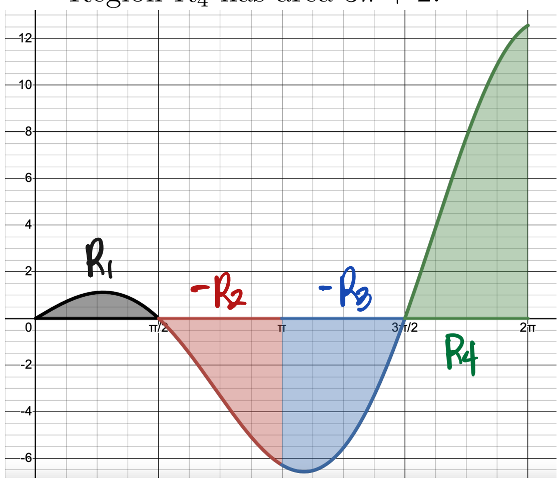
Instructions: Please work in small groups (3 or 4 students) on the following problems. You are expected to finish these problems even if there was not enough time in class to finish.

(1) The figure below shows the graph of $y = 2x\cos(x)$. The regions are R_1, R_2, R_3, and R_4, reading left to right. These correspond to the intervals $[0, \pi/2]$, $[\pi/2, \pi]$, $[\pi, 3\pi/2]$, and $[3\pi/2, 2\pi]$, respectively.

- Region R_1 has area $\pi - 2$.
- Region R_2 has area $\pi + 2$.
- Region R_3 has area $3\pi - 2$.
- Region R_4 has area $3\pi + 2$.

These are unsigned areas.

For convenience, let $f(x) = 2x\cos(x)$. Then,



$$A_1 = \int_0^{\frac{\pi}{2}} f(x)\,dx = \pi - 2 \quad ;$$

$$A_2 = \int_{\frac{\pi}{2}}^{\pi} f(x)\,dx = -(\pi+2) = -\pi - 2 \quad ;$$

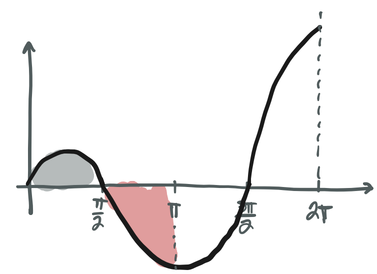
$$A_3 = \int_{\pi}^{\frac{3\pi}{2}} f(x)\,dx = -(3\pi - 2) = -3\pi + 2 \quad ;$$

$$A_4 = \int_{\frac{3\pi}{2}}^{2\pi} f(x)\,dx = 3\pi + 2 \quad ;$$

Use this information to evaluate the following integrals:

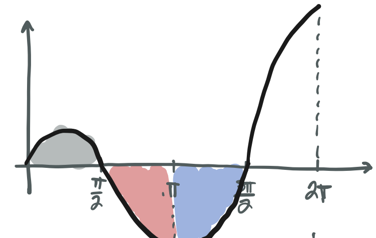
(a) $\int_0^{\pi} 2x\cos(x)dx$ $= \int_0^{\frac{\pi}{2}} f(x)dx + \int_{\frac{\pi}{2}}^{\pi} f(x)\,dx$

$= A_1 + A_2 = \pi - 2 + (-\pi - 2) = \boxed{-4}$



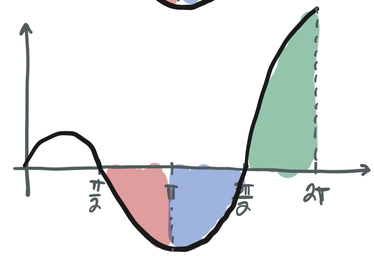
(b) $\int_0^{3\pi/2} 2x\cos(x)dx$ $= \cdots = A_1 + A_2 + A_3$

$= (\pi - 2) + (-\pi - 2) + (-3\pi + 2) = \boxed{-3\pi - 2}$



(c) $\int_{\pi/2}^{2\pi} 2x\cos(x)dx$ $= \cdots = A_2 + A_3 + A_4$

$= (-\pi - 2) + (-3\pi + 2) + (3\pi + 2) = \boxed{-\pi + 2}$



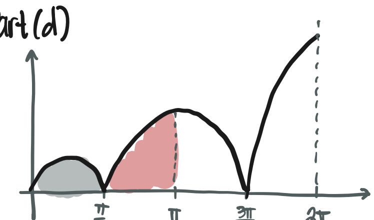
(d) $\int_0^{\pi} |2x\cos(x)|dx$

$= \int_0^{\frac{\pi}{2}} |f(x)|\,dx + \int_{\frac{\pi}{2}}^{\pi} |f(x)|\,dx$

$= \int_0^{\frac{\pi}{2}} f(x)\,dx + (-1)\int_{\frac{\pi}{2}}^{\pi} f(x)\,dx$ since $f(x) \geq 0$ on $x \in [0, \frac{\pi}{2}]$ and $f(x) \leq 0$ on $x \in [\frac{\pi}{2}, \pi]$

$= \pi - 2 + (-1)(-\pi - 2) = \boxed{2\pi}$

Part (d)



Note: for convenience, approximate to two decimal places.

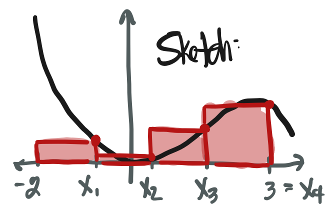
(2) Compute a **right** Riemann, R_4, sum for $4x^2 - x^3$ on $[-2, 3]$.

Let $f(x) = 4x^2 - x^3$, $[a,b] = [-2,3]$, $n = 4$; $\Delta x = \frac{b-a}{n} = \frac{3-(-2)}{4} = \frac{5}{4} = 1.25$;

i	$x_i = a + i\Delta x = -2 + i\left(\frac{5}{4}\right)$	$f(x_i) \approx$
1	$x_1 = -2 + (1)\left(\frac{5}{4}\right) = -0.75$	2.672
2	$x_2 = -2 + (2)\left(\frac{5}{4}\right) = 0.50$	0.875
3	$x_3 = -2 + (3)\left(\frac{5}{4}\right) = 1.75$	6.891
4	$x_4 = -2 + (4)\left(\frac{5}{4}\right) = 3$	9

$$R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x = 1.25\left(2.672 + 0.875 + 6.891 + 9\right)$$

$$= \boxed{24.30}$$



(3) Consider the definite integral

$$\int_{-4}^{8} 5x^4\, dx.$$

Fill in all the entries of the sigma notation needed to represent the definite integral as a right Riemann sum in terms of constants and n, where n represents the number of subintervals. **Note:** The only variable(s) in your summation should be n and the index of your summation. That is, there should be no x variables in your summation.

Reference: $R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$ with $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

$$\sum_{i=1}^{n} \overbrace{5\left(-4 + \frac{12i}{n}\right)^4}^{f(x_i)} \cdot \overbrace{\left(\frac{12}{n}\right)}^{\Delta x}$$

$[a,b] = [-4, 8]$, n arbitrary

$\Delta x = \frac{8-(-4)}{n} = \frac{12}{n}$, $x_i = a + i\Delta x = -4 + \frac{12i}{n}$

$f(x) = 5x^4$: $f(x_i) = 5\left(-4 + \frac{12i}{n}\right)^4$

(4) Joe gets a new pair of running shoes. If Joe runs 1 mile each day in week 1 and adds $\frac{1}{10}$ mile to his daily routine each week. Write a Riemann sum expressing the total mileage on Joe's shoes after 25 weeks.

On week i: Joe runs $1 + \frac{1}{10}(i-1)$ miles each day.

Weekly Mileage: $7\left[1 + \frac{1}{10}(i-1)\right]$

Total Mileage: $\boxed{\sum_{i=1}^{25} 7\left[1 + \frac{1}{10}(i-1)\right]}$

Alternatively: Let i correspond to Week $(i+1)$.
Then, weekly mileage: $7\left(1 + \frac{1}{10}i\right)$
from Week 1 $(i=0)$ to Week 25 $(i=24)$

Total Mileage: $\boxed{\sum_{i=0}^{24} 7\left(1 + \frac{1}{10}i\right)}$

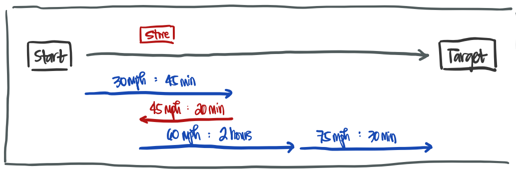
i	1	2	3	4	5		
time t	0	2	4	6	8	10	in seconds
velocity v(t)	0	2	7	5	2	4	in meters per second

(5) The above table shows the velocity of a dog running along a path towards a toy, where t is time in seconds. Approximate the net distance travelled, $\int_0^{10} v(t)dt$ by writing out and calculating the left endpoint approximation, L_5.

$[a,b]=[0,10]$, $n=5$; $\Delta t = \frac{10}{5} = 2$, $t_i = a+(i-1)\Delta x = 0+(i-1)(2) = 2i-2$;

$$\int_0^{10} v(t)\,dt \approx L_5 = \sum_{i=1}^{5} f(x_i)\Delta x = (2)\left[v(t_1)+v(t_2)+\dots+v(t_5)\right]$$
$$=(2)(0+2+7+5+2) = 32\,(s)\left(\frac{m}{s}\right) = \boxed{32\,m}$$

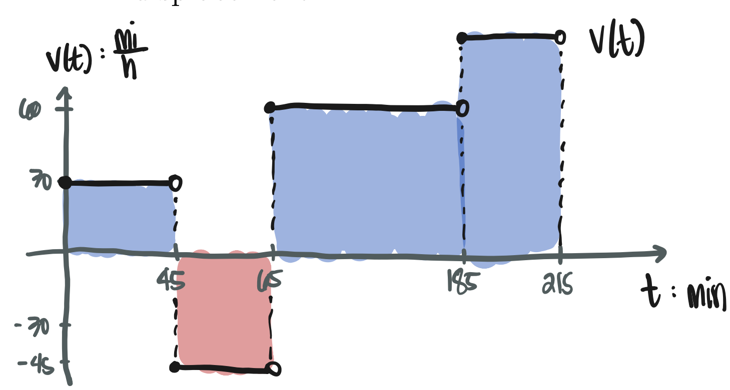
Assume the displacement is linear, as illustrated:



(6) You are on a road trip with some friends. You start out by driving at a speed of 30 miles per hour for 45 minutes, then realize you forgot toothpaste, so you travel back to the nearest convenience store at a speed of 45 miles per hour for 20 minutes. You then proceed by driving for 60 miles per hour for 2 hours, and then 75 miles per hour for 30 minutes.

(a) Sketch a graph and shade the regions on the graph that could represent the displacement.

Graph:



$$\text{let } v(t) = \begin{cases} 30 & \text{if } t\in[0,45) \\ -45 & \text{if } t\in[45,45+20) \\ 60 & \text{if } t\in[65,65+120) \\ 75 & \text{if } t\in[185,185+30) \end{cases};$$

Note: $\int_0^{215} v(t)\,dt$ has units $\left(\frac{mi}{h}\right)(min)$ $=\left(\frac{mi}{h}\right)(min)\left(\frac{1h}{60\,min}\right) = \frac{1}{60}mi$

(b) Use integrals to express the displacement, and total distance travelled.

Displacement: $S_{total} = \frac{1}{60}\int_0^{215} v(t)\,dt$ mi

Distance Travelled: $D_{total} = \frac{1}{60}\int_0^{215} |v(t)|\,dt$ mi

Note: The $\left(\frac{1}{60}\right)$ in front is for the unit conversion. You can also convert t-values into hours before the integral.

(c) Calculate the total distance travelled and displacement, using the integrals you found in the previous part.

Displacement: $S_{total} = \frac{1}{60}\int_0^{215} v(t)\,dt = \frac{1}{60}\left[(30)(45)+(-45)(20)+(60)(120)+(75)(30)\right] = \boxed{165\,mi}$

Distance Travelled: $D_{total} = \frac{1}{60}\int_0^{215} v(t)\,dt = \frac{1}{60}\left[\int_0^{45} v(t)\,dt + (-1)\int_{45}^{65} v(t)\,dt + \int_{65}^{215} v(t)\,dt\right]$

$$= \frac{1}{60}\left[(30)(45)+(45)(20)+(60)(120)+(75)(30)\right] = \boxed{195\,mi}$$