

Exercise 3

Consider Example 8.7. Suppose that there are three advertisers A;B; and C. There are three queries x; y; and z. Each advertiser has a budget of 2. Advertiser A only bids on x, B bids on x and y, and C bids on x; y, and z. Note that on the query sequence xxyyzz, the optimal offline algorithm would yield a revenue of 6, since all queries can be assigned.

1. Show that the greedy algorithm will assign at least 4 of the 6 queries xxyyzz.

Advertiser C bids the most. The first two 'x's will be allocated to C. Then, the query sequence will now become yyzz. Since advertiser B is now the one who bids the most, two 'y's will be allocated to B. Now, only zz is left, but advertiser A only bids on x, there is no one has an unexpended budget to pay for the two 'z's.

As a result, 'xxyy' in 'xxyyzz' is assigned, so the greedy algorithm assigns at least 4 of the 6 queries xxyyzz.

2. Find another sequence of queries such that the greedy algorithm can assign as few as half the queries that the optimal offline algorithm would assign to that sequence.

Given the same condition of previous question, queries 'yyzz' is another another sequence of queries such that the greedy algorithm can assign as few as half the queries that the optimal offline algorithm would assign to that sequence.

In optimal offline algorithm, zz will assign to advertiser C and yy will assign to advertiser B. 4 of the 4 queries are assigned.

In greedy algorithm, Advertiser C bids the most. The first two 'y's will be allocated to C. Then, the query sequence will now become zz. Now, only zz is left, but advertiser A and B do not bid on z, there is no one has an unexpended budget to pay for the two 'z's. The greedy algorithm assigns 2 of the 4 queries yyzz, which the half of the optimal offline algorithm (1/2).