

- 1.
- 2.
3. function dividePolynomials(z, y):
 - if y == 0: # can't divide by 0
 - return NaN
 -
 - a = 0 # future quotient
 - b = z # remainder initialized
 -
 - while degree(b) >= degree(y):
 - c = b[degree(b)] / y[degree(y)] # the division
 - e = degree(b) - degree(y) # calculate the difference in exponents to make x^e
 -
 - d = c * x^e # singular term created by the division
 - a = a + d # quotient equation adding next term
 - b = b - (d * y) # create the new reminder that i going to be used in the futured
 -
 - return a

Loop invariant

$z = (y * a) + b \wedge \text{degree}(b) \geq \text{degree}(y)$

Base case

$a = 0$

$b = z$

$z = (y * a) + b \wedge \text{degree}(b) \geq \text{degree}(y)$

$z = (y * 0) + b$

$z = (y * 0) + z$

$z = z$

$\text{degree}(b) \geq \text{degree}(y)$ has to be true in order for the loop to even run therefore the

base case is true

K and K+1 case

$z = (y * a_k) + b_k \wedge \text{degree}(b_k) \geq \text{degree}(y)$

$z = (y * a_{k+1}) + b_{k+1} \wedge \text{degree}(b_{k+1}) \geq \text{degree}(y)$

$z = (y * a_{k+1}) + b_{k+1}$

$z = (y * a_k + d) + b_k - (d * y)$

$z = ya_k + yd + b_k - dy$

$z = ya_k + b_k = z = (y * a_k) + b_k$

For the loop to run, $\text{degree}(b_k) \geq \text{degree}(y)$ and $\text{degree}(b_{k+1}) \geq \text{degree}(y)$ have to be true therefore this part is true.

K and K+1 were valid therefore this part of the loop invariant is true.