PROCYCLICAL PROMISES*

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April 15, 2018

Abstract

I explore how cyclicality affects debt capacity. Procyclicality has a well-known cost: procyclical firms make lower repayments in recessions. I point out that it can also have a benefit: procyclical firms make higher repayments on average. The reason is that their output is high when asset prices are high, and these high asset prices mitigate financial frictions, such as collateral constraints. This benefit is present across diverse models of financial frictions, and can be as important for debt capacity as the cost. It affects macroeconomic outcomes, generating aggregate fluctuations and a premium for assets used by procyclical firms.

*Thanks to Nittai Bergman, Mike Burkart, Jaewon Choi, Jon Danielsson, Amil Dasgupta, John Donaldson, Erik Eyster, Jack Favilukis, Daniel Ferreira, Stéphane Guibaud, Denis Gromb, Christian Julliard, Ohad Kadan, John Kuong, Mina Lee, Kai Li, Dong Lou, Igor Makarov, Ian Martin, Marc Martos-Vila, John Moore, Alan Morrison, Philippe Mueller, Francesco Nava, Bob Nobay, Clemens Otto, Daniel Paravisini, Giorgia Piacentino, Christopher Polk, Ronny Razin, Antoinette Schoar, Alan Schwatz, Joel Shapiro, Rob Shimer, Balazs Szentes, Anjan Thakor, Dimitri Vayanos, David Webb, Kathy Yuan, Kostas Zachariadis, Stefan Zeume, and Jean-Pierre Zigrand and seminar audiences at the Bank of Canada, the Federal Reserve Bank of Philadelphia, Indiana University, the London School of Economics, Oxford, Sciences Po, the University of British Columbia, and Washington University in St. Louis.

1 Introduction

What determines a firm's debt capacity? Williamson (1988) points to asset liquidation values. He explains that you should be more willing to lend to a firm if it has assets you can liquidate easily if it defaults. But, as Shleifer and Vishny (1992) emphasize, you should not care only about what assets the firm has on average. You should also care about when it has them. Is it better if it has them in booms or in recessions? One of the central ideas in asset pricing suggests recessions: that way, you get high repayments when they are most valuable (i.e. when marginal utility is high). In this paper, I argue that this is not the whole story for corporate finance.

My main point follows from an identity relating a firm's debt capacity to its cyclicality. To see where it comes from, consider a firm that borrows against a quantity y of an asset with price p. The most it ever repays is py. So its debt capacity, denoted by DC, is the present value of py:

$$\mathrm{DC} = \mathbb{E}\left[Mpy\right], \tag{1}$$

where M is a stochastic discount factor. By manipulating this expression, I find an identity that captures the intuition at the core of the paper:

$$\mathrm{DC} = \frac{1}{R_f}\mathbb{E}\left[y\right]\,\mathbb{E}\left[p\right] + \mathbb{E}\left[p\right]\mathrm{Cov}\left[M, y\right] + \frac{1}{R_f}\mathrm{Cov}\left[p, y\right] + \epsilon, \tag{DC}$$

where R_f is the risk-free rate and ϵ is a "nuisance term" that is typically small and can be calculated explicitly. The first term above is standard. It just says that firms that produce more/more valuable output can borrow more. The first covariance is also standard. It says that firms with positive covariance with the SDF can borrow more. Intuitively, you want to lend to a firm that makes repayments in recessions, when they are most valuable, in line with the asset pricing intuition. I refer to this as the "discount rate term." The second covariance reflects my new observation. It says that firms that produce more when their assets are valuable can make higher repayments on average. Intuitively, you want to lend to a firm that has assets that can be sold/liquidated at high prices. Thus, in contrast to the received view, there is a benefit to procyclicality: it can loosen financial constraints. I refer to this as the "procyclical promises term."

After imposing a bit more structure, I find that the discount rate term is close to minus Campbell and Vuolteenaho's (2004) discount-rate beta and the procyclical promises term is close to their cash-flow beta. Hence, I offer an explanation for the empirical finding that corporate leverage seems to be decreasing in discount-rate beta, but increasing in cash-flow beta (see Campbell, Polk, and Vuolteenaho (2010), Ellahie (2017), and Maia (2010)).

In the main analysis, I first show that the identity (DC) holds (up to an affine tranformation) in a broad set of models, including those based on collateral constraints as in Kiyotaki and Moore (1997) and on asset substitution as in Holmström and Tirole (1997). Despite its ubiquity, it seems that the procyclical promises channel is new to the literature. I think the reason is that the two things underlying it—the price of assets and the cyclicality of output—are not typically studied together in corporate finance (save a few exceptions discussed below).

Second, I analyze the trade-off between the discount rate term and the procyclical promises term. Which dominates? To address this question, I consider a neoclassical economy in which the SDF is proportional to the marginal utility of a representative consumer and the (rental) price of assets is proportional to the marginal productivity of a representative (unconstrained) firm. I show that the procyclical promises term can be just as important as the discount rate term. Indeed, with log utility and a single capital asset, the two terms cancel out, so procyclical firms can borrow just as much as acyclical firms (all else equal). I also consider a model with multiple capital assets and show that the procyclical promises term can actually dominate the discount rate term for some firms. These are firms that use assets with a volatile price p: increasing the variance of p increases the procyclical promises term (since $\mathrm{Cov}\,[\,p, y] \propto \sqrt{\mathrm{Var}\,[\,p\,]}$ in equation (DC)), hence increases the debt capacity.

Third, I turn this exercise around to dig deeper into the effects of procyclical promises. Rather than looking at a single constrained firm that takes aggregate outcomes as given in a larger economy, I model an economy in which constrained firms drive aggregate outcomes. How do procyclical promises affect aggregate investment, prices, output, and fluctuations? To address this question, I isolate the procyclical promises channel by switching off the discount rate channel entirely, and focusing on a risk-neutral economy ($M \equiv 1/R_f$). In this set-up, the more procyclical a firm is, the more it can lever up and, hence, the more capital it can invest. This high demand drives up the price of assets used in procyclical production: there is a collateral premium on "procyclical assets." Moreover, expected output is high when procyclical firms have more initial capital, since they are the firms that can lever it up. As a result, aggregate investment, prices, productivity, and output vary over time, even if production technologies do not (so the Arrow–Debreu benchmark allocation is constant). In other words, there are aggregate fluctuations, but they are pure "allocation cycles." They arise because aggregate productivity goes up when productive firms have high debt capacity, which allows them to buy assets and scale up their investments: it is who holds the capital that matters.

My analysis, which focuses on the procyclical promises channel, complements the few other models of the interaction between cyclicality and leverage, which focus on the discount rate channel. To my knowledge, these are Choi (2013), Shleifer and Vishny (1992), and Ross (1985), which connect it with the value premium, capital redeployment, and carried-forward tax deductions, respectively. In contrast, I connect it with the macroeconomy. Hence, I also contribute to the large literature on macro with financial frictions.[1] Although many papers in this literature speak to the cyclicality of firms' leverage—to how leverage changes over the cycle—they are largely silent on the leverage of firms with different degrees of cyclicality—on my question of how leverage varies in the cross-section of firms with different sensitivities to

[1]Some of the most related papers in this literature are Alvarez and Jermann (2000), Bernanke and Gertler (1989), Geanakoplos (1997), Geanakoplos and Kübler (2004), Kehoe and Levine (1993), Kiyotaki and Moore (1997), and Lorenzoni (2008).

the cycle.[2]

2 Procyclical Promises and Debt Capacity

I start by asking how the cyclicality of a firm's output affects its debt capacity. Most standard corporate finance models do not speak to this question directly, since they do not model how output varies with the aggregate state of the economy; hence, they abstract from cyclicality.[3] As such, they switch off two important things: (i) how much consumption goods are worth in terms of utility and (ii) how much assets are worth in terms of consumption goods. I switch these back on. I set up two standard models with different corporate finance frictions, collateral constraints and asset substitution/moral hazard. To capture (i), I include a stochastic discount factor M and, to capture (ii), I include the price of assets p. It turns out that the different frictions lead to the same conclusion: a firm with output y has debt capacity given by a simple formula, $\mathrm{DC} = \mathbb{E}\left[Mpy\right]$ (up to an affine transformation). This expression does speak to the effects of cyclicality on debt capacity, since it depends on how the firm's output y varies with the aggregate variables M and p. However, at this point, the effects are confounded in the product Mpy; M is countercyclical—marginal utility is high in recessions—whereas p is procyclical—asset prices are high in booms. Below, I derive a decomposition to disentangle these effects. But first I derive the primitive expression $\mathrm{DC} = \mathbb{E}\left[Mpy\right]$ in environments with collateral constraints, asset substitution/moral hazard, and a combination of the two.

[2]See, e.g., Chen (2010) and Hackbarth, Miao, and Morellec (2006) for models of firm financing over the cycle. Closer to me, Bhamra, Kuehn, and Strebulaev (2010a, 2010b) study a model that allows for heterogenous cyclicality; I go a step further by connecting cyclicality with endogenous financial constraints.

[3]Macroeconomic models with financial frictions typically do model how output varies with the aggregate state. But, as touched on in footnote 2, only a few include heterogenous firms with different degrees of cyclicality.

2.1 Collateral constraints. Consider the financing friction in Kiyotaki and Moore's (1997) model: repayments are limited by the value of collateral a firm has:

$$\text{repayment} \;\leq\; \text{value of its collateral} \tag{2}$$

(cf. p. 217 of their paper).[4] Thus, if firm has a quantity y of collateral assets with price p, it repays at most py. Taking the present value immediately gives the desired formula:

LEMMA 1. *In the environment with collateral constraints described above, the formula $DC = \mathbb{E}[Mpy]$ holds, where y is the quantity of collateral the firm has and p is its price.*

Note, however, that the formula has little bite in Kiyotaki and Moore's specific model, because firms do not produce collateral assets (they only use them to produce consumption goods). In other words, y is constant in their model. As a result, y factors out of the formula for debt capacity:

$$\mathrm{DC} = \mathbb{E}[Mpy] = \mathbb{E}[Mp]\,y. \tag{3}$$

Here, debt capacity does not depend how the output y co-moves with the aggregate variables M and p, since fixing the amount of collateral a firm has ruled out a role for cyclicality. However, just allowing firms to get more collateral in the course of production can rule it back in—if both the price and quantity of collateral can change, then it matters how they move together. This is the case that I focus on, which seems relevant empirically.[5] (Kiyotaki and Moore also assume everyone is risk neutral, so the SDF is constant: $M \equiv 1/R_f$. I do this too in my general equilibrium analysis in Section 4 below.)

2.2 Asset substitution/Moral hazard. Now consider the friction in Holmström and Tirole's (1997) model: repayments are limited because the firm must have incentive to continue its investment rather than do "asset substitution" (or, equivalently, just have incentive

[4]See, e.g., Benmelech and Bergman (2009) and Rampini and Viswanathan (2013) for empirical evidence on collateral values as a determinant of debt capacity.

[5]For example, Acharya et al. (2007) and Almeida and Philippon (2007) find variation in the proportion of assets creditors recover in bankruptcy, which is effectively variation in the amount of collateral firms have.

to work to complete its investment rather than shirk).

Consider a firm with outstanding debt with face value T and an investment that can succeed or fail. If it succeeds, it pays off a quantity y of an asset with price p. If it fails, it pays off zero. As in Holmström and Tirole (1997), the probability of success depends on whether the firm takes the (non-contractable) asset substitution action. The firm can take a "good" action, denoted by $e = 1$, in which case the probability of success is π_1. Or it can take a "bad" action, denoted by $e = 0$, in which case the probability of success is π_0. The bad action reduces the success probability, i.e. $\pi_0 < \pi_1$, but delivers private benefits B to the firm (or its manager). I assume that the probability of success depends only on e (not on the aggregate state) and that the probability of success given $e = 0$ is small (so $e = 1$ must be incentive compatible).

We can calculate that the firm has the incentive to take the good action only if the repayment T is not to large: the IC $e = 1 \succeq e = 0$ says

$$\begin{aligned} &\text{PV}\Big[\text{ value of output } - \text{ repayment } \Big| e = 1\Big] \\ &\geq \text{PV}\Big[B + \text{ value of output } - \text{ repayment } \Big| e = 0\Big], \end{aligned} \tag{4}$$

which can be rewritten[6] as

$$T \leq R_f \mathbb{E}\left[Mpy\right] - \frac{B}{\pi_1 - \pi_0}. \tag{5}$$

Intuitively, "shirking" ($e = 0$) gives private benefits, by definition; in contrast, "working" ($e = 1$) gives shared benefits, since it increases both output and repayments. The higher T is, the more of these benefits of working go to increasing the expected repayment, which

[6]Symbolically, the IC in equation (4) reads

$$\mathbb{E}\left[M\,\mathbb{1}_{\text{succ}}\left(py - T\right)\Big| e = 1\right] \geq \mathbb{E}\left[M\left(B + \mathbb{1}_{\text{succ}}\left(py - T\right)\right)\Big| e = 0\right],$$

where $\mathbb{1}_{\text{succ}}$ denotes the success indicator. Now, given that I have assumed that $\mathbb{1}_{\text{succ}}$ independent of everything but e, we can apply the Law of Iterated Expectations to write

$$\mathbb{E}\left[M\,\pi_1(py - T)\right] \geq \mathbb{E}\left[M\left(B + \pi_0(py - T)\right)\right].$$

Rearranging gives equation (5) in the text.

goes up by $(\pi_1 - \pi_0)T$. Hence, high T makes it more temping to shirk and get the private benefits. This leads to the upper bound on T above. The debt capacity formula follows from taking the present value of this expression.

LEMMA 2. *In the environment with asset substitution described above, the formula* $DC = \alpha_0 + \alpha_1 \mathbb{E}[Mpy]$ *holds for constants* $\alpha_0 = -\frac{\pi_1 B}{R_f(\pi_1 - \pi_0)}$ *and* $\alpha_1 = \pi_1$*, where* y *is the quantity of output produced and* p *is its price.*

2.3 Collateral and asset substitution. Now consider a combination of the frictions above, which is somewhat similar to Hart and Moore (1989/1998): repayments are limited both because a firm must back debt with collateral and because it must have incentive to continue its investment, rather than abandon it or "divert capital."

Consider a firm that has an investment k of a capital asset that produces output y in a consumption good. As above, suppose that only the capital asset serves as collateral, so the standard collateral constraint above must be satisfied (equation (2)). Unlike above, however, the amount of collateral the firm has depends on its continuing the investment, denoted by $e = 1$, rather than abandoning it, denoted by $e = 0$ (this can equivalently represent any "bad" asset substitution or shirking action). If $e = 1$, k stays in place and y is realized. If $e = 0$, k depreciates by δ and y is not realized, but the firm gets private benefit B.

To simplify the analysis, assume that the distribution of output y is binary: $y = Ak$ in the event of success and $y = 0$ in the event of failure. Now it is easy to see that the firm continues a successful investment but abandons a failing investment (at least as long as the productivity A is not too small and the rate of depreciation δ is not too large; see Appendix B).[7] Denoting the price of capital assets by p, we have that the firm's maximum repayment

[7] It may be worth emphasizing that here the firm chooses e before output is realized, but after learning whether the investment will succeed. For the results, it suffices that it gets some information about the likelihood of success. In the context of sales, this could come from observing market demand for a product; in the context of manufacturing, it could come from observing the efficiency of the machinery and workforce.

is

$$T^{\max} = \begin{cases} pk & \text{given success,} \\ \\ (1-\delta)pk & \text{given failure} \end{cases} \tag{6}$$

$$= \mathbb{1}_{\text{succ}}\, pk + (1 - \mathbb{1}_{\text{succ}})(1-\delta)pk \tag{7}$$

$$= (1-\delta)pk + \frac{\delta p y}{A}, \tag{8}$$

since $y \in \{0, Ak\}$ implies that $y = \mathbb{1}_{\text{succ}} Ak$. The debt capacity formula follows from taking the present value of this expression.

LEMMA 3. *In the environment with collateral constraints and asset substitution described above, the formula $DC = \alpha_0 + \alpha_1 \mathbb{E}[Mpy]$ holds for constants $\alpha_0 = (1-\delta)\mathbb{E}[Mp]\,k$ and $\alpha_1 = \delta/A$, where k is the quantity of collateral, p is its price, δ is the rate of depreciation (or the fraction of assets that can be diverted), and y is the output of the consumption good.*

This set-up will be useful below, since it can fit into the dynamic general equilibrium framework in Section 4 relatively easily. There, the set-up remains tractable despite its heterogenous firms with heterogenous financial constraints. One reason is that, since the firm does not produce the collateral asset here, I can find assumptions such that the total capital stock remains constant. (Specifically, I assume that the private benefits B constitute diverted capital, so $B \propto \delta k$ and $(1-\delta)k$ is left as collateral; cf. footnote 20.)

2.4 Decomposition. What does the expression $\text{DC} = \mathbb{E}[Mpy]$ have to say about the effect of cyclicality on debt capacity? To address this question, I decompose the expectation:

PROPOSITION 1.

$$\mathbb{E}[Mpy] = \frac{1}{R_f}\mathbb{E}[y]\ \mathbb{E}[p] + \mathbb{E}[p]\,\text{Cov}[M, y] + \frac{1}{R_f}\,\text{Cov}[p, y] + \epsilon, \tag{9}$$

where R_f is the risk-free rate and

$$\epsilon = \text{Cov}\left[\big(M - \mathbb{E}[M]\big)\big(p - \mathbb{E}[p]\big), y\right]. \tag{10}$$

Cyclicality is captured by the aggregate variables M and p: M is high in recessions when marginal utility is high (since consumption is low); p is high in booms when the demand for capital assets is high (since productivity is high).[8] Thus, the first term $\frac{1}{R_f}\mathbb{E}[y]\,\mathbb{E}[p]$ in Proposition 1, in which p and M decouple, does not depend on the cyclicality of y.[9] But the two covariance terms, $\mathbb{E}[p]\,\text{Cov}[M, y]$ and $\frac{1}{R_f}\,\text{Cov}[p, y]$, do. (I abstract from the "nuisance term" ϵ for now and show in Lemma 5 below that it is typically small anyway.)

I call $\mathbb{E}[p]\,\text{Cov}[M, y]$ the "discount rate term." It reflects the standard cost of procyclicality: a procyclical firm has low output in recessions, and hence it can repay little when M is high.[10]

I call $\frac{1}{R_f}\,\text{Cov}[p, y]$ the "procyclical promises term." It reflects the benefit of procyclicality that I emphasize in this paper: a procyclical firm has high output in booms, when prices are high, and hence it repays more on average. In the models above, this is because a procyclical firm produces collateral/continues its investments exactly when assets are most valuable. I.e. high output and high prices are complementary, because the expected (undiscounted) repayment is high when p and y move together.[11]

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[8] Empirical evidence that the values of collateral are procyclical is in Acharya, Bharath, and Srinivasan (2007).

[9] Indeed, in the benchmark in which collateral constraints do not depend on output, the debt capacity formula comprises only this term (equation (3)).

[10] This effect is ubiquitous in asset pricing—it is what the CAPM is all about. For corporate finance applications, see, e.g., Almeida and Philippon (2007).

[11] To see this another way, use the standard covariance formula to write $\mathbb{E}[py] = \mathbb{E}[p]\,\mathbb{E}[y] + \text{Cov}[p, y]$. Indeed, this is basically the decomposition in Proposition 1 for the special case of a constant SDF: if $M \equiv 1/R_f$,

$$\begin{aligned}\mathbb{E}[Mpy] &= \frac{1}{R_f}\mathbb{E}[py]\\ &= \frac{1}{R_f}\mathbb{E}[y]\;\mathbb{E}[p] + \frac{1}{R_f}\text{Cov}[p, y].\end{aligned}$$

3 Procyclical Promises vs. Discount Rates

The analysis above suggests that the effect of cyclicality on debt capacity involves a trade-off between the discount rate term and the procyclical promises term. Below, I explore how important each term is likely to be. To do this, I suppose that the SDF M and the price p are determined in a neoclassical economy, and I consider a "small" constrained firm with no impact on M or p. I ask whether it can borrow more if it is acyclical or procyclical (where cyclicality is defined as the correlation between its output y and aggregate productivity). I show that the procyclical promises term can be just as important as the discount rate term, and can be even more important in some circumstances. Along the way, I also show formally that the "nuisance term" ϵ in Proposition 1 is typically close to zero (Lemma 5).

3.1 A single capital asset. Here, I consider a neoclassical economy with a single durable capital asset that serves as the sole input of production. I assume that there is a representative firm that produces a perishable consumption good and a representative consumer who consumes it. To capture the changing state of the economy, I assume that the firm's productivity A_t is random: high A_t represents a boom and low A_t a recession. To understand how cyclicality affects debt capacity, I introduce a small constrained firm and I ask whether it can borrow more if its output moves with A_t or against it. I want to address this question directly with the debt capacity formula ($\text{DC} = \mathbb{E}\left[Mpy\right]$). To do so, I put some structure on the representative firm's production technology and the consumer's utility. That way, I can calculate the asset price p and the SDF M explicitly.

I assume the firm has a technology that uses K_t at date t to produce $A_{t+1}F(K_t)$ at date $t+1$. (It produces only the consumption good, so the capital stock is constant, $K_t \equiv K$.) As usual, the rental price of capital is given by its (discounted) marginal productivity,[12]

[12]The firm's problem is to maximize it's expected discounted profit:

$$\text{maximize } \mathbb{E}\left[\frac{A_{t+1}F(K_t)}{R_f} - R_{k,t}K_t\right],$$

$\mathbb{E}_t\left[A_{t+1}F'(K)\right]/R_f$.

I assume that the consumer has log utility and discounts the future at rate $1/R_f$. Since the consumption good is perishable, he consumes all the output, $C_t = Y_t$. These assumptions allow me to compute the SDF, which is his marginal rate of substitution,

$$M_{t,t+1} = \frac{u'(C_{t+1})}{R_f\, u'(C_t)} \tag{11}$$

$$= \frac{C_t}{R_f C_{t+1}} \tag{12}$$

$$= \frac{Y_t}{R_f Y_{t+1}}. \tag{13}$$

To solve for the asset prices and the SDF explicitly, I suppose that productivity is constant except for a (rationally anticipated) one-off shock at some date $t^*{+}1$. As I show in the proof of Lemma 4 below, this makes it easy to solve for asset prices and the SDF. Just the perpetuity formula and the expression in equation (13) imply that the price of assets is proportional to the productivity and the SDF is inversely proportional to it: $p = p_{t^*+1} = a_0 A_{t^*+1}$ and $M = M_{t^*,t^*+1} = a_1/A_{t^*+1}$, for constants a_0 and a_1.

Now, with p and M in hand, I turn to a small constrained firm that makes an investment at date t^* and produces output y at the next date. Is its debt capacity higher if it is procyclical—y and A_{t^*+1} are positively correlated—or acyclical—y and A_{t^*+1} are uncorrelated? The debt capacity formula $\mathrm{DC} = \mathbb{E}\left[Mpy\right]$ is designed to answer this question. Here, it says that the debt capacity does not depend on cyclicality at all, since Mp is constant ($Mp \equiv a_0 a_1$):

LEMMA 4. *In the neoclassical economy with log utility and a single capital asset described above, we have that*

$$\mathrm{DC} \equiv \mathbb{E}\left[Mpy\right] = a_0 a_1 \mathbb{E}\left[y\right], \tag{14}$$

where $R_{k,t}$ is the rental price of capital. The FOC gives the equation in the text. Cf., e.g., Acemoglu (2009), Ch. 2.

where

$$a_0 = \frac{F'(K)}{R_f - 1} \quad and \quad a_1 = \frac{Y_{t^*}}{R_f F(K)}. \tag{15}$$

I.e. DC does not depend on the cyclicality of output y.

Recall that the decomposition in Proposition 1 points to two offsetting effects of procyclicality. On the one hand, the SDF goes down as productivity goes up—higher productivity yields higher output and consumption, hence lower marginal utility. But, on the other hand, the price of capital goes up as productivity goes up—higher productivity means more valuable capital, hence higher prices. The result above implies that these effects cancel out perfectly in the set-up with log utility and permanent shocks.

This set-up also allows me to calculate the nuisance term ϵ explicitly and show that it is almost zero. We are not losing anything by focusing mainly on the discount rate and the procyclical promises terms in the decomposition:

Lemma 5. *In the neoclassical economy with log utility and a single capital asset described above, we have that*

$$\mathrm{DC} \approx \frac{1}{R_f}\mathbb{E}\left[y\right]\mathbb{E}\left[\,p\,\right] + \mathbb{E}\left[p\right]\mathrm{Cov}\left[M, y\right] + \frac{1}{R_f}\,\mathrm{Cov}\left[p, y\right]. \tag{16}$$

I.e., in Proposition 1, $\epsilon \approx 0$.

This set-up is also useful because it allows me to relate my decomposition to the beta decomposition in Campbell and Vuolteenaho (2004), which splits a firm's CAMP beta up into two betas, one reflecting its sensitivity to aggregate discount rate news and another to aggregate cash flow news. Here, aggregate discount rates are, of course, represented by M. Hence, my discount rate term $\mathrm{Cov}\left[M, y\right]$ captures the sensitivity to discount rate news. Here, aggregate cash flows are represented by the total output Y_{t+1}. Now, given a single shock, aggregate capital prices p are directly proportional to aggregate cash flows, as discussed above. Hence, my procyclical promises term $\mathrm{Cov}\left[p, y\right]$ captures the sensitivity to aggregate cash flow news. With this interpretation, my model says that firms with higher discount rate betas should have lower debt capacity, whereas firms with higher cash flow betas should have higher debt capacity. Hence, I offer an explanation for the empirical finding that corporate leverage seems to be decreasing in discount rate beta, but increasing in cash-flow beta (Campbell, Polk, and Vuolteenaho (2010), Ellahie (2017), and Maia (2010)). Further, since my results are about constraints on leverage (debt capacity), whereas these empirical findings are about leverage itself, my model suggests they are likely to be the result of financial constraints.

3.2 Many capital assets. Now consider a set-up in which everything is as above, except there is a large number I of capital assets, indexed by $i \in \{1, ..., I\}$. To simplify things, I assume that all assets have the same aggregate supply K and production functions have the same shape; they differ only in their productivity: in notation, asset i's output $Y^i_{t+1} = A^i_{t+1}F(K)$ depends on the asset i only via the productivity A^i_{t+1}. As above, there is a single productivity shock realized at date $t^* + 1$. Here, I assume each asset's productivity has aggregate and idiosyncratic components, i.e. $A^i_{t^*+1} = A_{t^*+1} + \varepsilon^i_{t^*+1}$, where $\varepsilon^i_{t^*+1}$ is independent noise. Finally, I assume that the assets are substitutes; each is used separately, and total output is just the sum of the output of each asset, $Y_{t+1} = \sum_{i=1}^{I} Y^i_{t+1}$. Under these assumptions, the expressions for the rental price of capital and the SDF are just as in the single-asset case above (with rental price of each asset indexed by its own expected productivity).

Now return to a small firm that makes a one-period investment at date t^*. Suppose that it uses a single capital asset j and that its output y is proportional to the productivity of this asset plus (independent) noise: $y = a(A^j_{t^*+1} + \varepsilon^y)$, for some constant a. Can the procyclical promises term ever dominate the discount rate term? Yes, if the asset j that the firm uses is sufficiently volatile:

LEMMA 6. *In the neoclassical economy with log utility and many capital assets described* *above, the discount rate term for a firm that uses asset j is approximately*[13]

$$\mathbb{E}\left[p^j_{t^*+1}\right] \mathrm{Cov}\left[M, y\right] \approx -\beta_0 \mathrm{Var}\left[A_{t^*+1}\right] \tag{17}$$

and the procyclical promises term for a firm that produces good j is

$$\frac{1}{R_f}\mathrm{Cov}\left[p^j, y\right] = \beta_1\Big(\mathrm{Var}\left[A_{t^*+1}\right] + \mathrm{Var}\left[\varepsilon^j_{t^*+1}\right]\Big) \tag{18}$$

where

$$\beta_0 = \frac{aY_{t^*}\mathbb{E}\left[A^j_{t^*+1}\right] F'(K)}{R_f(R_f-1)\mathbb{E}\left[A_{t^*+1}\right]^2 F(K) I} \quad \textit{and} \quad \beta_1 = \frac{aF'(K)}{R_f(R_f-1)}. \tag{19}$$

Hence, the procyclical promises term dominates the discount rate term whenever $\mathrm{Var}\left[\varepsilon^j_{t^*+1}\right]$ *is large, i.e. whenever the output of good j is volatile.*

Recall, yet again, that there is a trade-off to procyclicality. A procyclical firm makes repayments in booms when marginal utility is low (the discount rate term), but can commit to make larger repayments in these states since asset prices are high (the procyclical promises term). The last result says that if the price of a firm's assets moves around a lot—so the covariance between y and p^j is high—then the procyclical promises term can dominate the discount rate term. It also points to a subtlety in the procyclical promises term: it may not be the covariance with the aggregate price index that matters, but the covariance with the price of the specific capital asset used in production. This suggests that there could be a benefit to being in a volatile industry (in which the price of capital varies a lot over the cycle).

Whereas volatile prices make the procyclical promises term relatively more important, a volatile SDF makes the discount rate term relatively more important. Indeed, securities prices suggest that the SDF is volatile empirically (Hansen and Jagannathan (1991)). In representative-agent asset pricing models, capturing this typically requires a utility function

[13]The formula is not exact because I approximate the sample average of firms' output the expected value and I approximate $1/A$ with its first-order Taylor expansion.

with a lot of curvature (risk aversion). As such, it seems probable that my analysis with log utility could understate the importance of the discount rate term.[14] That said, since I use the SDF to price capital assets, not financial assets, it is not completely clear that this asset pricing literature is the right benchmark, especially in light of the equity volatility puzzle.[15] Indeed, I think log utility is one suitable benchmark to show how important the procyclical promises term can be. Moreover, Martin (2017) argues it is also a suitable benchmark for asset pricing. Taking the perspective of an unconstrained investor fully invested in the market, he concludes that log utility approximately rationalizes a fundamental relationship between options prices and realized returns.[16]

4 Procyclical Promises in Equilibrium

So far, I have stressed the trade-off between my procyclical promises term and the standard discount rate term. Now, I zero in on the procyclical promises term by assuming that the discount rate term is zero (i.e. investors are risk-neutral). Unlike in the previous section, in which I ask how a firm's cyclicality affects its borrowing constraints taking the aggregate outcomes as given, here I ask how these borrowing constraints affect aggregate outcomes themselves. To this end, I present a dynamic equilibrium model in which there are two types of firms (or "entrepreneurs"), one of which is more procyclical than the other. To understand how this difference in cyclicality affects asset prices, I assume that entrepreneurs

[14] Indeed, Schwert and Strubalaev (2014) find that firms with higher asset betas are more levered (after controlling for asset volatility). This could be because procyclical firms are able to borrow less, as would be the case if the discount rate term were more important than the procyclical promises term. But it could also be because they choose to borrow less, as would be the case if the discount rate term made it expensive for procyclical firms to borrow, even if unconstrained. (Distinguishing between these possibilities is a step beyond what I do here; I study leverage limits—debt capacity—but not yet optimal leverage choices.)

[15] That is, in the data, stock prices move around a lot more than consumption (see, e.g., Campbell (2003)). Hence, my simple set-up, in which capital asset prices move in lockstep with consumption by construction, is probably not well suited to quantitative stock pricing (the discussion in Subsection 4.10 below notwithstanding).

[16] Specifically, Martin (2017) calculates a bound on the market return in terms of a portfolio of equity options. He finds that the bound is approximately tight empirically, which is the case theoretically if a representative agent fully invested in the market has log utility.

make investments using different capital assets. Even though there is a lot of heterogeneity—heterogenous entrepreneurs lever up to invest in heterogeneous assets subject to heterogenous borrowing constraints—I manage to keep the model tractable by assuming that overlapping generations of short-lived entrepreneurs borrow from long-lived investors. Indeed, it admits an explicit solution in some configurations. I now proceed to investigate how procyclical promises affect aggregate investment, prices, and output in the time series and the cross section.

4.1 Environment. I set up the model in discrete time, $t \in \{..., -1, 0, 1, ...\}$. At each date, an i.i.d. state s_t is realized, assuming one of two equally likely realizations, a or b.

There is a single perishable consumption good and two durable capital assets, called α- and β-assets, each of which is in constant supply K. The state-s price of asset $\tau \in \{\alpha, \beta\}$ in terms of the consumption good is denoted by p_s^τ, and its average is denoted by $\bar{p}^\tau := \mathbb{E}\,[\,p^\tau] \equiv (p_a^\tau + p_b^\tau)/2$.

There are overlapping generations of two types of risk-neutral entrepreneurs, α- and β-entrepreneurs. At each date, a unit continuum of each type is born and lives for two dates. When they are born, they have an endowment that depends on the state: they have a unit of the consumption good in state a and nothing in state b. (This is the only exogenous difference between the states.) When young, they borrow and invest; when old, they produce and consume. Each α-entrepreneur uses the α-asset to do a risky investment that pays off only in state a and, symmetrically, each β-entrepreneur uses the β-asset to do a risky investment that pays off only in state b. The investments are both constant returns to scale with expected return A. I.e.

$$y^{\alpha}_{t+1} = \alpha(k)(s_{t+1}) = \begin{cases} 2Ak & \text{if } s_{t+1} = a, \\ \\ 0 & \text{if } s_{t+1} = b, \end{cases} \tag{20}$$

$$y^{\beta}_{t+1} = \beta(k)(s_{t+1}) = \begin{cases} 0 & \text{if } s_{t+1} = a, \\ \\ 2Ak & \text{if } s_{t+1} = b. \end{cases} \tag{21}$$

So "α" and "β" each denote three related things: investment technologies, the entrepreneurs who operate them, and the type of capital asset they employ.

There are also long-lived deep-pocketed investors in the background. They are risk-neutral and consume at each date, discounting the future at the risk free rate R_f, which I assume is not too large relative entrepreneurs' productivity:[17]

$$R_f < AK. \tag{22}$$

Investors can lend to entrepreneurs or use either asset (but not both) to invest in a deterministic technology γ that pays off at the next date. γ has decreasing returns to scale, $\gamma' > 0$ and $\gamma'' < 0$, but is at most as productive as entrepreneurs' investments, $\gamma'(0) = A$. Since each capital asset is in constant supply K, the quantity of τ-capital that investors hold is the quantity not held by τ-entrepreneurs: in state s, investors hold $K - k^{\tau}_s$, where k^{τ}_s denotes the amount of τ-asset that each τ-entrepreneur holds in state s.[18]

4.2 First-best/Arrow–Debreu. To focus on the effects of financial constraints, I set up the environment so that not much happens in the benchmark without them: in the Arrow–Debreu/first-best outcome, entrepreneurs hold all capital, since they have the most

[17] This will ensure that entrepreneurs exhaust their capacity; I use it only in the proof of Lemma 9 in the Appendix.

[18] Note that I give one index to entrepreneur's asset holding, representing both the type of asset and the entrepreneur who holds it. Rather than introducing a separate notation for investors' asset holdings, I just make use of the fact they hold whatever entrepreneurs do not by market clearing.

productive investments. Hence, the expected return A on their investments coincides with the aggregate productivity. And, since productivity does not change over time, asset prices are constant. So is realized output: exactly one type of entrepreneur produces $2AK$ at each date t, α-entrepreneurs if $s_t = a$ and β-entrepreneurs if $s_t = b$. In summary, there are no time-series fluctuations in productivity, prices, or output. Moreover, there are no ex ante differences in the cross section either: expected productivity, prices, and expected output coincide for α- and β-assets/entrepreneurs.

LEMMA 7. *In the Arrow–Debreu outcome, α-entrepreneurs hold all α-assets, $k^\alpha_s = K$, and β-entrepreneurs hold all β-assets, $k^\beta_s = K$. The equilibrium has the following properties.*

(i) Aggregate productivity[19] *is constant and equal to A.*

(ii) The prices of α- and β-assets are equal and constant; they equal the price of a perpetuity that pays coupon A,

$$p^\alpha_s = p^\beta_s = \frac{A}{R_f - 1}. \tag{23}$$

(iii) Aggregate output is constant and equal to $2AK$.

4.3 Borrowing constraints. I assume that an entrepreneur's debt capacity is given by $\mathrm{DC} = \alpha_0 + \alpha_1 \mathbb{E}[Mpy]$, where α_0 and α_1 are as in Lemma 3 above.[20] I.e. the loan ℓ an

[19] The aggregate productivity in state s is

$$\left.\frac{\text{total expected output}}{\text{asset supply}}\right|_s = \frac{\mathbb{E}\left[\ \alpha\big(k^\alpha_s\big) + \gamma\big(K - k^\alpha_s\big) + \beta\big(k^\beta_s\big) + \gamma\big(K - k^\beta_s\big)\ \right]}{2K}.$$

[20] As touched on in Subsection 2.3, I adopt the interpretation that the entrepreneur can divert a fraction δ of his assets, getting private benefits $B = \delta pk$ and leaving $(1 - \delta)k$ as collateral. This interpretation of δ—"diversion" rather than "depreciation"—is convenient just because it implies the supply of assets remains constant; it does not affect anything else. Many papers use capital diversion to generate borrowing constraints (e.g., Albuquerque and Hopenhayn (2004), DeMarzo and Fishman (2007b), Rampini and Viswanathan (2010)); indeed, as formalized in DeMarzo and Fishman (2007a), it is a useful catch-all for many agency problems. That said, flagrant diversion is a serious friction itself: Mironov (2013) calculates that Russian companies syphoned off upward of ten percent of GDP in both 2003 and 2004; Akerlof and Romer (1993) describe related problems of corporate "looting" in the US.

entrepreneur gets must satisfy the constraint

$$\ell \leq \alpha_0 + \alpha_1 \mathbb{E}\left[Mpy \right]. \tag{24}$$

After substituting in for α_0 and α_1, little manipulation[21] gives

$$\ell \leq \frac{(1-\delta)\mathbb{E}\left[p\right] + \frac{\delta}{2}\mathbb{E}\left[p \,|\, \text{success}\right]}{R_f} k. \tag{25}$$

Observe that an entrepreneur's borrowing limit depends not only on the average value of his assets, $\mathbb{E}\left[p\right]$, but also on the value given that his investment succeeds (i.e. does not pay off zero): the expectation of the price given success reflects the procyclical promises term. Now, recall that α-entrepreneurs succeed only in state a and β-entrepreneurs only state b. Thus, we can simplify the borrowing constraints, since for α-entrepreneurs, $\mathbb{E}\left[p_s^\alpha \,|\, \alpha\text{-success}\right] = p_a^\alpha$ and, likewise, for β-entrepreneurs, $\mathbb{E}\left[p_s^\beta \,|\, \beta\text{-success}\right] = p_b^\beta$:

$$\ell^\alpha \leq \frac{(1-\delta)\bar{p}^\alpha + \delta p_a^\alpha/2}{R_f} k^\alpha \quad \text{and} \quad \ell^\beta \leq \frac{(1-\delta)\bar{p}^\beta + \delta p_b^\beta/2}{R_f} k^\beta. \tag{26}$$

4.4 Equilibrium. I now move on to study the competitive Markov[22] equilibrium subject to the borrowing constraints above. An equilibrium thus constitutes a capital allocation k_s^τ, an amount borrowed ℓ_s^τ and associated repayments $\big(T_s^\tau(a), T_s^\tau(b)\big)$ for each type of entrepreneur $\tau \in \{\alpha, \beta\}$ in each state $s \in \{a, b\}$ at the next date[23] such that the following

[21]First, substitute in for α_0 and α_1 from Lemma 3, noting that A in Lemma 3 is replaced by $2A$, since the investment payoffs are defined slightly differently in this section; then, manipulate using $M \equiv 1/R_f$ (by risk neutrality) and $y = 2Ak\mathbb{1}_{\text{succ}}$ (by the definitions above):

$$\begin{aligned} \text{DC} &= (1-\delta)\mathbb{E}\left[Mp\right]k + \frac{\delta}{2A}\mathbb{E}\left[Mpy\right] \\ &= \frac{(1-\delta)\mathbb{E}\left[p\right]k}{R_f} + \delta\mathbb{E}\left[p\mathbb{1}_{\text{succ}}\right]k. \end{aligned}$$

From here, the Law of Total Expectation, $\mathbb{E}\left[p\mathbb{1}_{\text{succ}}\right] = \mathbb{P}\left[\text{success}\right]\mathbb{E}\left[p|\,\text{success}\right]$, gives equation (25).

[22]I have already implicitly restricted attention to Markov allocations, since I use the state s_t, rather than the entire history, to index the variables above.

[23]I focus on one-period contracts. This is without loss without of generality not only in my environment, in which entrepreneurs live for only two dates, but also in general environments with limited enforcement

hold.

(i) Entrepreneurs' maximize expected consumption subject to their borrowing constraints above (equation (26)) and their budget constraints

$$1 + \ell_a^\tau = p_a k_a^\tau \quad \text{and} \quad \ell_b^\tau = p_b k_b^\tau \tag{27}$$

(since the entrepreneurs' initial wealth is one in state a and zero in state b).

(ii) Investors are indifferent at the margin among consuming, lending, and investing (this is tantamount to market clearing here, given investors are deep pocketed and risk neutral):[24]

$$R_f = \frac{\mathbb{E}\left[\,T^\tau(s_{t+1}) \mid s_t\right]}{\ell_{s_t}^\tau} = \frac{\gamma'\big(K - k_{s_t}^\tau\big) + \mathbb{E}\left[\,p_{s_{t+1}}^\tau \mid s_t\right]}{p_{s_t}^\tau}. \tag{MC}$$

In words, this condition (MC) says that investors' marginal rate of substitution, which is just R_f here, is equal to marginal expected return from lending, and also equal to their marginal expected return from investing.

4.5 Time series fluctuations. Unlike the unconstrained entrepreneurs in the Arrow–Debreu benchmark, the constrained entrepreneurs here can invest more when they have more wealth to scale up. Thus, entrepreneurs' asset holdings are higher in state a, when they have some initial wealth, than in b, when they do not (in fact, they hold no capital at all in b, given their endowments are normalized to zero and hence they have nothing to pledge). This increased demand for capital drives prices up in state a, so asset prices are higher in a than b. Moreover, since entrepreneurs have the most productive investments, capital is better allocated when they have more of it. Hence, average productivity is higher in a than b. In these senses, state a is a "boom" and state b is a "recession." (Consistent with this interpretation, I also find that aggregate output is higher in a than b. I defer discussing this,

and no exclusion, as shown in Rampini and Viswanathan (2010). This suggests my results are not special to short-lived borrowers.

[24] I show this equivalence formally in an earlier version. I omit here it to streamline the analysis.

since it depends on the cross-sectional differences between α- and β-entrepreneurs, which I do not get into until the next subsection.)

PROPOSITION 2. *State a is a "boom" and state b is a "recession" in the sense that the following statements hold.*

(i) Aggregate productivity is higher in state a than in state b.

(ii) The prices of both capital assets are higher in state a than in state b, $p_a^\alpha > p_b^\alpha$ and $p_a^\beta > p_b^\beta$.

(iii) Aggregate output is higher in state a than in state b.

Since individual productivity and asset supply do not change over time, these fluctuations are entirely the result of assets being better allocated in state a than b—they are pure "allocation cycles." Aggregate productivity goes up in booms when assets go to their best use and down in recessions when they do not. So does output, even though larger individual investments are less productive (due to decreasing returns to scale). What Basu and Fernald (2001) call an "essential feature of business cycles" (p. 225)—output and productivity moving together—arises here just because asset allocation is procyclical, something Eisfeldt and Rampini (2006) find empirical support for. Moreover, Hsieh and Klenow (2009) find that asset allocation is a major driver of output, suggesting my allocation cycles could be a first-order contributor to real-world business cycles.

The procyclicality of aggregate asset allocation is the result of the procyclicality of individual firms' debt capacity. This is consistent with evidence on procyclical leverage in Begenau and Salomao (2014), Korajczyk and Levy (2003), and Korteweg and Strebulaev (2015).[25] Further, I find that procyclical debt capacity translates into procyclical investment, consistent with evidence in Dangl and Wu (2015).

[25]Halling, Yu, and Zechner (2016) point out that the empirical findings in Korteweg and Strebulaev (2015) and Korajczyk and Levy (2003) capture only the direct effect of the business cycle on leverage, but do not take into account how leverage determinants change over the business cycle.

4.6 Cross-sectional variation and the collateral premium. Since state a is a boom and state b a recession (Proposition 2), α-entrepreneurs are procyclical and β-entrepreneurs countercyclical (cf. their investment technologies (20)–(21)). Hence, due to procyclical promises, α-entrepreneurs can lever up and invest more than β-entrepreneurs (cf. the borrowing constraints (26)). This drives up the price of α-assets relative to β-assets. In other words, α-assets are more expensive than β-assets, because you can borrow more against them—they trade at a collateral premium.

PROPOSITION 3. *Procyclical (α-) assets trade at a premium over countercyclical (β-) assets: $p_s^\alpha > p_s^\beta$ for $s \in \{a, b\}$.*

The specific mechanism connecting entrepreneurs' demand to asset prices goes through investors' production function γ. Since entrepreneurs can lever up relatively more against procyclical assets, investors are left holding few of them. Thus, since γ has decreasing returns to scale, their marginal productivity of these procyclical assets is relatively high. This marginal productivity sets the price, resulting in a high price of procyclical assets relative to countercyclical assets. Thus, even though prices are set by investors (who are marginal), cross-sectional price differences reflect differences in the borrowing constraints of entrepreneurs (who are infra-marginal).

This argument hinges on the assumption that assets are not freely redeployable across sectors, as in, e.g., Shleifer and Vishny (1992):[26] here, investors use either α- or β-assets, but not both. If, to the contrary, investors used both types of assets—i.e. their output were a function $\tilde{\gamma}$ of their total asset holdings of both assets, rather than the function γ of each asset individually—then there would be no collateral premium, since in equilibrium both assets would have the same marginal productivity $\tilde{\gamma}'$. Hence, assets that are hard to redeploy could

[26] Shleifer and Vishny (1992) stress the empirical relevance of this assumption, saying that

> Unfortunately, most assets in the world are quite specialized and, therefore, are not redeployable. Oil rigs, brand name food products, pharmaceutical patents, and steel plants have no reasonable uses other than the one they are destined for. When such assets are sold, they have to be sold to someone who will use them in approximately the same way (p. 1344).

exhibit a higher collateral premium.

This finding is not in conflict with the widespread idea that redeployable assets are the best collateral. It just suggests that this might not lead to a premium in their prices, as their marginal valuation could already reflect their efficient use. To see why, consider the stylized example of a vineyard used to grow grapes for wine and a farm used to grow barely and hops for beer. Since wine is procyclical and beer is acyclical, the vineyard could represent an α-asset and the farm a β-asset. Now, since the vineyard is only useful to nearby winemakers, an increase in their debt capacity could increase demand enough to drive up its price, i.e. to generate the collateral premium. In contrast, since the farm is useful to many different kinds of farmers, an increase in brewers' debt capacity is unlikely to move its price. Hence, even if the farm is better collateral than the vineyard in absolute terms, the vineyard's price reflects its collateral value more. The reason is that financial constraints affect demand more for vineyards than for farmland.

4.7 A comment on policy. Since α-entrepreneurs' investments are larger than β-entrepreneurs', output is highest when they succeed, i.e. output is greater in state a than in state b, as I already mentioned (Proposition 2). This suggests one unusual policy implication: transferring wealth from countercyclical entrepreneurs to procyclical entrepreneurs, a policy that amplifies output fluctuations, can boost welfare.[27] It helps because the more procyclical an entrepreneur is, the more he can borrow to buy assets and scale up his investment, and hence the better assets are allocated. Of course, risk aversion would countervail agianst this result. Hence, I do not see it as something that policy makers should think about implementing off the shelf. Still, I think it points to benefits of procyclicality that could be worth

[27]Formally, a tax-subsidy scheme that transfers endowments from a countercyclical entrepreneur to a procyclical entrepreneur increases expected output, i.e. the entrepreneurs' expected output in state s,

$$A\big(k_s^\alpha(w_s+\epsilon)+k_s^\beta(w_s-\epsilon)\big)=2RA\left(\frac{w_s+\epsilon}{2Rp_s^\alpha-p_a^\alpha-(1-\delta)p_b^\alpha}+\frac{w_s-\epsilon}{2Rp_s^\beta-p_b^\beta-(1-\delta)p_a^\beta}\right), \tag{28}$$

is increasing in ϵ.

One caveat: this is for a transfer from one entrepreneur to another. A transfer from from all countercyclical entrepreneurs to all procyclical entrepreneurs would affect prices, something this analysis does not take into account.

taking into account, especially since the welfare costs of business cycle fluctuations appear to be quite small (e.g., Alvarez and Jermann (2004) and Lucas (1987)).

4.8 Explicit solution. Given a specific functional form for investors' production technology γ, the model admits an explicit solution:

LEMMA 8. *Let the investors' technology be $\gamma(k) = A\log(1+k)$. The equilibrium prices for $\tau \in \{\alpha, \beta\}$ are*

$$p_a^\tau = \frac{1 + K - k_a^\tau + A(2R_f - 1)(1+K)}{2R_f(R_f - 1)(1+K)(1+K-k_a^\tau)}, \tag{29}$$

$$p_b^\tau = \frac{(2R_f - 1)(1 + K - k_a^\tau) + A(1+K)}{2R_f(R_f - 1)(1+K)(1+K-k_a^\tau)}, \tag{30}$$

where, for each τ, the equilibrium asset holdings are $k_b^\tau = 0$ and

$$k_a^\tau = \frac{-a_1^\tau - \sqrt{(a_1^\tau)^2 - 4a_0 a_2^\tau}}{2a_2^\tau}, \tag{31}$$

where

$$a_0 = 4R_f^2(R_f - 1)(1+K)^2, \tag{32}$$

$$a_1^\alpha = -(1+K)\Big((2R_f - 1)\big(\delta + A(2R_f - 1)\big) - (1-\delta)A + 4R_f^2(R_f - 1)\Big), \tag{33}$$

$$a_2^\alpha = \delta(2R_f - 1), \tag{34}$$

$$a_1^\beta = -(1+K)\Big((2R_f - 1)\big(\delta + (2R_f - \delta)A + 4R_f^2(R_f - 1)\big)\Big), \tag{35}$$

$$a_2^\beta = \delta. \tag{36}$$

4.9 Comparative statics. I now turn to comparative to statics on δ, which can be interpreted as the rate of depreciation or the proportion of assets that can be diverted. The analysis implies that debt capacity is more sensitive to cyclicality for high δ (see equation (26)). Given the analysis in Section 3, this suggests the new testable prediction that the correlation between leverage and cash-flow beta should be increasing in δ, where increasing δ could be captured by (i) decreasing asset tangibility across firms or (ii) decreasing the strength of legal enforcement across countries. Further, in the model, asset prices and aggregate output are more volatile for low δ, consistent with evidence that aggregate fluctuations are negatively related to economic and financial development (King and Levine (1993), Koren and Tenreyro (2007), and Rajan and Zingales (1998)).[28]

4.10 Financial assets. So far, I have focused on how procyclical promises affect the prices of capital assets. Here, I try to say something about financial assets too. To do so, I model them in a stylized way, defining a "stock" in τ-entrepreneurs as a claim on the output of all their future generations. To find an expression for its price, recall that entrepreneurs invest only in state a (they have no wealth in state b). Hence, their investments pay off with probability 1/4—α-entrepreneurs' pay off in state a following a and β-entrepreneurs' in b following a. Thus, the average stock prices are given by the expected discounted sum of future cash flows as follows:

$$\bar{S}^{\alpha} = \mathbb{E}\left[\sum_{t=1}^{\infty} \frac{1}{4} M_t \, \alpha\left(k_a^{\alpha}\right)\right] \quad \text{and} \quad \bar{S}^{\beta} = \mathbb{E}\left[\sum_{t=1}^{\infty} \frac{1}{4} M_t \, \beta\left(k_a^{\beta}\right)\right], \tag{37}$$

where I have reintroduced the SDF M, to capture, e.g., the marginal utility of an (unmodeled) investor in the stock market. To sum the series, I separate out the time discounting by writing M in terms of the risk-neutral measure $\mathbb{Q}$. This gives

$$\bar{S}^{\alpha} = \frac{Ak_a^{\alpha}}{2(R-1)} \frac{d\mathbb{Q}}{d\mathbb{P}}(a) \quad \text{and} \quad \bar{S}^{\beta} = \frac{Ak_a^{\beta}}{2(R-1)} \frac{d\mathbb{Q}}{d\mathbb{P}}(b). \tag{38}$$

These expressions give us another way to see the trade-off between the procyclical promises and the discount rate terms. The procyclical promises term allows α-entrepreneurs to lever up, so $k_a^{\alpha} > k_b^{\beta}$ above. This pushes up the price $\bar{S}^{\alpha}$ of the α-stock. But the discount rate

[28] This echoes the results of models such as Cooley, Marimon, and Quadrini (2004), Kiyotaki and Moore (1997), and Rampini (2004) in which limited enforceability amplifies aggregate fluctuations. Unlike in these models, fluctuations in my model do not result from productivity shocks at any individual firm, but only from changes in capital allocation across firms. (Eisfeldt and Rampini (2008) explore a mechanism based on managers' private information that also leads to inefficient capital allocation in downturns.)

term should put more weight on cash flows in state a (when output is low) than in state b (when it is high), or

$$\frac{d\mathbb{Q}}{d\mathbb{P}}(a) < 1 < \frac{d\mathbb{Q}}{d\mathbb{P}}(b). \tag{39}$$

This pushes up the price $\bar{S}^{\beta}$ of the β-stock. As for debt capacity (Section 3), which term dominates for stock prices depends on the trade-off between these two effects. In this asset pricing context, the procyclical promises term may help to explain why procyclical assets do not trade at as much as a discount as models based on risk aversion alone typically suggest (Fama and French (2004)).

5 Conclusion

In this paper, I explore how cyclicality affects debt capacity. I start with the observation that there is a benefit of procyclicality, one present across a variety of models of financial frictions: procyclical firms can make higher repayments on average. The reason is that their output is high when asset prices are high, and high asset prices mitigate financial frictions, for example because they loosen collateral constraints. I show how to decompose a firm's debt capacity into a "procyclical promises term," capturing this benefit, and a "discount rate term," capturing the established cost of procyclicality, namely that procyclical firms make low repayments in recessions, when marginal utility is high and hence these repayments are most valuable. I go on to show that the procyclical promises term can be just as important as the discount rate term in some classical models. Then I consider a risk-neutral economy in which the discount rate term is zero. This allows me to study the equilibrium effects of the procyclical promises term in isolation. I find that they can matter for the macroeconomy, leading aggregate investment, prices, productivity, and output to fluctuate in the time series, even if production technologies do not. I.e. there are aggregate fluctuations, but they are pure "allocation cycles"—they arise entirely because investment goes up when procyclical firms are most productive, since procyclical firms are the firms that can lever up their investments the most. This leads to high demand for the assets they use, which generates a collateral premium on these "procyclical assets" in the cross section. My finding that procyclicality can help improve asset allocation gives a new perspective on macroeconomic policy. It suggests that making transfers to procyclical firms could actually increase welfare in some circumstances, even though it can amplify the business cycle.

A Proofs

Proof of Lemma 1

The result is immediate from the argument in the text.

Proof of Lemma 2

Debt capacity is highest if the repayment is highest in every state, i.e. if the inequality binds in equation (5). Since creditors receive that repayment only if the investment succeeds, we have that

$$\mathrm{DC} = \mathbb{E}\left[M\mathbb{1}_{\text{succ}}\left(R_f\mathbb{E}\left[Mpy\right] - \frac{B}{\pi_1 - \pi_0}\right) \middle| e = 1\right] \tag{40}$$

Now, since $\mathbb{1}_{\text{succ}}$ depends only on e (i.e. it is independent of M), we can write

$$\begin{aligned}\mathrm{DC} &= \mathbb{E}\left[\mathbb{1}_{\text{succ}}\right]\mathbb{E}\left[M\right]\left(R_f\mathbb{E}\left[Mpy\right] - \frac{B}{\pi_1 - \pi_0}\right) && (41)\\ &= \pi_1\left(\mathbb{E}\left[Mpy\right] - \frac{B}{R_f(\pi_1 - \pi_0)}\right) && (42)\end{aligned}$$

(since $\mathbb{E}\left[M\right] = 1/R_f$). This is the formula in the lemma.

Proof of Lemma 3

From equation (8), we have immediately that

$$\begin{aligned}\mathrm{DC} &= \mathbb{E}\left[M\left((1-\delta)pk + \frac{\delta py}{A}\right)\right] && (43)\\ &= (1-\delta)\mathbb{E}\left[Mp\right]k + \frac{\delta}{A}\mathbb{E}\left[Mpy\right] && (44)\\ &= \alpha_0 + \alpha_1\mathbb{E}\left[Mpy\right], && (45)\end{aligned}$$

where α_0 and α_1 are as stated in the lemma.

Proof of Proposition 1

The proof is by direct calculation. I use the following notation to streamline it: for a random variable X, $\bar{X} := \mathbb{E}[X]$ and $\Delta_X := X - \bar{X}$, so $X \equiv \bar{X} + \Delta_X$. Now:

$$\mathbb{E}[Mpy] = \mathbb{E}\left[(\bar{M} + \Delta_M)(\bar{p} + \Delta_p)y\right] \tag{46}$$

$$= \mathbb{E}\left[\bar{M}\bar{p}y + \bar{p}\Delta_M y + \bar{M}\Delta_p y + \Delta_M \Delta_p y\right] \tag{47}$$

$$= \bar{M}\bar{p}\,\mathbb{E}[y] + \bar{p}\,\mathbb{E}[\Delta_M y] + \bar{M}\,\mathbb{E}[\Delta_p y] + \mathbb{E}[\Delta_M \Delta_p y]. \tag{48}$$

From here, the lemma follows immediately from the fact that $\bar{M} = 1/R_f$ and the definition of the covariance, $\mathrm{Cov}[X, Y] \equiv \mathbb{E}[\Delta_X Y]$.[29] .

Proof of Lemma 4

The proof comprises calculating asset prices and the SDF, i.e. the constants a_0 and a_1 mentioned in the text and the lemma.

Asset prices. After the shock, the rental price of capital is constant. The price of assets is just the present value of the rental prices, and we can use the formula for a perpetuity:

$$p = p_{t^*+1} = \sum_{t=1}^{\infty} \frac{A_{t^*+1}F'(K)}{R_f^t} \tag{49}$$

$$= \frac{A_{t^*+1}F'(K)}{R_f - 1}. \tag{50}$$

So we can write $p_{t^*+1} = a_0 A_{t^*+1}$ for $a_0 := F'(K)/(R_f - 1)$.

SDF. After the shock, $t \geq t^*$, the output is $Y_t = A_{t^*+1}F(K)$ by assumption. Re-writing

[29] Note that this is equivalent to the definition $\mathrm{Cov}[X, Y] \equiv \mathbb{E}[\Delta_X \Delta_Y]$, which may be more common:

$$\begin{aligned}\mathbb{E}[\Delta_X Y] &= \mathbb{E}\left[\Delta_X(\bar{Y} + \Delta_Y)\right] \\ &= \mathbb{E}[\Delta_X]\,\bar{Y} + \mathbb{E}[\Delta_X \Delta_Y] \\ &= \mathbb{E}[\Delta_X \Delta_Y],\end{aligned}$$

since $\mathbb{E}[\Delta_X] = 0$.

equation (13) gives

$$M \equiv M_{t^*,t^*+1} = \frac{Y_{t^*}}{R_f A_{t^*+1} F(K)}. \tag{51}$$

So we can write $M = a_1/A_{t*+1}$ for $a_1 := Y_{t^*}/(R_f F(K))$.

Proof of Lemma 5

I start with the definition of ϵ in Proposition 1. I substitute $p = a_0 A$ and $M = a_1/A$ from the analysis in Section 3 to manipulate the expression for ϵ. Finally, I use the Taylor approximation to show that is close to zero.

From equation (10) and the expressions for p and M in Lemma 4, we have that

$$\begin{aligned}
\epsilon &= \mathrm{Cov}\left[\big(M - \mathbb{E}\,[M]\,\big)\big(p - \mathbb{E}\,[p]\,\big), y\right] && (52)\\
&= \mathrm{Cov}\left[\left(\frac{a_1}{A} - \mathbb{E}\left[\frac{a_1}{A}\right]\right)(a_0 A - \mathbb{E}\,[a_0 A])\,, y\right] && (53)\\
&= a_0 a_1 \mathrm{Cov}\left[\left(\frac{1}{A} - \mathbb{E}\,[1/A]\right)(A - \mathbb{E}\,[A])\,, y\right] && (54)\\
&= a_0 a_1 \mathrm{Cov}\left[1 - \mathbb{E}\,[1/A]\,A - \frac{1}{A}\mathbb{E}\,[A] + \mathbb{E}\,[1/A]\,\mathbb{E}\,[A]\,, y\right]. && (55)
\end{aligned}$$

By the linearity of the covariance and the fact that the covariance of anything and a constant is zero, this expression simplifies to

$$\epsilon = a_0 a_1 \Bigg(- \mathbb{E}\,[1/A]\,\mathrm{Cov}\,[A, y] - \mathbb{E}\,[A]\,\mathrm{Cov}\left[\frac{1}{A}\,, \,y\right]\Bigg). \tag{56}$$

Now, I use the first-order Taylor approximation of $1/A$ centered around the mean $\mathbb{E}\,[A]$:

$$\begin{aligned}
\frac{1}{A} &\approx \frac{1}{\mathbb{E}\,[A]} + \left(\frac{1}{A}\right)'\Bigg|_{A=\mathbb{E}[A]} (A - \mathbb{E}\,[A]) && (57)\\
&= \frac{1}{\mathbb{E}\,[A]} - \frac{1}{(\mathbb{E}\,[A])^2}\big(A - \mathbb{E}\,[A]\big) && (58)\\
&= \frac{2}{\mathbb{E}\,[A]} - \frac{A}{(\mathbb{E}\,[A])^2}. && (59)
\end{aligned}$$

This implies $\mathbb{E}[1/A] \approx 1/\mathbb{E}[A]$. Using this and returning to ϵ gives:

$$\epsilon \approx a_0 a_1 \left(-\frac{1}{\mathbb{E}[A]} \mathrm{Cov}[A, y] - \mathbb{E}[A] \, \mathrm{Cov}\left[\frac{2}{\mathbb{E}[A]} - \frac{A}{(\mathbb{E}[A])^2}, y \right] \right) = 0. \tag{60}$$

Proof of Lemma 6

I first compute the discount rate term and then the procyclical promises term. Below, I use the approximations that

$$\frac{1}{A} \approx \frac{2}{\mathbb{E}[A]} - \frac{A}{\mathbb{E}[A]^2} \tag{61}$$

(cf. the proof of Lemma 5 above) and

$$\frac{1}{I} \sum_{i=1}^{I} \varepsilon^i \approx 0, \tag{62}$$

where I have omitted the $t^* + 1$ subscripts, as I will do throughout.

I start with the discount rate term:

$$
\begin{aligned}
\mathbb{E}\left[p^j\right]\mathrm{Cov}\left[M,y\right] &= \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\mathrm{Cov}\left[M,y\right] && (63)\\
&= \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\mathrm{Cov}\left[\frac{Y_{t^*}}{R_f Y},y\right] && (64)\\
&= \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\cdot\frac{Y_{t^*}}{R_f}\mathrm{Cov}\left[\frac{1}{\sum_{i=1}^I A^i F(K)},a\big(A+\varepsilon^j+\varepsilon^y\big)\right] && (65)\\
&= \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\cdot\frac{Y_{t^*}}{R_f}\cdot\frac{a}{F(K)}\mathrm{Cov}\left[\frac{1}{\sum_{i=1}^I A^i},\big(A+\varepsilon^j+\varepsilon^y\big)\right] && (66)\\
&\approx \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\cdot\frac{Y_{t^*}}{R_f}\cdot\frac{a}{F(K)}\mathrm{Cov}\left[\frac{1}{IA},\big(A+\varepsilon^j+\varepsilon^y\big)\right] && (67)\\
&\approx \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\cdot\frac{Y_{t^*}}{R_f}\cdot\frac{a}{F(K)}\cdot\frac{1}{I}\mathrm{Cov}\left[\frac{2}{\mathbb{E}\left[A\right]}-\frac{A}{\mathbb{E}\left[A\right]^2},\big(A+\varepsilon^j+\varepsilon^y\big)\right] && (68)\\
&\approx \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\cdot\frac{Y_{t^*}}{R_f}\cdot\frac{a}{F(K)}\cdot\frac{1}{I}\left(-\frac{1}{\mathbb{E}\left[A\right]^2}\right)\mathrm{Cov}\left[A,\big(A+\varepsilon^j+\varepsilon^y\big)\right] && (69)\\
&= \frac{\mathbb{E}\left[A^j\right]F'(K)}{R_f-1}\cdot\frac{Y_{t^*}}{R_f}\cdot\frac{a}{F(K)}\cdot\frac{1}{I}\left(-\frac{1}{\mathbb{E}\left[A\right]^2}\right)\Big(\mathrm{Cov}\left[A,A\right]+\mathrm{Cov}\left[A,\varepsilon^j\right]+\mathrm{Cov}\left[A,\varepsilon^y\right]\Big) && (70)\\
&= -\beta_0\mathrm{Var}\left[A\right], && (71)
\end{aligned}
$$

having used $\mathrm{Cov}\left[A,\varepsilon^j\right]=\mathrm{Cov}\left[A,\varepsilon^y\right]=0$ and substituted β_0 as defined in equation (19).

Now I turn to the procyclical promises term:

$$
\begin{aligned}
\frac{1}{R_f}\mathrm{Cov}\left[p^j,y\right] &= \frac{1}{R_f}\mathrm{Cov}\left[\frac{A^jF'(K)}{R_f-1},a\big(A+\varepsilon^j+\varepsilon^y\big)\right] && (72)\\
&= \frac{1}{R_f}\cdot\frac{aF'(K)}{R_f-1}\mathrm{Cov}\left[A^j,A+\varepsilon^j+\varepsilon^y\right] && (73)\\
&= \frac{1}{R_f}\cdot\frac{aF'(K)}{R_f-1}\mathrm{Cov}\left[A+\varepsilon^j,A+\varepsilon^j+\varepsilon^y\right] && (74)\\
&= \frac{1}{R_f}\cdot\frac{aF'(K)}{R_f-1}\Big(\mathrm{Cov}\left[A,A\right]+2\mathrm{Cov}\left[A,\varepsilon^j\right]+\mathrm{Cov}\left[\varepsilon^j,\varepsilon^j\right]+\mathrm{Cov}\left[\varepsilon^j,\varepsilon^y\right]\Big) && (75)\\
&= \beta_1\left(\mathrm{Var}\left[A\right]+\mathrm{Var}\left[\varepsilon^j\right]\right), && (76)
\end{aligned}
$$

having used $\text{Cov}\left[A, \varepsilon^j\right] = \text{Cov}\left[\varepsilon^j, \varepsilon^y\right] = 0$ and substituted β_1 as defined in equation (19).

Proof of Lemma 7

The proof follows from the fact that in the first-best each capital asset must be put to its most productive use. Since entrepreneurs' are always more productive than investors, α-entrepreneurs hold all α-assets and β-entrepreneurs hold all β-assets or $k_s^\alpha = K$ and $k_s^\beta = K$ in both states. The three statements in the proposition all follow:

Statement (i). Given that $k_s^\alpha = K$ and $k_s^\beta = K$, the aggregate productivity (see footnote 19) is

$$\frac{\mathbb{E}\left[\alpha\big(k_{st}^\alpha\big) + \gamma\big(K - k_{st}^\alpha\big) + \beta\big(k_{st}^\beta\big) + \gamma\big(K - k_{st}^\beta\big)\right]}{2K} = \frac{\mathbb{E}\left[\alpha\big(K\big) + \beta\big(K\big)\right]}{2K} = A \tag{77}$$

since $\gamma(0) = 0$ (i.e. output is zero if investment is zero) and $\mathbb{E}\left[\tau(k)\right] = Ak$ for both investments $\tau \in \{\alpha, \beta\}$.

Statement (ii). The prices of α- and β-assets are determined such that investors are indifferent between consuming and buying assets (cf. the pricing equation (MC) in Subsection 4.4). Entrepreneurs hold all the assets, $k_s^\tau = K$, so $\gamma'(0) = A$; hence,

$$R_f = \frac{\gamma'\big(K - k_s^\tau\big) + \bar{p}^\tau}{p_s^\tau} \tag{78}$$

$$= \frac{A + \bar{p}^\tau}{p_s^\tau} \tag{79}$$

for both assets and both states. Substituting in $\bar{p}^\tau = (p_a^\tau + p_b^\tau)/2$ and rearranging implies that p_a^τ and p_b^τ solve the following system:

$$\begin{cases} (2R_f - 1)p_a^\tau = 2A + p_b^\tau, \\ (2R_f - 1)p_b^\tau = 2A + p_a^\tau. \end{cases} \tag{80}$$

Solving gives $p_s^\tau = A/(R_f - 1)$ for $\tau \in \{\alpha, \beta\}$ and $s \in \{a, b\}$.

Statement (iii). Expected output is

$$\text{total expected output} = \mathbb{E}\left[\alpha\big(k_s^\alpha\big) + \gamma\big(K - k_s^\alpha\big) + \beta\big(k_s^\beta\big) + \gamma\big(K - k_s^\beta\big)\right] = 2AK. \tag{81}$$

$\square$

Proof of Proposition 2

To prove the proposition, I first analyze entrepreneurs' borrowing and investment behavior. I prove that entrepreneurs borrow to capacity (Lemma 9). This allows me to solve for the entrepreneurs' asset holdings, k_s^τ for $\tau \in \{\alpha, \beta\}$ and $s \in \{a, b\}$ (Lemma 10).

LEMMA 9. *Entrepreneurs borrow to capacity, $\ell = DC$, i.e. the borrowing constraints in equation (26) bind.*

Proof. I prove this result in two steps. First, I show that the first-best outcome is not attained. This implies an upper bound on prices: $p_s^\tau < (A + \bar{p}^\tau)/R_f$. Second, I show that given prices are below this bound, entrepreneurs wish to scale up their investments as much as possible, so their borrowing constraints bind. Intuitively, since entrepreneurs' investments are highly productive, they borrow as much as they can to invest as much as they can. Note that the meat of the argument is in Step 2; Step 1 is all about making sure an inequality in Step 2 is strict.

Step 1: First best not attained. This says that entrepreneurs do not hold all the capital, as they would in the first best (Lemma 7): $k_s^\tau < K$ for both types of entrepreneurs $\tau \in \{\alpha, \beta\}$ in both states $s \in \{a, b\}$.

To prove the result, start by writing DC from equation (26) as

$$\text{DC}^\tau = \frac{(1-\delta)\bar{p}^\tau + \frac{\delta}{2} p_{\text{succ}}^\tau}{R_f} k^\tau, \tag{82}$$

where p^τ_{succ} denotes the price of τ-assets given τ-entrepreneurs succeed—i.e. $p^\alpha_{\text{succ}} = p^\alpha_a$ and $p^\beta_{\text{succ}} = p^\beta_b$.

Now, suppose (in anticipation of a contradiction) that an entrepreneur holds all the assets in state s, $k^\tau_s = K$ for $\tau \in \{\alpha, \beta\}$. Thus, a τ-entrepreneur's budget constraint implies

$$K = \frac{w_s + \ell^\tau_s}{p^\tau_s} \tag{83}$$

where w_s denotes his endowment in state s ($w_a = 1$ and $w_b = 0$). Since $\ell^\tau_s \leq \text{DC}$, by definition, we have

$$K \leq \frac{w_s + \text{DC}}{p^\tau_s}. \tag{84}$$

Substituting $k^\tau = K$ into equation (82) and rearranging gives

$$K \leq \frac{R_f w_s}{R_f p^\tau_s - (1-\delta)\bar{p}^\tau - \delta p^\tau_{\text{succ}}/2} \tag{85}$$

$$\leq \frac{R_f}{R_f p^\tau_s - \bar{p}^\tau} \tag{86}$$

since $\bar{p}^\tau = (p^\tau_a + p^\tau_b)/2 \geq p^\tau_{\text{succ}}/2$. Now, the pricing equation (MC) gives an expression for the denominator above,

$$R_f p^\tau_s - \bar{p}^\tau = \gamma'(K - k^\tau_s) = \gamma'(0) = A, \tag{87}$$

since $k^\tau_s = K$ by hypothesis. Substituting this into equation (86) implies

$$K \leq \frac{R_f}{A}. \tag{88}$$

This is contradicts the assumption in equation (22). We conclude that $k^\tau_s < K$ and hence $\gamma'(K - k^\tau_s) < A$. Given this, equation (MC) implies that

$$p^\tau_s < \frac{A + \bar{p}^\tau}{R_f}. \tag{89}$$

Step 2: Entrepreneurs scale up. In state s at date t, τ-entrepreneur borrows ℓ at rate R_f to invest k to maximize his expected payoff, i.e. his expected output plus the future value of his assets minus his repayments:

$$\mathbb{E}\left[\tau(k) + p^{\tau}_{s_{t+1}}k - T(s)\right] = Ak + \bar{p}^{\tau}k - R_f\ell \tag{90}$$

subject to the borrowing constraint in equation (26) and his budget constraint

$$p^{\tau}_{s}k = w_s + \ell, \tag{91}$$

(remember w_s is the entrepreneur's endowment, $w_a = 1$ and $w_b = 0$). Substituting from the budget constraint gives the objective function

$$\begin{aligned} Ak + \bar{p}^{\tau}k - R_f\ell &= (A + \bar{p}^{\tau})\frac{w+\ell}{p^{\tau}_{s}} - R_f\ell & (92)\\ &= \left(\frac{A + \bar{p}^{\tau}}{p^{\tau}_{s}} - R_f\right)\ell + \frac{A+\bar{p}^{\tau}}{p^{\tau}_{s}}w. & (93) \end{aligned}$$

This is strictly increasing in ℓ as long as $(A + \bar{p}^{\tau})/p^{\tau}_{s} > R_f$ or

$$p_s < \frac{A + \bar{p}}{R_f}, \tag{94}$$

which is satisfied by Step 1 above (equation (89)). Hence, entrepreneurs maximize ℓ, i.e. they borrow to capacity: $\ell = \mathrm{DC}$. □

I now solve for entrepreneurs' asset holdings.

Lemma 10. *Entrepreneurs' asset holdings are given by*

$$\begin{aligned} k^{\alpha}_{a} &= \frac{2R_f}{(2R_f - 1)p^{\alpha}_{a} - (1-\delta)p^{\alpha}_{b}}, & (95)\\ k^{\beta}_{a} &= \frac{2R_f}{(2R_f - 1 + \delta)p^{\beta}_{a} - p^{\beta}_{b}}, & (96) \end{aligned}$$

and $k_b^\alpha = k_b^\beta = 0$.

Proof. Having established that entrepreneurs borrow to capacity, $\ell = \text{DC}$ (Lemma 9). Given (both types of) entrepreneurs have a unit wealth in state a and nothing in state b, their budget constraints read

$$p_a^\tau k_a^\tau = 1 + \text{DC}_a^\tau, \tag{97}$$

$$p_b^\tau k_b^\tau = \text{DC}_b^\tau. \tag{98}$$

Now, we can find k_b^τ and k_a^τ.

To find $k^\tau - b$, observe that DC is proportional to k (cf. equation (82)). Hence, the state-b budget constraint above implies that it must be that $k_b^\tau = 0$.

To find k_a^τ, substitute the expression for DC from equation (82) into the state-a budget constraint:

$$p_a^\tau = 1 + \text{DC}_a^\tau = 1 + \frac{(1-\delta)\bar{p}^\tau + \frac{\delta}{2} p_{\text{succ}}^\tau}{R_f} k_a^\tau. \tag{99}$$

Rearranging gives

$$k_a^\tau = \frac{R_f}{R p_a^\tau - (1-\delta)\bar{p}^\tau - \frac{\delta}{2} p_{\text{succ}}^\tau}. \tag{100}$$

Substituting $p_{\text{succ}}^\alpha = p_a^\alpha$ and $p_{\text{succ}}^\beta = p_b^\beta$ gives the expressions in the lemma.

□

Given Lemma 9 and Lemma 10, I turn to the statements in the proposition.

Statement (i). The fact that productivity is higher in state a than in state b follows from Lemma 10 above, which implies that both types of entrepreneurs hold more assets in state a than in state b. Aggregate productivity is higher when entrepreneurs hold more assets, since they use them more productively than investors do.

Statement (ii). The fact that prices are higher in state a than in state b follows from equation (MC) and Lemma 10 above. Equation MC says that

$$p_s^\tau = \frac{\gamma'\left(K - k_s^\tau\right) + \bar{p}^\tau}{R_f}. \tag{101}$$

Since γ' is decreasing (i.e. $\gamma'' < 0$), $p_a^\tau > p_b^\tau$ if and only if $k_a^\tau > k_b^\tau$, as is the case by Lemma 10.

Statement (iii). For the proof of statement (iii), see the proof of Proposition 3 below. There, I establish that α-entrepreneurs invest more than β-entrepreneurs in each state, i.e. $k_s^\alpha > k_s^\beta$. As a result, output in state a is greater than output in state b, since the α-technology pays off in state a and the β-technology pays off in state b.

Proof of Proposition 3

The proof is by contradiction. I proceed in three steps. In Step 1, I show that it it sufficient to compare the state-a asset holdings, i.e. the price of α-assets is higher than the price of β-assets in each state if and only if $k_a^\alpha > k_a^\beta$. In Step 2, I define the variable Δ^τ as the difference in the prices of τ-assets across states, $\Delta^\tau := p_a^\tau - p_b^\tau$. I show that there is a collateral premium—$p_s^\alpha \geq p_s^\beta$—if and only if $\Delta^\alpha \geq \Delta^\beta$. (Note that this step connects the difference in prices across states with the difference across assets.) In Step 3, I suppose that $p_a^\beta \geq p_a^\alpha$ and show that it leads to a contradiction.

Step 1. Writing equation (MC) for each state $s \in \{a, b\}$ and rearranging, we get an expression for the average price:

$$\bar{p}^\tau = \frac{\gamma'(K - k_a^\tau) + \gamma'(K - k_b^\tau)}{2(R_f - 1)}. \tag{102}$$

Now, using $k_b^\alpha = k_b^\beta = 0$ from Lemma 10 and substituting back into equation (MC) gives

$$p_a^\tau = \frac{(2R_f - 1)\gamma'(K - k_a^\tau) + \gamma'(K)}{2R_f(R_f - 1)} \tag{103}$$

and

$$p_b^\tau = \frac{\gamma'(K - k_a^\tau) + (2R_f - 1)\gamma'(K)}{2R_f(R_f - 1)}. \tag{104}$$

The expressions for p_a^τ and p_b^τ in equations (103) and (104) both depend only on k_a^τ. The fact that $\gamma'' < 0$ implies that both p_a^τ and p_b^τ are increasing in k_a^τ. Thus, the following three statements are equivalent: (i) $p_a^\alpha > p_a^\beta$, (ii) $p_b^\alpha > p_b^\beta$, and (iii) $k_a^\alpha > k_a^\beta$.

Step 2. Define Δ^τ as the difference in price across states for a τ-entrepreneur:

$$\Delta^\tau := p_a^\tau - p_b^\tau. \tag{105}$$

Equation (MC) gives an expression for Δ^τ in terms of k_a^τ,

$$\Delta^\tau = \frac{\gamma'(K - k_a^\tau) - \gamma'(K)}{R_f}, \tag{106}$$

so

$$\gamma'(K - k_a^\tau) = R_f \Delta^\tau + \gamma'(K). \tag{107}$$

With this and equation (102) from Step 1, we can compute that

$$\begin{aligned} \bar{p}^\tau &= \frac{\gamma'(K - k_a^\tau) + \gamma'(K)}{2(R_f - 1)} && (108) \\ &= \frac{2\gamma'(K) + R_f \Delta^\tau}{2(R_f - 1)}. && (109) \end{aligned}$$

From this expression, we can see that there is a collateral premium—$\bar{p}^\alpha > \bar{p}^\beta$—if and only if $\Delta^\alpha > \Delta^\beta$.

Step 3. Suppose (in anticipation of a contradiction) that $p_s^\beta \geq p_s^\alpha$. By Step 1, it must be that $k_a^\beta \geq k_a^\alpha$. Using the expressions for k_a^β and k_a^α in Lemma 10, this says that

$$\frac{2R_f}{(2R_f - 1 + \delta)p_a^\beta - p_b^\beta} \geq \frac{2R_f}{(2R_f - 1)p_a^\alpha - (1 - \delta)p_b^\alpha} \tag{110}$$

or

$$(2R_f - 1)p_a^\alpha - (1-\delta)p_b^\alpha \geq (2R - 1 + \delta)p_a^\beta - p_b^\beta \tag{111}$$

Now, eliminate p_a^β and p_a^α from this inequality using $p_a^\beta = p_b^\beta + \Delta^\beta$ and $p_a^\alpha = p_b^\alpha + \Delta^\alpha$:

$$(2R_f - 1)(p_b^\alpha + \Delta^\alpha) - (1-\delta)p_b^\alpha \geq (2R_f - 1 + \delta)(p_b^\beta + \Delta^\beta) - p_b^\beta \tag{112}$$

or

$$\big(2(R_f - 1) + \delta\big)(p_b^\alpha - p_b^\beta) \geq (2R_f - 1)(\Delta^\beta - \Delta^\alpha) + (1-\theta)\Delta^\beta. \tag{113}$$

The right-hand side is strictly positive since $\Delta^\beta > 0$ and $\Delta^\beta \geq \Delta^\alpha$ by the hypothesis that $p_a^\beta \geq p_a^\alpha$. Thus, we have that

$$p_b^\alpha - p_b^\beta > 0, \tag{114}$$

contradicting the hypothesis that $p_b^\beta \geq p_b^\alpha$. We therefore conclude that $p_b^\beta > p_b^\alpha$. □

Proof of Lemma 8

α**-entrepreneurs.** Recall $k_b^\alpha = 0$, since entrepreneurs have no endowment in state b. Hence, we have a (non-linear) system of three equations in three unknowns:

$$k_a^\alpha = \frac{2R_f}{(2R_f - 1)p_a^\alpha - (1-\delta)p_b^\alpha}, \tag{115}$$

$$(2R_f - 1)p_a^\alpha = p_b^\alpha + \frac{2A}{1 + K - k_a^\alpha}, \tag{116}$$

$$(2R_f - 1)p_b^\alpha = p_a^\alpha + \frac{2}{1 + K}. \tag{117}$$

Equations (116) and (117) are linear linear in p_a^α and p_b^α. Solving them simultaneously gives

$$p_a^\alpha = \frac{1 + K - k_a^\alpha + A(2R_f - 1)(1 + K)}{2R_f(R_f - 1)(1 + K)(1 + K - k_a^\alpha)}, \tag{118}$$

$$p_b^\alpha = \frac{(2R_f - 1)(1 + K - k_a^\alpha) + A(1 + K)}{2R_f(R_f - 1)(1 + K)(1 + K - k_a^\alpha)}. \tag{119}$$

Substituting these expressions into equation (115) gives the following equation for k_a^α:

$$\begin{aligned} \Big((2R_f - 1)\big(1 + K - k_a^\alpha + A(2R_f - 1)(1 + K)\big)\Big)k_a^\alpha \\ -(1 - \delta)\big((2R_f - 1)(1 + K - k_a^\alpha) + A(1 + K)\big)k_a^\alpha \\ = 4R_f^2(R_f - 1)(1 + K)\big(1 + K - k_a^\alpha\big). \end{aligned} \tag{120}$$

This is a quadratic equation with coefficients a_0, a_1^α, and a_2^α given in the lemma. k_a^τ in equation (31) is its solution (it is easy so see that the larger root is greater than K, making the smaller root is the relevant one, since no one can hold more than the total supply of assets in equilibrium).

β-entrepreneurs. Again, $k_b^\beta = 0$, we have a (non-linear) system of three equations in three unknowns:

$$k_a^\beta = \frac{2R_f}{(2R_f - (1 - \delta))p_a^\beta - p_b^\beta}, \tag{121}$$

$$(2R_f - 1)p_a^\beta = p_b^\beta + \frac{2A}{1 + K - k_a^\beta}, \tag{122}$$

$$(2R_f - 1)p_b^\beta = p_a^\beta + \frac{2}{1 + K}. \tag{123}$$

Equations (122) and (123) has the same form as equations (116) and (117), and solving gives the same expressions (with the α-index replaced by β). Substituting these expressions into equation (121) the following equation for k_a^β:

$$\begin{aligned}\Big(\big(2R_f-(1-\delta)\big)\big(1+K-k_a^\beta+A(2R_f-1)(1+K)\big)\Big)k_a^\beta \\ -\Big((2R_f-1)(1+K-k_a^\beta)+A(1+K)\Big)k_a^\beta \\ =4R_f^2(R_f-1)(1+K)(1+K-k_a^\beta).\end{aligned} \tag{124}$$

This is a quadratic equation with coefficients a_0, a_1^β, and a_2^β given in the lemma. k_a^τ in equation (31) is its solution (it is easy so see that the larger root is greater than K, making the smaller root is the relevant one, since no one can hold more than the total supply of assets in equilibrium).

B Extensive Form for Subsection 2.3 and Generalized Collateral Constraints

Here I set up the model with asset substitution and collateral constraints explicitly more as an extensive form game (Subsection 2.3). In this formulation, the collateral constraint results from a formal renegotiation protocol, in which the firm and its creditor Nash bargain over output in the event of default. The collateral constraint in the text (equation (2)) corresponds to the case in which the firm has all the bargaining power. Allowing for general bargaining power not only affirms the baseline results, but also generates one more testable comparative-static prediction. Here, I am also explicit that contractual repayments can depend on the aggregate state; my results are not driven by any restriction to plain debt (or other ad hoc contractual restrictions).

Extensive form. A firm and its creditor play an extensive-form game with three dates, denoted by $t=0$, $t=1/2$, and $t=1$. A firm has an investment that uses capital k_0. At $t=1/2$, it can continue or abandon its investment. The investment has the chance of paying off if the firm continues; it delivers private benefits B to the firm if it abandons. At $t=1$, the firm's debt matures, and it either repays or it defaults and renegotiates with its creditor. In more formal detail, the timing is as follows.

t=0. The firm has assets k_0 and borrows from a creditor via a contingent contract promising $T(s)$ in state s at $t = 1$.[30]

t=1/2. The firm learns whether its investment will succeed or fail and decides whether to continue or abandon it. If it continues, its assets stay in place. If it abandons, it gets private benefits B, but its assets depreciate by an amount δ. I let k_1 denote the assets the firm has after this decision:

$$k_1 = \begin{cases} k_0 & \text{if continue,} \\ (1-\delta)k_0 & \text{if abandon.} \end{cases} \tag{125}$$

t=1. If the firm has continued and its investment is successful, the firm produces output $y = Ak_0$; otherwise it produces nothing. Either way, its assets k_1 stay in place. Their price p_s depends on the random state of the world s.

Next, the firm either repays $T(s)$ to its creditor or defaults. If it defaults, the creditor can seize its assets or renegotiate the repayment.

The division of surplus in renegotiation is determined by generalized Nash bargaining, where the creditor has bargaining power ρ and disagreement payoff $p_s k_1$, reflecting his outside option of seizing the defaulting firm's capital.[31]

I assume that the firm maximizes its expected payoff, including its private benefits. The creditor maximizes its expected payoff, and discounts the future using the SDF M (this

[30]I assume that the firm consumes whatever it borrows at $t = 0$; it does not use it to invest further, which would in turn affect the payoff y, and hence the equilibrium repayments. This is just for simplicity. It does not substantially affect the results. Indeed, I allow for this reinvestment in the analysis in Section 4; see, e.g., Lemma 10.

[31]Renegotiation is a first-order friction for real-world firms (Roberts and Sufi (2009)) and liquidation values are a first-order determinant of its outcome (Benmelech and Bergman (2008)). Although we will see below that the same repayments can be implemented with or without renegotiation on the equilibrium path (cf. equation 135), the threat of renegotiation is what determines the repayments in either case.

matters only for the computation of the debt capacity at the end of this analysis; it does not affect the equilibrium behavior).

Assumptions. I make three assumptions on technologies and preferences that streamline the analysis. First, I assume the investment payoff given success is large:

$$Ak_0 \geq \frac{B + (1-\delta)p_s k_0}{(1-\rho)} \tag{126}$$

for all s. Second, I assume that the private benefits are large too,

$$B > p_s \tag{127}$$

for all s. Third, I assume that the success of the investment is independent of the aggregate state, i.e. y is independent of s.

Solution. I find the subgame perfect equilibrium of this extensive-form game. In the steps below, I derive the equilibrium by backward induction.

1. **Creditor's choice of seizure or renegotiation at t=1.**

   - If the creditor seizes the assets, it gets $p_s k$.
   - If it chooses to renegotiate, it divides the surplus with the firm according to the Nash bargaining protocol, where the disagreement point is to seize the assets. Hence, denoting the creditor's bargaining power by ρ, we can write the renegotiation payoffs at $t = 1$ as follows:

$$\text{firm's payoff } = (1-\rho)y, \tag{128}$$

$$\text{creditor's payoff } = \rho y + p_s k_1. \tag{129}$$

   (Note that for simplicity I have written the firm's payoff net of the private benefit B, which is gets at $t = 1/2$ if it abandons the investment.)

- The creditor's payoff from renegotiation is always higher than its payoff from seizing assets, so there is always renegotiation if the firm defaults.

2. **Firm's choice to default or repay at t=1.**

- If the firm repays, it gets $y + p_s k_1 - T(s)$; the creditor gets the repayment $T(s)$.
- If it defaults, we know from above that the creditor renegotiates. Hence, the firm gets $(1-\rho)y$; the creditor gets $\rho y + p_s k_1$ (from equations (128) and (128)).
- Hence, the firm defaults if $(1-\rho)y > y + p_s k_1 - T(s)$. This can be rewritten as $T(s) < \rho y + p_s k_1$, so we can write the payoffs (net of the private benefits) as

$$\text{firm's payoff } = \max\left\{ y + p_s k_1 - T(s)\,,\, (1-\rho)y \right\}, \tag{130}$$

$$\text{creditor's payoff } = \min\left\{ T(s)\,,\, \rho y + p_s k_1 \right\}. \tag{131}$$

3. **Firm's choice to continue or abandon.**

- **Success.** If the firm learns its project is going to succeed, it knows its output is $y = Ak_0$ if it continues. Hence, its payoff given by

$$\text{firm's payoff } = \begin{cases} \mathbb{E}\left[\max\left\{ Ak_0 + p_s k_0 - T(s)\,,\, (1-\rho)Ak_0 \right\}\right] & \text{if continue,} \\ B + \mathbb{E}\left[\max\left\{ p_s(1-\delta)k_0 - T(s)\,,\, 0 \right\}\right] & \text{if abandon} \end{cases} \tag{132}$$

(Note that these are the payoffs from the point of view of $t = 1/2$; hence they are gross of the private benefits B.) Given $(1-\rho)Ak_0 > B + p_s(1-\delta)k_0$ by assumption (equation (126)), the continuation payoff it always greater than the abandonment payoff (no matter T).

Hence, the firm always continues if it learns it will succeed.

- **Failure.** If the firm learns its project is failing, it knows its output is $y = 0$.

Hence, its payoff is given by

$$\text{firm's payoff } = \begin{cases} \mathbb{E}\left[\max\left\{\, p_s k_0 - T(s)\,,\, 0\,\right\}\right] & \text{if continue,} \\ B + \mathbb{E}\left[\max\left\{\, p_s(1-\delta)k_0 - T(s)\,,\, 0\,\right\}\right] & \text{if abandon.} \end{cases} \tag{133}$$

Given $B > p_s(1-\delta)k_0$ by assumption (equation (127)), the abandonment payoff it always greater than the continuation payoff (no matter T).

Hence, the firm always abandons if it learns it is failing.

- Observe that the repayment T does not affect the firm's choice to continue/abandon at $t = 1/2$ (given the assumptions in equations (126) and (127)).

4. **Equilibrium repayments.** Since T does not affect the firm's choice to continue its investment, the repayment to the creditor is given by the creditor's payoff in equation (131), with $y = Ak_0$ and $k_1 = k_0$ in the event of success and $y = 0$ and $k_1 = (1-\delta)k_0$ in the event of failure:

$$\text{creditor's payoff } = \begin{cases} \min\left\{\, T(s)\,,\, \rho A k_0 + p_s k_0\,\right\} & \text{if success,} \\ \min\left\{\, T(s)\,,\, (1-\delta)p_s k_0\,\right\} & \text{if failure.} \end{cases} \tag{134}$$

5. **Maximum repayments.** Observe that for each state s, the equilibrium repayment in equation (135) is increasing in the promised repayment $T(s)$. (This is because increasing $T(s)$ does not affect the firm's choice at $t = 1/2$, as established above, and because default does not induce a deadweight cost at $t = 1$.) Thus, the maximum possible transfer in state s, which I denote by $T^{\max}(s)$, is given by the expression in equation (135) with $T(s) \equiv \infty$:

$$T^{\max}(s) = \begin{cases} \rho A k_0 + p_s k_0 & \text{if success,} \\ (1-\delta)p_s k_0 & \text{if failure.} \end{cases} \tag{135}$$

For $\rho = 0$, this expression coincides with that in the text (equation (6)). Now, by analogy with the analysis there (equation (8)), we can rewrite it as

$$T^{\max}(s) = \mathbb{1}_{\text{succ}}\big(\rho A k_0 + p_s k_0\big) + \big(1 - \mathbb{1}_{\text{succ}}\big)(1-\delta)p_s k_0 \tag{136}$$

$$= (1-\delta)pk_0 + \frac{\delta p y}{A} + \rho y. \tag{137}$$

6. **Debt capacity.** We can take the present value of $T^{\max}(s)$ to get the debt capacity:

$$\text{DC} = \alpha_0 + \alpha_1 \mathbb{E}\left[Mpy\right] + \rho \mathbb{E}\left[My\right], \tag{138}$$

where α_0 and α_1 are as in Lemma 3.

Summary. The baseline result in Lemma 3 is the special case of this set-up in which the creditor has no bargaining power, $\rho = 0$. For $\rho > 0$ the debt capacity formula is unchanged except for an extra additive term proportional to ρ (compare equation (138) with the expression in Lemma 3). The fact that the first terms are unchanged implies that procyclical promises can matter even when creditors have a lot of bargaining power (although $\rho \to 1$ is ruled out by the assumption in equation (126)). However, this extra term depends on M, which drives the discount rate term, but not on p, which drives the procyclical promises term. This suggests that the discount rate term becomes more important relative to the procyclical promises term as ρ increases. To the extent the that the ρ measures enforceability—e.g. because it reflects creditor rights—this is in line with the analysis in the text suggesting procyclical promises are more important when enforceability is limited; see, e.g., the comparative statics on δ in Subsection 4.9.

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