MIE 1622 Assignment 2

Risk-Based and Robust Portfolio Selection Strategies

REPORT

Qian Zhang

1000062736

1. **Introduction**

Assignment 2 builds upon the foundation of Assignment 1 by introducing 3 new strategies: Equal Risk Contribution, Leveraged Equal Risk Contribution, and Robust Mean Variance. To be used in the Equal Risk Contribution strategy, a gradient is calculated using the risk parity method. In the Leveraged Equal Risk Contribution strategy, the initial portfolio borrows 100% of its value with the same weights, and interests for the borrowed assets are paid at the end of each period. The Robust Mean-Variance Optimization method utilizes the return estimation error of the initial portfolio as well as the risk-free return acting as the target portfolio return.

For each period (2 months), the portfolio value is calculated for each of the seven strategies. In addition, two sets of data are tested, 2008 to 2009, and 2015 to 2016. Since the financial market during these two periods are drastically different, with the 2008 period being a financial crisis, the characteristics of each optimization strategy can be compared against each other.

A set of graphs are produced in the section below, including performance of each strategy in terms of daily portfolio value, and weight dynamic changes for strategies 3, 4, and 7. One set of graphs is produced for each set of data.

1. **MATLAB Code for Strategies 5, 6, and 7**
   1. **Strategy 5: Equal Risk Contribution (ERC)**

To begin, the initial portfolio weights is defined as the current weights on the day of rebalancing. As shown below

% Initial portfolio value

portf\_value\_erc\_init = cur\_prices \* curr\_positions + curr\_cash;

% Define initial portfolio weights

w0 = cur\_prices' .\* curr\_positions ./ portf\_value\_erc\_init;

Validation takes place to ensure that cash account is non-negative while accounting for a 5% transaction cost.

% calculating optimal x and cash based on weights calculated

x\_optimal = fix(((cur\_prices \* curr\_positions + curr\_cash) \* w\_erc) ./ transpose(cur\_prices));

cash\_optimal = cur\_prices \* curr\_positions - cur\_prices \* x\_optimal;

% Validation: Cash >=0 while accounting for transaction costs(0.5%)

while cash\_optimal - cur\_prices \* abs(curr\_positions - x\_optimal) \* 0.005 <= 0

% Subtracts one of each stock at a time until cash account is non-zero

x\_optimal = x\_optimal - 1;

% The new cash amount and weight after validation adjustment

cash\_optimal = cur\_prices \* curr\_positions - cur\_prices \* x\_optimal;

end

To compute the gradient function, the risk parity method is used. Mainly, a matrix col() is constructed to contain the elements of the gradient: (yi-yj)(dy/dx-dy/dx). Then, the columns of matrix col() is summed up and multiplied by 4 to become the gradient function gval. As seen below.

function gval = computeGradERC (x)

global Q

n = size(Q,1) ;

if(size(x,1)==1)

x = x';

end

% declaring matrices and variable useful in gradient calculations

gval = zeros(n,1);

col = zeros(n,n);

m = 1;

y = x.\* (Q \* x);

% i and j corresponds to each column index while k corresponds to each row

for i = 1:n

for j = i+1:n

for k = 1:n

% 3 scenarios of combinations of (yi-yj)(dy/dx-dy/dx)

% this loop computes an entire column of matrix col

if k == i

partial1 = Q(i,:) \* x + x(i) \* Q(i,i);

partial2 = x(j) \* Q(j,i);

elseif k == j

partial1 = x(i) \* Q(i,j);

partial2 = Q(j,:) \* x + x(j) \* Q(j,j);

else

partial1 = x(i) \* Q(i,k);

partial2 = x(j) \* Q(j,k);

end

% result is stored in the col matrix

% the col matrix contains all elements of the gradient

% before summation

col(k,m) = (y(i)-y(j)) \* (partial1 - partial2);

end

% next column

m = m+1;

end

end

% values of columns are summed up resulting in a 20 x 1 matrix gval

for l = 1:n

gval(l) = sum(col(l,:));

end

gval = 2 \* 2 \* gval;

To ensure that the risk contribution of each asset is equally distributed, a check is implemented to display the risk contribution of each period.

% Checking whether risks are equally distributed

fprintf('\n\nAsset risk contributions for ERC:\n')

[RC\_ERC]

For example, for the 12th period of 2016, the following risk distribution is produced:

Asset risk contributions for ERC:

RC\_ERC =

1.0e-03 \*

0.406587597828200

0.407238288646294

0.407801677446474

0.406834896894577

0.407650179553462

0.407679463159956

0.407204538398722

0.405329918850767

0.405129209859796

0.404602342372779

0.405596724497297

0.407101147649394

0.404525792330136

0.406724417931938

0.408135321594633

0.406527540071939

0.408117209119075

0.406843336954910

0.407039152934095

0.407385059350091

* 1. **Strategy 6: Leveraged Equal Risk Contribution**

This strategy borrows 100% of the initial portfolio value and weights and re-balances based on the equal risk contribution method. Since interest for the borrowed asset is paid at the end of each period based on the risk-free rate, the interest is defined as below, one for each time frame (2008/2015):

% two risk free rates for two sets of data (2015-2016 & 2008-2009)

if cur\_year == 2015 | 2016

% 2015-2016 risk free rate is 0.025/6 for each period

rf\_return = 0.025/6;

else

% 2008-2009 risk free rate is 0.045/6 for each period

rf\_return = 0.045/6;

end

Strategy 6 calls upon the ERC function directly, with twice the amount initial portfolio positions. Interest paid is calculated by multiplying the borrowed portfolio value by the risk-free rate.

if curr\_positions == init\_positions

% calling erc function with double the amount of current positions

[x\_lever cash\_lever] = strat\_equal\_risk\_contr(curr\_positions\*2, curr\_cash, mu, Q, cur\_prices);

% value of the borrowed stocks

portf\_value\_lever = cur\_prices \* curr\_positions + curr\_cash;

% interest is the risk free rate \* borrowed stocks

lever\_interest = portf\_value\_lever \* rf\_return;

Although transaction costs are already included in the ERC function, it is still required to validate that there are sufficient funds in the cash account to pay for interests.

% Cash validation to ensure enough cash to pay interest

% transaction costs are already accounted for in func\_erc

while cash\_optimal <= 0

% Subtracts one of each stock at a time until cash account is non-zero

x\_optimal = x\_optimal - 1;

% Cash increase after validation adjustment

cash\_optimal = cash\_optimal + sum(cur\_prices \* ones(20,1));

end

To calculate the portfolio value for strategy 6, the value of the borrowed assets is subtracted from the total value in the main file.

if strategy == 6

% Compute portfolio value for strategy 6 excluding borrowed assets

portf\_value{strategy}(day\_ind\_start:day\_ind\_end) = data\_prices(day\_ind\_start:day\_ind\_end,:) \* x{strategy,period} + cash{strategy,period} - init\_value;

else

% Compute portfolio value for other strategies

portf\_value{strategy}(day\_ind\_start:day\_ind\_end) = data\_prices(day\_ind\_start:day\_ind\_end,:) \* x{strategy,period} + cash{strategy,period};

end

* 1. **Strategy 7: Robust Mean-Variance Optimization**

To perform the RMVO, the target risk estimation error and target return are selected. The target risk estimation error is chosen to have minimal variance, selecting only the diagonal of the covariance matrix Q. This will ensure that assets variances are independent of each other, reducing the overall uncertainty and errors. The risk-free rate is chosen as the target return of the portfolio in order for the portfolio to yield at minimum the return of an risk-free asset.

Similar to strategy 5, the risk free rate is chosen based on the time frame. The daily risk-free rate is used for the RMVO.

% two risk free rates for two sets of data (2015-2016 & 2008-2009)

if cur\_year == 2015 | 2016

% 2015-2016 daily risk free rate

rf\_return = 0.025/252;

else

% 2008-2009 daily risk free rate

rf\_return = 0.045/252;

end

Target estimation error and return are defined as follows.

% Required portfolio robustness

var\_matr = diag(diag(Q));

% Target portfolio return estimation error is return estimation error of 1/n portfolio

rob\_init = w0' \* var\_matr \* w0; % return estimation error of initial portfolio

rob\_bnd = rob\_init; % target return estimation error

% Target portfolio return is the risk-free return for each period

Portf\_Retn = rf\_return;

Since it is possible that the RMVO cannot find an optimal solution depending on the financial performance of the market, the strategy will not rebalance the current portfolio if an optimal solution cannot be found.

% if optimal solution cannot be obtained, keep the portfolio unchanged

if strcmp(cplex\_rMV.Solution.statusstring,'optimal') == 0

x\_optimal = curr\_positions;

cash\_optimal = curr\_cash;

w\_rMW = transpose(cur\_prices) .\* x\_optimal / (cur\_prices \* x\_optimal + cash\_optimal);

1. **Plots and MATLAB outputs** 
   1. **Portfolio value corresponding to each strategy during 2015 and 2016**

Reading daily prices datafile - Daily\_closing\_prices.csv

Initial portfolio value = $ 1000002.12

Period 1: start date 1/2/2015, end date 2/27/2015

Strategy "Buy and Hold", value begin = $ 1000002.12, value end = $ 1043785.08

Strategy "Equally Weighted Portfolio", value begin = $ 1000002.12, value end = $ 1027293.47

Strategy "Minimum Variance Portfolio", value begin = $ 1000002.12, value end = $ 1024937.38

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1000002.12, value end = $ 1017243.04

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1000002.12, value end = $ 1025749.60

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1995837.56, value end = $ 2047536.09

Strategy "Robust Optimization Portfolio", value begin = $ 1000002.12, value end = $ 1024937.38

Period 2: start date 3/2/2015, end date 4/30/2015

Strategy "Buy and Hold", value begin = $ 1045234.09, value end = $ 1069877.19

Strategy "Equally Weighted Portfolio", value begin = $ 1031359.30, value end = $ 1011634.91

Strategy "Minimum Variance Portfolio", value begin = $ 1030374.03, value end = $ 1020571.98

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1025579.90, value end = $ 1064950.27

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1028326.22, value end = $ 1012367.38

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2050574.29, value end = $ 2018763.02

Strategy "Robust Optimization Portfolio", value begin = $ 1030374.03, value end = $ 1020571.98

Period 3: start date 5/1/2015, end date 6/30/2015

Strategy "Buy and Hold", value begin = $ 1085647.24, value end = $ 1027659.63

Strategy "Equally Weighted Portfolio", value begin = $ 1019720.49, value end = $ 985972.51

Strategy "Minimum Variance Portfolio", value begin = $ 1015575.91, value end = $ 976507.57

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1065116.75, value end = $ 1023037.33

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1018052.29, value end = $ 984840.36

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2026758.75, value end = $ 1960516.90

Strategy "Robust Optimization Portfolio", value begin = $ 1015575.91, value end = $ 973787.93

Period 4: start date 7/1/2015, end date 8/31/2015

Strategy "Buy and Hold", value begin = $ 1035245.91, value end = $ 947793.98

Strategy "Equally Weighted Portfolio", value begin = $ 989363.32, value end = $ 932219.27

Strategy "Minimum Variance Portfolio", value begin = $ 974906.69, value end = $ 933679.48

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1017088.58, value end = $ 927824.46

Strategy "Equal Risk Contributions Portfolio", value begin = $ 986091.36, value end = $ 932630.87

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1960729.83, value end = $ 1854332.60

Strategy "Robust Optimization Portfolio", value begin = $ 972270.27, value end = $ 919769.90

Period 5: start date 9/1/2015, end date 10/30/2015

Strategy "Buy and Hold", value begin = $ 912055.56, value end = $ 1027307.87

Strategy "Equally Weighted Portfolio", value begin = $ 900932.07, value end = $ 1018512.04

Strategy "Minimum Variance Portfolio", value begin = $ 902908.43, value end = $ 941525.26

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 880991.19, value end = $ 1099724.84

Strategy "Equal Risk Contributions Portfolio", value begin = $ 901488.53, value end = $ 1009478.01

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1789093.73, value end = $ 2003832.47

Strategy "Robust Optimization Portfolio", value begin = $ 886137.35, value end = $ 947335.61

Period 6: start date 11/2/2015, end date 12/31/2015

Strategy "Buy and Hold", value begin = $ 1039856.20, value end = $ 1003328.46

Strategy "Equally Weighted Portfolio", value begin = $ 1035051.05, value end = $ 1030450.33

Strategy "Minimum Variance Portfolio", value begin = $ 941555.52, value end = $ 955966.47

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1099134.74, value end = $ 1214223.44

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1024536.26, value end = $ 1018951.33

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2028245.93, value end = $ 2017070.66

Strategy "Robust Optimization Portfolio", value begin = $ 947269.43, value end = $ 956247.66

Period 7: start date 1/4/2016, end date 2/29/2016

Strategy "Buy and Hold", value begin = $ 994608.85, value end = $ 970570.87

Strategy "Equally Weighted Portfolio", value begin = $ 1008033.01, value end = $ 948182.62

Strategy "Minimum Variance Portfolio", value begin = $ 948934.96, value end = $ 945241.58

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1170858.20, value end = $ 1003157.46

Strategy "Equal Risk Contributions Portfolio", value begin = $ 999638.80, value end = $ 948462.28

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1971636.12, value end = $ 1870738.35

Strategy "Robust Optimization Portfolio", value begin = $ 945219.73, value end = $ 968946.89

Period 8: start date 3/1/2016, end date 4/29/2016

Strategy "Buy and Hold", value begin = $ 999683.25, value end = $ 975547.52

Strategy "Equally Weighted Portfolio", value begin = $ 974145.96, value end = $ 1043807.24

Strategy "Minimum Variance Portfolio", value begin = $ 955296.16, value end = $ 987780.96

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1029295.44, value end = $ 1000285.41

Strategy "Equal Risk Contributions Portfolio", value begin = $ 973840.12, value end = $ 1023862.25

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1914924.79, value end = $ 2013378.23

Strategy "Robust Optimization Portfolio", value begin = $ 983152.90, value end = $ 1012426.20

Period 9: start date 5/2/2016, end date 6/30/2016

Strategy "Buy and Hold", value begin = $ 982170.01, value end = $ 1000838.49

Strategy "Equally Weighted Portfolio", value begin = $ 1056485.22, value end = $ 1097862.98

Strategy "Minimum Variance Portfolio", value begin = $ 987711.66, value end = $ 1056901.42

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1002283.95, value end = $ 1090728.87

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1033107.00, value end = $ 1077836.35

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2024081.58, value end = $ 2111816.51

Strategy "Robust Optimization Portfolio", value begin = $ 1016046.71, value end = $ 1086989.25

Period 10: start date 7/1/2016, end date 8/31/2016

Strategy "Buy and Hold", value begin = $ 1003605.67, value end = $ 1067751.34

Strategy "Equally Weighted Portfolio", value begin = $ 1107024.12, value end = $ 1211950.95

Strategy "Minimum Variance Portfolio", value begin = $ 1056381.78, value end = $ 1041757.51

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1084443.78, value end = $ 1105255.48

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1085073.40, value end = $ 1146352.71

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2119057.48, value end = $ 2239138.25

Strategy "Robust Optimization Portfolio", value begin = $ 1085570.40, value end = $ 1070614.91

Period 11: start date 9/1/2016, end date 10/31/2016

Strategy "Buy and Hold", value begin = $ 1073361.15, value end = $ 1090939.15

Strategy "Equally Weighted Portfolio", value begin = $ 1213584.59, value end = $ 1212167.58

Strategy "Minimum Variance Portfolio", value begin = $ 1041393.40, value end = $ 1015666.25

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1106010.53, value end = $ 1168593.57

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1147750.67, value end = $ 1136217.94

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2234699.04, value end = $ 2212155.54

Strategy "Robust Optimization Portfolio", value begin = $ 1071293.13, value end = $ 1044875.63

Period 12: start date 11/1/2016, end date 12/30/2016

Strategy "Buy and Hold", value begin = $ 1077523.53, value end = $ 1173675.24

Strategy "Equally Weighted Portfolio", value begin = $ 1197891.88, value end = $ 1334312.14

Strategy "Minimum Variance Portfolio", value begin = $ 999347.27, value end = $ 1113683.76

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1143669.30, value end = $ 1520911.72

Strategy "Equal Risk Contributions Portfolio", value begin = $ 1124246.73, value end = $ 1233250.58

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 2181402.48, value end = $ 2393547.37

Strategy "Robust Optimization Portfolio", value begin = $ 1027917.30, value end = $ 1142044.37

* 1. **Plots for 2015 and 2016**

A close up of a map

Description automatically generated

Figure 1. Daily portfolio values of the 7 strategies during 2015 and 2016

A close up of a map

Description automatically generated

Figure 2. Dynamic changes of portfolio weights for the Minimum Variance Strategy during 2015 and 2016

A close up of a logo

Description automatically generated

Figure 3. Dynamic changes of portfolio weights for the Max. Sharpe Ratio Strategy during 2015 and 2016

A close up of a map

Description automatically generated

Figure 4. Dynamic changes of portfolio weights for the RMVO during 2015 and 2016

As seen in Figures 2, 3, and 4, the Robust MVO allows for a more diverse portfolio and less variations in assets weights (reduced trading).

As seen in Figure 1, the leveraged equal risk strategy outperforms the ERC and RMVO at the end of period 12. This is expected since the market towards the end of the 2016 is on an increasing trend, and since the leveraged method has extra portfolio value, the amount of increase of the portfolio is greater than the other strategies. It is worth noting that during times of poor performance of the market, as seen between approximately day 100 and 300 on Figure 1, the leveraged ERC method has the worst performance. The Maximum Sharpe Ratio strategy remains to have the best performance at the end of period 12, whereas the minimum variance strategy has the worst performance. The MSR strategy also has significantly better performance than the rest of the strategies during worse financial periods. During good financial times, the RMVO performs slightly better than the minimum variance method, placing it at second last. The equally weight strategy’s performance is similar to that of the leveraged method, and significantly better than the normal ERC method.

For normal financial times such as during 2015 and 2016, I would choose the Maximum Sharpe Strategy. Its performance is consistently better than the other 6 strategies and is less sensitive to the overall performance of the market.

* 1. **Portfolio value corresponding to each strategy during 2008 and 2009**

Reading daily prices datafile - Daily\_closing\_prices20082009.csv

Initial portfolio value = $ 548247.97

Period 1: start date 1/2/2008, end date 2/29/2008

Strategy "Buy and Hold", value begin = $ 548247.97, value end = $ 465217.72

Strategy "Equally Weighted Portfolio", value begin = $ 548247.97, value end = $ 474357.67

Strategy "Minimum Variance Portfolio", value begin = $ 548247.97, value end = $ 466229.19

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 548247.97, value end = $ 479567.84

Strategy "Equal Risk Contributions Portfolio", value begin = $ 548247.97, value end = $ 476057.28

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1094211.57, value end = $ 949607.11

Strategy "Robust Optimization Portfolio", value begin = $ 548247.97, value end = $ 460423.27

Period 2: start date 3/3/2008, end date 4/30/2008

Strategy "Buy and Hold", value begin = $ 462553.95, value end = $ 511257.48

Strategy "Equally Weighted Portfolio", value begin = $ 463495.66, value end = $ 509782.84

Strategy "Minimum Variance Portfolio", value begin = $ 457781.91, value end = $ 528369.81

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 468011.24, value end = $ 489130.22

Strategy "Equal Risk Contributions Portfolio", value begin = $ 465595.50, value end = $ 520488.31

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 928426.50, value end = $ 1038117.24

Strategy "Robust Optimization Portfolio", value begin = $ 453601.36, value end = $ 484293.01

Period 3: start date 5/1/2008, end date 6/30/2008

Strategy "Buy and Hold", value begin = $ 526490.95, value end = $ 486095.76

Strategy "Equally Weighted Portfolio", value begin = $ 526201.85, value end = $ 445754.92

Strategy "Minimum Variance Portfolio", value begin = $ 544114.28, value end = $ 501069.42

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 492323.31, value end = $ 426247.08

Strategy "Equal Risk Contributions Portfolio", value begin = $ 537726.93, value end = $ 461994.59

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1069723.85, value end = $ 918710.51

Strategy "Robust Optimization Portfolio", value begin = $ 493739.18, value end = $ 453184.76

Period 4: start date 7/1/2008, end date 8/29/2008

Strategy "Buy and Hold", value begin = $ 487307.50, value end = $ 485687.69

Strategy "Equally Weighted Portfolio", value begin = $ 445275.19, value end = $ 451392.73

Strategy "Minimum Variance Portfolio", value begin = $ 496690.13, value end = $ 499917.76

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 441282.69, value end = $ 440145.19

Strategy "Equal Risk Contributions Portfolio", value begin = $ 459517.12, value end = $ 463046.09

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 912470.46, value end = $ 919471.93

Strategy "Robust Optimization Portfolio", value begin = $ 452075.66, value end = $ 454918.57

Period 5: start date 9/2/2008, end date 10/31/2008

Strategy "Buy and Hold", value begin = $ 478985.24, value end = $ 369998.60

Strategy "Equally Weighted Portfolio", value begin = $ 449683.59, value end = $ 315308.85

Strategy "Minimum Variance Portfolio", value begin = $ 488586.08, value end = $ 376316.04

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 425693.18, value end = $ 299755.66

Strategy "Equal Risk Contributions Portfolio", value begin = $ 460532.34, value end = $ 333820.91

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 911767.50, value end = $ 660240.83

Strategy "Robust Optimization Portfolio", value begin = $ 444425.65, value end = $ 345578.51

Period 6: start date 11/3/2008, end date 12/31/2008

Strategy "Buy and Hold", value begin = $ 372792.12, value end = $ 338021.03

Strategy "Equally Weighted Portfolio", value begin = $ 313860.92, value end = $ 275719.11

Strategy "Minimum Variance Portfolio", value begin = $ 375688.03, value end = $ 337274.45

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 300887.07, value end = $ 259778.79

Strategy "Equal Risk Contributions Portfolio", value begin = $ 332186.60, value end = $ 293901.20

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 656681.88, value end = $ 580865.34

Strategy "Robust Optimization Portfolio", value begin = $ 348217.00, value end = $ 333985.71

Period 7: start date 1/2/2009, end date 2/27/2009

Strategy "Buy and Hold", value begin = $ 351630.52, value end = $ 325694.94

Strategy "Equally Weighted Portfolio", value begin = $ 286806.52, value end = $ 252955.85

Strategy "Minimum Variance Portfolio", value begin = $ 346291.91, value end = $ 326174.30

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 257972.33, value end = $ 211846.38

Strategy "Equal Risk Contributions Portfolio", value begin = $ 305101.15, value end = $ 270045.34

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 601801.65, value end = $ 532552.52

Strategy "Robust Optimization Portfolio", value begin = $ 339877.01, value end = $ 314244.38

Period 8: start date 3/2/2009, end date 4/30/2009

Strategy "Buy and Hold", value begin = $ 316048.57, value end = $ 392525.73

Strategy "Equally Weighted Portfolio", value begin = $ 241840.32, value end = $ 373042.80

Strategy "Minimum Variance Portfolio", value begin = $ 313245.00, value end = $ 424748.83

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 200745.24, value end = $ 281821.50

Strategy "Equal Risk Contributions Portfolio", value begin = $ 258960.54, value end = $ 379199.46

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 509527.53, value end = $ 746582.50

Strategy "Robust Optimization Portfolio", value begin = $ 302862.35, value end = $ 402930.97

Period 9: start date 5/1/2009, end date 6/30/2009

Strategy "Buy and Hold", value begin = $ 394998.62, value end = $ 426991.87

Strategy "Equally Weighted Portfolio", value begin = $ 372433.38, value end = $ 410801.17

Strategy "Minimum Variance Portfolio", value begin = $ 422205.71, value end = $ 427644.23

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 273217.55, value end = $ 297012.47

Strategy "Equal Risk Contributions Portfolio", value begin = $ 377944.87, value end = $ 406602.26

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 741901.08, value end = $ 798297.24

Strategy "Robust Optimization Portfolio", value begin = $ 401294.05, value end = $ 408437.56

Period 10: start date 7/1/2009, end date 8/31/2009

Strategy "Buy and Hold", value begin = $ 429930.17, value end = $ 467013.68

Strategy "Equally Weighted Portfolio", value begin = $ 411474.96, value end = $ 460447.30

Strategy "Minimum Variance Portfolio", value begin = $ 424649.32, value end = $ 449122.04

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 291831.29, value end = $ 288361.56

Strategy "Equal Risk Contributions Portfolio", value begin = $ 406800.28, value end = $ 445618.47

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 797077.81, value end = $ 873225.72

Strategy "Robust Optimization Portfolio", value begin = $ 406394.25, value end = $ 429856.21

Period 11: start date 9/1/2009, end date 10/30/2009

Strategy "Buy and Hold", value begin = $ 457407.27, value end = $ 489396.95

Strategy "Equally Weighted Portfolio", value begin = $ 445144.85, value end = $ 477078.42

Strategy "Minimum Variance Portfolio", value begin = $ 439095.04, value end = $ 461785.66

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 272076.58, value end = $ 274604.36

Strategy "Equal Risk Contributions Portfolio", value begin = $ 432414.97, value end = $ 462769.61

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 844976.31, value end = $ 904378.49

Strategy "Robust Optimization Portfolio", value begin = $ 420287.30, value end = $ 442079.96

Period 12: start date 11/2/2009, end date 12/31/2009

Strategy "Buy and Hold", value begin = $ 490582.55, value end = $ 542246.05

Strategy "Equally Weighted Portfolio", value begin = $ 478418.60, value end = $ 547587.03

Strategy "Minimum Variance Portfolio", value begin = $ 457311.96, value end = $ 511844.87

Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 273246.18, value end = $ 308790.10

Strategy "Equal Risk Contributions Portfolio", value begin = $ 462860.04, value end = $ 517819.65

Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 902741.25, value end = $ 1010118.87

Strategy "Robust Optimization Portfolio", value begin = $ 438402.84, value end = $ 490673.18

* 1. **Plots for 2008 and 2009**

A close up of a map

Description automatically generated

Figure 5. Daily portfolio values of the 7 strategies during 2008 and 2009

A close up of a map

Description automatically generated

Figure 6. Dynamic changes of portfolio weights for the Minimum Variance Strategy during 2008 and 09

A close up of a logo

Description automatically generated

Figure 7. Dynamic changes of portfolio weights for the Max. Sharpe Ratio Strategy during 2008 and 2009

A close up of a map

Description automatically generated

Figure 8. Dynamic changes of portfolio weights for the RMVO Strategy during 2008 and 2009

As seen in Figures 6, 7, and 8, the Robust MVO allows for a more diverse portfolio and less variations in assets weights (reduced trading).

The 2007-2008 financial crisis has generally been thought to have started recovery towards the end of 2008. As such, to enter an investment during this period is an ill-prepared decision. Nonetheless, it is important to know the potential consequences of such a decision, as the 7 strategies of portfolio optimization are implemented during the time period of 2008 and 2009. As seen in figure 5, the portfolio has entered the market as there was a steep downward trend. Every strategy has generally been affected with the same magnitude of fluctuation in value, with the exception of the leveraged equal risk contribution strategy, which at one point has lost all of its borrowed asset value and fails to recover completely towards the end of 2009. This outcome is the result of the leveraged strategy having 200% of the initial portfolio value as compared to the other strategies, as the leveraged strategy will be the most sensitive to market fluctuations. The maximum sharpe ratio strategy has the worst performing ending portfolio. The top performing strategies for 2008 and 2009 are the equally-weighted and buy-and-hold strategies, with the ending portfolio value approximately equal to their initial portfolio value when entering the market. This outcome is expected, since both strategies make minimal or no rebalancing to the portfolio assets, and mostly depend on the improvement of the market to grow in value. Since, the market is back on an upward trend towards the end of 2009, the two simplest strategies benefited the most. The equal risk contribution, minimum variance, and robust mean-variance strategies performed similarly to each other, slightly worse than the equally-weighted and buy-and-hold strategies. These strategies emphasize on risk distribution and minimizing variance, therefore they would perform relatively well during times of a financial crisis.

Comparing the 2008 data set to the data of 2015, the strategies with the greatest differences are the maximum sharpe ratio and the leveraged equal risk contribution strategies. The maximum sharpe ratio strategy performed the best during normal financial markets (2015-2016), and it performed the worst during the 2008 crisis. Although the leveraged equal risk contribution method was not the worst-performing strategy in the 2008 data, it has performed significantly worse than the other strategies, but also has recovered the most towards the end of 2009.