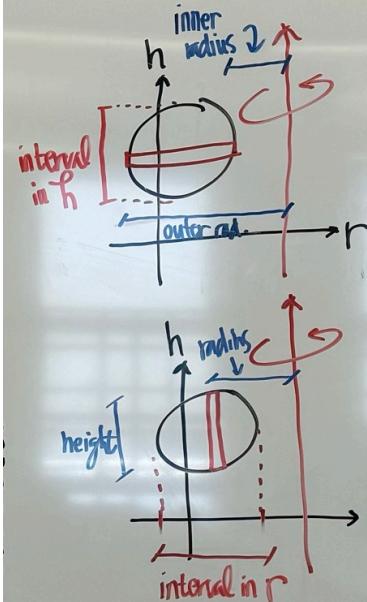


Reference for the Washer/Disk Method + Cylindrical Shells Method

1D AM Section.



Reference:

* Washer/Disk Method.

Slice PERPENDICULAR to the axis of rotation.

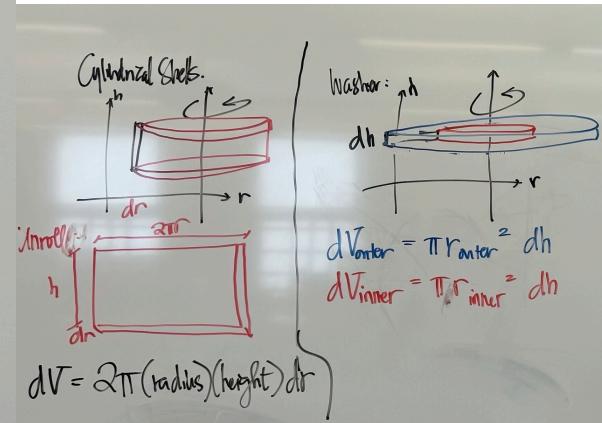
$$V = \int_{\text{interval in } h} \pi [(\text{outer radius})^2 - (\text{inner radius})^2] dh$$

* Cylindrical Shells Method.

Slice PARALLEL to the axis of rotation.

$$V = 2\pi \int_{\text{interval in } r} (\text{radius})(\text{height}) dr$$

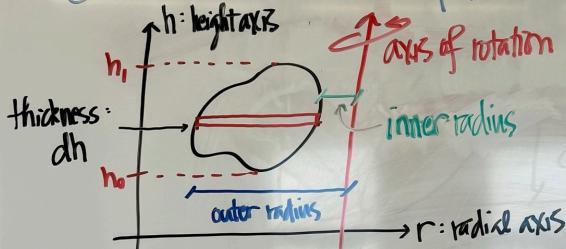
TIP: Always identify the axis of rotation in the drawing/sketch.



1D PM + 2D M Section.

* Washer/Disk Method.

Slice PERPENDICULAR to the axis of rotation.



$$V = \pi \int_{h_0}^{h_1} (\text{outer radius})^2 - (\text{inner radius})^2 dh$$

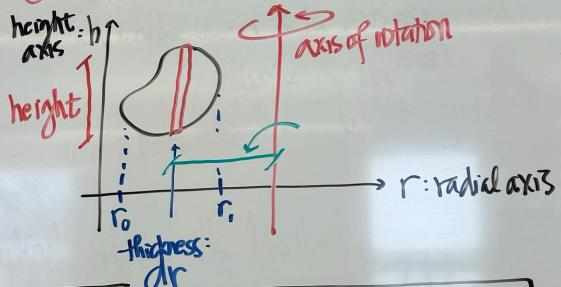
Side note: Volume of Disk: $dV = dV_{\text{outer}} - dV_{\text{inner}}$

with outer disk: $dV_{\text{outer}} = \pi (\text{outer radius})^2 dh$

and inner disk: $dV_{\text{inner}} = \pi (\text{inner radius})^2 dh$

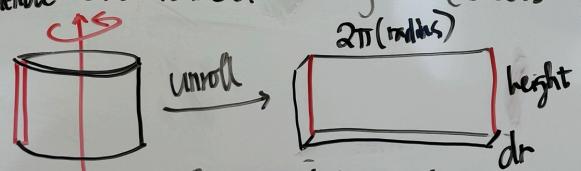
* Cylindrical Shells Method.

Slice PARALLEL to the axis of rotation.



$$V = \int_{r_0}^{r_1} (2\pi)(\text{radius})(\text{height}) dr$$

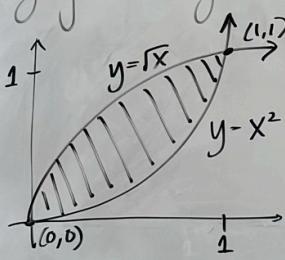
Side note: Slice looks like a hollow cylinder (shell)



$$\text{Volume of shell} = \text{Volume of rect. prism} \cdot dV = (2\pi)(\text{radius})(\text{height}) dr$$

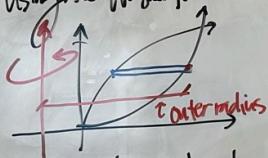
Example.

Let R be the region bounded by $y = x^2$ and $y = \sqrt{x}$:



A) Rotate R about the line $x = -1$.

Using the Washer Method:



thickness: dy , bounds: $y \in [0,1]$

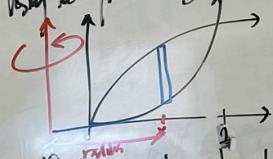
outer radius: $r_{\text{outer}} = x + 1$ with $y = x^2$, $x = \sqrt{y}$

$$r_{\text{outer}} = \sqrt{y} + 1$$

inner radius: $r_{\text{inner}} = x + 1$ with $y = \sqrt{x}$, $x = y^2$
 $= y^2 + 1$

$$V = \pi \int_0^1 (r_{\text{outer}}^2 - r_{\text{inner}}^2) dy$$

Using the Cylindrical Shells Method:



thickness: dx , bounds: $x \in [0,1]$

radius: $r = x + 1$

height: $h = y_{\text{high}} - y_{\text{low}}$

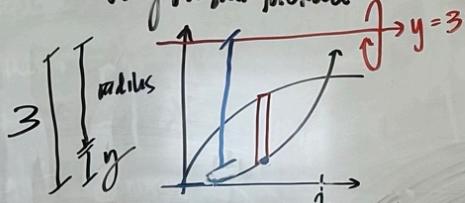
$$\text{with } y_{\text{high}} = y = \sqrt{x}, y_{\text{low}} = y = x^2$$

$$h = \sqrt{x} - x^2$$

$$V = \int_0^1 2\pi(r)(h) dx$$

B) Rotate R about the line $y = 3$.

Using Washer Method:



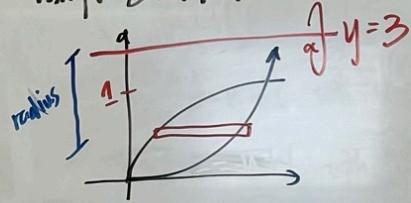
thickness: dx , bounds: $x \in [0,1]$

outer rad: $r_{\text{outer}} = 3 - y$ with $y = x^2$
 $= 3 - x^2$

inner radius: $r_{\text{inner}} = 3 - y$ with $y = \sqrt{x}$
 $= 3 - \sqrt{x}$

$$V = \pi \int_0^1 (r_{\text{outer}}^2 - r_{\text{inner}}^2) dx$$

Using the Shells Method.



thickness: dy , bounds: $y \in [0,1]$

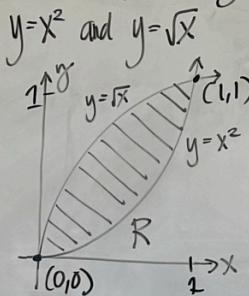
radius: $r = 3 - y$

height: $h = x_{\text{right}} - x_{\text{left}}$ with $\begin{cases} x_{\text{right}} = y = x^2, x = \sqrt{y} \\ x_{\text{left}} = y = \sqrt{x}, x = y^2 \end{cases}$
 $= \sqrt{y} - y^2$

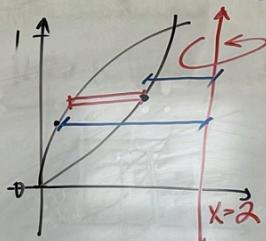
$$V = \int_0^1 2\pi(r)(h) dy$$

Example.

Ex. Let R be the region bounded by



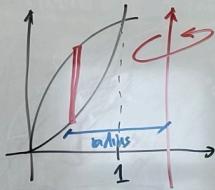
- (a1) Rotate R about the line $x=2$. Use Washer Method.



thickness: dy
 bounds: $y \in [0, 1]$
 $r_{\text{outer}} = 2 - x$ with $y = \sqrt{x}$, $x = y^2$; $r_{\text{outer}} = 2 - y^2$
 $r_{\text{inner}} = 2 - x$ with $y = x^2$, $x = \pm\sqrt{y}$; $r_{\text{inner}} = 2 - \sqrt{y}$

$$V = \int_0^1 \pi \left[(2-y)^2 - (2-\sqrt{y})^2 \right] dy$$

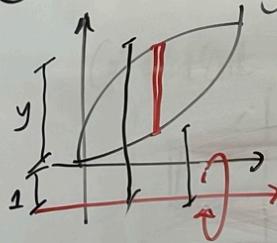
- (a2) Rotate R about $x=2$. Use Cylindrical Shells.



thickness: dx
 bounds: $x \in [0, 1]$
 radius: $2 - x$
 height: $y_{\text{high}} - y_{\text{low}}$
 with $y_{\text{high}} = y = \sqrt{x}$
 $y_{\text{low}} = y = x^2$

$$V = \int_0^1 2\pi(2-x)(\sqrt{x} - x^2) dx$$

- (b1) Rotate R about $y=-1$. Use Washer.

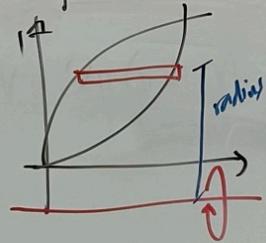


thickness: dx
 bounds: $x \in [0, 1]$
 outer radius: $r_{\text{outer}} = y + 1$ with $y = \sqrt{x}$; $r_{\text{outer}} = \sqrt{x} + 1$
 inner rad: $r_{\text{inner}} = y + 1$ with $y = x^2$; $r_{\text{inner}} = x^2 + 1$

$$V = \int_0^1 \pi \left[(\sqrt{x} + 1)^2 - (x^2 + 1)^2 \right] dx$$

- (b2) Rotate R about $y=-1$.

Use Cylindrical Shells.



thickness: dy
 bounds: $y \in [0, 1]$
 radius: $y + 1$
 height: $x_{\text{right}} - x_{\text{left}}$ with $\begin{cases} x_{\text{right}}: y - x^2, x = \pm\sqrt{y}, x = \sqrt{y} \\ x_{\text{left}}: y - \sqrt{x}, x = y^2 \end{cases}$

$$= \sqrt{y} - y \quad \boxed{V = \int_0^1 2\pi(y+1)(\sqrt{y} - y^2) dy}$$

