✩ Shortcut: u-substitution with a linear term

$$\boxed{\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C}$$ where $F(u)$ is the antiderivative of f with respect to $u = ax+b$

$\frac{1}{a}$ ← "Fudge Factor"

Examples:

$\int (10x+22)^{13}\,dx = \frac{1}{14}(10x+22)^{14}\left(\frac{1}{10}\right) + C$ with $a=10$ and $b=22$

$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$ with $a=3$ and $b=0$

$\int \sin(121x+1002)\,dx = \frac{1}{121}(-1)\cos(121x+1002) + C$ with $a=121$, $b=1002$

$\int \frac{1}{2x+3}\,dx = \frac{1}{2}\ln|2x+3| + C$ with $a=2$, $b=3$

$\int \sec^2\left(\frac{1}{2}x-2\right)dx = 2\tan\left(\frac{1}{2}x-2\right) + C$ with $a=\frac{1}{2}$, $b=-2$

$\int \frac{1}{1+4x^2}\,dx = \int \frac{1}{1+(2x)^2}\,dx = \frac{1}{2}\arctan(x) + C$ with $a=2$, $b=0$

$\int \frac{1}{4+9x^2}\,dx = \int \left(\frac{1}{4}\right)\left(\frac{1}{1+\frac{9}{4}x^2}\right)dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{3}{2}x\right)^2}\,dx = \frac{1}{4}\left(\frac{2}{3}\right)\arctan\left(\frac{3}{2}x\right)+C$

$= \frac{1}{6}\arctan\left(\frac{3}{2}x\right)+C$

✩ Trig. Antiderivative Rules

$\int \sin(x)\,dx = -\cos(x)+C$ and $\int \cos(x)\,dx = \sin(x)+C$ from derivative rules

$\int \sec^2(x)\,dx = \tan^2(x)+C$ and $\int \sec(x)\tan(x)\,dx = \sec(x)+C$ from antiderivative rules

$\int \frac{1}{1+x^2}\,dx = \arctan(x)+C$ from derivative rules

$\int \tan(x)\,dx = -\ln|\cos(x)|+C = \ln|\sec(x)|+C$

$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx \overset{u\text{-sub}}{=} \int (-1)\frac{1}{u}\,du = -\ln|u|+C = -\ln|\cos(x)|+C$

u-sub $\{u=\cos(x),\ du=-\sin(x)\,dx\}$

$\int \sec(x)\,dx = \ln|\tan(x)+\sec(x)|+C$

$\int \sec(x)\,dx = \int \sec(x)\left(\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\right)dx = \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{1}{u}\,du = \ln|u|+C$

u-sub $\{u=\sec(x)+\tan(x),\ du=\sec(x)\tan(x)+\sec^2(x)\,dx\}$

$= \ln|\sec(x)+\tan(x)|+C$

✯ Trigonometric Identities typically used in MTH 252.

Basic $\begin{cases} \sin\theta = \dfrac{1}{\csc\theta}, \quad \cos\theta = \dfrac{1}{\sec\theta}, \quad \tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta} \end{cases}$

Pythagorean
$$\begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \tan^2\theta + 1 = \sec\theta \\ 1 + \cot^2\theta = \csc^2\theta \end{cases}$$

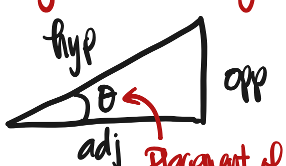
< divide both sides of $\sin^2(x) + \cos^2(x) = 1$ by $\cos^2(x)$

< divide both sides of $\sin^2(x) + \cos^2(x) = 1$ by $\sin^2(x)$

Power Reduction
$$\begin{cases} \sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta)) \\ \cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta)) \end{cases}$$

Double Angle
$$\begin{cases} \sin(2\theta) = 2\sin\theta\cos\theta \\ \cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \\ \tan(2\theta) = \dfrac{2\tan\theta}{1 - 2\tan^2\theta} \end{cases}$$

✯ Trig Functions on Right Triangles



Placement of θ is important!

SOH: $\sin\theta = \dfrac{opp}{hyp}$, CAH: $\cos\theta = \dfrac{adj}{hyp}$, TOA: $\tan\theta = \dfrac{opp}{adj}$;

Pythagorean Theorem

$$(hyp)^2 = (opp)^2 + (adj)^2$$