MANAGERIAL COST-CUTTING AND PRODUCT MARKET COMPETITION

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Abstract

This exercise uses Hotelling's model of duopolistic competition to demonstrate the connection between competition in the product market and managerial efficiency. It demonstrates that hiring new efficient cost-cutting managers may be inefficient for firms and consumers but still the unique equilibrium outcome.

Two firms compete in the product market on the Hotelling line. Leon is at the left ($x_L = 0$) and Roxy is at the right ($x_R = 1$). Consumers are distributed uniformly between them with utility from buying the good from firm $f \in \{L, R\}$ given by

$$u(x, f) = \bar{u} - p_f - \gamma \big| x_f - x \big| \tag{1}$$

for consumer $x \in [0, 1]$. Each consumer buys at most once, and assume that $\bar{u}$ is large enough that all consumers buy.

1. Hotelling (1929) interpreted these preferences in terms of transportation costs.
   (a) Explain Hotelling's interpretation.
   Solution
   - Competing retailers at either end of "Main Street" selling the same good (e.g. petrol, bread) to an market with a fixed demand dividend between those goods. Spatial competition.

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(b) Provide a more general interpretation of the preferences with examples where they apply. What does γ represent in your examples?

Solution

- Situations in which firms are competing in imperfect substitutes with fixed quality and a market in which preferences cross. For example a truck manufacture and a sports car manufacturer or an Italian restaurant and French restaurant. γ represents the degree of substitutability for the consumers. Probably γ is high in the car example: the builder needs a pick-up truck and the securities trader desires only a sports car while γ is lower in the restaurant example, many people are close to indifferent between French and Italian food but still have a preference for one or the other.

(c) Give two examples of duopolistic product market competition that this framework does not describe well.

Solution

- Two competing luxury bakeries might not be well-modelled with this set up, because their product quality is flexible and will change to meet the demands of the market. Competing Honda and Lexus dealerships might not be a good example, because few people's preferences cross. Most people prefer the Lexus; those who choose Hondas do so because of their lower prices.

2. (a) Demonstrate that either all the consumers prefer to buy from Leon, all from Roxy, or that there is an indifferent consumer x^* such that if $x < x^*$ then x prefers to buy from Leon and if $x > x^*$ then x prefers to buy from Roxy.

Solution

- That $u(x, 0) > u(x, 1)$ implies that $u(y, 0) > u(y, 1)$ whenever $y < x$ (and respectively that $u(x, 0) < u(x, 1)$ implies that $u(y, 0) < u(y, 1)$ whenever $y > x$) follows immediately from the form of the utility and implies the result.

(b) Express the demands for each seller's good $D_f(p_f, p_{\neg f})$ in terms of the indifferent consumer.

Solution

- For indifferent consumer $x^* = x^*(p_0, p_1)$,

$$D_0(p_0, p_1) = x^* \quad \text{and} \quad D_1(p_0, p_1) = 1 - x^* \tag{2}$$

where x^* solves
$$u\big(x^*, 0\big) = u\big(x^*, 1\big) \tag{3}$$
or
$$\bar{u} - p_0 - \gamma x^* = \bar{u} - p_1 - \gamma(1 - x^*) \tag{4}$$
so
$$x^* = \frac{\gamma - p_0 - p_1}{2\gamma}. \tag{5}$$
Leon and Roxy both have marginal cost C and no fixed cost, so firm f's profit is
$$\Pi_f = (p_f - C)D_f. \tag{6}$$

3. (a) Demonstrate that given $p_{\neg f}$, firm f's best response function is
$$p_f^*\big(p_{\neg f}\big) = \frac{\gamma + p_{\neg f} + C}{2} \tag{7}$$

Solution

- Demonstrate this only for Leon (Firm 0). Firstly, rewrite the profit in terms of the indifferent consumer:
$$\begin{aligned}\Pi_0 &= (p_0 - C)D_f &(8)\\ &= (p_0 - C)x^* &(9)\\ &= \frac{1}{2\gamma}(p_0 - C)(\gamma - p_0 + p_1) &(10)\end{aligned}$$
so the first-order condition
$$\frac{\partial}{\partial p_0}\bigg|_{p_0 = p_0^*(p_1)} \Pi_0(p_0, p_1) = 0 \tag{11}$$
reads
$$\gamma - p_0^* + p_1 = p_0^* - C \tag{12}$$
from the product rule. Thus
$$p_0^*(p_1) = \frac{\gamma + p_1 + C}{2}, \tag{13}$$
as desired.

(b) Use the Nash conditions
$$p_f^*\Big(p_{\neg f}\big(p_f\big)\Big) = p_f \tag{14}$$
for $f \in \{L, R\}$ to determine the equilibrium prices.
Solution

- For Leon, plugging in gives the equation

$$p_0 = \frac{\gamma + p_1^*(p_0) + C}{2} \tag{15}$$
$$= \frac{1}{2}\left(\gamma + \frac{\gamma + p_0 + C}{2} + C\right) \tag{16}$$
$$= \frac{3\gamma + p_0 + 3C}{4}, \tag{17}$$

so

$$p_0 = \gamma + C. \tag{18}$$

Likewise for Roxy.

(c) Write down the equilibrium profits.
Solution

- Compute Leon's profits by plugging in. Notice that since $p_0 = p_1$, $x^* = 1/2$ at equilibrium.

$$\Pi_0\big(\gamma + C, \gamma + C\big) = \big((\gamma + C) - C\big)x^* \tag{19}$$
$$= \frac{\gamma}{2}. \tag{20}$$

Likewise for Roxy.

(d) Comment on the prices and profits relative to the consumers' "transportation cost" γ and the firms' production costs C.
Solution

- Firms compete on the intensive margin. When their marginal costs increase they increase prices perfectly to compensate at equilibrium. Thus whenever their marginal costs coincide they make the same profit.
  γ, on the other hand, measures the substitutability of their goods. They prefer to act more like monopolists in local markets. When γ decreases they approach Bertrand competition, set price equal to marginal cost, and profits vanish.

Observe that despite their competing in prices, Leon and Roxy are still making a healthy profits as long as γ is not too small. Each, however, has the opportunity to bring in new management that will implement the latest cost-cutting practices. If a firm employs an expert manager, its marginal cost is reduced to $c = 0$, but due to labour market competition it must pay him a fixed wage $w > 0$.

4. (a) Suppose both Leon and Roxy hire new managers. Show that despite the increased efficiency at both enterprises, Leon and Roxy are both strictly worse off.

Solution

- This follows from the profits being independent of the marginal costs whenever they are the same and the firms' having to pay the managers. Since w is a fixed cost, it does not affect prices. Thus,

$$\Pi_f = \frac{\gamma}{2} - w. \tag{21}$$

(b) Now suppose firm f employs new management and firm $\neg f$ does not. Recover that firm f's profits are

$$\Pi_f = \frac{\left(3\gamma + C\right)^2}{18\gamma} - w \tag{22}$$

and

$$\Pi_{\neg f} = \frac{\left(3\gamma - C\right)^2}{18\gamma}. \tag{23}$$

Solution

- Without loss of generality suppose Leon employs new management and Roxy does not, so (off equilibrium) their profits are

$$\Pi_0 = p_0 x^* - w \tag{24}$$

$$= \frac{p_0}{2\gamma}\left(\gamma - p_0 + p_1\right) - w \tag{25}$$

and

$$\Pi_1 = \left(p_1 - C\right)\left(1 - x^*\right) \tag{26}$$

$$= \frac{1}{2\gamma}\left(p_1 - C\right)\left(\gamma - p_1 + p_0\right). \tag{27}$$

Since we the best-response functions can depend only on firms' own costs, by analogy with above we get

$$p_0^*(p_1) = \frac{\gamma + p_1}{2} \tag{28}$$

and

$$p_1^*(p_0) = \frac{\gamma + p_0 + C}{2}. \tag{29}$$

Plugging into the Nash conditions for Leon gives

$$p_0 = \frac{1}{2}\left(\gamma + \frac{\gamma + p_0 + C}{2}\right) \tag{30}$$

$$= \frac{3\gamma + p_0 + C}{4} \tag{31}$$

so

$$p_0 = \gamma + C/3. \tag{32}$$

And for Roxy

$$p_1 = \frac{1}{2}\left(\gamma + \frac{\gamma + p_1}{2} + C\right) \tag{33}$$

$$= \frac{3\gamma + p_1 + 2C}{4} \tag{34}$$

so

$$p_1 = \gamma + 2C/3. \tag{35}$$

Now find the indifferent consumer:

$$x^* = \frac{\gamma - p_0 + p_1}{2\gamma} \tag{36}$$

$$= \frac{1}{2\gamma}\big(\gamma - \big(\gamma - C/3\big) + \gamma + 2C/3\big) \tag{37}$$

$$= \frac{1}{2\gamma}\big(1 + C/3\big) \tag{38}$$

and so

$$1 - x^* = \frac{1}{2\gamma}\big(1 - C/3\big). \tag{39}$$

Plugging in for the profits gives

$$\Pi_0 = p_0 x^* - w \tag{40}$$

$$= \frac{1}{2\gamma}\big(\gamma + C/3\big)\big(\gamma + C/3\big) - w \tag{41}$$

$$= \frac{\big(3\gamma + C\big)^2}{18\gamma} - w \tag{42}$$

and

$$\Pi_1 = \frac{1}{2\gamma}\big(p_1 - C\big)\big(1 - x^*\big) \tag{43}$$

$$= \frac{1}{2\gamma}\big(\gamma + 2C/3 - C\big)\big(\gamma - C/3\big) \tag{44}$$

$$= \frac{\big(3\gamma - C\big)^2}{18\gamma}, \tag{45}$$

as desired.

Leon and Roxy's decisions to shake up management are strategic, dependent on the labour market decisions of their product market competitors.

5. (a) Demonstrate that the firms' employment decisions are represented by the two-by-two game below.

Solution

- Immediate from the payoffs above.

Roxy

		hire	don't
Leon	hire	$\frac{\gamma}{2} - w$, $\frac{\gamma}{2} - w$	$\frac{(3\gamma + C)^2}{18\gamma} - w,$, $\frac{(3\gamma - C)^2}{18\gamma}$
	don't	$\frac{(3\gamma - C)^2}{18\gamma}$, $\frac{(3\gamma + C)^2}{18\gamma} - w$	$\frac{\gamma}{2}$, $\frac{\gamma}{2}$

Hire expert managers?

Figure 1: For relatively high C, the firms are caught in a prisoners' dilemma in which they higher newly efficient management.

(b) Show that both firms hiring is the unique equilibrium whenever

$$6\gamma(3w - C) < C^2 < 6\gamma(C - 3w). \tag{46}$$

Explain the connection (equivalence) with the prisoners' dilemma. (For larger γ the game is a coordination game with multiple equilibria similar to Rousseau's stag hunt.)

Solution

- (hire, hire) payoff dominates (don't, don't), but for the parameters above the latter is the only equilibrium. The deviation from the equilibrium occurs if and only if

$$\frac{(3\gamma + C)^2}{18\gamma} - w > \frac{\gamma}{2} \tag{47}$$

or

$$9\gamma^2 - 18\gamma w < 9\gamma^2 + 6\gamma C + C^2 \tag{48}$$

which reduces to the left-hand condition above.

Likewise, in order for (hire, hire) to be an equilibrium it must be that

$$\frac{\gamma}{2} - w \geq \frac{(3\gamma - C)^2}{18\gamma}, \tag{49}$$

which reduces to the right-hand inequality exactly analogously.

6. Suppose the above condition on parameters so that that (hire, hire) is the unique equilibrium.
   (a) Who benefits from the increased efficiency caused by the changes of management?
   Solution
      - Perhaps the managers, as they may earn a higher wage and certainly not the firms who only suffer. The consumers, however, benefit from the cost savings as they're passed onto them directly via lower prices.

   (b) If firms could collude and retain their inefficiency they would. Should competition law regulate this?
   Solution
      - Seems like a good idea. Note it's not price collusion they would need to regulate but rather prohibiting them from not innovating, which could be impossible. Thus (to stretch the interpretation) firms have a way of colluding under the radar of regulators.

   (c) Comment on the social benefits of MBA training.
   Solution
      - It may be surprising that it's the "little guy" who's benefiting from increased managerial efficiency and not that oligopolistic firms. This might suggest the state should subsidize MBAs.