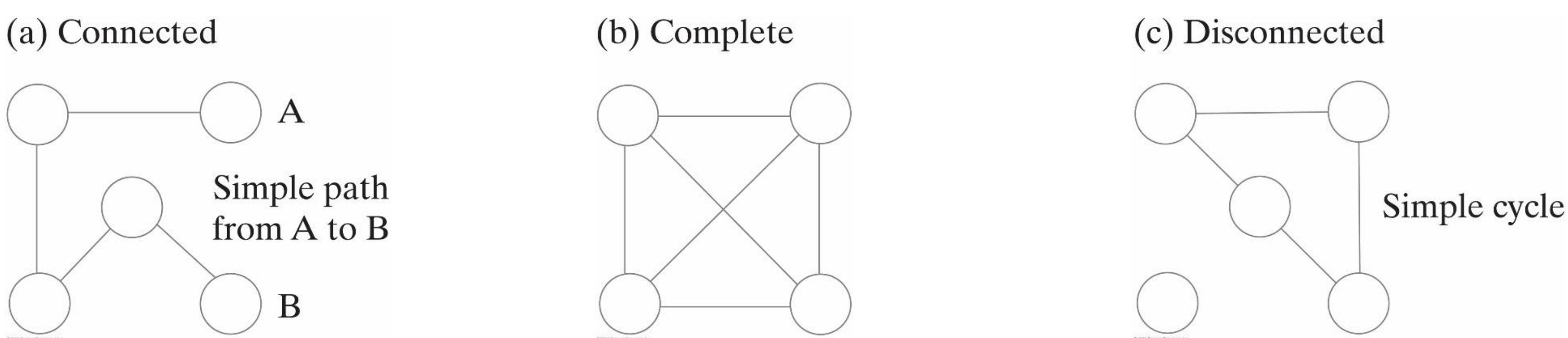
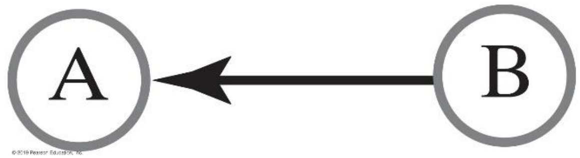
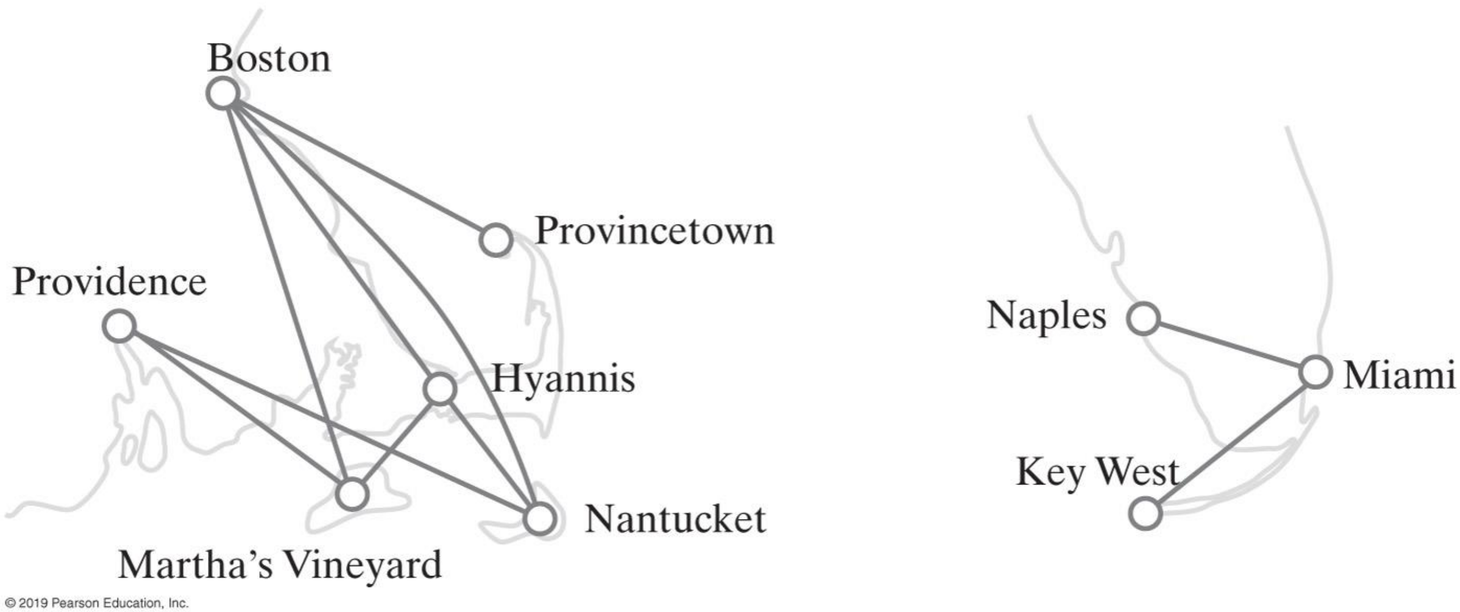


FIGURE 29-4 Undirected graphs

Directed Graphs

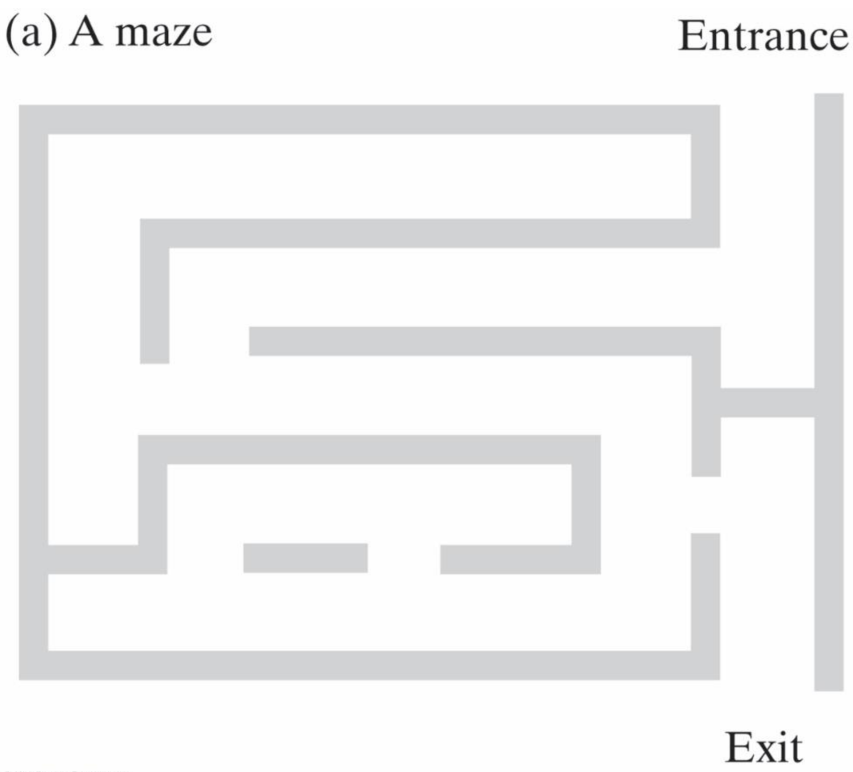
- In a directed graph vertex B is adjacent to A, if there is an edge leaving B and coming to A

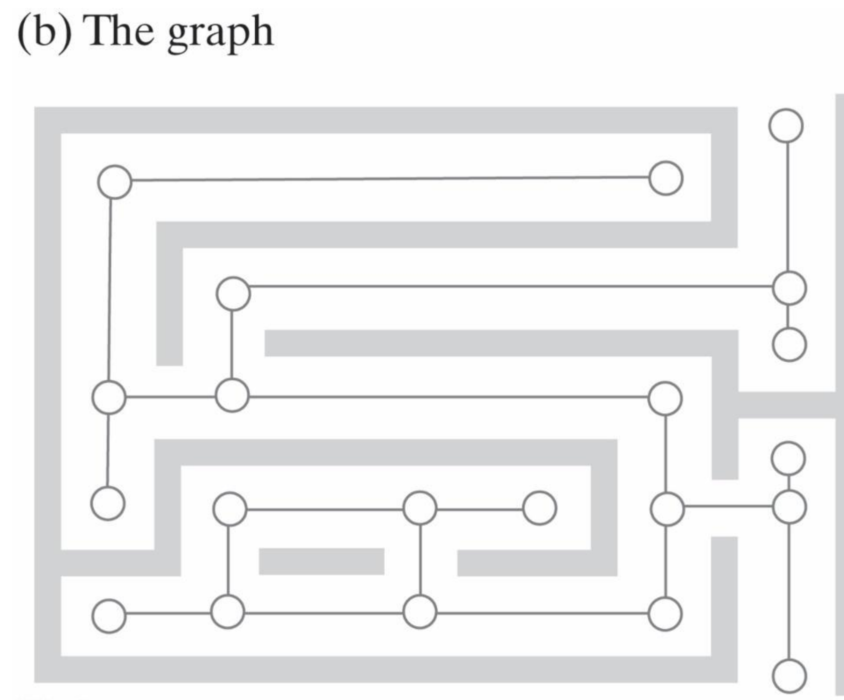




Airline routes

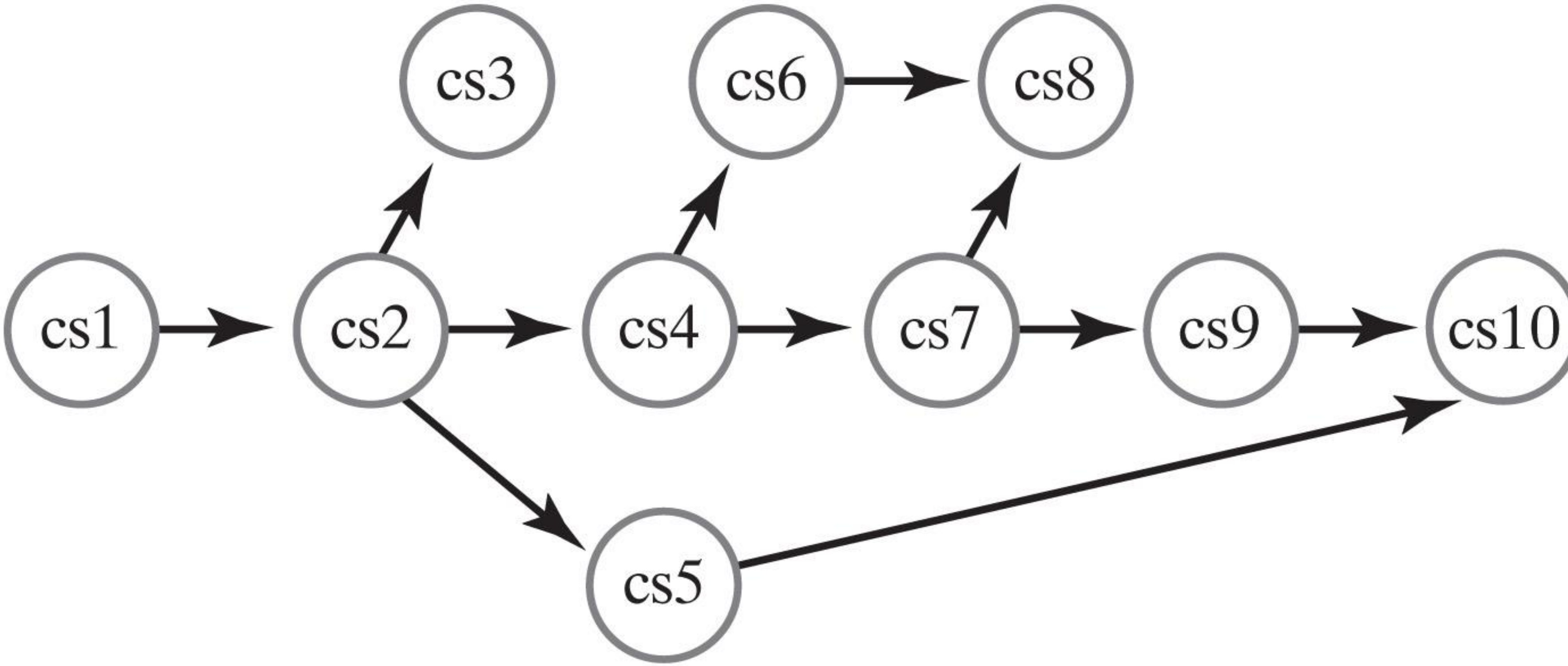
A maze and its representation as a graph





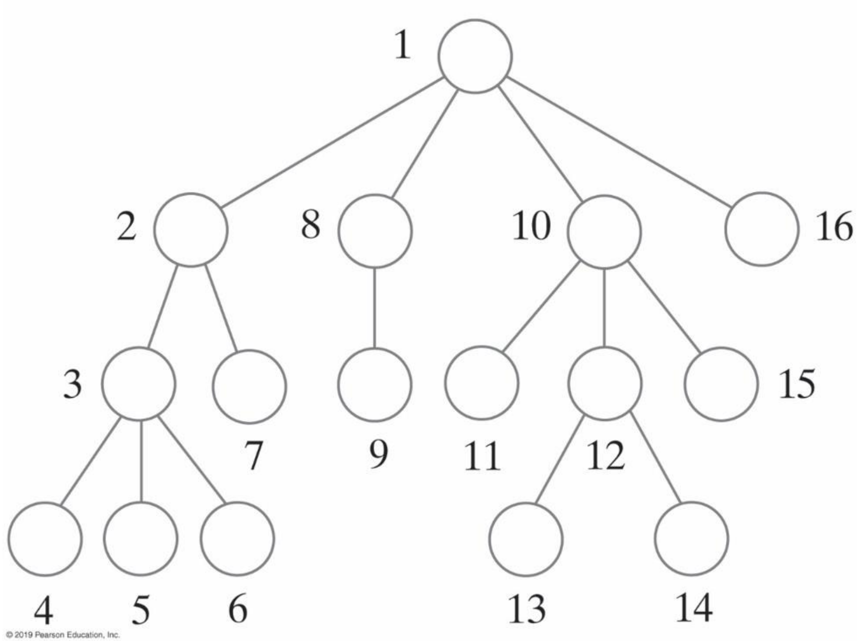
Directed Graphs

- The prerequisite structure for a selection of courses as a directed graph without cycles

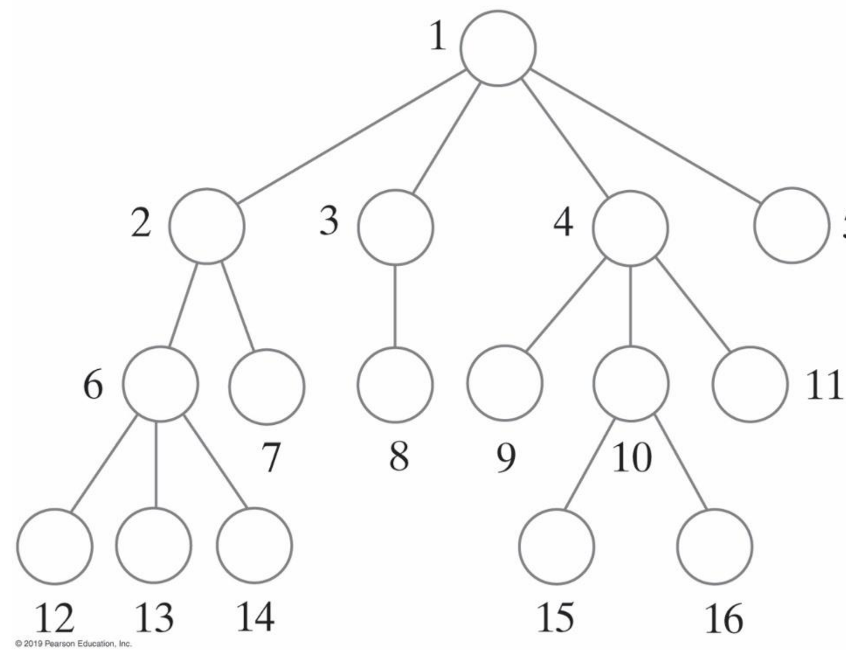


The visitation order of two traversals

(a) Depth-first traversal (preorder)



(b) Breadth-first traversal (level-order)

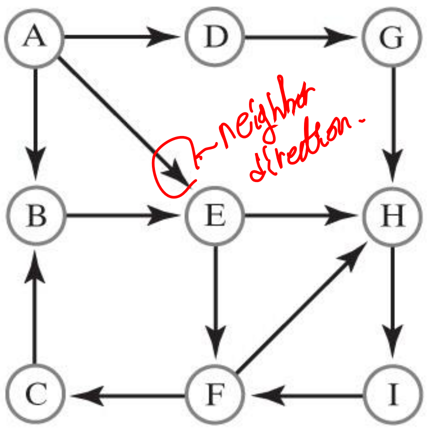


Breadth-First Traversal

- Algorithm that performs a breadth-first traversal of a nonempty graph beginning at a given vertex. This is similar to level order traversal, and use a queues.

```
Algorithm getBreadthFirstTraversal(originVertex)
traversalOrder = a new queue for the resulting traversal order
vertexQueue = a new queue to hold vertices as they are visited
Mark originVertex as visited
traversalOrder.enqueue(originVertex)
vertexQueue.enqueue(originVertex)
while (!vertexQueue.isEmpty())
{
        frontVertex = vertexQueue.dequeue()
        while (frontVertex has a neighbor)
        {
                nextNeighbor = next neighbor of frontVertex
                if (nextNeighbor is not visited)
                {
                        Mark nextNeighbor as visited
                        traversalOrder.enqueue(nextNeighbor)
                        vertexQueue.enqueue(nextNeighbor)
                }
        }
}
return traversalOrder
```

Breadth-First Traversal



frontVertex	nextNeighbor	Visited vertex	vertexQueue (front to back)	traversalOrder
		A	A	A
A			*empty*	
	B	B	B	A B
	D	D	B D	A B D
	E	E	B D E	A B D E
B	None.		D E	
D			E	
	G	G	E G	A B D E G
E			G	
	F	F	G F	A B D E G F
	H	H	G F H	A B D E G F H
G	None		F H	
F			H	
	C	C	H C	A B D E G F H C
H			C	
	I	I	C I	A B D E G F H C I
C			I	
I			*empty*	

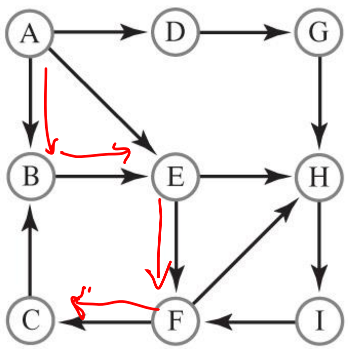
Depth-First Traversal

- Algorithm that performs a depth-first traversal of a nonempty graph, beginning at a given vertex. Like a preorder tree traversal using a stack.

```
Algorithm getDepthFirstTraversal(originVertex)
traversalOrder = a new queue for the resulting traversal order
vertexStack = a new stack to hold vertices as they are visited
Mark originVertex as visited
traversalOrder.enqueue(originVertex)
vertexStack.push(originVertex)
while  (!vertexStack.isEmpty())
{
	topVertex = vertexStack.peek()
	if (topVertex has an unvisited neighbor)
	{
		nextNeighbor = next unvisited neighbor of topVertex
		Mark nextNeighbor as visited
		traversalOrder.enqueue(nextNeighbor)
		vertexStack.push(nextNeighbor)
	}
	else // All neighbors are visited
		vertexStack.pop()
}
return traversalOrder
```

Depth-First Traversal

- A trace of a depth-first traversal beginning at vertex A of a directed graph



has to be unvisited also!

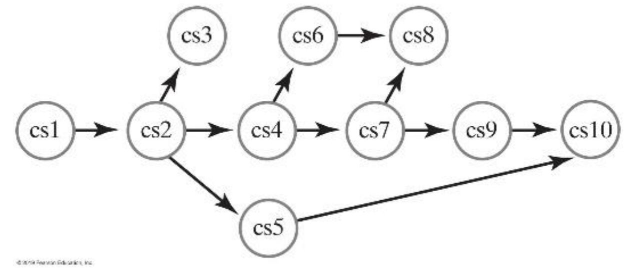
topVertex	nextNeighbor	Visited vertex	vertexStack (top to bottom)	traversalOrder (front to back)
		A	A	A
A			A	
	B	B	B A	A B
B			B A	
	E	E	E B A	A B E
E			E B A	
	F	F	F E B A	A B E F
F			F E B A	
	C	C	C F E B A	A B E F C
C			F E B A	
F			F E B A	
	H	H	H F E B A	A B E F C H
H			H F E B A	
	I	I	I H F E B A	A B E F C H I
I			H F E B A	
H			F E B A	
F			E B A	
E			B A	
B			A	
A			A	
	D	D	D A	A B E F C H I D
D			D A	
	G		G D A	A B E F C H I D G
G			D A	
D			A	
A			*empty*	

Topological Ordering

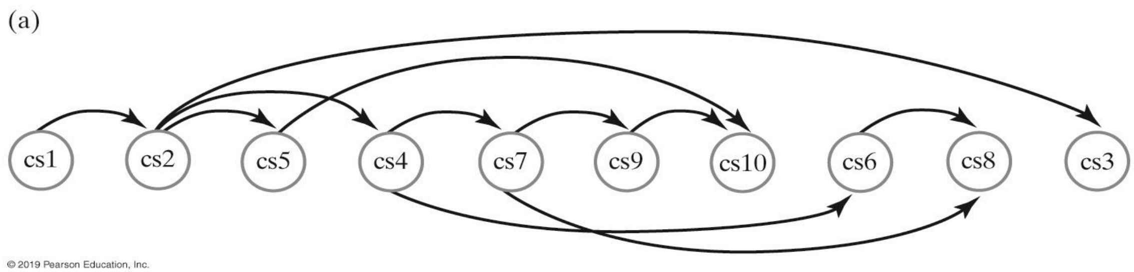
- An **impossible** prerequisite structure for three courses, as a directed graph with a cycle

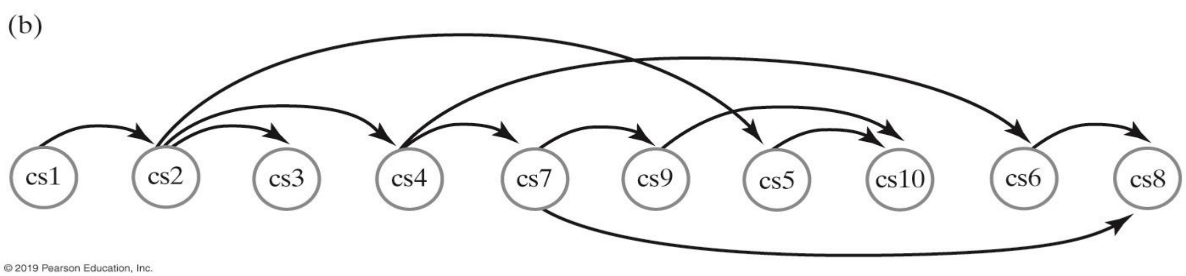
In a topological order of the vertices in a directed graph without cycles, vertex A precedes vertex B whenever a directed edge exists from A to B.

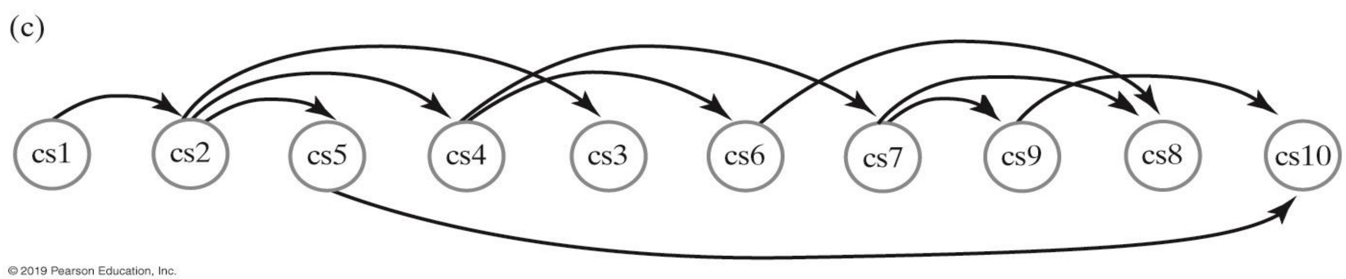
Topological Ordering



- Three topological orders for the graph to the right







Topological Order

- An algorithm that describes a topological sort

Algorithm **getTopologicalOrder()**

vertexStack = *a new stack to hold vertices as they are visited*

numberOfVertices = *number of vertices in the graph*

for (counter = 1 *to* numberOfVertices)

{

nextVertex = *an unvisited vertex whose neighbors, if any, are all visited*

Mark nextVertex *as visited*

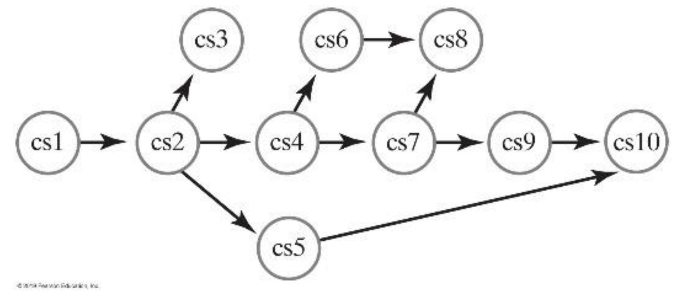
vertexStack.push(nextVertex)

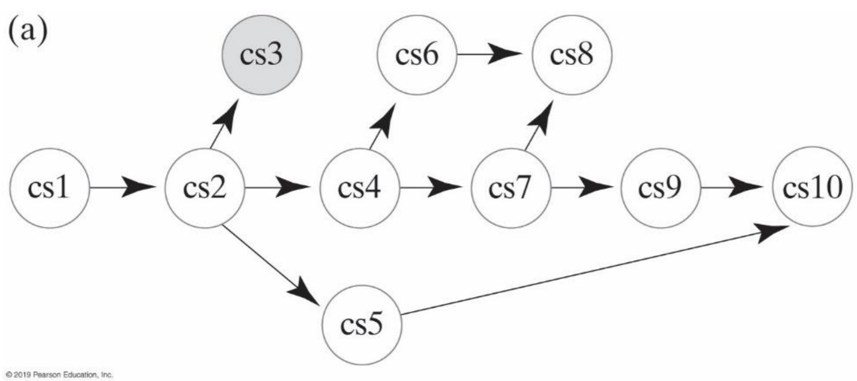
}

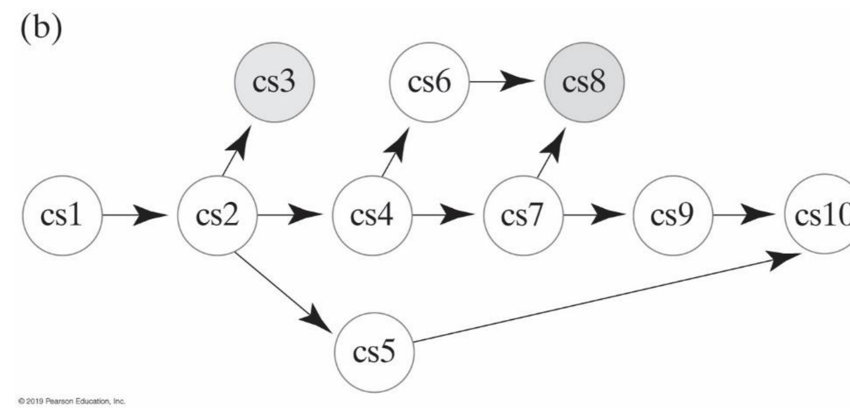
return vertexStack

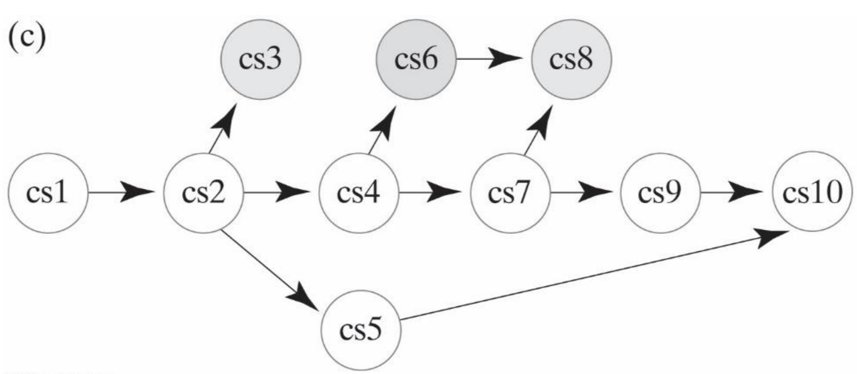
Topological Ordering (Part 1)

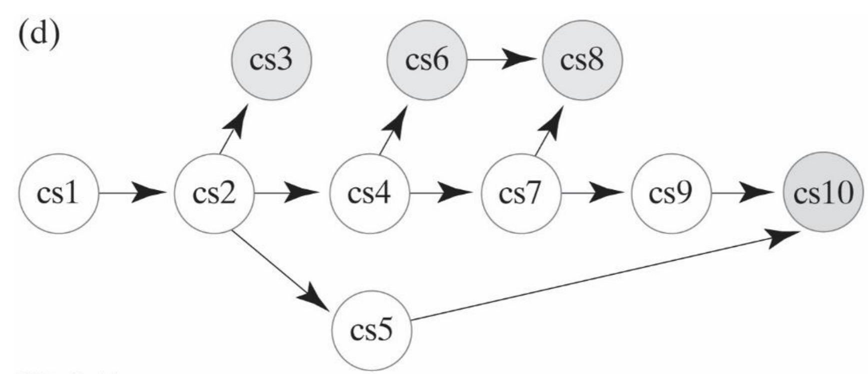
- Finding a topological order for the graph to the right. Use ordered list (1..10)
- Stack 10 6 8 3



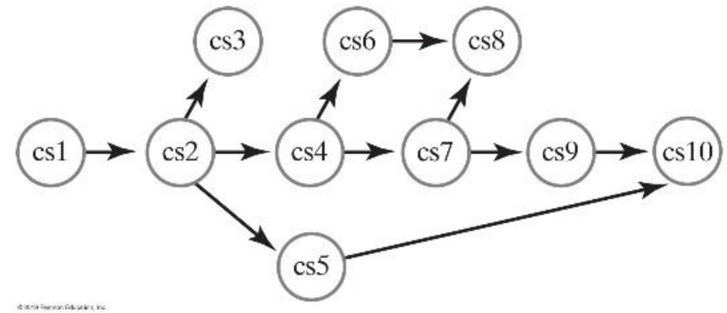




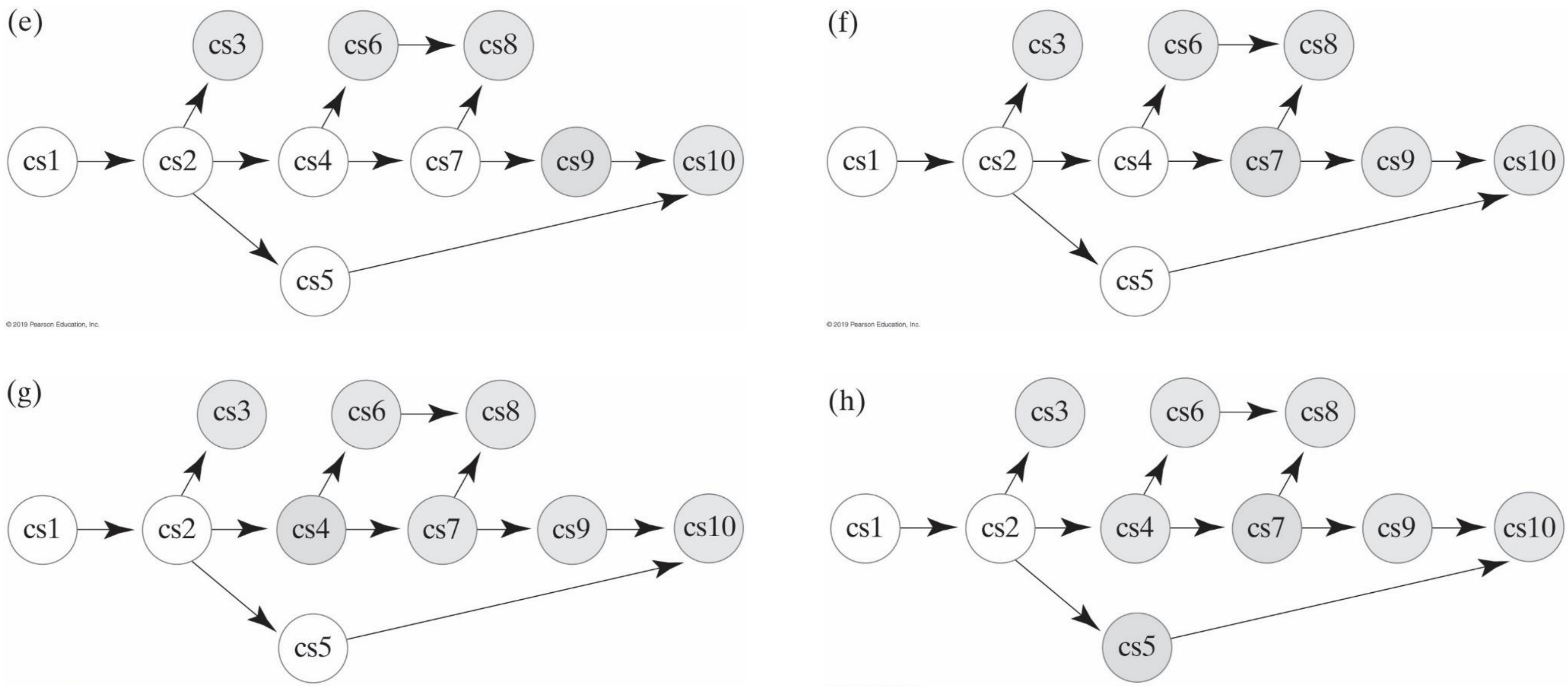




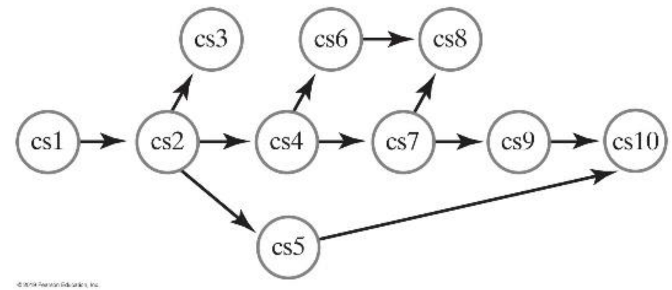
Topological Ordering (Part 2)



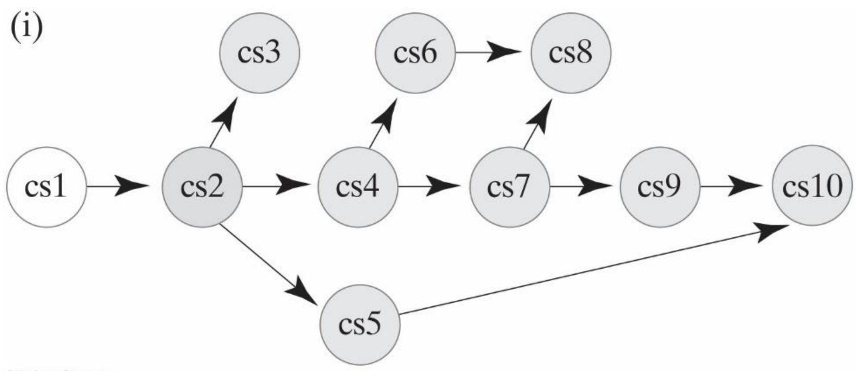
- Finding a topological order for the graph to the right
- Stack 4 7 9 5 10 6 8 3
  - Note difference from text (5 before 9)

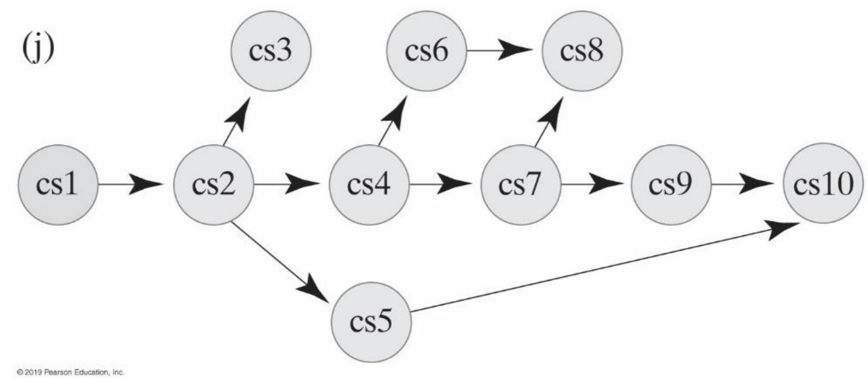


Topological Ordering (Part 3)



- Finding a topological order for the graph to the right
- Stack: 1 2 4 7 9 5 10 6 8 3

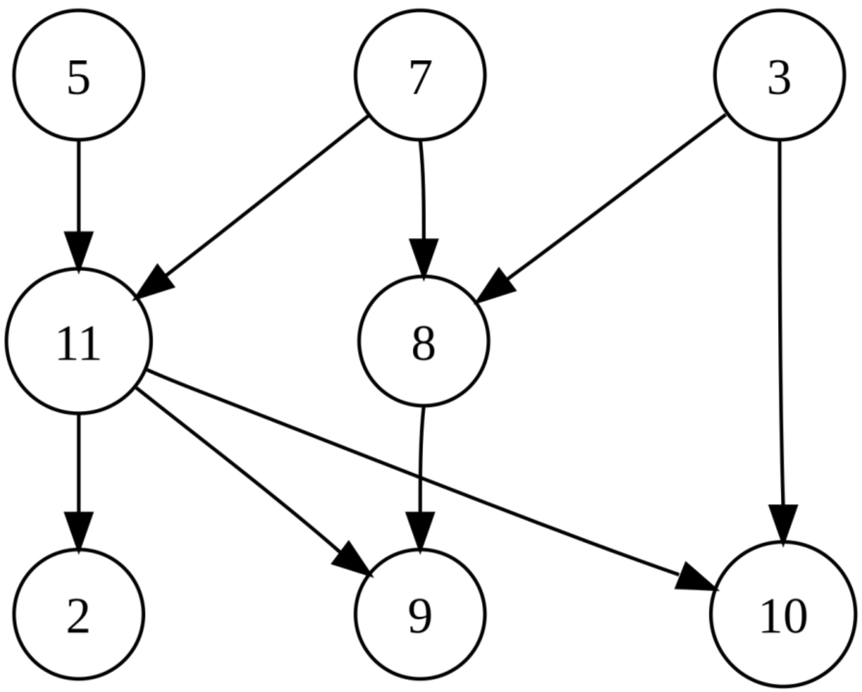




Another Topological Sort Example

Ignore this.

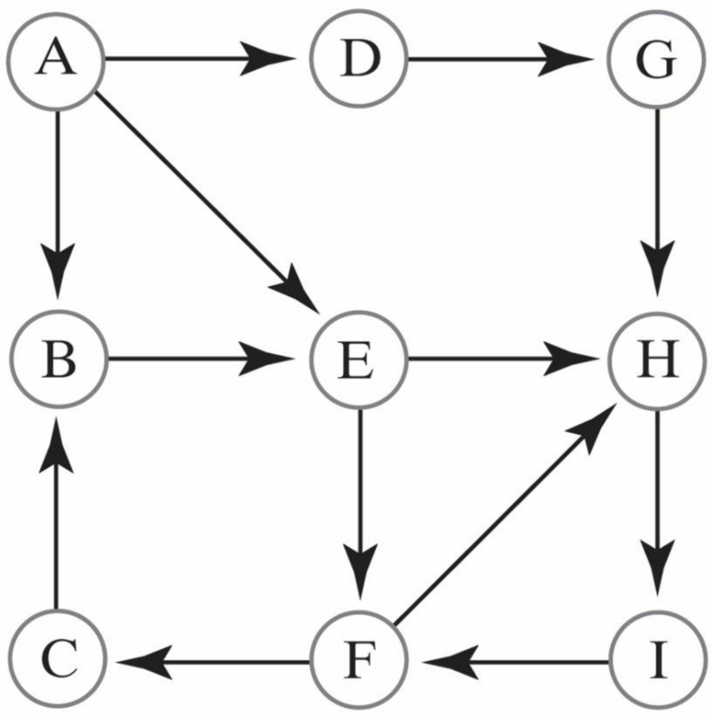
- Order of Entry: 3 8 10 5 11 7 9 2
- Topological Sort: 7 5 3 8 11 10 9 2



Shortest Path in Unweighted Graph

- An unweighted graph and the possible paths from vertex A to vertex H

(a) An unweighted graph



(b) Possible paths through the graph

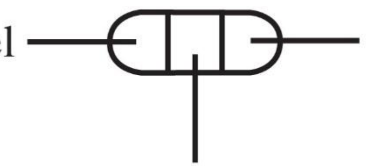
A → B → E → F → H
A → B → E → H
A → D → G → H
A → E → F → H
A → E → H

Shortest Path in an Unweighted Graph (Part 1)

```
Algorithm getShortestPath(originVertex, endVertex, path)
done = false
vertexQueue = a new queue to hold vertices as they are visited
Mark originVertex as visited
vertexQueue.enqueue(originVertex)
while (!done && !vertexQueue.isEmpty())
{
	frontVertex = vertexQueue.dequeue()
	while (!done && frontVertex has a neighbor)
	{
		nextNeighbor = next neighbor of frontVertex
		if (nextNeighbor is not visited)
		{
			Mark nextNeighbor as visited
			Set the length of the path to nextNeighbor to 1 + length of path to frontVertex
			Set the predecessor of nextNeighbor to frontVertex
			vertexQueue.enqueue(nextNeighbor)
		}
		if (nextNeighbor equals endVertex)
			done = true
	}
}
```

Shortest Path in an Unweighted Graph (Part 2)

```
// Traversal ends; construct shortest path
pathLength = length of path to endVertex
path.push(endVertex)
vertex = endVertex
while (vertex has a predecessor)
{
    vertex = predecessor of vertex
    path.push(vertex)
}
return pathLength
```

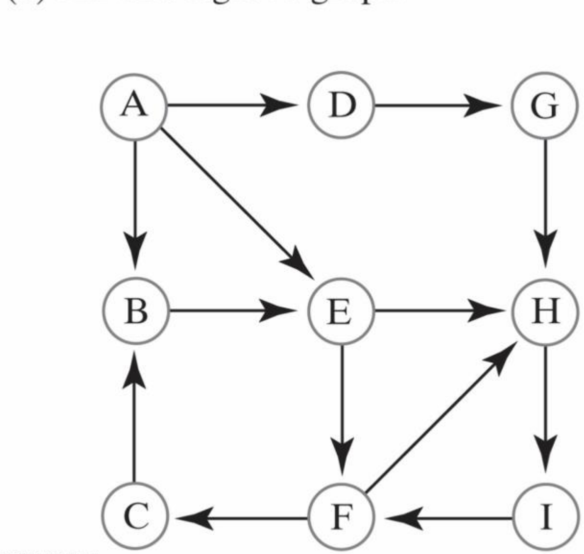


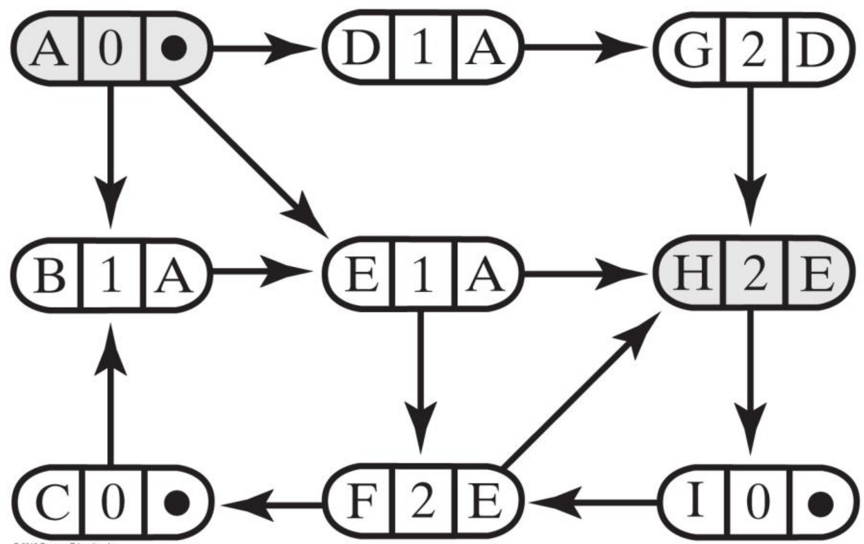
The data in a vertex

Shortest Path in a Weighted Graph

- The graph after the shortest-path algorithm has traversed from vertex A to vertex H

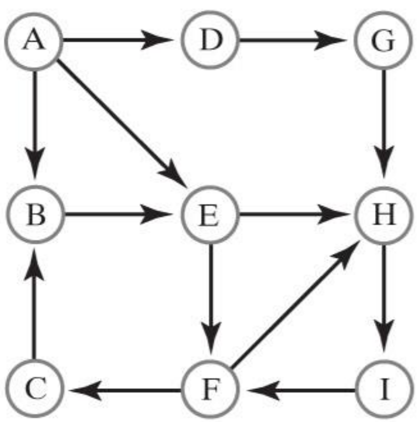
(a) An unweighted graph





Shortest Path in an Unweighted Graph

- A trace of the traversal in the algorithm to find the shortest path from vertex A to vertex H in an unweighted graph



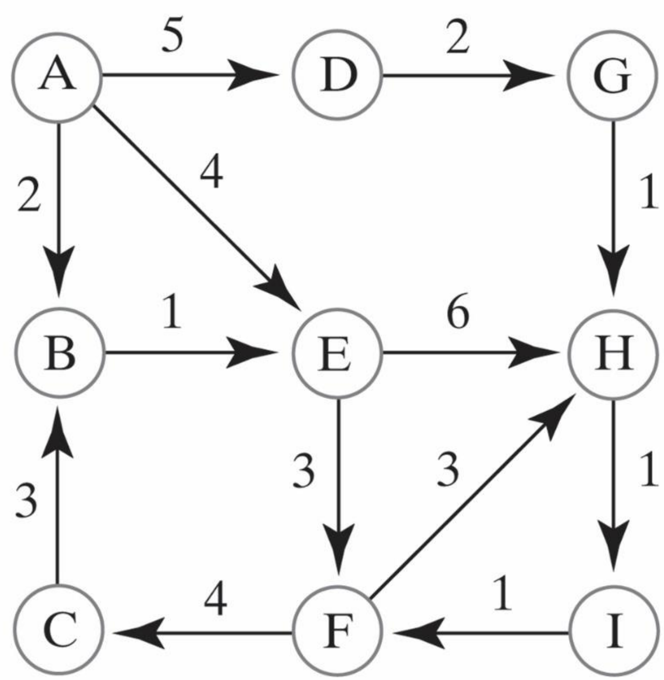
frontVertex	nextNeighbor	Visited vertex	vertexQueue (front to back)
		A	A 0 •
A 0 •			*empty*
	B	B	B 1 A
	D	D	B 1 A, D 1 A
	E	E	B 1 A, D 1 A, E 1 A
B 1 A			D 1 A, E 1 A
D 1 A			E 1 A
	G	G	E 1 A, G 2 D
E 1 A			G 2 D
	F	F	G 2 D, F 2 E
	H	H	G 2 D, F 2 E, H 2 E

Shortest Path in a Weighted Graph

- A weighted graph and the possible paths from vertex A to vertex H

higher number is faster
lower " is slower.

(a) A weighted graph

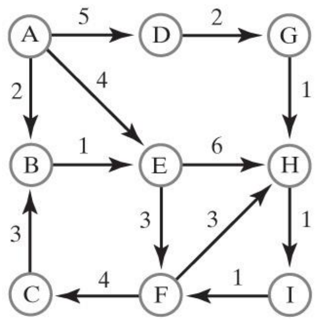


(b) Possible paths and their weights

Path	Weight
A → B → E → F → H	9
A → B → E → H	9
A → D → G → H	8
A → E → F → H	10
A → E → H	10

Shortest Path in a Weighted Graph

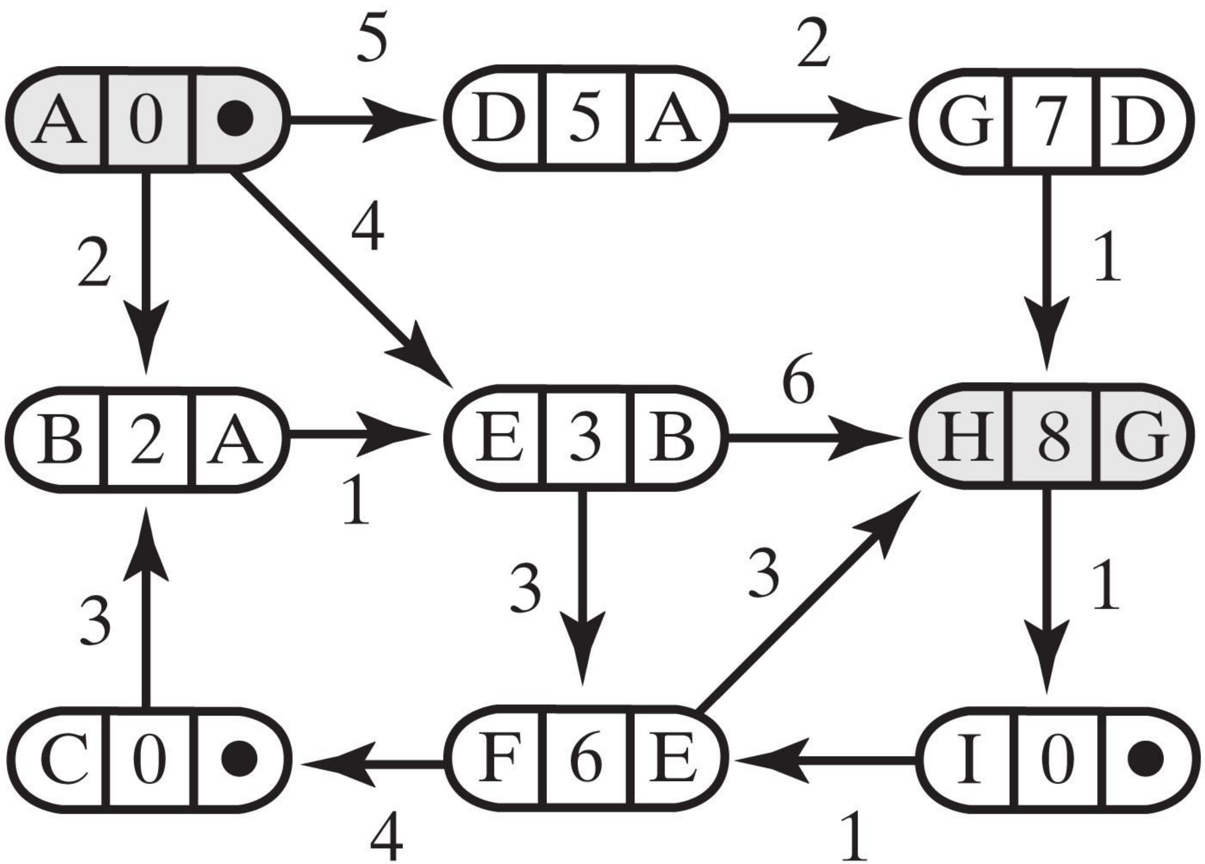
A trace of the traversal in the algorithm to find the cheapest path from vertex A to vertex H in a weighted graph



frontVertex	Visited vertex	nextNeighbor	Priority queue (front to back)
			A 0 •
A 0 •	A		*empty*
		B	B 2 A
		D	B 2 A, D 5 A
		E	B 2 A, E 4 A, D 5 A
B 2 A	B		E 4 A, D 5 A
		E	E 3 B, E 4 A, D 5 A
E 3 B	E		E 4 A, D 5 A
		F	E 4 A, D 5 A, F 6 E
		H	E 4 A, D 5 A, F 6 E, H 9 E
			D 5 A, F 6 E, H 9 E
D 5 A	D		F 6 E, H 9 E
		G	F 6 E, G 7 D, H 9 E
F 6 E	F		G 7 D, H 9 E
		H	G 7 D, H 9 E, H 9 F
		C	G 7 D, H 9 E, H 9 F, C 10 F
G 7 D	G		H 9 E, H 9 F, C 10 F
		H	H 8 G, H 9 E, H 9 F, C 10 F
H 8 G	H		H 9 E, H 9 F, C 10 F

Shortest Path in a Weighted Graph

- The graph after finding the cheapest path from vertex A to vertex H



The Shortest Path in a Weighted Graph

Q) How to traversal?

```
Algorithm getCheapestPath(originVertex, endVertex, path)
done = false
priorityQueue = a new priority queue
priorityQueue.add(new EntryPQ(originVertex, 0, null))
while (!done && !priorityQueue.isEmpty())
{
        frontEntry = priorityQueue.remove()
        frontVertex = vertex in frontEntry
        if (frontVertex is not visited)
        {
                Mark frontVertex as visited
                Set the cost of the path to frontVertex to the cost recorded in frontEntry
                Set the predecessor of frontVertex to the predecessor recorded in frontEntry
                if (frontVertex equals endVertex)
                done = true
                else
                {
                        while (frontVertex has a neighbor)
                        {
                                nextNeighbor = next neighbor of frontVertex
                                weightOfEdgeToNeighbor = weight of edge to nextNeighbor
                                if (nextNeighbor is not visited)
                                {
                                        nextCost = weightOfEdgeToNeighbor + cost of path to frontVertex
                                        priorityQueue.add(new EntryPQ(nextNeighbor, nextCost, frontVertex))
                                }
                        }
                }
        }
}
```

The Shortest Path in a Weighted Graph (Part 2)

```
// Traversal ends; now construct cheapest path
pathCost = cost of path to endVertex
path.push(endVertex)
vertex = endVertex
while (vertex has a predecessor)
{
		vertex = predecessor of vertex
		path.push(vertex)
}
return pathCost
```

Basic Graph Interface

```
public interface BasicGraphInterface<T> {
    /**
     * Adds a given vertex to this graph.
     *
     * @param vertexLabel An object that labels the new vertex and is distinct from
     *                    the labels of current vertices.
     * @return True if the vertex is added, or false if not.
     */
    public boolean addVertex(T vertexLabel);

    /**
     * Adds a weighted edge between two given distinct vertices that are currently
     * in this graph. The desired edge must not already be in the graph. In a
     * directed graph, the edge points toward the second vertex given.
     *
     * @param begin      An object that labels the origin vertex of the edge.
     * @param end        An object, distinct from begin, that labels the end vertex
     *                   of the edge.
     * @param edgeWeight The real value of the edge's weight.
     * @return True if the edge is added, or false if not.
     */
    public boolean addEdge(T begin, T end, double edgeWeight);

    /**
     * Adds an unweighted edge between two given distinct vertices that are
     * currently in this graph. The desired edge must not already be in the graph.
     * In a directed graph, the edge points toward the second vertex given.
     *
     * @param begin An object that labels the origin vertex of the edge.
     * @param end   An object, distinct from begin, that labels the end vertex of
     *              the edge.
     * @return True if the edge is added, or false if not.
     */
    public boolean addEdge(T begin, T end);

    /**
     * Sees whether an edge exists between two given vertices.
     *
     * @param begin An object that labels the origin vertex of the edge.
     * @param end   An object that labels the end vertex of the edge.
     * @return True if an edge exists.
     */
    public boolean hasEdge(T begin, T end);

    /**
     * Sees whether this graph is empty.
     *
     * @return True if the graph is empty.
     */
    public boolean isEmpty();

    /**
     * Gets the number of vertices in this graph.
     *
     * @return The number of vertices in the graph.
     */
    public int getNumberOfVertices();

    /**
     * Gets the number of edges in this graph.
     *
     * @return The number of edges in the graph.
     */
    public int getNumberOfEdges();

    /**
     * Removes all vertices and edges from this graph resulting in an empty graph.
     */
    public void clear();
}
```

Java Interfaces for the ADT Graph

- A portion of a flight map

GraphAlgorithmsInterface

```java
public interface GraphAlgorithmsInterface<T> {
		/**
		 * Performs a breadth-first traversal of this graph.
		 *
		 * @param origin An object that labels the origin vertex of the traversal.
		 * @return A queue of labels of the vertices in the traversal, with the label of
		 *         the origin vertex at the queue's front.
		 */
		public QueueInterface<T> getBreadthFirstTraversal(T origin);

		/**
		 * Performs a depth-first traversal of this graph.
		 *
		 * @param origin An object that labels the origin vertex of the traversal.
		 * @return A queue of labels of the vertices in the traversal, with the label of
		 *         the origin vertex at the queue's front.
		 */
		public QueueInterface<T> getDepthFirstTraversal(T origin);

		/**
		 * Performs a topological sort of the vertices in this graph without cycles.
		 *
		 * @return A stack of vertex labels in topological order, beginning with the
		 *         stack's top.
		 */
		public StackInterface<T> getTopologicalOrder();

		/**
		 * Finds the shortest-length path between two given vertices in this graph.
		 *
		 * @param begin An object that labels the path's origin vertex.
		 * @param end   An object that labels the path's destination vertex.
		 * @param path  A stack of labels that is empty initially; at the completion of
		 *              the method, this stack contains the labels of the vertices along
		 *              the shortest path; the label of the origin vertex is at the top,
		 *              and the label of the destination vertex is at the bottom
		 * @return The length of the shortest path.
		 */
		public int getShortestPath(T begin, T end, StackInterface<T> path);

		/**
		 * Finds the least-cost path between two given vertices in this graph.
		 *
		 * @param begin An object that labels the path's origin vertex.
		 * @param end   An object that labels the path's destination vertex.
		 * @param path  A stack of labels that is empty initially; at the completion of
		 *              the method, this stack contains the labels of the vertices along
		 *              the cheapest path; the label of the origin vertex is at the top,
		 *              and the label of the destination vertex is at the bottom
		 * @return The cost of the cheapest path.
		 */
		public double getCheapestPath(T begin, T end, StackInterface<T> path);
}
```

GraphInterface

- Combination of basic graph and algorithms interfaces

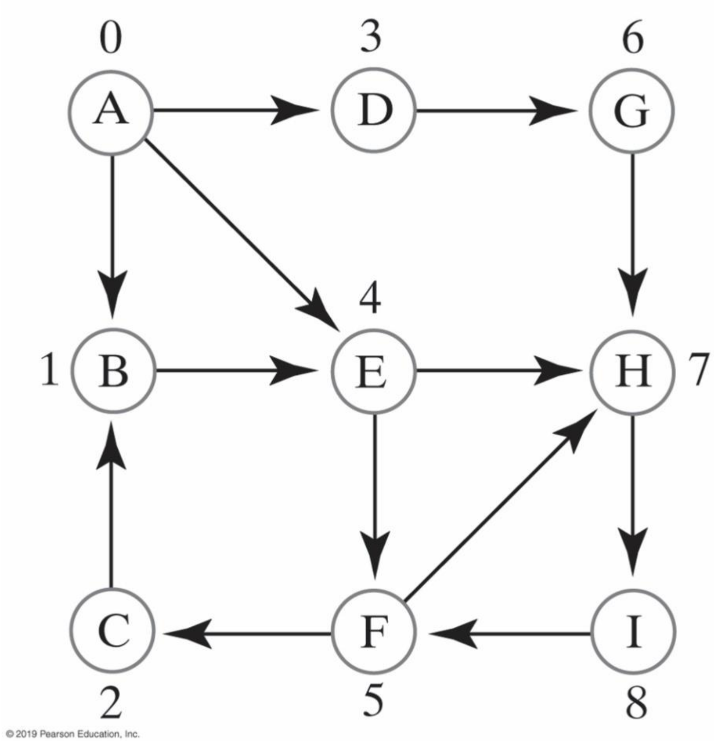
```
public interface GraphInterface<T> extends BasicGraphInterface<T>,GraphAlgorithmsInterface<T> {
}
```

Overview of Two Implementations

- Two common implementations of the ADT graph
  - Array:
    - Typically a two-dimensional array called an *adjacency matrix*
  - List:
    - Referred to as an *adjacency list*.

An unweighted, directed graph and its adjacency matrix

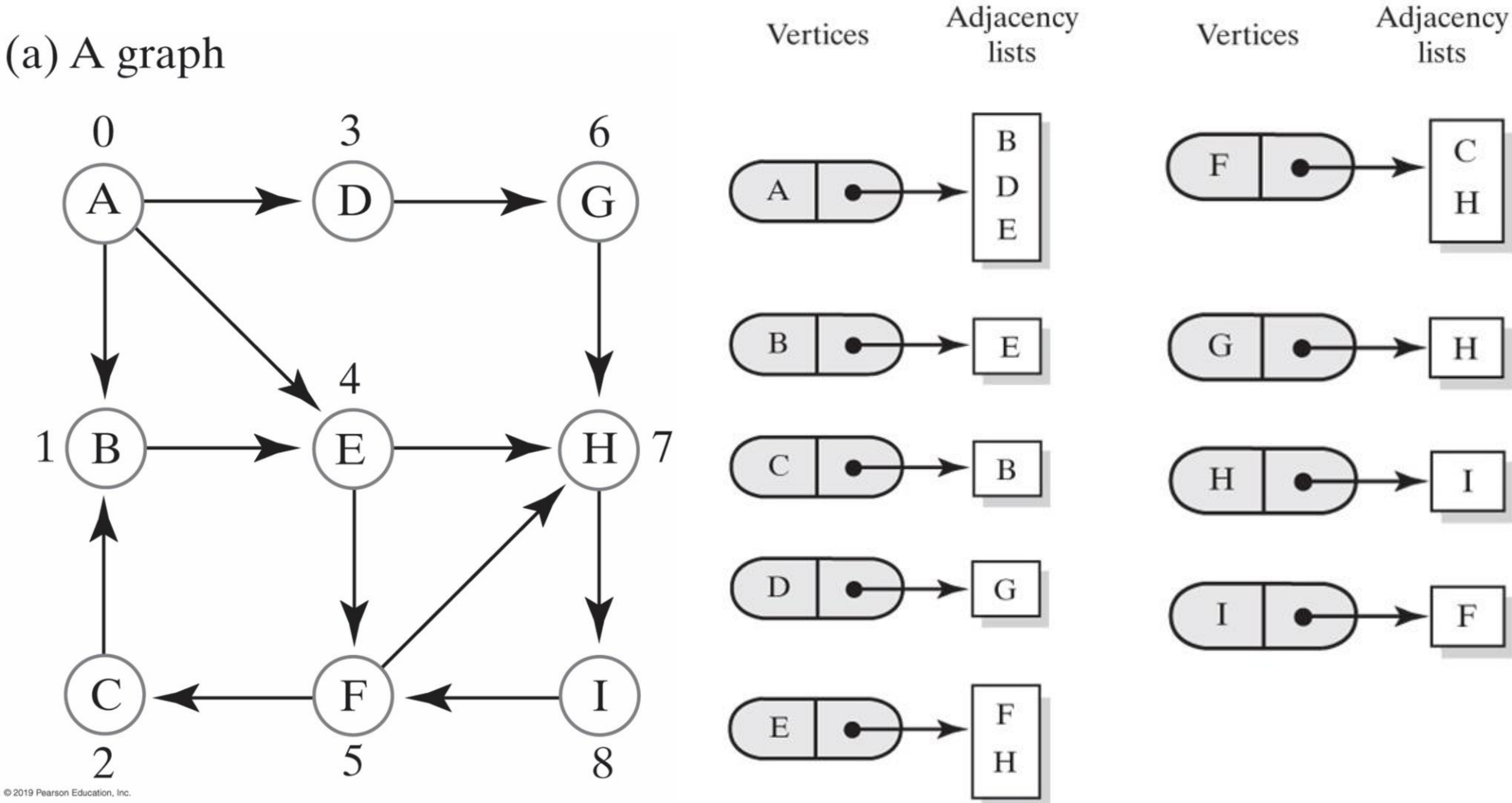
(a) A graph



(b) The graph's adjacency matrix

	A	B	C	D	E	F	G	H	I	
A		T		T	T					0
B					T					1
C		T								2
D							T			3
E						T		T		4
F			T					T		5
G								T		6
H									T	7
I						T				8
	0	1	2	3	4	5	6	7	8	

Adjacency lists for a directed graph



Vertices and Edges

- Specifying the Class **`Vertex`**
  - Identify vertices
  - Visit vertices
  - Adjacency list
  - Path operations

Vertex Interface

```java
/**
 * Gets this vertex's label.
 *
 * @return The object that labels the vertex.
 */
public T getLabel();

/** Marks this vertex as visited. */
public void visit();

/** Removes this vertex's visited mark. */
public void unvisit();

/**
 * Sees whether the vertex is marked as visited.
 *
 * @return True if the vertex is visited.
 */
public boolean isVisited();

/**
 * Connects this vertex and a given vertex with a weighted edge. The two
 * vertices cannot be the same, and must not already have this edge between
 * them. In a directed graph, the edge points toward the given vertex.
 *
 * @param endVertex  A vertex in the graph that ends the edge.
 * @param edgeWeight A real-valued edge weight, if any.
 * @return True if the edge is added, or false if not.
 */
public boolean connect(VertexInterface<T> endVertex, double edgeWeight);

/**
 * Connects this vertex and a given vertex with an unweighted edge. The two
 * vertices cannot be the same, and must not already have this edge between
 * them. In a directed graph, the edge points toward the given vertex.
 *
 * @param endVertex A vertex in the graph that ends the edge.
 * @return True if the edge is added, or false if not.
 */
public boolean connect(VertexInterface<T> endVertex);

/**
 * Creates an iterator of this vertex's neighbors by following all edges that
 * begin at this vertex.
 *
 * @return An iterator of the neighboring vertices of this vertex.
 */
public Iterator<VertexInterface<T>> getNeighborIterator();

/**
 * Creates an iterator of the weights of the edges to this vertex's neighbors.
 *
 * @return An iterator of edge weights for edges to neighbors of this vertex.
 */
public Iterator<Double> getWeightIterator();
```

Vertex Interface

```java
/**
 * Sees whether this vertex has at least one neighbor.
 *
 * @return True if the vertex has a neighbor.
 */
public boolean hasNeighbor();

/**
 * Gets an unvisited neighbor, if any, of this vertex.
 *
 * @return Either a vertex that is an unvisited neighbor or null if no such
 *         neighbor exists.
 */
public VertexInterface<T> getUnvisitedNeighbor();

/**
 * Records the previous vertex on a path to this vertex.
 *
 * @param predecessor The vertex previous to this one along a path.
 */
public void setPredecessor(VertexInterface<T> predecessor);

/**
 * Gets the recorded predecessor of this vertex.
 *
 * @return Either this vertex's predecessor or null if no predecessor was
 *         recorded.
 */
public VertexInterface<T> getPredecessor();

/**
 * Sees whether a predecessor was recorded for this vertex.
 *
 * @return True if a predecessor was recorded.
 */
public boolean hasPredecessor();

/**
 * Records the cost of a path to this vertex.
 *
 * @param newCost The cost of the path.
 */
public void setCost(double newCost);

/**
 * Gets the recorded cost of the path to this vertex.
 *
 * @return The cost of the path.
 */
public double getCost();
```

Edge class

```java
/**
 * Edge in a graph. An edge connects two vertices. We will
 * just keep the vertex at the end.
 *
 * Weight is also kept.
 */
protected class Edge implements Comparable<Edge> {
    private VertexInterface<T> vertex; // Vertex at end of edge
    private double weight;

    protected Edge(VertexInterface<T> endVertex, double edgeWeight) {
        vertex = endVertex;
        weight = edgeWeight;
    }

    protected Edge(VertexInterface<T> endVertex) {
        vertex = endVertex;
        weight = 0;
    }

    protected VertexInterface<T> getEndVertex() {
        return vertex;
    }

    protected double getWeight() {
        return weight;
    }

    @Override
    public int compareTo(Edge other) {

        // so we can reuse classes that support ListInterface

        throw new java.lang.UnsupportedOperationException("Comparison not supported.");
    }
}
```

Vertex class

- Implement with adjacency list (linked list)
- Could use ArrayList as well

```
class Vertex<T> implements VertexInterface<T> {
	private T label;
	private ListWithIteratorInterface<Edge> edgeList; // Edges to neighbors
	private boolean visited; // True if visited
	private VertexInterface<T> previousVertex; // On path to this vertex
	private double cost; // Of path to this vertex

	public Vertex(T vertexLabel) {
		label = vertexLabel;
		edgeList = new CompletedLListWithIterator<>();
//		edgeList = new LListWithIterator<>();
		visited = false;
		previousVertex = null;
		cost = 0;
	}
```

Vertex class basic operations

```
public T getLabel() {
      return label;
}

public boolean hasPredecessor() {
      return previousVertex != null;
}

public void setPredecessor(VertexInterface<T> predecessor) {
      previousVertex = predecessor;
}

public VertexInterface<T> getPredecessor() {
      return previousVertex;
}

public void visit() {
      visited = true;
}

public void unvisit() {
      visited = false;
}

public boolean isVisited() {
      return visited;
}

public double getCost() {
      return cost;
}

public void setCost(double newCost) {
      cost = newCost;
}

public String toString() {
      return label.toString();
}
```

Vertex class – connect()

- connects this object to another vertex
- make sure the other object is a different one!
    - Need to look at all of the neighbors as well.

```
public boolean connect(VertexInterface<T> endVertex, double edgeWeight) {
    boolean result = false;

    if (!this.equals(endVertex)) { // Vertices are distinct
        Iterator<VertexInterface<T>> neighbors = getNeighborIterator();
        boolean duplicateEdge = false;

        while (!duplicateEdge && neighbors.hasNext()) {
            VertexInterface<T> nextNeighbor = neighbors.next();
            if (endVertex.equals(nextNeighbor))
                duplicateEdge = true;
        }

        // this is a new unique vertex, so add an edge connecting it
        if (!duplicateEdge) {
            edgeList.add(new Edge(endVertex, edgeWeight));
            result = true;
        }
    }

    return result;
}

public boolean connect(VertexInterface<T> endVertex) {
    return connect(endVertex, 0);
}
```

Edge = neighbor.

Vertex class – Weight Iterator

- needed because Edge is not publicly accessible

```
public Iterator<Double> getWeightIterator() {
    return new WeightIterator();
}

private class WeightIterator implements Iterator<Double> {
    private Iterator<Edge> edges;

    private WeightIterator() {
        edges = edgeList.iterator();
    }

    public boolean hasNext() {
        return edges.hasNext();
    }

    public Double next() {
        Double edgeWeight = 0.0;
        if (edges.hasNext()) {
            Edge edgeToNextNeighbor = edges.next();
            edgeWeight = edgeToNextNeighbor.getWeight();
        } else
            throw new NoSuchElementException();

        return edgeWeight;
    }

    public void remove() {
        throw new UnsupportedOperationException();
    }
}
```

Vertex class – Neighbor iterator

```java
public Iterator<VertexInterface<T>> getNeighborIterator() {
    return new NeighborIterator();
}

private class NeighborIterator implements Iterator<VertexInterface<T>> {
    private Iterator<Edge> edges;

    private NeighborIterator() {
        edges = edgeList.iterator();
    }

    public boolean hasNext() {
        return edges.hasNext();
    }

    public VertexInterface<T> next() {
        VertexInterface<T> nextNeighbor = null;

        if (edges.hasNext()) {
            Edge edgeToNextNeighbor = edges.next();
            nextNeighbor = edgeToNextNeighbor.getEndVertex();
        } else
            throw new NoSuchElementException();

        return nextNeighbor;
    }

    public void remove() {
        throw new UnsupportedOperationException();
    }
}
```

Vertex class – Neighbor methods

- getUnvisitedNeighbor() is useful for topological ordering and shortest path

```
public boolean hasNeighbor() {
     return !edgeList.isEmpty();
}

public VertexInterface<T> getUnvisitedNeighbor() {
     VertexInterface<T> result = null;

     Iterator<VertexInterface<T>> neighbors = getNeighborIterator();
     while (neighbors.hasNext() && (result == null)) {
          VertexInterface<T> nextNeighbor = neighbors.next();
          if (!nextNeighbor.isVisited())
               result = nextNeighbor;
     }

     return result;
}
```

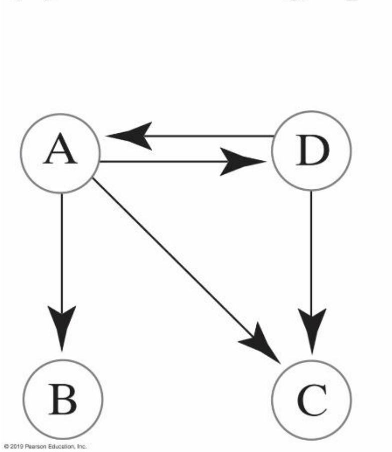
Vertex class – determining equality

- Note use of getClass() to make sure they are the same
- Simply checks the label (used as identifier)

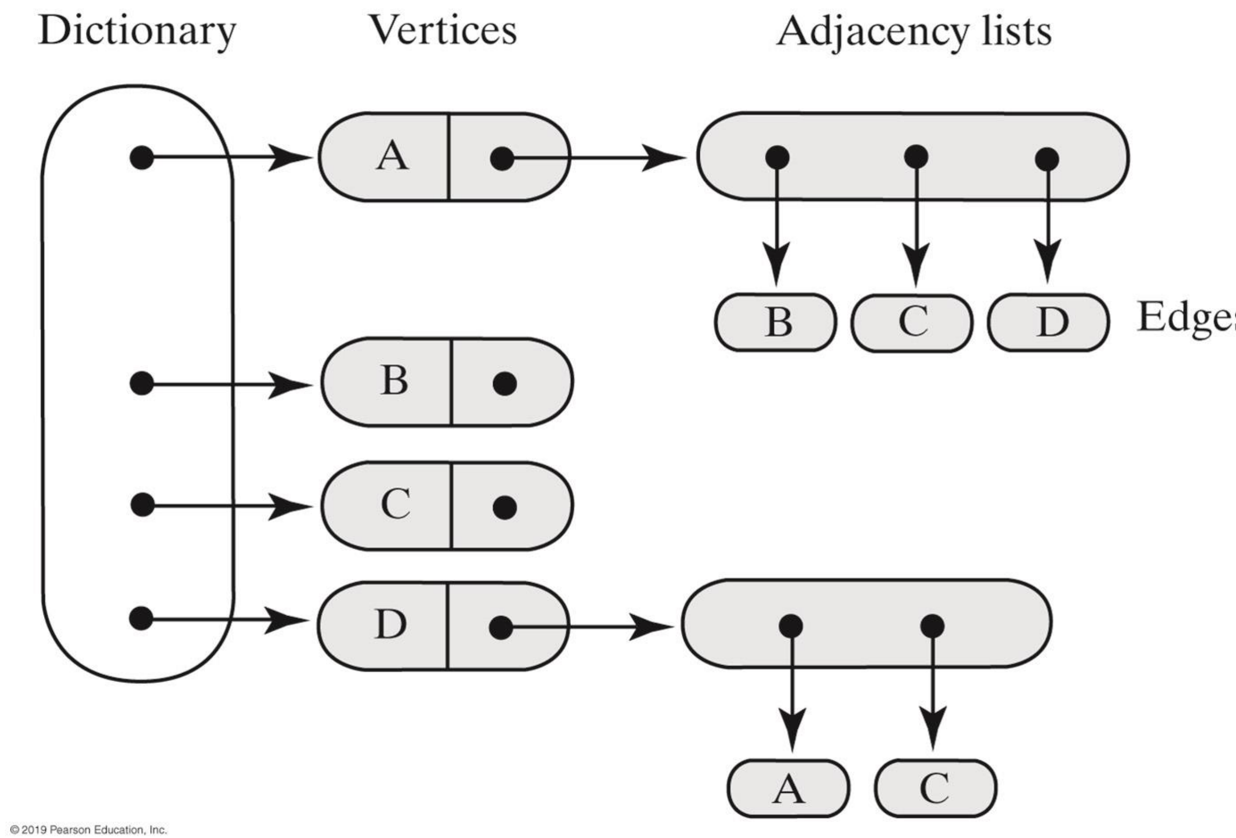
```
public boolean equals(Object other) {
    boolean result;

    if ((other == null) || (getClass() != other.getClass()))
        result = false;
    else {
        // The cast is safe within this else clause
        @SuppressWarnings("unchecked")
        Vertex<T> otherVertex = (Vertex<T>) other;
        result = label.equals(otherVertex.label);
    }

    return result;
}
```

Directed graph as a dictionary of vertices

(a) A directed graph



(b) An implementation of the graph using a dictionary



DirectedGraph implementation

- Use of linked dictionary, but could use others as well
  - ArrayDictionary, HashMapDictionary, …
- Need to added vertices and edges with other methods
- Dictionary key will be the label, and vertex the value.

```
public class CompletedDirectedGraph<T extends Comparable<? super T>> implements GraphInterface<T> {
    private DictionaryInterface<T, VertexInterface<T>> vertices;
    private int edgeCount;

    public CompletedDirectedGraph() {
        vertices = new CompletedLinkedDictionary<>();
        edgeCount = 0;
    }
```

DirectedGraph – addVertex()

- Add a vertex with a label.
- Could be a string, or any other identifying object

```
public boolean addVertex(T vertexLabel) {
    VertexInterface<T> addOutcome = vertices.add(vertexLabel, new Vertex<>(vertexLabel));
    return addOutcome == null; // Was addition to dictionary successful?
}
```

DirectedGraph – adding edges

- Take two labels and see if they exist
- If so, connect the begin to the end, which creates the edge

```
public boolean addEdge(T begin, T end, double edgeWeight) {
    boolean result = false;

    // look up begin and end vertices, and then connect them
    VertexInterface<T> beginVertex = vertices.getValue(begin);
    VertexInterface<T> endVertex = vertices.getValue(end);
    if ((beginVertex != null) && (endVertex != null))
        result = beginVertex.connect(endVertex, edgeWeight);
    if (result)
        edgeCount++;
    return result;
}

public boolean addEdge(T begin, T end) {
    return addEdge(begin, end, 0);
}
```

DirectedGraph – hasEdge()

- Similar to addEdge(), in that we check to see if begin and end exist
- If so see if begin has a neighbor that is the same as end

```
    public boolean hasEdge(T begin, T end) {
        boolean found = false;
        VertexInterface<T> beginVertex = vertices.getValue(begin);
        VertexInterface<T> endVertex = vertices.getValue(end);
        if ((beginVertex != null) && (endVertex != null)) {
            // look through all of the neighbors to find a match
            Iterator<VertexInterface<T>> neighbors = beginVertex.getNeighborIterator();
            while (!found && neighbors.hasNext()) {
                VertexInterface<T> nextNeighbor = neighbors.next();
                if (endVertex.equals(nextNeighbor))
                    found = true;
            }
        }

        return found;
    }
```

DirectedGraph – other methods

```java
public boolean isEmpty() {
    return vertices.isEmpty();
}

public void clear() {
    vertices.clear();
    edgeCount = 0;
}

public int getNumberOfVertices() {
    return vertices.size();
}

public int getNumberOfEdges() {
    return edgeCount;
}
```

DirectedGraph – resetVertices()

- make each vertex unvisited, and cost set to zero

```
protected void resetVertices() {
    Iterator<VertexInterface<T>> vertexIterator = vertices.getValueIterator();
    while (vertexIterator.hasNext()) {
        VertexInterface<T> nextVertex = vertexIterator.next();
        nextVertex.unvisit();
        nextVertex.setCost(0);
        nextVertex.setPredecessor(null);
    }
}
```

Efficiency of Basic Graph Operations

- The performance of basic operations of the ADT graph when implemented by using adjacency lists

Operation	
`addVertex`	O(n)
`addEdge`	O(n)
`hasEdge`	O(n)
`isEmpty`	O(1)
`getNumberOfVertices`	O(1)
`getNumberOfEdges`	O(1)
`Clear`	O(1)

DirectedGraph - BreadthFirstTraversal

- Note use of linked list queue, but any queue will do
- Queue of vertices
- Dequeue, then visit neighbors, enqueue each
- Record traversal order in separate queue

```
public QueueInterface<T> getBreadthFirstTraversal(T origin) {
    resetVertices();
    QueueInterface<T> traversalOrder = new CompletedLinkedQueue<>();
    QueueInterface<VertexInterface<T>> vertexQueue = new CompletedLinkedQueue<>();

    VertexInterface<T> originVertex = vertices.getValue(origin);
    originVertex.visit();
    traversalOrder.enqueue(origin); // Enqueue vertex label
    vertexQueue.enqueue(originVertex); // Enqueue vertex

    while (!vertexQueue.isEmpty()) {
        VertexInterface<T> frontVertex = vertexQueue.dequeue();
        Iterator<VertexInterface<T>> neighbors = frontVertex.getNeighborIterator();

        while (neighbors.hasNext()) {
            VertexInterface<T> nextNeighbor = neighbors.next();
            if (!nextNeighbor.isVisited()) {
                nextNeighbor.visit();
                traversalOrder.enqueue(nextNeighbor.getLabel());
                vertexQueue.enqueue(nextNeighbor);
            }
        }
    }

    return traversalOrder;
}
```

Lookup dictionary.

Levelorder.

Directed Graph – shortest path

- Use cost as hop count

```
public int getShortestPath(T begin, T end, StackInterface<T> path) {
    resetVertices();
    boolean done = false;
    QueueInterface<VertexInterface<T>> vertexQueue = new CompletedLinkedQueue<>();
    VertexInterface<T> originVertex = vertices.getValue(begin);
    VertexInterface<T> endVertex = vertices.getValue(end);
    originVertex.visit();

    vertexQueue.enqueue(originVertex);

    while (!done && !vertexQueue.isEmpty()) {
        VertexInterface<T> frontVertex = vertexQueue.dequeue();
        // look at each neighbor
        // if not visited, mark as such

        Iterator<VertexInterface<T>> neighbors = frontVertex.getNeighborIterator();
        while (!done && neighbors.hasNext()) {
            VertexInterface<T> nextNeighbor = neighbors.next();
            if (!nextNeighbor.isVisited()) {
                // mark as visited first
                // add the cost of the vertex being examined
                // to the neighbor, and set this vertex as
                // the neighbor's predecessor
                // finally add the neighbor to the queue
                nextNeighbor.visit();
                nextNeighbor.setCost(1 + frontVertex.getCost());
                nextNeighbor.setPredecessor(frontVertex);
                vertexQueue.enqueue(nextNeighbor);
            }

            if (nextNeighbor.equals(endVertex))
                done = true;
        }
    }

    // Traversal ends; construct shortest path
    // start at the end and push predecessors onto the stack

    int pathLength = (int) endVertex.getCost();
    path.push(endVertex.getLabel());

    VertexInterface<T> vertex = endVertex;
    while (vertex.hasPredecessor()) {
        vertex = vertex.getPredecessor();
        path.push(vertex.getLabel());
    }
    return pathLength;
}
```

they can't access traversal, so that push onto stack. (hand back to the user).

DirectedGraph – topological order

- Look for an unvisited terminal vertex through ALL vertices
- If found, push onto stack

```
public StackInterface<T> getTopologicalOrder() {
        resetVertices();

        // for each vertex in the graph, see if any of its
        // neighbors are terminal (unvisited and ONLY visited neighbors)
        // if so, push it onto the stack

        StackInterface<T> vertexStack = new CompletedLinkedStack<>();
        int numberOfVertices = getNumberOfVertices();
        for (int counter = 0; counter < numberOfVertices; counter++) {
                VertexInterface<T> nextVertex = findTerminal();
                nextVertex.visit();
                vertexStack.push(nextVertex.getLabel());
        }

        return vertexStack;
}

/**
 * Find the first vertex that is a terminal. That is, it has ONLY
 * visited neighbors.
 */
protected VertexInterface<T> findTerminal() {
        boolean found = false;
        VertexInterface<T> result = null;

        // search through all of the vertices in the graph.

        Iterator<VertexInterface<T>> vertexIterator = vertices.getValueIterator();

        while (!found && vertexIterator.hasNext()) {
                VertexInterface<T> nextVertex = vertexIterator.next();

                // If nextVertex is unvisited AND has only visited neighbors)
                // then we have found our terminal vertex

                if (!nextVertex.isVisited()) {
                        if (nextVertex.getUnvisitedNeighbor() == null) {
                                found = true;
                                result = nextVertex;
                        }
                }
        }
        return result;
}
```

UndirectedGraph

- Just a special case of DirectedGraph
- addEdge() adds edges in BOTH directions.
- numberOfEdges() is different
  - addEdge() will count the edge twice, so only return half of the number

```
public class UndirectedGraph<T extends Comparable<? super T>> extends CompletedDirectedGraph<T>
        implements GraphInterface<T>
//   extends DirectedGraph<T> implements GraphInterface<T>
{
     public UndirectedGraph() {
          super();
     }

     public boolean addEdge(T begin, T end, double edgeWeight) {
          return super.addEdge(begin, end, edgeWeight) && super.addEdge(end, begin, edgeWeight);
          // Assertion: edge count is twice its correct value due to calling addEdge twice
     }

     public boolean addEdge(T begin, T end) {
          return this.addEdge(begin, end, 0);
     }

     public int getNumberOfEdges() {
          return super.getNumberOfEdges() / 2;
     }

     public StackInterface<T> getTopologicalOrder() {
          throw new UnsupportedOperationException("Topological sort is illegal in an undirected graph.");
     }
}
```

Testing

- Implement DirectedGraph
  - Include
    - getShortestPath()
    - getCheapestPath()
    - getTopologicalOrder()
    - getBreadthFirstTraversal()
    - getDepthFirstTraversal()

    Study it!
  - UndirectedGraph simply extends DirectedGraph and is provided for you.
- Run all of the Graph*Demo programs
  - Undirected/Directed
  - Weighted/Unweighted
- Run other *Demo programs
  - ShortestPathDemo
  - TopologicalSortDemo
  - TraversalDemo
  - CheapestPathDemo