

# THE TRADE-OFF BETWEEN ALIGNMENT AND INTENSITY OF BUREAUCRATIC INCENTIVES

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## Abstract

This exercise is inspired by Prendergast’s “The Motivation and Bias of Bureaucrats” (*AER*, 2007). Under the assumption that bureaucratic workers who are passionate about their work are likely to disagree with their employers about which decision is best, it demonstrates that a principal will hire a biased agent in order to induce him to acquire information, even though she knows he’ll use it to implement a biased allocation. For a particular relationship between bias and preference intensity, it then finds the optimal level of bias from the principal’s point of view.

In what follows it will be useful to recall if  $\mathbf{X}$  and  $\varepsilon$  are Gaussian random variables with zero mean and variances unity and  $\sigma^2$ ,

$$\mathbf{X} \sim N[0, \sigma^2], \quad \varepsilon \sim N[0, \Sigma^2] \quad (1)$$

then if  $\mathbf{Y} = \mathbf{X} + \varepsilon = y$  then  $\mathbf{X}$  is conditionally distributed normally with mean  $y$  and precision  $\sigma^{-2} + \Sigma^{-2}$ ,

$$\mathbf{X} \mid \mathbf{Y} = y \sim N \left[ y, \frac{1}{\sigma^{-2} + \Sigma^{-2}} \right]. \quad (2)$$

Pamela is a principal in a government agency. Her job is to hire and fire bureaucrats who deal with public “clients”. The role of the bureaucrat is to make decisions on a case-by-case basis after learning as much as possible about clients’ circumstances.

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Pamela, embroiled in a bureaucracy, has no incentive instruments at her disposal, so she tries to find employees who wish to see their jobs done right themselves. However, her employees' definitions of "right" often differ from hers; in fact, she has observed a trend whereby the most motivated workers are also the most opinionated, and has found that it was not always for the best. On the other hand, workers with no ideological attachments tend to be lackadaisical in their field work and end up making less informed decisions.

Given uncertainty about the client's type  $\theta$ , model Pamela's preferences over the allocation with the quadratic loss utility function with preferred allocation  $a = \theta$ ,

$$u_P(\theta, a) = -(\theta - a)^2. \quad (3)$$

Now, describe a potential worker's type by a pair  $(b, B)$  where  $b$  is his bias and  $B$  represents the intensity of his motivation so that his payoff function implies the preferred allocation  $a = \theta + b$ ,

$$u_B(\theta, a) = -B^2(\theta - a + b)^2. \quad (4)$$

1. (a) Compare and contrast both the importance of a bureaucrat's private information and the potential for misalignment of incentives (among the principal, bureaucrat, and client) in the following government sectors:
  - i. Healthcare, where the bureaucrat is a doctor and Pamela is a hospital administrator.
  - ii. Airport security, where the bureaucrat is a boarder guard and Pamela is his coordinator.
  - iii. Tax auditing, where the bureaucrat is an auditor and Pamela is the IRS.
- (b) Comment on the symmetric nature of the quadratic loss form of utility. What does this mean in the examples above? When is it reasonable?
- (c) Who is the client in the examples above? In which cases are the client's interests aligned with Pamela's/the bureaucrats? How might this affect Pamela's employment decision? Comment with respect to (b).

Suppose that  $\tilde{\theta}$  is normally distributed with zero mean and precision  $h$ ,

$$\tilde{\theta} \sim N \left[ 0, \frac{1}{h} \right] \quad (5)$$

and that before determining his allocation  $a$ , a bureaucrat exerts effort  $e$  to observe a signal  $s$  about  $\theta$  confounded by noise with precision  $e$ ,

$$\tilde{s}_e = \theta + \varepsilon_e \quad (6)$$

where

$$\varepsilon_e \sim N \left[ 0, \frac{1}{e} \right]. \quad (7)$$

The agent bears a linear cost of effort  $c(e) = e$ .

2. This question solves for the bureaucrat's allocation choice and effort level by backward induction.

- (a) Show that the bureaucrat's allocation choice is a biased version of Pamela's, namely that

$$a^* = \mathbb{E} \left[ \tilde{\theta} \mid \tilde{s}_e = s \right] + b. \quad (8)$$

- (b) Show that that the bureaucrat's net utility is given by

$$\mathbb{E} \left[ u_B(\tilde{\theta}, a^*) \mid s \right] - e = -\text{Var} \left[ B\tilde{\theta} \mid s \right] - e. \quad (9)$$

Hint: Use the definition of the conditional variance backwards:

$$\text{Var} [\mathbf{X} \mid A] = \mathbb{E} \left[ \left( \mathbf{X} - \mathbb{E} [\mathbf{X} \mid A] \right)^2 \mid A \right]. \quad (10)$$

- (c) Compute the variance to see that the objective is in fact

$$\mathbb{E} \left[ u_B(\tilde{\theta}, a^*) \right] = -\frac{B^2}{h+e} - e. \quad (11)$$

- (d) Use the first-order approach to show that the equilibrium effort of a bureaucrat with type  $(b, B)$  is

$$e^* = \max \{ B - h, 0 \}. \quad (12)$$

Suppose  $B > h$  from now on.

- (e) Interpret the result:

- i. Why is  $e^*$  independent of the bias  $b$ ?
- ii. Why does  $e^*$  decrease in the prior precision  $h$ ?

3. This question determines the type of bureaucrat that Pamela hires.

- (a) Show that given she hires a bureaucrat of type  $(b, B)$ , Pamela's equilibrium utility is given by

$$\mathbb{E} \left[ u_P(\tilde{\theta}, a^*) \right] = -\mathbb{E} \left[ \text{Var} \left[ \tilde{\theta} \mid \tilde{\sigma}_{e^*} \right] \right] - b^2. \quad (13)$$

Hint: Use the law of iterated expectations to recover the conditional variance.

(b) Argue that Pamela's objective function is in fact

$$\mathbb{E} \left[ u_P(\tilde{\theta}, a^*) \right] = -\frac{1}{h + e^*} - b^2 \quad (14)$$

$$= -\frac{1}{B} - b^2. \quad (15)$$

(c) Suppose, in accordance with Pamela's observation that opinionated bureaucrats are also more motivated, that  $B$  is an increasing function of  $b$ ; in particular let  $B = \beta b/2$  for  $\beta > 0$ .

- i. What does  $\beta$  measure?
- ii. Show that Pamela employs the bureaucrat of type  $(b, \beta b/2)$  for

$$b^* = \beta^{-1/3}. \quad (16)$$

- iii. Interpret the final expression.