COLLATERAL CYCLICALITY AND SPECIFIC INVESTMENT*

DRAFT

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Abstract

A model of a borrowing secured by productive assets shows that firms' reliance on external finance determines the composition of the left-hand sides of their balance sheets. Financially constrained firms hoard collateral to make their repayment promises credible; asset countercyclicality intensifies the inefficiency, leading firms to overinvest further in pledgeable capital. Debtors are opportunistic ex post, but procyclical entrepreneurs take less advantage of their creditors because their cash flows are high in booms, exactly when their creditors' bargaining positions are strong: asset procyclicality gives borrowers commitment power and increases their debt capacity; sufficiently countercyclical firms are rationed and cannot undertake their projects no matter how much of their assets they hold as collateral.

Procyclicality's relaxing financial constraints is a potentially powerful mechanism of business cycle amplification.

1   Introduction

It is easier to do business in a boom than in a bust. The success of a firm's investments is complementary to its access to funding and thus to the liquidity of its present and potential creditors. The CAPM—a model which Harrison Hong refers to as "the only theory of asset pricing we really have"—asserts that asset countercyclicality (negative beta) demands a price premium, and thereby lowers funding costs. The model below asserts that considering the limits on borrowers' commitment turns the result on its head: procyclicality is valuable because comes with the power to make credible repayment promises—the ability to borrow. When a firm's cash flows correlate negatively with the business cycle, its creditors' redeployment value of collateral—the determinant of its outside option given default—is low

when the firm can afford to repay its debts. Outsiders assume a weak bargaining position exactly when insiders have the means to repay them. Lenders withhold credit when they anticipate borrowers' ex post opportunism. A procyclical project delivers an insider high cash flows when the market value of capital is high; thus an entrepreneur whose balance sheet moves with the market gives his creditors an effective seizure threat exactly when he has the strongest incentive to stay in business. Procyclical assets prevent insiders from exploiting outsiders ex post and thus they endow them with commitment power: they have the ability to keep promises ex post and consequently the ability to make promises ex ante: they have the ability to borrow. Entrepreneurs with less procyclical projects who still rely on outside investments find an alternative means to make their repayment promises credible by posting additional collateral, which often constitutes an unproductive investment. Procyclicality loosens financial constraints and allows debtors to use their assets efficiently, making specific investments that increase productivity but destroy liquidation values. Countercyclical firms must modify the left-hand sides of their balance sheets, hoarding collateral to loosen financial constraints imposed by the right-hand sides.

1.1 Mechanism

The mechanism relies on incomplete contracting fuelled by Williamsonian opportunism with the twist that, rather than default, debtors can sell in the market at their creditors' liquidation value and abscond, leading to the *continuation constraint*: an insider prefers to settle his debts and continue his project rather than abscond only if

$$\text{inside value} \; - \; \text{repayment} \; \geq \; \text{outside value} . \tag{CC}$$

The inside value is not verifiable, so creditors enforce repayment only via threat of seizure. In renegotiation they accept any repayment greater than their deployment value of the capital they can seize. The argument (originally in Hart and Moore (1989)) implies the *seizure constraint*:

$$\text{repayment} \; \leq \; \text{outside value} . \tag{SC}$$

While the seizure constraint says creditors require high collateral values to secure high repayments, the continuation constraint suggests that high collateral values incentivize debtors to abandon without repaying at all. A sufficiently high inside value, however, allows the continuation constraint to stay slack even when the outside value is high. All else equal, correlation between inside values and outside values mitigates the problem imposed by these countervailing forces: procyclicality increases debt capacity. (See section 3.1 for a more detailed treatment of the same two constraints.)

The model augments this mechanism by studying firms' ability to undertake specific investment: to transform assets to increase their productivity within the firm at the expense of their marketable value. Entrepreneurs require a fixed capital outlay on top of their endowments to implement their projects. Borrowers require security to pledge to their creditors. Procyclical firms obtain more funding per unit of collateral than their countercyclical counterparts, who must hoard collateral to access liquidity. Procyclical assets allow firms to make more efficient specific investments and still access outside liquidity.

2 Model

The model is an extensive game of complete information among two strategic players and nature.

2.1 The Entrepreneur

The entrepreneur has cash endowment e and requires capital I to implement his project. Thus, given price of capital p_0, he must borrow $p_0 I - e$ to fund his investment. The project yields $\mathbf{Y}(h, k)$ in present value at date 1, where h and k denote the quantities of specific and generic capital invested, respectively. Specific capital is more productive than generic capital, but has no recovery value in the event of failure, while generic capital can be liquidated at the market price $\mathbf{P} \in \{p_L, p_H\}$. The entrepreneur has a positive NPV project defined by the linear output technology

$$\mathbf{Y}(h, k) := \begin{cases} y_S = Ah + ak & \text{if success,} \\ y_F = 0 & \text{if failure.} \end{cases} \tag{1}$$

2.2 Liquidation Value

The outside, or liquidation, value depends on the marginal investor's marginal productivity of (generic) capital. When it is high, the price is $p_H > p_L$, the price when productivity is low.

Identify $\mathbf{P} = p_H$ with a boom and likewise $\mathbf{P} = p_L$ with a bust; each has unconditional probability half, $\mathbb{P}\{\mathbf{P} = p_H\} = \mathbb{P}\{\mathbf{P} = p_L\} = 1/2$. The outside value of a project utilizing generic capital k is

$$\mathbf{L}(k) := \mathbf{P}k. \tag{2}$$

Set the risk-free rate and the market price of risk to zero, so

$$\mathbb{E}[\mathbf{P}] = \frac{p_L + p_H}{2} = p_0. \tag{3}$$

The notation

$$\Delta \equiv \Delta p := p_H - p_L \tag{4}$$

so that

$$p_H = p_0 + \Delta/2 \tag{5}$$

and

$$p_L = p_0 - \Delta/2 \tag{6}$$

is convenient below.

2.3 The Project

The pair $\big(\mathbf{Y}(h, k), \mathbf{L}(k)\big)$ defines the project. The conditional probabilities

$$\mathbb{P}\{\mathbf{Y} = y_S \,|\, \mathbf{P} = p_H\} = \mathbb{P}\{\mathbf{Y} = y_F \,|\, \mathbf{P} = p_L\} =: q \tag{7}$$

give the joint distribution of output and liquidation values. Call the exogenous parameter q the *cyclicality* of the project as it measures the comovement of the inside value—specific to the entrepreneur—with the outside value—representing the

redeployment value of capital to an outsider, namely a reflection of the marginal TFP of capital, a macroeconomic variable.

Assume that specification is efficient even though generic capital has higher recovery value when the project fails, symbolically:

$$A - a > 2p_0 \tag{8}$$

(cf. equation (24)).

2.4 Contracts and Renegotiation

The entrepreneur borrows via a debt contract with face value D backed by the right to seize and liquidate the project. Only seizure rights enforce repayment. In the event of default, the entrepreneur renegotiates with his creditors according to the protocol in Hart and Moore (1989): if he fails to deliver D, the entrepreneur makes a take-it-or-leave-it offer of repayment. If the creditor accepts the offer, the entrepreneur pays and continues this project. If the creditor rejects the offer, he retains the right to seize the project. In equilibrium, the entrepreneur always repays the lesser of the face value of debt and the liquidation value of capital, $\min\{D, \mathbf{L}(k)\}$.

2.5 Timing

The entrepreneur first commits to a specific investment, then borrows to implement his project and invests. After the project succeeds or fails and the macroeconomic state realizes, the entrepreneur has the option to liquidate a proportion of his project in the market at λpk for $p \in \{p_L, p_H\}$; the constant $\lambda \in [0, 1]$ represents the fraction of value the entrepreneur can capture effectively. Given such internal liquidation, the entrepreneur absconds and leaves his creditor with a contractual claim without practical value. Otherwise, the entrepreneur continues and the creditor's seizure rights remain effective. To simplify the analysis, assume a sufficient condition for the entrepreneur never to abscond in the event of success via the requirement that his inside value given success be high:

$$a \geq (1 + \lambda)p_H. \tag{9}$$

The following timing describes the extensive form of the complete information game that constitutes the model:

1. The entrepreneur commits to specific investment h.

2. The entrepreneur offers a creditor a debt contract with face value D backed by the project to borrow $p_0 I$.

   - If the creditor rejects the offer, his payoff is zero and the entrepreneur's payoff is his capital value $p_0 k$.

3. If the creditor accepts the offer, the entrepreneur undertakes his project.

4. The economy booms or busts and the project succeeds of fails ($\mathbf{P}$ and $\mathbf{Y}$ realize).

5. The entrepreneur absconds or does not.

   - If the entrepreneur absconds his payoff is λpk and the creditor's is nil.

6. If the entrepreneur does not abscond, he repays $\min\{D, pk\}$ as per the renegotiation protocol; he continues his project.

3 Results

3.1 The Seizure and Continuation Constraints

Observe from the renegotiation protocol that the entrepreneur will always default and renegotiate if the face value of debt is greater than the creditor's seizure value. In particular, the repayment R is always less than the outside value of collateral or

$$R \leq pk. \tag{SC}$$

The inequality is the seizure constraint.

The more novel continuation constraint stems from the entrepreneur's decision as to whether to abscond. If he continues with inside value y and repays R, he obtains $y - R$, whereas if he absconds and repays nothing he obtains λpk. Thus he continues only if

$$y - R \geq \lambda pk \tag{10}$$

or

$$R \leq y - \lambda pk. \tag{CC}$$

The creditor's outside option implies that he will never pay the debtor ex post, but for any positive repayment the two constraints together,

$$R \leq \min\{pk, y - \lambda pk\}, \tag{11}$$

imply that the creditor cannot obtain high repayments neither when his collateral value is low—because he has a weak bargaining position—nor when it is high—because the entrepreneur will prefer to abscond rather than to continue the relationship. When y is sufficiently high, however, the creditor can obtain a high repayment because the continuation constraint stays slack.

3.2 Debt Capacity

Since the entrepreneur never repays more than D, an outside equity contract (namely $D = \infty$) maximizes the amount he can borrow against a project. Further, he always absconds if his project fails. Thus the most an entrepreneur can promise to repay in expectation, the project's debt capacity, is the expectation of cash-flow stream that returns nil to the creditor in the event of failure and the liquidation value in the event of success, namely

$$\text{DC}\big(\mathbf{Y}(h,k)\big) = \mathbb{P}\{\mathbf{Y} = y_S\}\,\mathbb{E}\left[\mathbf{L}(k) \mid \mathbf{Y} = y_S\right] \tag{12}$$

$$= \mathbb{P}\{\mathbf{Y} = y_S\}\,\mathbb{P}\{\mathbf{L}(k) = p_H k \mid \mathbf{Y} = y_S\}\,p_H k+ \tag{13}$$

$$+ \mathbb{P}\{\mathbf{Y} = y_S\}\,\mathbb{P}\{\mathbf{L}(k) = p_L k \mid \mathbf{Y} = y_S\}\,p_L k \tag{14}$$

$$= \frac{1}{2}\Big(p_L + (p_H - p_L)q\Big)k \tag{15}$$

$$= \frac{1}{2}\Big(p_L + \Delta\, q\Big)k, \tag{16}$$

having employed Bayes's theorem. The coefficient Δ of q is positive and the result that more cyclical entrepreneurs can borrow more is proved.

Proposition 3.2.1. *Debt capacity increases with cyclicality.*

3.3 Rationing

The entrepreneur can implement his project only if he can borrow $p_0 I - e$, or, since debt capacity is increasing in generic capital investment k,

$$\sup \left\{ \mathrm{DC}\big(\mathbf{Y}(h,k)\big) \; ; \; k \in [0, I] \right\} = \frac{1}{2}\Big(p_L + \Delta q\Big) I \geq p_0 I - e. \tag{17}$$

The expression implies that constrained entrepreneurs are unable to borrow unless q is sufficiently high, or that low-cyclicality entrepreneurs are rationed.

Proposition 3.3.1. *If*

$$q < p_H/\Delta - 2e/I \tag{18}$$

then the entrepreneur cannot borrow.

Proof. Inequality (17) says the entrepreneur can borrow only if

$$2p_0 - 2e/I \leq p_L + \Delta q \tag{19}$$
$$= p_0 - \Delta/2 + \Delta q \tag{20}$$

or

$$\Delta q \geq p_0 + \Delta/2 - 2e/I = p_H - 2e/I. \tag{21}$$

The negation of which is the result.

$\square$

3.4 The Optimal Debt Contract

Given specification h and thus collateral $k = I - h$, the optimal debt contract has the smallest face value such that the creditor accepts, namely D_k solves

$$\frac{1}{2}\Big(q \min\{D_k, p_H k\} + (1 - q) \min\{D_k, p_L k\}\Big) = p_0 I - e, \tag{22}$$

since, by assumption (9), the entrepreneur always continues his project when his project is successful.

3.5 Specification

The entrepreneur specifies to maximize his expected payoff subject to his funding constraint. He maximizes

$$\frac{1}{2}q\Big(Ah + ak - \min\{D_k, p_H k\}\Big) + \frac{1}{2}(1 - q)\Big(Ah + ak - \min\{D_k, p_L k\}\Big) + p_H k + p_L k \tag{23}$$

subject to the creditor's lending constraint and the technological restrictions that $h + k = I$ and $h, k \geq 0$. Substituting from equation (22) above implies the entrepreneur's objective is

$$\frac{1}{2}\Big(Ah + a(I - h)\Big) + \big(p_H + p_L\big)\big(I - h\big) - p_0 I + e = \frac{1}{2}\big(A - a - 2p_0\big)h + \big(p_0 + a/2\big)I + e, \tag{24}$$

which no longer depends on D_k and is increasing in h by assumption (8). Thus the entrepreneur chooses the highest h such that he can still obtain funding, in

particular such that the creditor is willing to lend for some D_k or that the project's debt capacity permits its implementation: in equilibrium

$$\mathrm{DC}\big(\mathbf{Y}(I-k,k)\big) = p_0 I - e \tag{25}$$

or

$$\frac{1}{2}\big(p_L + \Delta\, q\big)k = p_0 I - e. \tag{26}$$

Equivalently,

$$k = \frac{2(p_0 I - e)}{p_L + \Delta\, q}, \tag{27}$$

a decreasing function of q. The less cyclical a firm's project is, the more generic capital it must hold has collateral to secure funding.

Proposition 3.5.1. *Specification increases with cyclicality. More cyclical firms invest more efficiently while less cyclical firms hoard collateral.*

4 Conclusion

To secure repayment, creditors demand collateral from entrepreneurs, but anticipate that they may wish to liquidate and abscond to avoid repayment. The setup implies two constraints to repayment, the seizure constraint that requires high collateral values to give creditors bargaining power and the continuation constraint that requires low collateral values to mitigate the debtor's incentives for premature liquidation. The creditor weakens the conflict between these countervailing incentives by lending to procyclical entrepreneurs.

Collateral is endogenous. Entrepreneurs make specific investments to increase future productivity at the expense of outside value. Since countercyclical entrepreneurs must post more collateral than procyclical ones to promise outsiders the same repayments, they must hoard collateral to obtain funding. Despite having a good project, an entrepreneur may not be able obtain funding even if he holds all of his assets in generic capital, in which case he is rationed. Procyclical entrepreneurs avoid rationing while countercyclical entrepreneurs' investments go undone.

Procyclical entrepreneurs' ability to keep promises ex post allows them to make promises ex ante—to borrow. This loosening of financial constraints for procyclical projects amplifies exogenous cycles via two mechanisms: rationing and collateral hoarding.

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