Supplementary Materials: Turbulence modulation by suspended finite-sized particles - Towards physics-based multiphase subgrid modeling

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I. Collisional dissipation modeling

Inelastic particle collisions could contribute to energy dissipation when the particle volume fraction is high. The energy dissipation rate in the cell volume due to particle collisions can be evaluated as the average energy loss per unit collision multiplied by the collision rate. Considering a collision between two particles satisfying momentum conservation, the relationships between the pre and post-collision velocities are

$$\mathbf{v}_1+\mathbf{v}_2 = \tilde{\mathbf{v}}_1+\tilde{\mathbf{v}}_2, \quad \tilde{\mathbf{v}}_1-\tilde{\mathbf{v}}_2 = -e_{wet}\left(\mathbf{v}_1-\mathbf{v}_2\right), \quad (1)$$

where $\mathbf{v}_i$ and $\tilde{\mathbf{v}}_i$ ($i = 1, 2$) are the pre- and post-collision velocities, and e_{wet} represents the coefficient of restitution in viscous fluids. From those velocities, we can solve for the averaged energy loss per collision as

$$\langle \Delta E \rangle = \frac{1}{2} C_c \left(1 - e_{wet}^2\right) m_p \langle \mathbf{v} \rangle^2. \quad (2)$$

where $\langle \mathbf{v}^2 \rangle = \langle \mathbf{v}_1^2 \rangle = \langle \mathbf{v}_2^2 \rangle$, $\langle \cdots \rangle$ represent the essemble averaging. Note that we introduce a fitting parameter C_c to accommodate the possible correlation between the pre-collision velocities of the two particles.

The collision rate per unit volume N_c can be estimated from the collision kernel Γ as [1]

$$N_c = \frac{N^2}{2}\Gamma, \qquad \Gamma = 2\pi D^2 \langle |w_r| \rangle \, g_r(D), \quad (3)$$

where w_r is the radial relative velocity, $g_r(D)$ is the radial distribution function at distance D, which is induced to consider the effect of preferential concentration on the collision rate. Replacing the number of particles $N = 6\phi/\left(\pi D^3\right)$ by the particle volume fraction ϕ, we obtain

$$N_c = \frac{36\phi^2}{\pi D^4} \langle |w_r| \rangle g_r\left(D\right). \quad (4)$$

Combining this expression with the kinetic energy loss per collision, we have the additional energy dissipation due to particle collisions as

$$\epsilon_{coll} = N_c \langle \Delta E \rangle = 3 C_c' \rho \frac{\phi^2}{D} \langle |w_r| \rangle \langle \mathbf{v} \rangle^2, \quad (5)$$

where $C_c' = C_c\left(1 - e_{wet}^2\right) g_r(D)$.

Giving a short-hand notation for $\langle \mathbf{v}^2 \rangle = 2k_p$ and assuming the radial relative velocity is proportional to the slip velocity Δu, we can obtain the last term in the energy balance equation Eq. (1) in the paper. All the coefficients has been absorbed in C_{co}. The ratio between the collision-induced dissipation and the slip velocity-induced dissipation

$$R_c \sim \phi_p \rho \left(\frac{\eta_{k,sp}}{D}\right)^{-1}. \quad (6)$$

The inter-particle collision could significantly contribute to the energy balance for heavy, large-size particles with non-negligible volume fractions.

II. Relative velocity regimes

We examine the relative velocity regime into which the different cases considered fall. From Eq. (2) of the manuscript we define the regimes as follows: (i) Regime-1 corresponds to particles smaller in time scale than the Kolmogorov scale, (ii) Regime-2 corresponds to particles larger than the Kolmogorov scale, but smaller than the smallest resolved scale, (iii) Regime-3 corresponds to particles larger than the smallest resolved scale, and (iv) Regime-4 corresponds to particles whose relative velocity is dominated by the mean relative velocity. In the first three regimes, the relative velocity is determined by the particle's inertial response to the different scales of subgrid turbulence. Figures 1 and 2 present the regime of relative velocity for $u_r = 0$ and 1, for varying D/η, Re_Δ, ρ, and ϕ. The results are separated into different curves and frames as before. The results for $u_r = 10.0$ are not resented, since in all the cases the relative velocity is dominated by mean slip and therefore all the cases are in regime-4.

[1] L.-P. Wang, A. S. Wexler, and Y. Zhou, Journal of Fluid Mechanics **415**, 117 (2000).

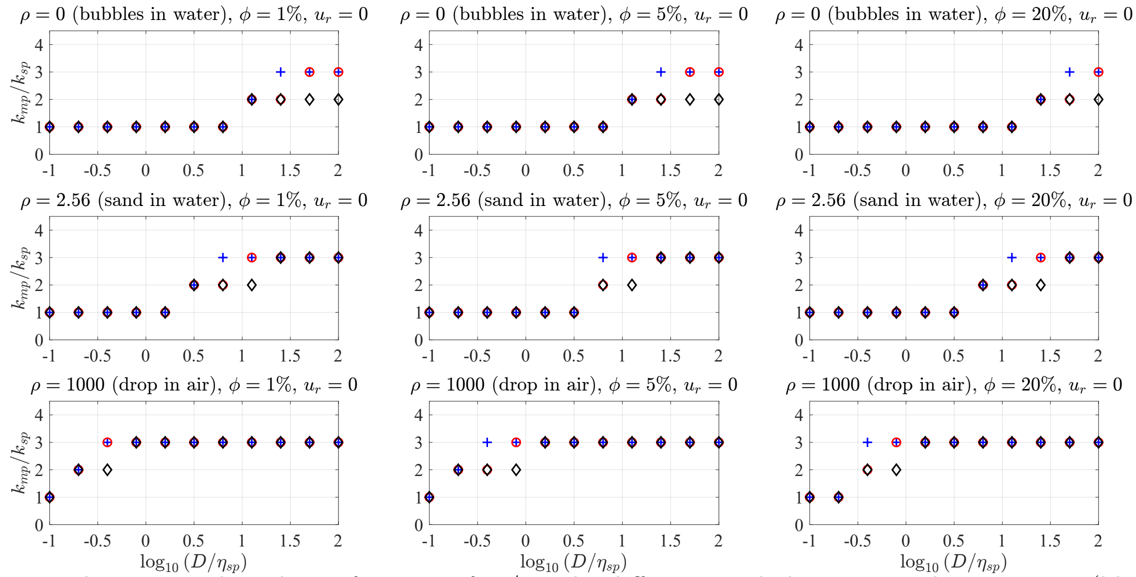

Figure 1: The regime plotted as a function of D/η. The different symbols correspond to $Re_\Delta = 5$ (blue +), 40 (red circle), and 400 (black diamond). This figure is for nondimensional mean relative velocity of $u_r = 0$

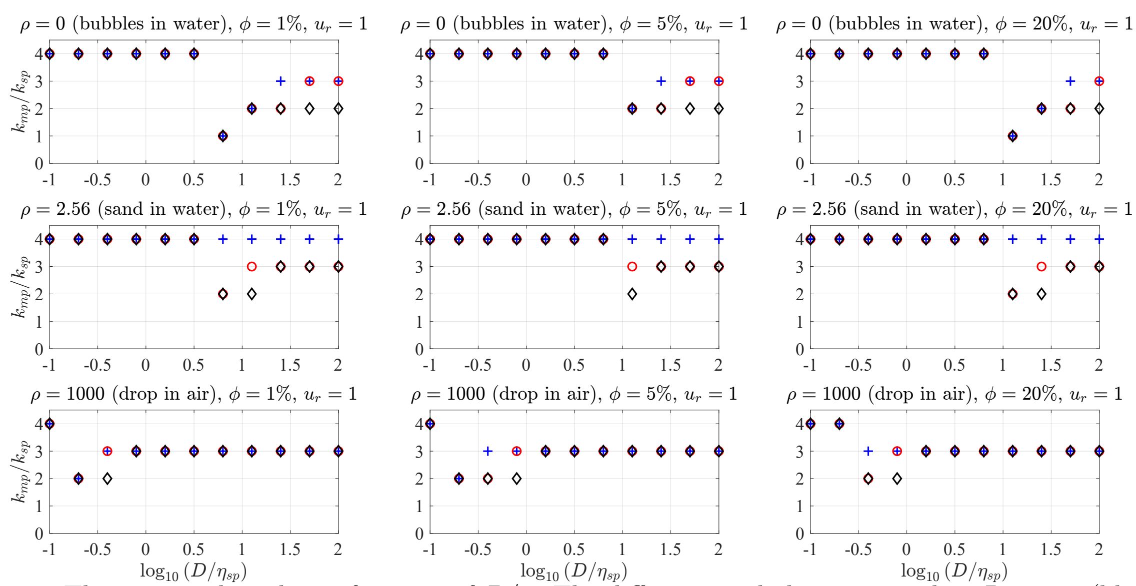

Figure 2: The regime plotted as a function of D/η. The different symbols correspond to $Re_\Delta = 5$ (blue +), 40 (red circle), and 400 (black diamond). This figure is for nondimensional mean relative velocity of $u_r = 1$