FIRM SIZE AND FINANCIAL CONSTRAINTS*

Jason Roderick Donaldson ☠ Alkis Georgiadis-Harris Ⓐ

DRAFT

October 29, 2012

Abstract

In a critical resource model of the firm, an entrepreneur has a project but insufficient capital and limited collateral. Taking on partners increases the seizure value of the assets but may hamper the organization's internal productivity. This trade-off determines the equilibrium size of the firm in a game constituting coalition-formation, secured borrowing, effort exertion, and ex post opportunism both between the firm and the creditor and among the inside partners. The theory predicts that a firm's size depends on the left-hand side of its balance sheet: the more effectively its assets can be transferred or liquidated, the smaller is the organization. When seizure is expensive the firm is large, as it is when it depends heavily on outside capital.

*Many thanks to Chris, Gio, and John.

Contents

1 Introduction

In 1937 Coase pioneered academic thinking on the boundaries of the firm, asserting that "a firm will tend to expand until the costs of organizing an extra transaction within the firm become equal to the costs of carrying out the same transaction by means of an exchange on the open market or the costs of organizing in another firm." Impelled by Rajan's 2012 American Finance Association presidential address, which asserts that the theory of the firm must examine further the effects of financing demands, we model a firm that needs to mortgage its productive assets to raise capital. Contrary to Coase, we suggest that firms grow till they can fund their projects, potentially at the expense of internal operational efficiency. Firm size thus depends on assets' quality as collateral, namely the ease with which an outsider can seize and liquidate its output.

Samuelson's chapter on "Business Organization and Income"—the first pages many of us read about the economics of the firm—focuses keenly on the connection between "big, small, and infinitesimal business" and the dependence on outside capital; he describes a hypothetical toothpaste company's expansion. That the dental entrepreneur hits a financing constraint since "ordinarily, a commercial bank will not provide 'venture capital'" is the main reason Samuelson provides for the firm's changing ownership structure and becoming a partnership. But apparently modern authors have been less influenced by Samuelson's chronicle of the oral hygiene start-up than by Smith's analysis of a pin factory, which gives precedence to the benefits of the division of labour as a guide to understanding the firm. For example, the generally comprehensive modern classics Williamson (1987) and Roberts (2004) make little reference to firms' need for external capital or their simple scale; like much of the recent literature—including that building on Grossman and Hart's 1986 paper that spearheaded the incomplete contracts formalism we employ below—these books study firm *scope* in detail. They analyze when firms integrate, often emphasizing the effect that assets have on investment, mainly in terms of residual control rights.

Our model addresses only firm *scale* (there is a single project) and, thus, to use Coase's term for more generic mergers, combination but not integration. Assets act as collateral to outside creditors; their quality as security determines the extent to which firms must expand in order to generate sufficient pledgeable output to obtain funding; cross-sectional predictions about the correlation between firms' scale and their balance sheet composition arise.

Samuelson imagines a toothpaste concern that, like our firm, grows to obtain finance, but the textbook treatment ignores the business's inner workings, discussing only benefits of scale in manufacturing and the perils of product-market monopoly. We model the organization's behaviour explicitly. In fact, workers are our only addition to the firm-in-corporate finance shell that Rajan describes as "a set of projects with a pattern of investments, cash flows, and liquidation values, with little explanation as to why the projects or people running them should be part of the same firm." Except, we suggest a theory for the missing explanation: the trade-off between the financing advantages and the organizational drawbacks of size.

1.1 Insiders, Outsiders, and Bargaining over Assets

Just over a decade ago Rajan and Zingales shook up the theory of intra-firm ownership set down ten years earlier in Hart and Moore (1990). They emphasized that, in the presence of Williamsonian opportunism ex post, not only property rights to physical assets affected agents' bargaining positions but also that an employee or manager might assume power via the ability to restrict *access* to other resources critical to the business—these include all productive interests within the firm, not only those, like machinery, over which a court can enforce claims to property rights, but also soft capital, like managerial expertise, over which no legal claims are upheld (note that whether ownership can be legally ascribed to a resource depends not only on its own properties but also on the legal system).

Rajan and Zingales's theory explains the structure and size of hierarchies within firms, using a model oriented around managers' attempts to prevent employees from stealing critical resources and starting up on their own, as, to cite a famous example from modern financial history, John Meriwether and his bond arbitrage desk at Solomon Brothers did when they incorporated Long Term Capital Management in 1994. The quants and traders had a flourishing sub-enterprise at Solomon where they, mainly newcomers to Wall Street, absorbed the markets, developed trading algorithms, formulated risk metrics, and, importantly, became friends, learning one another's strengths and weaknesses to exploit the complementarities in their skill sets and personalities. There are no patents on financial products and so the investment bank had only its bonus pool to try to keep the soft assets in place. Their "bribes" failed, but, as in Samuelson's virtual case study and our model below, the team had trouble raising capital for their offshoot and expanded first by bringing in more inside partners in order to raise sufficient outside capital to operate. The bigger team gave them more expertise to lever up with limited partners.

Hart and Moore anticipated Rajan and Zingales's generalization of their earlier theory with their 1994 article on an entrepreneur's "inalienability of human capital", in which he cannot commit not to renegotiate loans taken from a creditor with a low redeployment value for seizable assets. The model applies (roughly) to LTCM's relationship to its creditors when it was in perilous distress four years after having broken off from Solomon. Having lost its initial capital in a market flight to liquidity, the hedge fund could neither hedge nor fund its operations; the Federal Reserve and a consortium of banks bailed out the firm and took it over, leaving the general partners with a comparatively small settlement. Hart and Moore's set-up explains the outsiders' insistence on assuming control (and effective liquidation) with the result that despite having valuable human capital with which to run the business, Meriwether et. al could not commit not to take advantage of their creditors later on. In fact, less than a year later the same loyal group raised 250 million dollars in outside equity with which to employ the same strategy. In Hart and Moore's model, renegotiation dynamics lead to inefficient liquidation. Below, we simply assume that the project's maturity is longer than debt contract's—for example because all lenders are relocating permanently to a Greek island for retirement so the creditor has no one to sell his claim to and the firm cannot roll over its debt—but our motivation for including inefficient early liquidation comes from the earlier model's richer, dynamic micro-foundation.

In our model, a firm is a coalition of workers and a critical resource, which takes on the properties of a public good within the firm. It is non-rival among the workers and, while access is indeed excludable, a worker is productive only if he accesses it. Further, it is a rival good between firms, for example because they use it to compete in similar markets. Thus an entrepreneur with an idea faces a trade-off as he expands his firm: he acquires more productive labour input but must let his new partners in on his idea, weakening his bargaining position during renegotiation. In the language of cooperative game theory that Hart and Moore (1990) employs, the idea is essential to all agents and the entrepreneur is indispensable to the idea. For LTCM the idea was an arbitrage strategy and John Meriwether was the indispensable agent, since without his organization and vision the strategy was worthless.

Our measure of firm size is the cardinality of the coalition. The firm's project requires capital and labour to effect. The firm is long of labour and short of capital and borrows from an outsider against the promise to repay in capital later.

Three frictions hamper a creditor's threat to seize output ex post and thus his willingness to lend ex ante. Firstly, as in the models just described, only a fraction of the assets is transferable. Secondly, the project is specific to the firm and the portion of assets a creditor effectively seizes is still sold at a proportional discount, the size of which reflects the substitutability of the assets with other firms as well as the liquidity of secondary markets. Thirdly, we introduce a fixed cost of seizure, a simple addition that captures the effectiveness of creditors' seizure threats according to financial development, specialization, or the legal environment. As La Porta, Lopez-de Silanes, Shleifer, and Vishny (2000) put it, "One way to think about legal protection of outside investors is that it makes the expropriation technology less efficient." None of these barriers to commitment is necessary for the main result that firms grow to find finance, but we keep track of the distinction among them for the economic insight they offer, especially in comparative static analysis of firm size.

1.2 Explicit Preview

Section 2 formalizes the model in which one indispensable agent controls a critical resource but lacks the capital necessary to put it to use—the paradigmatic starting point of corporate finance theory: an entrepreneur has an idea but no money. He forms a partnership with other penniless workers, thereby making them productive by granting them access to the resource. The partnership is a coalition and the productive coalition is a firm. A coalition is stable if it implements an undominated imputation, where a payoff profile is feasible if a subgame perfect equilibrium of an ensuing extensive game between the firm and an outside creditor supports it. (In the model, the set of undominated imputations is exactly the solution of the game in the sense of von Neumann and Morgenstern (1944).)

Production demands a fixed investment of capital, after which effort exerted in conjunction with the critical resource yields output with decreasing returns. The firm requests a collateralized loan from a creditor who grants it only if the contract awards him sufficient property rights to secure redemption of his outlay, even though he knows that the firm is always opportunistic ex post.

If the firm raises enough cash to effect its technology, workers labour simultaneously and independently to realize output. The creditor demands repayment before the project's cash flows have arrived in the firm. If the firm repays the face value of the loan, then workers bargain over the residual surplus, in particular they divide it according to the Shapley value. If the debtors default, then the creditor threatens to seize his collateral; all parties, however, know not only that the collateral is worth more to the insiders than to the outsider, but also that seizure entails fixed costs. Thus the firm will bribe the creditor not to seize during renegotiation. In the model, the firm makes the lender a take-it-or-leave-it offer to revoke his rights to the collateral.

Section 3 solves the model, building up to the main result that the equilibrium firm size is the smallest firm that can obtain finance with a series of lemmata. Firstly, we prove that outside equity contracts—contracts which are defined only by collateral—implement any equilibrium effort and payoff profiles and thus restrict the contract space to them. Backward induction then yields the equilibrium effort, output, and utilities for any coalition. The key trade-off emerges from the public good nature of the critical resource: as firms become larger effort decreases but output increases. Thus bigger firms are less efficient internally but obtain external finance more easily.

While a small business may be unable to secure finance, if it takes on enough partners the outsider will lend to it. In particular, for each project there is a threshold number of workers above which the firm receives credit. As the size of a productive coalition increases, however, each worker in it obtains a lower payoff.

Subsection 3.3 formalizes the firm-formation process as a coalitional non-transferable utility game, defining the value function via the equilibrium payoff profiles of the extensive game played between the workers and the bank. Proportion 3.1 demonstrates that workers will stop expanding their enterprise as soon as they are many enough to fund their project, saying that all undominated payoff vectors are associated with the smallest coalitions that achieve funding; in fact, the unique solution of the game is the set of all such imputations.

The result yields a formula for the equilibrium size of a firm as a function of its assets, admitting easy comparative static analysis connecting the left-hand side of the firm's balance sheet with its number of workers/ownership structure. Corollary 3.1 states that when assets are more easily transferable or require smaller discounts during liquidation, firms are smaller, as they are when firms are more productive or less dependent on outside capital. Likewise, lower costs of labour and seizure reduce the size of businesses.

The model examines only one trade-off—between waning internal efficiency and the need to meet collateral constraints—and the final section discusses both the limitations of the analysis the the possible empirical insight the model might offer despite its simplicity.

2 Model

After the firm forms, its transactions with the lender and its workers' labour constitute a complete information extensive game with a unique subgame perfect equilibrium payoff. The size of the firm is then the unique cardinality of a stable coalition implementing the project conceived in anticipation of the equilibrium payoffs. The cooperative equilibrium concept is the solution in the von Neumann and Morgenstern (1944) sense, which concedes here with the set of all undominated imputations.

2.1 Firms: Projects and Workers

A firm is a set $\mathbf{N} \subset \mathbb{N}$ of workers with a technology that converts outside capital i and a nonnegative vector $e = (e_1, ..., e_{|\mathbf{N}|})$ of worker effort into firm output with value y,

$$y_{\mathbf{N}} : (i, e) \mapsto A\, \mathbb{1}_{\{i \geq I\}} \left(\sum_{n=1}^{|\mathbf{N}|} e_n \right)^{\beta}, \tag{1}$$

where the outside capital requirement $I > 0$, productivity $A > 0$, and returns to effort $\beta \in (0, 1)$ are production characteristics. Projects also differ in their ability to be pledged as collateral to an outsider; call the proportion of output that is transferable σ. Further, when a fraction L of a project is liquidated early, its value is discounted with a constant λ, i.e. it is worth $\lambda L y$.

Workers borrow i to fund their projects and repay R after implementing them. Each worker n has cost of effort $c(e_n) = c e_n^{\gamma}$ for $c > 0$ and $\gamma > 2$.[1]

Within the organization no contracts are enforceable. Workers choose effort individually and they bargain over the residual surplus, $S = y - R$, modelled by division according to the Shapley value, simply

$$V_n(S; \mathbf{N}) := \frac{S}{|\mathbf{N}|} \tag{2}$$

for each $n \in \mathbf{N}$, since all workers in the firm are identical ex post.

Finally, assume that Worker 1 is essential for the project—thereby restricting attention to firms including him, $1 \in \mathbf{N}$—which simplifies arguments and avoids hassles from multiple equilibria later on.

2.2 Finance: Creditors and Contracts

The firm obtains finance from a competitive lender called the bank. A loan contract $\big(i, (D|C)\big)$ specifies the face value D that the firm owes the bank and the collateral C that backs the promise for a loan of size $i \in \{0, I\}$; the bank has the right to seize C if the firm defaults, namely does not repay D.

[1] The assumption that cost is sufficiently convex eliminates preferences which lead to a free-rider problem so severe that firm expansion decreases not only individual effort but also aggregate effort. The model remains valid for general exponents, but returns to the workerless corporate finance entity: at equilibrium, the only firms are single entrepreneurs who obtain finance only if they can borrow alone.

If $D = \infty$ then the bank is an outside equity holder because the firm never repays in full and thus leaves the bank the residual claimant on C *non-contingently*. If the collateral is the firm itself, $C = y$, then the contract is the standard debt contract on an unencumbered firm. The only collateral the firm has is the project, but it can pledge any fraction F of it to secure its borrowing. Thus the feasible contracts coincide with all covered debt contracts $(D|Fy)$ for $D > 0$ and $F \in [0, 1]$.

The bank enforces repayment only via the threat of seizure. As Schelling (1960) reminds us, effective threats are those not acted upon, and even if it defaults the firm can offer the bank a repayment R as a bribe not to seize the collateral. During this renegotiation, the firm should consider the bank's liquidity benefits of the bribe over the contractually specified assets. If the bank does seize the assets, it takes effective control over only a fraction σ of them, since the remainder is nontransferable capital associated with the workers and suffers from the liquidation discount λ, since it cannot continue operations.

The competition assumption says that the bank will approve a loan whenever it anticipates that its repayment will exceed its outlay, $R \geq i$ (the bank's cost of capital is normalized to nil).

2.3 Timing

When Knight (1921) takes up our subject, terming it "Joint Production and Capitalization", he asserts that his ensuing analysis "will bring a greater semblance of reality into the imaginary, highly simplified economic system partially constructed above," but when he considers the boundaries of the firm he accepts that "violence is done to reality" in his argument, which anticipates coalitional game theory, verbally describing the deviations ("adjustments") of workers into equilibrium groups. Following Knight, we abstract from the intricacies of firm formation and development and consider a set of workers that frictionlessly and stably unites.

The model proceeds sequentially: the firm borrows secured capital from the bank to do its project and workers simultaneously exert effort toward production; output is illiquid, for example realized over a long period of time.

The loan comes due and the firm either repays its face value or defaults, in which case the bank can threaten to seize the assets and the firm can offer it a bribe not too. If the bank accepts the bribe, the firm, still cashless, liquidates a fraction of its assets to pay it. (For simplicity assume that the bank's and firm's liquidation discounts coincide; a wedge between them would only amplify the difference between inside and outside equity.)

Given the assumed absence of enforceable intra-firm contracts, the workers bargain over any residual surplus.

Formally, the timing is as follows (cf. figure 1):

0. Workers form stable coalitions among them **N**, the firm.
1. **N** offers the bank a contract $\big(I, (D|Fy)\big)$.
   - If the bank rejects the contract, the firm and bank receive nil; otherwise the bank give the firm i in exchange for the promise $(D|Fy)$.
2. Each worker n exerts effort e_n independently.
3. The firm either repays D or defaults.
   - If the firm does not default, the workers bargain over the surplus; otherwise,the firm renegotiates the loan.
4. The firm offers the bank a bribe R not to seize the collateral Fy.
   - If the bank rejects the bribe, it seizes Fy and liquidates it; the workers bargain over the remaining surplus. Otherwise, the firm liquidates a fraction L of its assets to pay the the bribe and the workers bargain over the remaining surplus.

Note that a noncooperative game theorist might (uncooperatively) criticize the model, suggesting that first the coalition formation process and second the actions of the entire firm should be modelled as collections of individual worker actions. We suggest that he mentally replace our assumptions with the protocol that Worker 1 makes decisions dictatorially about both the size of the firm and the aggregate firm actions, to obtain identical results.

3 Results

Solving the firm-bank subgame reveals that at equilibrium the effort exertion stage is a public good-provision game in which workers shade their efforts more as the firm gets larger, making production less efficient; however, total output increases with the size of the firm, making it better collateral to pledge for funding, and firms expand until they can credibly promise repayment to the bank. Our cross-sectional predictions follow from comparative statics on firm size, which subgame perfection and coalitional stability determine uniquely.

After simplifying the contract space with a debt-equity equivalence theorem, we employ backward induction to find the equilibrium efforts and utilities for each coalition. All undominated coalitions containing the essential

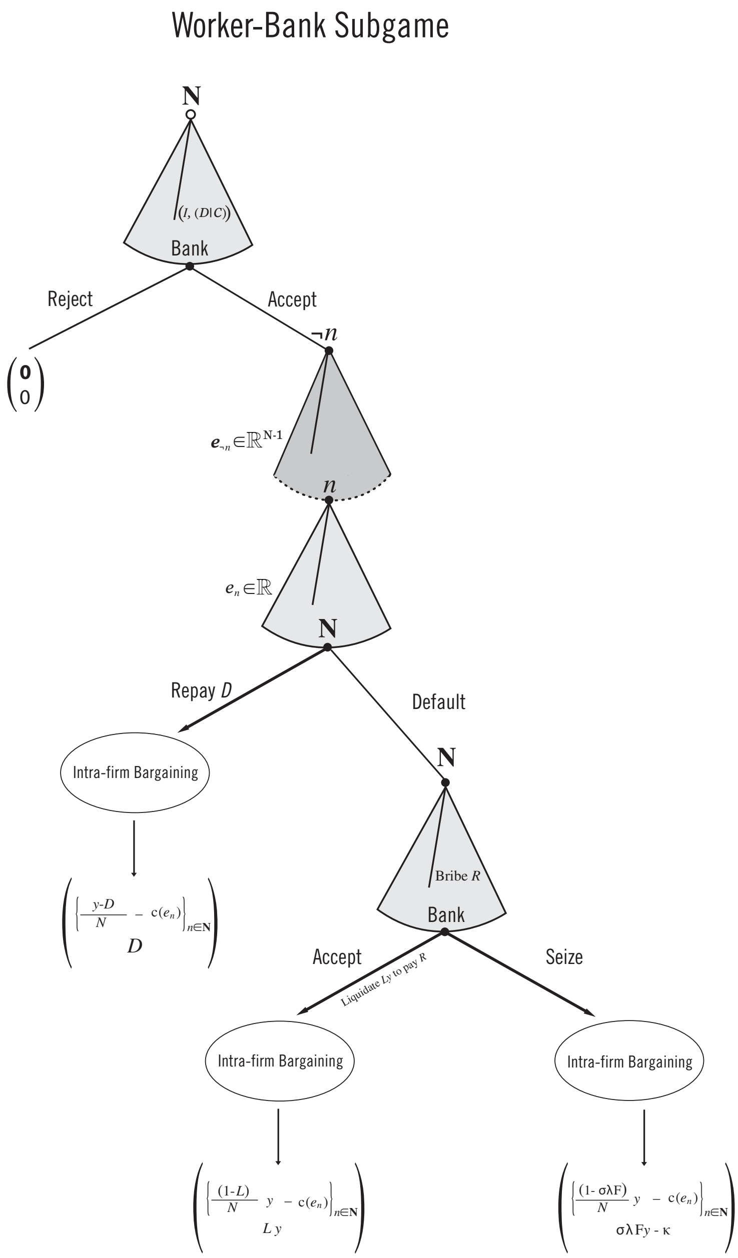

Figure 1: A pictorial representation of the game among the workers and the bank.

worker have the same cardinality, namely there is a unique equilibrium firm size.

3.1 Debt-(Outside) Equity Equivalence

The equivalence of debt and outside equity results from anticipated renegotiation without uncertainty, given that the two contracts ultimately induce the same deadweight loss from liquidation. The incremental difficulty over Hart and Moore's 1998 equivalence of the fastest (equity-like) and slowest (debt-like) loan contracts without randomness is showing that the workers' effort profile remains unchanged.

Lemma 3.1. *If $\big(I, (D|Fy)\big)$ is an equilibrium contract corresponding to equilibrium effort profile* $\mathbf{e}$ *and repayment I, then there is F' such that $\big(I, (\infty|F'y)\big)$ is also an equilibrium contract corresponding to effort profile e and repayment I.*

Proof. The proof, which simply demonstrates equivalence via iterative changes of face value and backing collateral is in appendix A.1

□

3.2 Backward Induction: Effort and Contracts

Focusing now only on contracts of the form $\big(\infty, (D|Fy)\big)$, we find the subgame perfect equilibrium given $\mathbf{N}$.

3.2.1 Renegotiation and Liquidation

The firm now always defaults. If the bank accepts the bribe R, then, since $\lambda < 1$ the firm liquidates the smallest fraction L possible to cover R, viz.

$$R = L\lambda y. \tag{3}$$

In order for the bank to accept the bribe, the repayment must exceed the value of its collateral Fy given seizure. Taking into account that the bank pays κ to appropriate only a fraction σ of the output, effectively, which it then liquidates, that is

$$R \geq \sigma\lambda Fy - \kappa. \tag{4}$$

The firm's optimal offer makes the equality bind. Thanks to the deadweight loss κ, the firm is always better off bribing and not renegotiating, and hence always anticipates liquidating the fraction

$$L = \sigma F - \frac{\kappa}{\lambda y}. \tag{5}$$

The equilibrium surplus over which the workers bargain is therefore

$$S = \big(1 - L\big)y \tag{6}$$
$$= \big(1 - \sigma F\big)y + \frac{\kappa}{\lambda}. \tag{7}$$

3.2.2 Effort Exertion

Worker n's utility is

$$u_n\big(e_n, e_{\neg n}; \mathbf{N}, F\,\big) = V_n\big(S;\, \mathbf{N}\big) - c(e_n) \tag{8}$$
$$= \frac{\big(1 - \sigma F\big)y + \kappa/\lambda}{|\mathbf{N}|} - c\, e^{\gamma} \tag{9}$$
$$= \frac{A\, \mathbb{1}_{\{i \geq I\}}\big(1 - \sigma F\big)}{|\mathbf{N}|} \left(\sum_{m=1}^{|\mathbf{N}|} e_m\right)^{\beta} - c\, e^{\gamma} + \frac{\kappa}{\lambda|\mathbf{N}|}. \tag{10}$$

If the bank does not lend to the firm, $i = 0$, then Worker n's best response to any effort profile $e_{\neg n} := (e_1, ..., e_{n-1}, e_{n+1}, ..., e_{|\mathbf{N}|})$ is $e_n\big(e_{\neg n};\, \mathbf{N}\big) = 0$; if the bank does lend, $i = I$, then Worker n's programme is convex and its solution is defined by the first-order condition

$$\frac{\partial u_n}{\partial e_n}\big(e_n, e_{\neg n}\big) = 0, \tag{11}$$

or

$$\gamma c\, |\mathbf{N}|\, e_n^{\gamma - 1} = A\beta\big(1 - \sigma F\big) \left(\sum_{m=1}^{|\mathbf{N}|} e_m\right)^{\beta - 1}. \tag{12}$$

Observe that the above Nash conditions for the one-shot effort-exertion game conditional on funding imply that any equilibrium is symmetric, $e_n = e_m$ for all $n, m \in \mathbf{N}$, since the right-hand sides coincide for all workers; at equilibrium

$$\sum_{m=1}^{|\mathbf{N}|} e_m = |\mathbf{N}|\, e_n \tag{13}$$

for any n, and therefore

$$\gamma c|\mathbf{N}|e_n^{\gamma - 1} = A\beta\big(1 - \sigma F\big)|\mathbf{N}|^{\beta - 1} e_n^{\beta - 1} \tag{14}$$

or

$$e_n\big(\mathbf{N}\big) = \left(\frac{A\beta\big(1 - \sigma F\big)}{\gamma c}\right)^{\frac{1}{\gamma - \beta}} |\mathbf{N}|^{-\frac{2 - \beta}{\gamma - \beta}} \tag{15}$$

for every worker n. The equilibrium output given funding is thus

$$y = A\left(\sum_{m=1}^{|\mathbf{N}|} e_m\right)^{\beta} \tag{16}$$

$$= \left(\frac{A^{\gamma/\beta}\beta}{\gamma c}\right)^{\frac{\beta}{\gamma-\beta}} \big(1-\sigma F\big)^{\frac{\beta}{\gamma-\beta}} \big|\mathbf{N}\big|^{\frac{\beta(\gamma-2)}{\gamma-\beta}} \tag{17}$$

$$= J_{|\mathbf{N}|}\big(1-\sigma F\big)^{\frac{\beta}{\gamma-\beta}} \tag{18}$$

where

$$J_N := \left(\frac{A^{\gamma/\beta}\beta}{\gamma c}\right)^{\frac{\beta}{\gamma-\beta}} N^{\frac{\beta(\gamma-2)}{\gamma-\beta}}. \tag{19}$$

3.2.3 Pledged Collateral

At the contracting stage, each worker's utility is (just plugging in)

$$u_n\big(F;\,\mathbf{N}\big) = K_{|\mathbf{N}|}\,\big(1-\sigma F\big)^{\frac{\gamma}{\gamma-\beta}} + \frac{\kappa}{\lambda|\mathbf{N}|} \tag{20}$$

conditionally on $i = I$, where K is a function depending only on fixed parameters and the size of the firm,

$$K_N := \left(\frac{A\beta}{\gamma\, c^{\beta/\gamma}}\right)^{\frac{\gamma}{\gamma-\beta}} N^{-\frac{\gamma\,(2-\beta)}{\gamma-\beta}} \left(\frac{\gamma N}{\beta} - 1\right) > 0. \tag{21}$$

Note that K is decreasing in firm size, $K' < 0$.

Since u_n is decreasing in the fraction F backing the loan and $u_n > 0$ for all $F \in [0,1]$, it offers the bank the contract $\big(I, (\infty|Fy)\big)$ with the minimum F that induces the bank to grant the loan, it if exists. The bank accepts the firm's request for credit whenever the equilibrium repayment exceeds the loan size, or

$$I \le L\lambda y = \left(\sigma F - \frac{\kappa}{\lambda y}\right)\lambda y \tag{22}$$

$$= \lambda\sigma F y - \kappa \tag{23}$$

$$= \lambda J_{|\mathbf{N}|}\sigma F\big(1-\sigma F\big)^{\frac{\beta}{\gamma-\beta}} - \kappa. \tag{24}$$

Now, define

$$F_0 := \inf\left\{F \in [0,1]\ ;\ \left(\frac{I+\kappa}{\sigma\lambda J_{|\mathbf{N}|}}\right)^{\frac{\gamma-\beta}{\beta}} = F^{\frac{\gamma-\beta}{\beta}}\,(1-\sigma F)\right\} \tag{25}$$

and keep in mind that the greatest lower bound of an empty set is infinite. The firm has enough collateral to secure a loan only when F_0 is finite; otherwise the bank will not lend.

Lemma 3.2. *If $F_0 = \infty$, then the firm obtains no funding; otherwise, $\big(I, (\infty | F_0\, y)\big)$ is an equilibrium contract.*

Proof. The proof in appendix A.2 simply demonstrates that the bank's constraint is satisfied on exactly a closed (possibly empty) subset of $(0, 1]$. Since the bank's objective is decreasing in F, the equilibrium contract is the smallest member of this set, if it exists. Otherwise the set of feasible contracts with $i = I$ satisfying the bank's constraint is empty and $i = 0$ or the bank rejects the firm's offer. $\square$

The finance condition in lemma 3.2 demonstrates the firm's need for collateral to fund its technology, and the condition for F_0's finiteness translates to say that firms with more workers have more pledgeable assets and establishes explicit bounds on firm size necessary and sufficient for funding; call N^* the number of workers in the smallest firm that obtains funding.

Lemma 3.3. *If $(1-\sigma)\gamma \geq \beta$, then $F_0 < \infty$ whenever*

$$\big|\mathbf{N}\big| \geq N^* := \left\lceil J^{-1}\left(\frac{I+\kappa}{\lambda\sigma\big(1-\sigma\big)^{\frac{\beta}{\gamma-\beta}}}\right)\right\rceil . \tag{26}$$

If $(1-\sigma)\gamma < \beta$, then $F_0 < \infty$ whenever

$$\big|\mathbf{N}\big| \geq N^* := \left\lceil J^{-1}\left(\frac{\gamma(I+\kappa)}{\lambda(\gamma-\beta)}\left(\frac{\gamma}{\beta}\right)^{\frac{\beta}{\gamma-\beta}}\right)\right\rceil . \tag{27}$$

Proof. The proof in appendix A.3 follows from considering when the maximum of the mapping

$$F \mapsto F^{\frac{\gamma-\beta}{\beta}}\big(1-\sigma F\big) \tag{28}$$

on $[0,1]$ is greater than the constant $(I+\kappa)^{\frac{\gamma-\beta}{\beta}}(\sigma\lambda J_{|\mathbf{N}|})^{-\frac{\gamma-\beta}{\beta}}$ (cf. the definition of F_0 in equation 25). The two cases divide between when the maximizer is interior and when it occurs at $F = 1$.

$\square$

3.3 Equilibrium Firm Size

Anticipating their welfare within different groups according to the equilibrium extensive game between the firms' workers and the bank, all workers arrange themselves in stable coalitions; a productive coalition defines a firm.

The absence of enforceable contracts within the firm implies that the value function $\mathfrak{v}$ of the coalitional non-transferable utility game[2] awards gives each player the equilibrium payoff $U_{|\mathbf{N}|}$ if he is a worker in the productive firm $\mathbf{N}$ and zero otherwise, i.e. for each $\mathbf{N} \in 2^{\mathbb{N}}$, $\mathfrak{v}(\mathbf{N}) \in \mathbb{R}^{|\mathbf{N}|}$ with

$$\big(\mathfrak{v}(\mathbf{N})\big)_n = \begin{cases} U_{|\mathbf{N}|} & \text{if } 1 \in \mathbf{N} \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

for each $n \in \mathbf{N}$. Critically, each worker in a productive firm would be worse off were his firm any larger (cf. figure 2).

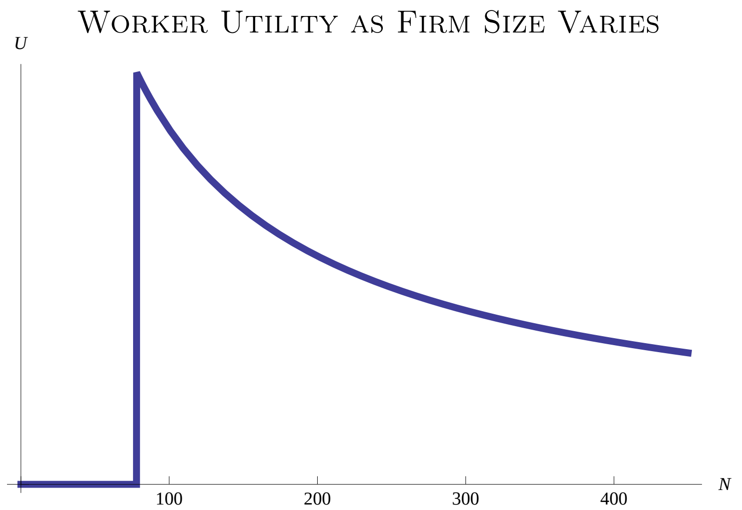

Figure 2: Plot of U_N for parameterization $\beta = 1/2$, $\gamma = 3$, $c = 1$, $I = 1$, $\sigma = 1/3$, $A = 2$, $\lambda = 3/4$, $\kappa = 1/4$.

Lemma 3.4. *Given a firm* $\mathbf{N}$*, if* $i = I$*, equilibrium utility* U_N *is decreasing in* $N = |\mathbf{N}|$ *for each* $n \in \mathbf{N}$*.*

Proof. The proof in appendix A.4 follows from direct computation of the derivative using implicit differentiation for F_0 and demonstrating that sufficient conditions for negativity hold.

[2]Typically, the value function of a coalitional non-transferable utility game is set-valued (see, for example, the text Peleg and Sudhölter (2007)), but, since here coalitions determine unique equilibrium payoffs, we write the function directly as a payoff profile rather than as a singleton set.

$\square$

Given that within funded firms all remaining workers are better off if their firm downsizes, at equilibrium firms are the smallest they can be while still being large enough to pledge sufficient collateral to borrow I: $|\mathbf{N}| = N^*$. In particular, any solution of the game—namely a set of internally and externally stable imputations—is the set of payoff profile with exactly N^* nonzero entries.[3]

Proposition 3.1. *The unique solution $\mathcal{S}$ of the firm-formation game is the set of payoff profiles for which exactly N^* workers obtain positive utility*[4]*,*

$$\mathcal{S} = \left\{ u = (u_n)_{n=1}^{\infty} \,\middle|\, |\mathrm{supp}(u)| = N^* \right\}. \tag{30}$$

Therefore, all productive firms have exactly N^* workers.

Proof. The proof makes use of the observation that since for $|\mathbf{N}| < N^*$ $\mathfrak{v}(\mathbf{N}) = 0$ and U_N is decreasing for $N > N^*$,

$$\sup \left\{ u_1 = \big(\mathfrak{v}(\mathbf{N})\big)_1 \,\middle|\, \mathbf{N} \in 2^{\mathbb{N}} \right\} = U_{N^*}. \tag{31}$$

We show, firstly, that if $|\mathrm{supp}(u)| = N^*$, then u is undominated, and therefore $u \in \mathcal{S}$ and, secondly, that if $|\mathrm{supp}(v)| \neq N^*$, then a payoff profile u with $|\mathrm{supp}(u)| = N^*$ dominates it, and therefore $v \notin \mathcal{S}$.

Suppose (in anticipation of a contraction) that $|\mathrm{supp}(u)| = N^*$ and a payoff profile v dominates u via some coalition $\mathbf{M}$. $1 \in \mathbf{M}$ since otherwise $\mathfrak{v}(\mathbf{M}) = 0$. But $v_1 \leq u_1$ since $u_1 = U_{N^*}$ is maximal. Thus no v can dominate u, a contraction.

Now suppose (again in anticipation of a contradiction) that $|\mathrm{supp}(v)| \neq N^*$ and $v \in \mathcal{S}$. $|\mathrm{supp}(v)| > N^*$ since otherwise $v = 0$. But, from above, there is $u \in \mathcal{S}$ with $|\mathrm{supp}(u)| = N^*$ and, since U_N is decreasing above N^*, $U_{|\mathrm{supp}(v)|} < U_{|\mathrm{supp}(u)|}$. Thus, for each $n \in \mathrm{supp}(u)$, $u_n > v_n$, or u dominates v via the coalition $\mathrm{supp}(u)$. By internal stability, $v \notin \mathcal{S}$, a contradiction.

The two contradictions above combine to assert that a payoff profile belongs to the solution of the game if and only if it is supported by a set of cardinality N^*. $\square$

[3]See, for example, Luce and Raiffa (1957) for the definition that a set of imputations $\mathcal{S}$ is a solution of a game if (i) for all $u, v \in \mathcal{S}$ no coalition exists with respect to which u dominates v and (ii) for all $w \in \mathcal{S}^c$ there is a a payoff profile in $\mathcal{S}$ that dominates w with respect to some coalition.

Note that u dominates v with respect to the coalition $\mathbf{N}$ if $v \in \mathfrak{v}(\mathbf{N})$ and $u_n > v_n$ for all $n \in \mathbf{N}$.

[4]Here the support of a sequence is the set of indices upon which the sequence assumes a nonzero value, for $u = \{u_n\}_{n=1}^{\infty}$, $\mathrm{supp}(u) := \{n \in \mathbb{N} \,|\, u_n \neq 0\}$.

3.4 Comparative Statics

Given proposition 3.1 implies an expression for the equilibrium firm size, comparative static analysis is on tap.

Corollary 3.1. *The size of the firm N^* exhibits the following comparative statics: N^* is increasing in κ, I, and c and deceasing in σ, λ, and A.*

Proof. The proof in appendix A.5 computes the derivatives of the lower bounds for funding directly.

□

Firm size comes from the need to expand to pledge collateral to raise funds to invest, so all of the comparative statics result from the effects of parameters on assets' quality as bank security. Firm size increases in κ because when lenders must pay higher seizure costs, their threats are less effective and they require more collateral backing. From the point of view of the lender, in fact, larger seizure costs have the same effect as a larger loan or higher I. When labour is more costly, i.e. c is higher, workers are less productive and thus output is lower, meaning again that the lender requires a bigger firm to generate enough security to fund I. The more transferable are assets, viz. the higher is σ, the better they can serve as collateral and thus the smaller are firms. However, when transferability is high, $\sigma > 1 - \beta/\gamma$, firm size no longer changes with σ since the firm's ability to pledge more capital replaces its need to grow (see appendix A.3). When liquidation value λ is high the collateral is simply more valuable to the bank so it needs less of it and increasing λ decreases firm size. Finally, when firms are productive, A is high, asset values are high in the future, meaning the bank is happy to lend against them.

4 Concluding Discussion

The model studies only one trade-off—between the need to expand to obtain finance and the costs of internal inefficiencies—and the noncontractibility assumptions are stark both within firms and between firms and banks, but it may still offer some insight to help explain some real world cross-sectional variation in firm size. That lower fixed costs of seizure lead to larger firms jives with Rajan and Zingales's observation that firms break up following periods of financial innovation, as well as abundant suggestions from regulators and the press that private equity firms engage in asset stripping (see, for example, Jones (2010)). Further, the Kumar, Rajan, and Zingales demonstrate that R&D intensive firms, capital intensive firms, and firms reliant on external funding are large, as are firms in our model that rely on human capital and large outside investments.

As Bloom, Mahajan, McKenzie, and Roberts (2010) summarizes, workers are unproductive in the developing world. The authors cite survey evidence that firms in poor countries, where weak legal systems grant less power to creditors, are likely to be financially constrained, especially if they are small. They then speculate that collateral constraints bias their investments inefficiently toward physical capital. While their findings add credibility to our results, our model may also add another finance-based explanation for the inefficiencies they discuss: entrepreneurs with insufficient collateral to fund their projects must join larger firms where they will be less productive. The labour market effect of credit constraints is to inhibit allocation of human capital.

Our model demonstrates that the need for finance can determine firm size, and reveals the interaction between the need to expand collateral and the left-hand side of firms' balance sheets as an explanatory factor of variation of average firm size across industries or financial regimes.

Since Coase (1937), economists have put forward many other theories to attempt to explain the size of the firm, highlighting various reasons that firms may be big or small. For example, those we build on most explicitly, due to Grossman and Hart (1986) and Rajan and Zingales (2001), focus on critical resources and relationship-specific investments between firms or within organizations. Others—such as Marcus (1982) and Giannetti (2001)—suggest risk sharing yields returns to scale. Incentive provision is a key function of the business enterprise, and papers like Baker and Hall (2004) and Fulghieri and Sevilir (2011) connect it with scale, while the role of diffuse information in decision making is the determinant of firm size in Radner and Van Zandt (1992) and in Meyer, Milgrom, and Roberts (1992). We aim not to refute any theory, but rather to suggest that the need to overcome financial constraints may be another relevant determinant of firm size.

A Appendices

A.1 Proof of Lemma 3.1

We divide the proof into six short steps, each now proved separately.

Step 1. $D \geq I$.

Proof. Suppose $D < I$, then

$$R = \min \{\sigma\lambda Fy - \kappa, D\} \leq D < I \tag{32}$$

and $R < I$ means the bank cannot break even and will always reject the offer.

□

Step 2. If $D > I$, then there is always default.

Proof. Suppose $D > I$ and there is no default, therefore the repayment is

$$R = \min \{\sigma\lambda FY(D) - \kappa, D\} = D > I \tag{33}$$

where $y = Y(D)$ emphasizes that y is endogenous.

But then there exists $D' \in (I, D)$ such that $Y(D') = Y(D)$ (note that the small change in debt level does not change effort provision incentives, since it's an additive term) and therefore

$$R' = D' > I \tag{34}$$

is a profitable deviation for the firm. □

Step 3. If there is always default, then it is without loss to say $D = \infty$ so either $(D = I|FY)$ or $(D = \infty|FY)$.

Step 4. $(I|FY)$ is an equilibrium then it is allocationally equivalent to the equilibrium $(I|F'Y)$ where

$$\sigma\lambda F'Y_{(I|FY)} - \kappa = I. \tag{35}$$

Proof. Suppose $(D = I|FY)$ is an equilibrium contract. Then, by the bank's break-even condition

$$\min \left\{\sigma\lambda FY_{(I|FY)} - \kappa, I\right\} \geq I, \tag{36}$$

it must be that

$$\sigma\lambda FY_{(I|FY)} - \kappa \geq I \tag{37}$$

and therefore either

$$\sigma\lambda F Y_{(I|FY)} - \kappa = I \tag{38}$$

in which case $F' = F$ or

$$\sigma\lambda F Y_{(I|FY)} - \kappa > I \tag{39}$$

in which case F does not affect the firm's payoff directly, and for any F' between the equilibrium F and the F which binds the constraint, incentives remain unchanged. That is, for any $F' \in [F, \frac{I+\kappa}{\sigma\lambda Y_{(I|FY)}}]$ nothing is changed.

□

Step 4 says that if $D = I$, we can restrict attention to contracts of the form

$$\left(I \;\middle|\; \left(\frac{I+\kappa}{\sigma\lambda Y_{(I|FY)}} \right) Y_{(I|FY)} \right) \tag{40}$$

Step 5. If $(I|FY)$ with

$$\sigma\lambda F Y_{(I|FY)} - \kappa = I \tag{41}$$

or

$$F = \frac{I+\kappa}{\sigma\lambda Y_{(I|FY)}} \tag{42}$$

at equilibrium, then default is weakly optimal and allocationally equivalent to no default .

Proof. Given F, after exerting effort and realizing Y, the firm's repayment given .

$$R_{\text{default}} = \sigma\lambda F Y_{(I|FY)} - \kappa \tag{43}$$

$$= I + \kappa - \kappa \tag{44}$$

$$= I \tag{45}$$

and the repayment given no default is by definition the face value I. Thus at any previous stage anticipating default and anticipating no default are equivalent.

□

Step 6. If $(I|FY)$ with

$$\sigma\lambda F Y_{(I|FY)} - \kappa = I \tag{46}$$

or

$$F = \frac{I+\kappa}{\sigma\lambda Y_{(I|FY)}} \tag{47}$$

at equilibrium, then $(\infty\,|FY)$ is an allocationally equivalent contract.

Proof. Immediate from Step 3 and Step 5. □

Steps 1 to 6 imply that it is without loss of generality to consider always the fastest contract, $D = \infty$.

A.2 Proof of Lemma 3.2

Since the firm's objective is decreasing in the fraction pledged F, the equilibrium F, if it exists, must be the smallest $F \in [0, 1]$ satisfying the constraint,

$$I \leq \lambda J_{|\mathbf{N}|}\sigma F\big(1 - \sigma F\big)^{\frac{\beta}{\gamma-\beta}} - \kappa \tag{48}$$

or

$$\left(\frac{I + \kappa}{\lambda J_{|\mathbf{N}|}}\right)^{\frac{\gamma-\beta}{\beta}} \frac{1}{1 - \sigma F} \leq (\sigma F)^{\frac{\gamma-\beta}{\beta}}, \tag{49}$$

which is clearly satisfied neither when $F = 0$ or as $F \to 1/\sigma > 1$. Since both sides of the inequality are continuous in F and the inequality is weak, the set upon which the inequality holds is closed. Further, if the inequality is satisfied for any F, then the equality holds for some F (cf. lemma 3.3).

A.3 Proof of Lemma 3.3

F_0 is finite when

$$\left(\frac{I + \kappa}{\sigma\lambda J_{|\mathbf{N}|}}\right)^{\frac{\gamma-\beta}{\beta}} \leq \sup\left\{F^{\frac{\gamma-\beta}{\beta}}(1 - \sigma F)\ ;\ F \in [0, 1]\right\}. \tag{50}$$

The mapping

$$F \mapsto F^{\frac{\gamma-\beta}{\beta}}\big(1 - \sigma F\big) \tag{51}$$

is increasing at $F = 0$ and has a unique global maximum on $[0, \infty)$ achieved at the solution to the first-order condition

$$\frac{\partial}{\partial F}\Big(F^{\frac{\gamma-\beta}{\beta}}\big(1 - \sigma F\big)\Big) = 0 \tag{52}$$

or

$$\frac{\gamma - \beta}{\beta}F^{\frac{\gamma-\beta}{\beta}-1} - \frac{\sigma\gamma}{\beta}F^{\frac{\gamma-\beta}{\beta}} = 0 \tag{53}$$

or

$$\gamma - \beta - \sigma\gamma F = 0, \tag{54}$$

which is to say that the maximizer for on $[0,1]$ is

$$F = \min\left\{\frac{\gamma-\beta}{\sigma\gamma}\,,\,1\right\}. \tag{55}$$

If $\gamma(1-\sigma) \geq \beta$, a solution exists whenever

$$\frac{I+\kappa}{\sigma\lambda J_{|\mathbf{N}|}} \leq \big(1-\sigma\big)^{\frac{\beta}{\gamma-\beta}} \tag{56}$$

or

$$J_{|\mathbf{N}|} \geq \frac{I+\kappa}{\lambda\sigma\big(1-\sigma\big)^{\frac{\beta}{\gamma-\beta}}}. \tag{57}$$

If $\gamma(1-\sigma) < \beta$, a solution exists whenever

$$\frac{I+\kappa}{\sigma\lambda J_{|\mathbf{N}|}} \leq \frac{\gamma-\beta}{\sigma\gamma}\left(1-\frac{\gamma-\beta}{\gamma}\right)^{\frac{\beta}{\gamma-\beta}} \tag{58}$$

or

$$J_{|\mathbf{N}|} \geq \frac{\gamma(I+\kappa)}{\lambda(\gamma-\beta)}\left(\frac{\gamma}{\beta}\right)^{\frac{\beta}{\gamma-\beta}}. \tag{59}$$

A.4 Proof of Lemma 3.4

Firstly, since F_0 solves

$$\left(\frac{I+\kappa}{\sigma\lambda}\right)^{\frac{\gamma-\beta}{\beta}} J_N^{-\frac{\gamma-\beta}{\beta}} = F_0^{\frac{\gamma-\beta}{\beta}}\big(1-\sigma F_0\big) \tag{60}$$

and note here that

$$\left(\frac{I+\kappa}{\sigma\lambda}\right)^{\frac{\gamma-\beta}{\beta}} J_N^{-\frac{\gamma-\beta}{\beta}} \geq \big(1-\sigma\big)F_0^{\frac{\gamma-\beta}{\beta}} \tag{61}$$

Implicit differentiation of equation 60 with respect to $N = |\mathbf{N}|$ gives

$$\begin{aligned}
-\frac{\gamma-\beta}{\beta}\left(\frac{I+\kappa}{\sigma\lambda}\right)^{\frac{\gamma-\beta}{\beta}}\frac{J'}{J^{\gamma/\beta}} &= \frac{\gamma-\beta}{\beta}F_0^{\frac{\gamma-\beta}{\beta}-1}\big(1-\sigma F_0\big)F_0' - \sigma F_0^{\frac{\gamma-\beta}{\beta}}\big(1-\sigma F_0\big)F_0' \\
&= F_0^{\frac{\gamma-\beta}{\beta}}\big(1-\sigma F_0\big)\left(\frac{\gamma-\beta}{\beta F_0}-\sigma\right)F_0' \\
&= \left(\frac{I+\kappa}{\sigma\lambda}\right)^{\frac{\gamma-\beta}{\beta}} J^{-\frac{\gamma-\beta}{\beta}}\left(\frac{\gamma-\beta}{\beta F_0}-\sigma\right)F_0'
\end{aligned} \tag{62}$$

so

$$F_0' = \frac{F_0\, J'/J}{\beta\sigma F_0/(\gamma-\beta) - 1}. \tag{63}$$

From equation 19,

$$\frac{J'}{J} = \frac{\beta(\gamma-2)}{(\gamma-\beta)N} \tag{64}$$

and thus

$$F_0' = \frac{(\gamma-2)F_0}{1+\sigma F_0 - \gamma/\beta}\,\frac{1}{N}. \tag{65}$$

The magnitude of F_0' is highest if $F_0 = 1$, so

$$\begin{aligned}
\left|F_0'\right| &\le \left|\frac{\gamma-2}{1+\sigma-\gamma/\beta}\right|\,\frac{1}{N} && (66)\\
&= \left|\frac{\gamma-2}{\gamma/\beta-1-\sigma}\right|\,\frac{1}{N} && (67)\\
&\le \left|\frac{\gamma-2}{\gamma/\beta-2}\right|\,\frac{1}{N} && (68)\\
&= \left|\frac{\gamma-2}{\gamma-2\beta}\right|\,\frac{\beta}{N} && (69)\\
&< \frac{\beta}{N}. && (70)
\end{aligned}$$

Now, from equation 20, u to be decreasing in N when

$$\frac{\partial}{\partial N}\left(K_N\big(1-\sigma F_0\big)^{\frac{\gamma}{\gamma-\beta}}\right) < \frac{\kappa}{\lambda N^2}. \tag{71}$$

A sufficient condition for this is:

$$K_N'\big(1-\sigma F_0\big)^{\frac{\gamma}{\gamma-\beta}} - \frac{\sigma\gamma}{\gamma-\beta}K_N\big(1-\sigma F_0\big)^{\frac{\gamma}{\gamma-\beta}-1}F_0' < 0 \tag{72}$$

since $K_N' < 0$, $F_0' < 0$, and $\sigma F_0 < 1$, the inequality is satisfied if

$$\frac{K_N'}{K_N} < \frac{\sigma\gamma}{\gamma-\beta}\,\frac{F_0'}{1-\sigma F_0} \tag{73}$$

or

$$\left|\frac{K_N'}{K_N}\right| > \frac{\sigma\gamma}{\gamma-\beta}\frac{\left|F_0'\right|}{1-\sigma F_0} \tag{74}$$

which is satisfied if

$$\left|\frac{K_N'}{K_N}\right| > \frac{\sigma\beta\gamma/N}{(\gamma-\beta)(1-\sigma F_0)} \tag{75}$$

equation 60 says that $F_0 \to 0$ for N large.

A.5 Proof of Corollary 3.1

Define

$$\hat{N}_1 = \left[\frac{I+\kappa}{\lambda\sigma(1-\sigma)^{\beta/(\gamma-\beta)}}\right]^{\frac{\gamma-\beta}{\beta(\gamma-2)}} \left(\frac{A^{\gamma/\beta}}{\gamma c}\right)^{-\frac{1}{\gamma-2}} \tag{76}$$

and

$$\hat{N}_2 = \left[\frac{\gamma(I+\kappa)}{\lambda(\gamma-\beta)}\right]^{\frac{\gamma-\beta}{\beta(\gamma-2)}} \left(\frac{\gamma}{\beta}\right)^{\frac{1}{\gamma-2}} \left(\frac{A^{\gamma/\beta}}{\gamma c}\right)^{-\frac{1}{\gamma-2}} \tag{77}$$

so that

$$N^* = \begin{cases} \lceil \hat{N}_1 \rceil & \text{if } (1-\sigma)\gamma \geq \beta \\ \lceil \hat{N}_2 \rceil & \text{otherwise.} \end{cases} \tag{78}$$

To demonstrate the comparative statics on the (nondifferentiable) N^*, show that the $\hat{N}_1$ and $\hat{N}_2$ are monotonic in all their arguments and therefore the analysis carries over to N^*. Further all first partial derivatives of $\hat{N}_1$ and $\hat{N}_2$ have the same signs.

For $\hat{N}_1$:

$$\frac{\partial \hat{N}_1}{\partial \kappa} = \left(\frac{\gamma-\beta}{\beta(\gamma-2)(I+\kappa)}\right) \hat{N}_1 > 0, \tag{79}$$

$$\frac{\partial \hat{N}_1}{\partial I} = \frac{\partial \hat{N}_1}{\partial \kappa} > 0, \tag{80}$$

$$\frac{\partial \hat{N}_1}{\partial \sigma} = \frac{-(\gamma-\beta)(1-\sigma)+\beta\lambda\sigma}{\beta\lambda\sigma(\gamma-2)(1-\sigma)} \hat{N}_1 < 0, \tag{81}$$

$$\frac{\partial \hat{N}_1}{\partial \lambda} = -\left(\frac{\gamma-\beta}{\beta(\gamma-2)\lambda}\right) \hat{N}_1 < 0, \tag{82}$$

$$\frac{\partial \hat{N}_1}{\partial c} = \frac{1}{(\gamma-2)c} \hat{N}_1 > 0, \tag{83}$$

$$\frac{\partial \hat{N}_1}{\partial A} = -\frac{\gamma}{\beta(\gamma-2)A} \hat{N}_1 < 0. \tag{84}$$

For $\hat{N}_2$:

$$\frac{\partial N_2^*}{\partial \kappa} = \left(\frac{\gamma - \beta}{\beta(\gamma - 2)(I + \kappa)} \right) N_2^* > 0, \tag{85}$$

$$\frac{\partial N_2^*}{\partial I} = \frac{\partial N_2^*}{\partial \kappa} > 0, \tag{86}$$

$$\frac{\partial N_2^*}{\partial \lambda} = -\left(\frac{\gamma - \beta}{\beta(\gamma - 2)\lambda} \right) N_2^* < 0, , \tag{87}$$

$$\frac{\partial N_2^*}{\partial c} = \frac{1}{(\gamma - 2)c} N_2^* > 0, \tag{88}$$

$$\frac{\partial N_2^*}{\partial A} = -\frac{\gamma}{\beta(\gamma - 2)A} N_2^* < 0. \tag{89}$$

References

Baker, G. P. and B. J. Hall (2004, October). CEO incentives and firm size. *Journal of Labor Economics 22*(4), 767–798.

Bloom, N., A. Mahajan, D. McKenzie, and J. Roberts (2010, May). Why do firms in developing countries have low productivity? *American Economic Review 100*(2), 619–23.

Bolton, P. and D. S. Scharfstein (1990, March). A theory of predation based on agency problems in financial contracting. *American Economic Review 80*(1), 93–106.

Coase, R. H. (1937). The nature of the firm. *Economica 4*(16), 386–405.

Fulghieri, P. and M. Sevilir (2011). Mergers, spinoffs, and employee incentives. *Review of Financial Studies 24*(7), 2207–2241.

Giannetti, M. (2001, November). Risk sharing and firm size: theory and international evidence. Working paper series in economics and finance, Stockholm School of Economics.

Grossman, S. J. and O. D. Hart (1986, August). The costs and benefits of ownership: A theory of vertical and lateral integration. *Journal of Political Economy 94*(4), 691–719.

Hart, O. and J. Moore (1989, May). Default and renegotiation: A dynamics model of debt. Working Paper 520, MIT.

Hart, O. and J. Moore (1990, December). Property rights and the nature of the firm. *Journal of Political Economy 98*(6), 1119–58.

Hart, O. and J. Moore (1994, November). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics 109*(4), 841–79.

Hart, O. and J. Moore (1998, February). Default and renegotiation: A dynamic model of debt. *The Quarterly Journal of Economics 113*(1), 1–41.

Jones, H. (2010, September). Private equity faces EU curbs on asset stripping.

Knight, F. (1921). *Risk, Uncertainty and Profit.* Hart, Schaffner & Marx prize essays. Houghton Mifflin.

Kumar, K. B., R. G. Rajan, and L. Zingales (1999, July). What determines firm size? NBER Working Papers 7208, National Bureau of Economic Research, Inc.

La Porta, R., F. Lopez-de Silanes, A. Shleifer, and R. Vishny (2000). Investor protection and corporate governance. *Journal of Financial Economics 58*(1-2), 3–27.

Luce, R. and H. Raiffa (1957). *Games and Decisions: Introduction and Critical Survey*. Wiley.

Marcus, A. J. (1982, Autumn). Risk sharing and the theory of the firm. *Bell Journal of Economics 13*(2), 369–378.

Meyer, M., P. Milgrom, and J. Roberts (1992, Spring). Organizational prospects, influence costs, and ownership changes. *Journal of Economics & Management Strategy 1*(1), 9–35.

Peleg, B. and P. Sudhölter (2007). *Introduction to the Theory of Cooperative Games*. Theory and decision library. v. 34: Game theory, mathematical programming, and operations research. Springer-Verlag Berlin Heidelberg.

Radner, R. and T. Van Zandt (1992). Information processing in firms and returns to scale. *Annals of Economics and Statistics/Annales d'Économie et de Statistique* (25/26), 265–298.

Rajan, R. (2012, January). The corporation in finance. NBER Working Papers 17760, National Bureau of Economic Research, Inc.

Rajan, R. G. and L. Zingales (1998, May). Power in a theory of the firm. *The Quarterly Journal of Economics 113*(2), 387–432.

Rajan, R. G. and L. Zingales (2001, May). The influence of the financial revolution on the nature of firms. *American Economic Review 91*(2), 206–211.

Roberts, J. (2004). *The Modern Firm: Organizational Design for Performance and Growth*. Clarendon Lectures in Management Studies. OUP Oxford.

Samuelson, P. A. (1980). *Economics* (11 ed.). New York: McGraw-Hill.

Schelling, T. (1960). *The strategy of conflict*. Harvard University Press.

Smith, A. (1961). *An inquiry into the nature and causes of the wealth of nations: representative selections.* Number v. 1 in Library of liberal arts. Bobbs-Merrill.

von Neumann, J. and O. Morgenstern (1944). *Theory of games and economic behavior.* Princeton university press.

Williamson, O. (1987). *The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting.* Free Press.