PROCYCLICAL PROMISES*

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Abstract

I explore how the cyclicality of firms' output affects their debt capacity. I point out that, although firms with procyclical output make low repayments in recessions (when marginal utility is high), they can make higher repayments on average. The reason is that their output is high when asset prices are high, and these high asset prices mitigate financial frictions, such as collateral constraints. This benefit is present across diverse models of financial frictions, and can be a first-order determinant of debt capacity. It affects macroeconomic outcomes, generating aggregate fluctuations and a premium for assets used by procyclical firms.

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1 Introduction

What determines a firm's debt capacity? Received theory suggests that one thing that matters is the cyclicality of its output. The argument goes that you should be reluctant to lend to a firm that has procyclical output, which is high in booms and low in recessions, since its repayments are low exactly when they are most valuable: in recessions, when marginal utility is high. However, there is limited empirical evidence supporting this idea. It fact, some papers suggest that, on the contrary, firms borrow more when they have more procyclical output.[1] Why?

In this paper, I identify a theoretical mechanism that gives a new perspective on how cyclicality affects debt capacity, and explains why firms with relatively procyclical output may sometimes borrow more. In line with the received view, I find that these firms make low repayments in recessions, making you reluctant to lend to them. However, I find that there is another effect that arises in the presence of financial frictions. In contrast to the received view, procyclical firms make higher repayments on average, making you relatively more willing to lend to them. So the effect of cyclicality on debt capacity can be decomposed into a cost—low repayments in high marginal utility states—and a benefit—high average repayments thanks to decreased financial frictions. I show that this novel benefit can be as important as the established cost in standard benchmark environments.

To explore the implications of the mechanism for the aggregate economy, I embed it in a general equilibrium model. I show that it gives rise to fluctuations in aggregate output, productivity, and prices, even in an environment in which there is no time-series variation whatsoever in the frictionless benchmark. Moreover, the mechanism leads to an endogenous price premium on assets used in procyclical production—there is a collateral premium for "procyclical assets" since these assets allow firms to lever up.

[1]For example, whereas Schwert and Strubalaev (2014) find evidence in support of the received view, Campbell, Polk, and Vuolteenaho (2010) and Ellahie (2017) find evidence to the contrary. Maia (2010) finds different results for different notions of cyclicality, exactly in line with my theoretical results (see Section 3).

Debt capacity decomposition. In the first part of the paper, I study how the cyclicality of a firm's output affects its debt capacity. I address this question in the context of several standard models of corporate finance frictions. Most of these models do not speak to it directly, however, since they do not take into account how a firm's output varies with the aggregate state of the economy. As such, they rule out a role for cyclicality. To rule it back in, I allow for cyclical variation in investors' marginal utility, captured by a stochastic discount factor M, and in the value of the firm's assets, captured by their price p. I incorporate M and p into models based on collateral constraints as in Kiyotaki and Moore (1997) and on asset substitution as in Holmström and Tirole (1997).

I find that these different models give rise to similar conclusions about how cyclicality affects debt capacity. In fact, I derive a decomposition that holds robustly across them. It shows that the effect of cyclicality on debt capacity can be divided into two terms. The first term says that a firm can borrow more if its output covaries positively with the SDF. Intuitively, you want to lend to a firm that makes repayments in recessions, when they are most valuable, in line with received theory. I refer to this as the *discount rate term.* The second term is my new observation. It says that a firm can borrow more if its output covaries positively with asset prices. Intuitively, you want to lend to a firm that has assets that can be sold/liquidated at high prices. Since assets have high prices in booms, this suggests, in contrast to received theory, that there is a benefit to procyclicality: it can loosen financial constraints. I refer to this as the *procyclical promises term*—you are willing to lend to a procyclical firm because it can credibly promise to make high repayments on average.

Procyclical promises vs. discount rates. In the second part of the paper, I analyze the relative importance of the discount rate and procyclical promises terms. To do so, I consider a standard neoclassical economy in which I can calculate the SDF M and asset prices p explicitly. I show that the procyclical promises term can be just as important as the discount rate term. Indeed, with log utility and a single capital asset, the two terms cancel out, so procyclical firms can borrow just as much as acyclical firms (all else equal). I also consider a model with multiple capital assets and show that the procyclical promises term can actually dominate the discount rate term for some firms.

The additional structure in this part also allows me to compare my theoretical findings with empirical results in the literature. In particular, I show that my discount rate and procyclical promises terms are closely related to two notations of cyclicality introduced in Campbell and Vuolteenaho (2004), called the discount rate and cash flow betas. My results imply that leverage should be decreasing in the discount rate beta and increasing in the cash flow beta, in line with empirical findings (see Maia (2010) as well as Campbell, Polk, and Vuolteenaho (2010) and Ellahie (2017)).

Procyclical promises in equilibrium. In the third part of the paper, I dig deeper into the effects of procyclical promises for the aggregate economy. Here, I no longer stress the trade-off between my procyclical promises term and the standard discount rate term. Rather, I zero in on the procyclical promises term by assuming that the discount rate term is zero (i.e. investors are risk-neutral). Unlike in the previous part, in which I ask how a firm's cyclicality affects its borrowing constraints taking aggregate outcomes as given, here I ask how these borrowing constraints affect aggregate outcomes themselves. To this end, I present a dynamic equilibrium model in which there are two types of firms (or "entrepreneurs"), one of which is more procyclical than the other. To understand how this difference in cyclicality affects asset prices, I assume that entrepreneurs make investments using different capital assets. Even though there is a lot of heterogeneity—heterogenous entrepreneurs lever up to invest in heterogeneous assets subject to heterogenous borrowing constraints—I manage to keep the model tractable by assuming that overlapping generations of short-lived entrepreneurs borrow from long-lived investors. Indeed, it admits an explicit solution in some configurations.

Due to the procyclical promises channel, the model generates macroeconomic fluctuations that resonate with empirical evidence, even though it includes a minimal amount of variation in exogenous variables. Indeed, I set it up so that there is next to nothing going on in the Arrow–Debreu benchmark. I assume that there is no capital accumulation and there are no technology shocks, so output, productivity, and asset prices are all constant in the Arrow–Debreu allocation. But I show that this is far from the case in the model with borrowing constraints. With the discount rate term switched off, the procyclical promises term is the sole determinant of debt capacity: the more procyclical a firm is, the more it can lever up and the more it can invest. As a result, expected output is high when procyclical firms have more initial capital—they lever it up to invest efficiently. In contrast to the Arrow–Debreu benchmark, aggregate investment, prices, productivity, and output vary over time. In other words, there are aggregate fluctuations, even absent technology shocks. These are pure "allocation cycles"—they are driven entirely by fluctuations in who has capital. This is in line the empirical fact that capital allocation is a major driver of output (Hsieh and Klenow (2009)), and the results are also consistent with evidence on the dynamics of aggregate capital allocation, output, and productivity (see, e.g., Basu and Fernald (2001) and Eisfeldt and Rampini (2006)).

The model also generates endogenous variation in the cross-section of asset prices: procyclical firms' high demand drives up the price of the assets they use in production, generating a collateral premium on these "procyclical assets." This premium reflects the procyclical promises term, which says that making procyclical investments helps you to borrow—procyclical assets are good collateral. In an extension, I suggest that this may also affect the prices of financial assets, helping to explain why stocks with high CAPM beta do not trade at as much of a discount as theory suggests they should (Fama and French (2004)).

Contribution to the literature. Overall, my results suggest that the procyclical promises channel could help us to understand corporate leverage and asset prices in the cross-section and capital allocation and output fluctuations in the time series. Identifying this channel and its implications is the main contribution of this paper. It adds to the small set of models on the interaction between output cyclicality and leverage, which all focus on the discount rate channel: Choi (2013) connects it with the value premium, Shleifer and Vishny (1992) with capital redeployment, and Ross (1985) with carried forward tax deductions. By connecting it with the macroeconomy, I also contribute to the large literature on macro with financial frictions.[2] Although many papers in this literature speak to how leverage changes over the cycle, they are largely silent on my question of how leverage varies in the cross-section of firms with different sensitivities to the cycle.[3]

2 Debt Capacity Decomposition

I start by asking how the cyclicality of a firm's output affects its debt capacity. Most standard corporate finance models do not speak to this question directly, since they do not model how output varies with the aggregate state of the economy; hence, they abstract from cyclicality.[4] As such, they switch off two important things: (i) how much consumption goods are worth in terms of utility and (ii) how much assets are worth in terms of consumption goods. I switch these back on. To capture (i), I include a stochastic discount factor M and, to capture (ii), I include the price of assets p. I embed M and p in models based on two different corporate finance frictions, collateral constraints and asset substitution/moral hazard. It turns out that the different frictions lead to the same conclusion: a firm with output y has debt capacity given by a simple formula, $\mathrm{DC} = \mathbb{E}[Mpy]$ (up to an affine transformation). This expression does speak to the effects of cyclicality on debt capacity, since it depends on how the firm's output y varies with the aggregate variables M and p. However, at this point, the effects are confounded in the product Mpy; M is countercyclical—marginal utility is high in recessions—whereas p is procyclical—asset prices are high in booms. Below, I derive

[2]Some of the most related papers in this literature are Alvarez and Jermann (2000), Bernanke and Gertler (1989), Geanakoplos (1997), Geanakoplos and Kübler (2004), Kehoe and Levine (1993), Kiyotaki and Moore (1997), and Lorenzoni (2008).

[3]See, e.g., Chen (2010) and Hackbarth, Miao, and Morellec (2006) for models of firm financing over the cycle. Closer to me, Bhamra, Kuehn, and Strebulaev (2010a, 2010b) study a model that allows for heterogenous cyclicality; I go a step further by connecting cyclicality with endogenous financial constraints.

[4]Macroeconomic models with financial frictions typically do model how output varies with the aggregate state. But, as touched on in footnote 3, only a few include heterogenous firms with different degrees of cyclicality.

a decomposition to disentangle these effects. But first I derive the primitive expression $\text{DC} = \mathbb{E}[Mpy]$ in environments with collateral constraints, asset substitution/moral hazard, and a combination of the two.

2.1 Collateral constraints. Consider the financing friction in Kiyotaki and Moore's (1997) model: repayments are limited by the value of collateral a firm has:

$$\text{repayment} \;\leq\; \text{value of its collateral} \tag{1}$$

(cf. p. 217 of their paper).[5] Thus, if firm has a quantity y of collateral assets with price p, it repays at most py. The present value of the maximum the firm can repay in the future is the maximum it can borrow today, i.e. its debt capacity. Indeed, computing the present value of the maximum repayment py gives the desired formula:

Lemma 1. *In the environment with collateral constraints described above, the formula DC =* $\mathbb{E}[Mpy]$ *holds, where* y *is the quantity of collateral the firm has and* p *is its price.*

Note, however, that the formula has little bite in Kiyotaki and Moore's specific model, because firms do not produce collateral assets (they only use them to produce consumption goods). In other words, y is constant in their model. As a result, y factors out of the formula for debt capacity:

$$\text{DC} = \mathbb{E}[Mpy] = \mathbb{E}[Mp]\, y. \tag{2}$$

Here, debt capacity does not depend on how the output y co-moves with the aggregate variables M and p, since fixing the amount of collateral a firm has rules out a role for cyclicality. However, just allowing firms to get more collateral in the course of production can rule it back in—if both the price and quantity of collateral can change, then it matters how they move together. This is the case that I focus on, which seems relevant empirically.[6]

[5] See, e.g., Benmelech and Bergman (2009) and Rampini and Viswanathan (2013) for empirical evidence on collateral values as a determinant of debt capacity.

[6] For example, Acharya, Bharath, and Srinivasan (2007) and Almeida and Philippon (2007) find variation in the proportion of assets creditors recover in bankruptcy, which is effectively variation in the amount of collateral firms have.

(Kiyotaki and Moore also assume everyone is risk neutral, so the SDF is constant: $M \equiv 1/R_f$. I do this too in my general equilibrium analysis in Section 4 below.)

2.2 Asset substitution/Moral hazard. Now consider the friction in Holmström and Tirole's (1997) model: repayments are limited because the firm (or its manager) must have incentive to continue its investment rather than do "asset substitution" (or, equivalently, just have incentive to work to complete its investment rather than shirk and abandon it).

Consider a firm with outstanding debt with face value T and an investment that can succeed or fail. If it succeeds, it pays off a quantity y of an asset with price p. If it fails, it pays off zero. As in Holmström and Tirole (1997), the probability of success depends on whether the firm takes the (non-contractable) asset substitution action. The firm can take a "good" action, denoted by $e = 1$, in which case the probability of success is π_1. Or it can take a "bad" action, denoted by $e = 0$, in which case the probability of success is π_0. The bad action reduces the success probability, i.e. $\pi_0 < \pi_1$, but delivers private benefits B to the firm/manager. I assume that the probability of success depends only on e (not on the aggregate state) and that the probability of success given $e = 0$ is small (so $e = 1$ must be incentive compatible).

We can calculate that the firm has the incentive to take the good action only if the repayment T is not too large: the IC $e = 1 \succeq e = 0$ says

$$\begin{aligned} &\mathrm{PV}\Big[\text{ value of output } - \text{ repayment } \Big| e = 1\Big] \\ &\geq \mathrm{PV}\Big[B + \text{ value of output } - \text{ repayment } \Big| e = 0\Big], \end{aligned} \tag{3}$$

which can be rewritten[7] as

$$T \leq R_f \mathbb{E}[Mpy] - \frac{B}{\pi_1 - \pi_0}. \tag{4}$$

Intuitively, "shirking" ($e = 0$) gives private benefits by definition; in contrast, "working" ($e = 1$) gives shared benefits, since it increases both output and repayments. The higher T is, the more of these benefits of working go to increasing the expected repayment, which goes up by $(\pi_1 - \pi_0)T$. Hence, high T makes it more temping to shirk and get the private benefits. This leads to the upper bound on T above. The debt capacity formula follows from taking the present value of this expression.

LEMMA 2. *In the environment with asset substitution described above, the formula* $DC = \alpha_0 + \alpha_1 \mathbb{E}[Mpy]$ *holds for constants* $\alpha_0 = -\frac{\pi_1 B}{R_f(\pi_1 - \pi_0)}$ *and* $\alpha_1 = \pi_1$*, where* y *is the quantity of output produced and* p *is its price.*

2.3 Collateral and asset substitution. Now consider a combination of the frictions above, which is somewhat similar to Hart and Moore (1989/1998): repayments are limited both because a firm must back debt with collateral and because it must have incentive to continue its investment, rather than abandon it or "divert capital."

Consider a firm that has an investment k of a capital asset that can both serve as collateral and produce output y. Here, I assume that the distribution of output y is binary: the investment either succeeds, in which case $y = Ak$, or fails, in which case $y = 0$. And I assume the firm privately observes whether the investment will succeed at an interim date, before it pays off.[8] At this point, the firm can either continue the investment, denoted by

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[7]Symbolically, the IC in equation (3) reads

$$\mathbb{E}\left[M\,\mathbb{1}_{\text{succ}}(py - T)\,\middle|\,e = 1\right] \geq \mathbb{E}\left[M\left(B + \mathbb{1}_{\text{succ}}(py - T)\right)\,\middle|\,e = 0\right],$$

where $\mathbb{1}_{\text{succ}}$ denotes the success indicator. Now, given that I have assumed that $\mathbb{1}_{\text{succ}}$ is independent of everything but e, we can apply the Law of Iterated Expectations to write

$$\mathbb{E}\left[M\,\pi_1(py - T)\right] \geq \mathbb{E}\left[M\left(B + \pi_0(py - T)\right)\right].$$

Rearranging gives equation (4) in the text.

[8] For simplicity, I assume that the firm observes success or failure perfectly. However, for the qualitative

$e = 1$, or abandon it, denoted by $e = 0$. Choosing $e = 0$ yields B to the firm, but decreases the value of the capital assets by δ. Thus, "abandonment" is a catchall for any action that delivers private benefits at the expense of asset value, of which numerous examples are standard in the literature, e.g., asset substitution, capital diversion, shirking, tunneling, under-maintenance, and asset stripping.

Now it is easy to see that the firm continues a successful investment but abandons a failing investment (as long as the productivity A is not too small and the rate of depreciation δ is not too large; see Appendix B for details). Denoting the price of capital assets by p and the success indicator by $\mathbb{1}_{\text{succ}}$, we have that the firm's maximum repayment is

$$T^{\max} = \begin{cases} pk & \text{given success,} \\ (1-\delta)pk & \text{given failure} \end{cases} \tag{5}$$

$$= \mathbb{1}_{\text{succ}}\, pk + (1 - \mathbb{1}_{\text{succ}})(1-\delta)pk \tag{6}$$

$$= (1-\delta)pk + \frac{\delta p y}{A}, \tag{7}$$

since $y \in \{0, Ak\}$ implies that $y = \mathbb{1}_{\text{succ}} Ak$. The debt capacity formula follows from taking the present value of this expression.

LEMMA 3. *In the environment with collateral constraints and asset substitution described above, the formula $DC = \alpha_0 + \alpha_1 \mathbb{E}[Mpy]$ holds for constants $\alpha_0 = (1-\delta)\mathbb{E}[Mp]\,k$ and $\alpha_1 = \delta/A$, where k is the quantity of collateral, p is its price, δ is the rate of depreciation (or the fraction of assets that can be diverted), and y is the output of the consumption good.*

This set-up will be useful below, since it can fit into the dynamic general equilibrium framework in Section 4 relatively easily. There, the set-up remains tractable despite its heterogenous firms with heterogenous financial constraints. One reason is that, since the

results it suffices that it gets some information about the likelihood of success. In the context of sales, this could come from observing market demand for a product; in the context of manufacturing, it could come from observing the efficiency of the machinery and workforce.

firm does not produce the collateral asset here, I can find assumptions such that the total capital stock remains constant. (Specifically, I assume that the private benefits B constitute diverted capital: the firm can divert $B = \delta k$, leaving $(1-\delta)k$ as collateral; cf. footnote 21.)

2.4 Decomposition. What does the expression $\mathrm{DC} = \mathbb{E}[Mpy]$ have to say about the effect of cyclicality on debt capacity? To address this question, I decompose the expectation:

PROPOSITION 1.

$$\mathbb{E}[Mpy] = \frac{1}{R_f} f_p \, \mathbb{E}[y] + \mathbb{E}[p] \operatorname{Cov}[M, y] + \frac{1}{R_f} \operatorname{Cov}[p, y] + \epsilon, \tag{8}$$

where R_f is the risk-free rate, f_p is the forward price of assets ($= R_f \mathbb{E}[Mp]$), and

$$\epsilon = \operatorname{Cov}\left[\big(M - \mathbb{E}[M]\big)\big(p - \mathbb{E}[p]\big), y\right]. \tag{9}$$

Cyclicality is captured by the aggregate variables M and p: M is high in recessions when marginal utility is high (since consumption is low); p is high in booms when the demand for capital assets is high (since productivity is high).[9] Thus, the first term $\frac{1}{R_f}\mathbb{E}[y] f_p$ in Proposition 1, in which y decouples, does not depend on the cyclicality of y.[10] But the two covariance terms, $\mathbb{E}[p] \operatorname{Cov}[M, y]$ and $\frac{1}{R_f} \operatorname{Cov}[p, y]$, do. (I abstract from the "nuisance term" ϵ for now and show in Lemma 5 below that it is typically small anyway.)

I call $\mathbb{E}[p] \operatorname{Cov}[M, y]$ the "discount rate term." It reflects the standard cost of procyclicality: a procyclical firm has low output in recessions, and hence it can repay little when M is high.[11]

I call $\frac{1}{R_f} \operatorname{Cov}[p, y]$ the "procyclical promises term." It reflects the benefit of procyclicality that I emphasize in this paper: a procyclical firm has high output in booms, when prices are

[9] Empirical evidence that the values of collateral are procyclical is in Acharya, Bharath, and Srinivasan (2007).

[10] Indeed, in the benchmark in which collateral constraints do not depend on output, the debt capacity formula comprises only this term (equation (2)).

[11] This effect is ubiquitous in asset pricing—it is what the CAPM is all about. For corporate finance applications, see, e.g., Almeida and Philippon (2007).

high, and hence it repays more on average. In the models above, this is because a procyclical firm produces collateral/continues its investments exactly when assets are most valuable. I.e. high output and high prices are complementary, because the expected (undiscounted) repayment is high when p and y move together.[12]

3 Procyclical Promises vs. Discount Rates

The analysis above suggests that the effect of cyclicality on debt capacity involves a trade-off between the discount rate term and the procyclical promises term. Below, I explore how important each term is likely to be. To do this, I embed a "small" constrained firm in larger economies in which I can calculate these terms explicitly. I start with the natural baseline of a neoclassical economy with log utility and a single capital asset. There, the two terms both move as a function of the same aggregate variable, productivity. Hence, I move on to a richer specification with many capital assets, in which the discount rate term still depends on aggregate productivity, but the procyclical promises term depends on the productivity of the assets a firm uses. Overall, I find that the procyclical promises term is just as important as the discount rate term, and even more important for some firms in the many-asset specification. Along the way, I use this section's additional structure to show formally that the "nuisance term" ϵ in the debt capacity decomposition is typically close to zero (Lemma 5).

3.1 A single capital asset. Here, I consider a neoclassical economy with a single durable capital asset that serves as the sole input of production. I assume that there is

[12]To see this another way, use the standard covariance formula to write $\mathbb{E}[py] = \mathbb{E}[p]\,\mathbb{E}[y] + \mathrm{Cov}[p,y]$. Indeed, this is basically the decomposition in Proposition 1 for the special case of a constant SDF: if $M \equiv 1/R_f$,

$$\begin{aligned}\mathbb{E}[Mpy] &= \frac{1}{R_f}\mathbb{E}[py] \\ &= \frac{1}{R_f}\mathbb{E}[p]\;\mathbb{E}[y] + \frac{1}{R_f}\mathrm{Cov}[p,y]\,.\end{aligned}$$

a representative firm that produces a perishable consumption good and a representative consumer who consumes it. To capture the changing state of the economy, I assume that there is a random shock to the firm's productivity A_t: high A_t represents a boom and low A_t a recession. And to understand how cyclicality affects debt capacity, I introduce a small constrained firm and I ask whether it can borrow more if its output moves with A_t or against it. I want to address this question directly with the debt capacity formula ($\text{DC} = \mathbb{E}\left[Mpy\right]$). To do so, I put some structure on the representative firm's production technology and the consumer's utility. That way, I can calculate the asset price p and the SDF M explicitly.

I assume the firm has a technology that uses K_t at date t to produce $A_{t+1}F(K_t)$ at date $t+1$. (It produces only the consumption good, so the capital stock is constant, $K_t \equiv K$.) As usual, the rental price of capital is given by its (discounted) marginal productivity,[13] $\mathbb{E}_t\left[A_{t+1}F'(K)\right]/R_f$.

I assume that the consumer has log utility and discounts the future at rate $1/R_f$. Since the consumption good is perishable, he consumes all the output, $C_t = Y_t$. These assumptions allow me to compute the SDF, which is his marginal rate of substitution,

$$M_{t,t+1} = \frac{u'(C_{t+1})}{R_f\, u'(C_t)} \tag{10}$$
$$= \frac{C_t}{R_f C_{t+1}} \tag{11}$$
$$= \frac{Y_t}{R_f Y_{t+1}}. \tag{12}$$

To solve for the asset prices and the SDF explicitly, I suppose that productivity is constant except for a (rationally anticipated) one-off shock at some date t^*+1. As I show in the proof of Lemma 4 below, this makes it easy to solve for asset prices and the SDF. Just the perpetuity

[13] The firm's problem is to maximize its expected discounted profit:

$$\text{maximize } \mathbb{E}\left[\frac{A_{t+1}F(K_t)}{R_f} - R_{k,t}K_t\right],$$

where $R_{k,t}$ is the rental price of capital. The FOC gives the equation in the text. Cf., e.g., Ch. 2 of Acemoglu's (2009) textbook.

formula and the expression in equation (12) imply that the price of assets is proportional to the productivity and the SDF is inversely proportional to it: $p = p_{t^*+1} = a_0 A_{t^*+1}$ and $M = M_{t^*,t^*+1} = a_1/A_{t^*+1}$, for constants a_0 and a_1.

Now, with p and M in hand, I turn to a small constrained firm that makes an investment at date t^* and produces output y at the next date. Is its debt capacity higher if it is procyclical—y and A_{t^*+1} are positively correlated—or acyclical—y and A_{t^*+1} are uncorrelated? The debt capacity formula $\mathrm{DC} = \mathbb{E}\left[Mpy\right]$ is designed to answer this question. Here, it says that the debt capacity does not depend on cyclicality at all, since Mp is constant ($Mp \equiv a_0 a_1$):

Lemma 4. *In the neoclassical economy with log utility and a single capital asset described above, we have that*

$$\mathrm{DC} \equiv \mathbb{E}\left[Mpy\right] = a_0 a_1 \mathbb{E}\left[y\right], \tag{13}$$

where

$$a_0 = \frac{F'(K)}{R_f - 1} \quad \text{and} \quad a_1 = \frac{Y_{t^*}}{R_f F(K)}. \tag{14}$$

I.e. DC does not depend on the cyclicality of output y.

Recall that the decomposition in Proposition 1 points to two offsetting effects of procyclicality. On the one hand, the SDF goes down as productivity goes up—higher productivity yields higher output and consumption, hence lower marginal utility. But, on the other hand, the price of capital goes up as productivity goes up—higher productivity means more valuable capital, hence higher prices. The result above implies that these effects cancel out perfectly in the set-up with log utility and permanent shocks.

This set-up also allows me to calculate the nuisance term ϵ explicitly and show that it is almost zero. We are not losing anything by focusing mainly on the discount rate and the procyclical promises terms in the decomposition:

Lemma 5. *In the neoclassical economy with log utility and a single capital asset described* *above, we have that*

$$\text{DC} \approx \frac{1}{R_f} f_p \, \mathbb{E}\left[y\right] + \mathbb{E}\left[p\right] \text{Cov}\left[M, y\right] + \frac{1}{R_f} \text{Cov}\left[p, y\right]. \tag{15}$$

I.e. in Proposition 1, $\epsilon \approx 0$.

This set-up is also useful because it allows me to relate my decomposition to the beta decomposition in Campbell and Vuolteenaho (2004), which splits a firm's CAMP beta up into two betas, one reflecting its sensitivity to aggregate discount rate news and another to aggregate cash flow news. Here, aggregate discount rates are, of course, represented by M. Hence, my discount rate term $\text{Cov}\left[M, y\right]$ captures the sensitivity to discount rate news. Here, aggregate cash flows are represented by the total output Y_{t+1}. Now, given a single shock, aggregate capital prices p are directly proportional to aggregate cash flows, as discussed above. Hence, my procyclical promises term $\text{Cov}\left[p, y\right]$ captures the sensitivity to aggregate cash flow news. With this interpretation, my model says that firms with higher discount rate betas should have lower debt capacity, whereas firms with higher cash flow betas should have higher debt capacity. Hence, I offer an explanation for the empirical finding that corporate leverage is decreasing in discount rate beta, but increasing in cash-flow beta (see Maia (2010) as well as Campbell, Polk, and Vuolteenaho (2010), Ellahie (2017)). Further, since my results are about constraints on leverage (debt capacity), whereas these empirical findings are about leverage itself, my model suggests they are likely to be the result of financial constraints.

3.2 Many capital assets. Now consider a set-up in which everything is as above, except there is a large number I of capital assets, indexed by $i \in \{1, ..., I\}$. To simplify things, I assume that the assets are substitutes; each is used separately, and total output is just the sum of the output of each asset, $Y_{t+1} = \sum_{i=1}^{I} Y_{t+1}^{i}$. And, futher, I assume that all assets have the same aggregate supply K and production functions have the same shape; they differ only in their productivity: in notation, asset i's output $Y_{t+1}^{i} = A_{t+1}^{i} F(K)$ depends on the asset i

only via the productivity A^i_{t+1}. As above, there is a single productivity shock realized at date t^*+1. Here, I assume each asset's productivity has aggregate and idiosyncratic components, i.e. $A^i_{t^*+1} = A_{t^*+1} + \varepsilon^i_{t^*+1}$, where $\varepsilon^i_{t^*+1}$ is independent noise. Under these assumptions, the expressions for the rental price of capital and the SDF are just as in the single-asset case above (with rental price of each asset indexed by its own expected productivity).

Now return to a small firm that makes a one-period investment at date t^*. Suppose that it uses a single capital asset i^* and that its output y is proportional to the productivity of this asset plus (independent) noise: $y = a(A^{i^*}_{t^*+1} + \varepsilon^y)$, for some constant a. Can the procyclical promises term ever dominate the discount rate term? Yes, if the asset i^* that the firm uses is sufficiently volatile:

Lemma 6. *In the neoclassical economy with log utility and many capital assets described above, the discount rate term for a firm that uses asset i^* is approximately*[14]

$$\mathbb{E}\left[p^{i^*}_{t^*+1}\right]\operatorname{Cov}\left[M,y\right] \approx -\beta_0 \operatorname{Var}\left[A_{t^*+1}\right] \tag{16}$$

and the procyclical promises term for a firm that produces good i^* is

$$\frac{1}{R_f}\operatorname{Cov}\left[p^{i^*},y\right] = \beta_1\Big(\operatorname{Var}\left[A_{t^*+1}\right] + \operatorname{Var}\left[\varepsilon^{i^*}_{t^*+1}\right]\Big) \tag{17}$$

where

$$\beta_0 = \frac{aY_{t^*}\mathbb{E}\left[A^{i^*}_{t^*+1}\right]F'(K)}{R_f(R_f-1)\mathbb{E}\left[A_{t^*+1}\right]^2 F(K)I} \quad \textit{and} \quad \beta_1 = \frac{aF'(K)}{R_f(R_f-1)}. \tag{18}$$

Hence, the procyclical promises term dominates the discount rate term whenever $\operatorname{Var}\left[\varepsilon^{i^*}_{t^*+1}\right]$ *is large, i.e. whenever the output of good i^* is volatile.*

Recall, yet again, that there is a trade-off to procyclicality. A procyclical firm makes repayments in booms when marginal utility is low (the discount rate term), but can commit to make larger repayments in these states since asset prices are high (the procyclical promises

[14]The formula is not exact because I approximate the sample average of firms' output with the expected value and I approximate $1/A$ with its first-order Taylor expansion.

term). The last result says that if the price of a firm's assets moves around a lot—so the covariance between y and p^{i^*} is high—then the procyclical promises term can dominate the discount rate term. It also points to a subtlety in the procyclical promises term: it may not be the covariance with the aggregate price index that matters, but the covariance with the price of the specific capital asset used in production. This suggests that there could be a benefit to being in a volatile industry (in which the price of capital varies a lot over the cycle).

Whereas volatile prices make the procyclical promises term relatively more important, a volatile SDF makes the discount rate term relatively more important. Indeed, securities' prices suggest that the SDF is volatile empirically (Hansen and Jagannathan (1991)). In representative-agent asset pricing models, capturing this typically requires a utility function with a lot of curvature (risk aversion). As such, it seems probable that my analysis with log utility could understate the importance of the discount rate term.[15] That said, since I use the SDF to price capital assets, not financial assets, it is not completely clear that this asset pricing literature is the right benchmark, especially in light of the equity volatility puzzle.[16] And log utility is a natural starting point—it reflects the very origins of utility theory (Bernoulli (1954)), and is used even in quantitative macro models (e.g., Bernanke, Gertler, and Gilchrist (1996)). Hence, I think it is suitable benchmark to show how important the procyclical promises term can be.[17]

[15] Indeed, Schwert and Strubalaev (2014) find that firms with higher asset betas are more levered (after controlling for asset volatility). This could be because procyclical firms are able to borrow less, as would be the case if the discount rate term were more important than the procyclical promises term. But it could also be because they choose to borrow less, as would be the case if the discount rate term made it expensive for procyclical firms to borrow, even if unconstrained. (Distinguishing between these possibilities is a step beyond what I do here; I study leverage limits—debt capacity—but not yet optimal leverage choices.)

[16] That is, in the data, stock prices move around a lot more than consumption (see, e.g., Campbell (2003)). Hence, my simple set-up, in which capital asset prices move in lockstep with consumption by construction, is probably not well suited to quantitative stock pricing (the discussion in Subsection 4.10 below notwithstanding).

[17] Martin (2017) argues that log utility could actually be a suitable benchmark for asset pricing too. Taking the perspective of an unconstrained investor fully invested in the market, he concludes that log utility approximately rationalizes a fundamental relationship between options prices and realized returns. Specifically, he calculates a bound on the market return in terms of a portfolio of equity options. He finds that the bound is approximately tight empirically, which is the case theoretically if a representative agent fully invested in the market has log utility.

4 Procyclical Promises in Equilibrium

So far, I have stressed the trade-off between my procyclical promises term and the standard discount rate term. Now, I zero in on the procyclical promises term by assuming that the discount rate term is zero (i.e. investors are risk-neutral). Unlike in the previous section, in which I ask how a firm's cyclicality affects its borrowing constraints taking the aggregate outcomes as given, here I ask how these borrowing constraints affect aggregate outcomes themselves. To this end, I present a dynamic equilibrium model in which there are two types of firms (or "entrepreneurs"), one of which is more procyclical than the other. To understand how this difference in cyclicality affects asset prices, I assume that entrepreneurs make investments using different capital assets. Even though there is a lot of heterogeneity—heterogenous entrepreneurs lever up to invest in heterogeneous assets subject to heterogenous borrowing constraints—I manage to keep the model tractable by assuming that overlapping generations of short-lived entrepreneurs borrow from long-lived investors. Indeed, it admits an explicit solution in some configurations. I now proceed to investigate how procyclical promises affect aggregate investment, prices, and output in the time series and the cross section.

4.1 Environment. I set up the model in discrete time, $t \in \{..., -1, 0, 1, ...\}$. At each date, an i.i.d. state s_t is realized, assuming one of two equally likely realizations, a or b.

There is a single perishable consumption good, which serves as numeraire, and two durable capital assets, called α- and β-assets, which are each in constant supply K. The state-s price of asset $\tau \in \{\alpha, \beta\}$ is denoted by p_s^τ, and its average by $\bar{p}^\tau := \mathbb{E}\left[p^\tau\right] \equiv (p_a^\tau + p_b^\tau)/2$.

There are overlapping generations of two types of risk-neutral entrepreneurs, α- and β-entrepreneurs. At each date, a unit continuum of each type is born and lives for two dates. When they are born, they have an endowment that depends on the state: they have a unit of the consumption good in state a and nothing in state b. (This is the only exogenous difference between the states.) When young, they borrow and invest; when old, they produce and consume. Each α-entrepreneur uses the α-asset to do a risky investment that pays off only in state a and, symmetrically, each β-entrepreneur uses the β-asset to do a risky investment that pays off only in state b. The investments are both constant returns to scale with expected return A. I.e.

$$y_{t+1}^{\alpha} = \alpha(k)(s_{t+1}) = \begin{cases} 2Ak & \text{if } s_{t+1} = a, \\ 0 & \text{if } s_{t+1} = b, \end{cases} \tag{19}$$

$$y_{t+1}^{\beta} = \beta(k)(s_{t+1}) = \begin{cases} 0 & \text{if } s_{t+1} = a, \\ 2Ak & \text{if } s_{t+1} = b. \end{cases} \tag{20}$$

So "α" and "β" each denote three related things: investment technologies, the entrepreneurs who operate them, and the type of capital asset they employ.

There are also long-lived deep-pocketed investors in the background. They are risk-neutral and consume at each date, discounting the future at the risk free rate R_f, which I assume is not too large relative to entrepreneurs' productivity:[18]

$$R_f < AK. \tag{21}$$

Investors can lend to entrepreneurs or use either asset (but not both) to invest in a deterministic technology γ that pays off at the next date, where, as usual, nothing is produced if nothing is invested, $\gamma(0) = 0$, and more is produced if more is invested, $\gamma' > 0$. Further, I assume that γ has decreasing returns to scale, $\gamma'' < 0$, and is at most as productive as entrepreneurs' investments, $\gamma'(0) = A$. Since each capital asset is in constant supply K, the quantity of τ-capital that investors hold is the quantity not held by τ-entrepreneurs: in state s, investors hold $K - k_s^{\tau}$, where k_s^{τ} denotes the amount of the τ-asset that each

[18]This will ensure that entrepreneurs exhaust their capacity; I use it only in the proof of Lemma 9 in the Appendix.

τ-entrepreneur holds in state s.[19]

4.2 First-best/Arrow–Debreu. To focus on the effects of financial constraints, I have set up the environment so that not much happens in the benchmark without them: in the Arrow–Debreu/first-best outcome, entrepreneurs hold all of the assets, since they have the most productive investments. Hence, the expected return A on their investments coincides with the aggregate productivity. And, since productivity does not change over time, asset prices are constant. So is realized output: exactly one type of entrepreneurs produces $2AK$ at each date t, these are the α-entrepreneurs if $s_t = a$ and β-entrepreneurs if $s_t = b$. In summary, there are no time-series fluctuations in productivity, prices, or output. Moreover, there are no ex ante differences in the cross section either: expected productivity, prices, and expected output coincide for α- and β-assets/entrepreneurs.

LEMMA 7. *In the Arrow–Debreu outcome, α-entrepreneurs hold all α-assets, $k_s^\alpha = K$, and β-entrepreneurs hold all β-assets, $k_s^\beta = K$. The equilibrium has the following properties.*

(i) Aggregate productivity[20] *is constant and equal to A.*

(ii) The prices of α- and β-assets are equal and constant; they equal the price of a perpetuity that pays coupon A,

$$p_s^\alpha = p_s^\beta = \frac{A}{R_f - 1}. \tag{22}$$

(iii) Aggregate output is constant and equal to $2AK$.

4.3 Borrowing constraints. I assume that an entrepreneur's debt capacity is given by $\mathrm{DC} = \alpha_0 + \alpha_1 \mathbb{E}[Mpy]$, where α_0 and α_1 are as in Lemma 3 above.[21] I.e. the loan ℓ an

[19]Note that I give one index to entrepreneurs' asset holding, representing both the type of asset and the entrepreneur who holds it. Rather than introducing a separate notation for investors' asset holdings, I just make use of the fact they hold whatever entrepreneurs do not by market clearing.

[20]To avoid ambiguity, let me state the definition of the aggregate productivity in state s formally:

$$\text{aggregate productivity}\Big|_s = \frac{\text{total expected output}}{\text{asset supply}}\Bigg|_s = \frac{\mathbb{E}\Big[\ \alpha\big(k_s^\alpha\big) + \gamma\big(K - k_s^\alpha\big) + \beta\big(k_s^\beta\big) + \gamma\big(K - k_s^\beta\big)\ \Big]}{2K}.$$

[21]As touched on in Subsection 2.3, I adopt the interpretation that the entrepreneur can divert a fraction δ of his assets, getting private benefits $B = \delta pk$ and leaving $(1-\delta)k$ as collateral. This interpretation of

entrepreneur gets must satisfy the constraint

$$\ell \leq \alpha_0 + \alpha_1 \mathbb{E}\left[Mpy\right]. \tag{23}$$

After substituting in for α_0 and α_1, a little manipulation[22] gives

$$\ell \leq \frac{(1-\delta)\mathbb{E}\left[p\right] + \frac{\delta}{2}\,\mathbb{E}\left[p \,|\, \text{success}\right]}{R_f}\, k. \tag{24}$$

Observe that an entrepreneur's borrowing limit depends not only on the average value of his assets, $\mathbb{E}[p]$, but also on another term, namely the expectation of the price given his investment succeeds (i.e. does not pay off zero); this term reflects the procyclical promises term in the debt capacity decomposition (Proposition 1). Now, recall that α-entrepreneurs succeed only in state a and β-entrepreneurs only in state b. Thus, for α-entrepreneurs, $\mathbb{E}\left[p_s^\alpha \,|\, \alpha\text{-success}\right] = p_a^\alpha$ and, likewise, for β-entrepreneurs, $\mathbb{E}\left[p_s^\beta \,|\, \beta\text{-success}\right] = p_b^\beta$, which helps us to simplify the borrowing constraints:

$$\ell^\alpha \leq \frac{(1-\delta)\bar{p}^\alpha + \delta p_a^\alpha/2}{R_f}\, k^\alpha \quad \text{and} \quad \ell^\beta \leq \frac{(1-\delta)\bar{p}^\beta + \delta p_b^\beta/2}{R_f}\, k^\beta \tag{25}$$

(where I have added back the sub- and superscripts to distinguish between states and assets).

δ—"diversion" rather than "depreciation"—is convenient just because it implies the supply of assets remains constant; it does not affect anything else.

Many papers use capital diversion to generate borrowing constraints (e.g., Albuquerque and Hopenhayn (2004), DeMarzo and Fishman (2007b), Rampini and Viswanathan (2010)); indeed, as formalized in DeMarzo and Fishman (2007a), it is a useful catch-all for many agency problems. That said, flagrant diversion is a serious friction itself: Mironov (2013) calculates that Russian companies siphoned off upward of ten percent of GDP in both 2003 and 2004; Akerlof and Romer (1993) describe related problems of explicit "looting" at US firms.

[22] First, substitute in for α_0 and α_1 from Lemma 3, noting that A in Lemma 3 is replaced by $2A$, since the investment payoffs are defined slightly differently in this section; then, manipulate using $M \equiv 1/R_f$ (by risk neutrality) and $y = 2Ak\mathbb{1}_{\text{succ}}$ (by the definitions above):

$$\begin{aligned} \text{DC} &= (1-\delta)\mathbb{E}\left[Mp\right]k + \frac{\delta}{2A}\mathbb{E}\left[Mpy\right] \\ &= \frac{(1-\delta)\mathbb{E}\left[p\right]k}{R_f} + \delta\mathbb{E}\left[p\mathbb{1}_{\text{succ}}\right]k. \end{aligned}$$

From here, the Law of Total Expectation, $\mathbb{E}\left[p\mathbb{1}_{\text{succ}}\right] = \mathbb{P}\left[\text{success}\right]\mathbb{E}\left[p|\,\text{success}\right]$, gives equation (24).

4.4 Equilibrium. I now move on to study the competitive Markov[23] equilibrium subject to the borrowing constraints above. An equilibrium constitutes a profile $\left\langle k_s^\tau, \ell_s^\tau, \left(T_s^\tau(a), T_s^\tau(b)\right)\right\rangle$ for each type of entrepreneur $\tau \in \{\alpha, \beta\}$ in each state $s \in \{a, b\}$, where k_s^τ is the amount of assets that τ-entrepreneurs hold, ℓ_s^τ is the amount they borrow, and $\left(T_s^\tau(a), T_s^\tau(b)\right)$ are their state-contingent repayments at the next date.[24] The profile is an equilibrium if it is consistent with everyone optimizing and markets clearing:

(i) Entrepreneurs maximize expected consumption subject to their borrowing constraints above (equation (25)) and their budget constraints

$$1 + \ell_a^\tau = p_a^\tau k_a^\tau \quad \text{and} \quad \ell_b^\tau = p_b^\tau k_b^\tau \tag{26}$$

(where I have used the fact that the entrepreneurs' initial wealth is one in state a and zero in state b).

(ii) Investors are indifferent at the margin among consuming, lending, and investing (this is tantamount to market clearing here, given investors are deep pocketed and risk neutral):[25]

$$R_f = \frac{\mathbb{E}\left[\,T^\tau(s_{t+1})\,|\,s_t\right]}{\ell_{s_t}^\tau} = \frac{\gamma'\big(K - k_{s_t}^\tau\big) + \mathbb{E}\left[\,p_{s_{t+1}}^\tau\,|\,s_t\right]}{p_{s_t}^\tau}. \tag{MC}$$

In words, this condition (MC) says that investors' marginal rate of substitution, which is just R_f here, is equal to their marginal expected return from lending, and also equal to their marginal expected return from investing.

4.5 Time series fluctuations. Unlike the unconstrained entrepreneurs in the Arrow–Debreu benchmark, the constrained entrepreneurs here can invest more when they have more

[23] I have already implicitly restricted attention to Markov allocations, since I use the state s_t, rather than the entire history, to index the variables above.

[24] I focus on one-period contracts. This is without loss of generality not only in my environment, in which entrepreneurs live for only two dates, but also in general environments with limited enforcement and no exclusion, as shown in Rampini and Viswanathan (2010). This suggests my results are not special to short-lived borrowers.

[25] I show this equivalence formally in an earlier version. I omit here it to streamline the analysis.

wealth to scale up. Thus, entrepreneurs' asset holdings are higher in state a, when they have some initial wealth, than in b, when they do not (in fact, they hold no capital at all in b, given their endowments are normalized to zero and hence they have nothing to pledge). This increased demand for capital assets drives prices up in state a, so asset prices are higher in a than b. Moreover, since entrepreneurs have the most productive investments, capital is better allocated when they have more of it. Hence, average productivity is higher in a than b. In these senses, state a is a "boom" and state b is a "recession." (Consistent with this interpretation, I also find that aggregate output is higher in a than b. I defer discussing this, since it depends on the cross-sectional differences between α- and β-entrepreneurs, which I do not get into until the next subsection.)

PROPOSITION 2. *State a is a "boom" and state b is a "recession" in the sense that the following statements hold.*

(i) Aggregate productivity is higher in state a than in state b.

(ii) The prices of both capital assets are higher in state a than in state b, $p_a^\alpha > p_b^\alpha$ and $p_a^\beta > p_b^\beta$.

(iii) Aggregate output is higher in state a than in state b.

Since individual productivity and asset supply do not change over time, these fluctuations are entirely the result of assets being better allocated in state a than b—they are pure "allocation cycles." Aggregate productivity goes up in booms when assets go to their best use and down in recessions when they do not. So does output, even though individual investments become less productive when investment goes up (due to decreasing returns to scale). What Basu and Fernald (2001) call an "essential feature of business cycles" (p. 225)—output and productivity moving together—arises here just because asset allocation is procyclical, something Eisfeldt and Rampini (2006) find empirical support for. Moreover, Hsieh and Klenow (2009) find that asset allocation is a major driver of output, suggesting my allocation cycles could be a first-order contributor to real-world business cycles.

The procyclicality of aggregate asset allocation is the result of the procyclicality of individual firms' debt capacity. This is consistent with evidence on procyclical leverage in Begenau and Salomao (2014), Korajczyk and Levy (2003), and Korteweg and Strebulaev (2015).[26] Further, I find that procyclical debt capacity translates into procyclical investment, consistent with evidence in Dangl and Wu (2015).

4.6 Cross-sectional variation and the collateral premium. Since state a is a boom and state b a recession (Proposition 2), α-entrepreneurs are procyclical and β-entrepreneurs countercyclical (cf. their investment technologies (19)–(20)). Hence, due to procyclical promises, α-entrepreneurs can lever up and invest more than β-entrepreneurs (cf. the borrowing constraints (25)). This drives up the price of α-assets relative to β-assets. In other words, α-assets are more expensive than β-assets, because you can borrow more against them—they trade at a collateral premium.

PROPOSITION 3. *Procyclical (α-) assets trade at a premium over countercyclical (β-) assets: $p_s^\alpha > p_s^\beta$ for $s \in \{a, b\}$.*

The specific mechanism connecting entrepreneurs' demand to asset prices goes through investors' production function γ. Since entrepreneurs can lever up relatively more against procyclical assets, investors are left holding few of them. Thus, since γ has decreasing returns to scale, their marginal productivity is relatively high. This marginal productivity sets the price, resulting in a high price of procyclical assets relative to countercyclical assets. Thus, even though prices are set by investors (who are marginal), cross-sectional price differences reflect differences in the borrowing constraints of entrepreneurs (who are infra-marginal).

This argument hinges on the assumption that assets are not freely redeployable across sectors, in line with, e.g., Shleifer and Vishny (1992):[27] here, investors use either α- or β-

[26] Halling, Yu, and Zechner (2016) point out that the empirical findings in Korteweg and Strebulaev (2015) and Korajczyk and Levy (2003) capture only the direct effect of the business cycle on leverage, but do not take into account how leverage determinants change over the business cycle.

[27] Shleifer and Vishny (1992) stress the empirical relevance of this assumption, saying that

> Unfortunately, most assets in the world are quite specialized and, therefore, are not redeployable. Oil rigs, brand name food products, pharmaceutical patents, and steel plants have no

assets, but not both. If, to the contrary, investors used both types of assets—i.e. their output were a function $\tilde{\gamma}$ of their total asset holdings of both assets, rather than the function γ of each asset individually—then there would be no collateral premium, since in equilibrium both assets would have the same marginal productivity $\tilde{\gamma}'$. This suggests that specific assets that are hard to redeploy could exhibit a higher collateral premium.

This finding is not in conflict with the widespread idea that redeployable assets are the best collateral. It just suggests that this might not lead to a premium in their prices, as their marginal valuation could already reflect their efficient use. To see why, consider the stylized example of a vineyard used to grow grapes for wine and a farm used to grow barley and hops for beer. Since wine is procyclical and beer is acyclical, the vineyard could represent an α-asset and the farm a β-asset. Now, since the vineyard is only useful to nearby winemakers, an increase in their debt capacity could increase demand enough to drive up its price, i.e. to generate the collateral premium. In contrast, since the farm is useful to many different kinds of farmers, an increase in brewers' debt capacity is unlikely to move its price. Hence, even if the farm is better collateral than the vineyard in absolute terms, the vineyard's price reflects its collateral value more. The reason is that financial constraints affect demand more for vineyards than for farmland.

4.7 A comment on policy. Since α-entrepreneurs' investments are larger than β-entrepreneurs', output is highest when they succeed, i.e. output is greater in state a than in state b, as stated above (Proposition 2). This suggests one unusual policy implication: transferring wealth from countercyclical entrepreneurs to procyclical entrepreneurs, a policy that amplifies output fluctuations, can boost welfare.[28] It helps because the more procyclical

reasonable uses other than the one they are destined for. When such assets are sold, they have to be sold to someone who will use them in approximately the same way (p. 1344).

[28]Formally, a tax-subsidy scheme that transfers endowments from a countercyclical entrepreneur to a procyclical entrepreneur increases expected output, i.e. the entrepreneurs' expected output in state s,

$$A\big(k_s^\alpha(w_s+\epsilon)+k_s^\beta(w_s-\epsilon)\big)=2RA\left(\frac{w_s+\epsilon}{2Rp_s^\alpha-p_a^\alpha-(1-\delta)p_b^\alpha}+\frac{w_s-\epsilon}{2Rp_s^\beta-p_b^\beta-(1-\delta)p_a^\beta}\right), \tag{27}$$

is increasing in ϵ.

an entrepreneur is, the more he can borrow to buy assets and scale up his investment, and hence the better asset allocation is. Of course, risk aversion would countervail agianst this result. Hence, I do not see it as something that policy makers should think about implementing off the shelf. Still, I think it points to benefits of procyclicality that could be worth taking into account, especially since the welfare costs of business cycle fluctuations appear to be quite small (e.g., Alvarez and Jermann (2004) and Lucas (1987)).

4.8 Explicit solution. Given a specific functional form for investors' production technology γ, the model admits an explicit solution:

LEMMA 8. *Let the investors' technology be $\gamma(k) = A\log(1+k)$. The equilibrium prices for $\tau \in \{\alpha, \beta\}$ are*

$$p_a^\tau = \frac{1 + K - k_a^\tau + A(2R_f - 1)(1+K)}{2R_f(R_f - 1)(1+K)(1+K-k_a^\tau)}, \tag{28}$$

$$p_b^\tau = \frac{(2R_f - 1)(1 + K - k_a^\tau) + A(1+K)}{2R_f(R_f - 1)(1+K)(1+K-k_a^\tau)}, \tag{29}$$

where, for each τ, the equilibrium asset holdings are $k_b^\tau = 0$ and

$$k_a^\tau = \frac{-a_1^\tau - \sqrt{(a_1^\tau)^2 - 4a_0 a_2^\tau}}{2a_2^\tau}, \tag{30}$$

One caveat: this is for a transfer from one entrepreneur to another. A transfer from from all countercyclical entrepreneurs to all procyclical entrepreneurs would affect prices, something this analysis does not take into account.

where

$$a_0 = 4R_f^2(R_f-1)(1+K)^2, \tag{31}$$

$$a_1^\alpha = -(1+K)\Big((2R_f-1)\big(\delta + A(2R_f-1)\big) - (1-\delta)A + 4R_f^2(R_f-1)\Big), \tag{32}$$

$$a_2^\alpha = \delta(2R_f-1), \tag{33}$$

$$a_1^\beta = -(1+K)\Big((2R_f-1)\big(\delta + (2R_f-\delta)A + 4R_f^2(R_f-1)\big)\Big), \tag{34}$$

$$a_2^\beta = \delta. \tag{35}$$

4.9 Comparative statics. I now turn to comparative statics on δ, which can be interpreted as the rate of depreciation or the proportion of assets that can be diverted. The analysis implies that debt capacity is more sensitive to cyclicality for high δ (see equation (25)). Given the analysis in Section 3, this suggests the new testable prediction that the correlation between leverage and cash-flow beta should be increasing in δ, where increasing δ could be captured by (i) decreasing asset tangibility across firms or (ii) decreasing the strength of legal enforcement across countries. Further, in the model, asset prices and aggregate output are more volatile for low δ, consistent with evidence that aggregate fluctuations are negatively related to economic and financial development (King and Levine (1993), Koren and Tenreyro (2007), and Rajan and Zingales (1998)).[29]

4.10 Financial assets. So far, I have focused on how procyclical promises affect the prices of capital assets. Here, I try to say something about financial assets too. To do so, I model them in a stylized way, defining a "stock" in τ-entrepreneurs as a claim on the output of all their future generations. To find an expression for its price, recall that entrepreneurs invest only in state a (they have no wealth in state b). Hence, their investments pay off with probability 1/4—α-entrepreneurs' pay off in state a following a and β-entrepreneurs'

[29]This echoes the results of models such as Cooley, Marimon, and Quadrini (2004), Kiyotaki and Moore (1997), and Rampini (2004) in which limited enforceability amplifies aggregate fluctuations. Unlike in these models, fluctuations in my model do not result from productivity shocks at any individual firm, but only from changes in capital allocation across firms. (Eisfeldt and Rampini (2008) explore a mechanism based on managers' private information that also leads to inefficient capital allocation in downturns.)

in b following a. Thus, the average stock prices are given by the expected discounted sum of future cash flows as follows:

$$\bar{S}^{\alpha} = \mathbb{E}\left[\sum_{t=1}^{\infty} \frac{1}{4} M_t\, \alpha\left(k_a^{\alpha}\right)\right] \quad \text{and} \quad \bar{S}^{\beta} = \mathbb{E}\left[\sum_{t=1}^{\infty} \frac{1}{4} M_t\, \beta\left(k_a^{\beta}\right)\right], \tag{36}$$

where I have reintroduced the SDF M, to capture, e.g., the marginal utility of an (unmodeled) investor in the stock market. To sum the series, I separate out the time discounting by writing M in terms of the risk-neutral measure $\mathbb{Q}$. This gives

$$\bar{S}^{\alpha} = \frac{Ak_a^{\alpha}}{2(R-1)} \frac{d\mathbb{Q}}{d\mathbb{P}}(a) \quad \text{and} \quad \bar{S}^{\beta} = \frac{Ak_a^{\beta}}{2(R-1)} \frac{d\mathbb{Q}}{d\mathbb{P}}(b). \tag{37}$$

These expressions give us another way to see the trade-off between the procyclical promises and the discount rate terms. The procyclical promises term allows α-entrepreneurs to lever up, so $k_a^{\alpha} > k_b^{\beta}$ above. This pushes up the price $\bar{S}^{\alpha}$ of the α-stock. But the discount rate term should put more weight on cash flows in state a (when output is low) than in state b (when it is high), or

$$\frac{d\mathbb{Q}}{d\mathbb{P}}(a) < 1 < \frac{d\mathbb{Q}}{d\mathbb{P}}(b). \tag{38}$$

This pushes up the price $\bar{S}^{\beta}$ of the β-stock. Like in the analysis of debt capacity above (Section 3), which term dominates for stock prices depends on the trade-off between these two effects. In this asset pricing context, the procyclical promises term may help to explain why procyclical assets do not trade at as much of a discount as models based on risk aversion alone typically suggest they should (Fama and French (2004)).

5 Conclusion

In this paper, I explore how cyclicality affects debt capacity. I start with the observation that there is a benefit of procyclicality, one present across a variety of models of financial frictions:

procyclical firms can make higher repayments on average. The reason is that their output is high when asset prices are high, and high asset prices mitigate financial frictions, for example because they loosen collateral constraints. I derive a robust decomposition of a firm's debt capacity into a "procyclical promises term," capturing this new benefit of procyclicality, and a "discount rate term," capturing its established cost. I go on to show that the procyclical promises term can be just as important as the discount rate term in some classical models.

To explore the implications of the mechanism for the aggregate economy, I embed it in a general equilibrium model. I show that it gives rise to fluctuations in aggregate output, productivity, and prices, even in an environment in which there is no time-series variation whatsoever in the frictionless benchmark. I.e. there are aggregate fluctuations, but they are pure "allocation cycles"—they arise entirely because investment goes up when procyclical firms are most productive, since procyclical firms are the firms that can lever up their investments the most. This leads to high demand for the assets they use, and hence generates a collateral premium on these "procyclical assets" in the cross section.

These results emphasize that allocating more capital to procyclical firms can improve capital allocation by loosening financial constraints. Thus, within the model, an unusual policy can improve welfare, even though it amplifies the business cycle: make transfers to procyclical firms. This points to a possible trade-off between improving capital allocation and smoothing aggregate fluctuations.

A Proofs

Proof of Lemma 1

The result is immediate from the argument in the text.

Proof of Lemma 2

Debt capacity is highest if the repayment is highest in every state, i.e. if the inequality binds in equation (4). Since creditors receive the repayment only if the investment succeeds, we have that

$$\mathrm{DC} = \mathbb{E}\left[M \mathbb{1}_{\text{succ}} \left(R_f \mathbb{E}\left[Mpy\right] - \frac{B}{\pi_1 - \pi_0} \right) \Bigg| \, e = 1 \right], \tag{39}$$

where I have conditioned on $e = 1$, as is ensured by the incentive constraint. Now, since $\mathbb{1}_{\text{succ}}$ depends only on e (i.e. it is independent of M) and $\mathbb{E}\left[M\right] = 1/R_f$, we can compute the debt capacity:

$$\begin{aligned}
\mathrm{DC} &= \mathbb{E}\left[\mathbb{1}_{\text{succ}} \,|\, e = 1\right] \mathbb{E}\left[M\right] \left(R_f \mathbb{E}\left[Mpy\right] - \frac{B}{\pi_1 - \pi_0} \right) && (40)\\
&= \pi_1 \left(\mathbb{E}\left[Mpy\right] - \frac{B}{R_f(\pi_1 - \pi_0)} \right) && (41)\\
&= \alpha_0 + \alpha_1 \mathbb{E}\left[Mpy\right], && (42)
\end{aligned}$$

where α_0 and α_1 are as stated in the lemma.

Proof of Lemma 3

From equation (7), we have immediately that

$$\begin{aligned}
\mathrm{DC} &= \mathbb{E}\left[M\left((1-\delta)pk + \frac{\delta py}{A}\right)\right] && (43)\\
&= (1-\delta)\mathbb{E}\left[Mp\right]k + \frac{\delta}{A}\mathbb{E}\left[Mpy\right] && (44)\\
&= \alpha_0 + \alpha_1\mathbb{E}\left[Mpy\right], && (45)
\end{aligned}$$

where α_0 and α_1 are as stated in the lemma.

Proof of Proposition 1

The proof is by direct calculation. I start with the covariance formula:

$$\mathbb{E}\left[Mpy\right] = \mathbb{E}\left[Mp\right]\mathbb{E}\left[y\right] + \mathrm{Cov}\left[Mp, y\right]. \tag{46}$$

Recall that the forward price of an asset with price p is $f_p = R_f\mathbb{E}\left[Mp\right]$, so we can write

$$\mathbb{E}\left[Mpy\right] = \frac{1}{R_f} f_p\,\mathbb{E}\left[y\right] + \mathrm{Cov}\left[Mp, y\right]. \tag{47}$$

From here, most of the proof is just the computation of the second term on the RHS. I use the following notation to streamline it: for a random variable X, $\bar{X} := \mathbb{E}\left[X\right]$ and $\Delta_X := X - \bar{X}$, so $X \equiv \bar{X} + \Delta_X$. Now:

$$\begin{aligned}
\mathrm{Cov}\left[Mp, y\right] &= \mathrm{Cov}\left[(\bar{M} + \Delta_M)(\bar{p} + \Delta_p), y\right] && (48)\\
&= \mathrm{Cov}\left[\bar{M}\bar{p} + \bar{p}\,\Delta_M + \bar{M}\Delta_p + \Delta_M\Delta_p, y\right] && (49)\\
&= \bar{p}\,\mathrm{Cov}\left[\Delta_M, y\right] + \bar{M}\mathrm{Cov}\left[\Delta_p, y\right] + \mathrm{Cov}\left[\Delta_M\Delta_p, y\right]. && (50)
\end{aligned}$$

Now, recall that for any random variables X and Y we have that $\mathrm{Cov}[X,Y] = \mathrm{Cov}[\Delta_X, Y]$. With this and the fact that $\bar{M} = 1/R_f$, we can substitute back for $\mathrm{Cov}[Mp, y]$ in the original expression and write

$$\mathbb{E}[Mpy] = \frac{1}{R_f} f_p \, \mathbb{E}[y] + \bar{M}\mathrm{Cov}[\Delta_p, y] + \bar{p}\mathrm{Cov}[\Delta_M, y] + \mathrm{Cov}[\Delta_M \Delta_p, y] \tag{51}$$

$$= \frac{1}{R_f} f_p \, \mathbb{E}[y] + \frac{1}{R_f}\mathrm{Cov}[p, y] + \bar{p}\,\mathrm{Cov}[M, y] + \epsilon, \tag{52}$$

where $\epsilon = \mathrm{Cov}[\Delta_M \Delta_p, y]$. This is the expression in the proposition.

Proof of Lemma 4

The proof comprises calculating asset prices and the SDF, i.e. the constants a_0 and a_1 mentioned in the text and the lemma.

Asset prices. After the shock, the rental price of capital is constant. The price of assets is just the present value of the rental prices, and we can use the formula for a perpetuity:

$$p = p_{t^*+1} = \sum_{t=1}^{\infty} \frac{A_{t^*+1} F'(K)}{R_f^t} \tag{53}$$

$$= \frac{A_{t^*+1} F'(K)}{R_f - 1}. \tag{54}$$

So we can write $p_{t^*+1} = a_0 A_{t^*+1}$ for $a_0 := F'(K)/(R_f - 1)$.

SDF. After the shock, $t \geq t^*$, the output is $Y \equiv A_{t^*+1} F(K)$ by assumption. Re-writing equation (12) gives

$$M \equiv M_{t^*, t^*+1} = \frac{Y_{t^*}}{R_f A_{t^*+1} F(K)}. \tag{55}$$

So we can write $M = a_1 / A_{t*+1}$ for $a_1 := Y_{t^*}/(R_f F(K))$.

Proof of Lemma 5

I start with the definition of ϵ in Proposition 1. I substitute $p = a_0 A$ and $M = a_1/A$ as in Section 3 (see Lemma 4) to manipulate the expression for ϵ. Finally, I use the Taylor approximation to show that it is close to zero.

From equation (9) and the expressions for p and M in Lemma 4, we have that

$$\epsilon = \mathrm{Cov}\left[\big(M - \mathbb{E}[M]\big)\big(p - \mathbb{E}[p]\big), y\right] \tag{56}$$

$$= \mathrm{Cov}\left[\left(\frac{a_1}{A} - \mathbb{E}\left[\frac{a_1}{A}\right]\right)\left(a_0 A - \mathbb{E}[a_0 A]\right), y\right] \tag{57}$$

$$= a_0 a_1 \mathrm{Cov}\left[\left(\frac{1}{A} - \mathbb{E}[1/A]\right)\left(A - \mathbb{E}[A]\right), y\right] \tag{58}$$

$$= a_0 a_1 \mathrm{Cov}\left[1 - \mathbb{E}[1/A]\, A - \frac{1}{A}\mathbb{E}[A] + \mathbb{E}[1/A]\,\mathbb{E}[A], y\right]. \tag{59}$$

By the linearity of the covariance and the fact that the covariance of anything and a constant is zero, this expression simplifies to

$$\epsilon = a_0 a_1 \left(- \mathbb{E}[1/A]\, \mathrm{Cov}[A, y] - \mathbb{E}[A]\, \mathrm{Cov}\left[\frac{1}{A}, y\right]\right). \tag{60}$$

Now, I use the first-order Taylor approximation of $1/A$ centered around the mean $\mathbb{E}[A]$:

$$\frac{1}{A} \approx \frac{1}{\mathbb{E}[A]} + \left(\frac{1}{A}\right)'\Bigg|_{A=\mathbb{E}[A]} \left(A - \mathbb{E}[A]\right) \tag{61}$$

$$= \frac{1}{\mathbb{E}[A]} - \frac{1}{(\mathbb{E}[A])^2}\big(A - \mathbb{E}[A]\big) \tag{62}$$

$$= \frac{2}{\mathbb{E}[A]} - \frac{A}{(\mathbb{E}[A])^2}. \tag{63}$$

This implies $\mathbb{E}[1/A] \approx 1/\mathbb{E}[A]$. Using this and returning to ϵ gives:

$$\epsilon \approx a_0 a_1 \left(- \frac{1}{\mathbb{E}[A]} \mathrm{Cov}[A, y] - \mathbb{E}[A]\, \mathrm{Cov}\left[\frac{2}{\mathbb{E}[A]} - \frac{A}{(\mathbb{E}[A])^2}, y\right]\right) = 0. \tag{64}$$

Proof of Lemma 6

I first compute the discount rate term and then the procyclical promises term. Below, I use the approximations that

$$\frac{1}{A} \approx \frac{2}{\mathbb{E}\left[A\right]} - \frac{A}{\mathbb{E}\left[A\right]^2} \tag{65}$$

(cf. the proof of Lemma 5 above) and

$$\frac{1}{I}\sum_{i=1}^{I} \varepsilon^i \approx 0, \tag{66}$$

where I have omitted the $t^* + 1$ subscripts, as I will do throughout.

I start with the discount rate term:

$$
\begin{aligned}
\mathbb{E}\left[p^{i^*}\right] \mathrm{Cov}\left[M, y\right] &= \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \mathrm{Cov}\left[M, y\right] && (67)\\
&= \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \mathrm{Cov}\left[\frac{Y_{t^*}}{R_f Y}, y\right] && (68)\\
&= \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \cdot \frac{Y_{t^*}}{R_f} \mathrm{Cov}\left[\frac{1}{\sum_{i=1}^{I} A^i F(K)}, a\left(A + \varepsilon^{i^*} + \varepsilon^y\right)\right] && (69)\\
&= \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \cdot \frac{Y_{t^*}}{R_f} \cdot \frac{a}{F(K)} \mathrm{Cov}\left[\frac{1}{\sum_{i=1}^{I} A^i}, \left(A + \varepsilon^{i^*} + \varepsilon^y\right)\right] && (70)\\
&\approx \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \cdot \frac{Y_{t^*}}{R_f} \cdot \frac{a}{F(K)} \mathrm{Cov}\left[\frac{1}{IA}, \left(A + \varepsilon^{i^*} + \varepsilon^y\right)\right] && (71)\\
&\approx \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \cdot \frac{Y_{t^*}}{R_f} \cdot \frac{a}{F(K)} \cdot \frac{1}{I} \mathrm{Cov}\left[\frac{2}{\mathbb{E}\left[A\right]} - \frac{A}{\mathbb{E}\left[A\right]^2}, \left(A + \varepsilon^{i^*} + \varepsilon^y\right)\right] && (72)\\
&\approx \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \cdot \frac{Y_{t^*}}{R_f} \cdot \frac{a}{F(K)} \cdot \frac{1}{I} \left(-\frac{1}{\mathbb{E}\left[A\right]^2}\right) \mathrm{Cov}\left[A, \left(A + \varepsilon^{i^*} + \varepsilon^y\right)\right] && (73)\\
&= \frac{\mathbb{E}\left[A^{i^*}\right] F'(K)}{R_f - 1} \cdot \frac{Y_{t^*}}{R_f} \cdot \frac{a}{F(K)} \cdot \frac{1}{I} \left(-\frac{1}{\mathbb{E}\left[A\right]^2}\right) \left(\mathrm{Cov}\left[A, A\right] + \mathrm{Cov}\left[A, \varepsilon^{i^*}\right] + \mathrm{Cov}\left[A, \varepsilon^y\right]\right) && (74)\\
&= -\beta_0 \mathrm{Var}\left[A\right], && (75)
\end{aligned}
$$

having used $\mathrm{Cov}\left[A, \varepsilon^{i^*}\right] = \mathrm{Cov}\left[A, \varepsilon^y\right] = 0$ and substituted β_0 as defined in equation (18).

Now I turn to the procyclical promises term:

$$\frac{1}{R_f}\text{Cov}\left[p^{i^*}, y\right] = \frac{1}{R_f}\text{Cov}\left[\frac{A^{i^*}F'(K)}{R_f-1}, a\big(A+\varepsilon^{i^*}+\varepsilon^{y}\big)\right] \tag{76}$$

$$= \frac{1}{R_f}\cdot\frac{aF'(K)}{R_f-1}\text{Cov}\left[A^{i^*}, A+\varepsilon^{i^*}+\varepsilon^{y}\right] \tag{77}$$

$$= \frac{1}{R_f}\cdot\frac{aF'(K)}{R_f-1}\text{Cov}\left[A+\varepsilon^{i^*}, A+\varepsilon^{i^*}+\varepsilon^{y}\right] \tag{78}$$

$$= \frac{1}{R_f}\cdot\frac{aF'(K)}{R_f-1}\Big(\text{Cov}\left[A, A\right] + 2\text{Cov}\left[A, \varepsilon^{i^*}\right] + \text{Cov}\left[\varepsilon^{i^*}, \varepsilon^{i^*}\right] + \text{Cov}\left[\varepsilon^{i^*}, \varepsilon^{y}\right]\Big) \tag{79}$$

$$= \beta_1\left(\text{Var}\left[A\right] + \text{Var}\left[\varepsilon^{i^*}\right]\right), \tag{80}$$

having used $\text{Cov}\left[A, \varepsilon^{i^*}\right] = \text{Cov}\left[\varepsilon^{i^*}, \varepsilon^{y}\right] = 0$ and substituted β_1 as defined in equation (18).

Proof of Lemma 7

The proof follows from the fact that in the first-best each capital asset must be put to its most productive use. Since entrepreneurs are always more productive than investors, α-entrepreneurs hold all α-assets and β-entrepreneurs hold all β-assets or $k_s^\alpha = K$ and $k_s^\beta = K$ in both states. The three statements in the proposition all follow:

Statement (i). Given that $k_s^\alpha = K$ and $k_s^\beta = K$, the aggregate productivity (see footnote 20) is

$$\frac{\mathbb{E}\left[\alpha\big(k_{s_t}^\alpha\big) + \gamma\big(K - k_{s_t}^\alpha\big) + \beta\big(k_{s_t}^\beta\big) + \gamma\big(K - k_{s_t}^\beta\big)\right]}{2K} = \frac{\mathbb{E}\left[\alpha\big(K\big) + \beta\big(K\big)\right]}{2K} = A \tag{81}$$

since $\gamma(0) = 0$ and $\mathbb{E}\left[\tau(k)\right] = Ak$ for $\tau \in \{\alpha, \beta\}$.

Statement (ii). The prices of α- and β-assets are determined such that investors are indifferent between consuming and buying assets (cf. the pricing equation (MC) in Subsection

4.4). Entrepreneurs hold all the assets, $k_s^\tau = K$, so $\gamma'(K - k_s^\tau) = \gamma'(0) = A$; hence,

$$R_f = \frac{\gamma'\big(K - k_s^\tau\big) + \bar{p}^\tau}{p_s^\tau} \tag{82}$$
$$= \frac{A + \bar{p}^\tau}{p_s^\tau} \tag{83}$$

for both assets and both states. Substituting in $\bar{p}^\tau = (p_a^\tau + p_b^\tau)/2$ and rearranging implies that p_a^τ and p_b^τ solve the following system:

$$\begin{cases} (2R_f - 1)p_a^\tau = 2A + p_b^\tau, \\ (2R_f - 1)p_b^\tau = 2A + p_a^\tau. \end{cases} \tag{84}$$

Solving gives $p_s^\tau = A/(R_f - 1)$ for $\tau \in \{\alpha, \beta\}$ and $s \in \{a, b\}$.

Statement (iii). Expected output is

$$\text{total expected output} = \mathbb{E}\left[\alpha\big(k_s^\alpha\big) + \gamma\big(K - k_s^\alpha\big) + \beta\big(k_s^\beta\big) + \gamma\big(K - k_s^\beta\big)\right] = 2AK. \tag{85}$$

□

Proof of Proposition 2

To prove the proposition, I first analyze entrepreneurs' borrowing and investment behavior. I prove that entrepreneurs borrow to capacity (Lemma 9). This allows me to solve for the entrepreneurs' asset holdings, k_s^τ for $\tau \in \{\alpha, \beta\}$ and $s \in \{a, b\}$ (Lemma 10).

Lemma 9. *Entrepreneurs borrow to capacity, $\ell = DC$, i.e. the borrowing constraints in equation (25) bind.*

Proof. I prove this result in two steps. First, I show that the first-best outcome is not attained. This implies an upper bound on prices: $p_s^\tau < (A + \bar{p}^\tau)/R_f$. Second, I show that given prices are below this bound, entrepreneurs wish to scale up their investments as much as possible, so their borrowing constraints bind. Intuitively, since entrepreneurs' investments are highly productive, they borrow as much as they can to invest as much as they can. Note that the meat of the argument is in Step 2; Step 1 is all about making sure an inequality in Step 2 is strict.

Step 1: First-best not attained. This says that entrepreneurs do not hold all the capital, as they would in the first-best (Lemma 7): $k_s^\tau < K$ for both types of entrepreneurs $\tau \in \{\alpha, \beta\}$ in both states $s \in \{a, b\}$.

To prove the result, start by writing DC from equation (25) as

$$\mathrm{DC}^\tau = \frac{(1-\delta)\bar{p}^\tau + \frac{\delta}{2} p_{\mathrm{succ}}^\tau}{R_f} k^\tau, \tag{86}$$

where p_{succ}^τ denotes the price of τ-assets given τ-entrepreneurs succeed—i.e. $p_{\mathrm{succ}}^\alpha = p_a^\alpha$ and $p_{\mathrm{succ}}^\beta = p_b^\beta$.

Now, suppose (in anticipation of a contradiction) that an entrepreneur holds all the assets in state s, $k_s^\tau = K$ for $\tau \in \{\alpha, \beta\}$. Thus, a τ-entrepreneur's budget constraint implies

$$K = \frac{w_s + \ell_s^\tau}{p_s^\tau} \tag{87}$$

where w_s denotes his endowment in state s ($w_a = 1$ and $w_b = 0$). Since $\ell_s^\tau \leq \mathrm{DC}$, by definition, we have

$$K \leq \frac{w_s + \mathrm{DC}}{p_s^\tau}. \tag{88}$$

Substituting $k^\tau = K$ into equation (86) and rearranging gives

$$K \leq \frac{R_f w_s}{R_f p_s^\tau - (1-\delta)\bar{p}^\tau - \delta p_{\mathrm{succ}}^\tau / 2} \tag{89}$$

$$\leq \frac{R_f}{R_f p_s^\tau - \bar{p}^\tau} \tag{90}$$

since $\bar{p}^\tau = (p_a^\tau + p_b^\tau)/2 \geq p_{\mathrm{succ}}^\tau/2$. Now, the pricing equation (MC) gives an expression for the denominator above,

$$R_f p_s^\tau - \bar{p}^\tau = \gamma'(K - k_s^\tau) = \gamma'(0) = A, \tag{91}$$

since $k_s^\tau = K$ by hypothesis. Substituting this into equation (90) implies

$$K \leq \frac{R_f}{A}. \tag{92}$$

This contradicts the assumption in equation (21). We conclude that $k_s^\tau < K$ and hence $\gamma'(K - k_s^\tau) < A$. Given this, equation (MC) implies that

$$p_s^\tau < \frac{A + \bar{p}^\tau}{R_f}. \tag{93}$$

Step 2: Entrepreneurs scale up. In state s at date t, a τ-entrepreneur borrows ℓ at rate R_f to invest k to maximize his expected payoff, i.e. to maximize his expected output plus the future value of his assets minus his repayments:

$$\mathbb{E}\left[\tau(k) + p_{s_{t+1}}^\tau k - T(s)\right] = Ak + \bar{p}^\tau k - R_f \ell \tag{94}$$

subject to the borrowing constraint in equation (25) and his budget constraint

$$p_s^\tau k = w_s + \ell, \tag{95}$$

(remember w_s is the entrepreneur's endowment, $w_a = 1$ and $w_b = 0$). Substituting from the budget constraint gives the objective function

$$\begin{aligned} Ak + \bar{p}^\tau k - R_f \ell &= (A + \bar{p}^\tau)\frac{w + \ell}{p_s^\tau} - R_f \ell && (96) \\ &= \left(\frac{A + \bar{p}^\tau}{p_s^\tau} - R_f\right)\ell + \frac{A + \bar{p}^\tau}{p_s^\tau} w. && (97) \end{aligned}$$

This is strictly increasing in ℓ as long as $(A+\bar{p}^\tau)/p_s^\tau > R_f$ or

$$p_s^\tau < \frac{A+\bar{p}}{R_f}, \tag{98}$$

which is satisfied by Step 1 above (equation (93)). Hence, entrepreneurs maximize ℓ, i.e. they borrow to capacity: $\ell = \mathrm{DC}$. □

I now solve for entrepreneurs' asset holdings.

Lemma 10. *Entrepreneurs' asset holdings are given by*

$$k_a^\alpha = \frac{2R_f}{(2R_f-1)p_a^\alpha - (1-\delta)p_b^\alpha}, \tag{99}$$

$$k_a^\beta = \frac{2R_f}{(2R_f-1+\delta)p_a^\beta - p_b^\beta}, \tag{100}$$

and $k_b^\alpha = k_b^\beta = 0$.

Proof. Given (both types of) entrepreneurs have a unit wealth in state a and nothing in state b, their budget constraints read

$$p_a^\tau k_a^\tau = 1 + \mathrm{DC}_a^\tau, \tag{101}$$

$$p_b^\tau k_b^\tau = \mathrm{DC}_b^\tau. \tag{102}$$

Now, we can find k_b^τ and k_a^τ given that entrepreneurs borrow to capacity, $\ell = \mathrm{DC}$ (Lemma 9).

To find k_b^τ, observe that DC is proportional to k (cf. equation (86)). Hence, the state-b budget constraint above implies that it must be that $k_b^\tau = 0$ for $\tau \in \{\alpha, \beta\}$.

To find k_a^τ, substitute the expression for DC from equation (86) into the state-a budget constraint:

$$p_a^\tau k_a^\tau = 1 + \mathrm{DC}_a^\tau = 1 + \frac{(1-\delta)\bar{p}^\tau + \frac{\delta}{2}p_{\mathrm{succ}}^\tau}{R_f} k_a^\tau. \tag{103}$$

Rearranging gives

$$k_a^\tau = \frac{R_f}{Rp_a^\tau - (1-\delta)\bar{p}^\tau - \frac{\delta}{2}p_{\text{succ}}^\tau}. \tag{104}$$

Substituting $p_{\text{succ}}^\alpha = p_a^\alpha$ and $p_{\text{succ}}^\beta = p_b^\beta$ gives the expressions in the lemma.

□

Given Lemma 9 and Lemma 10, I turn to the statements in the proposition.

Statement (i). The fact that productivity is higher in state a than in state b follows from Lemma 10 above, which implies that both types of entrepreneurs hold more assets in state a than in state b. Aggregate productivity is higher when entrepreneurs hold more assets, since they use them more productively than investors do.

Statement (ii). The fact that prices are higher in state a than in state b follows from equation (MC) and Lemma 10 above. Equation MC says that

$$p_s^\tau = \frac{\gamma'\left(K - k_s^\tau\right) + \bar{p}^\tau}{R_f}. \tag{105}$$

Since γ' is decreasing (i.e. $\gamma'' < 0$), $p_a^\tau > p_b^\tau$ if and only if $k_a^\tau > k_b^\tau$, as is the case by Lemma 10.

Statement (iii). For the proof of statement (iii), see the proof of Proposition 3 below. There, I establish that α-entrepreneurs invest more than β-entrepreneurs in each state, i.e. $k_s^\alpha > k_s^\beta$. As a result, output in state a is greater than output in state b, since the α-technology pays off in state a and the β-technology pays off in state b.

Proof of Proposition 3

The proof is by contradiction. I proceed in three steps. In Step 1, I show that it it sufficient to compare the state-a asset holdings, i.e. the price of α-assets is higher than the price of β-assets in each state if and only if $k_a^\alpha > k_a^\beta$. In Step 2, I define the variable Δ^τ as the difference in the prices of τ-assets across states, $\Delta^\tau := p_a^\tau - p_b^\tau$. I show that there is a collateral premium—$p_s^\alpha > p_s^\beta$—if and only if $\Delta^\alpha > \Delta^\beta$. (Note that this step connects the difference in prices across states with the difference across assets.) In Step 3, I suppose that $p_a^\beta \geq p_a^\alpha$ and show that it leads to a contradiction.

Step 1. Writing equation (MC) for each state $s \in \{a, b\}$ and rearranging, we get an expression for the average price:

$$\bar{p}^\tau = \frac{\gamma'(K - k_a^\tau) + \gamma'(K - k_b^\tau)}{2(R_f - 1)}. \tag{106}$$

Now, using $k_b^\alpha = k_b^\beta = 0$ from Lemma 10 and substituting back into equation (MC) gives

$$p_a^\tau = \frac{(2R_f - 1)\gamma'(K - k_a^\tau) + \gamma'(K)}{2R_f(R_f - 1)} \tag{107}$$

and

$$p_b^\tau = \frac{\gamma'(K - k_a^\tau) + (2R_f - 1)\gamma'(K)}{2R_f(R_f - 1)}. \tag{108}$$

The expressions for p_a^τ and p_b^τ in equations (107) and (108) both depend only on k_a^τ. The fact that $\gamma'' < 0$ implies that both p_a^τ and p_b^τ are increasing in k_a^τ. Thus, the following three statements are equivalent: (i) $p_a^\alpha > p_a^\beta$, (ii) $p_b^\alpha > p_b^\beta$, and (iii) $k_a^\alpha > k_a^\beta$.

Step 2. Define Δ^τ as the difference in price across states for a τ-entrepreneur:

$$\Delta^\tau := p_a^\tau - p_b^\tau. \tag{109}$$

Equation (MC) gives an expression for Δ^τ in terms of k_a^τ,

$$\Delta^\tau = \frac{\gamma'(K - k_a^\tau) - \gamma'(K)}{R_f}, \tag{110}$$

so

$$\gamma'(K - k_a^\tau) = R_f \Delta^\tau + \gamma'(K). \tag{111}$$

With this and equation (106) from Step 1, we can compute that

$$\bar{p}^{\tau} = \frac{\gamma'(K - k_a^{\tau}) + \gamma'(K)}{2(R_f - 1)} \tag{112}$$

$$= \frac{2\gamma'(K) + R_f \Delta^{\tau}}{2(R_f - 1)}. \tag{113}$$

From this expression, we can see that there is a collateral premium—$\bar{p}^{\alpha} > \bar{p}^{\beta}$—if and only if $\Delta^{\alpha} > \Delta^{\beta}$.

Step 3. Suppose (in anticipation of a contradiction) that $p_s^{\beta} \geq p_s^{\alpha}$. By Step 1, it must be that $k_a^{\beta} \geq k_a^{\alpha}$. Given the expressions for k_a^{β} and k_a^{α} in Lemma 10, this says that

$$\frac{2R_f}{(2R_f - 1 + \delta)p_a^{\beta} - p_b^{\beta}} \geq \frac{2R_f}{(2R_f - 1)p_a^{\alpha} - (1 - \delta)p_b^{\alpha}} \tag{114}$$

or

$$(2R_f - 1)p_a^{\alpha} - (1 - \delta)p_b^{\alpha} \geq (2R - 1 + \delta)p_a^{\beta} - p_b^{\beta} \tag{115}$$

Now, eliminate p_a^{β} and p_a^{α} from this inequality using $p_a^{\beta} = p_b^{\beta} + \Delta^{\beta}$ and $p_a^{\alpha} = p_b^{\alpha} + \Delta^{\alpha}$:

$$(2R_f - 1)(p_b^{\alpha} + \Delta^{\alpha}) - (1 - \delta)p_b^{\alpha} \geq (2R_f - 1 + \delta)(p_b^{\beta} + \Delta^{\beta}) - p_b^{\beta} \tag{116}$$

or

$$\big(2(R_f - 1) + \delta\big)(p_b^{\alpha} - p_b^{\beta}) \geq (2R_f - 1)(\Delta^{\beta} - \Delta^{\alpha}) + (1 - \theta)\Delta^{\beta}. \tag{117}$$

The right-hand side is strictly positive since $\Delta^{\beta} > 0$ and, by Step 2, $\Delta^{\beta} \geq \Delta^{\alpha}$ under the hypothesis that $p_a^{\beta} \geq p_a^{\alpha}$. Thus, we have that

$$p_b^{\alpha} - p_b^{\beta} > 0, \tag{118}$$

contradicting the hypothesis that $p_b^{\beta} \geq p_b^{\alpha}$. We therefore conclude that $p_b^{\beta} > p_b^{\alpha}$. $\square$

Proof of Lemma 8

α-entrepreneurs. Recall $k_b^\alpha = 0$, since entrepreneurs have no endowment in state b. Hence, we have a (non-linear) system of three equations in three unknowns:

$$\begin{aligned} k_a^\alpha &= \frac{2R_f}{(2R_f-1)p_a^\alpha - (1-\delta)p_b^\alpha}, && (119)\\ (2R_f-1)p_a^\alpha &= p_b^\alpha + \frac{2A}{1+K-k_a^\alpha}, && (120)\\ (2R_f-1)p_b^\alpha &= p_a^\alpha + \frac{2}{1+K}. && (121) \end{aligned}$$

Equations (120) and (121) are linear in p_a^α and p_b^α. Solving them simultaneously gives

$$\begin{aligned} p_a^\alpha &= \frac{1+K-k_a^\alpha + A(2R_f-1)(1+K)}{2R_f(R_f-1)(1+K)(1+K-k_a^\alpha)}, && (122)\\ p_b^\alpha &= \frac{(2R_f-1)(1+K-k_a^\alpha) + A(1+K)}{2R_f(R_f-1)(1+K)(1+K-k_a^\alpha)}. && (123) \end{aligned}$$

Substituting these expressions into equation (119) gives the following equation for k_a^α:

$$\begin{aligned} &\Big((2R_f-1)\big(1+K-k_a^\alpha + A(2R_f-1)(1+K)\big)\Big)k_a^\alpha \\ &-(1-\delta)\big((2R_f-1)(1+K-k_a^\alpha)+A(1+K)\big)k_a^\alpha \qquad (124)\\ &= 4R_f^2(R_f-1)(1+K)\big(1+K-k_a^\alpha\big). \end{aligned}$$

This is a quadratic equation with coefficients a_0, a_1^α, and a_2^α given in the lemma. k_a^τ in equation (30) is its solution (it is easy so see that the larger root is greater than K, making the smaller root the relevant one, since no one can hold more than the total supply of assets in equilibrium).

β-entrepreneurs. Again, $k_b^\beta = 0$, we have a (non-linear) system of three equations in three unknowns:

$$k_a^\beta = \frac{2R_f}{(2R_f-(1-\delta))p_a^\beta - p_b^\beta}, \tag{125}$$

$$(2R_f-1)p_a^\beta = p_b^\beta + \frac{2A}{1+K-k_a^\beta}, \tag{126}$$

$$(2R_f-1)p_b^\beta = p_a^\beta + \frac{2}{1+K}. \tag{127}$$

Equations (126) and (127) have the same form as equations (120) and (121), and solving gives the same expressions (with the α-index replaced by β). Substituting these expressions into equation (125) gives the following equation for k_a^β:

$$\begin{aligned}\Big(\big(2R_f-(1-\delta)\big)\big(1+K-k_a^\beta+A(2R_f-1)(1+K)\big)\Big)k_a^\beta \\ -\Big((2R_f-1)(1+K-k_a^\beta)+A(1+K)\Big)k_a^\beta \\ = 4R_f^2(R_f-1)(1+K)(1+K-k_a^\beta).\end{aligned} \tag{128}$$

This is a quadratic equation with coefficients a_0, a_1^β, and a_2^β given in the lemma. k_a^τ in equation (30) is its solution (it is easy so see that the larger root is greater than K, making the smaller root the relevant one, since no one can hold more than the total supply of assets in equilibrium).

B Extensive Form for Subsection 2.3 and Generalized Collateral Constraints

Here I set up the model with asset substitution and collateral constraints in Subsection 2.3 as an extensive form game in which collateral constraints result from ex post renegotiation. I do this for two reasons: first, to be rigorous about the optimality of the actions I describe in Subsection 2.3 and, second, to allow for general bargaining power between the firm and its creditor in the event of renegotiation. This not only affirms the baseline results—the collateral constraint in the text (equation (1)) corresponds to the case in which the firm has all the bargaining power—but also also generates a new testable comparative-static prediction via the new bargaining power parameter. Note that here I am also explicit that contractual repayments can depend on the aggregate state; this ensures that my results are not driven by any restriction to plain debt (or other ad hoc contractual restrictions).

Extensive form. A firm and its creditor play an extensive-form game with three dates, denoted by $t = 0$, $t = 1/2$, and $t = 1$. A firm has an investment that uses capital k_0. At $t = 1/2$, it can continue or abandon its investment. The investment has the chance of paying off if the firm continues; it delivers private benefits B to the firm if it abandons. At $t = 1$, the firm's debt matures, and it either repays or it defaults and renegotiates with its creditor. In more formal detail, the timing is as follows.

t=0. The firm has assets k_0 and borrows from a creditor via a contingent contract promising $T(s)$ in state s at $t = 1$.[30]

t=1/2. The firm learns whether its investment will succeed or fail and decides whether to continue or abandon it. If it continues, its assets stay in place. If it abandons, it gets private benefits B, but its assets depreciate by an amount δ. I let k_1 denote the assets the firm has after this decision:

$$k_1 = \begin{cases} k_0 & \text{if continue,} \\ (1-\delta)k_0 & \text{if abandon.} \end{cases} \tag{129}$$

t=1. If the firm has continued and its investment is successful, the firm produces output $y = Ak_0$; otherwise it produces nothing. Either way, its assets k_1 stay in place. Their price p_s depends on the random state of the world s.

[30] I assume that the firm consumes whatever it borrows at $t = 0$; it does not use it to invest further, which would in turn affect the payoff y, and hence the equilibrium repayments. This is just for simplicity. It does not substantially affect the results. Indeed, I allow for this reinvestment in the analysis in Section 4; see, e.g., Lemma 10.

Next, the firm either repays $T(s)$ to its creditor or defaults. If it defaults, the creditor can seize its assets (in which case any output y is destroyed) or renegotiate the repayment.

The division of surplus in renegotiation is determined by generalized Nash bargaining, where the creditor has bargaining power ρ and disagreement payoff $p_s k_1$, reflecting his outside option of seizing the defaulting firm's capital.[31]

I assume that the firm maximizes its expected payoff, including its private benefits. The creditor maximizes its expected payoff, and discounts the future using the SDF M (this matters only for the computation of the debt capacity at the end of this analysis; it does not affect the equilibrium behavior).

Assumptions. I make three assumptions on technologies and preferences that streamline the analysis. First, I assume the investment payoff given success is large:

$$Ak_0 \geq \frac{B + (1-\delta)p_s k_0}{(1-\rho)} \tag{130}$$

for all s. Second, I assume that the private benefits are large too,

$$B > p_s k_0 \tag{131}$$

for all s. Third, I assume that the success of the investment is independent of the aggregate state, i.e. y is independent of s.

Solution. I find the subgame perfect equilibrium of this extensive-form game. In the steps below, I derive the equilibrium by backward induction.

1. **Creditor's choice of seizure or renegotiation at t=1.**

[31] Renegotiation is a first-order friction for real-world firms (Roberts and Sufi (2009)) and liquidation values are a first-order determinant of its outcome (Benmelech and Bergman (2008)). Although we will see below that the same repayments can be implemented with or without renegotiation on the equilibrium path (cf. equation 139), the threat of renegotiation is what determines the repayments in either case.

- If the creditor seizes the assets, it gets $p_s k$.
- If it chooses to renegotiate, it divides the surplus with the firm according to the Nash bargaining protocol, where the disagreement point is to seize the assets. Hence, we can write the renegotiation payoffs at $t = 1$ as follows:

$$\text{firm's payoff } = (1-\rho)y, \tag{132}$$

$$\text{creditor's payoff } = \rho y + p_s k_1. \tag{133}$$

(Note that for simplicity I have written the firm's payoff net of the private benefit B, which it gets at $t = 1/2$ if it abandons the investment.)

- The creditor's payoff from renegotiation is always higher than its payoff from seizing assets, so there is always renegotiation if the firm defaults.

2. **Firm's choice to default or repay at t=1.**

- If the firm repays, it gets $y + p_s k_1 - T(s)$; the creditor gets the repayment $T(s)$.
- If it defaults, we know from above that the creditor renegotiates. Hence, the firm gets $(1-\rho)y$; the creditor gets $\rho y + p_s k_1$ (from equations (132) and (132)).
- Hence, the firm defaults if $(1-\rho)y > y + p_s k_1 - T(s)$. This can be rewritten as $T(s) < \rho y + p_s k_1$, so we can write the payoffs (net of the private benefits) as

$$\text{firm's payoff } = \max\{\, y + p_s k_1 - T(s)\,,\, (1-\rho)y \,\}, \tag{134}$$

$$\text{creditor's payoff } = \min\{\, T(s)\,,\, \rho y + p_s k_1 \,\}. \tag{135}$$

3. **Firm's choice to continue or abandon.**

- **Success.** If the firm learns its project is going to succeed, it knows its output is

$y = Ak_0$ if it continues. Hence, its payoff is given by

$$\text{firm's payoff} \;=\; \begin{cases} \mathbb{E}\left[\max\left\{\, Ak_0 + p_s k_0 - T(s)\,,\,(1-\rho)Ak_0\,\right\}\right] & \text{if continue,} \\ B + \mathbb{E}\left[\max\left\{\, p_s(1-\delta)k_0 - T(s)\,,\,0\,\right\}\right] & \text{if abandon.} \end{cases} \tag{136}$$

(Note that these are the payoffs from the point of view of $t = 1/2$; hence they are gross of the private benefits B.) Given $(1-\rho)Ak_0 > B + p_s(1-\delta)k_0$ by assumption (equation (130)), the continuation payoff is always greater than the abandonment payoff (no matter T).

Hence, the firm always continues if it learns it will succeed.

- **Failure.** If the firm learns its project is failing, it knows its output is $y = 0$. Hence, its payoff is given by

$$\text{firm's payoff} \;=\; \begin{cases} \mathbb{E}\left[\max\left\{\, p_s k_0 - T(s)\,,\,0\,\right\}\right] & \text{if continue,} \\ B + \mathbb{E}\left[\max\left\{\, p_s(1-\delta)k_0 - T(s)\,,\,0\,\right\}\right] & \text{if abandon.} \end{cases} \tag{137}$$

Given $B > p_s k_0$ by assumption (equation (131)), the abandonment payoff is always greater than the continuation payoff (no matter T).

Hence, the firm always abandons if it learns it is failing.

- Observe that the repayment T does not affect the firm's choice to continue/abandon at $t = 1/2$ (given the assumptions in equations (130) and (131)).

4. **Equilibrium repayments.** Since T does not affect the firm's choice to continue its investment, the repayment to the creditor is given by the creditor's payoff in equation (135), with $y = Ak_0$ and $k_1 = k_0$ in the event of success and $y = 0$ and $k_1 = (1-\delta)k_0$ in the event of failure:

$$\text{creditor's payoff} \;=\; \begin{cases} \min\left\{\, T(s)\,,\,\rho Ak_0 + p_s k_0\,\right\} & \text{if success,} \\ \min\left\{\, T(s)\,,\,(1-\delta)p_s k_0\,\right\} & \text{if failure.} \end{cases} \tag{138}$$

5. **Maximum repayments.** Observe that for each state s, the equilibrium repayment in equation (139) is increasing in the promised repayment $T(s)$. (This is because increasing $T(s)$ does not affect the firm's choice at $t = 1/2$, as established above, and because default does not induce a deadweight cost at $t = 1$.) Thus, the maximum possible transfer in state s, which I denote by $T^{\max}(s)$, is given by the expression in equation (139) with $T(s) \equiv \infty$:

$$T^{\max}(s) = \begin{cases} \rho A k_0 + p_s k_0 & \text{if success,} \\ (1-\delta) p_s k_0 & \text{if failure.} \end{cases} \tag{139}$$

For $\rho = 0$, this expression coincides with that in the text (equation (5)). Now, by analogy with the analysis there (equation (7)), we can rewrite it as

$$T^{\max}(s) = \mathbb{1}_{\text{succ}}\big(\rho A k_0 + p_s k_0\big) + \big(1 - \mathbb{1}_{\text{succ}}\big)(1-\delta) p_s k_0 \tag{140}$$

$$= (1-\delta) p_s k_0 + \frac{\delta p_s y}{A} + \rho y. \tag{141}$$

6. **Debt capacity.** We can take the present value of $T^{\max}(s)$ to get the debt capacity:

$$\text{DC} = \alpha_0 + \alpha_1 \mathbb{E}\left[Mpy\right] + \rho \mathbb{E}\left[My\right], \tag{142}$$

where α_0 and α_1 are as in Lemma 3.

Summary. The baseline result in Lemma 3 is the special case of this set-up in which the creditor has no bargaining power, $\rho = 0$. For $\rho > 0$ the debt capacity formula is unchanged except for an extra additive term proportional to ρ (compare equation (142) with the expression in Lemma 3). The fact that the first terms are unchanged implies that procyclical promises can matter even when creditors have a lot of bargaining power (although $\rho \to 1$ is ruled out by the assumption in equation (130)). However, this extra term depends on M, which drives the discount rate term, but not on p, which drives the procyclical promises term. This suggests that the discount rate term becomes more important relative to the procyclical promises term as ρ increases. To the extent the that the ρ measures enforceability—e.g., because it reflects creditor rights—this is in line with the analysis in the text suggesting procyclical promises are more important when enforceability is limited; see, e.g., the comparative statics on δ in Subsection 4.9.

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