


# Seats arrangement problem with SAT

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# Problem description

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$r$	$c$				
	1	2	3	4	5
1					
2					
3					
4					
5					

- Each student can choose their preferred zone
- Each student can choose
  1. Who he want to seat with
  2. Who he don't want to seat with

# Modeling

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A 5x5 grid representing a seating arrangement. The columns are labeled 1 through 5, and the rows are labeled 1 through 5. The grid is empty.

	1	2	3	4	5
1					
2					
3					
4					
5					

$$x_{i,r,c} \equiv \begin{cases} 1 & i_{th} \text{ person seat at } (r, c) \\ 0 & i_{th} \text{ person not seat at } (r, c) \end{cases}$$

$$O(NRC) = O(R^2C^2)$$

# Constraints



$r$	$c$				
	1	2	3	4	5
1					
2					
3					
4					
5					

Each seat has only one student

// number of student at each seat  $\geq 1$

$$(x_{1,r,c} + x_{2,r,c} + \dots + x_{25,r,c}) = 1$$

// number of student at each seat  $< 2$

$$(ab = 0 \rightarrow a' + b' = 1)$$

$$(\overline{x_{1,r,c}} + \overline{x_{2,r,c}}) = 1$$

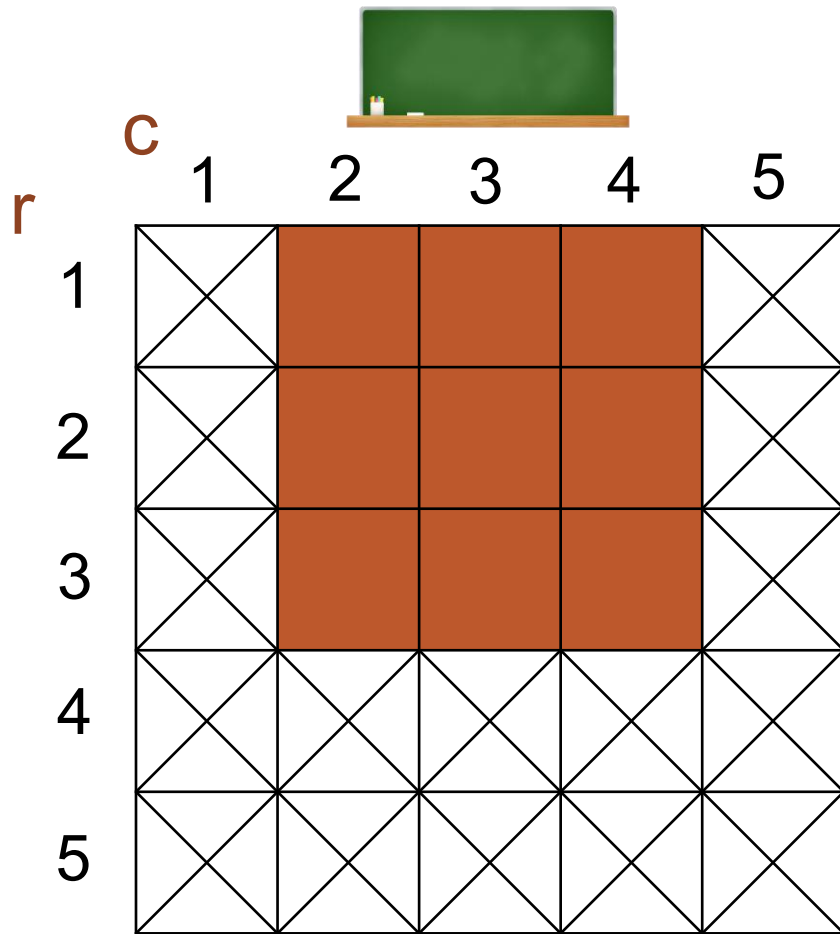
$$(\overline{x_{1,r,c}} + \overline{x_{3,r,c}}) = 1$$

$\vdots$

$$(\overline{x_{24,r,c}} + \overline{x_{25,r,c}}) = 1$$

$$O(N^2RC) = O(R^3C^3)$$

# Constraints



Each student has only one seat in their preferred zone

// #selected seat in preferred zone  $\geq 1$

$$(x_{i,1,2} + x_{i,1,3} + \dots + x_{i,3,4}) = 1$$

// #selected seat in preferred zone  $< 2$

$$(ab = 0 \rightarrow a' + b' = 1)$$

$$(\overline{x_{i,1,2}} + \overline{x_{i,1,3}}) = 1$$

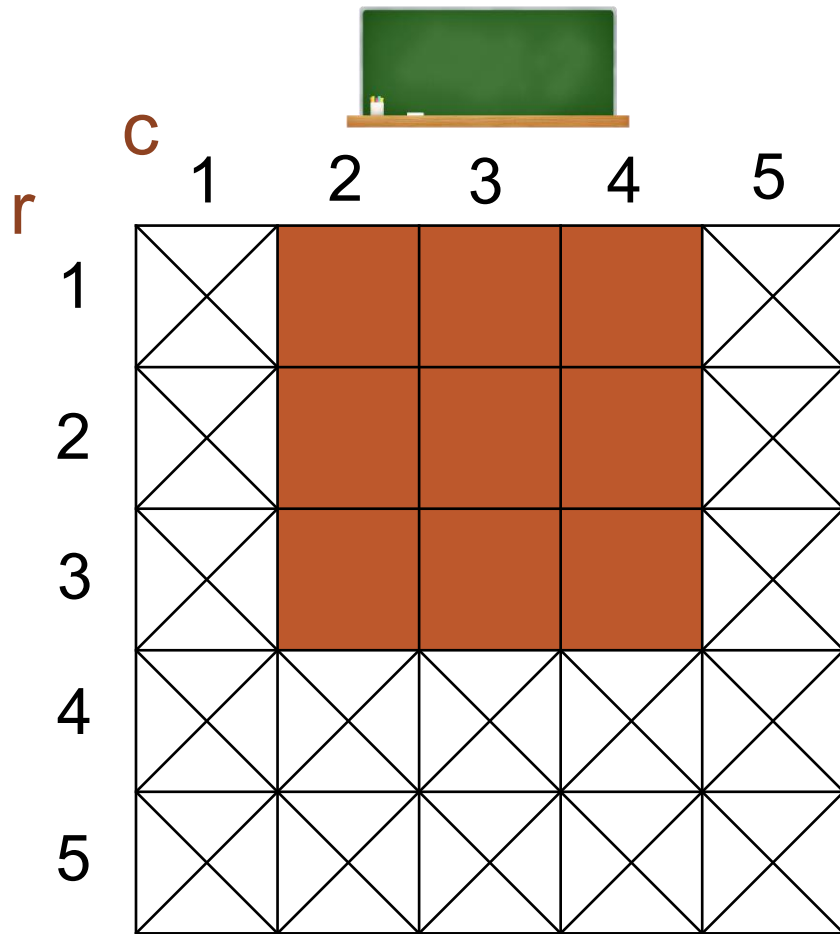
$$(\overline{x_{i,1,2}} + \overline{x_{i,1,4}}) = 1$$

$\vdots$

$$(\overline{x_{i,3,3}} + \overline{x_{i,3,4}}) = 1$$

$$O(N(RC)^2) = O(R^3C^3)$$

# Constraints



No selected seat in unpreferred zone:

// every  $x_{i,r,c}$  in unpreferred zone = 0

$$\overline{x_{i,1,1}} = 1$$

$$\overline{x_{i,2,1}} = 1$$


$$\overline{x_{i,3,1}} = 1$$

$\vdots$

$$\overline{x_{i,1,5}} = 1$$

$$O(NRC) = O(R^2C^2)$$

# Neighboring problem



	<b>c</b>	1	2	3	4	5
<b>r</b>	1					
	2			<i>j</i>		
	3		<i>j</i>	<i>i</i>	<i>j</i>	
	4			<i>j</i>		
	5					

Suppose *i* wishes *j* seating beside him

$$\Rightarrow x_{i,r,c} \rightarrow (x_{j,r-1,c} \vee x_{j,r+1,c} \vee x_{j,r,c-1} \vee x_{j,r,c+1})$$

$$// a \rightarrow b \Leftrightarrow \bar{a} + b = 1$$

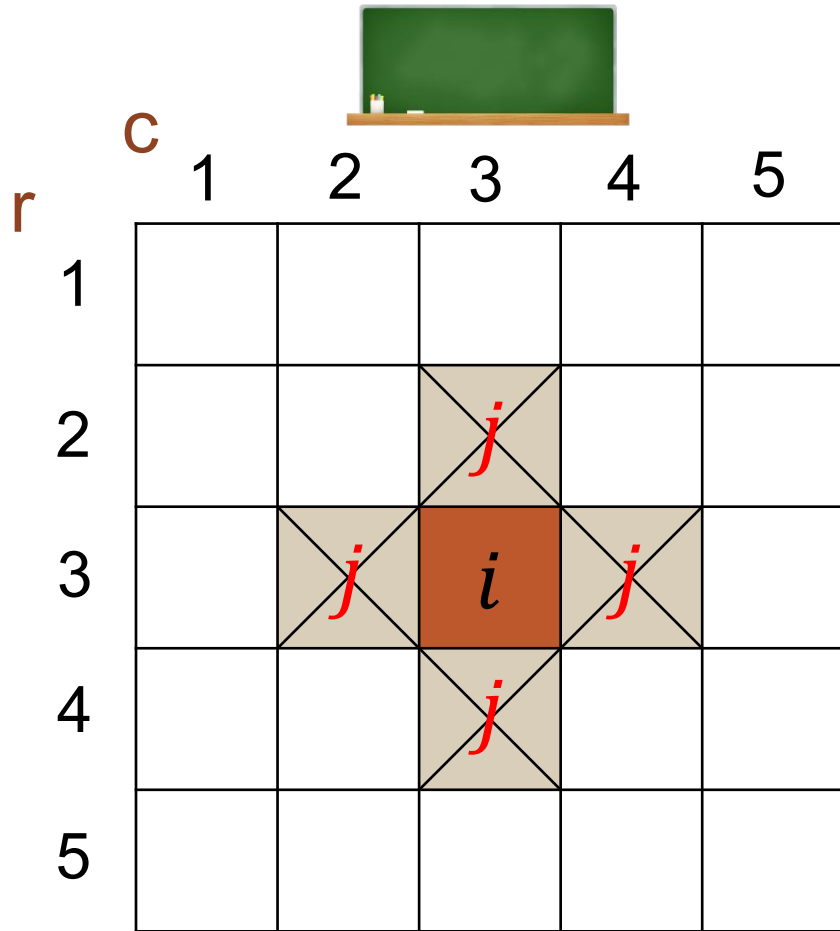
$$\Rightarrow \overline{x_{(i,r,c)}} + x_{j,r-1,c} + x_{j,r+1,c} + x_{j,r,c-1} + x_{j,r,c+1} = 1$$

where  $(r, c) \in i$ 's preferred zone

(make sure that seat exist)

$$O(N^2RC) = O(R^3C^3)$$

# Neighboring problem



Suppose  $i$  **does not** wish  $j$  seating beside him

$$\Rightarrow x_{i,r,c} \rightarrow (\overline{x_{j,r-1,c}} \wedge \overline{x_{j,r+1,c}} \wedge \overline{x_{j,r,c-1}} \wedge \overline{x_{j,r,c+1}})$$

$$// a \rightarrow b \Leftrightarrow \bar{a} + b = 1$$

$$\Rightarrow \overline{x_{(i,r,c)}} + (\overline{x_{j,r-1,c}} * \overline{x_{j,r+1,c}} * \overline{x_{j,r,c-1}} * \overline{x_{j,r,c+1}}) = 1$$

$$\Rightarrow (\overline{x_{(i,r,c)}} + \overline{x_{j,r-1,c}}) = 1$$

$$(\overline{x_{(i,r,c)}} + \overline{x_{j,r-1,c}}) = 1$$

$$(\overline{x_{(i,r,c)}} + \overline{x_{j,r-1,c}}) = 1$$

$$(\overline{x_{(i,r,c)}} + \overline{x_{j,r-1,c}}) = 1$$

where  $(r, c) \in i$ 's preferred zone

\*(make sure the seats exist)

$$O(N^2RC) = O(R^3C^3)$$



# 目前流程

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生成座位ans\_seat(給定N, R, C, 生成一個隨機的座位分布,  $N \leq RC$ )



生成學生選擇student\_choice(此選擇必定包含一解)



根據學生選擇生成cnf file



miniSAT 求解



Check miniSAT的答案是否符合student\_choice

# About Student \_choice

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對每一個學生(S)

1. Preferred zone:

- 必定包含ans\_seat中S的座位
- 在教室中隨機選擇其他1/3~2/3個座位

2. Want\_seat\_with

- ans\_seat中每個S身邊的同學有1/4的機率被加進Seat\_with名單中

3. Don't\_want\_Seat\_with

- 從ans\_seat中隨機挑選1~( $\#(\text{others})/10$ )個不坐在S旁邊的人

# 測試結果

ans\_seat

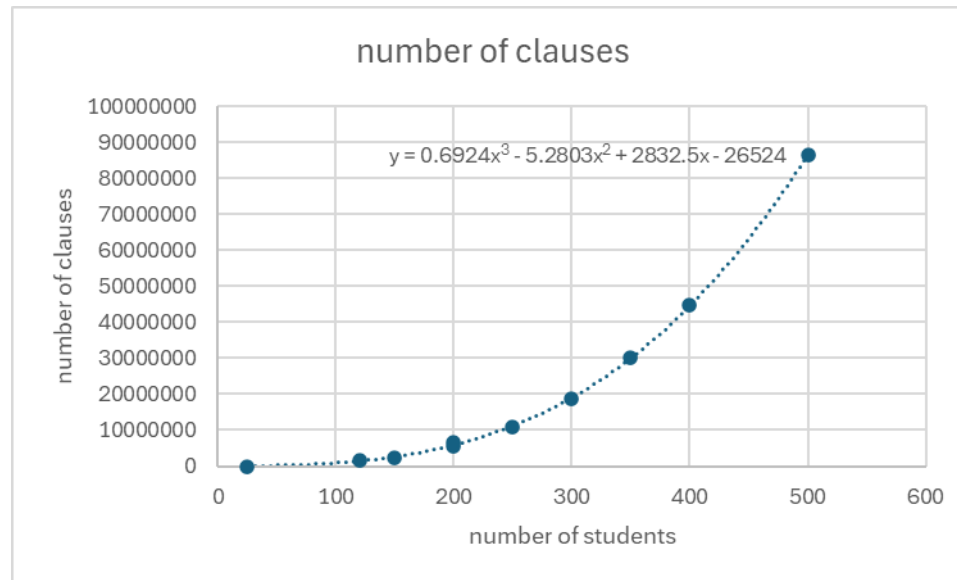
21	-1	44	54	22	10	17	74	38	24
6	59	32	58	69	15	-1	63	64	8
35	-1	53	-1	-1	76	3	47	67	79
-1	56	68	9	-1	42	-1	80	1	14
49	78	72	43	57	29	75	-1	4	50
61	66	-1	62	30	36	52	20	-1	-1
37	27	40	60	71	65	-1	-1	48	73
55	-1	39	-1	-1	33	11	18	28	46
-1	45	51	70	25	16	41	-1	-1	19
13	77	31	23	5	12	7	34	26	2

Arrangement found by minisat

```
SATISFIABLE
Found a solution:
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 21| 52|   | 39| 71| 53| 73| 24| 38| 55|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 6| 20|   | 40| 60| 62| 46|   | 64| 8|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|   |   | 75| 26|   | 34| 3| 47| 67| 79|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 68| 56|   | 2|   | 42|   | 80| 1| 14|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 49| 78| 72| 43| 57| 29| 48| 35| 4| 50|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 61| 66|   | 9| 30| 36| 28| 7|   |   |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 37| 27|   | 76|   |   | 18| 12| 70|   |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 69| 58| 54| 22| 65| 33| 11|   | 51| 5|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 77| 45| 15| 10| 25| 16| 41|   | 31| 23|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| 13| 59|   | 19| 17| 74| 63| 32| 44|   |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
the result is also in seat_assignment.txt
begin to check the result
check result: true
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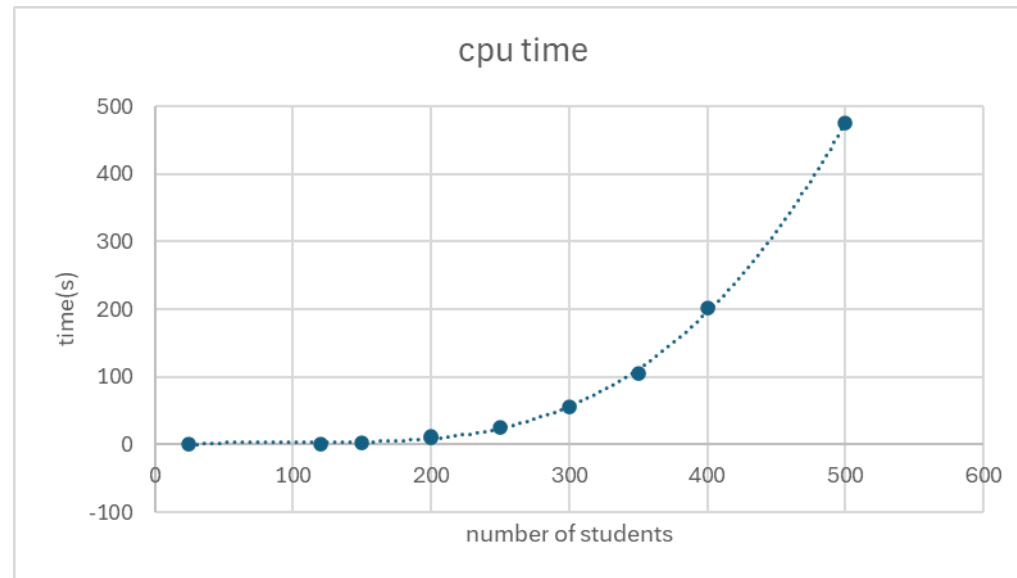
# 測試結果

Number of students(N)	25	120	150	200	200	250	300	350	400	500
#Row(R)	5	15	15	20	15	25	20	25	20	25
#Column(C)	5	10	10	10	15	10	15	14	20	20
#Variable	625	18000	22500	40000	45000	62500	90000	122500	160000	250000
#clause	10878	1608631	2402055	5587683	6504631	10850762	18876845	30187591	44643499	86578692
generate_cnf_time(s)	0.1	1.69	2.53	5.662	7.16	10.95	16.1	28.2198	45.663	83.881
cpu time(s)	0.01	1.76	3.3	10.1147	11.9	25.7	55.0189	104.898	202.396	474.892
simplification time(s)	0.01	1.46	3	9.3	10	23.9	51.82	99.66	194.27	457.4
search time(s)	0	0.3	0.3	0.8147	1.9	1.8	3.1989	5.238	8.126	17.492



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