## MANAGERIAL COST-CUTTING AND PRODUCT MARKET COMPETITION

Jason Roderick Donaldson\*



Keywords: Hotelling duopoly, cost-cutting

This version: November 19, 2012

## Abstract

This exercise uses Hotelling's model of duopolistic competition to demonstrate the connection between competition in the product market and managerial efficiency. It demonstrates that hiring new efficient costcutting managers may be inefficient for firms and consumers but still the unique equilibrium outcome.

Two firms compete in the product market on the Hotelling line. Leon is at the left  $(x_L = 0)$  and Roxy is at the right  $(x_R = 1)$ . Consumers are distributed uniformly between them with utility from buying the good from firm  $f \in \{L, R\}$  given by

$$u(x,f) = \bar{u} - p_f - \gamma |x_f - x| \tag{1}$$

for consumer  $x \in [0, 1]$ . Each consumer buys at most once, and assume that  $\bar{u}$  is large enough that all consumers buy.

- 1. Hotelling (1929) interpreted these preferences in terms of transportation costs.
  - (a) Explain Hotelling's interpretation.
  - (b) Provide a more general interpretation of the preferences with examples where they apply. What does  $\gamma$  represent in your examples?
  - (c) Give two examples of duopolistic product market competition that this framework does not describe well.

<sup>\*</sup>Finance Department, LSE, contact: j.r.donaldson@lse.ac.uk

- 2. (a) Demonstrate that either all the consumers prefer to buy from Leon, all from Roxy, or that there is an indifferent consumer  $x^*$  such that if  $x < x^*$  then x prefers to buy from Leon and if  $x > x^*$  then x prefers to buy from Roxy.
  - (b) Express the equilibrium  $x^*$  in terms of the prices  $p_0$  and  $p_1$
  - (c) Express the demands for each seller's good  $D_f(p_0, p_1)$  in terms of  $x^* = x^*(p_0, p_1)$ .

Leon and Roxy both have marginal cost C and no fixed cost, so firm f's profit is

$$\Pi_f = (p_f - C)D_f. \tag{2}$$

3. (a) Demonstrate that given  $p_{\neg f}$ , firm f's best response function is

$$p_f^*(p_{\neg f}) = \frac{\gamma + p_{\neg f} + C}{2} \tag{3}$$

(b) Use the Nash conditions

$$p_f^* \Big( p_{\neg f} \big( p_f \big) \Big) = p_f \tag{4}$$

for  $f \in \{L, R\}$  to determine the equilibrium prices.

- (c) Write down the equilibrium profits.
- (d) Comment on the prices and profits relative to the consumers' "transportation cost"  $\gamma$  and the firms' production costs C.

Observe that despite their competing in prices, Leon and Roxy are still making a healthy profits as long as  $\gamma$  is not too small. Each, however, has the opportunity to bring in new management that will implement the latest cost-cutting practices. If a firm employs an expert manager, its marginal cost is reduced to c = 0, but due to labour market competition it must pay him a fixed wage w > 0.

- 4. (a) Suppose both Leon and Roxy hire new managers. Show that despite the increased efficiency at both enterprises, Leon and Roxy are both strictly worse off.
  - (b) Now suppose firm f employs new management and firm  $\neg f$  does not. Recover that firm f's profits are

$$\Pi_f = \frac{\left(3\gamma + C\right)^2}{18\gamma} - w \tag{5}$$

and

$$\Pi_{\neg f} = \frac{\left(3\gamma - C\right)^2}{18\gamma}.\tag{6}$$

Leon and Roxy's decisions to shake up management are strategic, dependent on the labour market decisions of their product market competitors.

5. (a) Demonstrate that the firms' employment decisions are represented by the two-by-two game below.

Leon hire  $\frac{\gamma}{don't} = \frac{\gamma}{2} - w, \frac{\gamma}{2} - w = \frac{\left(3\gamma + C\right)^2}{18\gamma} - w, \frac{\left(3\gamma - C\right)^2}{18\gamma} = \frac{\gamma}{2}, \frac{\gamma}{2}$ 

Hire expert managers?

Roxy

Figure 1: For relatively high C, the firms are caught in a prisoners' dilemma in which they higher newly efficient management.

(b) Show that both firms hiring is the unique equilibrium whenever

$$6\gamma(3w - C) < C^2 < 6\gamma(C - 3w). \tag{7}$$

Explain the connection (equivalence) with the prisoners' dilemma. (For larger  $\gamma$  the game is a coordination game with multiple equilibria similar to Rousseau's stag hunt.)

- 6. Suppose the above condition on parameters so that that (hire, hire) is the unique equilibrium.
  - (a) Who benefits from the increased efficiency caused by the changes of management?
  - (b) If firms could collude and retain their inefficiency they would. Should competition law regulate this?
  - (c) Comment on the social benefits of MBA training.