## RESALEABLE DEBT AND SYSTEMIC RISK\*

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#### Abstract

Many debt claims, such as bonds, are resaleable, whereas others, such as repos, are not. There was a fivefold increase in repo borrowing before the 2008 crisis. Why? Did banks' dependence on non-resaleable debt precipitate the crisis? In this paper, we develop a model of bank lending with credit frictions. The key feature of the model is that debt claims are heterogenous in their resaleability. We find that decreasing credit market frictions leads to an increase in borrowing via non-resaleable debt. Borrowing via non-resaleable debt has a dark side: it causes credit chains to form, since if a bank makes a loan via non-resaleable debt and needs liquidity, it cannot sell the loan but must borrow via a new contract. These credit chains are a source of systemic risk, since one bank's default harms not only its creditors but also its creditors' creditors. Overall, our model suggests that reducing credit market frictions may have an adverse effect on the financial system and may even lead to the failures of financial institutions.

**Keywords:** resaleable debt, systemic risk, bankruptcy, repos, securities law

JEL Classification Numbers: G21, G28, G33, K12, K22

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## 1 Introduction

Credit frictions decreased substantially in the decades leading up to the 2008 financial crisis.<sup>1</sup> This coincided with the expansion of repo markets, which grew fivefold between 1990 and 2007. Before the crisis, the value of outstanding repos in the US exceeded five trillion USD.<sup>2</sup> The markets appeared to be functioning well, allowing banks to find cheap, short-term liquidity. However, they were harboring systemic risk, because banks were exposed to one another in credit chains. This meant that if one bank defaulted, it harmed not only its immediate creditors, but potentially its creditors' creditors as well. This systemic risk manifested itself in the financial crisis, in which shocks to a relatively small set of assets threatened to bring down the entire financial system. Did the buildup of systemic risk relate to the decrease in credit frictions? In general, can a decrease in credit frictions cause an increase in systemic risk?

In this paper, we construct a corporate finance-style model to address this question. We find that the answer is yes. Our main result is that a decrease in credit frictions increases systemic risk. This is because the decrease in credit frictions leads credit chains to become more widespread, and these credit chains harbor systemic risk.

The key novel ingredient in our model is the heterogeneous resaleability of debt claims. For concreteness, consider the salient examples of bonds and repos. Bonds are resaleable, whereas repos are not.<sup>3</sup> As a result, lending via repos leads to credit chains, whereas lending via bonds does not. To see this, suppose you are a lender—you have a loan on the asset side of your balance sheet—and you suddenly need liquidity. Your options for raising this liquidity are different if you hold a bond than if you hold a repo. If you hold a bond you can sell it in the market. In contrast, if you hold a repo, you cannot sell it. Hence, you obtain liquidity by borrowing via a new repo. This creates a credit chain, because you are now not only a creditor in the original repo, but a debtor in the new repo as well. In summary, when you hold a non-resalebale instrument such as a repo, the result is a credit chain. This brings with it systemic risk, since defaults can transmit through the chain.

<sup>&</sup>lt;sup>1</sup>Low credit market frictions in the US before the crisis reflected a number of factors, including advanced information technology for execution and settlement, low transaction costs (Domowitz, Glen, and Madhavan (2001), Jones (2002)), relatively low information asymmetries (Bai, Philippon, and Savov (2012), Greenwood, Sanchez, and Wang (2013)), and a number of potential legal factors, such as privileged bankruptcy treatment of some bank liabilities (Morrison, Roe, and Sontchi (2014)) and required financial disclosure (La Porta, Lopez-De-Silanes, and Shleifer (2006)).

<sup>&</sup>lt;sup>2</sup>See Homquist and Gallin (2014).

<sup>&</sup>lt;sup>3</sup>That bonds are resaleable and repos are not is a formal legal property of these claims. Other financial claims, such as derivatives, are also not resaleable; we comment on our model's applicability to derivative markets in Subsection 1.1.

How does a change in credit frictions affect your choice whether to lend with a bond or a repo? In our model, a decrease in credit frictions makes you relatively more likely to lend via a repo. This is due to the fact that when you are an intermediate link in a credit chain, there are two contracts that must be enforced, one between you and your creditor and another between you and your debtor. Thus, you bear the costs of credit frictions twice, once for each contract. If frictions are high, you have a strong incentive to avoid these double costs. To do this you lend via resaleable debt like bonds. In this case, no credit chain is formed and systemic risk is low. On the other hand, if credit frictions are low, you have a weaker incentive to avoid the costs of credit chains. You may prefer to lend via non-resaleable debt like repos, as repos may come with other advantages, such as preferential treatment in bankruptcy or lower issuance costs. In this case, credit chains form and systemic risk is high. This is the essence of our main result: decreasing credit market frictions can increase systemic risk. The reason is that decreasing credit frictions makes it is less likely that banks issue resalable debt and, hence, more likely that credit chains form.

Model preview. We now describe our model and results in more detail. We model the interbank market within a classical corporate finance framework. At the core of the model is one financial institution, which we call Bank A, that needs to raise finance in order to scale up a project. Bank A borrows from a competitive creditor, which we call Bank B. Bank A can borrow via one of two instruments, a bond or a repo.<sup>4</sup> As discussed above, a bond is resaleable whereas a repo is not. The amount that a bank can borrow is limited by the assets it can pledge, via a standard limit to pledgeabity. Specifically, the repayment a bank makes to its creditor cannot exceed a fixed fraction  $\theta$  of the bank's assets. This fraction  $\theta$ , which we refer to as the "enforceability" in the economy, captures credit frictions. An increase in enforceability  $\theta$  corresponds to a decrease in credit frictions. At an interim date, after Bank B has made the loan to Bank A, it may suffer a "liquidity shock," i.e. it may suddenly need cash. If Bank B suffers a liquidity shock, it raises liquidity in the interbank market from a third financial institution, which we call Bank C. Specifically, Bank B raises this liquidity either by selling Bank A's bond to Bank C or by entering a new repo agreement with Bank C.

Bonds are attractive relative to repos because they are resaleable. However, repos are attractive relative to bonds for another reason: they are effectively senior to bonds in bankruptcy; in fact, they are exempt from bankruptcy stays in reality. Thus, when

<sup>&</sup>lt;sup>4</sup>We use the labels repo and bond throughout for non-resaleable and resaleable instruments, respectively. Note that when we think about short-term bank funding, the kind of bond we have in mind is commercial paper. We discuss the applicability of our model to short-term bank funding further in Subsection 1.1 and to more general abstract settings in Subsection 4.1.

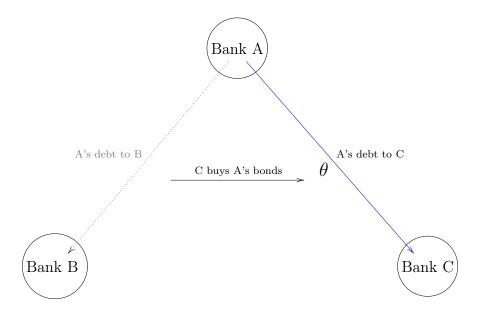


Figure 1: Because bonds are resaleable, Bank B obtains liquidity by selling Bank A's bonds to Bank C. No credit chain emerges.

Bank A borrows in the interbank market, it trades off the resaleability benefit of bonds against the seniority benefit of repos.

For most of our analysis, we focus on this trade-off in the interbank market, but we generalize our model in Subsection 4.1. There, we relax the assumption that non-resaleable debt claims (repos) are senior to resaleable debt claims (bonds). Specifically, we model general debt markets following Kiyotaki and Moore (2005) and show that our main results are broadly applicable. For example, this analysis may cast light on a borrower's choice whether to fund itself via a bank loan (non-resaleable) or long-term bonds (resaleable).

Results preview. First consider the case in which Bank A borrows from Bank B via a bond. In this case, when Bank B suffers a liquidity shock, it sells Bank A's bond to Bank C. This sale is depicted in Figure 1. Observe that Bank A now has a debt to Bank C directly. There is no credit chain. There is only one contract to be enforced, the debt from Bank A to Bank C. Credit frictions kick in only once and Bank A's debt capacity is (roughly) proportional to the enforceability  $\theta$  of this contract.

Now turn to the case in which Bank A borrows from Bank B via a repo. In this case, when Bank B suffers a liquidity shock, it must enter into a new contract to find liquidity—because Bank A's repo debt is not resaleable, Bank B cannot liquidate it

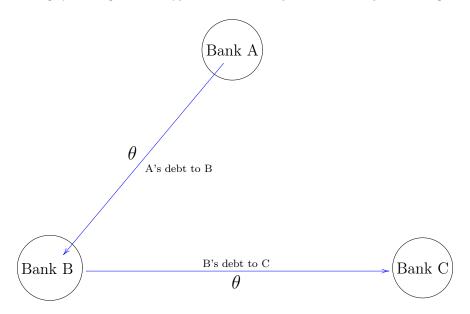


Figure 2: A credit chain emerges when Bank A borrows from Bank B via repos.

in the market. Thus, Bank A borrows from Bank C via a new repo contract. This is depicted in Figure 2. Observe that Bank A has debt to Bank B and Bank B has debt to Bank C. There is a credit chain. There are two contracts to be enforced. Credit frictions kick in twice, once at each link in the credit chain, and Bank A's debt capacity is (roughly) proportional to the enforceability squared or  $\theta \times \theta$ . Intuitively, there is one  $\theta$  for each of the two contracts.

Now consider how an increase in enforceability affects Bank A's choice between bonds and repos. As  $\theta$  increases, the amount Bank A can borrow with bonds increases linearly and the amount Bank A can borrow with repos increases quadratically. In other words, the sensitivity of Bank A's debt capacity to enforceability is higher when it borrows via repos than when it borrowers via bonds. Thus, as credit frictions decrease, Bank A switches from bond borrowing to repo borrowing.

What are the implications of increasing enforceability for systemic risk? We have just established that increasing enforceability leads Bank A to borrow via repos and that this, in turn, leads to credit chains. Credit chains harbor systemic risk because if Bank A defaults on its debt to Bank B, Bank B may default on its debt to Bank C. In our model, such default cascades can arise only when enforceability is high, because that is when Bank A funds itself with repos and credit chains emerge. Note that even though increasing enforceability improves the functioning of each market individually, it may have an adverse effect on the system as a whole, causing an increase in systemic

risk.

Further results. We also explore six extensions of our baseline model. analysis affirms the robustness of our main findings and provides new results. First, we show that our results hold generally, not only in the interbank market. In particular, our main results are robust to relaxing the assumption that non-resaleable debt (repos) is senior to resaleable debt (bonds). Second, we consider Bank A's maturity choice in the presence of roll-over risk. We show conditions under which Bank A matches the maturity of its liabilities to the maturity of its project, as we assume exogenously in the baseline model. Third, we discuss how to incorporate the rehypothecation of liquid collateral into our setup and argue that our main findings are robust. Fourth, we consider the possibility that credit chains may have more than two links. We show that that longer chains make repo borrowing relatively less attractive. However, our qualitative findings do not change. Fifth, we ask how systemic risk is affected by a relatively short-term stay on repos, rather than a full exemption. We show that a short-term stay is preferable to an exemption in our setting, but that still longer stays for repos are even better. Finally, we do a reduced-form welfare analysis. We explain that if bank default is socially costly, then increasing systemic risk corresponds to decreasing social welfare.

Policy. Our model is stylized, but may still cast light on policy debate. Should repos maintain their special treatment in bankruptcy? The exemption from automatic stays for repos makes repos more desirable to Bank A. Thus, the exemption leads Bank A to undertake more repo borrowing and, hence, leads to more credit chains. Since these credit chains are the source of systemic risk in the model, the exemption from the stay exacerbates systemic risk. This finding contrasts with the arguments advanced by proponents of the exemption, who suggest that the safe harbors are "effective in...limiting [counterparties'] exposure to possibly catastrophic losses from the failure of the debtor. This is the very reason why Congress enacted he safe harbors in the first place" (Exploring Chapter 11 Reform (2014)).

Our findings also affirm that regulators must take a macro-prudential approach, as decreasing credit frictions makes every market function better individually, but makes the system as a whole more dangerous.

**Layout.** The remainder of the paper is organized as follows. There are two remaining subsections in the Introduction, first, a discussion of the realism of our assumptions and the empirical relevance of our results and, second, a review of related literature. Section 2 presents the model. Section 3 contains the formal analysis. In Section 4, we derive further results by extending the model to include more general instruments, rollover risk, rehypothecation of liquid collateral, longer chains, short-term stays for

repos, and social costs of bank default. Section 5 concludes. Appendix A contains omitted derivations and proofs.

#### 1.1 Realism and Empirical Evidence

While our model is stylized, we believe that our baseline model provides a useful approximation of the interbank market, with reasonable assumptions and predictions. Here we discuss these briefly in connection with empirical work. First, we point out that repos and asset-backed commercial paper (a type of bond) are relatively substitutable instruments for short-term bank funding. This is because they both have relatively short maturities and they are often secured by similar collateral (Krishnamurthy, Nagel, and Orlov (2014)). Second, we suggest that the bankruptcy advantage of repos is important, as repo volume increased after Congress introduced the safe harbor provision (Garbade (2006)). Third, we emphasize that credit chains are an important feature of the repo market (repo chains are typically associated with the so-called "rehypothecation" of collateral, see Singh and Aitken (2010) and Singh (2010)). Banks assume offsetting long and short repo positions, even though many repos are very short-term and it may seem that they should be "self-liquidating." This may be because banks manage liquidity over very short time horizons, taking offsetting positions within each day. Another reason for this may be that many repos are of longer maturities, with an estimated thirty percent of repos having maturity longer than a month (Comotto (2015)). Finally, many repos have "open" tenors, with no specified maturity. These are typically thought about as overnight contracts, but a lender in an open repo must give its counterparty notice before closing the contract; sometimes several weeks' notice is required (Comotto (2014)).

We would also like to point out that our model also applies to financial derivatives. Like repos, derivatives are non-resaleable instruments that enjoy special treatment in bankruptcy. Further, derivatives markets grew even more dramatically than repo markets in the years before the 2008 crisis. The notional value of all financial derivatives contracts was estimated at 766 trillion USD in 2009, a three hundred-fold increase from thirty years earlier (Stulz (2009)). Repos and derivatives often constitute a larger fraction of banks' balance sheets than bonds of all maturities combined. For example, in 2009 over forty-five percent of Barclay's liabilities were listed as "repurchase agreements and stock lending" or "derivatives" on its balance sheet.<sup>6</sup>

<sup>&</sup>lt;sup>5</sup>Since a repo contract is formally the sale and repurchase of assets, not the pledging (or "hypothecating") of collateral, the term "rehypothecation" is not favored by lawyers.

 $<sup>^6</sup>$ Barclay's annual reports are available online here <https://www.home.barclays/barclays-investor-relations/results-and-reports/annual-reports.html>. The Royal Bank of Scotland reports similar numbers (see

Our application to the interbank market depends on the assumption that there are frictions in the interbank market. In particular, we assume that there is limited enforceability of contracts or, equivalently, limited pledgeability of cash flows. The assumption is standard in the theory literature—for example, Homstrom and Tirole (2011) make the assumption and provide a list of "several reasons why this [limited enforceability] is by and large reality" (p. 3). We think that the realism of the assumption for our application is demonstrated by the importance of collateral in interbank contracts (Bank for International Settlements (2013))—if there were no pledgeability frictions, banks would not need to post collateral at all. In addition, the years-long bankruptcy proceedings of Lehman Brothers demonstrated that bank creditors can face severe frictions when trying to claim repayment. Further, we point out that our model does not rely on the assumption that contractual enforceability is weak, but only on the assumption that it is imperfect, which we believe it is for all contracts in practice.

Finally, to emphasize the empirical importance of the problem we study, we remark that several papers suggest that the systemic risk that built up in the repo market may have played an important role in the financial crisis of 2008–2009 (Copeland, Martin, and Walker 2014, Gorton and Metrick (2010), Gorton and Metrick (2012), Krishnamurthy, Nagel, and Orlov (2014)).

#### 1.2 Related Literature

Kiyotaki and Moore (2000) also analyze how the resaleability of debt claims can mitigate the allocational inefficiencies that stem from limits to enforceability.<sup>8</sup> They demonstrate that a small amount of resaleability (or "multilateral commitment") can substitute for a substantial lack of enforceability (or "bilateral commitment") in a deterministic, infinite horizon economy. Rather than focus on allocational efficiency as they do, we study borrowers' endogenous choice of instruments and analyze the implications for systemic risk. Our analysis points to a potential dark side of enforceability that is not present in Kiyotaki and Moore's deterministic setting.

In another 2001 paper, Kiyotaki and Moore study credit chains. Rather than study the transferability of debt, that paper shows how chains of bilateral borrowing can

<sup>&</sup>lt;a href="http://investors.rbs.com/annual-report-subsidiary-results/2010.aspx">http://investors.rbs.com/annual-report-subsidiary-results/2010.aspx</a>). The corresponding figures are hard to find for US banks, since they classify their derivatives holdings as risk management instruments and, therefore, are not required to list them on their balance sheets.

<sup>&</sup>lt;sup>7</sup>Note that these papers differ in their conclusions about the *way* in which repos contributed to the crisis. 
<sup>8</sup>In this paper, Kiyotaki and Moore develop a framework that they explore further in subsequent work, including Kiyotaki and Moore (2001a), Kiyotaki and Moore (2005), and Kiyotaki and Moore (2012).

emerge and, as such, it constitutes an early contribution to the growing literature on financial networks. Many papers in this literature study systemic risk, including Acemoglu, Ozdaglar, and Thabaz-Salehi (2013), Allen, Babus, and Carletti (2012), Allen and Gale (2000), Blumh, Faia, and Krahnen (2013), Cabrales, Gottardi, and Vega-Redondo (2013), Elliott, Golub, and Jackson (2014), Gale and Kariv (2007), Glode and Opp (2013), Rahi and Zigrand (2013), and Zawadowski (2013). In only a few of these papers, however, is the equilibrium network endogenous. An emerging theory literature takes a detailed approach to modeling credit chains in the repo market specifically, including Kahn and Park (2015), Infante (2015), and Lee (2015).

Numerous other papers study the circulation of private debt, including Gorton and Pennacchi (1990), Gu, Mattesini, Monnet, and Wright (2013), Kahn and Roberts (2007), and Townsend and Wallace (1987). These papers typically do not consider debt resaleability as a choice of the borrower and, therefore, they do not study the implications of this choice for systemic risk.

We also hope to contribute to the debate surrounding the bankruptcy seniority of repos and derivatives. Relevant papers in this literature include Antinolfi, Carapella, Kahn, Martin, Mills, and Nosal (2014), Bliss and Kaufman (2006), Duffie and Skeel (2006), Edwards and Morrison (2005), Lubben (2009), Roe (2011), and Skeel and Jackson (2012). Notably, Bolton and Oehmke (2014) bring a corporate finance model to bear on the question of bankruptcy seniority, but they focus on the exemptions for derivatives.

## 2 Model

In this section we set up the model, outlining the players and their technologies, the debt instruments by which they can borrow, the specific nature of limited enforcement, and the timing of moves. We also include a subsection describing several restrictions that we impose on parameters.

#### 2.1 Players and Technologies

There is one good called cash. There are three dates Date 0, Date 1, and Date 2. The time between Date t and Date t+1 is called "overnight." Cash is the input of production, the output of production, and the consumption good. The main actor in the model is a risk-neutral bank called Bank A. Bank A has an endowment of e pounds and a risky constant-returns-to-scale technology. The technology takes two periods to produce, starting at Date 0 and terminating at Date 2. It has random gross return  $\tilde{R}$ ,

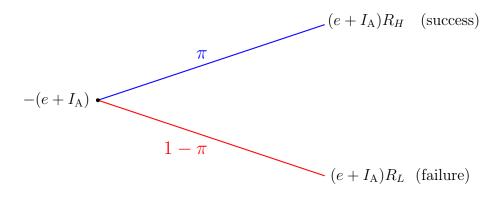


Figure 3: Depiction of Bank A's technology.

which is  $R_H$  with probability  $\pi$  and  $R_L < R_H$  with probability  $1 - \pi$ . Figure 3 depicts the technology. We call the event that  $\tilde{R} = R_H$  "success" and the event that  $\tilde{R} = R_L$  "failure." Denote the expected return by  $\bar{R} := \pi R_H + (1 - \pi) R_L$ .

Bank A funds its investment by borrowing capital I from a competitive market of risk-neutral banks. The project is scaleable, so the quantity I is determined in equilibrium. We model the competitive market in reduced form by having Bank A make a take-it-or-leave-it offer to borrow from a second risk-neutral bank, Bank B. Bank B breaks even in expectation but its preferences are uncertain: with probability  $1 - \mu$  Bank B values consumption only at Date 1 and with probability  $\mu$  it values consumption only at Date 2 (all random variables are pairwise independent). To be more specific, with probability  $1 - \mu$  Bank B lexicographically prefers Date 1 consumption to Date 2 consumption; with probability  $\mu$  Bank B lexicographically prefers Date 2 consumption to Date 1 consumption. We call the event that a bank wishes to consume at Date 1 a "liquidity shock." The inclusion of the possibility that a bank is hit by a liquidity shock is a simple way to generate a motive to trade in a secondary market before Bank A's

<sup>&</sup>lt;sup>9</sup>Note that we think about  $\pi$  as rather large so that failure is an extreme event. In the repo market, failure should be interpreted as the joint event in which Bank A's project fails and the value of its pledged collateral is not sufficient to cover its loan. We do not model this collateral explicitly here, but we discuss it in the extension in Subsection 4.3.

<sup>&</sup>lt;sup>10</sup>The lexicographic preferences are just a modelling device that induces Bank B to have well-defined preference for more to less at Date 2 even if it is hit by a liquidity shock at Date 1; this is important only in the details of micro-founding enforcement constraints (see Subsection 2.3 below).

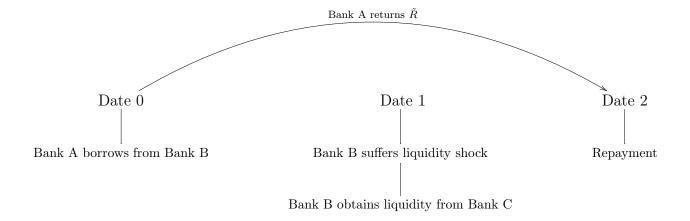


Figure 4: Timeline when Bank B suffers a liquidity shock.

debt matures—when hit by a liquidity shock, Bank B wishes either to resell Bank A's debt or to borrow against Bank A's debt to satisfy its liquidity needs at Date 1. Rather than viewing Date 1 as a fixed point time that the banks know in advance, we interpret it as a random time between at which Bank B needs liquidity. Thus, since the arrival time of Bank B's liquidity shock is uncertain at Date 0, Bank A cannot borrow with a contract that matures exactly when Bank B suffers the liquidity shock. We discuss this further in Subsection 4.2.

For simplicity, we assume that Bank B has deep pockets at Date 0. By "deep pockets" we mean that it has sufficient cash to fund Bank A at Date 0 so that Bank A does not need to find a second creditor. If Bank B is hit by a liquidity shock, it uses all this cash to generate liquidity at Date 1. We discuss the role of assets in place further in Subsection 4.3.

There is a competitive interbank market open at Date 1, in which banks buy and sell bonds in the secondary market as well as borrow and lend among themselves. We model this by allowing Bank B to obtain funds from a third risk-neutral bank, Bank C. Bank B can either sell Bank A's debt or borrow against it. Again, competition is captured by assuming that Bank B makes Bank C a take-it-or-leave it offer, whether to sell bonds or to borrow against repos.

Figure 4 depicts the timing described here for the case in which Bank B suffers a liquidity shock. Subsection 2.4 below gives a more formal description of the timing.

## 2.2 Borrowing Instruments

The crux of the model is the trade-off between borrowing via a bilateral contract called a repo and borrowing via a resaleable instrument called a bond. In the model, two features distinguish repos from bonds. The first feature is that bonds are resaleable. A bank that buys a bond can sell it to another bank in the Date-1 market. The issuer of the bond repays its bearer at maturity, regardless of whether this bearer was the original owner at Date 0. Repos, in contrast, are not resaleable. A repo must be settled by the writer and its counterparty. The second feature that distinguishes repos from bonds is that repos are not stayed in bankruptcy. 11 The counterparty to a repo recoups its debt immediately, even if its counterparty defaults. The counterparty to a bond, in contrast, must wait to liquidate until it is awarded the assets in the bankruptcy proceedings. To capture the costs of waiting to liquidate, we normalize bondholders' liquidation value to zero in the event of default. We assume that the realization of  $\tilde{R}$  is not verifiable, so state-contingent contracts are impossible. Thus, as in reality, both bonds and repos are debt contracts, i.e. promises to repay a state-independent face value in the future in exchange for cash today. We summarize the dimensions along which repos and bonds differ in Figure 5.

A main question we ask is under what conditions Bank A will fund its Date 0 investment via repos as opposed to bonds. When Bank A determines its funding instrument, it will face a trade-off in borrowing costs. Repos decrease borrowing costs because creditors have higher recovery values in the event of default; in contrast, bonds reduce borrowing costs because they may come with a *liquidity premium*. This liquidity premium is a result of the fact that lenders can sell them at Date 1 to meet their liquidity needs when they suffer liquidity shocks. That is to say, borrowers trade off bonds' resaleability against repos' super-seniority.

#### 2.3 Limited Enforcement

The key friction in the economy is limited enforcement. We assume that creditors cannot extract all of a project's surplus when they collect on their debts. In particular,

<sup>&</sup>lt;sup>11</sup>As mentioned in the Introduction, this specific assumption of seniority is not essential for our main results, as we discuss in Subsection 4.1.

<sup>&</sup>lt;sup>12</sup>We make this assumption following Bolton and Oehmke (2014), because it provides an easy way to model bankruptcy costs. In our model, it will also imply that the value of the bond in the event of default is independent of enforcement frictions. In Subsection 4.1, we relax this assumption to ensure that it is not driving our results.

<sup>&</sup>lt;sup>13</sup>We make this assumption only to add realism to our application to the interbank market. Our main mechanism does not depend on it. In particular, in the generalization in Subsection 4.1, the analysis does not depend on the fact that Bank A's cash flows are not verifiable.

# TWO DIMENSIONS OF LEGAL ASYMMETRY: TRANSFERABILITY AND BANKRUPTCY TREATMENT

	super-senior	less senior	
resaleable		e.g. bonds, stock	
not resaleable	e.g. repos, derivatives		

Figure 5: A table that depicts the two dimensions of legal asymmetry we focus on, transferability and bankruptcy treatment. Bonds and stock are resaleable, but they are junior in bankruptcy to non-resaleable instruments such as repos and derivatives.

there is an exogenous number  $\theta \in (0,1)$  that gives an upper bound on the proportion of assets that a creditor can extract from its debtor, heuristically

repayment 
$$\leq \theta \times \text{assets}$$
. (1)

Note that this proportion  $\theta$  is the same for all debts in the economy. We refer to  $\theta$  as the *enforceability* in the economy.  $\theta$  represents creditors' power to extract repayment from debtors; developments that we would expect to increase  $\theta$  include efficient liquidation procedures, strong creditor rights, standardized contracts, technological development for improved recored keeping, and increased accounting transparency.

The formal micro-foundation we provide for the constraint above (inequality (1)) comes from borrowers' incentives to divert assets and abscond. Specifically,  $\theta$  is the pledgeable proportion of assets. We assume that this fraction  $\theta$  is not divertable. In other words, a borrower with assets A has the option to divert  $(1 - \theta)A$  and then default. Thus a borrower will repay debt with face value F only if the residual value net of repayment exceeds its gain from diverting, or

$$A - F \ge (1 - \theta)A$$
.

This inequality can be rewritten as

$$F < \theta A$$
,

which is simply inequality (1) restated symbolically. With this formalism, an increase in enforceability is an increase in the collateralizability or securitizability of assets, which

makes it harder for borrowers to divert.

Note Subsection 2.4 formalizes this diversion motive which leads endogenously to the constraint in inequality (1).

#### 2.4 Timing

We now specify the timing of the extensive game we use to model the economy. This section serves mainly to formalize the sequencing that we have already sketched above. Since bonds are resaleable but repos are not, we outline the timing for these two cases separately. We describe first what can happen when Bank A issues bonds at Date 0 and then what can happen when Bank A borrows via repos at Date 0. The repo case is slightly more complicated because credit chains can emerge.

The first move is Banks A's choice of financing instrument:

#### Date 0

0.0 Bank A chooses either bonds or repos

We write the subsequent moves separately for the cases in which Bank A chooses bonds and in which Bank A chooses repos.

Several of the moves below involve one bank making a take-it-or-leave-it offer to another bank. Should the second bank reject the offer, it forgoes the relationship. This captures the idea that the credit market is competitive.

Bank A Borrows via Bonds. If Bank A issues bonds, the game proceeds as follows:

#### Date 0

- 0.1 Bank A offers Bank B face value  $F_A$  to borrow  $I_A$ 
  - Bank B accepts or rejects

#### Date 1

- 1.1 Bank B is hit by a liquidity shock or not
- 1.2 If Bank B is hit by a liquidity shock
  - Bank B offers Bank C a resale price to sell its claim to  $F_A$  from Bank A
    - Bank C accepts or rejects

#### Date 2

- 2.1 The return  $\tilde{R}$  on Bank A's project realizes
- 2.2 Bank A either repays F to the bondholder or diverts and defaults

Recall from Subsection 2.2 that if the debtor defaults the bondholder's payoff is normalized to zero, reflecting the costs of bankruptcy stays.

Bank A Borrows via Repos. If Bank A borrows via repos, the game proceeds as follows:

#### Date 0

- 0.1 Bank A offers Bank B face value  $F_{\rm A}$  to borrow  $I_{\rm A}$ 
  - Bank B accepts or rejects

#### Date 1

- 1.1 Bank B is hit by a liquidity shock or not
- 1.2 If Bank B is hit by a liquidity shock
  - Bank B offers Bank C  $F_{\rm B}$  to borrow  $I_{\rm B}$  from Bank C
    - Bank C accepts or rejects

## Date 2

- 2.1 The return  $\tilde{R}$  on Bank A's project realizes
- 2.2 Bank A either repays  $F_A$  to Bank B or diverts and defaults
- 2.3 If Bank B has borrowed from Bank C
  - Bank B either repays  $F_{\rm B}$  to Bank C or diverts and defaults

## 2.5 Assumptions

In this section we make three restrictions on parameters. The first assumption implies that Bank A's project is a good investment, even if all revenues are lost due to bankruptcy costs when  $\tilde{R} = R_L$ . Thus there is no question as to whether the project should go ahead.

Assumption 2.5.1.

$$1 < \pi R_H$$
.

The second assumption, in contrast, says that the returns on Bank A's project are not so high that it can lever up infinitely. Specifically, it says that limits to enforcement are severe enough ( $\theta$  is low enough) that Bank A's credit is rationed according to the amount of its own capital it invests in its project.<sup>14</sup>

<sup>&</sup>lt;sup>14</sup>Note that the alternative assumption that Bank A's project has decreasing-returns-to-scale would also prevent its project from becoming infinitely big; we choose the constant-returns-to-scale set-up because it is a particularly tractable way to capture the economic mechanism we wish to study.

Assumption 2.5.2.

$$\theta \bar{R} < 1.$$

Finally, the third assumption says that the return  $R_L$  that realizes in the event of failure is relatively low. The assumption suffices to ensure that Bank A will default in equilibrium whenever its project fails ( $\tilde{R} = R_L$ ).

Assumption 2.5.3.

$$R_L < \frac{(\pi R_H - 1)R_H}{R_H - 1}.$$

#### 2.6 Equilibrium Concept

The equilibrium concept is subgame perfect equilibrium. We solve the model by backward induction.

## 3 Results

In this section we solve the model. We analyze first the case when Bank A borrows via bonds, then, separately, the case in which Bank A borrows via repos. We then compare Bank A's payoffs from borrowing via each instrument and solve for the equilibrium borrowing instrument. Finally, we study the implications for systemic risk; here we show our main result that increasing enforceability increases systemic risk.

#### 3.1 Borrowing via Bonds

We now solve for the equilibrium of the subgame in which Bank A issues bonds. In particular we wish to calculate its loan size  $I_A^b$  and its Date 0 PV  $\Pi_A^b$ , where the superscript "b" indicates that the quantities correspond to the subgame in which Bank A has borrowed via bonds.

In order to find the amount  $I_A^b$  that Bank B is willing to lend to Bank A against a promise to repay  $F_A^b$ , we solve the game backward. We begin with the case in which Bank B is not hit by a liquidity shock. In this case, it recovers the expected value of Bank A's debt. If there is no default, then Bank B recovers  $F_A^b$  and, if there is default, it recovers zero. Bank A defaults exactly when it prefers to repay rather than to divert capital, or when  $\theta(e+I_A^b)R < F_A^b$  for  $R \in \{R_L, R_H\}$ . It repays zero when it defaults due to the stay in bankruptcy and it repays  $F_A$  otherwise. This is summarized in the

expression below.

expected bond repayment 
$$\begin{cases} F_A & \text{if } \theta(e+I_{\mathrm{A}}^{\mathrm{b}})R_L \geq F_{\mathrm{A}}^{\mathrm{b}}, \\ \pi F_A & \text{if } \theta(e+I_{\mathrm{A}}^{\mathrm{b}})R_L < F_{\mathrm{A}}^{\mathrm{b}} \leq \theta(e+I_{\mathrm{A}}^{\mathrm{b}})R_H, \\ 0 & \text{otherwise} \end{cases}$$

$$= \pi \mathbf{1}_{\left\{\theta(e+I_{\mathrm{A}}^{\mathrm{b}})R_H \geq F_{\mathrm{A}}^{\mathrm{b}}\right\}} F_{\mathrm{A}}^{\mathrm{b}} + (1-\pi) \mathbf{1}_{\left\{\theta(e+I_{\mathrm{A}}^{\mathrm{b}})R_L \geq F_{\mathrm{A}}^{\mathrm{b}}\right\}} F_{\mathrm{A}}^{\mathrm{b}}.$$

Now turn to the case in which Bank B is hit by a liquidity shock. Now it sells Bank A's bonds to Bank C in a competitive market. Bank C demands its break-even value, which is the expected value of Bank A's debt. This coincides with the expression above for Bank A's expected repayment, i.e.

bond resale price = 
$$\pi \mathbf{1}_{\{\theta(e+I_{\Delta}^{b})R_{H} \geq F_{\Delta}^{b}\}} F_{A}^{b} + (1-\pi) \mathbf{1}_{\{\theta(e+I_{\Delta}^{b})R_{L} \geq F_{\Delta}^{b}\}} F_{A}^{b}$$
.

Thus, when Bank A issues bonds, Bank B's payoff is independent of whether Bank B itself is hit by a liquidity shock. Bank B's condition for accepting Bank A's bond offer, i.e. the contract  $(F_A, I_A)$ , reduces to the participation constraint that Bank B must make a positive NPV investment. This (ex ante) participation constraint takes into account the (ex post) limits to enforcement captured by  $\theta$ . Hence, we can rewrite the first round of the game in which Bank A determines how much to borrow and invest as a constrained optimization program. Bank A maximizes its profits subject to its borrowing constraints. The next lemma states this problem.

Lemma 3.1.1.  $F_{\rm A}^{\rm b}$  and  $I_{\rm A}^{\rm b}$  are determined to maximize

$$\Pi_{\mathcal{A}}^{\mathbf{b}} = \mathbb{E}\left[\max\left\{(e+I)\tilde{R} - F, (1-\theta)(e+I)\tilde{R}\right\}\right]$$

over F and I subject to

$$\pi \mathbf{1}_{\{\theta(e+I)R_H \ge F\}} F + (1-\pi) \mathbf{1}_{\{\theta(e+I)R_L \ge F\}} F \ge I.$$

The program has a convex objective with a piecewise linear constraint, so it has a corner solution. There are three possible solutions: (i) Bank A does not borrow at all, (ii) Bank A borrows as much as it can while ensuring it will never default—i.e. ensuring it can repay F even when  $\tilde{R} = R_L$ —or (iii) Bank A borrows as much as it can, accepting that it will default when it fails but that it will still be able to repay when it succeeds—i.e. ensuring it can repay F when  $\tilde{R} = R_H$ . The next lemma states that, given the assumptions in Subsection 2.5, this third possibility obtains in equilibrium,

BANK B'S BALANCE SHEET COMPOSITION WHEN IT SELLS BANK A'S BONDS

$\underline{\text{Date } 0}$			$\underline{\text{Date } 1}$		
assets	liabilities		assets	liabilities	
debt from A	all equity	7	cash	all equity	

Figure 6: When Bank B sells Bank A's bonds to Bank C, it does not assume a new liability.

i.e. Bank A will always lever up so much that it will default when its project fails.

Lemma 3.1.2.

$$F_{\rm A}^{\rm b} = \theta (e + I_{\rm A}^{\rm b}) R_H.$$

*Proof.* See Appendix A.1

Because competition is perfect in the Date 1 market, Bank B sells Bank A's bonds at fair value if it suffers a liquidity shock at Date 1. As a result, Bank B's Date 1 payoff is unaffected by the liquidity shock and Bank B's Date 0 break-even condition reads

$$I_{\mathbf{A}}^{\mathbf{b}} = \pi F_{\mathbf{A}}^{\mathbf{b}}$$
$$= \pi \theta (e + I_{\mathbf{A}}^{\mathbf{b}}) R_{H},$$

having taken into account that the recovery value for Bank B is zero due to the stay in bankruptcy. This says that

$$I_{\rm A}^{\rm b} = \frac{\pi \theta e R_H}{1 - \pi \theta R_H}.$$

Before Bank B is hit by a liquidity shock, Bank B has Bank A's debt on the assets side of its balance sheet. In response to the liquidity shock, Bank B sells Bank A's bonds, replacing this asset with cash on its balance sheet. This is depicted in Figure 6. Note that Bank B only ever has equity on the right-hand side of its balance sheet—when Bank B funds Bank A via bonds, its balance sheet does not expand.

Now we can calculate Bank A's expected equity value when it issues bonds. With probability  $\pi$  it succeeds and repays  $F_{\rm A}^{\rm b} = \theta \left(e + I_{\rm A}^{\rm b}\right) R_H$ . With probability  $1 - \pi$  it fails

and diverts capital  $(1-\theta)(e+I_{\rm A}^{\rm b})$ . Thus,

$$\Pi_{A}^{b} = \pi \left( (e + I_{A}^{b}) R_{H} - F_{A}^{b} \right) + (1 - \pi)(1 - \theta) \left( e + I_{A}^{b} \right) R_{L} 
= \pi (1 - \theta) \left( e + I_{A}^{b} \right) R_{H} + (1 - \pi)(1 - \theta) \left( e + I_{A}^{b} \right) R_{L} 
= (1 - \theta) \left( e + I_{A}^{b} \right) \bar{R} 
= \frac{(1 - \theta) e \bar{R}}{1 - \pi \theta R_{H}}.$$
(2)

## 3.2 Borrowoing via Repos

We now solve for the equilibrium of the subgame in which Bank A issues repos. In particular, we wish to calculate its loan size  $I_{\rm A}^{\rm r}$  and its Date 0 PV  $\Pi_{\rm A}^{\rm r}$ , where the superscript "r" indicates that the quantities correspond to the subgame in which Bank A has borrowed via repos.

Again we solve the game backward to determine the amount  $I_A^r$  that Bank B is willing to lend to Bank A against the promise to repay  $F_A^r$ . We begin with the case in which Bank B is not hit by a liquidity shock. In this case, it holds Bank A's repos to maturity and recovers the expected value of Bank A's debt. If there is no default, Bank B receives  $F_A^r$  and if there is default, it recovers  $\theta(e+I_A^r)R$  for  $R \in \{R_L, R_H\}$ . As before, Bank A defaults exactly when it prefers to repay than to divert capital, or when  $\theta(e+I_A^r)R < F_A^b$ . In contrast to the case of bonds, when Bank A defaults, its repo creditors are not subject to the bankruptcy stay and, hence, they recover the fraction of assets that Bank A does not divert. This is summarized in the expression below.

expected repo repayment 
$$= \pi \left[ \mathbf{1}_{\left\{\theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{H} \geq F_{\mathrm{A}}^{\mathrm{r}}\right\}} F_{\mathrm{A}}^{\mathrm{r}} + \mathbf{1}_{\left\{\theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{H} < F_{\mathrm{A}}^{\mathrm{r}}\right\}} \theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{H} \right] +$$

$$+ (1-\pi) \left[ \mathbf{1}_{\left\{\theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{L} \geq F_{\mathrm{A}}^{\mathrm{r}}\right\}} F_{\mathrm{A}}^{\mathrm{r}} + \mathbf{1}_{\left\{\theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{L} < F_{\mathrm{A}}^{\mathrm{r}}\right\}} \theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{L} \right]$$

$$= \pi \min \left\{ \theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{H}, F_{\mathrm{A}}^{\mathrm{r}} \right\} + (1-\pi) \min \left\{ \theta(e+I_{\mathrm{A}}^{\mathrm{r}})R_{L}, F_{\mathrm{A}}^{\mathrm{r}} \right\}$$

$$= \mathbb{E} \left[ \min \left\{ \theta(e+I_{\mathrm{A}}^{\mathrm{r}})\tilde{R}, F_{\mathrm{A}}^{\mathrm{r}} \right\} \right].$$

Now turn to the case in which Bank B is hit by a liquidity shock. At Date 1, Bank B now must find liquidity in the interbank market. In contrast to the case of bond-borrowing considered in Subsection 3.1, Bank A's debt to Bank B is not resaleable. Instead of liquidating Bank A's bond in the interbank market as before, now Bank B must borrow from Bank C to obtain liquidity. It does so by borrowing  $I_B$  in exchange for the promise to repay  $F_B$ . But now Bank C must anticipate the enforcement frictions

BANK B'S BALANCE SHEET EXPANDS WHEN IT HOLDS BANK A'S REPOS

Date 0		Date 1		
			assets	liabilities
assets	liabilities			
debt from A	all equity	$\longrightarrow$	cash	debt to C
			debt from A	equity

Figure 7: If Bank A borrows via repos and Bank B needs liquidity at the interim date, then Bank B borrows from Bank C. Bank B's balance sheet thus expands, as it holds debt on both sides of its balance sheet.

it faces with Bank B: Bank B will divert if its promised repayments to Bank C are too high. Specifically, Bank B diverts if it profits more from diverting its repayment from Bank A than it profits from making its promised repayment  $F_{\rm B}$  to Bank C. This gives the condition that Bank B diverts whenever

$$\theta \min \left\{ \theta(e + I_{A}^{r})\tilde{R}, F_{A}^{r} \right\} < F_{B}.$$

If Bank B does divert and default on its debt to Bank C, then Bank C seizes Bank B's assets and recovers  $\theta \min\{\theta(e+I_A^r)\tilde{R}, F_A^r\}$ . Note, now, that since the interbank market is competitive (Bank B makes Bank C a take-it-or-leave-it offer), Bank B will always borrow an amount  $I_B$  equal to its expected repayment (given the face value  $F_B$ ), so

$$I_{\rm B} = \text{ expected repayment from B to C} = \mathbb{E}\left[\min\left\{\theta \min\left\{\theta(e+I_{\rm A}^{\rm r})\tilde{R}, F_{\rm A}^{\rm r}\right\}, F_{\rm B}\right\}\right].$$

Further, since Bank B has been hit by a liquidity shock, it values Date 1 consumption infinitely more than Date 2 consumption. Thus, it sets  $F_{\rm B}$  to maximize  $I_{\rm B}$  in the equation above. Since the expectation is weakly increasing in  $F_{\rm B}$ , it is without loss of generality to set  $F_{\rm B}=\infty$ . Thus,

$$I_{\mathrm{B}} = \theta \, \mathbb{E} \left[ \min \left\{ \theta(e + I_{\mathrm{A}}^{\mathrm{r}}) \tilde{R}, F_{\mathrm{A}}^{\mathrm{r}} \right\} \right].$$

Before Bank B is hit by a liquidity shock, it has Bank A's debt on the assets side of its balance sheet. In response to the liquidity shock, Bank B borrows from Bank C,

adding cash as an asset on its balance sheet. This is depicted in Figure 7. Note that, in contrast with the bond case depicted in Figure 6, Bank B now has debt on both sides of its balance sheet—it has debt from Bank A on the assets side and debt to Bank C on the liabilities side. In other words, Bank B is a link in a *credit chain*. When Bank B lends via repos, its balance sheet blows up when it needs liquidity.

We now calculate Bank B's expected payoff given it holds Bank A's repo with face value  $F_A$ . To do so, we take the expectation of the value of the repo to Bank B across the two cases above—the case in which it is not hit by a liquidity shock and holds Bank A's repo till maturity and the case in which it is hit by a liquidity shock and borrows from Bank C,

value of A's repo 
$$= \mu \mathbb{E}\left[\min\left\{\theta(e + I_{A}^{r})\tilde{R}, F_{A}^{r}\right\}\right] + (1 - \mu)\theta \mathbb{E}\left[\min\left\{\theta(e + I_{A}^{r})\tilde{R}, F_{A}^{r}\right\}\right]$$
$$= \left(\mu + (1 - \mu)\theta\right) \mathbb{E}\left[\min\left\{\theta(e + I_{A}^{r})\tilde{R}, F_{A}^{r}\right\}\right]. \tag{3}$$

Bank A determines its repo contract  $(F_A^r, I_A^r)$  to maximize its present value  $\Pi_A^r$ . It does by making Bank B a take-it-or-leave-it offer such that the value of the contract expressed above just induces Bank B to accept the offer. Thus, we can rewrite Bank A's choice of contract as a constrained maximization problem in which the objective is Bank A's PV and the constraint is that Bank B must (weakly) prefer the repo promise  $F_A^r$  to its cash  $I_A^r$ . We can now rewrite Bank A's choice of repo contract as an optimization program. Lemma 3.2.1 summarizes.

Lemma 3.2.1.  $F_{\rm A}^{\rm r}$  and  $I_{\rm A}^{\rm r}$  are determined to maximize

$$\Pi_{\mathcal{A}}^{\mathbf{r}} = \mathbb{E}\left[\max\left\{(e+I)\tilde{R} - F, (1-\theta)(e+I)\tilde{R}\right\}\right]$$

over F and I subject to

$$(\mu + (1 - \mu)\theta)\mathbb{E}\left[\min\left\{\theta(e + I)\tilde{R}, F\right\}\right] \ge I.$$

As in the program in Lemma 3.1.1 above for the bond borrowing case, there will be a corner solution. Lemma 3.2.2 now states that in equilibrium Bank A either does not borrow at all or it exhausts its debt capacity completely, promising the maximum repayment.

Lemma 3.2.2. In equilibrium, Bank A either does not borrow,  $F_{\rm A}^{\rm r}=I_{\rm A}^{\rm r}=0$ , or sets  $F_{\rm A}^{\rm r}$  large enough to induce the maximum repayment, <sup>15</sup>

<sup>&</sup>lt;sup>15</sup>Whenever  $F_{\rm A}^{\rm r} > \theta(e + I_{\rm A}^{\rm r})R_H$ , the repayment does not depend on  $F_{\rm A}^{\rm r}$ , i.e.  $\min\{\theta(e + I_{\rm A}^{\rm r})R, F_{\rm A}^{\rm r}\} =$ 

$$F_{\rm A}^{\rm r} = \theta (e + I_{\rm A}^{\rm r}) R_H.$$

*Proof.* See Appendix A.2.

If Bank A borrows (i.e. if  $I_A^r \neq 0$ ), then we can plug  $F_A^r = \theta(e+I_A^r)R_H$  from Lemma 3.2.2 into the binding constraint in Lemma 3.2.1 to recover the following equation for  $I_A^r$ :

$$I_{\mathbf{A}}^{\mathbf{r}} = \theta \left( \mu + (1 - \mu)\theta \right) \left( e + I_{\mathbf{A}}^{\mathbf{r}} \right) \bar{R}. \tag{4}$$

The enforceability parameter  $\theta$  appears in this equation twice, because enforceability kicks in twice, once at each link in the credit chain—Bank B has to enforce its contract with Bank A and Bank C has to enforce its contract with Bank B. We can solve this equation for  $I_A^r$  to recover

$$I_{\rm A}^{\rm r} = \frac{\theta \left(\mu + (1 - \mu)\theta\right) e\bar{R}}{1 - \theta \left(\mu + (1 - \mu)\theta\right)\bar{R}},$$

which allows us to write down an expression for the PV of Bank A when it funds itself with repos. When  $\tilde{R} = R_H$ , Bank A repays its debt  $F_A^r = \theta(e + I_A^r)R_H$ , whereas when  $\tilde{R} = R_L$ , Bank A diverts a proportion  $1 - \theta$  of its assets. Thus, if Bank A borrows,

$$\Pi_{A}^{r} = \pi \left( (e + I_{A}^{r}) R_{H} - F_{A}^{r} \right) + (1 - \pi)(1 - \theta) \left( e + I_{A}^{r} \right) R_{L} 
= \pi \left( (e + I_{A}^{r}) R_{H} - \theta \left( e + I_{A}^{r} \right) R_{H} \right) + (1 - \pi)(1 - \theta) \left( e + I_{A}^{r} \right) R_{L} 
= (1 - \theta) \left( e + I_{A}^{r} \right) \bar{R} 
= \frac{(1 - \theta) e \bar{R}}{1 - \theta \left( \mu + (1 - \mu) \theta \right) \bar{R}}.$$
(5)

Recall that Bank A may prefer not to borrow and therefore prefer just to invest its inside equity e into its project, in which case  $\Pi_{A}^{r} = e\bar{R}$ . Thus, the value of borrowing via repos is the greater of the value of not borrowing and borrowing with face value  $F_{A}^{r} = \theta(e + I_{A}^{r})R_{H}$ , or

$$\Pi_{\mathbf{A}}^{\mathbf{r}} = \max \left\{ e\bar{R}, \frac{(1-\theta)e\bar{R}}{1-\theta(\mu+(1-\mu)\theta)\bar{R}} \right\}.$$
(6)

 $\theta(e+I_{\rm A}^{\rm r})R$ . Hence, any face value  $F_{\rm A}^{\rm r}>\theta(e+I_{\rm A}^{\rm r})R_H$  is equivalent to  $F_{\rm A}^{\rm r}=\theta(e+I_{\rm A}^{\rm r})R_H$  in the sense that it induces the same transfers for each realization of  $\tilde{R}$ . If  $I_{\rm A}^{\rm r}\neq0$ , we focus on  $F_{\rm A}^{\rm r}=\theta(e+I_{\rm A}^{\rm r})R_H$  without loss of generality.

## 3.3 The Equilibrium Borrowing Instrument

This section presents our main theoretical result that increasing enforceability  $\theta$  leads Bank A to favor repos and thereby leads to credit chains—Bank C lends to Bank B, which lends to Bank A.

To determine when Bank A borrows via bonds and when it borrows via repos, we compare its PV in each case by comparing the expression for  $\Pi_A^b$  in equation (2) with the expression for  $\Pi_A^r$  in equation (5). This comparison is illustrated in Figure 8. Bank A borrows via bonds whenever  $\Pi_A^b \geq \Pi_A^r$  or

$$\frac{(1-\theta)e\bar{R}}{1-\pi\theta R_H} \ge \frac{(1-\theta)e\bar{R}}{1-\theta(\mu+(1-\mu)\theta)\bar{R}},$$

which can be written as

$$\pi R_H \ge (\mu + (1 - \mu)\theta)\bar{R}.$$

With the above equation, we have derived that increased enforceability leads Bank A to prefer repos. We now state this as Proposition 3.3.1.

Proposition 3.3.1. Bank A borrows via bonds only if

$$\theta \le \theta^* := \frac{\pi R_H - \mu \bar{R}}{(1 - \mu)\bar{R}}$$

and borrows via repos otherwise.

This result is the key result behind our main finding that increasing enforceability can increase systemic risk, since more enforceability leads banks to rely more on non-resaleable instruments—on repos—and borrowing via non-resaleable instruments leads to credit chains.

## 3.4 Implications for Systemic Risk

In this section we analyze the effect of increasing enforceability on risk in the financial system as a whole. Here, we analyze when risk on the balance sheet of a single institution can spread beyond that institution's immediate creditors, in particular, when one bank's default causes the default of other banks. This is our notion of systemic risk, which we call a *default cascade* and restate in the next definition.

DEFINITION 3.4.1. A <u>default cascade</u> is an event in which a bank fails as a consequence of another bank's failure. In the model, this occurs whenever Bank B fails (which occurs only because its debtor, Bank A, has failed).

BANK A'S PV FROM ISSUING BONDS AND REPOS AS A FUNCTION OF ENFORCEABILITY

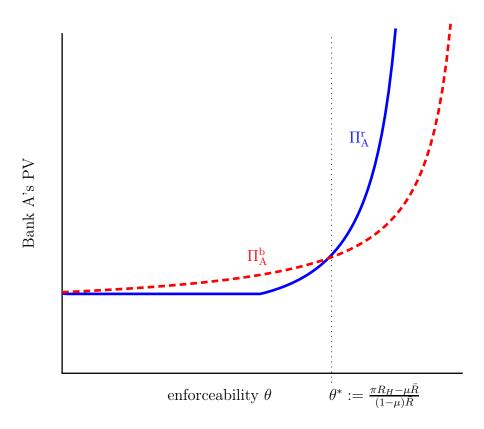


Figure 8: When enforceability is low  $(\theta \le \theta^*)$  Bank A's PV is higher from issuing bonds; when enforceability is high  $(\theta > \theta^*)$  Bank A's PV is higher from issuing repos. The parameters used to make the plot are  $e = 1, \bar{R} = 1.4, \pi R_H = 1.2$ , and  $\mu = 1/2$ .

Bank B can fail only when it has debt to default on. Bank B has debt only when it borrows from Bank C to satisfy its liquidity needs. This occurs only when Bank A borrows via repos. In this case, since repos are not resaleable, Bank B cannot find liquidity by selling Bank A's debt in the market; as a result, Bank B borrows from Bank C creating a credit chain. Hence, Bank A's default can lead to Bank B's default—i.e. default cascades can occur only when Bank A borrows via repos. The next result is that default cascades only happen when enforceability is high. This follows as a corollary of Proposition 3.3.1.

COROLLARY 3.4.1. Default cascades occur only when enforceability is high, specifically when

$$\theta > \theta^* \equiv \frac{\pi R_H - \mu \bar{R}}{(1 - \mu)\bar{R}}.$$

This result says that increasing enforceability increases systemic risk in the sense that increasing enforceability can cause default cascades. Specifically, with repo borrowing, a credit chain emerges in which Bank A borrows from Bank B and Bank B borrows from Bank C. When Bank A's project fails it defaults on its debt to Bank B. This depletes the left-hand side of Bank B's balance sheet, so Bank B cannot cover its debt to Bank C and Bank B also defaults.

## 4 Generalizations, Extensions, and Robustness

In this section, we extend the analysis in six ways. First, we argue that our main result that increasing enforceability increases systemic risk is broadly applicable, not only in the interbank market. Second, we consider the possibility that Bank A borrows via one-period debt and rolls over at Date 1. Third, we discuss the role of additional collateral within the application to the interbank market and we argue that including it would not overturn our qualitative results Fourth, we allow for credit chains with more than two links. Fifth, we consider a short-term stay for repos, rather than an all-out exemption. Finally, we consider implications for social welfare, not just systemic risk. Our results are robust to all these extensions.

## 4.1 More General Instruments

So far, we have focused on the trade-off between borrowing via bonds (commercial paper) and repos in the interbank market. In this section, we argue that our main result—that increasing enforceability leads to credit chains and, therefore, increases systemic risk—generalizes to other markets. In fact, the basic mechanism may be at

work in nearly all debt markets, even absent the formal, legal differences in resaleability and bankruptcy seniority that exist between repos and bonds. The reason is as follows. In addition to legal non-resaleability, fundamental economic frictions such as adverse selection can inhibit the resaleability of debt. A debt issuer may mitigate these frictions at a cost—for example by using securitization to combat the lemons problem—and thereby make debt resaleable or "liquid" in secondary markets. When enforceability increases, however, the relative benefits of resaleability decrease and, as a result, issuers are not willing to pay the cost to issue resaleable debt. Thus, for high enforceability, creditors, unable to sell their assets, may enter into new debt contracts to meet liquidity needs. This is the creation of a credit chain, which harbors systemic risk, just as in our baseline analysis. We formalize this argument below. The analysis in this section does not depend on (i) the assumption that bankruptcy is costly or (ii) the assumption that the outcome R of Bank A's project is non-contractable. We make these assumptions above only for realism of the application to interbank loan markets.

Here we abstract from legal asymmetries. Rather, we follow Kiyotaki and Moore (2005) and assume that adverse selection frictions inhibit the resale of debt in the secondary market, but that an issuer can pay an upfront cost to mitigate these frictions.<sup>17</sup> Specifically, we modify the model above in the following way. When Bank A borrows from Bank B, it can pay a proportional cost c to securitize its project. That is, if Bank A securitizes its project, its returns are decreased by the proportion c to (1-c)R,  $R \in \{R_L, R_H\}$ . Securitization circumvents the adverse selection friction, making Bank A's debt resaleable. There are no bankruptcy costs. We now analyze when Bank A will choose to securitize its project, forfeiting some returns but making its debt liquid/resaleable.

Consider first the case in which Bank A does *not* securitize its project. Here its PV is simply the repo PV expression in equation (6):

$$\Pi_{\mathbf{A}}^{\text{no sec.}} = \max \left\{ e\bar{R}, \frac{(1-\theta)e\bar{R}}{1-\theta(\mu+(1-\mu)\theta)\bar{R}} \right\}.$$

Now turn to the case in which Bank A securitizes its project. Securitization lowers the returns on its project, but eliminates the cost associated with the liquidity shock. This observation allows us to write Bank A's PV in this no-securitization case immediately.

 $<sup>^{16}</sup>$ See Kiyotaki and Moore (2002) for a list of reasons that "between the date of issue and the date of delivery, an initial creditor C may not be able to resell [the debtor] D's paper on to a third party...insofar as D gets locked in with C ex post." (p. 62)

<sup>&</sup>lt;sup>17</sup>See page 703 of Kiyotaki and Moore (2005) for a discussion of this adverse-selection-based microfoundation. Kiyotaki and Moore (2000, 2001a, 2005, 2012) make similar assumptions.

We simply scale down the returns by a factor 1-c and replace the probability  $1-\mu$  of a liquidity shock with zero:

$$\Pi_{\mathbf{A}}^{\text{sec.}} = \max \left\{ e(1-c)\bar{R}, \frac{(1-\theta)e(1-c)\bar{R}}{1-\theta(1-c)\bar{R}} \right\}.$$

Now, Bank A securitizes only when  $\Pi_A^{\text{sec.}} \geq \Pi_A^{\text{no sec.}}$ . This inequality leads to the main result of this section, that Bank A securitizes only below a threshold level of enforceability  $\theta^{**}$ . Thus, credit chains emerge only for high levels of enforceability and, therefore, increasing enforceability increases systemic risk as in Subsection 3.4 above. We summarize this in Proposition 4.1.1 below.

PROPOSITION 4.1.1. Bank A securitizes its debt only if enforceability  $\theta$  is below a threshold, i.e. if  $\theta \leq \theta^{**}$  where

$$\theta^{**} := \frac{1}{2} \left( -1 + \sqrt{1 + \frac{4c}{(1-\mu)(1-c)\bar{R}}} \right).$$

Thus, credit chains emerge and default cascades can occur for only high levels of enforceability.

Proof. See Appendix A.3. 
$$\square$$

This result demonstrates that our finding that increasing enforceability can increase systemic risk is not specific to the interbank market. Rather, the interbank market is just an environment in which systemic risk arising from credit chains is especially important and in which formal legal asymmetries make the trade-offs between resaleable debt like commercial paper and non-resaleable debt like repos especially stark.

## 4.2 Rollover and the Timing of the Liquidity Shock

In the baseline model, we assume that Bank A can borrow only via a two-period debt. Thus, when Bank B suffers a liquidity shock at Date 1, there is still one period before Bank A's debt matures and Bank B must raise liquidity from Bank C. If Bank A could borrow via one-period debt, would it do away with the frictions in our model? I.e., would Bank A borrow via one-period debt from Bank B from Date 0 to Date 1 and from a new creditor, Bank B', from Date 1 to Date 2, eliminating the need for Bank B to resell Bank A's debt at Date 1? In this subsection, we briefly analyze two extensions to argue that such an arrangement may be infeasible. First, we explain how "Date 1" should represent the random arrival time of Bank B's liquidity shock, not a fixed point in time (as mentioned in Subsection 2.1). With this interpretation, Bank A cannot

write debt maturing at Date 1, because it does not know when Date 1 will arrive. Second, we extend the model to include *rollover risk* for Bank A. Specifically, Bank A may fail to find credit from Bank B' at Date 1 and be forced to liquidate its project.

Random arrival of liquidity shock. Consider the following twist on the baseline model: time is continuous in the interval between Date 0 and Date 2 and Bank B's liquidity shock arrives at a Poisson rate  $(1 - \mu)/2$ . Thus, the probability that Bank B suffers a liquidity shock at some time  $\tau \in [0,2]$  is  $1-\mu$ . In other words, Bank B suffers a liquidity shock with the same probability as in the baseline model, and the time  $\tau$  corresponds to Date 1 in the baseline model. Since  $\tau$  is random, Bank A cannot borrow from Bank B via a debt contract with maturity  $\tau$ . Thus, Bank A cannot employ a rollover strategy, whereby it borrows short-term twice first from Bank B and then from Bank B'.

In practice, many repo contracts are indeed short-term. This may reflect banks' attempting to allow their creditors to close contracts easily if they need liquidity at an unexpected date. However, in practice it seems like overnight is the shortest possible debt maturity and, as a result, banks use offsetting repos to manage liquidity intraday. Further, as discussed in Subsection 1.1, many repos have much longer maturities. This likely reflects the fact that rolling debt over is risky, as we now discuss.

Rollover risk. Consider the following alternative twist on the baseline model in which Bank A may face rollover risk at Date 1. At Date 0, Bank A can borrow from Bank B via a one-period repo. In this case, Bank A must borrow from Bank B' at Date 1, to fulfill its obligation to Bank B. With probability  $\rho$  Bank A borrows from Bank B' successfully and rolls over its repo position. But with probability  $1-\rho$ , Bank A is not able to borrow form Bank B' in time to pay its debt to Bank B, so it must liquidate its project to pay Bank B. We assume that the Date-1 liquidation value of Bank A's project is a fraction  $\theta$  of its expected payoff, i.e. exactly the pledgeable fraction  $\theta$  of the project is recoverable in liquidation. This assumption simplifies the analysis, because it implies that if Bank A fails to roll over its debt, Bank B is still repaid in full, but Bank A is left with no assets. As a result, if Bank A borrows via one-period repos, it captures the entire NPV of its project with probability  $\rho$ —with one-period repos, it borrows at the fair price because it avoids the cost of both credit chains and of bankruptcy—but with probability  $1-\rho$ , it receives payoff zero—it fails to roll over its debt and liquidates. Thus, the PV of borrowing via one-period repos is  $\rho$  times the PV of the project as if it were funded with repos, but the probability of a liquidity shock were zero ( $\mu = 1$ ). From the expression for  $\Pi_A^r$  in equation (6), we have the PV of Bank A's project from

borrowing via one-period repos with rollover risk:

$$\Pi_{\mathbf{A}}^{\text{rollover}} = \rho \Pi_{A}^{\mathbf{r}}|_{\mu=1} = \rho \max \left\{ e\bar{R}, \frac{(1-\theta)e\bar{R}}{1-\theta\bar{R}} \right\}.$$

Comparing this expression with the  $\Pi_A^r$  gives the next proposition.

Proposition 4.2.1. As long as rollover risk is not too small, Bank A prefers to borrow via two-period repos than one-period repos. I.e. as long as  $\rho$  is below a threshold  $\rho^*$ , Bank A never borrows via one-period repos, where  $\rho^*$  is given by

$$\rho^* := \frac{1 - \theta \bar{R}}{1 - \theta^2 \bar{R}}.$$

This proposition says that the analysis of our baseline model is valid even if Bank A has the option of borrowing via one-period repos, as long as one-period repos come with some rollover risk.

Proof. See Appendix A.5. 
$$\Box$$

## 4.3 The Role of Collateral and Rehypothecation

Repos and asset-backed commercial paper are collateralized by financial securities. In our model, we have assumed that Bank A's project serves as collateral for its debt. In this subsection, we briefly discuss the consequences of using other securities as collateral and argue that our main results are robust, even if Bank A can rehypothecate its repo collateral. Our result that credit chains emerge only when Bank A borrows via repos is a direct result of repos' non-resaleability, so the inclusion of further collateral would not affect that result. Here we explain that the inclusion of collateral does not affect our finding that increasing enforceability increases the value of repos relatively more than the value of bonds. Thus, our result that credit chains and default cascades are more likely to arise when enforceability is high is robust to the inclusion of collateral.

Suppose that Bank A borrowed from Bank B via a repo collateralized by securities. Denote the value of these securities at maturity by the random variable  $\tilde{s}$ . When Bank B is hit by a liquidity shock, it rehypothecates the securities to borrow from Bank C, creating a credit chain. Note that Bank C holds the securities.

Now we argue that as long as there is some risk that the securities will not cover all debts, the credit chain leads (with some probability) to *two* contracts having to be enforced. Thus, the limited enforcement frictions kick in twice with repos, giving our result that the value of repos is more sensitive than the value of bonds to increases in enforceability.

Suppose that  $\tilde{s} < F_{\rm B}$ . In this case, even after Bank C has liquidated its collateral  $\tilde{s}$ , it must claim on Bank B for the remainder of its debt. This claim is subject to limited enforceability. Further, Bank B is now without collateral because it was liquidated by Bank C. Bank B now has to claim on Bank A for its repayment. This claim is subject to limited enforceability. In summary, when  $\tilde{s} < F_{\rm B}$ , limited enforcement frictions kick in at each link in the credit chain.

Thus, our mechanism is robust to the inclusion of financial securities as collateral as long as their value  $\tilde{s}$  is not perfectly riskless, as no asset value is.

#### 4.4 Longer Chains

So far, we have focused on credit chains with only two links—between Bank A and Bank B and between Bank B and Bank C. In this section, we extend the model to include longer credit chains. Here we consider the possibility that liquidity shocks hit not only Bank A's immediate creditor, but also its creditor's creditor, and its creditor's creditor's creditor and so on. Thus, in this section, credit chains may become arbitrarily long.

We extend the model above to include a sequence of competitive creditors. We refer to the nth creditor in the sequence as Bank  $B_n$ , so Bank  $B_1$  is Bank A's immediate creditor and, generally, Bank  $B_{n+1}$  is Bank  $B_n$ 's creditor. With this notation in the baseline model, Bank B would be called Bank  $B_1$  and Bank C would be called Bank  $B_2$ . Each Bank  $B_n$  suffers a liquidity shock with probability  $1 - \mu$ , in which case it borrows from Bank  $B_{n+1}$ . For simplicity, we assume that all liquidity shocks occur between Date 1 and Date 2, but that Bank  $B_n$  borrows from Bank  $B_{n+1}$  before Bank  $B_{n+1}$  suffers a liquidity shock. As above, we consider the possibility of borrowing either via repos or via bonds. Since the problem is stationary—Bank  $B_n$ 's problem of borrowing from Bank  $B_{n+1}$  coincides with  $B_{n+1}$ 's problem of borrowing from  $B_{n+2}$ —we restrict attention to the cases in which all banks borrow via bonds and all banks borrow via repos.

Consider first the case in which banks borrow via bonds. Since the bond is perfectly resaleable, the credit chain does not affect its value. Thus, Bank A's PV is simply the bond PV expression in equation (2):

$$\Pi_{\mathcal{A}}^{\mathbf{b},\infty} = \frac{(1-\theta)e\bar{R}}{1-\pi\theta R_H}.$$
(7)

Now turn to the case in which banks borrow via repos. In this case, if Bank  $B_n$  needs liquidity, it cannot sell Bank  $B_{n-1}$ 's debt, but must borrow from Bank  $B_{n+1}$ , extending

the credit chain. Each bank extends the chain if and only if it suffers a liquidity shock, which occurs with probability  $\mu$ . Thus, the length of the credit chain is geometrically distributed with mean  $1/\mu$ , i.e. the chain has length one with probability  $\mu$ , length two with probability  $(1 - \mu)\mu$ , length three with probability  $(1 - \mu)^2\mu$ , and length n with probability  $(1 - \mu)^{(n-1)}\mu$ .

Repos are senior in bankruptcy and thus, given the analysis in Subsection 3.2 above, Bank A repays Bank B a fraction  $\theta$  of the value of its asset value in every state. Likewise, Bank B<sub>n</sub> repays Bank B<sub>n+1</sub> a fraction  $\theta$  of its own value, which is a fraction  $\theta$ <sup>n</sup> of Bank A's asset value. Now, this observation allows us to write down the analogy of equation (3) for the value of Bank A's repo.

Assuming that Bank A borrows and denoting the amount Bank A borrows from Bank B<sub>1</sub> by  $I_{\rm A}^{\rm r,\infty}$ , we have the following:

value of A's repo 
$$= \left(\mu + (1-\mu)\mu\theta + (1-\mu)^2\mu\theta^2 + \cdots\right)\theta\left(e + I_{\rm A}^{\rm r,\infty}\right)\bar{R}$$

$$= \mu\theta\left(e + I_{\rm A}^{\rm r,\infty}\right)\bar{R}\sum_{n=0}^{\infty}\left((1-\mu)\theta\right)^n$$

$$= \frac{\mu\theta\left(e + I_{\rm A}^{\rm r,\infty}\right)\bar{R}}{1 - (1-\mu)\theta}.$$

To find the PV of Bank A's project when it borrows via repos, we proceed by analogy with the baseline model (Subsection 3.2). Bank B<sub>1</sub>'s indifference condition implies that the amount of capital it provides equals the repo value of Bank A's repo above, or

$$I_{\mathbf{A}}^{\mathbf{r},\infty} = \frac{\mu\theta\left(e + I_{\mathbf{A}}^{\mathbf{r},\infty}\right)\bar{R}}{1 - (1 - \mu)\theta}$$

which implies

$$I_{\rm A}^{\rm r,\infty} = \frac{\mu\theta\bar{R}e}{1 - (1 - \mu)\theta - \mu\theta\bar{R}},$$

if Bank A borrows and  $I_{\rm A}^{\rm r,\infty}=0$  otherwise. This allows us to express the PV of Bank A's project as follows:

$$\Pi_{\mathbf{A}}^{\mathbf{r},\infty} = \max\left\{e\bar{R}, (1-\theta)\left(e+I_{\mathbf{A}}^{\mathbf{r},\infty}\right)\bar{R}\right\} 
= \max\left\{e\bar{R}, \frac{(1-\theta)\left(1-(1-\mu)\theta\right)e\bar{R}}{1-\left(1-\mu(1+\bar{R})\right)\theta}\right\}.$$
(8)

We now turn to the equilibrium borrowing instrument. Bank A borrows via bonds as long as  $\Pi_A^{r,\infty} \geq \Pi_A^{b,\infty}$ . This inequality leads to the main result of this section, that

Bank A borrows via bonds only above a threshold level of enforceability  $\theta^{\infty}$ , which we summarize in 4.4.1 below.

PROPOSITION 4.4.1. Bank A borrows via bonds only if enforceability  $\theta$  is below a threshold, i.e. Bank A borrows via bonds if  $\theta \leq \theta^{\infty}$  where

$$\theta^{\infty} := \frac{\pi R_H - \mu \bar{R}}{\pi (1 - \mu) R_H}$$

and borrows via repos otherwise. Thus, credit chains emerge and default cascades can occur for only high levels of enforceability.

This result confirms our main finding of the baseline model: credit chains emerge only for high levels of enforceability and, therefore, increasing enforceability increases systemic risk as in Subsection 3.4 above. It also leads to an additional result. Comparison of the threshold  $\theta^{\infty}$  above and the threshold  $\theta^*$  in the baseline model reveals that the possibility of longer credit chains make repo borrowing relatively less attractive. We state this as a corollary.

COROLLARY 4.4.1. In the extension with longer credit chains, the threshold above which Bank A borrows via repos is higher than the threshold in the baseline model,

$$\theta^{\infty} > \theta^*$$
,

as defined in Proposition 3.3.1 and Proposition 4.4.1.

*Proof.* See Appendix A.7. 
$$\Box$$

The intuition behind this result is that if Bank A borrows via repos, credit frictions  $\theta$  kick in at each link in the chain. If chains become arbitrarily long, this amplifies credit frictions, making bond borrowing relatively more attractive.

## 4.5 Short-term Stay for Repos

Currently, repos and derivatives are exempt from bankruptcy stays, whereas bonds and bank loans are subject to a stay of indefinite length, determined by the bankruptcy court. In this subsection, we consider the effects of a fixed-term stay for repos. Specifically, we suppose that in the default state, repo creditors receive a proportion  $\lambda$  of the

value of the debtor's pledgeable assets.<sup>18</sup>  $\lambda$  represents the inverse length of the stay:  $\lambda = 1$  represents the case in which repos are exempt from the stay and  $\lambda = 0$  represents the stay of indefinite length, as in the case of bonds above. We show that increasing the length of the stay for repos, decreases the likelihood that credit chains form.

To find the PV of Bank A's project if it borrows via repos with a short-term stay, we proceed by analogy with the analysis of repo borrowing in Subsection 3.2. In particular, the amount  $I^{\lambda}$  that Bank A can borrow via repos with the "inverse length" of stay  $\lambda$  is given by the analog of equation (4). We simply replace  $R_L$  with  $\lambda R_L$ , since creditor's recovery value is reduced by  $\lambda$  in the default state:

$$I_{\mathbf{A}}^{\lambda} = \theta \left( \mu + (1 - \mu)\theta \right) \left( e + I_{\mathbf{A}}^{\mathbf{r}} \right) \left( \pi R_H + (1 - \pi)\lambda R_L \right)$$
$$= \theta \left( \mu + (1 - \mu)\theta \right) \left( e + I_{\mathbf{A}}^{\mathbf{r}} \right) R^{\lambda},$$

where  $R^{\lambda} := \pi R_H + (1 - \pi) \lambda R_L$ . Thus we have that

$$I_{\mathcal{A}}^{\lambda} = \frac{\theta(\mu + (1-\mu)\theta)eR^{\lambda}}{1 - \theta(\mu + (1-\mu)\theta)R^{\lambda}}.$$

Now we compute the PV of Bank A's project. As in the baseline analysis, Bank A retains a fraction  $1 - \theta$  of the asset value, so

$$\Pi_{\mathcal{A}}^{\lambda} = \pi (1 - \theta)(e + I_{\mathcal{A}}^{\lambda})R_{H} + (1 - \pi)(1 - \theta)(e + I_{\mathcal{A}}^{\lambda})R_{L}$$

$$= \frac{(1 - \theta)e\bar{R}}{1 - \theta(\mu + (1 - \mu)\theta)R^{\lambda}}.$$
(9)

Comparison of this expression with the expression for the PV of Bank A's project if it borrows via bonds (equation (2)), gives the threshold  $\theta^{\lambda}$  above which Bank A borrows via repos. Thus, we have the analog of Proposition 3.3.1 given a stay of inverse length  $\lambda$ .

PROPOSITION 4.5.1. Bank A borrows via bonds only if enforceability  $\theta$  is below a threshold, i.e. if  $\theta \leq \theta^{\lambda}$  where

$$\theta^{\lambda} = \frac{\pi R_H - \mu R^{\lambda}}{(1 - \mu)R^{\lambda}}.$$

Thus, credit chains emerge and default cascades can occur for only high levels of enforceability.

Proof. See Appendix A.8. 
$$\square$$

<sup>&</sup>lt;sup>18</sup>Throughout this analysis, we assume that Bank A chooses to borrow enough that it defaults if  $\tilde{R} = R_L$ , i.e. Lemma 3.2.2 holds. This is the case as long as  $\pi$  is sufficiently large.

This result confirms that our main result is robust to the possibility that repos are subject to a short-term stay. It also allows us to analyze how Bank A's choice of borrowing instrument depends on the length of the stay.

COROLLARY 4.5.1. The threshold above which Bank A borrows via repos is decreasing in the length of the stay on repo assets, i.e.  $\theta^{\lambda}$  is increasing in  $\lambda$ ,

$$\frac{\partial \theta^{\lambda}}{\partial \lambda} > 0.$$

*Proof.* See Appendix A.9.

This corollary suggests that the length of the stay for repos can mitigate systemic risk. Thus, policy makers should be concerned not only with the question of whether repos should be stayed in bankruptcy, but also with how long they should be stayed for. In particular, in the context of the model, a short-term stay for repo collateral can decrease systemic risk, while still maintain repos' effective seniority over other instruments such as bonds.

#### 4.6 Welfare Consequences of Systemic Risk

Our analysis has focused on systemic risk and how to mitigate it. Whereas many regulations aim expressly to decrease systemic risk, we believe that it is important to acknowledge that decreasing systemic risk is just one component of a regulator's objective function, and some policies that reduce systemic risk may have other costs. In this section, we argue that, in our model, decreasing systemic risk increases social welfare under reasonable assumptions.

We assume that there is a fixed social cost of each bank's default.

Assumption 4.6.1. Each bank's default has social cost D.

This assumption leads immediately to the result that the social costs of bank default are higher when Bank A borrows via repos than when Bank A borrows via bonds.

Lemma 4.6.1. The social costs of default are higher when Bank A has borrowed via repos than when Bank A has borrowed via bonds, i.e.

$$(1-\pi)(2-\mu)D > (1-\pi)D,\tag{10}$$

where  $(1-\pi)D$  is the expected social cost of bank default when Bank A borrows via bonds and  $(1-\pi)(2-\mu)D$  is the expected social cost of bank default when Bank A borrows via repos.

Viewed in conjunction with Proposition 3.3.1, this proposition implies that decreasing credit market frictions can decrease welfare, <sup>19</sup> as we state formally in the next corollary. Figure 9 depicts the social costs of default as a function of enforceability  $\theta$ .

COROLLARY 4.6.1. Increasing credit frictions can decrease welfare. Specifically, increasing enforceability from below  $\theta^*$  to above  $\theta^*$  leads to an increase in the social costs of default from  $(1-\pi)D$  to  $(1-\pi)(2-\mu)D$ .

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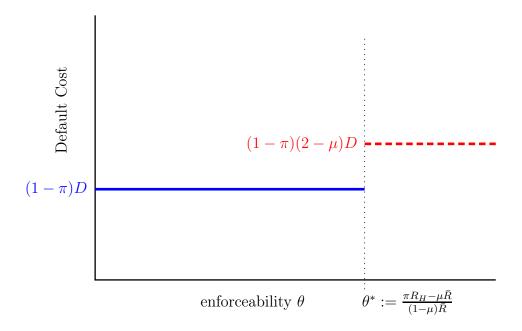


Figure 9: When enforceability is low, Bank A funds itself via bonds and the social costs of default are low; when enforceability is high, Bank A funds itself via repos and the social costs of default are high. The parameters used to create the plot are  $(1 - \pi)D = 100$ , and  $\mu = 1/2$ .

<sup>&</sup>lt;sup>19</sup>Note that decreasing credit frictions also has a positive effect on welfare. It allows Bank A to scale up its project further. Thus, away from the cutoff  $\theta^*$  increasing enforceability has the standard positive effect. However, we emphasize here the negative effect of increasing enforceability around  $\theta^*$ .

## 5 Conclusions

In this paper, we have developed a model to analyze the connection between credit market frictions and systemic risk. We argued that a decrease in credit market frictions can lead to an increase in systemic risk and a decrease in welfare—even though a decrease in credit market frictions makes each market function better in isolation, it can harm the financial system as a whole. The reason is that in markets with low credit market frictions, financial institutions are likely to borrow via non-resaleable debt (e.g. repos) rather than resaleable debt (e.g. bonds) and borrowing via non-resaleable debt leads to credit chains, which harbor systemic risk.

Our model is stylized, but we hope that it draws attention to some features of debt claims and financial markets that may deserve more attention in the policy debate. Most notably, borrowing via resalable instruments mitigates systemic risk. Therefore, a regulator aiming to combat systemic risk should encourage financial institutions to use resaleable debt to fund themselves. However, improvements in financial markets that mitigate credit frictions (e.g., improving creditor rights) may have unintended consequences. Specifically, lowering credit frictions may induce financial institutions to borrow via non-resaleable debt, increasing systemic risk. In particular, the exemption to the automatic stay for repos appears to have had unintended consequences, increasing repo borrowing, which lead to credit chains, consistent with the predictions of the model.

## A Omitted Derivations and Proofs

## A.1 Proof of Lemma 3.1.2

Since the program in Lemma 3.1.1 is linear, it must have a corner solution. Thus, there are three possible solutions: the Bank A either borrows nothing, borrows the maximum so that it never defaults, or borrows the maximum so that it defaults only when it fails. The case in which it borrows the maximum so that it defaults only when it fails is analyzed in the main text and yields expected equity value given in equation (2),

$$\Pi_{\rm A}^{\rm b}\Big|_{\rm repay \ if \ \tilde{R} = R_H} = \frac{(1-\theta)e\bar{R}}{1-\pi\theta R_H}.$$

If it borrows nothing its expected equity value is

$$\Pi_{\rm A}^{\rm b}\Big|_{\rm borrow\ nothing} = e\bar{R}.$$

Now,  $\Pi_{\rm A}^{\rm b}\big|_{\rm repay\ if\ \tilde{R}=R_H}>\Pi_{\rm A}^{\rm b}\big|_{\rm borrow\ nothing}$  if and only if  $\pi R_H>1$ , which is guaranteed by Assumption 2.5.1. Thus, it remains only to compare the case in which Bank A defaults only when it fails with the case in which Bank A never defaults.

If Bank A never defaults, it borrows as much as it can given that it does not default in the event that  $\tilde{R} = R_L$ . Thus, it borrows

$$I_{\rm A} = F_{\rm A} = \theta(e + I_{\rm A})R_{I,\rm A}$$

and its expected equity value is

$$\begin{split} \Pi_{\rm A}|_{\rm never\ default} &= \pi \big( (e + I_{\rm A}) R_H - F_{\rm A} \big) + (1 - \pi) (1 - \theta) (e + I_{\rm A}) R_L \\ &= \Big( \pi (R_H - \theta R_L) + (1 - \pi) (1 - \theta) R_L \Big) (e + I_{\rm A}) \\ &= \Big( \pi \big( (1 - \theta) R_H - (1 - \theta) R_H + R_H - \theta R_L ) \big) + (1 - \pi) (1 - \theta) R_L \Big) (e + I_{\rm A}) \\ &= \Big( (1 - \theta) \bar{R} + \pi \theta (R_H - R_L) \Big) (e + I_{\rm A}) \\ &= \frac{\big( (1 - \theta) \bar{R} + \pi \theta (R_H - R_L) \big) e}{1 - \theta R_L}. \end{split}$$

Assumption 2.5.3 ensures that this expression is always smaller than  $\Pi_{\rm A}^{\rm b}|_{\rm repay\ if\ \tilde{R}=R_H}$  from equation (2). Therefore, Bank A always sets  $F_{\rm A}=\pi\theta(e+I_{\rm A})R_H$ , as in the case analyzed in the main text.

#### A.2 Proof of Lemma 3.2.2

Since there are no inefficiencies from default in the repo case, if Bank A borrows it is without loss of generality to assume that Bank A defaults whenever it borrows, i.e. that  $F = \infty$  if I > 0 or, alternatively, since  $R \le R_H$ , that  $F = \theta(e+I)R_H$  whenever I > 0. Thus it suffices to consider  $F = \theta(e+I)R_H$  and F = 0, as stated in the lemma.

Note that a more explicit computational proof could also be done in exact analogy with Lemma 3.1.2, but we omit it here.

## A.3 Proof of Lemma 4.1.1

Recall that Bank A borrows via non-securitized debt if and only if  $\Pi_A^{\text{no sec.}} \geq \Pi_A^{\text{sec.}}$ . Recalling the expressions for  $\Pi_A^{\text{no sec.}}$  and  $\Pi_A^{\text{sec.}}$  in Subsection 4.1 from Subsection 4.1, we see that a necessary condition for this is that the

$$\frac{(1-\theta)e\bar{R}}{1-\theta(\mu+(1-\mu)\theta)\bar{R}} \ge \frac{(1-\theta)e(1-c)\bar{R}}{1-\theta(1-c)\bar{R}}$$

or, rewriting, that

$$\theta^2 + \theta - \frac{c}{(1-\mu)(1-c)\bar{R}} \ge 0.$$

Thus, Bank A securitizes only if

$$\frac{1}{2}\left(-1-\sqrt{1+\frac{4c}{(1-\mu)(1-c)\bar{R}}}\right) \le \theta \le \frac{1}{2}\left(-1+\sqrt{1+\frac{4c}{(1-\mu)(1-c)\bar{R}}}\right).$$

Since the lower root is negative whenever it exists and  $\theta \in (0,1)$ , this is equivalent to saying that Bank A securitizes only if

$$\theta \le \theta^{**} := \frac{1}{2} \left( -1 + \sqrt{1 + \frac{4c}{(1-\mu)(1-c)\bar{R}}} \right).$$

The proposition follows.

#### A.4 Proof of Lemma 4.6.1

Bank A defaults with probability  $1 - \pi$ . Since no other bank ever defaults if Bank A has borrowed via bonds, the expected social costs of default are simply  $(1 - \pi)D$  if Bank A has borrowed via bonds.

If Bank A has borrowed via repos, and only if Bank A has borrowed via repos, Bank B defaults if and only if Bank A defaults and Bank B itself has been hit by a liquidity shock. This liquidity shock occurs with independent probability  $1 - \mu$ . Thus, the expected social costs of default are

$$(1-\pi)D + (1-\pi)(1-\mu)D = (1-\pi)(2-\mu)D.$$

Since  $\mu < 1$ , the social costs are greater when Bank A has borrowed via repos.

## A.5 Proof of Proposition 4.2.1

As long as  $\Pi_A^{\text{rollover}} < \Pi_A^{\text{r}}$ , Bank A never borrows via one-period repos. From the expression for  $\Pi_A^{\text{rollover}}$  in Proposition 4.2.1 and the expression for  $\Pi_A^{\text{r}}$  in equation (6), this says Bank A never borrows via one-period repos as long as

$$\frac{\rho(1-\theta)e\bar{R}}{1-\theta\bar{R}} < \frac{(1-\theta)e\bar{R}}{1-\theta(\mu+(1-\mu)\theta)\bar{R}}$$

or

$$\rho < \frac{1 - \theta \bar{R}}{1 - \theta (\mu + (1 - \mu)\theta)\bar{R}}.$$

The left-hand side above is decreasing in  $\mu$ , so if it holds for  $\mu = 0$  it holds for all  $\mu$ . This implies that Bank A never borrows via one-period repos as long as

$$\rho < \frac{1 - \theta \bar{R}}{1 - \theta^2 \bar{R}} \equiv \rho^*,$$

as in the statement of the proposition.

## A.6 Proof of Proposition 4.4.1

The result follows directly from the comparison of  $\Pi_A^{b,\infty}$  and  $\Pi_A^{r,\infty}$  in equations (7) and (8).

## A.7 Proof of Corollary 4.4.1

The result follows directly from the comparison of  $\theta^*$  in Proposition 3.3.1 and  $\theta^{\infty}$  in Proposition 4.4.1, using the fact that  $\pi R_H < \bar{R} \equiv \pi R_H + (1-\pi)R_L$ .

## A.8 Proof of Proposition 4.5.1

Given repos are subject to a stay with inverse length  $\lambda$ , Bank A borrows via bonds whenever  $\Pi_A^b$  is greater than  $\Pi_A^{\lambda}$  or, substituting from equations (2) and (9),

$$\frac{(1-\theta)e\bar{R}}{1-\pi\theta R_H} \le \frac{(1-\theta)e\bar{R}}{1-\theta(\mu+(1-\mu)\theta)R^{\lambda}}.$$

Simplifying this inequality gives

$$\theta \ge \frac{\pi R_H - \mu R^{\lambda}}{(1 - \mu)R^{\lambda}} \equiv \theta^{\lambda}$$

as stated in the proposition.

## A.9 Proof of Corollary 4.5.1

We can restrict attention to the case in which  $\theta^{\lambda} > 0$ . Otherwise, Bank A always borrow via bonds. Differentiating gives

$$\frac{\partial \theta^{\lambda}}{\partial \lambda} = -\frac{(1-\mu)(\mu R^{\lambda} + (\pi R_H - \mu R^{\lambda}))}{(1-\mu)^2 (R^{\lambda})^2} \frac{\partial R^{\lambda}}{\partial \lambda}.$$

This is negative since  $\theta^{\lambda} > 0$  implies  $\pi R_H - \mu R^{\lambda} > 0$  and

$$\frac{\partial R^{\lambda}}{\partial \lambda} = (1 - \pi)R_L > 0. \tag{11}$$

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