

EC476 Part IV

Class 4

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Principal 1 and Principal 2 are middle managers in medium-sized firm. They work on different projects but share a common assistant, Eeyore. The big boss is wondering whether it is a good idea to hire an additional assistant so the managers no longer have to share Eeyore between them.

The environment at the firm is competitive, and the principals are always trying to get Eeyore to spend relatively more time on their own projects. If Principal i 's project goes ahead, it generates A_i for him from which he offers Eeyore benefits b_i in the form of contingent perks. Each principal has no other incentive instrument at his disposal.

Managers generate value for the firm only by offering incentives to assistants, so identify the probability of a Project i 's going ahead with Eeyore's effort e_i exerted for Principal i . Unfortunately, as usual Eeyore doesn't like to work. His cost of effort is given by the quadratic form

$$c_E(e_1, e_2) = \frac{1}{2} (e_1^2 + 2\gamma e_1 e_2 + e_2^2). \quad (1)$$

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The principals are risk-neutral and utility is transferable.

Suppose as usual throughout that parameters are such that the first-order approach is valid.

1. (a) Give an example of a firm or industry in which different employees may share common inputs to produce unrelated outputs.
- (b) Give an example of a firm or industry in which different employees may use distinct inputs to contribute to common output.
- (c) Give an example of a firm or industry in which some employees neither share any inputs nor contribute to the same outputs.
- (d) Comment on why each of the types of firms above exists. What do you think of as the typical scenario in the theory of the firm?
2. (a) Conditional on the benefits b_1 and b_2 the principals offer, solve for Eeyore's optimal effort $e(b_1, b_2) \in [0, 1] \times [0, 1]$. Comment on the role γ plays.
- (b) Solve for Principal i 's best-response function $b_i(b_{-i})$ (remember that subgame perfection implies that $e(b_1, b_2)$ is anticipated from backward induction), so maximize the objective

$$\mathbb{E}[\Pi_i] = (A_i - b_i)e_i(b_1, b_2) \quad (2)$$

over b_i .

- (c) Find the Nash equilibrium of the principals' game via the fixed-point (Nash) condition

$$b_i(b_{-i}(B)) = B \quad (3)$$

for $i \in \{1, 2\}$.

- (d) Find the Eeyore's equilibrium effort and utility levels.

- (e) Compute the total expected profit of each division and of the “whole firm”, $\mathbb{E}[\Pi_1 + \Pi_2]$.
 - (f) Comment on the roles of A_1 , A_2 and γ .
3. Now suppose that the firm does indeed hire another assistant and separates Principal 1’s office from Principal 2’s so neither manager can offer the other’s assistant perks.
- (a) Find e_i , the effort Assistant i exerts toward project i .
 - (b) Find the benefit b_i that Principal i offers his assistant.
 - (c) Find the Assistants’ equilibrium effort and utility levels.
 - (d) Find the principals’ expected profits and the expected profits of the whole firm as above.
4. When should the firm hire the second assistant. Comment. Who benefits from the hiring?