Formula Reference Library

1 Mathematics

1.1 Arithmetic & Number Theory

1.1.1 Sequences

- **math_arith_sequence_01** – Arithmetic Sequence (n-th Term):

$$a_n = a_1 + (n-1)d$$

  Here n is the term number, a_1 is the first term, and d is the common difference. This formula gives the n-th term of an arithmetic sequence (constant difference between consecutive terms).

- **math_arith_sequence_02** – Geometric Sequence (n-th Term):

$$a_n = a_1 \cdot r^{n-1}$$

  In this formula, a_1 is the first term and r is the common ratio between terms. It provides the n-th term of a geometric sequence (each term is obtained by multiplying the previous term by r).

1.1.2 Series (Summations)

- **math_arith_series_01** – Sum of First n Integers:

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

  Here n is a positive integer. It yields the sum of the first n natural numbers.

- **math_arith_series_02** – Sum of First n Squares:

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

  In this formula, n is a positive integer. It calculates the sum of the squares of the first n natural numbers.

- **math_arith_series_03** – Sum of First n Cubes:

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

  Here n is a positive integer. It gives the sum of the cubes of the first n natural numbers (which equals the square of the sum of the first n numbers).

- **math_arith_series_04** – Arithmetic Series (Finite Sum):

$$S_n = \frac{n}{2}\big(2a_1 + (n-1)d\big)$$

Here n is the number of terms, a_1 is the first term, and d is the common difference. This formula computes the sum S_n of the first n terms of an arithmetic progression.

- **math_arith_series_05** – Geometric Series (Finite Sum):

$$S_n = a_1 \frac{r^n - 1}{r - 1} \quad (\text{for } r \neq 1)$$

In this formula, n is the number of terms, a_1 is the first term, and r is the common ratio. It gives the sum of the first n terms of a geometric series.

- **math_arith_series_06** – Infinite Geometric Series:

$$S_\infty = \frac{a_1}{1 - r} \quad (\text{valid if } |r| < 1)$$

Here a_1 is the first term and r is the common ratio in absolute value less than 1. This formula provides the sum of an infinite geometric series that converges.

1.1.3 Number Theory

- **math_arith_number_theory_01** – GCD–LCM Relationship:

$$\text{lcm}(a, b) = \frac{a \cdot b}{\gcd(a, b)}$$

Here a and b are positive integers, $\gcd(a, b)$ is their greatest common divisor, and $\text{lcm}(a, b)$ is their least common multiple. This formula relates the product of two integers to the product of their GCD and LCM.

1.2 Algebra

1.2.1 Quadratic Equations

- **math_algebra_quad_01** – Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here a, b, and c are coefficients of the quadratic equation $ax^2 + bx + c = 0$, and x represents the variable. This formula gives the two solutions (roots) for x in a quadratic equation.

- **math_algebra_quad_02** – Discriminant:

$$D = b^2 - 4ac$$

In this expression, a, b, c are coefficients of a quadratic $ax^2 + bx + c = 0$. The discriminant D indicates the nature of the roots: if $D > 0$ (two distinct real roots), $D = 0$ (one real double root), or $D < 0$ (two complex conjugate roots).