Idaho Congressional Redistricting Plan Proposal

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I. Executive Summary Letter

Congressional redistricting in the United States is an essential process of redrawing each state's boundaries for legislative and congressional districts after the U.S. census every ten years. Despite the challenges posed by COVID-19 in 2020, the Idaho Independent Redistricting Commission approved a new congressional map. This updated map helped ensure equal voting power amongst citizens. From the models studied by the project team in IEM 4013 by Professor Austin Buchannan, the project team will apply to this report using integer programming to create optimal congressional districts in Idaho.

The team proposed a solution for redistricting Idaho based on data collected from the 2020 census. This proposed solution follows both federal redistricting and state requirements with one slight caveat regarding irregularity in the shape of the districts outlined in Idaho's Statues, but this stipulation is mitigated by this not being a strictly imposed requirement. Also, due to the largely uniform population present in Idaho this constraint becomes less important. The team decided that equal population was more important as this criterion is outlined in both federal and state requirements. However, the team also provided a redistricting proposal that avoids any irregularity but adds a slight population deviation (of around 0.00169%) between districts.

The group's main proposed solution aims to minimize edge cuts between counties while maintaining an equal population for both districts. The other proposed solution is supported by a minimize edge cut, minimize edge of perimeter cut, and moment of inertia model.

II. Introduction

Updated population statistics for every US state are provided by the census every ten years. Based on population size, the government uses this data to determine how many representatives each state will have in the House of Representatives. Each state is then responsible for allocating its people into voting districts, each of which will choose one representative. The purpose of these districts is to guarantee that every state resident has equal representation. However, a number of federal and state-specific regulations must be followed throughout the creation of these districts. Idaho has 44 counties with divided into 2 congressional districts (Figure 1 will shows the counties and congressional districts of Idaho)

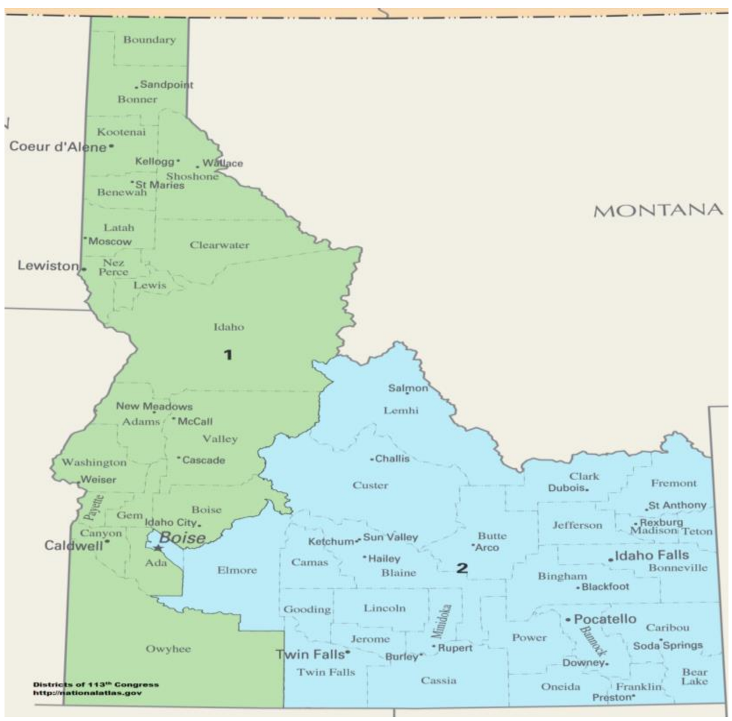


Figure 1: Idaho Counties & Congressional District Map

III. Criteria for Congressional Redistricting

a. Federal Criteria

The federal law and judicial precedents govern congressional redistricting in the United States. One of the key federal criteria for redistricting includes the requirement for equal population. Congressional districts are required to have nearly equal populations as highlighted by the "one person, one vote" principle which was established in Wesberry v. Sanders (1964) and Article 1, section 2 of the United States' constitution (U.S. Supreme Court, n.d.). The Voting Rights Act (VRA) also prohibits states from implementing any voting practice, procedure or disqualification that denies people the right to vote based on their color, race or membership in a language minority. VRA also prohibits states from coming up with district maps that reduce minority voting strength. According to the United States' Supreme Court ruling in Karcher v. Daggett, 462 U.S. 725 (1983), congressional districts should have population deviations of less than 1%.

b. State Criteria

Idaho's criteria for congressional redistricting mainly prioritizes factors such as geographic integrity, public participation and fairness. Idaho's Constitution requires legislative districts to be contiguous and that counties should only be divided to comply with the U.S. Constitution (Article III, Section 5). Additional criteria for congressional redistricting are outlined in Idaho Statutes. Specifically, Title 72, Chapter 15 section 72-1506 establishes criteria that are pertinent to our project. These relevant criteria are as follows: (a) "districts shall preserve traditional neighborhoods and local communities of interest", (b) "Districts shall be substantially equal in population and should seek to comply with all applicable federal standards and statutes." (c) "To the maximum extent possible, the plan should avoid drawing districts that are oddly shaped." (d) "Division of counties shall be avoided whenever possible. If a county must be divided, the number of such divisions, per county, should be kept to a minimum." (e) "To the extent that counties must be divided to create districts, such districts shall be composed of contiguous counties." Overall, these criteria ensure preserving communities of interests, contiguity, compactness and minimizing country splits. Other Idaho State criteria about redistricting are outlined in Title 72, Chapter 15 but are not important for our project. Sections 72-1501, 72-1502, 72-1505, and parts of 72-1506 discuss criteria about the redistricting process that detail legislative intent, appointment process for commission in reapportionment, and the necessity of commission to ensure transparency to the public. These criteria are not specifically relevant to the project as we are providing a map and not an appointed commission. Moreover, some of the common measures used by the state to define compactness includes the Polsby-Popper test for analyzing the area to perimeter ratio, "reock degree of compactness test" for comparing the area of a district to the smallest possible enclosing circle. Idaho State also utilizes the Convex Hull Test to determine how closely a district can fix within the smallest convex shape enclosing it.

IV. Problem Statement

Our goal is to minimize the difference in the population of the two congressional districts of Idaho while adhering to federal and state requirements.

V. Operation Research Model

Due to minimizing the difference in the population of districts with constraints imposed in Idaho, the team used several different models based on the IP Model (Integer Programming) to create a redistricting plan for the state. Each model (along with its different objectives and constraints) is presented below:

a. **Minimum Moment of Inertia Model**

Sets:

V = set of counties/nodes

N(i) = set of neighbors of vertex i ∈ V

Indices:

i = county number i ∈ {1, 2,...44}

j = district number j ∈ {1,2}

Parameters:

U = Upper Population Bound

L = Lower Population Bound

k = Number of Districts, k = 2

n = number of counties, n = 44

$w_{ij} = p_i^{d}\,{}_{ij}^{2}$ = the penalty for moment-of-inertia objective

p_i = Population of node i

d_{ij} = Distance from node i to node j

Decision Variables:

x_{ij} = {1 - vertex i is assigned to the districted centered at j,

0 - otherwise}

f_{ij}^{v} = amount of flow originating at district center v, that is sent across edge {i,j}

Integer Programming Model (Moment-of-Inertia)

Objective: This integer programming model seeks to minimize the penalty of the moment-of-inertia. Essentially, this model seeks to make districts as compact as possible while still following the constraints listed below.

minimize $\sum_{i \epsilon V} \sum_{j \epsilon V} w_{ij} \, x_{ij}$

Constraints

such that

$\sum_{j \epsilon V} \quad x_{ij} \ = 1 \qquad \forall i \epsilon V \qquad (1a)$

$\sum_{j \epsilon V} \quad x_{jj} \ = \ k \qquad \forall i \epsilon V \qquad (1b)$

$L x_{jj} \leq \sum_{i \epsilon V} \quad p_i x_{ij} \ \leq \ U x_{jj} \qquad \forall j \epsilon V \qquad (1c)$

$x_{ij} \epsilon \{0,1\} \qquad \forall i, \, j \epsilon V \qquad (1d)$

Constraints explanation: (1a) Ensures that each county is assigned to exactly one district. (1b) Ensures that both districts in Idaho (k) are chosen. (1c) Ensures that the population in both districts in Idaho are between the allowed U and L bounds. (1d) Binary constraint that ensures each county is either assigned to a district or is not.

Contiguity Constraints

$\sum_{u \epsilon N(i)} \quad \left(f_{iu}^{i} \ - \ f_{iu}^{i}\right) = \ x_{ij} \qquad \forall i \epsilon V \, \{j\}, \, \forall j \epsilon V \qquad (2a)$

$\sum_{u \epsilon N(i)} \quad f_{ui}^{i} \leq x_{ij} \, (n-1) \qquad \forall i \epsilon V \, \{j\}, \, \forall j \epsilon V \qquad (2b)$

$\sum_{u \epsilon N(i)} \quad f_{uj}^{j} = \ 0 \qquad \forall j \epsilon V \qquad (2c)$

$f_{ij'}^{v} \, f_{ji}^{v} \ \geq \ 0 \qquad \forall \{i, j\} \epsilon E \, , \, \forall v \epsilon V \qquad (2d)$

<u>Contiguity constraints explanation</u>: (2a) Ensures that county i consumes a unit of flow type j if it is assigned to district j; otherwise, it consumes none. (2b) Ensures that vertex i can only receive flow of type j if county i is assigned to district j. (2c) Ensures that there is no circulation of flow in model. (2d) Ensures nonnegativity of flows variables.

b. Minium Cut Edges Model

Sets:

V – set of counties/nodes

N(i) = set of neighbors of vertex i ∈ V

E - set of edges {i,j}

Indices:

i = county number i ∈ {1, 2,...44}

j = district number j ∈ {1,2}

e = edge number connecting nodes on map

Parameters:

s_e = shared perimeter between two nodes in e ϵE

k = number of districts, k = 2

n = number of counties, n = 44

M = n – k +1

Decision Variables:

x_{ij} = {1 - vertex i is assigned to the districted centered at j,

0 - otherwise}

y_e = {1 – edge e is cut,

0 - otherwise}

r_{ij} = {1 – node i is assigned to the root districted centered at j,

0 - otherwise}

f_{ij} = flow on edge from i to j

<u>Integer Programming Model (Minimum Cut Edges Model)</u>

<u>Objective:</u> This integer programming model seeks to minimize the number of cut edges

minimize $\sum_{e \in E} y_e$

<u>Constraints</u>

such that

$x_{iv} - x_{jv} \leq y_e \qquad \forall e = \{u, v\} \in E, \forall j \in \{1,2\}$ (3a)

$\sum_{j=1}^{k} x_{ij} = 1 \qquad \forall i \in V$ (3b)

$L \leq \sum_{i \in V} p_i \, x_{ij} \leq U \qquad \forall j \in \{1,2\}$ (3c)

$x_{ij} \in \{0,1\} \qquad \forall i \in V, \forall j \in \{1,2\}$ (3d)

$y_e \in \{0,1\} \qquad \forall \{i, j\} \in E$ (3e)

<u>Constraints explanation:</u> (3a) Ensures that edges are cut if county u is in district j but v is not in district j. (3b) Ensures that each county is assigned to exactly one district. (3c) Ensures that the population in both districts in Idaho is between the allowed U and L bounds. (3d) Binary constraint that ensures each county is either assigned to a district or is not. (3e) Binary constraint that ensures that each edge either has a cut or does not have a cut.

<u>Contiguity Constraints</u>

$\sum_{i \in V} r_{ij} = 1 \qquad \forall j \in \{1,2\}$ (4a)

$r_{ij} \leq x_{ij}$ $\forall i \in V, \forall j \in \{1,2\}$ (4b)

$\sum_{u \in N(i)}(f_{ui} - f_{iu}) \leq 1 - M \sum_{j=1}^{k} r_{ij}$ $\forall i \in V$ (4c)

$f_{ij} + f_{ji} \leq M(1 - y_e)$ $\forall e = \{i,j\} \in E$ (4d)

$f_{ij}, f_{ji} \geq 0$ $\forall \{i,j\} \in E$ (4e)

$r_{ij} \in \{0,1\}$ $\forall i \in V, \forall j \in \{1,2\}$ (4f)

Contiguity constraints explanation: (4a) Ensure each district has one root county that it can be flowed to. (4b) Ensure each county i can't be root county to a district j that it does not belong to. (4c) Ensure County i consumes the flow if it is not a root county. (4d) Ensure that there is no flow across edges that have been cut. (4e) Ensure that the flow is nonnegative. (4f) Binary constraint that each county is either a root county or is not a root county.

c. Minimum Perimeter Model

Parameters:

s_e= shared perimeter between two nodes in e

The only additional parameter of this model is to minimize the shared perimeter between the counties in cuts e. This would change the objective function to the following:

minimize $\sum_{e \in E} s_e y_e$

The following constraints would be the same with the addition as the minimize cut edges model shown above.

VI. Python/Gurobi Code

The project team has submitted the code for this project in separate files, and there should be an additional file with the project code. This code is based on Professor Austin Buchanan's work from a previous project, and the team included a link to the source. The following link will show the source: https://github.com/AustinLBuchanan/Districting-Examples-2020

All the population data files (including database file, .json file, projection file, shape file, and shape index file) are also given by Professor Austin Buchanan in the student's course canvas, as well as all code models coded by the project team members, will also be included in the team project's GitHub Repository with the following link: https://github.com/jasonptm05/IEM-4013-Idaho-Redistricting

VII. Experiments

When the team set the standard deviation to 1%, the minimum cut edge objective value found was 1.100000000000e+01, the minimum cut perimeter objective value was 6.859607089463e+00, and the minimum moment of inertia model objective value was 2.815441927327e+10 with solved times of 0.32 seconds, 1.15 seconds, and 1.17 seconds correspondingly.

Given this quick solve time, the team continuously lowered the standard deviations until they were able to achieve a standard deviation of zero for the minimum cut edge model. The objective value found 2.900000000000e+01 with a solve time of 1012.69 seconds.

The attributes of the team's computer are presented below:

Processor - 12th Gen Intel(R) Core (TM) i7-12700H 2.30 GHz

- Computer's baseline speed 2.3GHz-4.7GHz

RAM – 32.0 GB (31.7 GB usable)

Graphics Card - NVIDIA RTX A2000 8GB Laptop GPU (8 GB), Intel(R) Iris(R) Xe Graphics (128 MB)

System Type - 64-bit operating system, x64-based processor

The attributes of the solver are presented below:

Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (win64 - Windows 10.0 (19045.2))

VIII. Plans and Maps

The project team created a minimum moment of inertia plan, a minimum perimeter plan, and a minimum edge cut plan with an allowable standard deviation of 1% between both districts using Python/Gurobi as well as used davesredistrict.org for creating maps from another four different methods (same as above) shown below:

Minimum Moment of Inertia Model

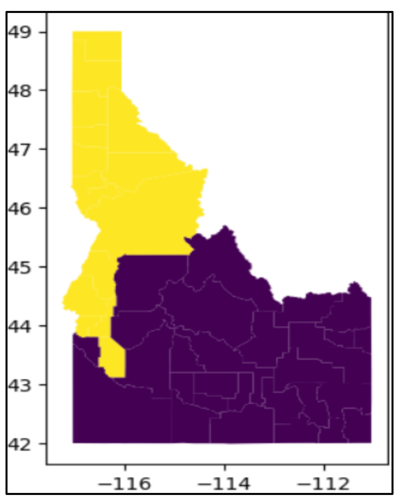


Figure 2A: Idaho's District Plan generated by using Python/ Gurobi

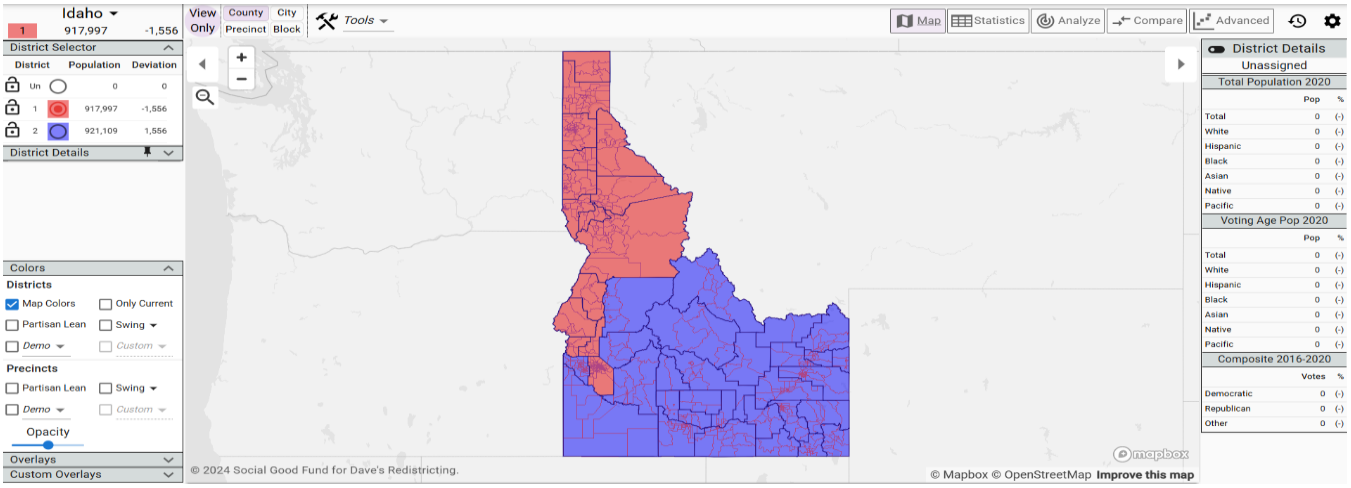


Figure 2B: Idaho's District Plan generated by using daveredistricting.org

The project team members used two colors (red and blue) to represent Idaho's two congressional districts and plotted our model using davedistricting.org. The first district (red) has a population of 917,997 with a deviation of "-1,556", and the second district (blue) has a population of 921,109 with a deviation of "+1,556". These two districts' population number and deviation will remain the same for the minimum of moment inertia and minimum cut edge with a deviation of 0.01.

<u>Minimum Cut Edge Perimeter</u>

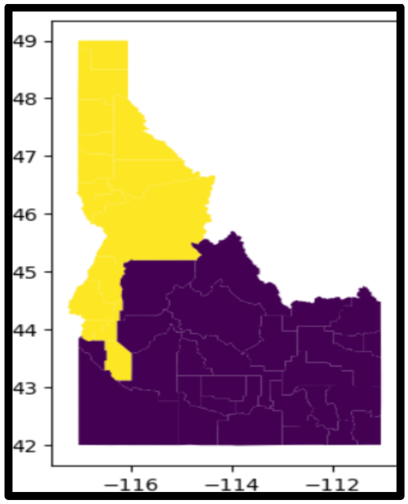


Figure 3A: Idaho's District Plan generated by using Python/ Gurobi

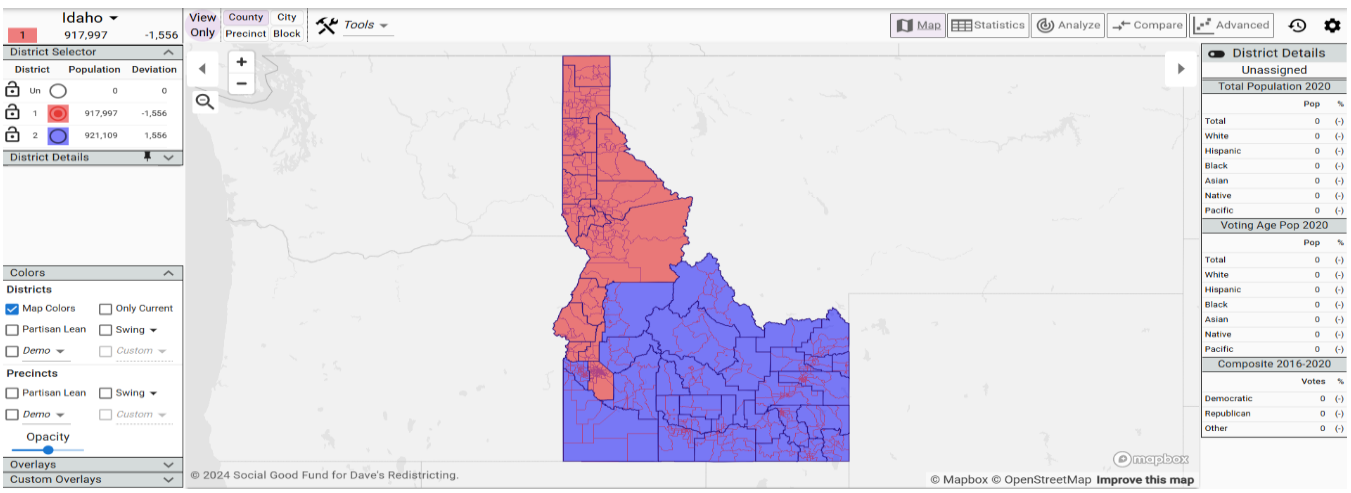
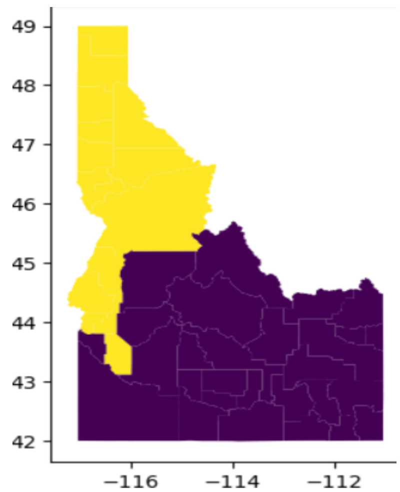


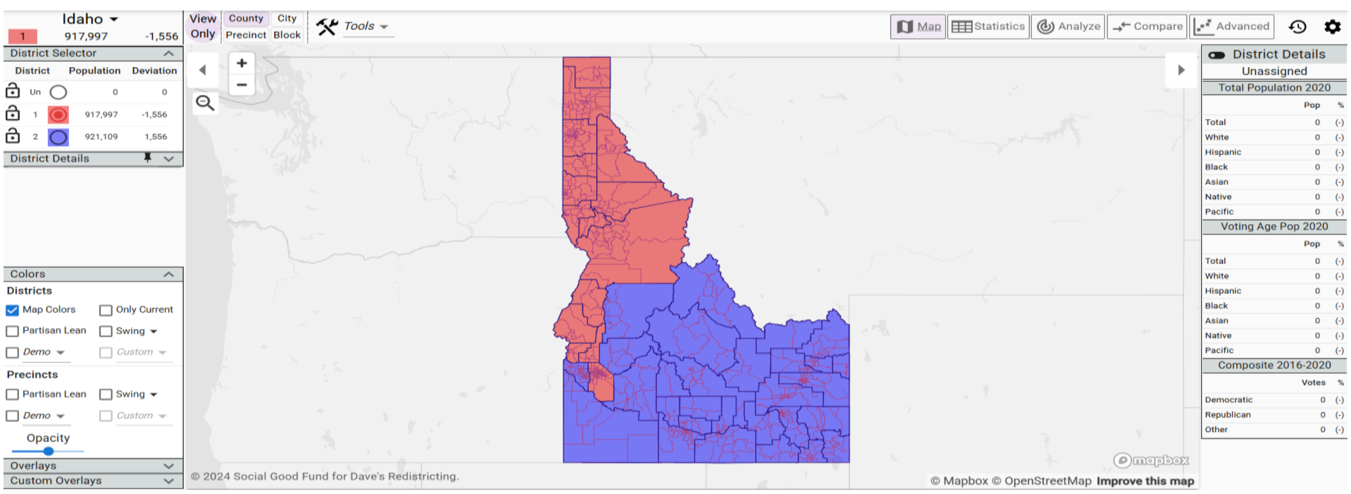
Figure 3B: Idaho's District Plan generated by using daveredistricting.org

The project team members used two colors (red and blue) to represent Idaho's two congressional districts and plotted our model using davedistricting.org. The first district (red) has a population of 917,997 with a deviation of "-1,556", and the second district (blue) has a population of 921,109 with a deviation of "+1,556". These two districts' population number and deviation will remain the same for the minimum of moment inertia and minimum cut edge with a deviation of 0.01.

<u>Minimum Cut Edge (with deviation = 0.01)</u>



<u>Figure 4A: Idaho's District Plan generated by using Python/ Gurobi</u>



<u>Figure 4B: Idaho's District Plan generated by using daveredistricting.org</u>

The project team members used two colors (red and blue) to represent Idaho's two congressional districts and plotted our model using davedistricting.org. The first district (red) has a population of 917,997 with a deviation of "-1,556", and the second district (blue) has a population of 921,109 with a deviation of "+1,556". These two districts' population number and deviation will remain the same for the minimum cut edge perimeter and minimum moment of inertia.

Minimum Cut Edge (optimal with deviation = 0)

The project team also created a map where the standard deviation between the maps was zero using the minimized cut edge model with a standard deviation of 0.

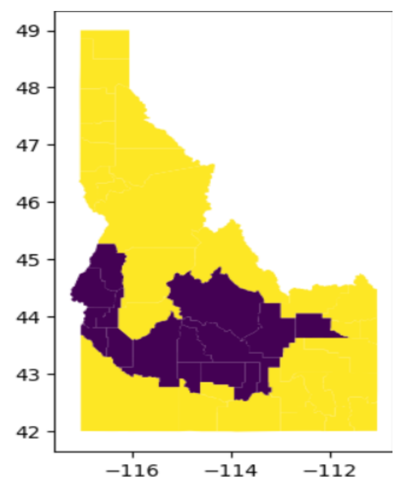


Figure 5A: Idaho's District Plan generated by using Python/ Gurobi

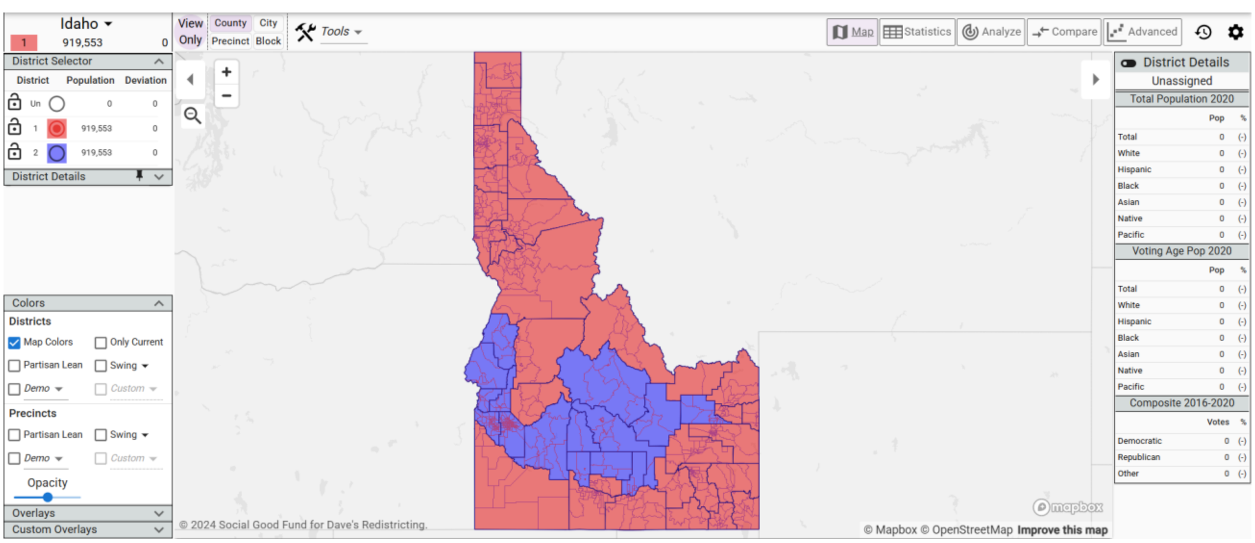


Figure 5B: Idaho's District Plan generated by using daveredistricting.org

The project team members colored two different colors (red and blue) along with two congressional districts that Idaho State has generating from davedistricting.org, the first district (red) has 919,553 in population with a deviation of "0", as well as the second district (blue) has 919,553 with a deviation of “0”.

IX. Evaluation of plans

The three models at a standard deviation of one percent all proposed the same map. These maps had a percentage difference in population of ≈ 0.0016921265% between districts. This aligns with the federal criteria of a population having a less than 1% standard deviation between districts and does a fairly good job of equal population in each of the districts. The federal criteria established by the Voting Rights Act is also inherently followed due to the largely white and conservative population present in Idaho. Similar state level criteria on protecting political parties, preserving communities of interest are followed for the same reason. State contiguity criteria are also followed due to the constraints imposed through in modeling. The model also follows the state requirement of trying to keep out irregular county arrangements.

The minimum cut edge model with a standard deviation of zero finds a solution with the population of both districts being the exact same. This map follows the equal population for districts perfectly and as previously mentioned due to Idaho’s largely conservative and white population, gerrymandering is not a necessary concern for, so the map follows all federal requirements. On a state level, the map maintains contiguity like the previous models. The only state criteria that this map challenges are avoiding oddly shaped districts; however, this is a subjective criterion, and this requirement is not strictly imposed in state law.

X. Conclusion

The team used multiple models to try and most accurately follow state and federal requirements for congressional districting. Due to Idaho's small size, the team was able to find a model with zero population difference between the two congressional districts using a minimum cut model. The team located this model as the best redistricting plan based on the importance of equal population that is present in both the state and federal criteria in redistricting. While the shape proposed using this model is somewhat "irregular," due to the largely homogenous aspects of Idaho's population (for example majority white and conservative) there should be little issue on gerrymandering for a specific political party or race. If, however, the shape of the districts is deemed too "irregular," to be utilized, the map created for the other models is a suitable redistricting solution as it follows all state and federal criteria as well.

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