Exercises

Single Index Model

Reading and Program Downloads

- Please read the course material on compass.
- For R exercises, the R code (SImodel) in Compass will be helpful. Make sure you understand the output rather than just following the hint.

R Exercises

The following questions require R. On Compass is the R script files SImodel.R. The file contains hints for completing the R exercises. Copy and paste all statistical results and graphs into a MS Word document (or your favorite word processor) while you work, add any comments and answer all questions in this document. Start MS Word and open a blank document. You will save all of your work in this document. PLEASE UNDERSTAND THE OUTPUT.

Using the monthly return data on the four Northwest stocks (Boeing, Microsoft, Nordstrom and Starbucks) as well as the S&P 500 index (as the market return) over the period November 1998 – October 2003, you will estimate the single index model.

1. For the four Northwest stocks (Boeing, Microsoft, Nordstrom and Starbucks) estimate the single index model

$$\begin{aligned} R_{it} &= \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}, \\ \varepsilon_{it} &\sim iid\ N(0,\sigma_{\varepsilon i}^2) \end{aligned}$$

(a) For each stock, create a scatterplot of the returns against the market returns, and show the estimated regression line.

(b) What does the estimated value of β_i say about the risk of each asset in terms of portfolio theory?

(c) What proportion of the risk of each asset is due to the market and what portion is firm specific?

(d) What is the estimated value of for $\sigma^2_{\varepsilon i}$ each stock?

(e) Compute the 95% confidence intervals for β_i. Is β_i estimated very precisely?

(f) For each stock, test the hypotheses $H_0 : \beta_i = 1$ using a 5% significance level. What do you conclude?

1. For an equally weighted portfolio of the stocks, estimate the single index model.

   (a) Verify that the estimated β_p for the portfolio is equal to an equally weighted average of the individual stock β_i values.

   (b) Comment on the 95% confidence intervals for the portfolio β_p. Is the portfolio β_p estimated more precisely than the individual stock β_i values?

   (c) Is the R-square value for the portfolio larger than the R-square values for each stock? Would you expect it to be?

2. Compute the sample covariance matrix and sample correlation matrix of the asset returns. Next, using the estimates of β_i, $\sigma^2_{\varepsilon i}$ and σ^2_M compute the covariance matrix and correlation matrix implied by the single index model. That is, compute

$$\begin{aligned}\sigma_i^2 &= \beta_i^2\sigma_M^2 + \sigma^2_{\varepsilon i},\ \sigma_{ij} = \beta_i\beta_j\sigma^2_M, \\ \rho_{ij} &= \frac{\sigma_{ij}}{\sigma_i\sigma_j}.\end{aligned}$$

   Compare the sample covariance to the single index covariance. Are they similar?

Review Questions

1. **(Single Index Model)** SI model for asset returns has the form

$$\begin{aligned} R_{it} &= \alpha_i + \beta_i R_{Mt} + \varepsilon_{it},\ \varepsilon_{it} \sim iid\ N\left(0, \sigma^2_{\varepsilon i}\right) \\ i &= 1, ..., N \text{ assets}, \\ t &= 1, ..., T \text{ time periods}, \end{aligned}$$

   where R_{it} denotes the cc return on asset i at time t, and R_{Mt} denotes the return on a market index portfolio at time t.

1.1. In the SI model, what are the interpretations of β_i and $\sigma_{\varepsilon i}$?

1.2. SI model with large portfolios can be constructed as

$$\sum_{i=1}^{N} \frac{1}{N} R_{it} = R_{p,t} = \bar{\alpha} + \bar{\beta} R_{Mt} + \bar{\varepsilon}_t,$$

   where

$$\bar{\alpha} = \frac{1}{N}\sum_{i=1}^{N} \alpha_i,\ \bar{\beta} = \frac{1}{N}\sum_{i=1}^{N} \beta_i \text{ and } \bar{\varepsilon}_t = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_{it}.$$

   In a large "well diversified" portfolio, we could diversify away all non-market variance, hence $R_{p,t} \approx \bar{\alpha} + \bar{\beta} R_{Mt}$. Explain how this is possible.

From the earlier discussion, we may use the S&P 500 index as the "market return" for our SI model. The following represents R linear regression output from estimating the SI model for the four Northwest stocks (Boeing, Microsoft, Nordstrom and Starbucks) using monthly continuously compounded return data over the the period November 1998 – October 2003.

> summary(boeing.fit)							
Coefficients:							
	Estimate	Std. Error	t value	Pr(>|t|)			
(Intercept)	0.00216	0.01366	0.16	0.875			
sp500	0.63862	0.27354	2.33	0.023	*		
Residual standard error: 0.106 on 58 degrees of freedom							
Multiple R-squared: 0.0859, Adjusted R-squared: 0.0701							
F-statistic: 5.45 on 1 and 58 DF, p-value: 0.023							
> summary(nord.fit)							
Coefficients:							
	Estimate	Std. Error	t value	Pr(>|t|)			
(Intercept)	0.00432	0.01414	0.31	0.76			
sp500	1.50799	0.28312	5.33	1.70E-06	***		
Residual standard error: 0.11 on 58 degrees of freedom							
Multiple R-squared: 0.328, Adjusted R-squared: 0.317							
F-statistic: 28.4 on 1 and 58 DF, p-value: 1.7e-06							

> summary(msft.fit)					
Coefficients:					
	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.0012	0.014	0.09	0.93	
sp500	1.6971	0.2808	6.04	1.20E-07	***
Residual standard error: 0.109 on 58 degrees of freedom					
Multiple R-squared: 0.386, Adjusted R-squared: 0.376					
F-statistic: 36.5 on 1 and 58 DF, p-value: 1.16e-07					
> summary(sbux.fit)					
Coefficients:					
	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.0183	0.0171	1.07	0.288	
sp500	0.6666	0.3418	1.95	0.056	.
Residual standard error: 0.132 on 58 degrees of freedom					
Multiple R-squared: 0.0615, Adjusted R-squared: 0.0454					
F-statistic: 3.8 on 1 and 58 DF, p-value: 0.056					
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1					

1.3. Make a table showing the estimated values of β_i, its estimated standard error, the estimate of $\sigma^2_{\varepsilon i}$, and the R_i^2 values from the four regression equations.

Asset	$\hat{\beta}_i$	$\mathrm{SE}\left(\hat{\beta}_i\right)$	$\hat{\sigma}^2_{\varepsilon i}$	R_i^2

(1)

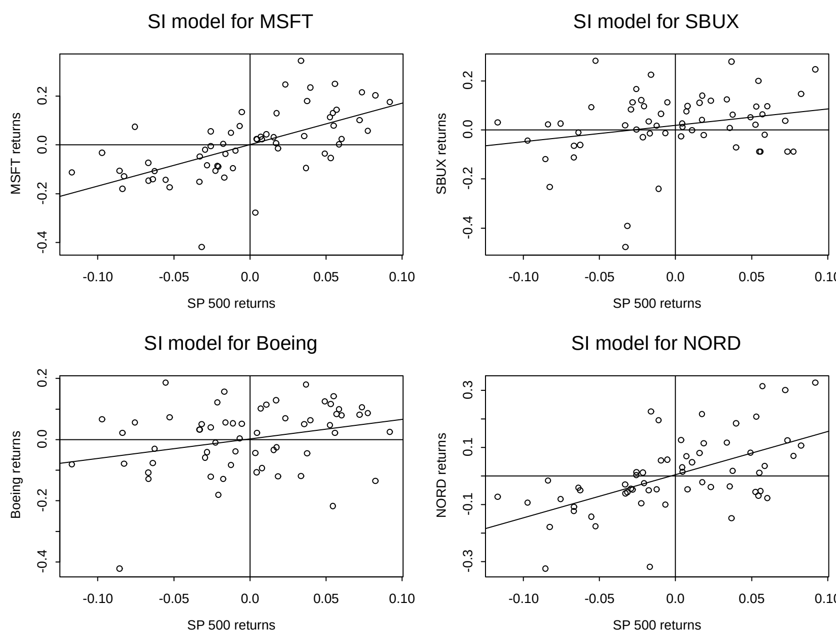
1.4. Characterize the relation among $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$, $R_i^2 = \left(\frac{\sigma_{iM}}{\sigma_i \sigma_M}\right)^2$ and the correlation coefficient ρ_{iM}. Can $|\beta_i| > 1$ or $|R_i| > 1$?

$$\beta_i = (\quad) \times (R_i) \quad .$$
$$\rho_{iM} = (\quad) \times (R_i) \quad .$$

$\lvert\beta_i\rvert > 1$?	possible	( )
	not possible	( )
$\lvert R_i\rvert > 1$?	possible	( )
	not possible	( )

1.5. From (1), which asset appears to be most correlated to "Market"? What does it mean in terms of risk diversification?

1.6. The followings are estimated regression lines for each SI model.



For MSFT stock, write down the test statistics for the hypotheses $H_0 : \beta_i = 1$ vs. $H_1 : \beta_i \neq 1$. Which asset has a (statistically) non-zero intercept (with 5% significance level)?