CS4243

AY23/24 Sem 2

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05. Segmentation

- Goal: Separate image into coherent regions
- Idea: **Clustering** - Group similar data points together
- Challenges: What makes 2 points same/different? Choice of features (e.g. Color, Intensity, Position), Which clustering algorithm?
- **k-Means Clustering** - Iteratively re-assign points to nearest cluster center

1. Randomly initialize the cluster centers $c_1, \dots, c_K$
2. For each point p_i, find the closest c_j to put p_i in
3. Set c_j to be mean of points in cluster j
4. Repeat, if c_j have changed up to some threshold

- Pros: Simple, Converges to local min.
- Cons: Setting K, Sensitive to initial centers (Since k-means converges to local min.), Sensitive to outliers (Can add more clusters), Assumes spherical clusters
- **Simple Linear Iterative Clustering (SLIC) Superpixels**
  - **Superpixel** - Group of pixels that share common traits
    - Application: Inputs to other CV algo. since more compact representation with perceptual meaning
  - Num. of pixels: n_{tp}; Target num. of superpixels: n_{sp}
  - Initial width of each superpixel: $s = \sqrt{n_{tp}/n_{sp}}$
  - Features: $z = [r, g, b, x, y]$
  - Color distance: $d_c = ||\langle r_j, g_j, b_j\rangle - \langle r_i, g_i, b_i\rangle||$
  - Spatial distance: $d_s = ||\langle x_j, y_j\rangle - \langle x_i, y_i\rangle||$
  - Scaling factors: d_{cm} and $d_{sm} = s$ set as max. expected values of d_c and d_s respectively
  - $D = \sqrt{(\frac{d_c}{d_{cm}})^2 + (\frac{d_s}{d_{sm}})^2} = \sqrt{d_c^2 + (\frac{d_s}{s})^2 c^2}$

1. Split img. into grid of size $s \times s$. Set cluster centers as lowest gradient position in 3×3 neighborhood from superpixel center to speed up convergence since initialize on value common to surrounding.
2. For each cluster center, check distance to all pixels within $2s \times 2s$ neighborhood. Assign pixels to closest checked center.
3. Update cluster centers using mean and repeat if not converged (Same as k-Means)
4. Optional: Replace superpixel region by average value to create stained glass effect

  - Modification of k-Means: Not random initialization, Compute pixel's distance only to closest set of cluster centers

- **Mean-Shift Clustering** - Find local density maxima in feature space
  - **Attraction basin** - Region in feature space for which all trajectories of centroids lead to same mode
  - **Cluster** - All data points in attraction basin of a mode

1. For each data point:

(1) Define window around and get centroid

(2) Shift window to centroid

(3) Repeat until window centroid stops moving

  - Segmentation with Mean Shift: Do mean shift and merge pixels in same attraction basin
  - Choosing window size: Trial and error, Sample points and use avg. dist. to knn. (Num. of neighbors needs to be large enough to ensure increase in density)
    - Larger window size $\rightarrow$ Fewer clusters
  - Pros: No assumptions on cluster shape, 1 parameter, Finds variable num. of modes (vs. specified k in k-Means), Robust to outliers
  - Cons: Choosing h, Slow, Scales poorly with feature space dimension
  - Optimizations:
    - After each run of mean shift, assign all points within radius r of end point to same cluster
    - Assign points in radius $c < r$ of search path to mode

06. Texture

- **Texture** - Pattern with repeating elements
- **Filter Bank** - Measures variety of structures in local neighborhood and generates multi-dimensional features
  - Goal: How to represent texture?
  - Idea: Apply filters with small windows to generate statistics that summarize local patterns. Dist. in feature space bet. windows $\rightarrow$ Pixel's texture similarity
  - d filters $\rightarrow$ d-dimensional feature vector
  - Choosing window size: Try many sizes and look for one where statistic does not change much
  - Choose filters in different scales and orientations (to solve window size problem)
  - **Gabor Filter** - Represent filter banks mathematically by combining sinusoids with exp. (Gaussian) envelope
- **Texton** - Characterizes texture by replacing each pixel with integer representing **texture type**

1. Apply filter bank to training image
2. Cluster in feature space and store cluster centers (Texton dictionary)
3. For test image, filter image with same filter bank to get feature vector for each pixel. Assign each pixel to nearest cluster. Cluster ID = Texton ID.
4. For a given region, compute **texton histograms**

  - Classification: Given new img., compare hist. to other trng. samples and assign to label with most similarity
  - Segmentation: Use texton histograms as a feature

- **Perceived Boundaries** - Segmentation by human
  - Idea: Texture gradients indicate boundaries well

1. For each pixel, consider a disk that is split into 2 halves with some orientation
2. Measure texture diff. bet. 2 halves via texton hist.
3. Try all orientations. Orientation with high difference suggests boundaries.

07. Keypoints

- Motivation: How to stitch 2 images (e.g. Panaroma)?

1. Keypoints: Find locations
2. Descriptors: Rep. surrounding regions with math
3. Do the matching

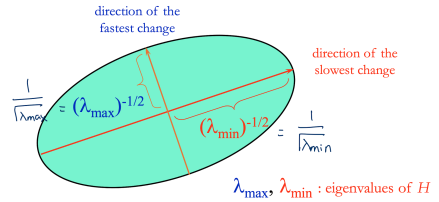
- Good keypoints are **repeatable** and **distinct**
- **Harris Corner Detection**
  - Significance: Corners have big changes in all directions when shifting window
  - Given window W shifted by offset (u, v):

$$E(U, v) = \sum_{(x,y)\in W} (I(x+u, y+v) - I(x,y))^2$$

- Assuming only small shifts (for Taylor Series Exp.):

$$E(u,v) = Au^2 + 2Buv + Cv^2 = \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

  - $A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_x I_y$
  - $C = \sum_{(x,y)\in W} I_y^2$
  - **2nd Moment Matrix (H)** - Middle matrix
- $E = k$ visualized as ellipse, where H controls shape
  - Eigenvectors of H $\rightarrow$ Axes orientation
  - Eigenvalues of H $\rightarrow$ Axes length



- **Eigenvectors** of A are vectors $\mathbf{x}$ that: $A\mathbf{x} = \lambda\mathbf{x}$
- **Eigenvalue** (λ) corresponds to $\mathbf{x}$: $\det(A - \lambda I) = 0$
- Since $A = H$ is 2×2: $\lambda_{\pm} = \frac{1}{2}((h_{11} + h_{22}) \pm \sqrt{2h_{12}h_{21} + (h_{11} - h_{22})^2})$
- After getting λs, find $\mathbf{x}$: $(A - \lambda I)\mathbf{x} = 0$
- Both λ_{max} and λ_{min} are large $\rightarrow$ Corner
- 'Cornerness' Score: $R = \min(\lambda_1, \lambda_2)$ (But getting λ is slow)
- **Harris Operator**: $R = \det(H) - \kappa(\text{trace}(H))^2$
  - $\det(H) = AC - B^2 = \lambda_1\lambda_2$
  - $\text{trace}(H) = A + C = \lambda_1 + \lambda_2$
  - $R > 0 \rightarrow$ Corner, $R < 0 \rightarrow$ Edge, $R \approx 0 \rightarrow$ Flat

1. Compute gradient for each point in image
2. Compute H matrix for each image window and get 'correctness' score
3. Find points with window where $R >$ threshold
4. Take points of local maxima

- **Non-Max. Suppression** - Iteratively search for max. values, then zero everything in surrounding window
  - Window size important
  - Adaptive: To prevent uneven distri. of keypoints in areas of higher contrast, pick corners which are both local max. and whose response is greater than all neighboring local max.
- In practice: $H = \sum_{(x,y)\in W} w_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$ (e.g. Convolve with Gaussian)
- **Harris Corner Invariances**
  - Purpose: If img. transf., how repeatable is detection?
  - **Equivariance** - Image transformed, and detection location undergoes similar transformation
  - **Invariance** - Image tranf., but no detection score change
  - Translation: Equivariant and invariant
  - Rotation: Equivariant and invariant
  - Photometric transformation (Assume $I' = aI + b$): Invariant to $b \neq 0$, but not invariant to $a \neq 1$
  - Scaling: Not equivariant and invariant
    - Scale of window can determine if location is keypoint $\rightarrow$ Need to scale up window by image scale
    - **Auto. Scale Selection** - When looking for keypoints, try window sizes and find scale that gives local max.
- **Laplacian of Gaussian** - Alternative keypoint detector which detects 'blobs' and is scale-sensitive
  - $\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$
  - Practice: Approx. by Difference of Gaussian for speed
  - Idea: Convolution with LoG has highest response when signal has same scale as Gaussian. Built-in scale sensitivity by varying scale σ.
  - Implementation: Fix window and kernel size; rescale img. with Gaussian blurring and downsampling

08. Descriptors

- Goal: Get feature vector surrounding each keypoint and measure similarity between feature vectors for matching
- Desc. should be invariant/equivariant and unique. E.g.:
  - Raw intensity: Good for exact template matching, but sensitive to lighting
  - Image gradient: Invariant to raw intensity (i.e. Lighting), but sensitive to transformations
  - Color histogram: Invariant to scale and rotation, but not sensitive to spatial layout
  - Spatial histogram: Compute color histograms over spatial cells. But not invariant to large rotations.
    - Orientation normalization: Normalize orientation of patch based on dominant image gradient
    - Save orien. angle θ w/ keypoint (e.g. Mean, mode)
  - **GIST Descriptor** - Rough spatial distribution of image gradients that is rotation invariant

1. Divide image into 4×4 grid
2. Apply Gabor filters (All dir. edge; N filters)
3. Compute filter response averages for each cells
4. Size of descriptor: $4 \times 4 \times N$

- **SIFT** - Keypoint detector and descriptor
  - Detector: Uses multi-scale LoG to get scale invariance, orientation normalization for rotation invariance, and threshold for removing low-contrast and low-curvature keypoints
- **SIFT Descriptor**

1. Take 16×16-pixel window around keypoint. Partition window into 4×4 grid.
2. Compute **gradient orientations and magnitudes** for each pixel. Reweight magnitudes using Gaussian and discard pixels with low magnitude.
3. For each 4×4-pixel cell, make **histogram with 8 orientation bins**. Shift histogram binning by dominant orien. (i.e. Subtract by dom. orien.) for rotation invariance. Collapse into 1×128 vector.
4. Normalize vector to unit length

  - Invariant to scale, rotation, and lighting
  - Partially invariant to viewpoint (Up to $60°$)
  - Quick and efficient

- **Feature Matching** - Given feature in I_1, how to find best match in I_2?

1. Define distance function that compares desc.
   - Euclidean distance: $||f_1 - f_2||$ (Can give small distances for incorrect matches)
   - Ratio distance bet. best vs. next-best: $\frac{||f_1 - f_2||}{||f_1 - f_2'||}$
2. Nested loop: Find vector with min. distance in I_2
   - Or ratio of best vs. next-best $<$ threshold

- Evaluation: **ROC curve** by varying threshold
  - Recall vs. 1 - Specificity

- Area Under the Curve (AUC): 1 is the best
- **Recall** - $\frac{TP}{TP+FN}$ **Specificity** - $\frac{TN}{TN+FP}$
- **Precision** - $\frac{TP}{TP+FP}$

09. Homography

- Goal: Stitch images from diff. viewpoints via projection
- When to use: Scene is planar, approx. planar (i.e. Small depth variation), or only camera rotation
- Problem: Given set of matched keypoints $\{p_i, p_i'\}$, get transformation $p' = f(p; H)$ where H are parameters
- Given homography function: Convert to homogeneous coord., multiply by homo. matrix, and convert back

$$p = \begin{bmatrix} x \\ y \end{bmatrix} P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}; P' = HP; P' = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} p' = \begin{bmatrix} \frac{x'}{w'} \\ \frac{y'}{w'} \end{bmatrix}$$

- **Direct Linear Transform** - Find best estimate of H

1. For each matching, create 2×9 matrix A_i
   - $A_i = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix}$
2. Concatenate into $2n \times 9$ matrix A
3. Compute SVD of $A = U \sum V^T$
4. Store vector of smallest singular value $h = v_{\hat{i}}$
5. Reshape to get H

- Assumptions: Projective model with linear transf.
- Cons: Sensitive to scaling (i.e. P: High res.; P': Low res.) $\to$ Normalize, Outliers $\to$ Poor est. of H

- **RANSAC** - More robust method for est. homographies
  - Motivation: DLT easily corrupted by outliers

1. Loop N times
   (1) Sample randomly num. of pts. required to fit model
   (2) Solve for model params. using samples
   (3) Score by frac. of inliers in threshold δ of model
2. Fit model to samples with most inliers

1. RANSAC loop
   (1) Sample 4 matches (H has 8 deg. of freedom)
   (2) Compute H using DLT
   (3) Inliers: Get P'' using H and check distance to P'
   (4) Keep H if largest number of inliers
2. Using best H with most inliers, recompute using all inliers

- δ: Impacts if inliers are kept (Trial and error)
- $N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$ where p is prob. that ≥ 1 set of samples does not contain outliers, e is prob. that point is outlier, and s is num. of samples per iter.
- Can loop N times or stop early when expected prob. of inliers reached, but both need prob. of outliers
- Integrate RANSAC with feature matching: Compute matches as before, add RANSAC loop and eliminate some matches that do not fit H

- **Warping** - Moves pixels of an image
  - Mapping: Transf. from source to destination via f
  - Resampling: **Splat** if map to bet. pixels, avg. if receive > 1 source

10. Optical Flow

- **Flow** - Displacement of pixels bet. frames (Vector field)
- Assumptions:
  - Color constancy: $I(x, y, t) = C$
  - Small motion: $I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$

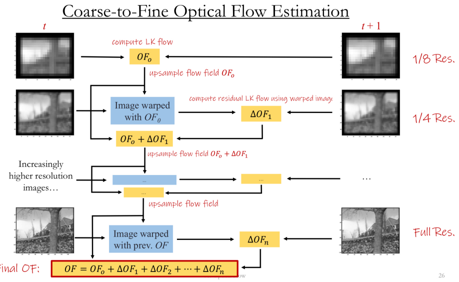
$$I_x u + I_y v + I_t = 0$$

  - $I_t = I(x, y, t+1) - I(x, y, t)$
- Problem: 1 equation, 2 unknowns
- **Lucas-Kanade** - Assumes constant flow in small region
  - Given $m \times n$ patch: $Ax = b; x = (A^T A)^{-1} A^T b$

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ \vdots & \vdots \\ I_x(p_{m \times n}) & I_y(p_{m \times n}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ \vdots \\ I_t(p_{m \times n}) \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}^{-1} \begin{bmatrix} \sum_{p \in P} I_x I_t \\ \sum_{p \in P} I_y I_t \end{bmatrix}$$

  - Requirements: $A^T A$ is invertible $\to \det(A^T A) = \lambda_1 \lambda_2$ should be big, $A^T A$ is well-conditioned $\to \frac{\lambda_{\max}}{\lambda_{\min}}$ should be small
    - Produces **sparse flow**: Only for some features
    - Similar to corner detector: Corners good for flow
  - Aperture Problem: Given small window over an edge, hard to tell which direction line is moving
    - Solution: Get windows with diff. gradients (Corner)
  - Aliasing: Undersampling of frames $\to$ Nearest match based on intensity is incorrect
    - Similar example: Image motion is large
    - Solution: Reduce res. to reduce apparent movement



- **Horn-Schunck** - Assumes smooth flow field
  - Minimization problem: Use gradient descent

$$\min_{u,v} \sum_{i,j} (E_s(i,j) + \lambda E_d(i,j))$$

  - $E_s(i,j) = \frac{1}{4}((u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{v,j+1})^2)$
  - $E_d(i,j) = (I_x u_{ij} + I_y v_{ij} + I_t)^2$

1. Compute I_x, I_y, I_t and initialize flow $u = v = \mathbf{0}$
2. Do until converge: $\hat{u}_{kl} = \bar{u}_{kl} - kI_x; \hat{v}_{kl} = \bar{v}_{kl} - kI_y$

$$k = \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} \text{ where } \bar{u}_{kl} \text{ is local avg.}$$

  - Choice of λ: When small, maximize smoothness
  - Produces **dense flow**: Flow for all pixels
  - Good for when image motion is small
- Evaluation: Euclidean distance, Cosine similarity

11. Tracking

- **Tracking** - Estimates parameters (location) of dynamic system (Feature points, target objects) over time
- Use optical flow for tracking? Flow only reliable for small motions, Occlusions and textureless regions (i.e. No corners) get bad est., LK gets sparse fields
- How to match template:
  - Template matching with normalized cross-correlation: Brute force, slow
    - $x_{ij} = \frac{1}{||F||||w_{ij}||} \sum_{u=-k}^{k} \sum_{v=-k}^{k} f_{uv} \cdot p_{i+u,j+v}$
  - Multi-scale template matching: Start search with coarser res., Faster
  - Local refinement based on some initial guess (e.g. Previous frame): Fastest, but sensitive to guess
- Notation:
  - 2D image transformation: $W(\mathbf{x}; \mathbf{p})$ where $\mathbf{x} = [x \ \ y]$ and $\mathbf{p} = \{p_1, \ldots, p_N\}$
  - Brightness constancy assump.: $I(x') = I(W(x; p))$
  - Translation: $W(\mathbf{x}; \mathbf{p}) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & p_1 \\ 0 & 1 & p_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
  - Affine: $W = \begin{bmatrix} p_1 x + p_2 y + p_3 \\ p_4 x + p_5 y + p_6 \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
- Image alignment problem: Given template $T(\mathbf{x})$ and source img. $I(\mathbf{x})$, solve for transformation parameters $\mathbf{p}$ that maps coordinates of img. to template such that: $\min_{\mathbf{p}} \sum_{\mathbf{x}} (I(W(\mathbf{x}; \mathbf{p})) - T(\mathbf{x}))^2$
- **Lucas-Kanade Image Alignment**
  - Assumption: Good initial guess of $\mathbf{p}$ and increment $\triangle \mathbf{p}$ from guess $\mathbf{p}$ is small $\to$ Linearize with Taylor Series and solve for $\triangle \mathbf{p}$

1. Warp image with initial $\mathbf{p}$: $I(W(\mathbf{x}; \mathbf{p}))$
2. Get error image: $T(\mathbf{x}) - I(W(\mathbf{x}; \mathbf{p}))$
3. Get gradient: $\nabla I(x')$ where x': Warped img. coord.
4. Get Jacobian: $\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \frac{\partial(\mathbf{W_1}, \mathbf{W_2})}{\partial(p_1, \ldots, p_N)}$
5. Get Hessian approx.: $H = \sum_{\mathbf{x}} (\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}})^T (\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}})$
6. $\triangle \mathbf{p} = H^{-1} \sum_{\mathbf{x}} (\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}})^T (T(\mathbf{x}) - I(W(\mathbf{x}; \mathbf{p})))$
7. $\mathbf{p} = \mathbf{p} + \triangle \mathbf{p}$

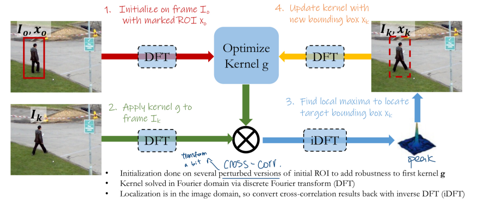
- **KLT Tracker** - Use LK img. alignment to track **feature points**
  - Good features to track: Hessian is well-conditioned matrix $\to$ Corners!

1. Find corners where $\min(\lambda_1, \lambda_2) > \lambda$ for Hessian
2. Loop over corners:
   (1) Get displacement to next frame using LK alignment
   (2) Store disp. of each corner, update corner position
   (3) Optional: Add corners every M frames via step 1
3. Return long trajectories for each corner point

  - Challenges: Determining features to track, How to track efficiently, Changing appearance of tracked points, **Drift** - Accumulation of small errors as model updates, Handle addition/removal of tracked points
  - Feature tracking vs. Optical flow: Duality
- **Template-based Tracking** - General algorithm to track **target objects**

1. Mark bounding box around target obj. in 1st frame
2. From target object template, get template descriptor: $\mathbf{q} = \{q_1, \ldots, q_M\}$
3. In next frame, find similar descriptor in neighborhood
   - Candidate descriptor: $\mathbf{p}(\mathbf{y})$ centered at location $\mathbf{y}$
   - Find peak similarity in region: $\max_{\mathbf{y}} \rho(\mathbf{p}(\mathbf{y}), \mathbf{q})$
4. Update target and descriptor $\mathbf{q}$ and repeat

  - What features to use? (Naive: Raw intensity; Better: Gradient, hist.)
  - Search for candidates (Naive: Brute force image; Better: Use prev. results)
  - Finding best candidate (Naive: Simple global max; Better: Use prev. locations)
  - Update target's template (Naive: Keep fixed; Better: Update)
- Cross-correlation as Naive Tracker: Crop region of interest as template and apply norm. cross-correlation with template as filter
  - Problem: False local max. when target differs a lot from template $\to$ Template need to dynamically adapt
- **MOSSE Filter** - Given target x, get unknown filter g to get response y modelled as Gaussian with small σ
  - $g = \arg\min_g \frac{1}{N} \sum_{i=1}^{N} (g \otimes x_i - y_i)^2 + \lambda ||g||^2$
  - $g = (X^T X + \lambda I)^{-1} X^T y$
  - N: Total number of target samples
  - X: $N \times n_g$



  - Incr. update kernel: Discrete Fourier Transform
    - Use multiplication, instead of X-corr. $\to$ Faster
    - Kernel remains additive across targets $\to$ Recently detected target gets higher weight to update kernel
  - Issues: Init., Occlusions, Exit frame, Clutter, Drift
  - Accuracy: How well does tracker bounding box overlap with ground truth box? $\frac{\text{Area}(B_{gt} \cap B_p)}{\text{Area}(B_{gt} \cup B_p)}$
  - Robustness: How many times does tracker fail?

12. Deep Learning

- **Neural Network** - Interconnected perceptrons with biases and weights
- Images as inputs: 1 fully-connected perceptron contains 1 weighted combo. of all input pixels $\to$ Ignores spatial info. (Neighboring pixels are more correlated) and slow
  - Insight: Stationarity (Statistics are similar at diff. locations) $\to$ Use same weights $\to$ Same as convolution
- **Convolution** - Apply multiple filters/kernels to image
  - Single filter $\to$ 1 2D-Feature map/channel
  - **Pooling** - Reduce large distortions $\to$ More invariance
    - Interleaving conv. with pooling causes later conv. to capture more of the image with same kernel size
    - **Receptive Field** - Set of img. pixels that intermediate output pixel depends on
    - Convolutions after pooling increase receptive field
  - **Activation Func.** - Introduces non-linearities
    - ReLU$(x) = \max(0, x)$
- Principles behind CNN:
  - Local interactions: Spatial locality included and layers will expand from local region to global region
  - Parameter sharing: Stationarity assumption $\to$ Equivariant representation