THE TRADE-OFF BETWEEN ALIGNMENT AND INTENSITY OF BUREAUCRATIC INCENTIVES **SOLUTION**

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Abstract

This exercise is inspired by Prendergast's "The Motivation and Bias of Bureaucrats" (*AER*, 2007). Under the assumption that bureaucratic workers who are passionate about their work are likely to disagree with their employers about which decision is best, it demonstrates that a principal will hire a biased agent in order to induce him to acquire information, even though she knows he'll use it to implement a biased allocation. For a particular relationship between bias and preference intensity, it then finds the optimal level of bias from the principal's point of view.

In what follows it will be useful to recall if $\mathbf{X}$ and ε are Gauissian random variables with zero mean and variances unity and σ^2,

$$\mathbf{X} \sim \mathrm{N}\big[0, \sigma^2\big], \quad \varepsilon \sim \mathrm{N}\big[0, \Sigma^2\,\big] \tag{1}$$

then if $\mathbf{Y} = \mathbf{X} + \varepsilon = y$ then $\mathbf{X}$ is conditionally distributed normally with mean y and precision $\sigma^{-2} + \Sigma^{-2}$,

$$\mathbf{X} \,\Big|\, \mathbf{Y} = y \;\sim\; \mathrm{N}\left[y\ ,\ \frac{1}{\sigma^{-2} + \Sigma^{-2}}\right]. \tag{2}$$

Pamela is a principal in a government agency. Her job is to hire and fire bureaucrats who deal with public "clients". The role of the bureaucrat is

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to make decisions on a case-by-case basis after learning as much as possible about clients' circumstances.

Pamela, embroiled in a bureaucracy, has no incentive instruments at her disposal, so she tires to find employees who wish to see their jobs done right themselves. However, her employees' definitions of "right" often differ from hers; in fact, she has observed a trend whereby the most motivated workers are also the most opinionated, and has found that it was not always for the best. On the other hand, workers with no ideological attachments tend to be lackadaisical in their field work and end up making less informed decisions.

Given uncertainty about the client's type θ, model Pamela's preferences over the allocation with the quadratic loss utility function with preferred allocation $a = \theta$,

$$u_P(\theta, a) = -\big(\theta - a\big)^2. \tag{3}$$

Now, describe a potential worker's type by a pair (b, B) where b is his bias and B represents the intensity of his motivation so that his payoff function implies the preferred allocation $a = \theta + b$,

$$u_B(\theta, a) = -B^2\big(\theta - a + b\big)^2. \tag{4}$$

1. (a) Compare and contrast both the importance of a bureaucrat's private information and the potential for misalignment of incentives (among the bureaucrat, principal, and client) in the following government sectors:

   i. Healthcare, where the bureaucrat is a doctor and Pamela is a hospital administrator.

      - Here the doctor's private information about his patients is essential, assessing patients' conditions is the main part of many doctors' jobs.
        Incentives for major operations are aligned all round: when a patient needs heart surgery, the doctor suggests it only when it is necessary, which is when the patient wants it. For perceptions this may not be the case, as the principal must take budget considerations into consideration. Likely the principal favours cost-cutting more than the doctor more than the patient.

   ii. Airport security, where the bureaucrat is a boarder guard and Pamela is his coordinator.

      - The security guard's private information is essential, but by nature randomization is an important part of security. Cf. Becker v. Dostoyevsky.

Experience (at least in the US) tells us that airport security guards with tough personalities may be hired. While every individual client (traveller) wishes to have a lax security check for himself, those with criminal intensions suffer more than those without—if you are a terrorist the consequences of investigation are worse than if you're not, while they're inconvenient for all of us.

iii. Tax auditing, where the bureaucrat is an auditor and Pamela is the IRS.

- While the auditor's job is also to acquire information, differently from the doctor, he must rely as much on effort as expertise.
  Like the airport, the clients who are cheating on their taxes suffer more from an audit than those who are, while the interests of the principal are aligned against those of the client.

(b) Comment on the symmetric nature of the quadratic loss form of utility. What does this mean in the examples above? When is it reasonable?

- In most cases taking a drug when you don't need it is not as bad as not taking one when you do and being searched when you shouldn't be is not as bad for the principal as not searching a criminal. I think auditing exists mainly for preventative purposes (perhaps so does airport security), so the question is less clear in the last example.
  The symmetry of the preferences, in any case, is an assumption for convenience, not realism, but may not be outrageous in many circumstances, for example night club security does not suffer greatly if party goers sneak alcohol or drugs into their establishments.

(c) Who is the client in the examples above? In which cases are the client's interests aligned with Pamela's/the bureaucrats? How might this affect Pamela's employment decision? Comment with respect to (b).

- Mainly covered above.

Suppose that $\tilde{\theta}$ is normally distributed with zero mean and precision h,

$$\tilde{\theta} \sim \mathrm{N}\left[0\ ,\ \frac{1}{h}\right] \tag{5}$$

and that before determining his allocation a, a bureaucrat exerts effort

e to observe a signal s about θ confounded by noise with precision e,

$$\tilde{s}_e = \theta + \varepsilon_e \tag{6}$$

where

$$\varepsilon_e \sim \mathrm{N}\left[0\ ,\ \frac{1}{e}\right]. \tag{7}$$

The agent bears a linear cost of effort $c(e) = e$.

2. This question solves for the bureaucrat's allocation choice and effort level by backward induction.
   (a) Show that the bureaucrat's allocation choice is a biased version of Pamela's, namely that

$$a^* = \mathbb{E}\left[\tilde{\theta} \,\middle|\, \tilde{s}_e = s\right] + b. \tag{8}$$

Solution

- The bureaucrat maximizes

$$-B^2\mathbb{E}\left[\left(\tilde{\theta} - a + b\right)^2 \,\middle|\, \tilde{s}_e = s\right] \tag{9}$$

giving first-order condition

$$\mathbb{E}\left[\tilde{\theta} - a + b \,\middle|\, \tilde{\sigma}_e = s\right] = 0. \tag{10}$$

The result follows.

   (b) Show that the bureaucrat's net utility is given by

$$\mathbb{E}\left[u_B\left(\tilde{\theta}, a^*\right) \,\middle|\, s\right] - e = -\mathrm{Var}\left[B\tilde{\theta} \,\middle|\, s\right] - e. \tag{11}$$

Hint: Use the definition of the conditional variance backwards:

$$\mathrm{Var}\left[\mathbf{X} \mid A\right] = \mathbb{E}\left[\left(\mathbf{X} - \mathbb{E}\left[\mathbf{X} \mid A\right]\right)^2 \,\middle|\, A\right]. \tag{12}$$

Solution

- Simply plug in for the optimal action and compute:

$$
\begin{aligned}
\mathbb{E}\left[u_B(\tilde{\theta}, a^*) - e \,\middle|\, s\right] &= -B^2 \mathbb{E}\left[\left(\tilde{\theta} - a^* + b\right)^2 \,\middle|\, s\right] - e \\
&= -B^2 \mathbb{E}\left[\left(\tilde{\theta} - \left(\mathbb{E}\left[\tilde{\theta} \,\middle|\, s\right] + b\right) + b\right)^2 \,\middle|\, s\right] - e \\
&= -B^2 \mathbb{E}\left[\left(\tilde{\theta} - \left(\mathbb{E}\left[\tilde{\theta} \,\middle|\, s\right] + b\right) + b\right)^2 \,\middle|\, s\right] - e \\
&= -B^2 \mathbb{E}\left[\left(\tilde{\theta} - \mathbb{E}\left[\tilde{\theta} \,\middle|\, s\right]\right)^2 \,\middle|\, s\right] - e \\
&\equiv -B^2 \mathrm{Var}\left[\tilde{\theta} \,|\, s\right] - e \\
&= -\mathrm{Var}\left[B\tilde{\theta} \,|\, s\right] - e
\end{aligned}
\tag{13}
$$

(c) Compute the variance to see that the objective is in fact

$$
\mathbb{E}\left[u_B\left(\tilde{\theta}, a^*\right)\right] = -\frac{B^2}{h+e} - e. \tag{14}
$$

Solution

- This is immediate from the normal updating formula in equation 2 above.

(d) Use the first-order approach to show that the equilibrium effort of a bureaucrat with type (b, B) is

$$
e^* = \max\{B - h, 0\}. \tag{15}
$$

Suppose $B > h$ from now on.

Solution

- Suppose and interior solution and compute the first-order condition:

$$
\frac{\partial}{\partial e}\left(-\frac{B^2}{h+e} - e\right) = 0 \tag{16}
$$

or

$$
\frac{B^2}{(h+e)^2} - 1 = 0 \tag{17}
$$

so

$$
e^* = B - h, \tag{18}
$$

as above.

Of course $e > 0$ and when $B - h < 0$

$$\frac{\partial}{\partial e}\bigg|_{e=0}\left(-\frac{B^2}{h+e} - e\right) < 0 \tag{19}$$

so when $B - h < 0$, $e^* = 0$ so the optimum is the maximum of 0 and $B - h$.

(e) Interpret the result:

i. Why is e^* independent of the bias b?

- The bureaucrat's action corrects for the bias perfectly, so his payoff doesn't depend on the bias, only Pamela's does.

ii. Why does e^* decrease in the prior precision h?

- When prior information is good, the signal has less influence on the action and is therefore less valuable.

3. This question determines the type of bureaucrat that Pamela hires.

(a) Show that given she hires a bureaucrat of type (b, B), Pamela's equilibrium utility is given by

$$\mathbb{E}\left[u_P(\tilde{\theta}, a^*)\right] = -\mathbb{E}\left[\mathrm{Var}\left[\tilde{\theta} \,\middle|\, \tilde{s}_{e^*}\right]\right] - b^2. \tag{20}$$

Hint: Use the law of iterated expectations to recover the conditional variance.

Solution

- Just compute plugging in the bureaucrat's actions:

$$\mathbb{E}\left[u_P(\tilde{\theta}, a^*)\right] = -\mathbb{E}\left[\left(\tilde{\theta} - a^*\right)^2\right] \tag{21}$$

$$= -\mathbb{E}\left[\tilde{\theta} - \left(\mathbb{E}\left[\tilde{\theta}\,\middle|\, \tilde{s}_{e^*}\right] + b\right)^2\right] \tag{22}$$

$$= -\mathbb{E}\left[\mathbb{E}\left[\left(\tilde{\theta} - \left(\mathbb{E}\left[\tilde{\theta}\,\middle|\, \tilde{s}_{e^*}\right] + b\right)\right)^2 \,\middle|\, \tilde{s}_{e^*}\right]\right] \tag{23}$$

$$= -\mathbb{E}\left[\mathbb{E}\left[\left(\tilde{\theta} - \left(\mathbb{E}\left[\tilde{\theta}\,\middle|\, \tilde{s}_{e^*}\right]\right)\right)^2 \,\middle|\, \tilde{s}_{e^*}\right]\right] \tag{24}$$

$$+ 2b\,\underbrace{\mathbb{E}\left[\tilde{\theta} - \mathbb{E}\left[\tilde{\theta}\,\middle|\, \tilde{s}_{e^*}\right]\right]}_{=0} - b^2 \tag{25}$$

$$= -\mathbb{E}\left[\mathrm{Var}\left[\tilde{\theta} \,\middle|\, s_{e^*}\right]\right] - b^2 \tag{26}$$

(b) Argue that Pamela's objective function is in fact

$$\mathbb{E}\left[u_P(\tilde{\theta}, a^*)\right] = -\frac{1}{h+e^*} - b^2 \tag{27}$$

$$= -\frac{1}{B} - b^2. \tag{28}$$

Solution

- This follows immediately from the observation that conditional variance in the normal-linear case is independent of the conditioning:

$$\text{Var}\left[\tilde{\theta} \,\middle|\, \tilde{s}_e = s\right] = \text{Var}\left[\tilde{\theta} \,\middle|\, \tilde{s}_e = s'\right] \tag{29}$$

for any s and s'. Or, here,

$$\text{Var}\left[\tilde{\theta} \,\middle|\, \tilde{s}_e = s\right] = \frac{1}{h+e}. \tag{30}$$

Since $e^* = B - h$,

$$\text{Var}\left[\tilde{\theta} \,\middle|\, \tilde{s}_{e^*} = s\right] = \frac{1}{e^*+h} = \frac{1}{B}. \tag{31}$$

(c) Suppose, in accordance with Pamela's observation that opinionated bureaucrats are also more motivated, that B is an increasing function of b; in particular let $B = \beta b/2$ for $\beta > 0$.

i. What does β measure?

- The trade-off between bias and intrinsic incentives; the higher is β the more motivation a bureaucrat has for a given level of bias. For low β, only highly biased agents are motivated.

ii. Show that Pamela employs the bureaucrat of type $(b, \beta b/2)$ for

$$b^* = \beta^{-1/3}. \tag{32}$$

Solution

- Pamela's FOC are

$$\left.\frac{\partial}{\partial \beta}\right|_{b=b^*} \left(-\frac{2}{\beta b} - b^2\right) = 0 \tag{33}$$

or

$$\frac{2}{\beta b^{*2}} - 2b^* = 0 \tag{34}$$

or

$$b^{*3} = \frac{1}{\beta}. \tag{35}$$

iii. Interpret the final expression.

- From the figure below, you see that when β is small Pamela hires very biased bureaucrats; otherwise she could not get them to exert reasonable effort levels. For high β she hires bureaucrats who are hardly biased at all. For low β, Pamela's choice of agent is very sensitive to β, while for high β it is not.
  If we take the model literally, we should expect to see more biased bureaucrats in less-rewarding jobs.

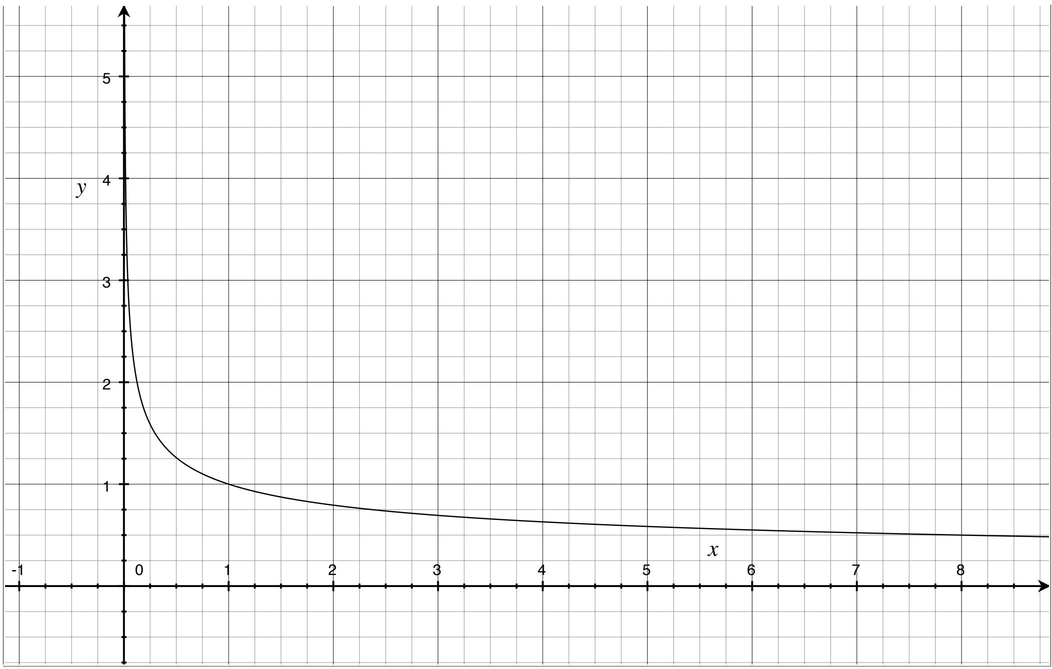

Figure 1: b^* as a function of β.