$fractional$ **How does the height of water level affect the flow rate of water out of a container?**

Introduction

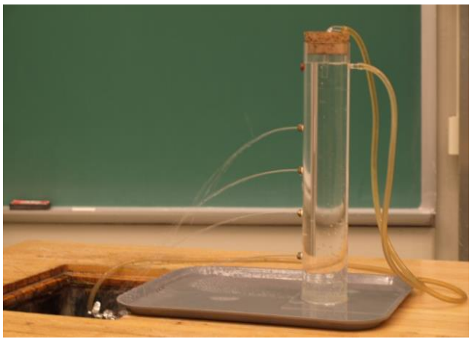

Figure 1 - Torricelli's Law (physics demo manual)

One of the highlights of my IGCSE physics course was a lesson where my teacher showcased a water column with holes of varying heights. What followed after the plugs were removed was a series of water jets shooting through the openings and I learned that the lower the depth of the water, the further the jet reaches due to pressure from the liquid above. Physics is only as fun when you learn a certain idea empirically which is why that was one of the more memorable lessons for me. However, that experiment left me with more questions than what was answered, *what if I raise the temperature of the water? How about changing the shape of the orifice?* I thought of all the possibilities as I quietly observed the simple yet fascinating phenomenon.

I decided to revisit this concept in greater depth, since fluid dynamics is a part of the HL optional syllabus, I had to conduct independent research and discovered the link between Torricelli's law and Bernoulli's principle. As I understood more about the relationship between changing height and the exit velocity of a liquid through the orifice, I began to wonder how height plays a factor in how quickly water is displaced as shown from the experiment.

Purpose

My exploration focus is established as ***how does the height of water level affect the flow rate of water out of a container?*** The aim of the investigation is to explore whether the experiment backs up the theory and establish a clear relationship between the two variables.

Background Information

As mentioned, there exists a link between Bernoulli's principle and Torricelli's law which provides a relationship between the velocity of liquid exiting an orifice at the respective water level. Bernoulli's principle can be considered to be a statement of the conservation of energy appropriate for flowing fluids (Connor). Torricelli's formula can be derived by beginning with Bernoulli's equation as following (Boundless):

$where:$
$P_x = pressure\ at\ a\ certain\ level$
$\rho = density\ of\ the\ liquid$
$h_x = water\ level$
$g = acceleration\ due\ to\ gravity$
$v_x = velocity\ at\ the\ water\ level$

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho\, g\, h_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho\, g\, h_2$$

**note: Bernoulli's equation also applies to liquid flowing in the horizontal direction.*

Due to conservation of energy, the pressure added with total energy at h_1 — gravitational and kinetic — is equal to the total energy and pressure at h_2. We can assume that the value for h_2 is 0 as it corresponds to the level of the orifice and h_1 can be any point the opening where there is liquid.

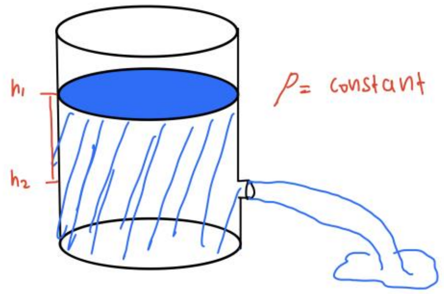

Figure 2 - Visual diagram of an open water tank

Starting from the left-hand side, v_1 should be 0 as only gravitational potential energy is present at that level.

$$P_1 + \rho\, g\, h_1 = P_2 + \frac{1}{2}\rho\, {v_2}^2 + \rho\, g\, h_2$$

Meanwhile, at the right-hand side, as the height is 0, there is only kinetic energy which leaves us with:

$$P_1 + \rho\, g\, h_1 = P_2 + \frac{1}{2}\rho\, {v_2}^2$$

At h_2, the water is exposed to the atmosphere through the opening which means that the pressure at h_1 — assuming it is an open tank — should be the same. As the density of the liquid should also be the same, the equation is the following:

$$g\, h_1 = \frac{1}{2}\, {v_2}^2$$

After some rearrangement, we are left with the equation that is represented by Torricelli's law.

$$v_2 = \sqrt{2\, g\, h_1}$$

The formula displays a relationship between the exit velocity of liquid at point h_2 and the current level of water which is represented by $v_2 \propto \sqrt{h_1}$.

$$V = A\,.\,h$$

The equation of the flow rate is the total volume displaced divided by the time taken which should give the loss in volume of water every second (Libretexts). We can substitute the equation for volume into the following flow rate equation:

$$Q = \frac{V}{t} = \frac{A\,.\,h}{t}$$

As distance in h is divided by the time, we get the velocity of water flowing out.

$$Q = A\,.\,v$$

This makes sense as Q measures the volume of water discharged from the bottle every second and it is indicated with a negative sign as the liquid leaves the container. The volume discharged should be the cross-sectional area of the orifice multiplied by the velocity at which the liquid exits through it.

Substituting the Torricelli's equation, we get the following relationship:

$$\boldsymbol{Q} = \boldsymbol{A} . \sqrt{\boldsymbol{2\, g\, h}}$$

<u>Equation to predict the time taken to reach each interval</u>

The data can also be compared to the theoretical time taken to reach each water level. It is known that the volume of a particular cylinder can be found by multiplying the cross-sectional area by the height. Due to the water flowing out of the container, the only variable that is changing is the height itself. Thus, taking the derivative of the volume with respect to time — also known the rate of water flowing out of the container— should give the following (McEvoy):

$$\frac{dV}{dt} = A . \frac{dh}{dt} = -Q$$

The relationship between the change in water in the container with the volume discharged per second through the opening is:

$$A . \frac{dh}{dt} = -a . v$$

Factoring in the Torricelli's equation:

$$A . \frac{dh(t)}{dt} = -a . \sqrt{2\, g\, h(t)}$$

$$\frac{dh(t)}{dt} = \frac{-a}{A} . \sqrt{2\, g} . \sqrt{h(t)}$$

To simplify the equation, a variable k can be used to represent the constant values.

$$\frac{dh(t)}{dt} = -k . \sqrt{h(t)}$$

The function of height at a given time can be found by integrating both sides of the equation.

$$\int_{h(0)}^{h(t)} \frac{1}{\sqrt{h(t)}} dh(t) = -k \int_{0}^{t} dt$$

We now have 2 multiplied by the change in height where h_0 represents the initial water level within the container. The following equation is useful for finding the theoretical data for comparison.

$$2\,(\sqrt{h} - \sqrt{h_0}) = -k\, t$$

$$\boldsymbol{t} = -\frac{\boldsymbol{2\,(\sqrt{h} - \sqrt{h_0})}}{\boldsymbol{k}}$$

where
k = constant
a = area of the orifice
A = area of the bottle
d = diameter of the orifice
D = diameter of the bottle
g = acceleration due to gravity

<u>Simplification of the k constant</u>

$$k = \frac{a}{A}\sqrt{2\, g} = \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} D^2} \sqrt{2\, g} = \frac{\boldsymbol{d^2}}{\boldsymbol{D^2}} \sqrt{\boldsymbol{2\, g}}$$

Hypothesis

As the water level decreases, the time taken to displace a certain volume of water increases. This is shown by Torricelli's law which states that the velocity of water exiting an orifice is proportional to the square root of the height. As the height decreases, the velocity will also decrease in the manner of the relationship described. Similarly, the flow rate over a certain period of time is also proportional to the square root of h. This gives us the relationship as shown:

$$Q \propto \sqrt{h}$$

$$Q = k \,.\, \sqrt{h}$$

Methodology

Independent Variable

For this experiment, I will be recording the time taken to reach a certain water level from the previous level. The length of the bottle from the top to the orifice is 25 cm and 10 intervals are needed which means the time will be recorded at 22.5, 20, 17.5, 15, 12.5, 10, 7.5, 5, and 2.5 cm. However, the data will be displayed as the midpoints between the height intervals as flow rate is representative of the flow of water throughout the time period, rather than instantaneous.

Dependent Variable

Torricelli's law states that the efflux velocity of water out of the orifice is affected by the height of the water level, the dependent variable would be the time taken to reach each of the pre-determine water levels. The experiment would be recorded using a stopwatch, with an uncertainty of 0.001 seconds by factoring in reaction time, and repeated 5 times to reduce random errors.

Control Variable

There are numerous variables to keep constant to prevent false or inaccurate results.

1) Temperature of liquid
   a) Justification: After some research, the temperature of liquid affects its viscosity — the time taken for liquid to flow through a viscometer tube. This indicates that as liquid gain more temperature, the particles within the container gains more kinetic energy and it gains the ability to overcome the weak forces binding the molecules together (Gillespie)
   b) Method: Conduct the experiment in the same environment and water source to prevent any significant influence on the flow rate by temperature.
2) Bottle/container
   a) Justification: Different container could have different cross-sectional area despite having the same volume. For example, the material of a bottle could affect the pressure exerted within a bottle.
3) Diameter:
   a) Justification: Diameter of the bottle must be kept the same as various diameters affects the total volume at a point which influences the speed at which volume of water is ejected.
4) Height intervals
   a) Justification: Use the same intervals (2.5 cm) between all the heights to ensure the consistency of data.
5) Use the same type of liquid

a) Justification: different types of liquids have different viscosity which could influence the flow rate and the efflux velocity through the orifice.

Apparatus

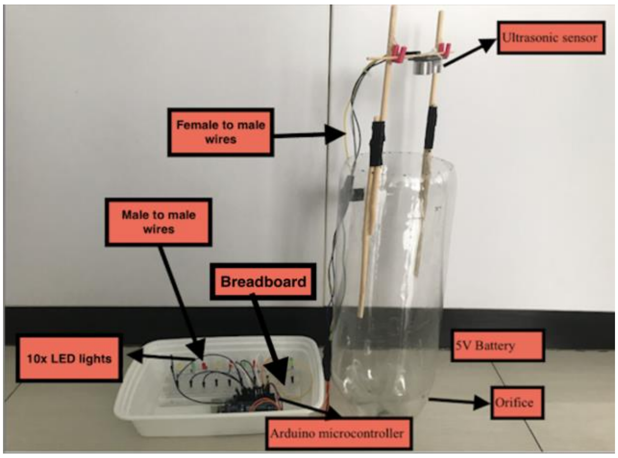

Figure 3 - General setup and apparatus

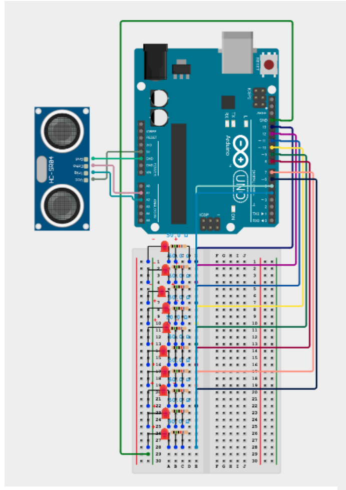

Figure 4 – Arduino Setup

Methodology

<u>Setup Explanation</u>

The breadboard includes 10 LED lights which are programmed to light up whenever the water level reaches the respective level (see appendix 1 for the code). The ultrasonic sensor emits a high frequency sound and it records the time taken to receive the reflected sound wave when an object or surface is detected.

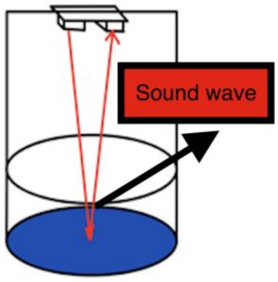

Figure 5 – Sound Waves

$$Distance = \ speed \times time$$

Since the speed of sound is 343 m s^{-1} (0.0343 cm s^{-1}), the distance between the sensor and the surface level can be found by:

$$Distance = \ 0.343 \times \frac{duration\ (time)}{2}$$

In order to minimize the interference of anomalies to the validity of the data, the reading is outputted in the form of the average of the last 10 readings. The distance between the sensor and the top of the bottle surface was also considered by subtracting 8.8 cm $\pm\ 0.1\ \mathrm{cm}$ from the calculated distance value.

<u>Experimental Procedures</u>

1. The apparatus is setup as shown in *figure 3*, the water is filled to the top while the orifice is held closed. (orifice diameter is 0.75 cm)
2. When the orifice is released, the timer is started and the time is recorded whenever an LED lights up.
3. The time is stopped when the water level reaches the top of the orifice
4. Steps 2 and 3 are repeated 5 times

Data Collection and Processing

Height (± 0.0005 m)	Time Taken to Reach the Level (± 0.01 s)					Average Time (± 0.0005 m)	Absolute Uncertainty (± s)	Flow rate ($m^3 s^{-1}$)	Absolute Uncertainty (± $m^3 s^{-1}$)
0.2375	3.47	3.51	3.57	3.58	3.79	3.6	0.2	0.000078	0.000005
0.2125	3.82	3.70	3.90	3.75	3.79	3.8	0.1	0.000074	0.000003
0.1875	4.01	4.22	3.81	3.98	3.90	4.0	0.2	0.000070	0.000004
0.1625	4.37	4.36	4.42	4.25	4.32	4.3	0.1	0.000065	0.000002
0.1375	4.64	4.59	4.87	4.75	4.27	4.6	0.3	0.000061	0.000005
0.1125	4.94	5.16	5.05	5.25	5.67	5.2	0.4	0.000054	0.000005
0.0875	6.15	5.64	5.68	5.69	6.03	5.8	0.3	0.000048	0.000003
0.0625	6.43	7.26	7.46	6.57	6.57	6.9	0.5	0.000041	0.000003
0.0375	9.81	9.61	10.2	10.2	9.76	9.9	0.3	0.000028	0.000001
0.0125	14.7	15.2	14.8	14.6	15.2	14.9	0.3	0.000012	0.000001

Table 1 – Table showing height midpoints (m) in relation to time (s) and flow rate ($m^3 s^{-1}$)

Flow Rate Calculation

$$Measured\ circumference = 37.50\ cm \pm 0.05\ cm$$

$$Radius = \frac{Circumference}{2\pi} = 5.968cm \pm 0.008\ cm$$

$$Area\ of\ bottle\ cross-section = \pi r^2 = \pi\, 5.968^2 = 111.9\ cm^2 \pm 0.9\ cm^2$$

$$Volume\ lost\ at\ every\ interval = area\ of\ bottle \times height$$

$$= 111.9 \times 2.50 = 280\ cm^3 \pm 3\ cm^3$$

$$\boldsymbol{Flow\ rate} = \frac{volume\ lost}{time\ taken} = \frac{280}{3.6} = 77.78\ cm^3\ s^{-1} = 0.000078\ \mathrm{m}^3\ s^{-1} \pm 0.000005\ \mathrm{m}^3\ s^{-1}$$

Absolute Uncertainty for Time

Data for 0.238 height interval

3.47	3.51	3.57	3.58	3.79

$$\text{Uncertainty} = \frac{\max - min}{2} = \frac{3.79-3.47}{2}$$

$$= \pm 0.160\ seconds$$

$$\approx \pm 0.2\ seconds\ (1\ s.f)$$

Measurement Uncertainties

$$Radius\ uncertainty = \frac{0.05}{2\pi} = \pm 0.008\ cm$$

$$Fractional\ \ Unc.(Bottle\ area) = \frac{0.008}{5.968} + \frac{0.008}{5.968}$$

$$= 0.00268$$

$$Absolute\ Unc. = 0.00268 \times 111.9 \times \pi$$

$$\approx \pm 0.9\ cm^2\ \ (1\ s.f)$$

<table>
<tr><td></td><td>$$Fractional\ \ Unc.(volume\ lost) = \frac{0.9}{111.9} + \frac{0.05}{2.50}$$ $$= \ 0.03$$ $$Absolute\ Unc.\ \ = \ 0.03\ \times\ 111.9 \approx \pm\, 3\ cm^3\ \ (1\ s.f)$$</td></tr>
<tr><td colspan="2">Absolute Uncertainty for Flow Rate
$$Fractional\ uncertainty\ for\ flow\ rate\ \ = \frac{3}{280} + \frac{0.2}{3.6}\ = \ 0.0662$$ $$Absolute\ uncertainty\ \ = \ 0.0662\ \times\ 77.78\ = \pm\, 5.14\ cm^3$$ $$\approx \pm\, 5\ cm^3\ (1\ s.f)$$ $$\approx \pm\ 0.000005\ \ \mathrm{m}^3\ s^{-1}$$</td></tr>
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Data Analysis and Conclusion

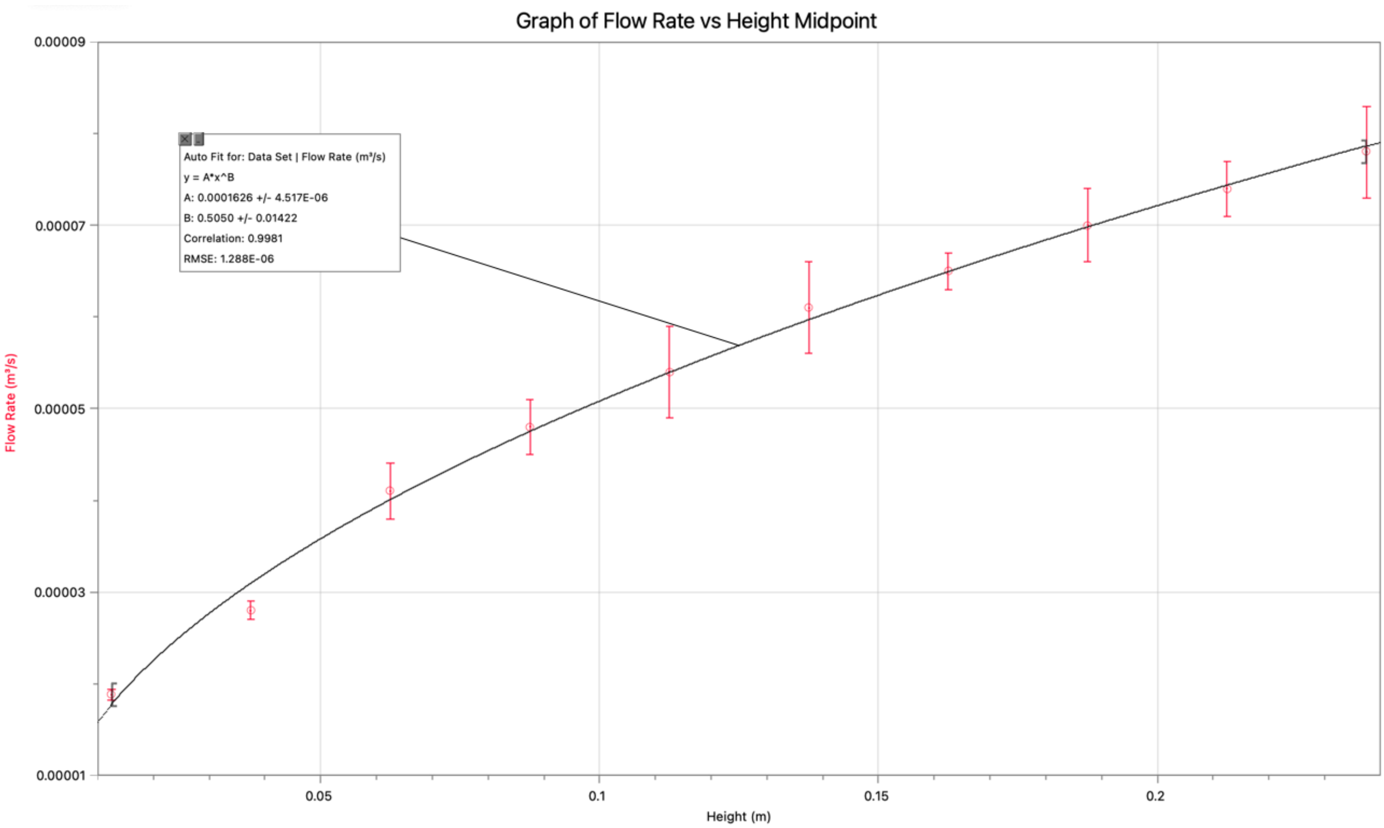

Figure 6 - Graph of Flow Rate ($m^3\ s^{-1}$) against height (m), best fit line generated using logger pro

<u>Graph Analysis</u>

The data was fitted against the power model which resulted in a correlation of **0.9981** indicating a great fit for the data points. One thing that can be observed is that the equation $\mathbf{0.0001622x^{0.5034}}$ is very similar to the relationship established within the hypothesis which was $\boldsymbol{Q = k\sqrt{h}}$**.** This suggests that the hypothesis where **Q**, the flow rate, is proportional to the $\sqrt{\boldsymbol{h}}$ was correct. An observation of graph shows that the gap of the data in the y-axis decreases as the height approaches to the top of the bottle, thus, establishing a graph that has a low rate of change when the container first begins to displace water and high rates of change as the water reaches the orifice level, this is due to a lack of pressure exerted by the water above the orifice level, as shown from the initial research.

Linearized Graph

The linearized graph further backs the relationship that $\boldsymbol{Q} \propto \sqrt{\boldsymbol{h}}$ as it produces a straight-line graph with a correlation value of **0.997**, suggesting a strong fit for the linear data. The line from *figure 7* shows that the slope is **0.00016 ± 0.00001**—a percentage uncertainty of **6.25%**. Meanwhile the y-intercept is **0.00000014 ± 0.00000002**—a percentage uncertainty of **14.3%**. Resulting in a straight-line graph of equation $(\mathbf{0.00016} \pm \mathbf{0.00001})\sqrt{\boldsymbol{h}} + (\mathbf{0.00000013} \pm \mathbf{0.00000002})$. The value indicates that as the linearized height, $\sqrt{\boldsymbol{h}}$**,** move towards the bottle opening by 1 m, the flow rate increases by **0.00016 ± 0.00001**, confirming the relationship that as height decreases, the flow rate decreases. An anomaly—which was the flow rate for midpoint **0.0375 m**— was removed as it did not fit the trend of best fit line. The percentage error suggest that are errors within the experimental data, but they are relatively small and could possibly be the result of random errors. The existence of a y-intercept is the result of the use of height midpoints, which means the flow rate at height 0 m is not considered. Another possible reason that water continues to flow at the top of the orifice point.

Flow rate ($m^3\ s^{-1}$)	Absolute Uncertainty (± $m^3\ s^{-1}$)	$\sqrt{Height}$	Absolute Uncertainty (±m)
0.000078	0.000005	0.487	0.001
0.000074	0.000003	0.461	0.001
0.000070	0.000004	0.433	0.001
0.000065	0.000002	0.403	0.002
0.000061	0.000005	0.371	0.002
0.000054	0.000005	0.335	0.002
0.000048	0.000003	0.300	0.003
0.000041	0.000003	0.250	0.004
0.000028	0.000001	0.194	0.007
0.000012	0.000001	0.11	0.02

Table 2 – Table showing height midpoints (m) in relation to time (s) and flow rate ($m^3\ s^{-1}$)

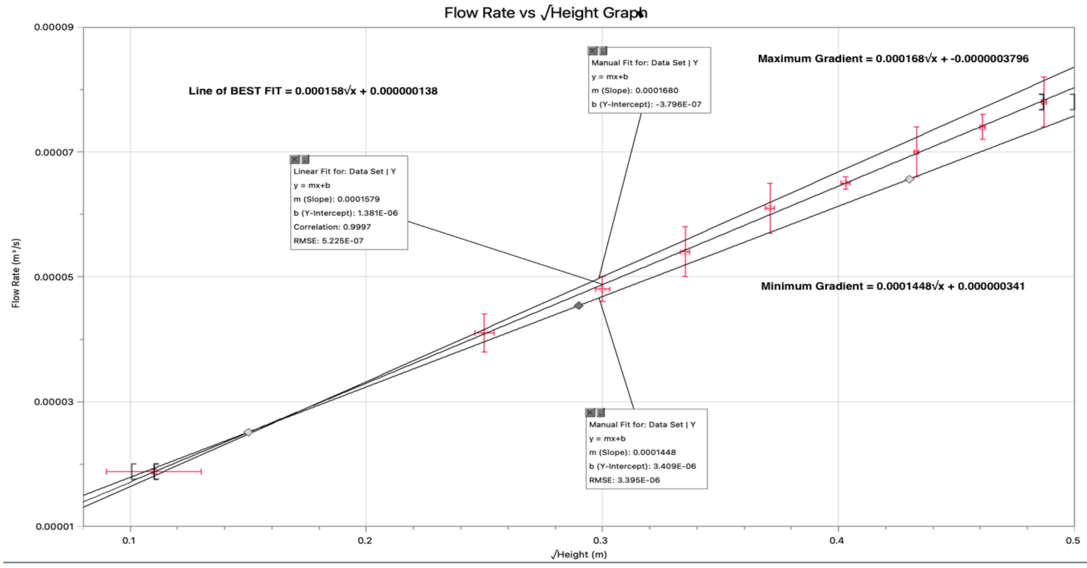

Figure 7 - Graph of flow rate (m^3 s^{-1}) against √height (±m)

Confirmation of Relationship

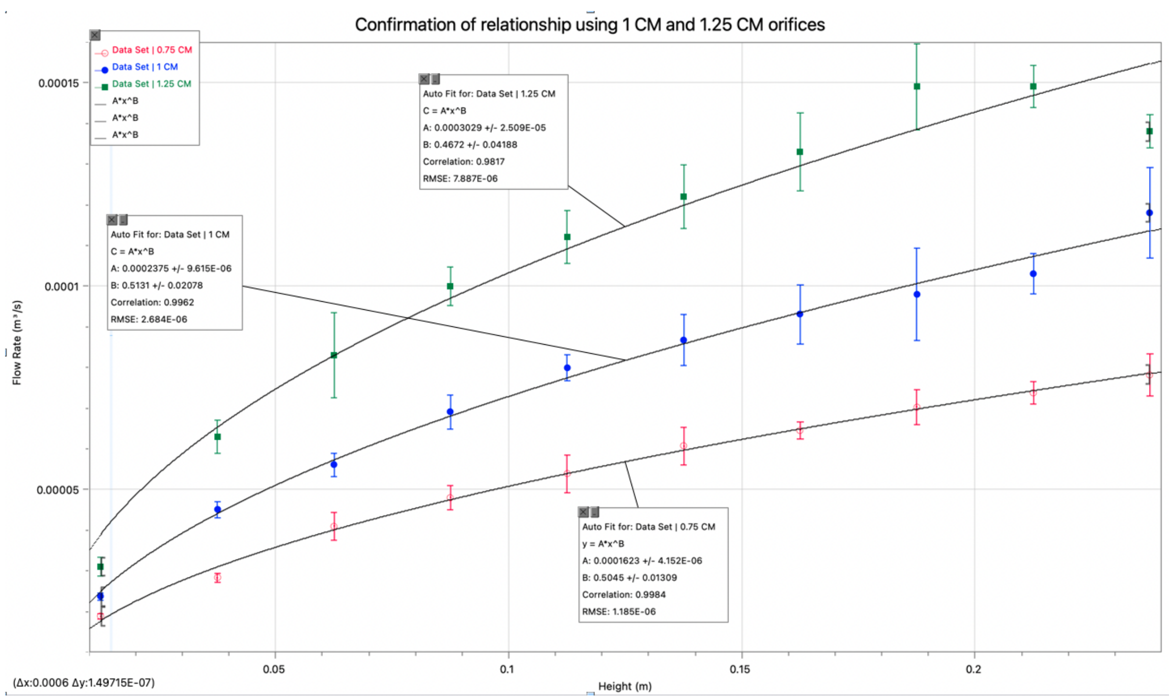

Figure 8 – Graph of Flow Rate (m^3 s^{-1}) against height (m), (different diameters)

In order the confirm the relationship, the experiment was also conducted on orifice diameters of 1 and 1.25 cm. As shown from the graph, the data from varying hole sizes support the current data with the major difference being that large orifice sizes have a much higher average flow rate at each interval as water flows out of the container much quicker. (see **appendix 3** and **4** for the table of results).

Theoretical vs experimental data points

Sample Calculation – first interval

Using the previously derived function of t, we can predict the values for time taken to reach each of the determined intervals, the sample calculation is shown as the following:

$$t = -\frac{2\left(\sqrt{h} - \sqrt{h_0}\right)}{k}$$

$$k\ (constant) = \frac{0.00075^2}{0.1194^2}\sqrt{2\,.\,9.81} = 0.0175$$

$$t = -\frac{2\left(\sqrt{22.5} - \sqrt{25}\right)}{0.175} = 2.94\ seconds$$

$$Flow\ rate = \frac{total\ volume}{seconds} = \frac{280}{2.94}$$

$$(convert\ to\ \mathrm{m}^3/\mathrm{s}) = \frac{95.3}{1000000} = 0.0000953\ \mathrm{m}^3\ s^{-1}$$

Height (± 0.0005 m)	Time taken to reach the water level (s)	Flow Rate (m^3/s)
0.2375	2.94	0.0000953
0.2125	3.10	0.0000902
0.1875	3.31	0.0000847
0.1625	3.55	0.0000788
0.1375	3.86	0.0000725
0.1125	4.27	0.0000655
0.0875	4.85	0.0000577
0.0625	5.75	0.0000487
0.0375	7.49	0.0000373
0.0125	18.1	0.0000155

Table 3 – Table of theoretical data

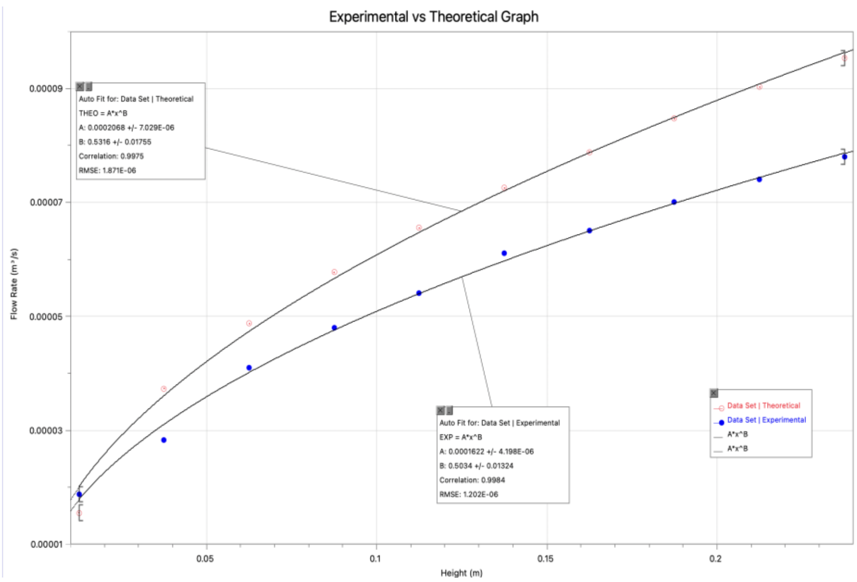

Figure 9 – Graph of theoretical against experimental flow rate (m^3 s^{-1})

Sample calculation of the mean percentage error between experiemntal and theoretical data

$$MAPE = \frac{1}{10}\sum_{i=1}^{10}\frac{|A_i - F_i|}{A_i}$$

$Where:$
$A_i = Experimental\ value$
$F_i = Theoretical\ value$

When i = 1

$$Percentage\ error = \frac{|0.000078 - 0.0000953|}{0.000078} = 0.219 \times 100 = 21.9\ \%$$

The process is repeated for all 10 variables and the average is found.

$$\boldsymbol{Mean\ Absolute\ Percentage\ Error\ =\ 21.9\%}$$

Both the experimental and theoretical data show a similar shape and trend in terms of the change in flow rate throughout the heights. However, the theoretical graph has a much higher flow rate on average which could suggest inaccuracies or other variables that presented problems during the experimentation. The result of **21.9%** seems like a big margin of error, however, as the data concerns small data values, the difference in flow rate doesn't indicate a major problem with the experimentation.

Conclusion

The aim of the investigation was to find out the relationship between the flow rate of water inside a container as a result of the change in height caused by water leaking out of the orifice; As well as, establishing a relationship between the two variables. Using the theoretical background, the hypothesis stated the relationship should be $\boldsymbol{Q} \propto \sqrt{\boldsymbol{h}}$ due to Torricelli's law being factored into the flow rate equation.

The relationship of $\boldsymbol{Q} = \boldsymbol{A} \,.\sqrt{\boldsymbol{2\ g\ h}}$ was confirmed through the data collected which fits the power model and produced an equation of $\boldsymbol{0.000162x^{0.505}}$. To answer the question of "how does the water level affect the flow rate of water out of a container?", firstly, the rate of change of flow rate increases as the water level decreases. Through qualitative observation, it can be seen from *figure* 6 that the flow rate changes drastically at the lower intervals of heights, suggesting that it takes much longer when there is less water above the orifice level and the change is much more significant. The straight-line graph produced positive and linear line by square rooting the height which shows that as the height increases the flow rate also increases and further confirms the existing relationship. This is due to more pressure acting on the water flowing out of the orifice at higher water levels, which suggests that as the height decreases, the flow rate decreases. Both models depict a correlation of more than 0.9 suggesting a strong fit for the data.

In order to confirm this phenomenon, the experimentation was tested with two other diameters which depicts the same trend, further confirming the validity of the results. The experimental results were also evaluated against the theoretical data sets using the mean average percentage error with a result of **21.9%**. Although the difference in values were small, the comparison suggests that other variables that exist within the real-world experimentation altered the results by a small margin. One **major limitation** of the experiment is the use of midpoints to represent the flow rate throughout an interval which means that data and graphs are only approximations and not instantaneous values.

Evaluation

Random Errors

Type	How it affects the experiment	Improvements
Reaction time	As the data was recorded by pressing the lap button in the timer app, the human reaction time could influence the precision of the results.	The methodology can be improved by recording the data and inputting them into a spreadsheet which would rely less on human reactions.
Water turbulence	It was observed that ultrasonic sensor is highly sensitive to changes in the environment. As the water flowing through the orifice experiences a strong kinetic energy, the water level can often be disturbed. The sensor is able to pick up on this and could lead to the wrong readings.	Inserting pins that are programmed to detect the presence and absence of water at various levels of the bottle—the surface of water has less effect on the results.

Systematic Errors

Type	How it affects the experiment	Improvements
Bottle material	The bottle was made from a flexible and thin plastic where the shape can be altered slightly by the change in condition within the container. This could lead to systematic shift in flow rate as was seen from the comparison between theoretical and experiment results.	Conduct the experiment using a stronger and rigid material where the shape can't be influenced by external forces or perform the experiment using different bottles of the same type to confirm the validity of the data. The anomaly at 0.0375 meters could potentially be fixed with a different bottle.
Measurement uncertainties	Several measurements could have altered the accuracy of the results: - Distance between the sensor and the top of the water level - Volume loss at every interval - Diameter of the orifice	The uncertainties can be reduced by using a more precise measuring instrument such as Vernier calliper. For example, the calculation of radius from the measured circumference for this experiment introduced a large uncertainty into the data and using a device that can directly measure the radius can reduce this.

Possible extension works:

- The experiment can be conducted using different liquid types with different viscosity and density levels to see how the relationship varies
- Using larger orifice sizes to see whether the flow rate follows the same pattern or whether friction losses and water turbulence leads to significant measurement errors

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Appendices

Appendix 1:

```
const int numReadings = 17;


float readings[numReadings];      // the readings from the analog input
```

```
int readIndex = 0;              // the index of the current reading
float total = 0;                  // the running total
float average = 0;

int trig = 3;
int echo = 2;

int LED1 = 13;
int LED2 = 12;
int LED3 = 11;
int LED4 = 10;
int LED5 = 9;
int LED6 = 8;
int LED7 = 7;
int LED8 = 6;
int LED9 = 5;
int LED10 = 4;

void setup() {
  // put your setup code here, to run once:
  Serial.begin(9600);
  pinMode(LED1,OUTPUT);
  pinMode(LED2,OUTPUT);
  pinMode(LED3,OUTPUT);
  pinMode(LED4,OUTPUT);
  pinMode(LED5,OUTPUT);
  pinMode(LED6,OUTPUT);
  pinMode(LED7,OUTPUT);
  pinMode(LED8,OUTPUT);
  pinMode(LED9,OUTPUT);
  pinMode(LED10,OUTPUT);
  turneverythingoff();

  for (int thisReading = 0; thisReading < numReadings; thisReading++) {
    readings[thisReading] = 0;
  }
```

```
}

void loop() {
  total = total - readings[readIndex];

  float duration, cm;
  pinMode(trig, OUTPUT);
  digitalWrite(trig, LOW);
  delayMicroseconds(2);
  digitalWrite(trig, HIGH);
  delayMicroseconds(10);
  digitalWrite(trig, LOW);
  pinMode(echo, INPUT);
  duration = pulseIn(echo, HIGH);
  readings[readIndex] = microsecondsToCentimeters(duration)-8.8;
   total = total + readings[readIndex];
   readIndex = readIndex + 1;

   if (readIndex >= numReadings) {
   // ...wrap around to the beginning:
   readIndex = 0;
  }
  average = total / numReadings;

Serial.println(average);

turneverythingoff();

 if(average>2.5&&average<5){
   digitalWrite(LED1,HIGH);
  }
  else if(average>5&&average<7.5){
   digitalWrite(LED2,HIGH);
  }
  else if(average>7.5&&average<10){
```

```
    digitalWrite(LED3,HIGH);
  }
  else if(average>10&&average<12.5){
    digitalWrite(LED4,HIGH);
  }
  else if(average>12.5&&average<15){
    digitalWrite(LED5,HIGH);
  }
  else if(average>15&&average<17.5){
    digitalWrite(LED6,HIGH);
  }
  else if(average>17.5&&average<20){
    digitalWrite(LED7,HIGH);
  }
  else if(average>20&&average<22.5){
    digitalWrite(LED8,HIGH);
  }
  else if(average>22.5&&average<25){
    digitalWrite(LED9,HIGH);
  }
  else if (average>25){
    digitalWrite(LED10,HIGH);
  }
  delay(100);
}
float microsecondsToCentimeters(float microseconds) {
   return (microseconds /2) *0.0343;
}
void turneverythingoff(){
  digitalWrite(LED1,LOW);
  digitalWrite(LED2,LOW);
  digitalWrite(LED3,LOW);
  digitalWrite(LED4,LOW);
  digitalWrite(LED5,LOW);
  digitalWrite(LED6,LOW);
  digitalWrite(LED7,LOW);
```

```
  digitalWrite(LED8,LOW);
  digitalWrite(LED9,LOW);
  digitalWrite(LED10,LOW);
}
```

Appendix 2 – Flow rate and height table for 0.75 cm:

Height (± 0.0005 m)	Time Taken to Reach the Level (s)					Average Time (s)	Max	Min	Absolute Uncertainty (± s)	Flow rate ($m^3\ s^{-1}$)	Absolute Uncertainty ($\pm m^3\ s^{-1}$)
0.2375	3.47	3.51	3.57	3.58	3.79	3.6	3.79	3.47	0.2	0.000078	0.000005
0.2125	3.82	3.70	3.90	3.75	3.79	3.8	3.90	3.70	0.1	0.000074	0.000003
0.1875	4.01	4.22	3.81	3.98	3.90	4.0	4.22	3.81	0.2	0.000070	0.000004
0.1625	4.37	4.36	4.42	4.25	4.32	4.3	4.42	4.25	0.1	0.000065	0.000002
0.1375	4.64	4.59	4.87	4.75	4.27	4.6	4.87	4.27	0.3	0.000061	0.000005
0.1125	4.94	5.16	5.05	5.25	5.67	5.2	5.67	4.94	0.4	0.000054	0.000005
0.0875	6.15	5.64	5.68	5.69	6.03	5.8	6.15	5.64	0.3	0.000048	0.000003
0.0625	6.43	7.26	7.46	6.57	6.57	6.9	7.46	6.43	0.5	0.000041	0.000003
0.0375	9.81	9.61	10.2	10.2	9.76	9.9	10.21	9.61	0.3	0.000028	0.000001
0.0125	14.7	15.2	14.8	14.6	15.2	14.9	15.20	14.60	0.3	0.000012	0.000001

Appendix 3 – Flow rate and height table for 1 cm:

Height (± 0.0005 m)	Time Taken to Reach the Level (s)					Average Time (s)	Max	Min	Absolute Uncertainty (± s)	Flow rate ($m^3\ s^{-1}$)	Absolute Uncertainty ($\pm m^3\ s^{-1}$)
0.2375	2.40	2.53	2.49	2.23	2.28	2.39	2.53	2.23	0.2	0.000118	0.000011
0.2125	2.80	2.75	2.57	2.76	2.76	2.73	2.80	2.57	0.1	0.000103	0.000005
0.1875	2.63	3.15	2.90	2.86	2.77	2.86	3.15	2.63	0.3	0.000098	0.000011
0.1625	3.08	2.79	3.05	3.09	3.01	3.00	3.09	2.79	0.2	0.000093	0.000007
0.1375	3.13	3.43	3.32	2.98	3.33	3.24	3.43	2.98	0.2	0.000087	0.000006
0.1125	3.44	3.47	3.50	3.52	3.54	3.49	3.54	3.44	0.1	0.000080	0.000003
0.0875	4.27	3.97	4.10	4.02	3.98	4.07	4.27	3.97	0.2	0.000069	0.000004
0.0625	4.93	4.98	4.73	5.17	5.01	4.96	5.17	4.73	0.2	0.000056	0.000003
0.0375	6.03	6.35	6.31	6.08	6.11	6.18	6.35	6.03	0.2	0.000045	0.000002
0.0125	12.04	11.96	11.86	11.60	11.52	11.80	12.04	11.52	0.3	0.000024	0.000001

Appendix 4 – Flow rate and Height table for 1.25 cm:

Height (± 0.0005 m)	Time Taken to Reach the Level (s)					Average Time (s)	Max	Min	Absolute Uncertainty (± s)	Flow rate ($m^3\ s^{-1}$)	Absolute Uncertainty ($\pm m^3\ s^{-1}$)
0.2375	1.97	1.98	2.08	2.09	2.05	2.03	2.09	1.97	0.1	0.000138	0.000008
0.2125	1.86	1.85	1.98	1.85	1.87	1.88	1.98	1.85	0.1	0.000149	0.000010
0.1875	1.86	1.9	1.75	1.91	2.02	1.89	2.02	1.75	0.1	0.000149	0.000009
0.1625	2.02	2.25	2.1	2.2	1.95	2.10	2.25	1.95	0.2	0.000133	0.000014
0.1375	2.29	2.22	2.33	2.2	2.5	2.31	2.5	2.2	0.2	0.000122	0.000012
0.1125	2.65	2.44	2.53	2.57	2.36	2.51	2.65	2.36	0.1	0.000112	0.000006
0.0875	2.76	2.8	2.7	2.97	2.84	2.81	2.97	2.7	0.1	0.000100	0.000005
0.0625	3.22	3.59	3.93	3.08	3.16	3.40	3.93	3.08	0.4	0.000083	0.000011
0.0375	4.37	4.31	4.36	4.49	4.9	4.49	4.9	4.31	0.3	0.000063	0.000005
0.0125	9.87	8.55	8.64	8.55	9.5	9.02	9.87	8.55	0.7	0.000031	0.000003