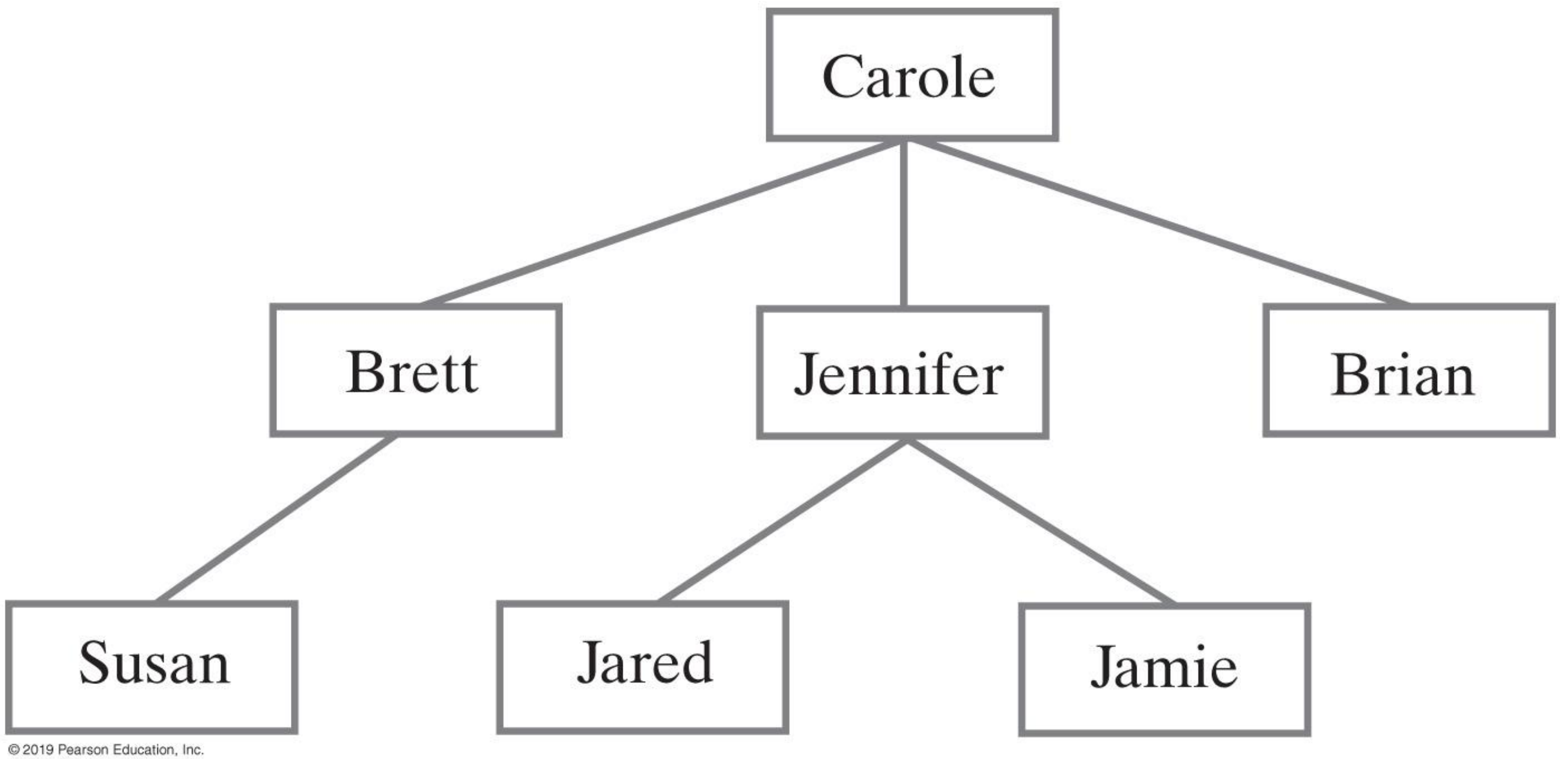
Class 10 - Trees

CSIS 3475 Data Structures and Algorithms

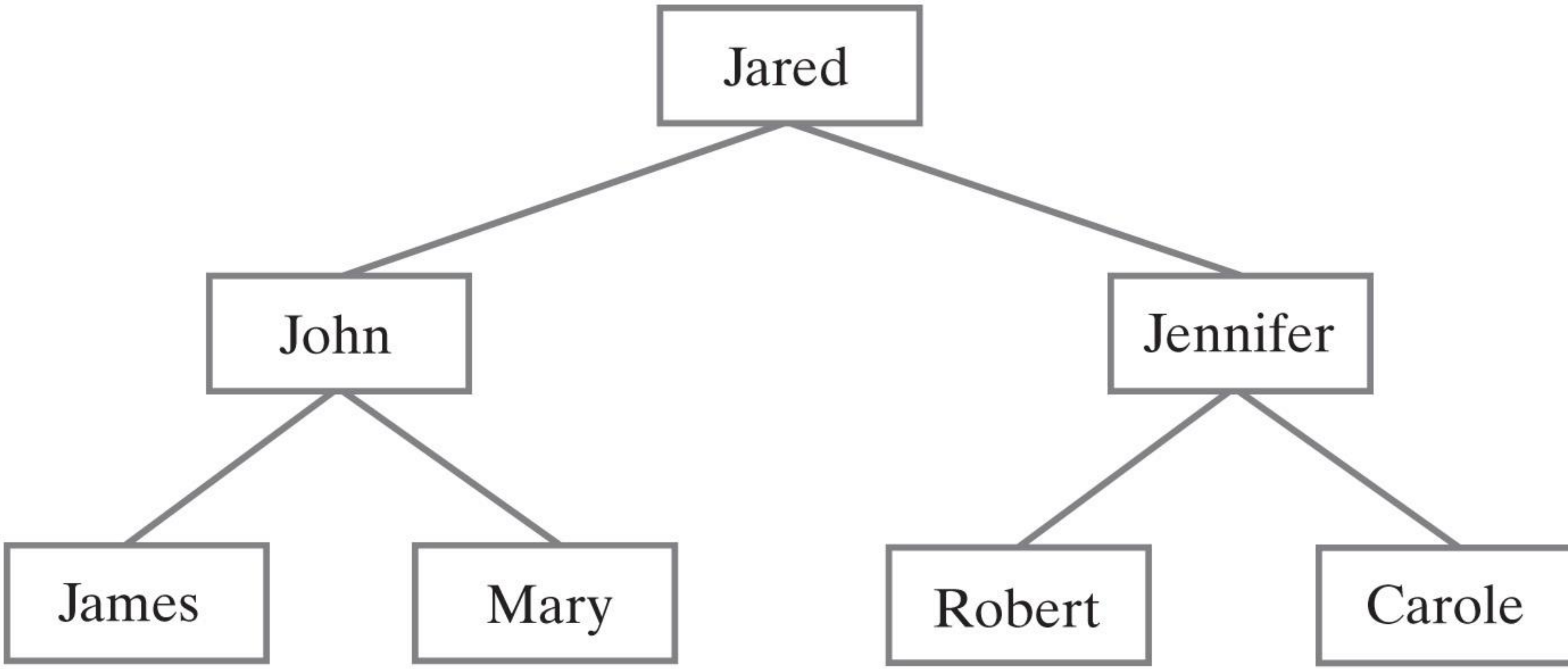
Hierarchical Organizations

- Carole’s children and grandchildren



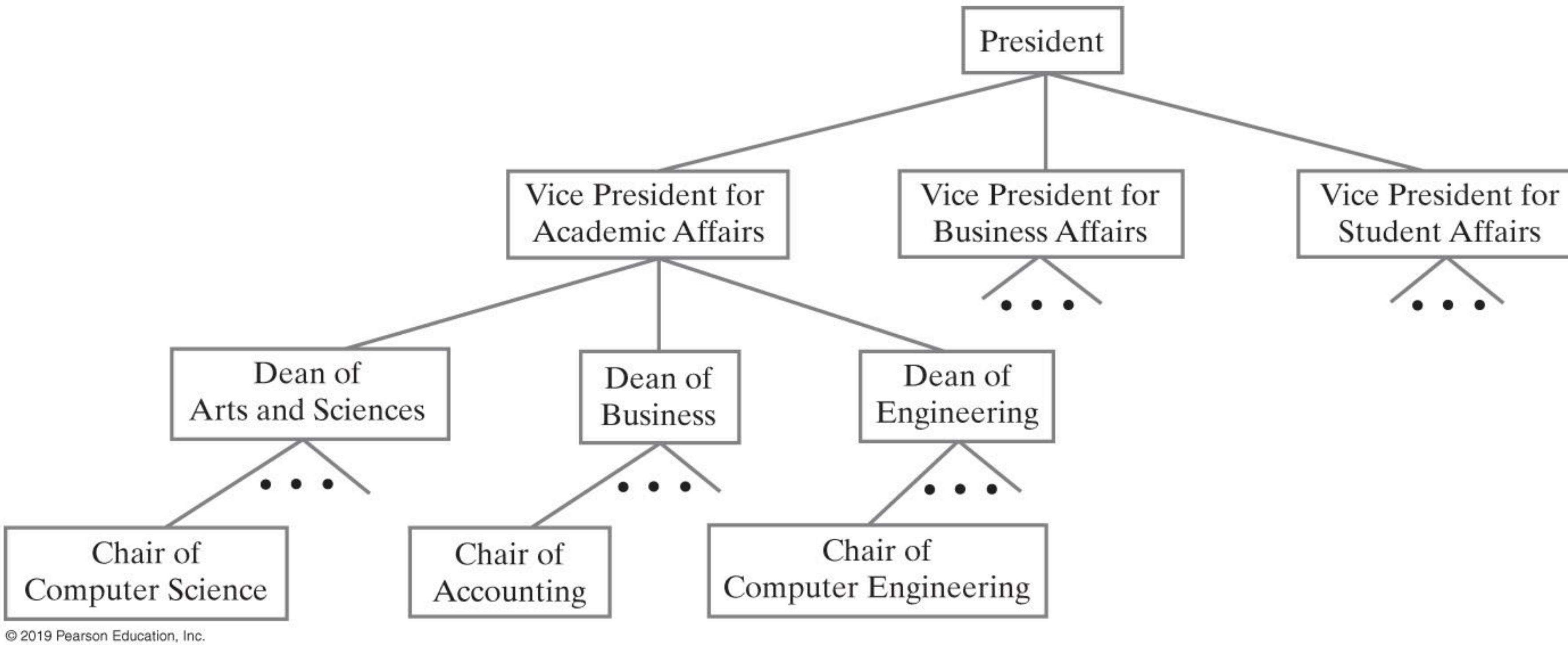
Hierarchical Organizations

- Jared's parents and grandparents



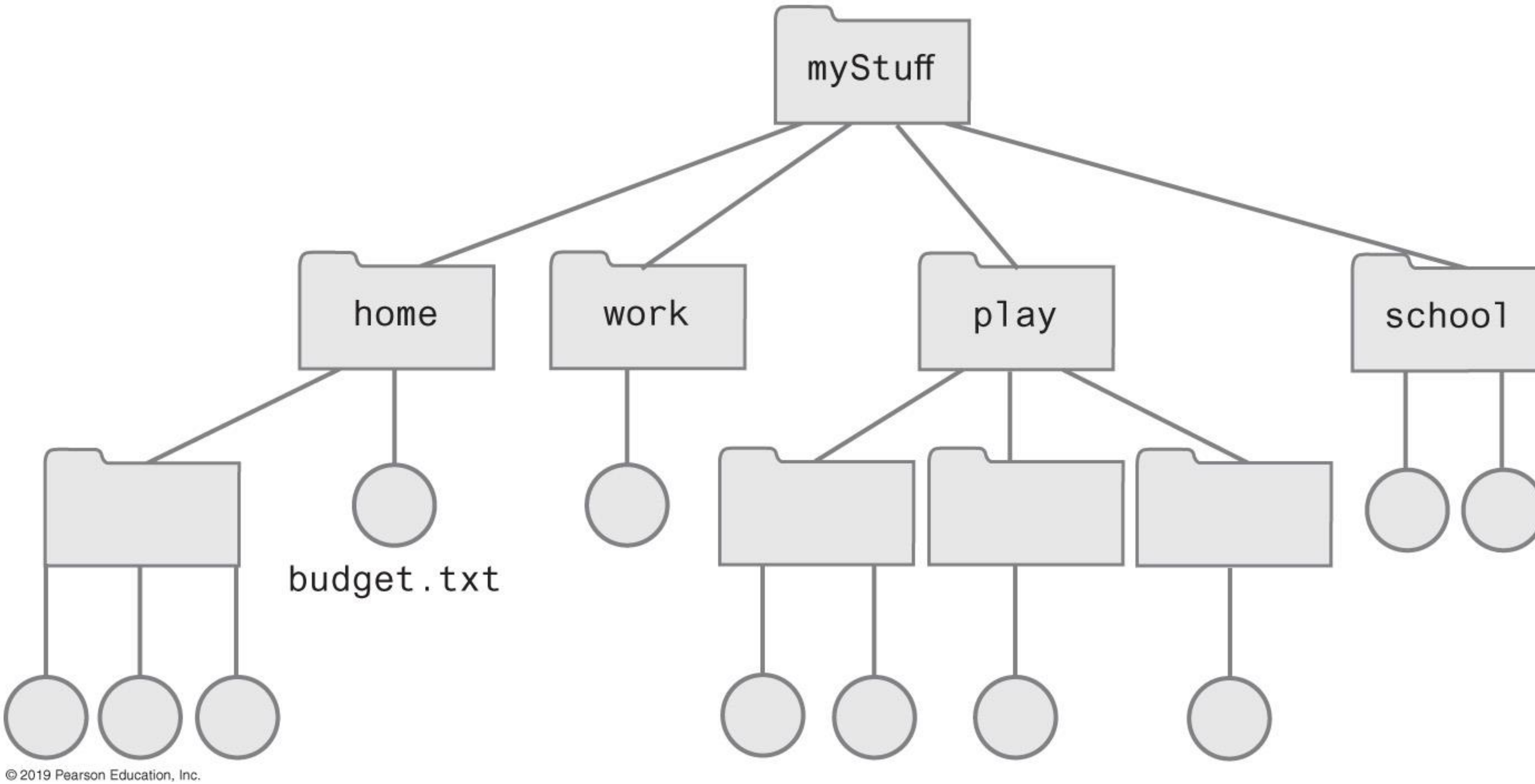
Hierarchical Organizations

- A portion of a university's administrative structure



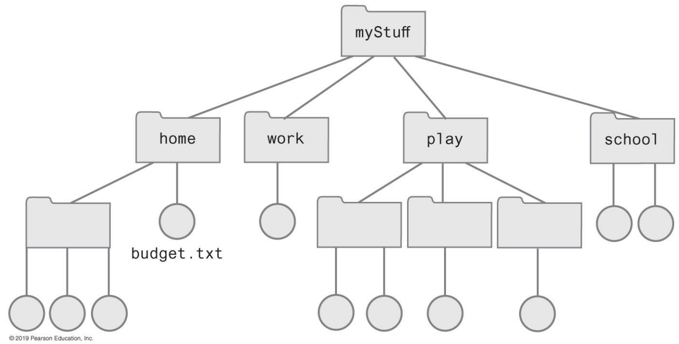
Hierarchical Organizations

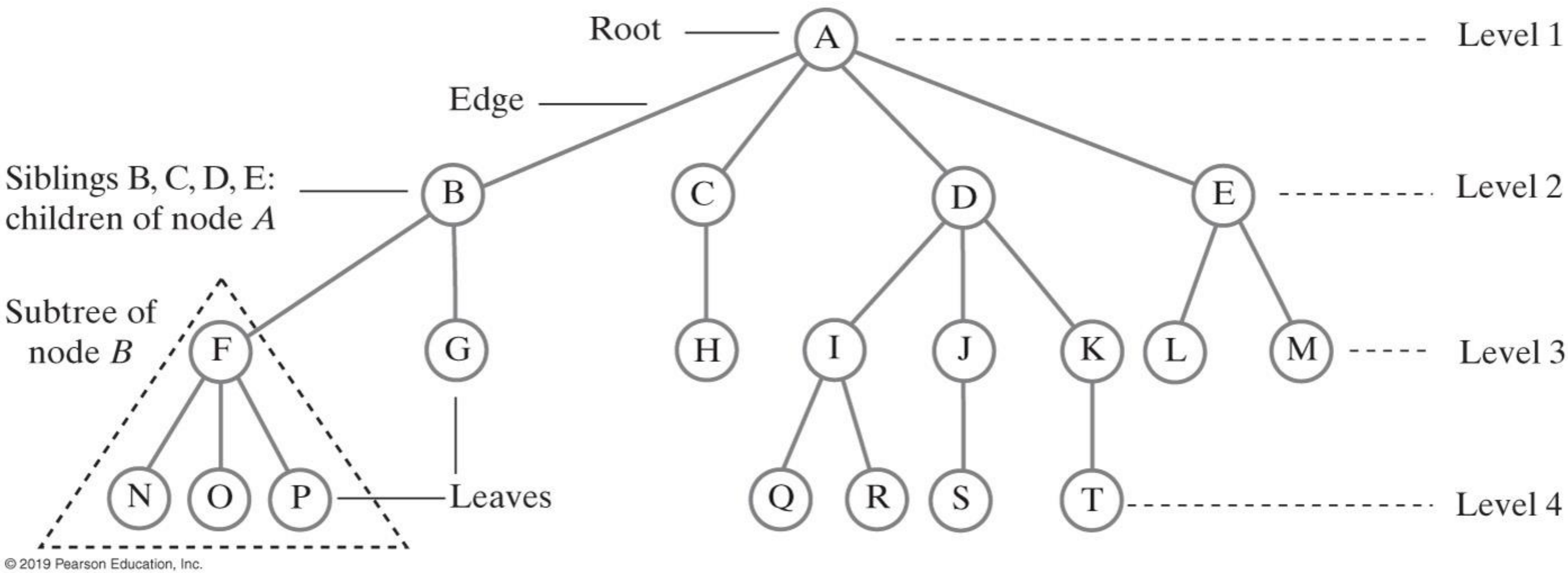
- Computer files organized into folders



Tree Terminology

- A tree equivalent to the file organization tree

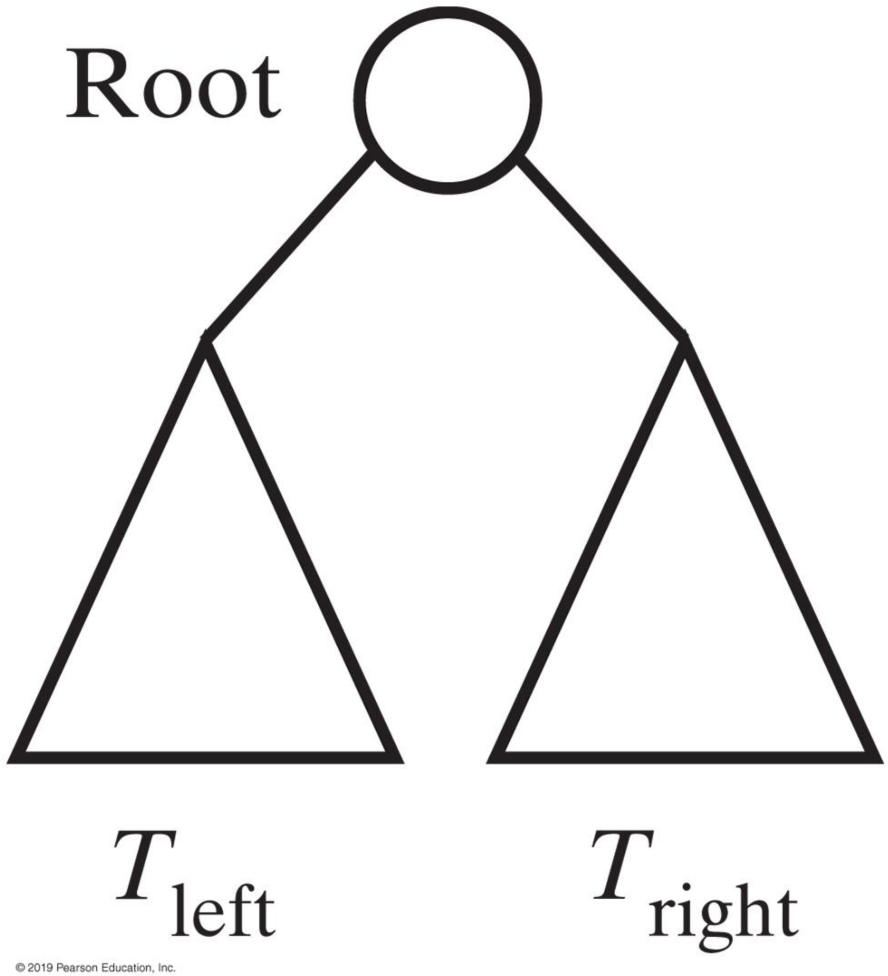




Tree Terminology

- Contrast plants with root at bottom
  - ADT tree with root at top
  - Root is only node with no parent
- A tree can be empty
- Any node and its descendants form a subtree of the original tree
- The height of a tree is the number of levels in the tree

Binary trees

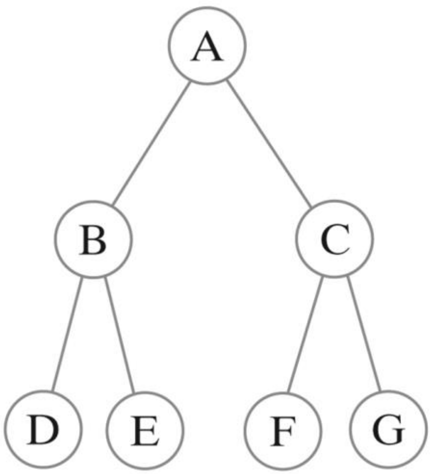


Tree types

- Tree is said to be **full** when a a binary tree of height h has all of its leaves at level h and every parent has exactly two children
- Tree is said to be **complete** when all levels but the last contain as many nodes as possible, and the nodes on the last level are filled in from left to tight. (Not full, but complete)
- When each node in a binary tree has two subtrees who's heights are exactly the same the tree is said to be **completely balanced**
- Completely balanced trees are full
- A tree is **height balanced** or simply **balanced** if the subtrees of a node differ by no more than one

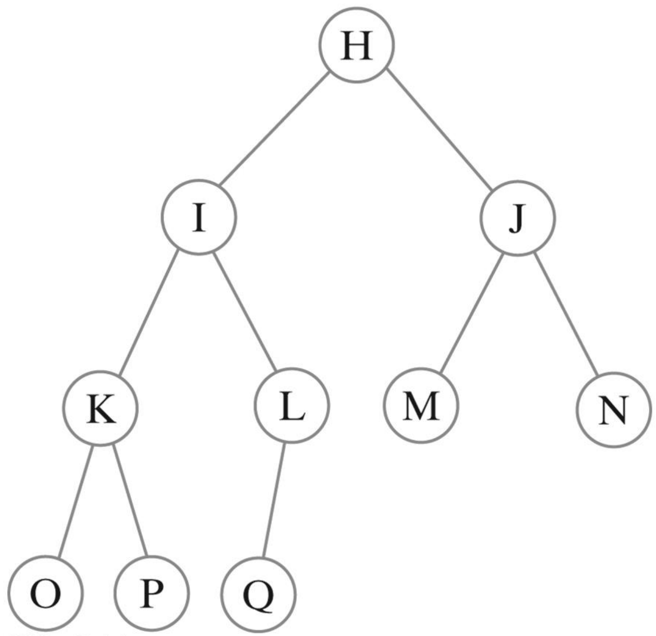
Three binary trees

(a) Full tree

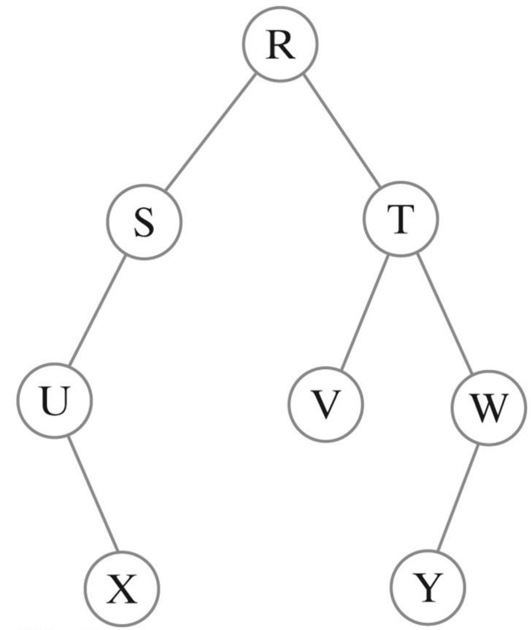


Left children: B, D, F
Right children: C, E, G

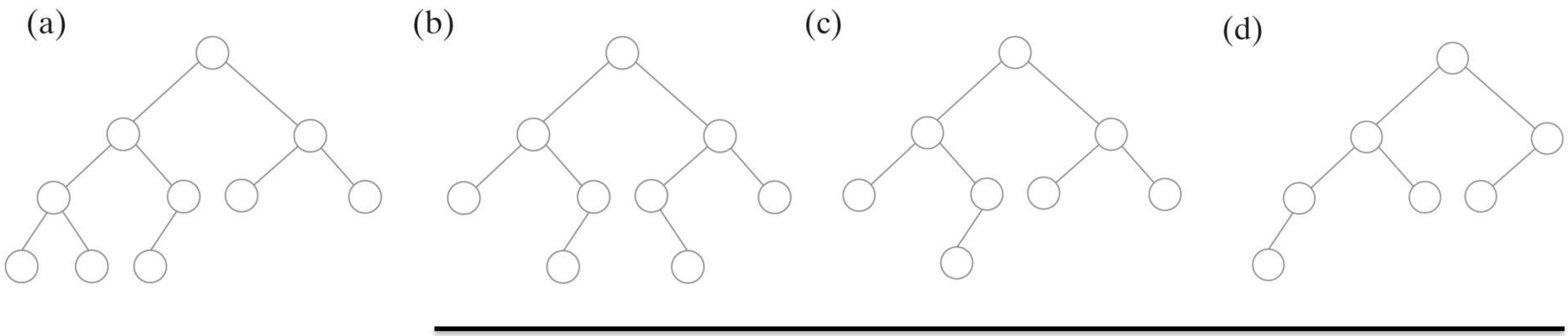
(b) Complete tree



(c) Tree that is not full and not complete



Height balanced Binary Trees

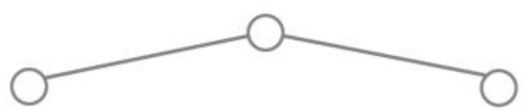
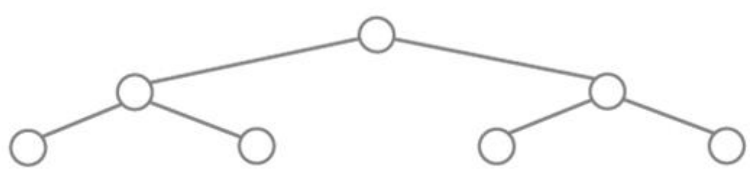


Balanced and complete

Balanced, but not complete

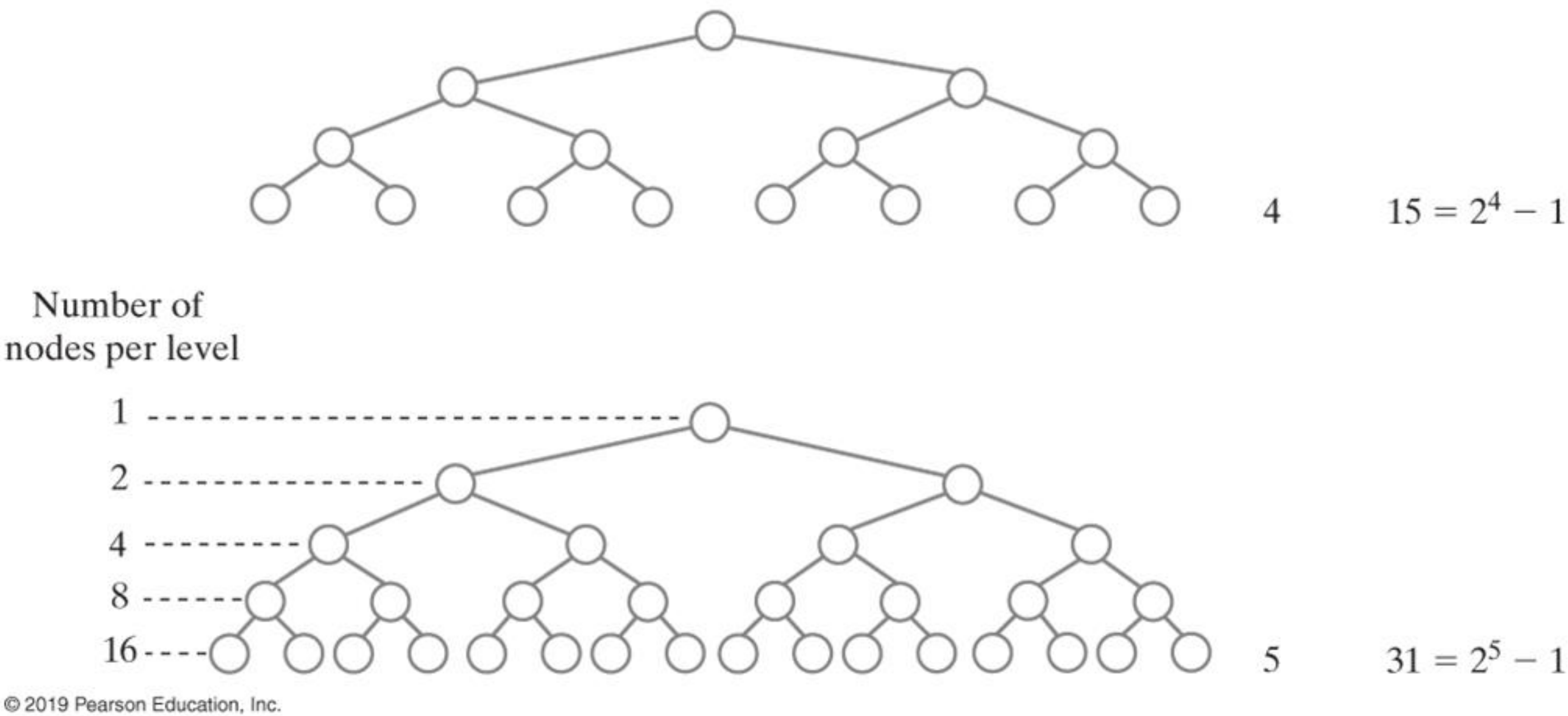
Binary Tree Height (Part 1)

- The number of nodes in a full binary tree as a function of the tree's height.
- Number of Nodes = $2^N - 1$, where N is height.

Full Tree	Height	Number of Nodes
	1	$1 = 2^1 - 1$
	2	$3 = 2^2 - 1$
	3	$7 = 2^3 - 1$

Binary Tree Height (Part 2)

- The number of nodes in a full binary tree as a function of the tree's height



Traversals of A Tree

- Traversal:
  - Visit, or process, each data item exactly once
- We will say that traversal can pass through a node without visiting it at that moment.
- Order in which we visit items is not unique
- Traversals of a binary tree are somewhat easy to understand

Traversals of a Binary Tree

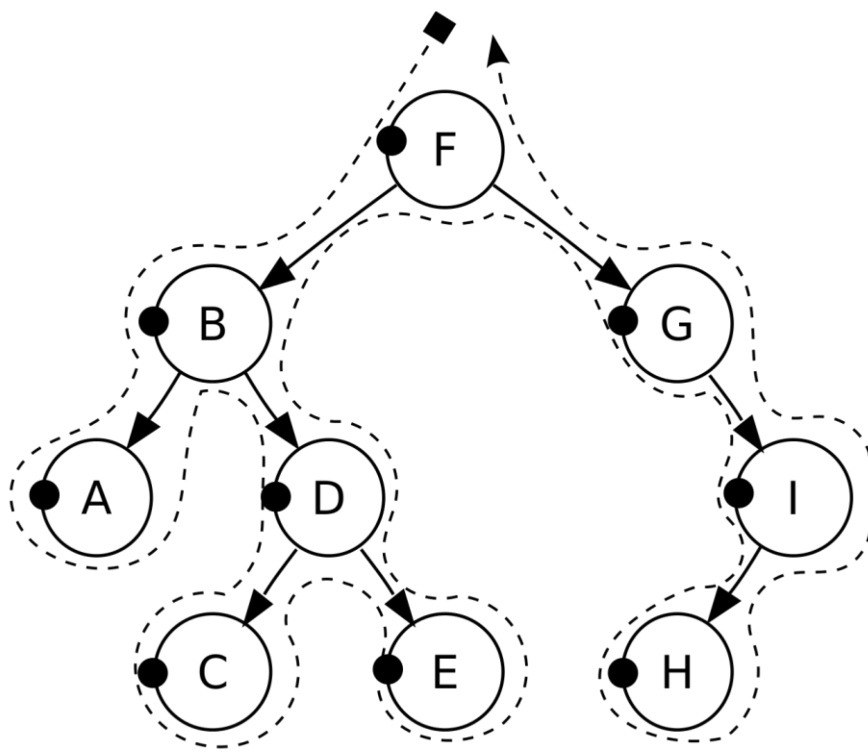
- We use recursion
- To visit all the nodes in a binary tree, we must
  - Visit the root
  - Visit all the nodes in the root's left subtree
  - Visit all the nodes in the root's right subtree

Traversals of a Binary Tree

- **Preorder traversal** — Root left Right
  - Visit root before we visit root's subtrees
- **Inorder traversal** — left Root Right
  - Visit root of a binary tree between visiting nodes in root's subtrees.
- **Postorder traversal** — left Right Root
  - Visit root of a binary tree after visiting nodes in root's subtrees
- **Level-order traversal**
  - Begin at root and visit nodes one level at a time

Preorder traversal

- Check if the current node is empty or null.
- Display the data part of the root (or current node).
- Traverse the left subtree by recursively calling the pre-order function.
- Traverse the right subtree by recursively calling the pre-order function.
- Follow the dots and dashed lines below: FBADCEGIH



Source: Wikipedia
https://en.wikipedia.org/wiki/Tree_traversal

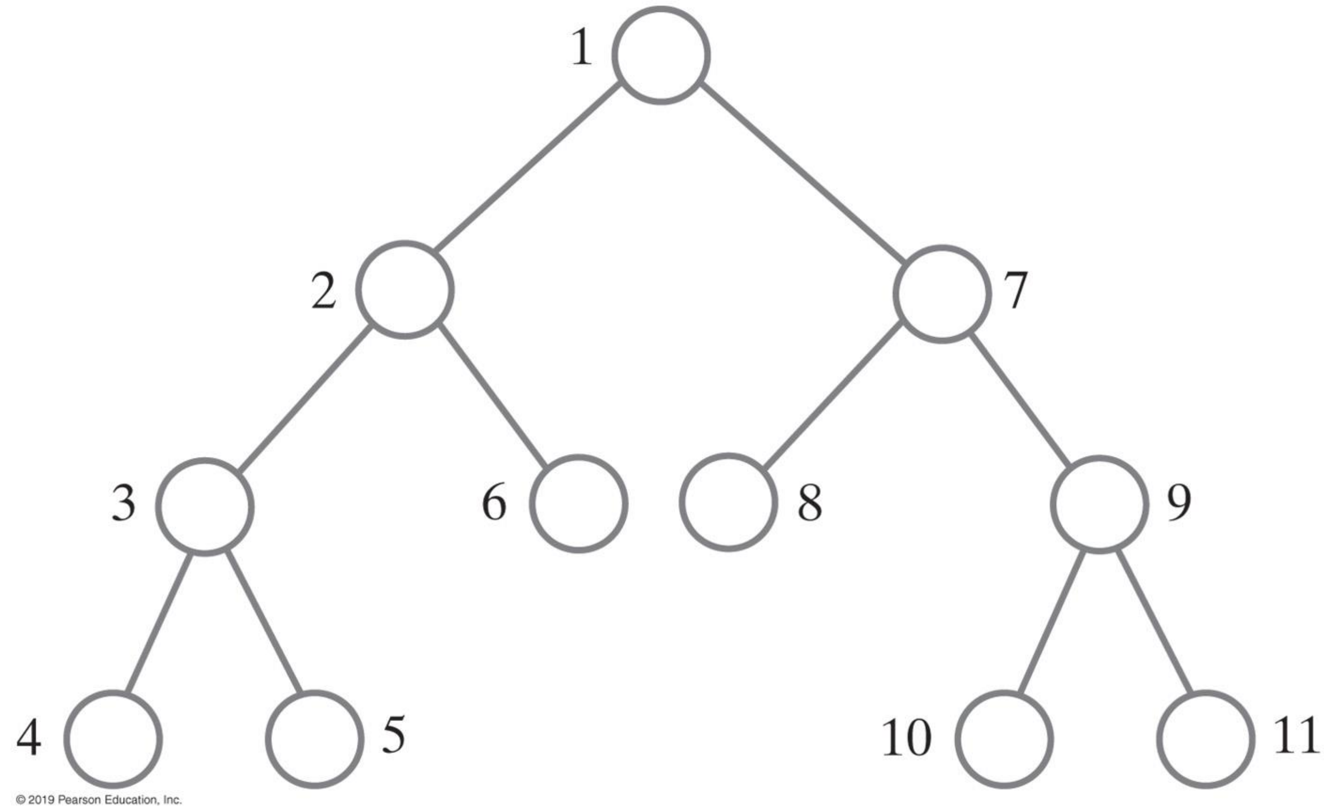
Preorder traversal algorithm

- https://en.wikipedia.org/wiki/Tree_traversal

```
preorder(node)
  if (node = null)
    return
  visit(node)
  preorder(node.left)
  preorder(node.right)
```

```
iterativePreorder(node)
  if (node = null)
    return
  s ← empty stack
  s.push(node)
  while (not s.isEmpty())
    node ← s.pop()
    visit(node)
    //right child is pushed first so that left is processed first
    if (node.right ≠ null)
      s.push(node.right)
    if (node.left ≠ null)
      s.push(node.left)
```

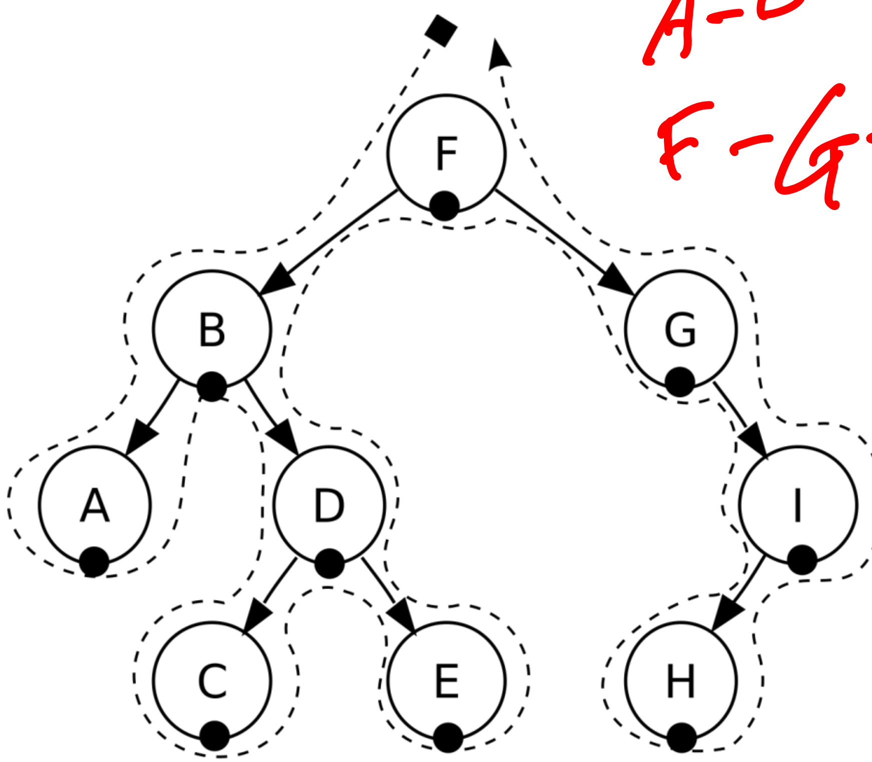
Preorder traversal



Inorder traversal

- Check if the current node is empty or null.
- Traverse the left subtree by recursively calling the in-order function.
- Display the data part of the root (or current node).
- Traverse the right subtree by recursively calling the in-order function.
- In a binary search tree, in-order traversal retrieves data in sorted order.
- Order: ABCDEFGHI

A-B-C-D-E
F-G-H-I



Source: Wikipedia
https://en.wikipedia.org/wiki/Tree_traversal

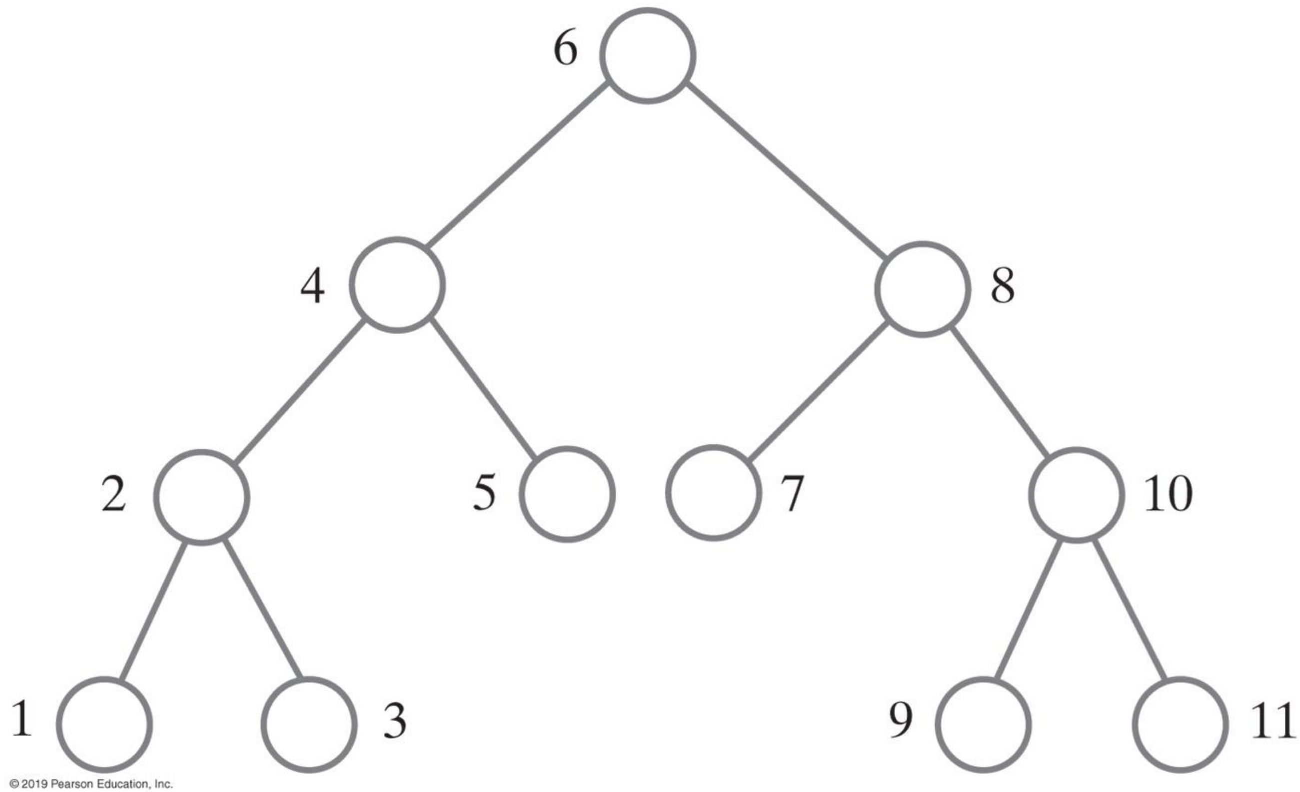
Inorder traversal algorithm

- https://en.wikipedia.org/wiki/Tree_traversal

```
inorder(node)
  if (node = null)
    return
  inorder(node.left)
  visit(node)
  inorder(node.right)
```

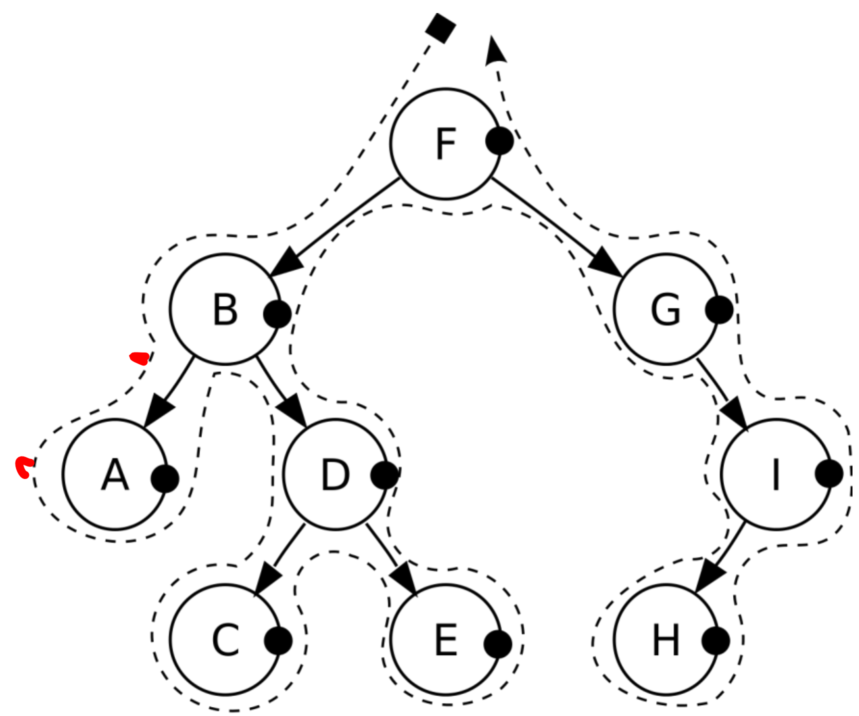
```
iterativeInorder(node)
  s ← empty stack
  while (not s.isEmpty() or node ≠ null)
    if (node ≠ null)
      s.push(node)
      node ← node.left
    else
      node ← s.pop()
      visit(node)
      node ← node.right
```

Inorder traversal



Postorder traversal

- Check if the current node is empty or null.
- Traverse the left subtree by recursively calling the post-order function.
- Traverse the right subtree by recursively calling the post-order function.
- Display the data part of the root (or current node).
- Order: ACEDBHIGF



Source: Wikipedia
https://en.wikipedia.org/wiki/Tree_traversal

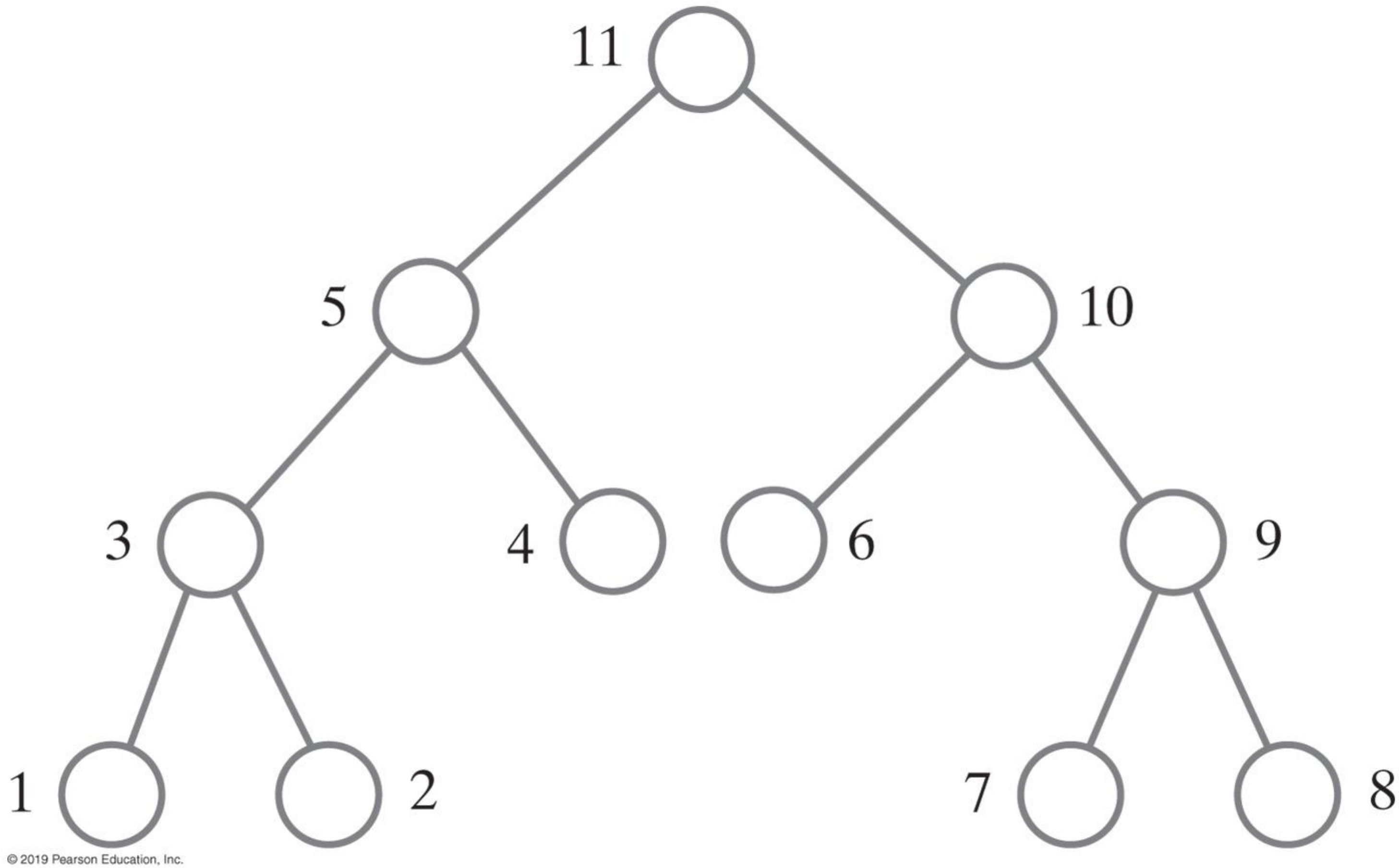
Postorder traversal algorithm

- https://en.wikipedia.org/wiki/Tree_traversal

```
postorder(node)
  if (node = null)
    return
  postorder(node.left)
  postorder(node.right)
  visit(node)
```

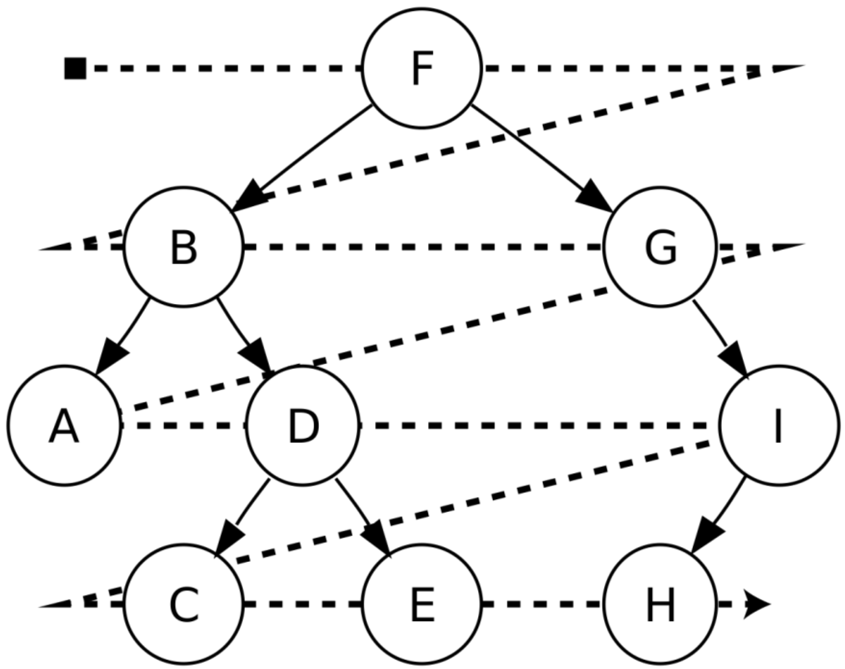
```
iterativePostorder(node)
  s ← empty stack
  lastNodeVisited ← null
  while (not s.isEmpty() or node ≠ null)
    if (node ≠ null)
      s.push(node)
      node ← node.left
    else
      peekNode ← s.peek()
      // if right child exists and traversing node
      // from left child, then move right
      if (peekNode.right ≠ null and lastNodeVisited ≠ peekNode.right)
        node ← peekNode.right
      else
        visit(peekNode)
        lastNodeVisited ← s.pop()
```

Postorder traversal



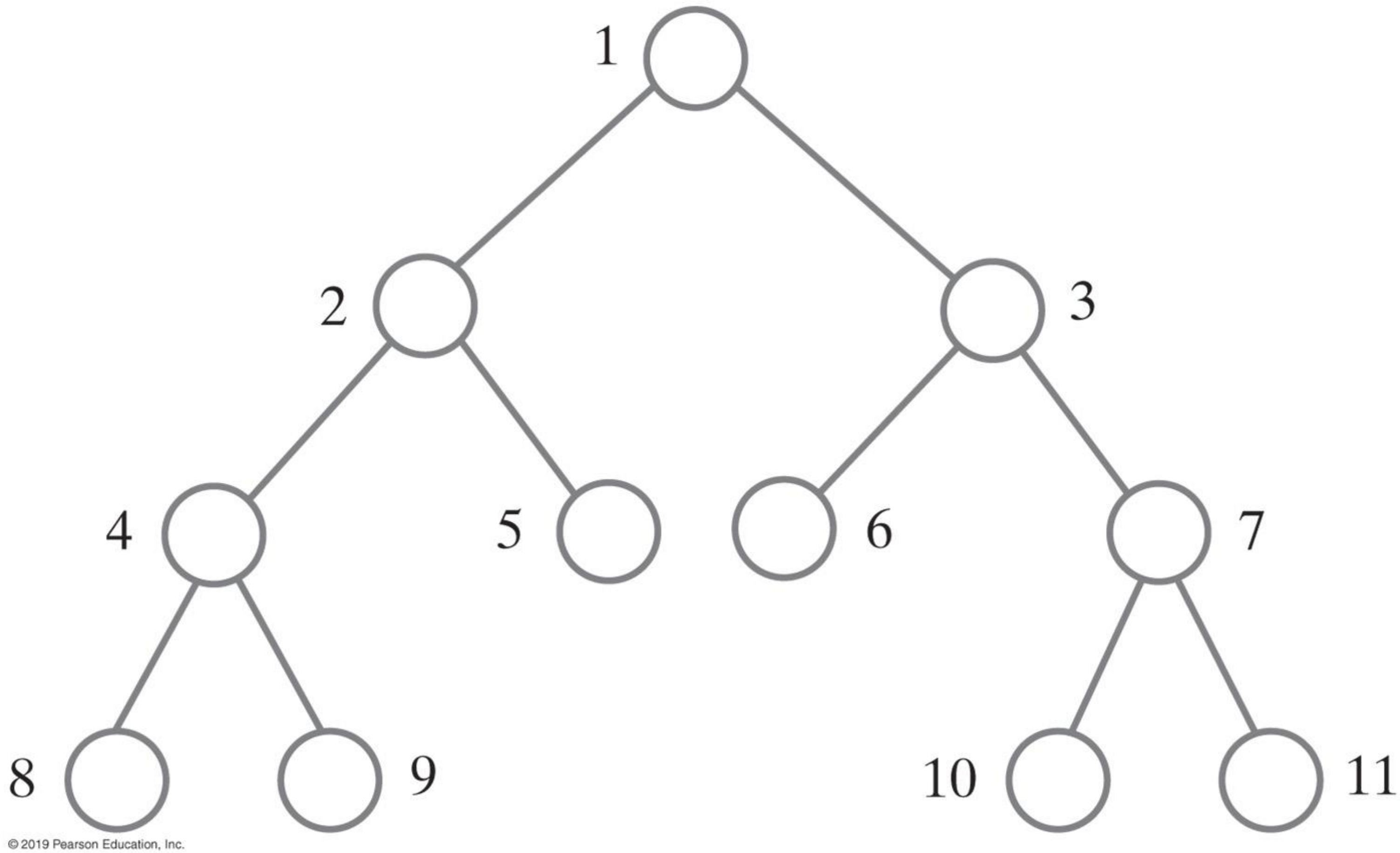
Level order traversal

- Visits each node at each level
- FBGADICEH



Source: Wikipedia
https://en.wikipedia.org/wiki/Tree_traversal

Level-order traversal



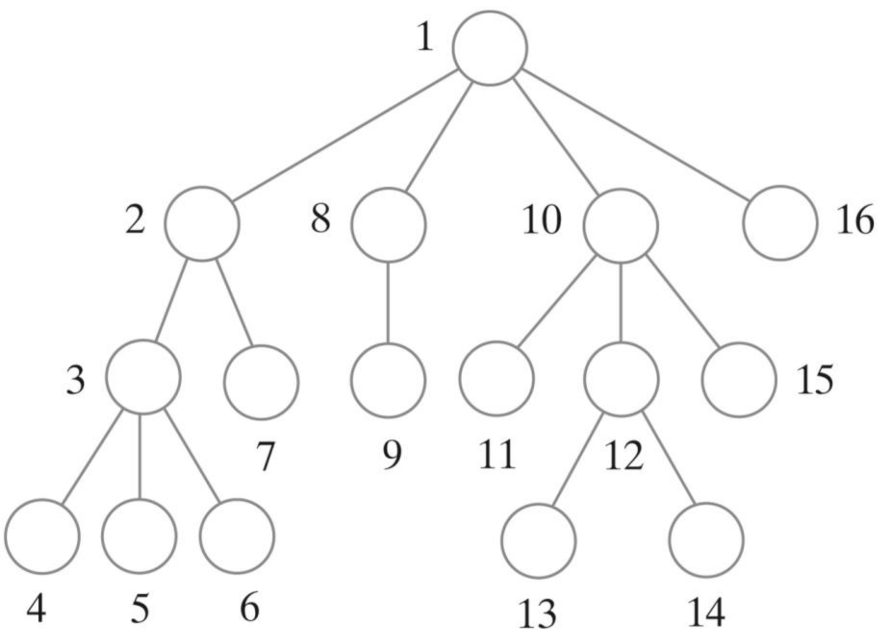
Traversals of a General Tree

- Types of traversals for general tree
  - Level order
  - Preorder
  - Postorder
- Not suited for general tree traversal
  - Inorder

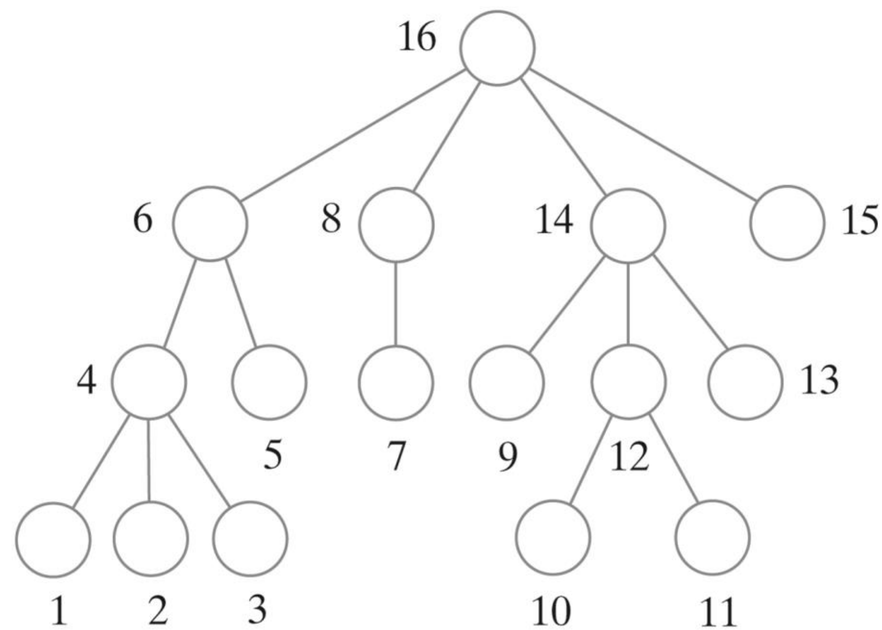
We don't know when we visit the root.

Traversals of a General Tree

- The visitation order of two traversals of a general tree



(a) Preorder traversal



(b) Postorder traversal

Basic Tree Interface

```
public interface TreeInterface<T> {

	/**
	 * Gets the data at the root node of the tree
	 * @return data of type T in the node
	 */
	public T getRootData();

	/**
	 * Get the height of the tree
	 * @return tree height
	 */
	public int getHeight();

	/**
	 * Get the number of nodes in the tree from the root
	 * @return number of nodes
	 */
	public int getNumberOfNodes();

	/**
	 * Checks to see if the tree has any nodes
	 * @return true if there are no nodes in the tree
	 */
	public boolean isEmpty();

	/**
	 * Clears all nodes in the tree, leaving it empty
	 */
	public void clear();
}
```

Traversals

- Iterator interface specifies iterators for each type of tree traversal

```java
public interface TreeIteratorInterface<T> {
      public Iterator<T> getPreorderIterator();

      public Iterator<T> getPostorderIterator();

      public Iterator<T> getInorderIterator();

      public Iterator<T> getLevelOrderIterator();
}
```

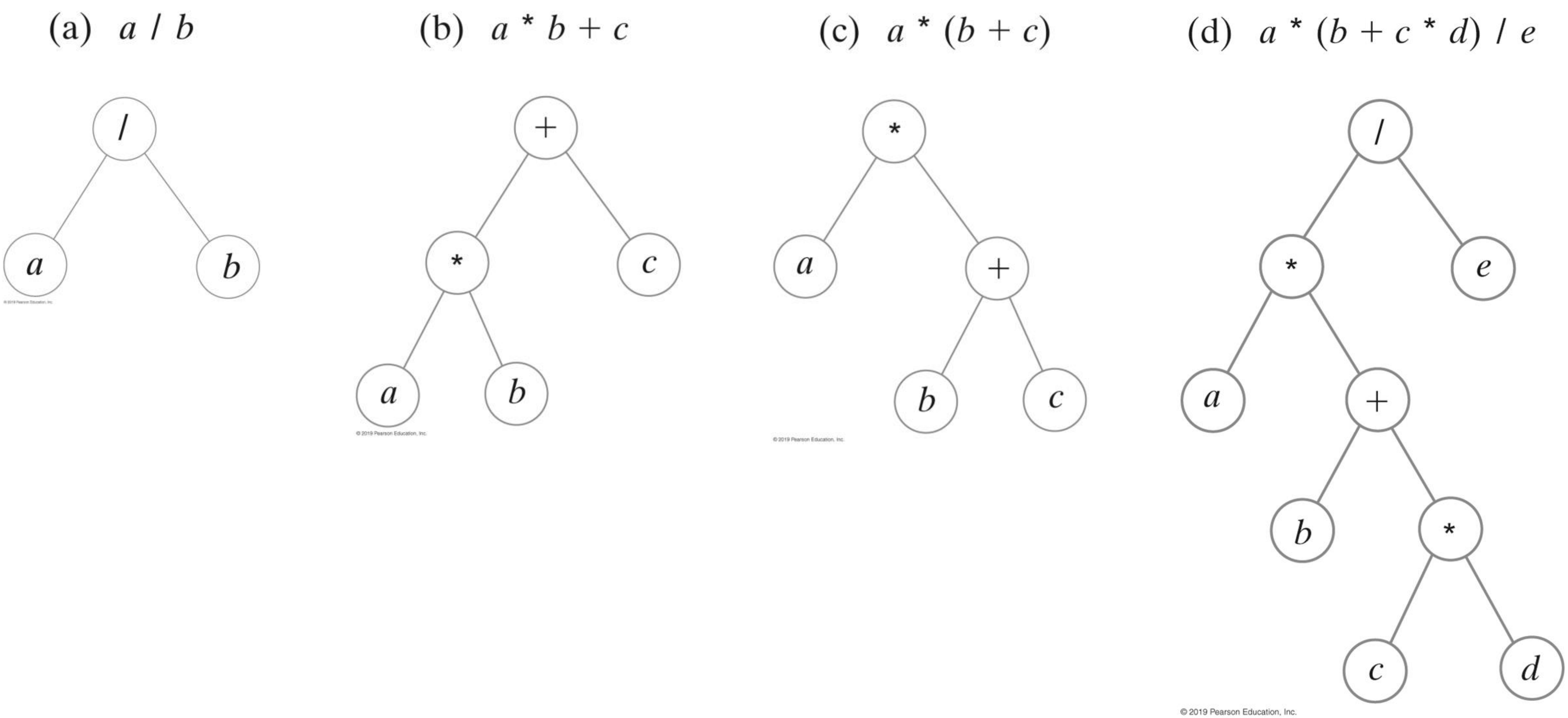
Binary Tree Interface

- In a binary tree, nodes only have two children (left and right).
- Interface adds setRootData() method

```
public interface BinaryTreeInterface<T> extends TreeInterface<T>, TreeIteratorInterface<T> {
    /**
     * Sets the data in the root of this binary tree.
     *
     * @param rootData The object that is the data for the tree's root.
     */
    public void setRootData(T rootData);

    /**
     * Sets this binary tree to a new binary tree.
     *
     * @param rootData  The object that is the data for the new tree's root.
     * @param leftTree  The left subtree of the new tree.
     * @param rightTree The right subtree of the new tree.
     */
    public void setTree(T rootData, BinaryTreeInterface<T> leftTree, BinaryTreeInterface<T> rightTree);
}
```

Expression Trees

- Expression trees for four algebraic expressions



Expression Trees

- Algorithm for postorder traversal of an expression tree.

Algorithm **evaluate(expressionTree)**

if (expressionTree *is empty*)

    return 0

else

{

    firstOperand = evaluate(*left subtree of* expressionTree)

    secondOperand = evaluate(*right subtree of* expressionTree)

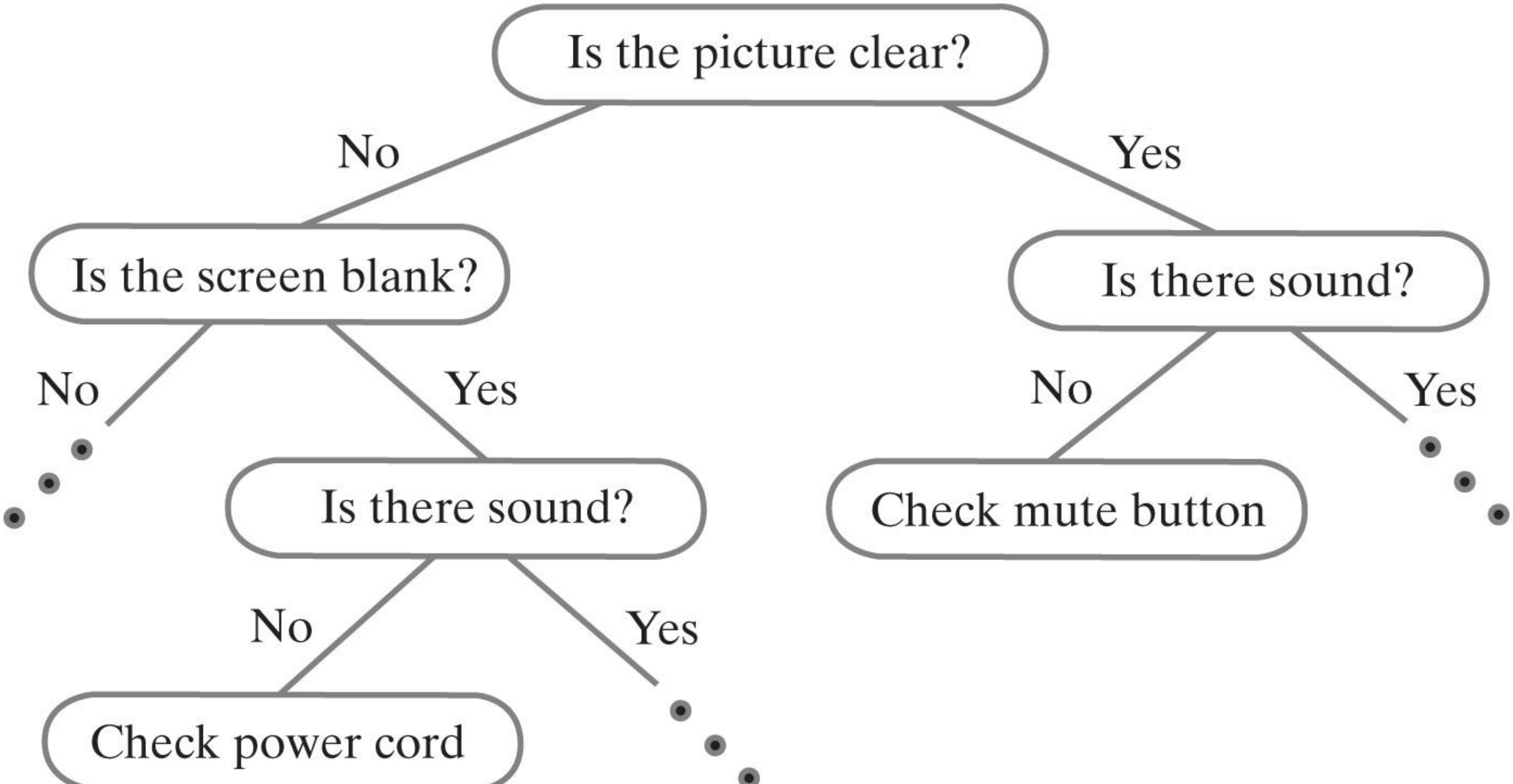
    operator = *the root of* expressionTree

    return *the result of the operation* operator *and its operands* firstOperand

        and secondOperand

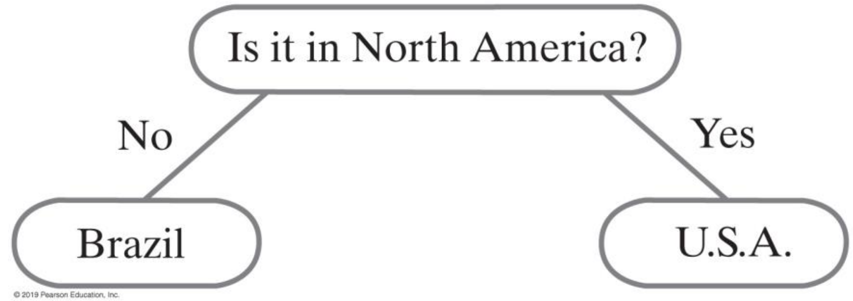
}

Expert System Using A Decision Tree



Guessing Game

- An initial decision tree for a guessing game



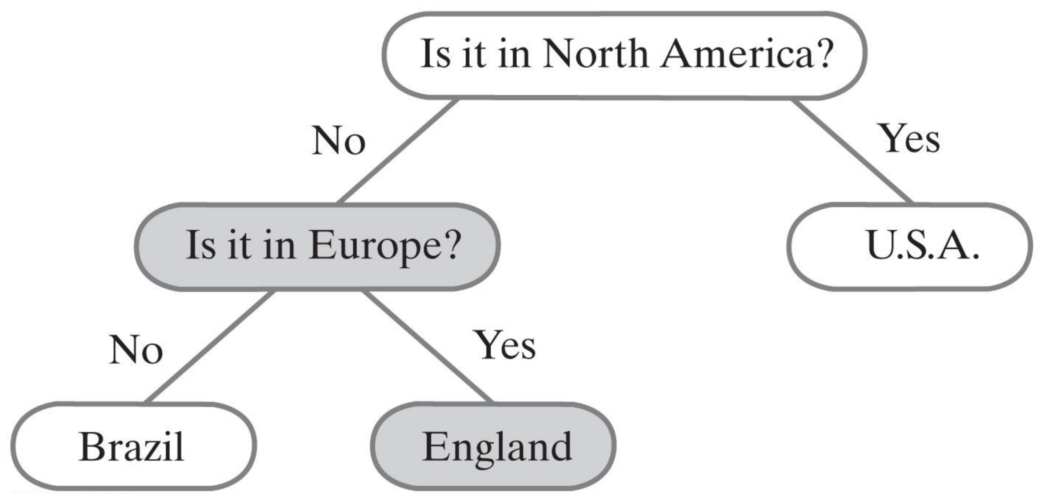


FIGURE 24-18 The decision tree for a guessing game after acquiring another fact

Decision Tree Interface

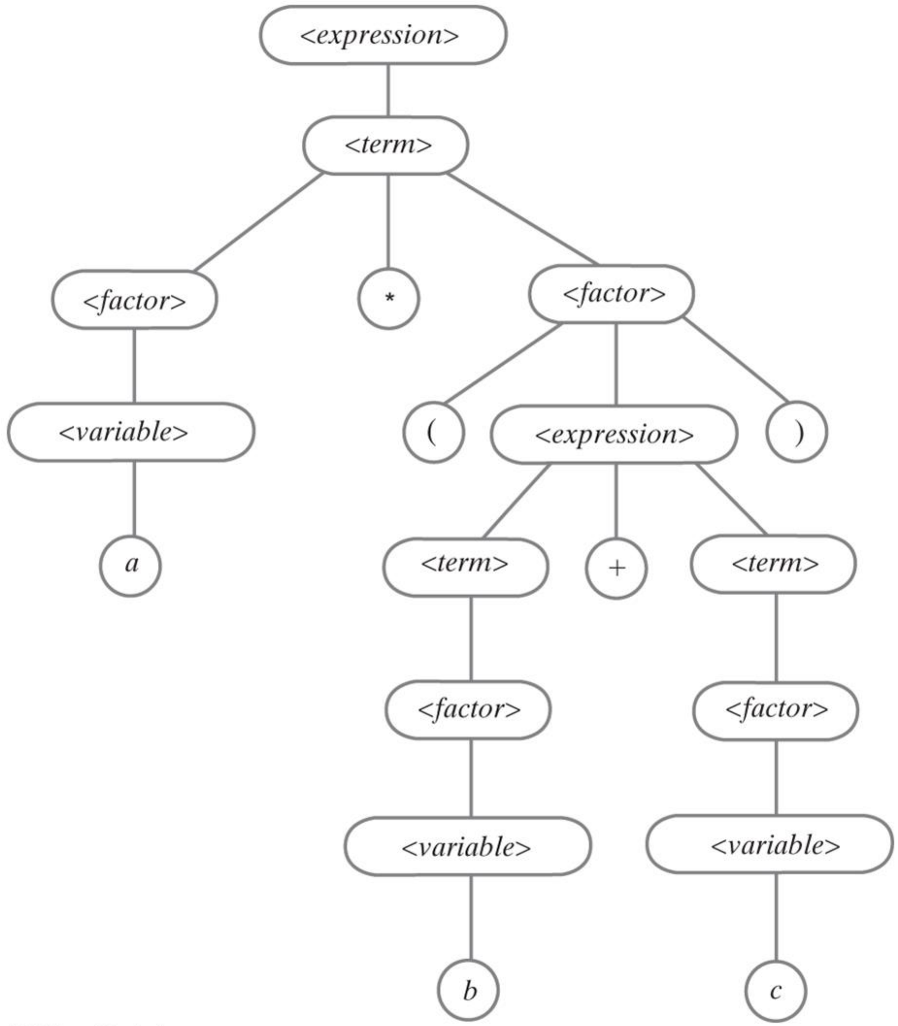
```java
public interface DecisionTreeInterface<T> extends BinaryTreeInterface<T> {
                /**
                 * Gets the data in the current node.
                 *
                 * @return The data object in the current node, or null if the current node is
                 *         null.
                 */
                public T getCurrentData();

                /**
                 * Sets the data in the current node. Precondition: The current node is not
                 * null.
                 *
                 * @param newData The new data object.
                 */
                public void setCurrentData(T newData);

                /**
                 * Sets the data in the children of the current node, creating them if they do
                 * not exist. Precondition: The current node is not null.
                 *
                 * @param responseForNo  The new data object for the left child.
                 * @param responseForYes The new data object for the right child.
                 */
                public void setResponses(T responseForNo, T responseForYes);

                /**
                 * Sees whether the current node contains an answer.
                 *
                 * @return True if the current node is a leaf, or false if it is a nonleaf.
                 */
                public boolean isAnswer();

                /**
                 * Sets the current node to its left child. If the child does not exist, sets
                 * the current node to null.
                 */
                public void advanceToNo();

                /**
                 * Sets the current node to its right child. If the child does not exist, sets
                 * the current node to null.
                 */
                public void advanceToYes();

                /** Sets the current node to the root of the tree. */
                public void resetCurrentNode();
}
```

Parse Trees

- Parse tree
  - Check syntax of a string for valid algebraic expression
  - If valid can be expressed as a parse tree
- Parse tree must be a general tree
  - So it can accommodate any expression
- Compilers use parse trees
  - Check syntax, produce code

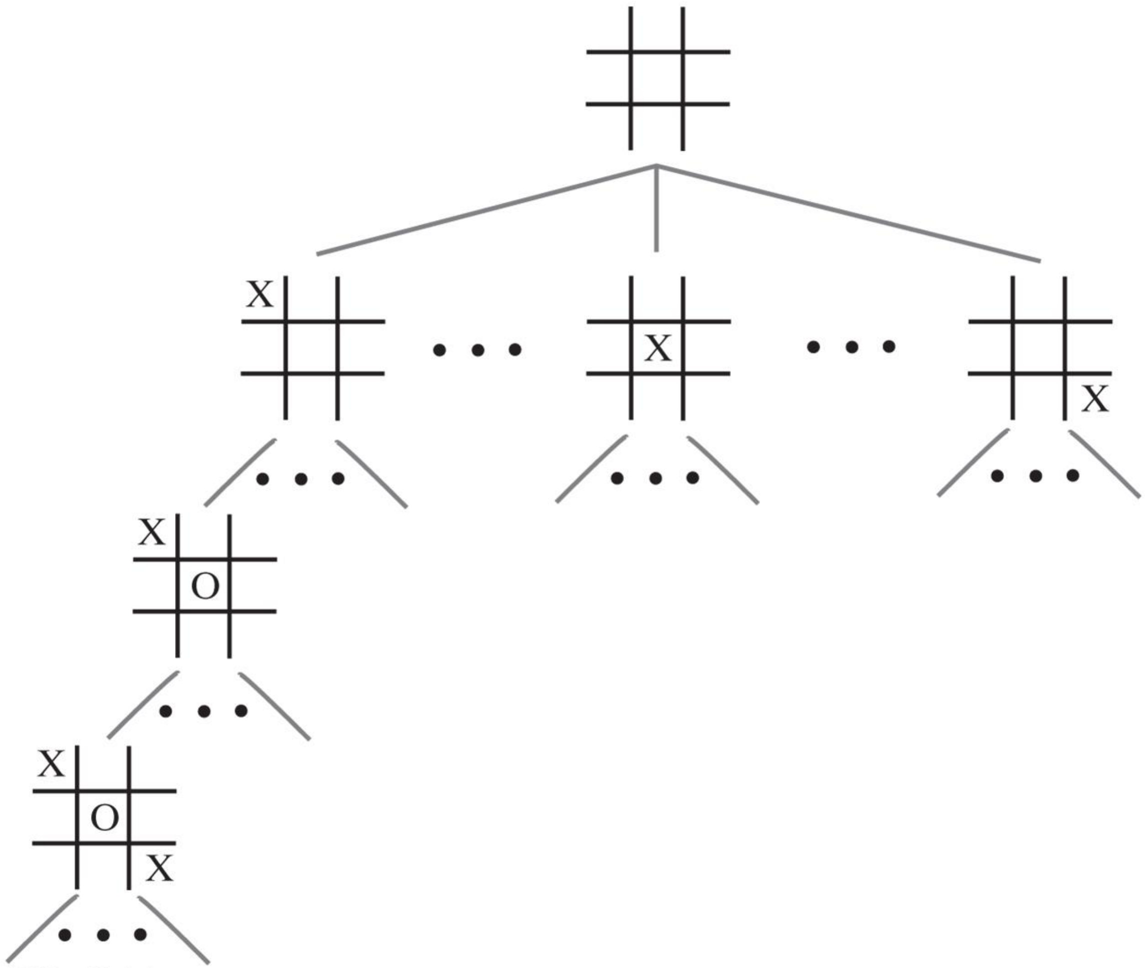
Parse Tree for an Equation

- A parse tree for the algebraic expression `a * (b + c)`



Parse Tree for a Game

- A portion of a game tree for tic-tac-toe



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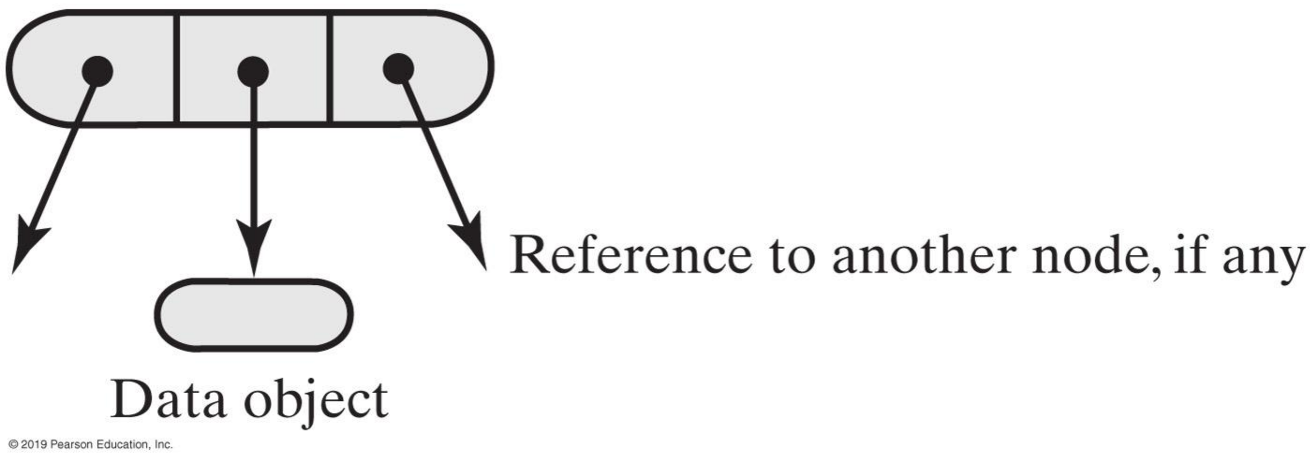
Binary Tree

- Define a BinaryTreeInterface
  - be able to set the root of the tree and retrieve data
  - in addition to TreeInterface
- BinaryNode class
  - Able to use left/right children
  - Get other characteristics of the node
    - Height of tree from this node down
    - Copy all nodes to another tree
    - Get number of nodes from this node down

Nodes in a Binary Tree



BinaryNode class – constructors and data

```
public class BinaryNode<T> {
    private T data;
    private BinaryNode<T> leftChild; // Reference to left child
    private BinaryNode<T> rightChild; // Reference to right child

    public BinaryNode() {
        this(null); // Call next constructor
    }

    public BinaryNode(T dataPortion) {
        this(dataPortion, null, null); // Call next constructor
    }

    public BinaryNode(T dataPortion, BinaryNode<T> newLeftChild, BinaryNode<T> newRightChild) {
        data = dataPortion;
        leftChild = newLeftChild;
        rightChild = newRightChild;
    }

    /**
     * Retrieves the data portion of this node.
     *
     * @return The object in the data portion of the node.
     */
    public T getData() {
        return data;
    }

    /**
     * Sets the data portion of this node.
     *
     * @param newData The data object.
     */
    public void setData(T newData) {
        data = newData;
    }
```

BinaryNode – child methods

```java
/**
 * Retrieves the left child of this node.
 *
 * @return A reference to this node's left child.
 */
public BinaryNode<T> getLeftChild() {
      return leftChild;
}

/**
 * Sets this node’s left child to a given node.
 *
 * @param newLeftChild A node that will be the left child.
 */
public void setLeftChild(BinaryNode<T> newLeftChild) {
      leftChild = newLeftChild;
}

/**
 * Detects whether this node has a left child.
 *
 * @return True if the node has a left child.
 */
public boolean hasLeftChild() {
      return leftChild != null;
}
```

BinaryNode – child methods

```java
/**
 * Retrieves the right child of this node.
 *
 * @return A reference to this node's right child.
 */
public BinaryNode<T> getRightChild() {
    return rightChild;
}

/**
 * Sets this node’s right child to a given node.
 *
 * @param newRightChild A node that will be the right child.
 */
public void setRightChild(BinaryNode<T> newRightChild) {
    rightChild = newRightChild;
}

/**
 * Detects whether this node has a right child.
 *
 * @return True if the node has a right child.
 */
public boolean hasRightChild() {
    return rightChild != null;
}
```

BinaryNode utility methods

- use recursive traversal to get the tree height and number of nodes from the current node.

```
    /**
     * Counts the nodes in the subtree rooted at this node.
     * Do this by recursively descending the tree, counting as we go.
     *
     * @return The number of nodes in the subtree rooted at this node.
     */
    public int getNumberOfNodes() {
        int leftNumber = 0;
        int rightNumber = 0;
        if (leftChild != null)
            leftNumber = leftChild.getNumberOfNodes();
        if (rightChild != null)
            rightNumber = rightChild.getNumberOfNodes();
        return 1 + leftNumber + rightNumber;
    }

    /**
     * Computes the height of the subtree rooted at this node.
     *
     * @return The height of the subtree rooted at this node.
     */
    public int getHeight() {
        return getHeight(this); // Call private getHeight
    }

    /**
     * recursively descend the tree, counting as we go
     * @param node
     * @return tree height
     */
    private int getHeight(BinaryNode<T> node) {
        int height = 0;
        if (node != null)
            height = 1 + Math.max(getHeight(node.getLeftChild()), getHeight(node.getRightChild()));
        return height;
    }
```

BinaryNode utility methods

- Can make a new copy of a node, and recursively copy the rest of the children

```
public boolean isLeaf() {
    return (leftChild == null) && (rightChild == null);
}

/**
 * Copies the subtree rooted at this node.
 *
 * @return The root of a copy of the subtree rooted at this node.
 */
public BinaryNode<T> copy() {
    BinaryNode<T> newRoot = new BinaryNode<>(data);
    if (leftChild != null)
        newRoot.setLeftChild(leftChild.copy());

    if (rightChild != null)
        newRoot.setRightChild(rightChild.copy());

    return newRoot;
}
```

BinaryTreeInterface

- Make sure that all binary trees can set a root node and data

```
public interface BinaryTreeInterface<T> extends TreeInterface<T>, TreeIteratorInterface<T>
{
    /**
     * Sets the data in the root of this binary tree.
     *
     * @param rootData The object that is the data for the tree's root.
     */
    public void setRootData(T rootData);

    /**
     * Sets this binary tree to a new binary tree.
     *
     * @param rootData  The object that is the data for the new tree's root.
     * @param leftTree  The left subtree of the new tree.
     * @param rightTree The right subtree of the new tree.
     */
    public void setTree(T rootData, BinaryTreeInterface<T> leftTree,
    BinaryTreeInterface<T> rightTree);
}
```

Binary Tree class

- Uses a private BinaryNode as the root
- Options to set up the tree with two existing subtrees

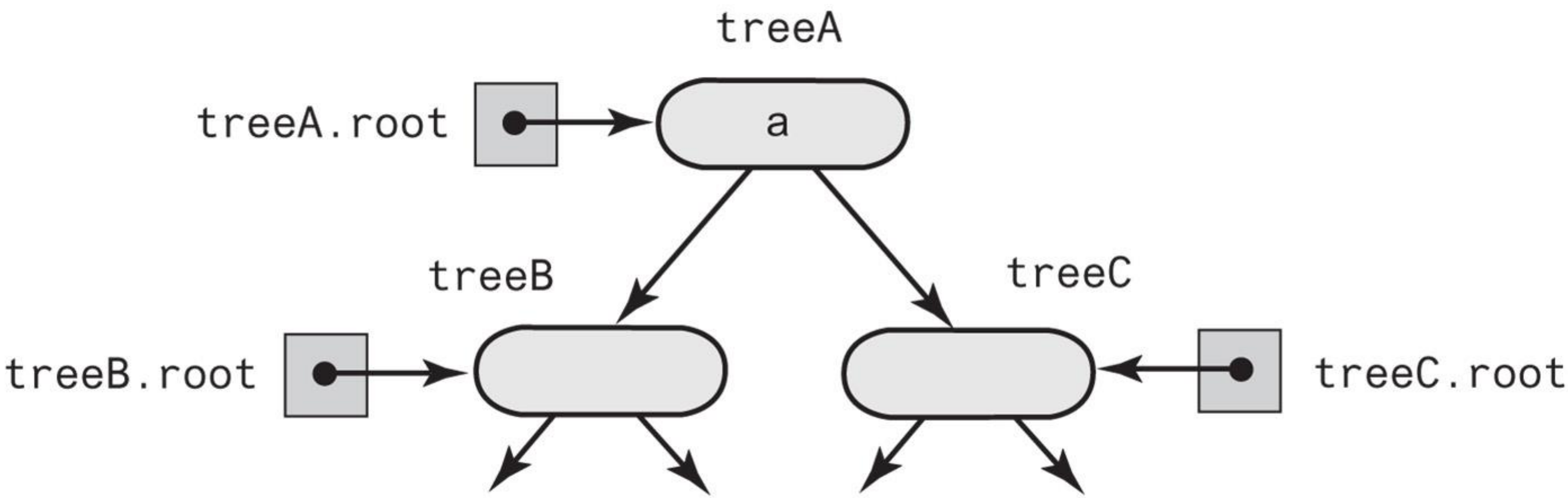
```
public class BinaryTree<T> implements BinaryTreeInterface<T> {
     private BinaryNode<T> root;

     public BinaryTree() {
          root = null;
     } // end default constructor

     /**
      * Create the tree with a single root node from data
      * @param rootData
      */
     public BinaryTree(T rootData) {
          root = new BinaryNode<>(rootData);
     } // end constructor

     /**
      * Create the tree from a new root node and two other trees that will now become subtrees from root.
      * @param rootData
      * @param leftTree
      * @param rightTree
      */
     public BinaryTree(T rootData, BinaryTree<T> leftTree, BinaryTree<T> rightTree) {
          initializeTree(rootData, leftTree, rightTree);
     } // end constructor

     public void setTree(T rootData, BinaryTreeInterface<T> leftTree, BinaryTreeInterface<T> rightTree) {
          initializeTree(rootData, (BinaryTree<T>) leftTree, (BinaryTree<T>) rightTree);
     } // end setTree
```

Initialize the tree

```java
/**
 * Combine two subtrees into a single tree with a new root.
 * @param rootData
 * @param leftTree
 * @param rightTree
 */
private void initializeTree(T rootData, BinaryTree<T> leftTree, BinaryTree<T> rightTree) {
     // < FIRST DRAFT - See Segments 25.4 - 25.7 for improvements. >
     root = new BinaryNode<T>(rootData);

     if (leftTree != null)
          root.setLeftChild(leftTree.root);

     if (rightTree != null)
          root.setRightChild(rightTree.root);
}
```

Creating a Binary Tree

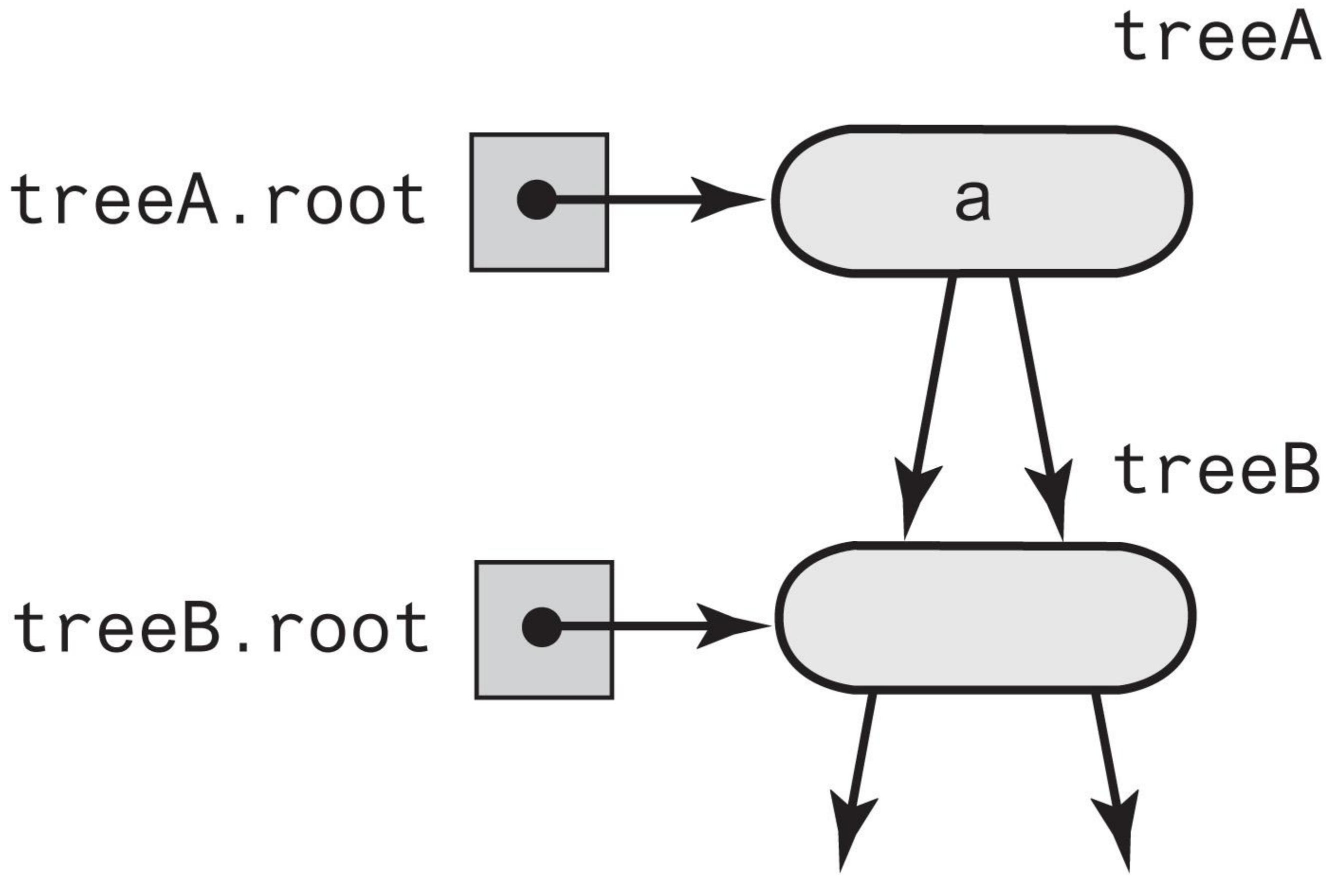
- The binary tree `treeA` shares nodes with `treeB` and `treeC`



treeA.setTree(a, treeB, treeC);

Additional Challenges

- `treeA` has identical subtrees
- We will want these to be distinct



Method `initializeTree` Solution

- If left subtree exists and not empty,
    - attach root node to **`r`** as left child.
- Create root node **`r`** containing given data.
- If right subtree exists, not empty, and distinct from left subtree,
    - attach root node to **`r`** as a right child.
- But if right and left subtrees are same,
    - attach copy of right subtree to **`r`** instead.
- If the left subtree exists and differs from the tree object used to call **`initializeTree`**,
    - set the subtree's data field root to null.
- If right subtree exists and differs from the tree object used to call **`initializeTree`**,
    - set subtree's data field root to null.

Revised initialize method

- Copy all nodes to new tree, then clear the old subtrees.
- Make sure only one copy of data exists

```
private void initializeTree(T rootData, BinaryTree<T> leftTree, BinaryTree<T> rightTree)
  {
     root = new BinaryNode<>(rootData);

     if ((leftTree != null) && !leftTree.isEmpty())
        root.setLeftChild(leftTree.root);

     if ((rightTree != null) && !rightTree.isEmpty())
     {
        if (rightTree != leftTree)
           root.setRightChild(rightTree.root);
        else
           root.setRightChild(rightTree.root.copy());
     } // end if

     if ((leftTree != null) && (leftTree != this))
        leftTree.clear();

     if ((rightTree != null) && (rightTree != this))
        rightTree.clear();
  }
```

Binary Tree accessor/mutator methods

```
public void setRootData(T rootData)
   {
      root.setData(rootData);
   } // end setRootData

   public T getRootData()
   {
      if (isEmpty())
         return null;
      else
         return root.getData();
   } // end getRootData

   public boolean isEmpty()
   {
      return root == null;
   } // end isEmpty

   public void clear()
   {
      root = null;
   } // end clear

   protected void setRootNode(BinaryNode<T> rootNode)
   {
      root = rootNode;
   } // end setRootNode

   protected BinaryNode<T> getRootNode()
   {
      return root;
   }
```

Binary Tree – counting and height

- These methods use recursive BinaryNode methods.

```
public int getHeight()
  {
     int height = 0;
     if (root != null)
        height = root.getHeight();
     return height;
  } // end getHeight

  public int getNumberOfNodes()
  {
     int numberOfNodes = 0;
     if (root != null)
        numberOfNodes = root.getNumberOfNodes();
     return numberOfNodes;
  }
```

Creating trees

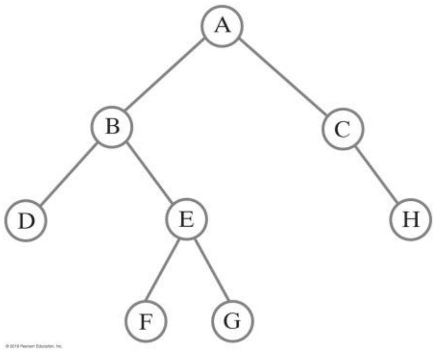
- Create a set of trees and place in a HashMap for easy retrieval
- See BinaryTreeTestDriver
- Uncomment the implementation to be tested

```
    public static HashMap<String, BinaryTreeInterface<String>> createTrees() {
        HashMap<String, BinaryTreeInterface<String>> trees = new HashMap<>();
        String[] treeNames = { "A", "B", "C", "D", "E", "F", "G", "H", "empty" };

        // for each tree name, create a new tree and add it to the map with
        //  its name as a key

        for(String treeName : treeNames) {
            trees.put(treeName, new CompletedArrayBinaryTree<String>(treeName));
//          trees.put(treeName, new CompletedBinaryTree<String>(treeName));
//          trees.put(treeName, new ArrayBinaryTree<String>(treeName));
//          trees.put(treeName, new BinaryTree<String>(treeName));
        }

        // an empty tree was added to the map.
        //  clear all data in this tree, which will also set its name to null
        //  the key will remain in the HashMap so we can still retrieve it

        // there is a difference between a tree that is set to null
        //   and an empty tree

        trees.get("empty").clear();
        return trees;
    }
```

Building a binary tree

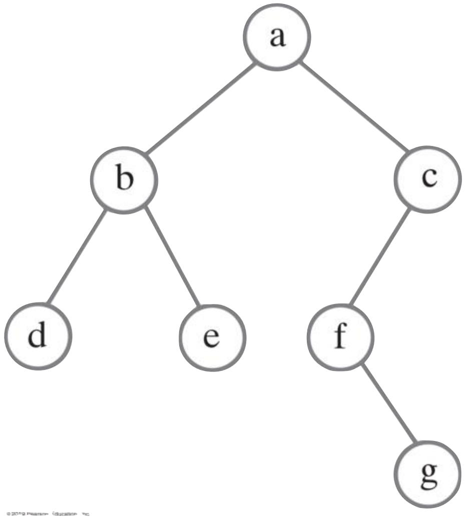
```java
// create each tree with no left and right subtrees
// need to recreate all of the subtrees for each test
//   so a simple reset doesn't work
// because BinaryTree implementation *copies* the subtree then
// destroys the original. So the existing HashMap is then invalid

// this is NOT the case for ArrayBinaryTree :-(

HashMap<String, BinaryTreeInterface<String>> trees = createTrees();

// use the HashMap to get each tree
// start at the bottom of the tree to set subtrees

// set E node subtrees to F and G
trees.get("E").setTree("E", trees.get("F"), trees.get("G"));

// set B node subtrees to D and E
trees.get("B").setTree("B", trees.get("D"), trees.get("E"));

// set C node subtrees to empty and H. Empty has no data at all (root is null)
trees.get("C").setTree("C", trees.get("empty"), trees.get("H"));

// finally, set A node subtrees to B and C
trees.get("A").setTree("A", trees.get("B"), trees.get("C"));

// A is now the root and contains links to all other nodes in the tree
// return A's tree to the caller as the root
return trees.get("A");
```

A binary tree whose nodes contain one-letter strings (Segment 24.21)



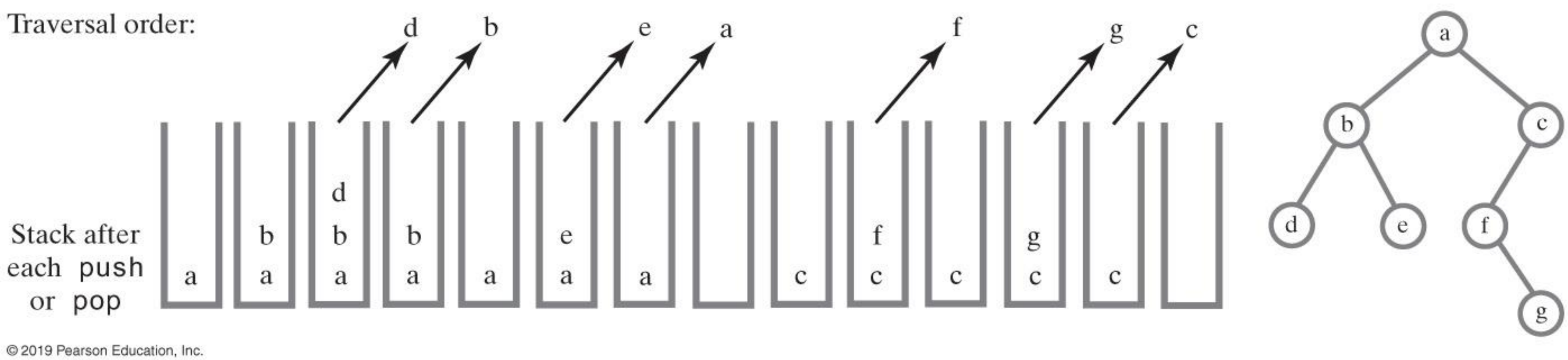
Traversing a binary tree recursively

```
public void recursiveInorderTraverse() {
    recursiveInorderTraverse(root);
}

private void recursiveInorderTraverse(BinaryNode<T> node) {

    if (node != null) {
        recursiveInorderTraverse(node.getLeftChild());
        System.out.print(node.getData() + " ");
        recursiveInorderTraverse(node.getRightChild());
    }
}
```



Non-recursive Traversal

- Using a stack to perform an in-order traversal of a binary tree



Non-recursive Inorder traversal

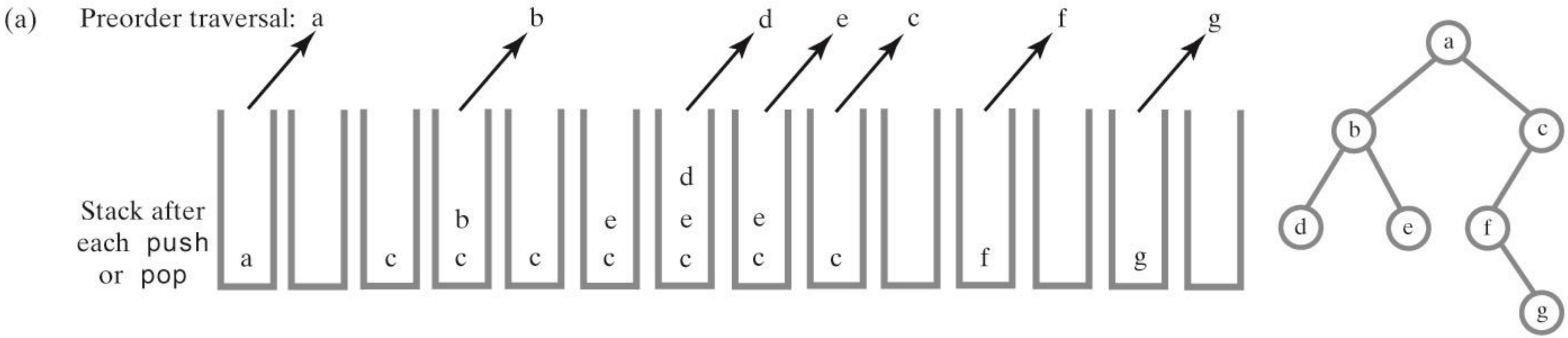
- Uses a Stack

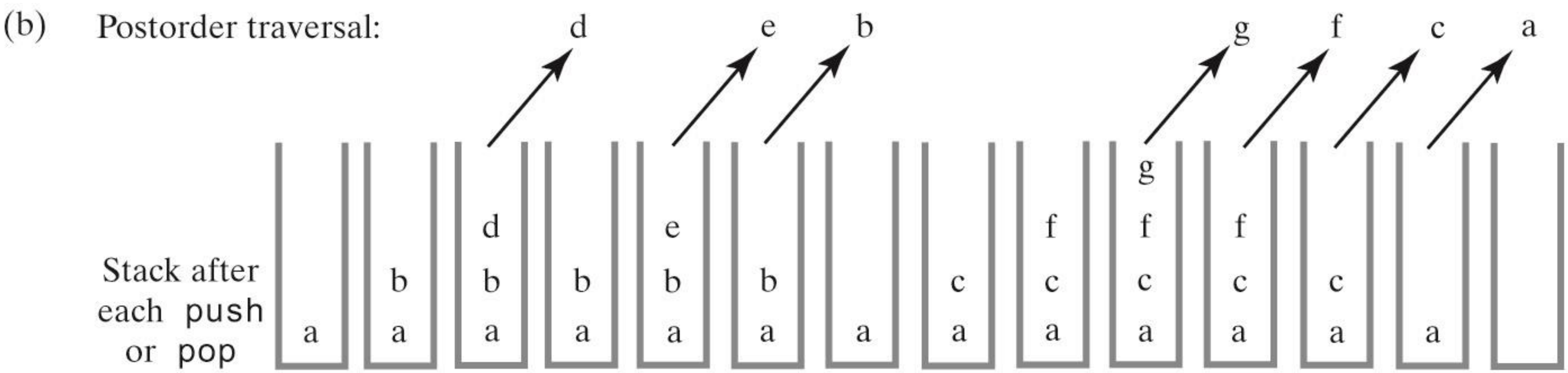
```
public void iterativeInorderTraverse() {
    StackInterface<BinaryNode<T>> nodeStack = new CompletedLinkedStack<>();
    BinaryNode<T> currentNode = root;

    while (!nodeStack.isEmpty() || (currentNode != null)) {
        // Find leftmost node with no left child
        while (currentNode != null) {
            nodeStack.push(currentNode);
            currentNode = currentNode.getLeftChild();
        }

        // Visit leftmost node, then traverse its right subtree
        if (!nodeStack.isEmpty()) {
            BinaryNode<T> nextNode = nodeStack.pop();
            // Assertion: nextNode != null, since nodeStack was not empty
            // before the pop
            System.out.print(nextNode.getData() + " ");
            currentNode = nextNode.getRightChild();
        }
    }
}
```

Inorder traverse using Iterator

```
private class InorderIterator implements Iterator<T> {
     private StackInterface<BinaryNode<T>> nodeStack;
     private BinaryNode<T> currentNode;

     public InorderIterator() {
          nodeStack = new CompletedLinkedStack<>();
          currentNode = root;
     }

     public boolean hasNext() {
          return !nodeStack.isEmpty() || (currentNode != null);
     }

     public T next() {
          BinaryNode<T> nextNode = null;

          // Find leftmost node with no left child
          while (currentNode != null) {
               nodeStack.push(currentNode);
               currentNode = currentNode.getLeftChild();
          }

          // Get leftmost node, then move to its right subtree
          if (!nodeStack.isEmpty()) {
               nextNode = nodeStack.pop();
               // Assertion: nextNode != null, since nodeStack was not empty
               // before the pop
               currentNode = nextNode.getRightChild();
          } else
               throw new NoSuchElementException();

          return nextNode.getData();
     }

     public void remove() {
          throw new UnsupportedOperationException();
     }
}
```

Using a Stack for Preorder/Postorder Traversal





Preorder traverse using Iterator

```
private class PreorderIterator implements Iterator<T> {
    private StackInterface<BinaryNode<T>> nodeStack;

    public PreorderIterator() {
        nodeStack = new CompletedLinkedStack<>();
        if (root != null)
            nodeStack.push(root);
    }

    public boolean hasNext() {
        return !nodeStack.isEmpty();
    }

    public T next() {
        BinaryNode<T> nextNode;

        if (hasNext()) {
            nextNode = nodeStack.pop();
            BinaryNode<T> leftChild = nextNode.getLeftChild();
            BinaryNode<T> rightChild = nextNode.getRightChild();

            // Push into stack in reverse order of recursive calls
            if (rightChild != null)
                nodeStack.push(rightChild);

            if (leftChild != null)
                nodeStack.push(leftChild);
        } else {
            throw new NoSuchElementException();
        }

        return nextNode.getData();
    }
}
```

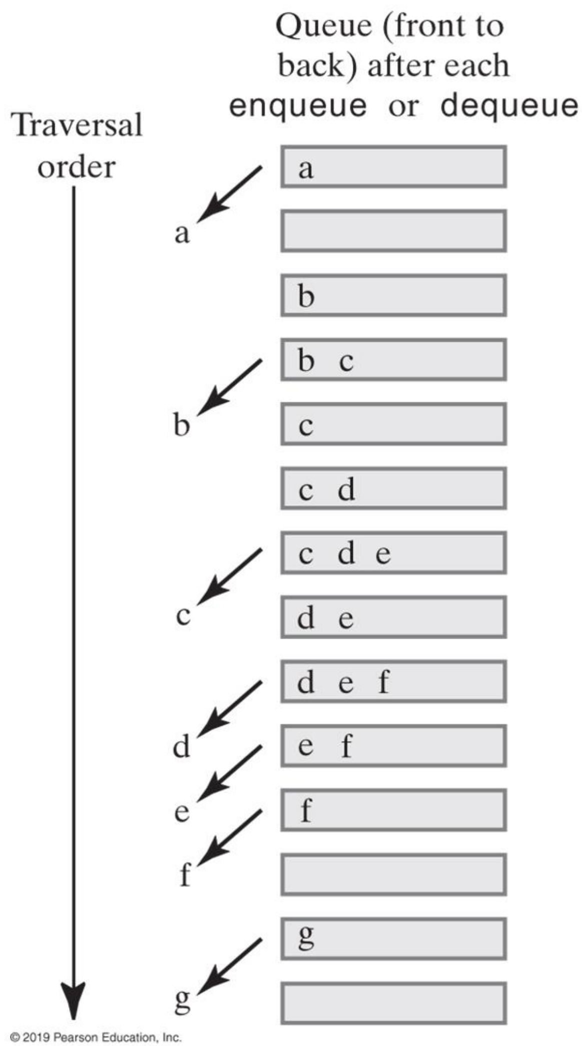
Postorder traverse using Iterator

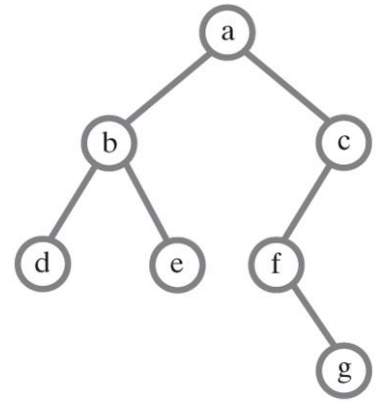
```
private class PostorderIterator implements Iterator<T> {
      private StackInterface<BinaryNode<T>> nodeStack;
      private BinaryNode<T> currentNode;

      public PostorderIterator() {
            nodeStack = new CompletedLinkedStack<>();
            currentNode = root;
      }

      public boolean hasNext() {
            return !nodeStack.isEmpty() || (currentNode != null);
      }

      public T next() {
            BinaryNode<T> leftChild, nextNode = null;

            // Find leftmost leaf
            while (currentNode != null) {
                  nodeStack.push(currentNode);
                  leftChild = currentNode.getLeftChild();
                  if (leftChild == null)
                        currentNode = currentNode.getRightChild();
                  else
                        currentNode = leftChild;
            }

            // Stack is not empty either because we just pushed a node, or
            // it wasn't empty to begin with since hasNext() is true.
            // But Iterator specifies an exception for next() in case
            // hasNext() is false.

            if (!nodeStack.isEmpty()) {
                  nextNode = nodeStack.pop();
                  // nextNode != null since stack was not empty before pop

                  BinaryNode<T> parent = null;
                  if (!nodeStack.isEmpty()) {
                        parent = nodeStack.peek();
                        if (nextNode == parent.getLeftChild())
                              currentNode = parent.getRightChild();
                        else
                              currentNode = null;
                  } else
                        currentNode = null;
            } else {
                  throw new NoSuchElementException();
            }

            return nextNode.getData();
      }
}
```

Inorder traverse using Iterator

```
private class InorderIterator implements Iterator<T> {
     private StackInterface<BinaryNode<T>> nodeStack;
     private BinaryNode<T> currentNode;

     public InorderIterator() {
          nodeStack = new CompletedLinkedStack<>();
          currentNode = root;
     }

     public boolean hasNext() {
          return !nodeStack.isEmpty() || (currentNode != null);
     }

     public T next() {
          BinaryNode<T> nextNode = null;

          // Find leftmost node with no left child
          while (currentNode != null) {
               nodeStack.push(currentNode);
               currentNode = currentNode.getLeftChild();
          }

          // Get leftmost node, then move to its right subtree
          if (!nodeStack.isEmpty()) {
               nextNode = nodeStack.pop();
               // Assertion: nextNode != null, since nodeStack was not empty
               // before the pop
               currentNode = nextNode.getRightChild();
          } else
               throw new NoSuchElementException();

          return nextNode.getData();
     }
}
```

Using a Queue for Level-Order Traversal





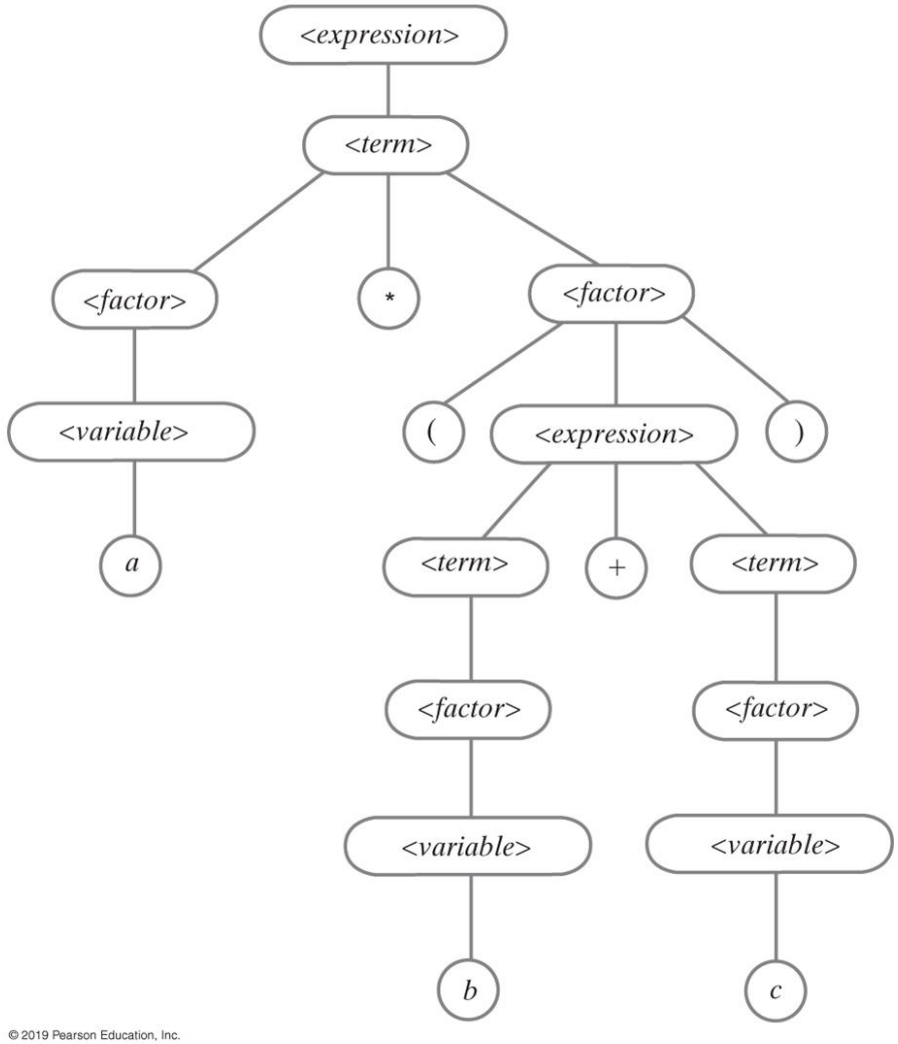
Levelorder traverse using Iterator

- Note use of Queue

```
private class LevelOrderIterator implements Iterator<T> {
     private QueueInterface<BinaryNode<T>> nodeQueue;

     public LevelOrderIterator() {
          nodeQueue = new CompletedLinkedQueue<>();
          if (root != null)
               nodeQueue.enqueue(root);
     }

     public boolean hasNext() {
          return !nodeQueue.isEmpty();
     }

     public T next() {
          BinaryNode<T> nextNode;

          if (hasNext()) {
               nextNode = nodeQueue.dequeue();
               BinaryNode<T> leftChild = nextNode.getLeftChild();
               BinaryNode<T> rightChild = nextNode.getRightChild();

               // Add to queue in order of recursive calls
               if (leftChild != null)
                    nodeQueue.enqueue(leftChild);

               if (rightChild != null)
                    nodeQueue.enqueue(rightChild);
          } else {
               throw new NoSuchElementException();
          }

          return nextNode.getData();
     }
}
```

Parse Tree for an Equation

- A parse tree for the algebraic expression `a * (b + c)`



Expression Tree interface

- Just add an evaluation method to the Binary Tree

```java
public interface ExpressionTreeInterface extends BinaryTreeInterface<String> {
/**
 * Computes the value of the expression in this tree.
 *
 * @return The value of the expression.
 */
  public double evaluate();
}
```

Expression Tree – evaluate()

- evaluate() walks through expression tree recursively
  - at leaf, gets value of the data and returns.
  - If the node has children, it is an operator, and invoke compute()

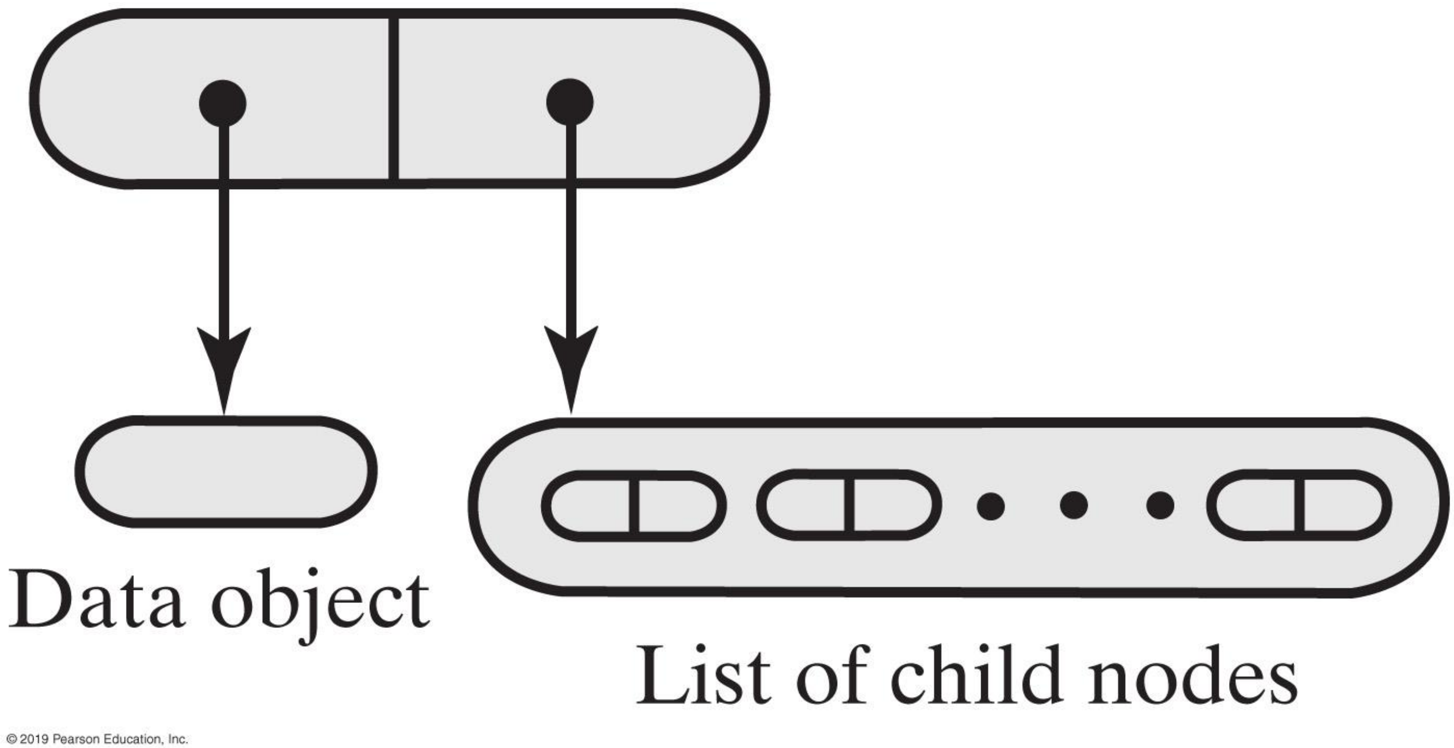
```
public class CompletedExpressionTree extends CompletedBinaryTree<String>
     implements ExpressionTreeInterface {

     public CompletedExpressionTree() {
     }

     public CompletedExpressionTree(String rootData) {
          super(rootData);
     }

     public double evaluate() {
          return evaluate(getRootNode());
     }

     private double evaluate(BinaryNode<String> rootNode) {
          double result;

          if (rootNode == null)
               result = 0;
          else if (rootNode.isLeaf()) {
               String variable = rootNode.getData();
               result = getValueOf(variable);
          } else {
               double firstOperand = evaluate(rootNode.getLeftChild());
               double secondOperand = evaluate(rootNode.getRightChild());
               String operator = rootNode.getData();
               result = compute(operator, firstOperand, secondOperand);
          }

          return result;
     }
```

Expression Tree – values and compute

- We would typically set values in other methods, but this demonstrates how values could be intrinsic
- compute() looks at the operator
  - decides on the operation, and then operates on the operands

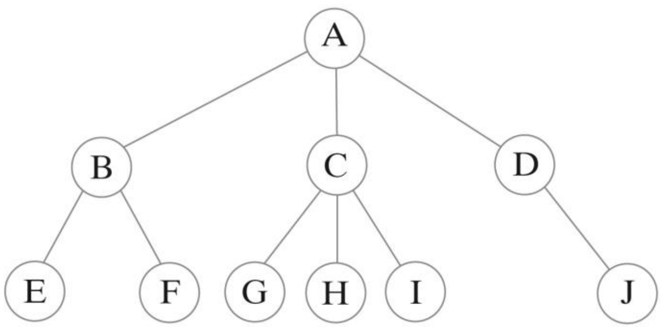
```
private double getValueOf(String variable) { // Strings allow multicharacter variables

	double result = 0;

	if (variable.equals("a"))
		result = 2;
	else if (variable.equals("b"))
		result = 3;
	else if (variable.equals("c"))
		result = 4;
	else if (variable.equals("d"))
		result = 5;
	else if (variable.equals("e"))
		result = 2;

	return result;
}

private double compute(String operator, double firstOperand, double secondOperand) {
	double result = 0;

	if (operator.equals("+"))
		result = firstOperand + secondOperand;
	else if (operator.equals("-"))
		result = firstOperand - secondOperand;
	else if (operator.equals("*"))
		result = firstOperand * secondOperand;
	else if (operator.equals("/"))
		result = firstOperand / secondOperand;

	return result;
}
```

Representing General Trees

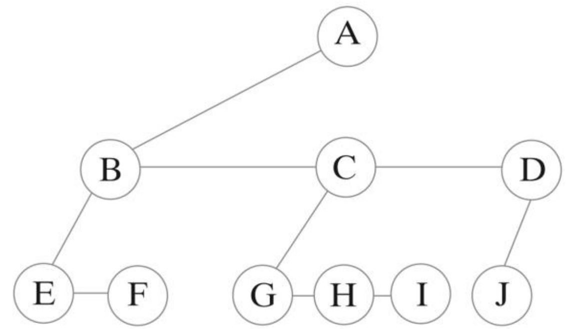


General Trees vs Binary Trees

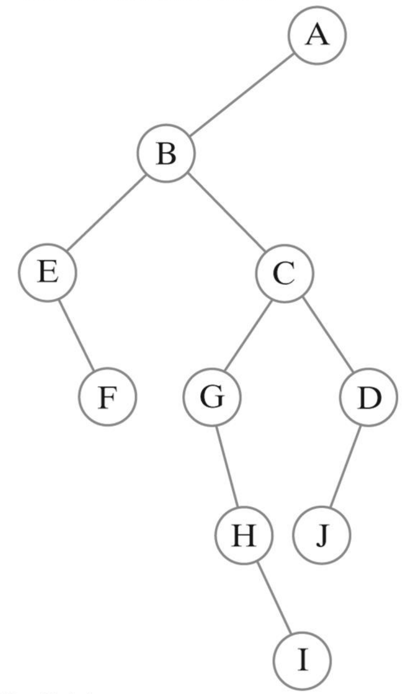
(a) A general tree



(b) An equivalent binary tree



(c) The same binary tree in a conventional form



General Tree Node interface

- General tree node can have many children, not just two

```
interface GeneralNodeInterface<T> {
	/**
	 * Get the data from a node
	 * @return
	 */
	public T getData();

	/**
	 * Set the data in a node
	 * @param newData
	 */
	public void setData(T newData);

	/**
	 * Tests to see if this node is a leaf node
	 * @return true if a leaf node with no children
	 */
	public boolean isLeaf();

	/**
	 * Gets and iterator to traverse all child nodes
	 * @return
	 */
	public Iterator<GeneralNodeInterface<T>> getChildrenIterator();

	/**
	 * Adds a child to this node
	 * @param newChild
	 */
	public void addChild(GeneralNodeInterface<T> newChild);
}
```

Testing - TreeDriverPackage

- Has drivers for BinaryTree, DecisionTree, and ExpressionTree
- You must complete the three classes for the test drivers to work.
- Much of the code can be found in the lecture notes and the textbook.